Gravitational form factor of soliton in 1+1 dimensional $\phi^4$ model
Hiroaki Ito, Kitazawa Masakiyo

$T_{\mu\nu}$ : Noether current for translational symmetry

$$T_{\mu\nu} = \begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{01} & T_{11} & T_{12} & T_{13} \\ T_{02} & T_{21} & T_{22} & T_{23} \\ T_{03} & T_{23} & T_{32} & T_{33} \end{pmatrix}$$

Energy density | Momentum density

Advantage
✓ Gauge invariance
✓ Conserved quantity
✓ Local interaction

Motivate theoretical analysis of quantum correction to
EMT distribution in $Q\bar{Q}$ system

Distribution of EMT in $Q\bar{Q}$

Lattice QCD
Stress distribution in $Q\bar{Q}$

Forming fluxtube
Pulling force (parallel to field)
Pulling force (vertical to field)

Yanagihara, et al. 2019

Quantum effect in $Q\bar{Q}$
Theoretical model
Fattening due to string vibration
$L^2 \sim \log r$

Lüscher, Münster, and Weisz 1981
**Purpose:** Analysis of quantum correction to EMT distribution around a soliton in 1+1d real scalar $\phi^4$ model

\[ S = \int dx^2 \left\{ \frac{1}{2} \partial^\mu \phi \partial^\mu \phi - \frac{\lambda}{4} \left( \phi^2 - \frac{m^2}{\lambda} \right)^2 \right\} \]

Classical

- Kink solution of EOM
  \[ \phi_{cl} = + \frac{m}{\sqrt{\lambda}} \tanh \left( \frac{m(x-x)}{\sqrt{2}} \right) \]

X: free parameter

- $T_{00}$ under kink solution
  \[ T_{00} = \frac{m^4}{\lambda} \cosh^{-4} \left( \frac{m(x-X)}{\sqrt{2}} \right) \]

QM

\[ E_{LO} \sim O(\lambda^0) \quad T_{00}^{LO} \sim O(\lambda^0) \]


We calculated $T_{00}^{LO}(x)$ and $T_{11}^{LO}(x)$!

**Eigenmode of quantum fluctuation**

Expanding $\phi^4$ action $S[\phi]$ around a soliton

Substituting $\phi(x) = \phi_{cl} + \eta(x)$

\[ S[\eta] = S_{cl} + \int dx^2 \left[ \frac{1}{2} \left( \partial_0 \eta \right)^2 - \frac{1}{2} \eta \left( - \frac{\partial^2}{\partial x^2} - m^2 + \frac{3 \phi_{cl}^2}{\lambda} \right) \eta + O(\lambda^{1/2}) \right] \]

**Eigenvalues**

\[ \omega_q^2 = q^2 + 2m^2 \quad \eta_q(x) \xrightarrow{x \to \pm \infty} e^{iqx} \delta(q) \]

**Eigenfunctions**

Phase shift

\[ \omega_1^2 = \frac{3}{2} m^2 \quad \eta_1(x) \]

\[ \omega_0^2 = 0 \quad \eta_0(x) = \partial_x \phi_{cl} \]

Translational mode $\Rightarrow$ IR divergence
Collective coordinate method

Rewriting Lagrangian
Remove translational mode($\eta_0(x)$) and IR divergence

$$\phi(x, t) = \phi_{cl}(x - X) + \eta(x - X)$$

$X$ is promoted to a dynamical variable

$$X \rightarrow X(t) \quad \text{Dynamical variable}$$

$$P \leftrightarrow X \quad \text{Canonical conjugate momentum}$$

$$L[\pi, \phi] \rightarrow L[\tilde{\pi}, \tilde{\eta}, X, P]$$

$\tilde{\pi}, \tilde{\eta}$: without translational mode

Convert
Degree of freedom of $\eta_0(x)$

→
Degree of freedom of
Soliton center of motion

Rewriting EMT

- Translational invariance
- Lorentz symmetry

$\phi^4$ Model in 1+1 d

$$T^\mu_\nu[\phi] = \frac{\partial \mathcal{L}}{\partial (\partial^\mu \phi)} \partial_\nu \phi - \mathcal{L} \delta^\mu_\nu$$

Collective coordinate method

$$T^\mu_\nu[\tilde{\pi}, \tilde{\eta}, X, P] = T^\mu_\nu[X, P] + T^\mu_\nu[\tilde{\pi}, \tilde{\eta}]$$

$O(\lambda^1)$

We calculate $\langle T^\mu_\nu \rangle$

$$\langle T^\mu_\nu \rangle \equiv \frac{1}{Z} \int D\tilde{\pi} D\tilde{\eta} T^\mu_\nu[\tilde{\pi}, \tilde{\eta}] e^{-i \int dt L[\tilde{\pi}, \tilde{\eta}]}$$
Mode number cutoff (MNC)

- Finite system
- Phase shift
- "Mode number" is the same for soliton and vacuum

\[
\sum_{n=-\infty}^{n=\infty} \sqrt{q_n^2 + 2m^2} - \sum_{n=-\infty}^{n=\infty} \sqrt{k_n^2 + 2m^2} = 0
\]

Mass renormalization

- Only 1-loop mass renormalization
- Mass renormalization in vacuum sector

Vacuum sector

\[
\delta m^2 + \lambda = 0
\]

Soliton sector

\[
\delta m^2 \phi_{cl} = \text{finite}
\]

\[
\delta m^2 \phi_{cl}^2 = \text{finite}
\]
Result, summary, and future work

**In a space of finite size $R$**

\[
T_{00}^R(x) = -\frac{3\sqrt{2m}}{2\pi} \frac{1}{R} + T_{00}^\infty(x)
\]

\[
T_{11}^R = -\frac{3\sqrt{2m}}{2\pi} \frac{1}{R}
\]

**Summary**

1-loop calculation of EMT distribution around a soliton in 1+1d real scalar $\phi^4$ model

- Removing IR divergences by collective coordinate method
- Removing UV divergences by vacuum subtraction (Mode number cutoff) and mass renormalization

- The spatial integrals of obtained $T_{00}$ reproduce the known total energies. $T_{11}$ are consistent with the EMT conservation laws.

**Future work**

- 2+1d $\phi^4$ model
- finite temperature
- sine-Gordon model