Gravitational form factor of soliton in 1+1 dimensional ϕ^4 model

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- $T_{\mu\nu}$: Noether current for translational symmetry

Energy density

Momentum density

$$T_{\mu\nu} = \begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{01} & T_{11} & T_{12} & T_{13} \\ T_{02} & T_{21} & T_{22} & T_{23} \\ T_{03} & T_{23} & T_{32} & T_{33} \end{pmatrix}$$

stress

Advantage

✓ Gauge invariance

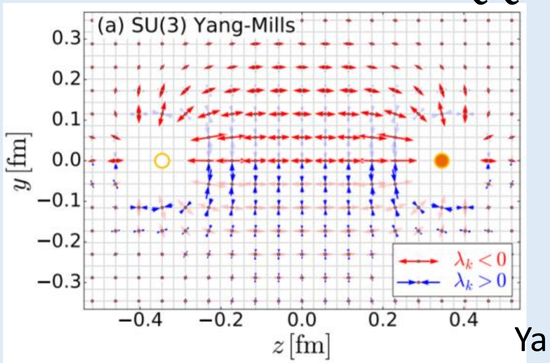
✓ Conserved quantity

✓ Local interaction

- **Distribution of EMT** in $Q\bar{Q}$

Lattice QCD

Stress distribution in $Q\bar{Q}$



Yanagihara, et al. 2019

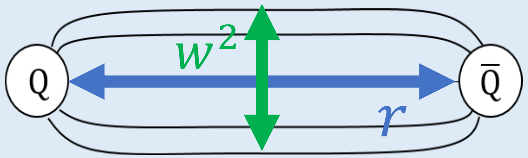
Forming fluxtube

Pulling force (parallel to field)

Pulling force (vertical to field)

- **Quantum effect** in $Q\bar{Q}$

Theoretical model



Fattening due to string vibration

$$w^2 \sim \log r$$

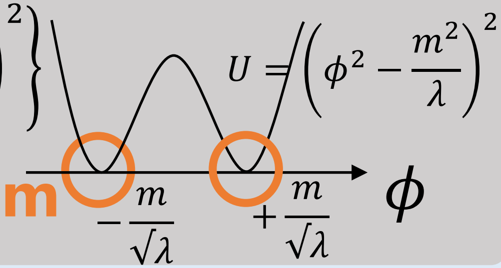
Lüscher, Münster, and Weisz 1981

Motivate theoretical analysis of quantum correction to EMT distribution in $Q\bar{Q}$ system

ϕ^4 Model in 1+1 d

Purpose: Analysis of quantum correction to EMT distribution around a soliton in 1+1d real scalar ϕ^4 model

$$S = \int dx^2 \left\{ \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2 \right\}$$

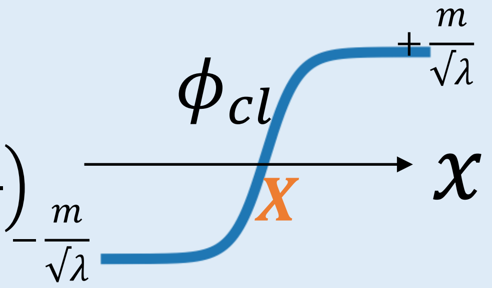


Classical

- Kink solution of EOM

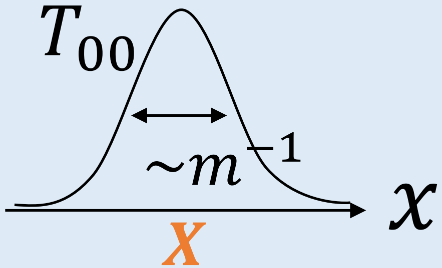
$$\phi_{cl} = +\frac{m}{\sqrt{\lambda}}\tanh\left(\frac{m(x-X)}{\sqrt{2}}\right)$$

X: free parameter



- T_{00} under kink solution

$$T_{00} = \frac{m^4}{\lambda}\cosh^{-4}\left(\frac{m(x-X)}{\sqrt{2}}\right)$$



QM

$E_{LO} \sim O(\lambda^0)$ $\quad T_{00}^{LO} \sim O(\lambda^0)$

Dashen, et al 1974 | Goldhaber, et al. 2003

We calculated $T_{00}^{LO}(x)$ and $T_{11}^{LO}(x)$!

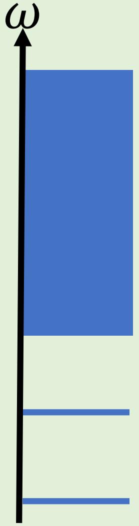
Eigenmode of quantum fluctuation

Expanding ϕ^4 action $S[\phi]$ around a soliton

Substituting $\phi(x) = \phi_{cl} + \eta(x)$

$$S[\eta] = S_{cl} + \int dx^2 \left[\frac{1}{2}(\partial_0\eta)^2 - \frac{1}{2}\eta\left(-\frac{\partial^2}{\partial x^2} - m^2 + \frac{3\phi_{cl}^2}{\lambda}\right)\eta + O(\lambda^{\frac{1}{2}})\right.$$

$$\left(-\frac{\partial^2}{\partial x^2} - m^2 + \frac{3\phi_{cl}^2}{\lambda}\right)\eta_n = \omega_n^2\,\eta_n$$



Eigenvalues	Eigenfunctions
$\omega_q^2 = q^2 + 2m^2$	$\eta_q(x) \xrightarrow{x\to\pm\infty} e^{\left[i\left(qx \pm \frac{1}{2}\delta(q)\right)\right]}$ Phase shift
$\omega_1^2 = \frac{3}{2}m^2$	$\eta_1(x)$
$\omega_0^2 = 0$	$\eta_0(x) = \partial_x\phi_{cl}$

Translational mode ➡ IR divergence

Collective coordinate method

Gervais, Jevicki, Sakita 1975 Tomboulis 1975

Rewriting Lagrangian

Remove translational mode($\eta_0(x)$) and IR divergence

$$\phi(x,t) = \phi_{cl}(x - X) + \eta(x - X)$$

X is promoted to a dynamical variable

$X \to X(t)$ **Dynamical variable**

$P \leftrightarrow X$ **Canonical conjugate momentum**

$$L[\pi,\phi] \to L[\tilde{\pi},\tilde{\eta},X,P]$$

$\tilde{\pi},\tilde{\eta}$:without translational mode

Convert

Degree of freedom of $\eta_0(x)$

➡ **Degree of freedom of Soliton center of motion**

Rewriting EMT

- Translational invariance
- Lorentz symmetry

Gervais, Jevicki, Sakita 1975
Goldstone and Jackiw 1975
Tomboulis 1975

ϕ^4 Model in 1+1 d

$$T^{\mu}{}_{\nu}[\phi] = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\partial_\nu\phi - \mathcal{L}\delta^{\mu}{}_{\nu}$$

↓ **Collective coordinate method**

$$T^{\mu}{}_{\nu}[\tilde{\pi},\tilde{\eta},X,P] = T^{\mu}{}_{\nu}[X,P] + T^{\mu}{}_{\nu}[\tilde{\pi},\tilde{\eta}]$$

$O(\lambda^1)$

We calculate $\langle T^{\mu}{}_{\nu}\rangle$

$$\langle T^{\mu}{}_{\nu}\rangle \equiv \frac{1}{Z}\int \mathcal{D}\tilde{\pi}\mathcal{D}\tilde{\eta}\, T^{\mu}{}_{\nu}[\tilde{\pi},\tilde{\eta}]\, e^{-i\int dt\, L[\tilde{\pi},\tilde{\eta}]}$$

EMT regularization

Rebhan and Nieuwenhuizen 1997
Dashen, et al 1974

Counter terms

$$\langle T_{\mu\nu}\rangle_{soliton} - \langle T_{\mu\nu}\rangle_{vac} + (\langle \delta T_{\mu\nu}\rangle_{soliton} - \langle \delta T_{\mu\nu}\rangle_{vac}) = \text{finite}$$

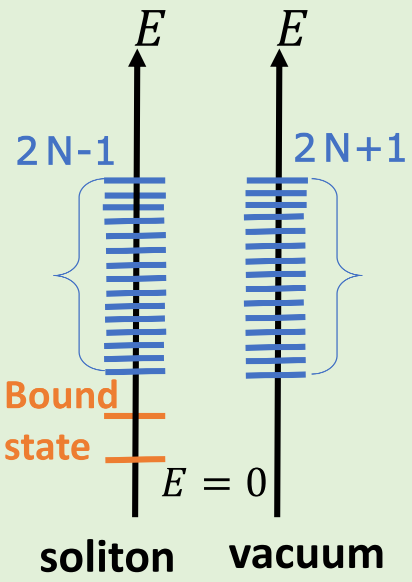
Mode number cutoff (MNC)

- ✓ Finite system
- ✓ Phase shift
- ✓ "Mode number" is the same for soliton and vacuum

Soliton continuous mode / vacuum

$$\sum_{n=-\infty}^{n=\infty} \sqrt{q_n^2 + 2m^2} - \sum_{n=-\infty}^{n=\infty} \sqrt{k_n^2 + 2m^2}$$

$$= \sum_{n=-(N-1)}^{N-1} \sqrt{q_n^2 + 2m^2} - \sum_{n=-N}^{N} \sqrt{k_n^2 + 2m^2}$$



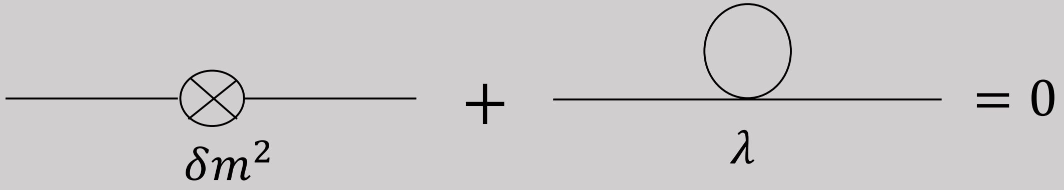
subtraction $\xrightarrow{R \to \infty}$ **Infinite system**

Mass renormalization

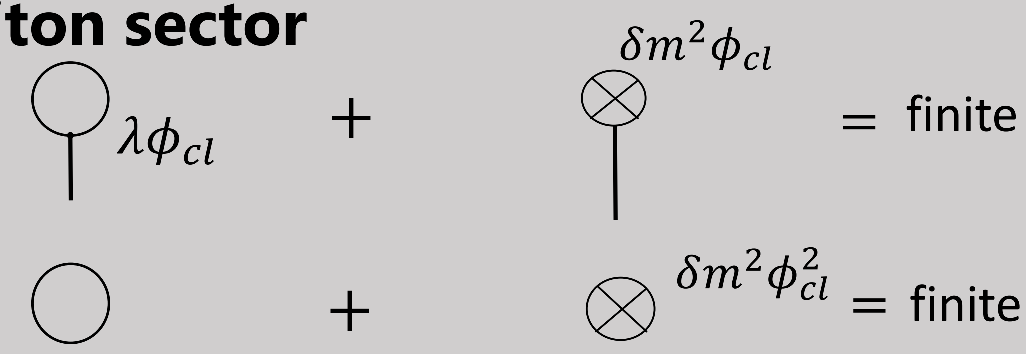
- ✓ Only 1-loop mass renormalization
- ✓ Mass renormalization in **vacuum** sector

➡Counter terms appear in **soliton** sector

Vacuum sector $m^2 \to m^2 + \delta m^2$



Soliton sector

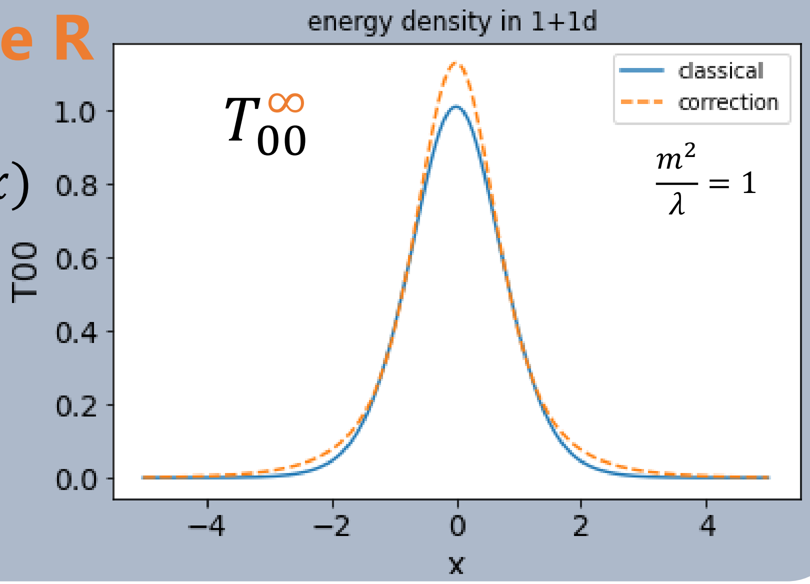


Result, summary, and future work

In a space of finite size R

$$T_{00}^R(x) = -\frac{3\sqrt{2}m}{2\pi}\frac{1}{R} + T_{00}^{\infty}(x)$$

$$T_{11}^R = -\frac{3\sqrt{2}m}{2\pi}\frac{1}{R}$$



- T_{00} distribution

$$\lim_{R\to\infty}\int_{-R/2}^{R/2} dx\, T_{00}^R = E \quad \times \quad \int_{-\infty}^{\infty} dx \lim_{R\to\infty} T_{00}^R$$

reproduce the E_{MNC} calculated by Dashen et al.

- T_{11} distribution

T_{11}^R is constant ➡ $\partial_0 T^{01} + \partial_1 T^{11} = 0$

EMT conservation law is satisfied

Summary

1-loop calculation of EMT distribution around a soliton in 1+1d real scalar ϕ^4 model

-Removing IR divergences by collective coordinate method

-Removing UV divergences by vacuum subtraction(Mode number cutoff) and mass renormalization

-The spatial integrals of obtained T_{00} reproduce the known total energies. T_{11} are consistent with the EMT conservation laws.

Future work

- **2+1d ϕ^4model**
- **finite temperature**
- **sine-Gordon model**