Pseudo-gauge dependence of spin polarization in heavy-ion collisions

SQM 2022
The 20th International Conference on Strangeness in Quark Matter
13-17 June 2022 Busan, Republic of Korea

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Based on
[MB, PRC 105, 044907 (2022); 2109.12084]

- Measurements of Lambda spin polarization in heavy-ion collision has opened the possibility for new phenomenological investigations of spin physics in relativistic fluids.
- Predictions of spin polarization are based on the Local Thermal Equilibrium (LTE) description of the QGP.
- Out-of-equilibrium phenomena are pseudo-gauge dependent (decomposition of angular momentum).
- As a result we have different predictions of the spin polarization vector in heavy-ion collision depending on the choice of the pseudo-gauge.

Pseudo-gauge transformation

Noether current, canonical form:

$$\partial_\mu \widehat{T}_{\rm C}^{\mu\nu} = 0 \qquad \widehat{J}_{\rm C}^{\lambda,\mu\nu} = x^\mu \widehat{T}_{\rm C}^{\lambda\nu} - x^\nu \widehat{T}_{\rm C}^{\lambda\mu} + \widehat{\mathcal{S}}_{\rm C}^{\lambda,\mu\nu}$$

Energy momentum tensor (EMT) and spin tensor are not unique in special relativity

Pseudo-gauge transformations

F. W. Hehl, Rept. Math. Phys. 9 (1976) 55

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\nabla_\lambda \left(\widehat{\Phi}^{\lambda,\mu\nu} - \widehat{\Phi}^{\mu,\lambda\nu} - \widehat{\Phi}^{\nu,\lambda\mu} \right)$$

$$\widehat{\mathcal{S}}'^{\lambda,\mu\nu} = \widehat{\mathcal{S}}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu} + \partial_\rho \widehat{Z}^{\mu\nu,\lambda\rho}$$

$$\widehat{\Phi}^{\lambda,\mu\nu} = -\widehat{\Phi}^{\lambda,\nu\mu}, \quad \widehat{Z}^{\mu\nu,\lambda\rho} = -\widehat{Z}^{\nu\mu,\lambda\rho} = -\widehat{Z}^{\mu\nu,\rho\lambda}$$

They leave the conservation equations and spacial integrals (=generators, or total energy, momentum and angular momentum operators) invariant

Belinfante EMT and spin tensor

From canonical choosing $\widehat{\Phi} = \widehat{\mathcal{S}}_{\rm C},\ \widehat{Z} = 0$ ⟹ $\widehat{T}_{\rm B}^{\mu\nu} - \widehat{T}_{\rm B}^{\nu\mu} = 0 \qquad \widehat{\mathcal{S}}_{\rm B}^{\lambda,\mu\nu} = 0$

Experiments cannot actually measure densities in space!

Local equilibrium density operator

F. Becattini, W. Florkowski, and E. Speranza, Phys. Lett. B 789, 419 (2019)
E. Speranza and N. Weickgenannt, Eur. Phys. J. A 57, 155 (2021)

For Belinfante decomposition:

$$\widehat{T}_{\rm B}^{\mu\nu},\ \widehat{\mathcal{S}}_{\rm B}^{\lambda,\mu\nu}=0$$

$$\widehat{\rho}_{\rm LTE}^{\rm B}=\frac{1}{\mathcal{Z}}\exp\left[-\int {\rm d}\Sigma_\mu\left(\widehat{T}_{\rm B}^{\mu\nu}\beta_\nu-\widehat{j}^\mu\zeta\right)\right]$$

For a general decomposition:

$$\widehat{\rho}_{\rm LTE}^{\Phi}=\frac{1}{\mathcal{Z}}\exp\left[-\int {\rm d}\Sigma_\mu\left(\widehat{T}_{\Phi}^{\mu\nu}\beta_\nu-\frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}_{\Phi}^{\mu,\lambda\nu}-\widehat{j}^\mu\zeta\right)\right]$$

$$\widehat{T}_{\Phi}^{\mu\nu}=\widehat{T}_{\rm B}^{\mu\nu}+\frac{1}{2}\nabla_\lambda\left(\widehat{\Phi}^{\lambda,\mu\nu}-\widehat{\Phi}^{\mu,\lambda\nu}-\widehat{\Phi}^{\nu,\lambda\mu}\right),\ \widehat{\mathcal{S}}_{\Phi}^{\lambda,\mu\nu}=-\widehat{\Phi}^{\lambda,\mu\nu}+\partial_\rho\widehat{Z}^{\mu\nu,\lambda\rho}$$

We can highlight the difference:

$$\widehat{\rho}_{\rm LTE}^{\Phi}=\frac{1}{\mathcal{Z}}\exp\left\{-\int {\rm d}\Sigma_\mu\left[\widehat{T}_{\rm B}^{\mu\nu}\beta_\nu-\frac{1}{2}\left(\varpi_{\lambda\nu}-\Omega_{\lambda\nu}\right)\widehat{\Phi}^{\mu,\lambda\nu}+\xi_{\lambda\nu}\widehat{\Phi}^{\lambda,\mu\nu}-\frac{1}{2}\Omega_{\lambda\nu}\nabla_\rho\widehat{Z}^{\lambda\nu,\mu\rho}-\widehat{j}^\mu\zeta\right]\right\}$$

$$\varpi_{\mu\nu}=-\frac{1}{2}\left(\partial_\mu\beta_\nu-\partial_\nu\beta_\mu\right)$$ Thermal vorticity

$$\xi_{\mu\nu}=\frac{1}{2}\left(\partial_\mu\beta_\nu+\partial_\nu\beta_\mu\right)$$ Thermal shear

What spin-tensor should we use?

- Belinfante

$$\widehat{\mathcal{S}}_{\mathrm{B}}^{\lambda,\mu\nu} = 0$$

- Canonical spin tensor

F. W. Hehl and Y. N. Obukhov,Fundam. Theor. Phys. 199 (2020), 217-252

$$\widehat{\mathcal{S}}_{\mathrm{C}}^{\lambda,\mu\nu} = \frac{\mathrm{i}}{8}\bar{\Psi}\left\{\gamma^{\lambda},[\gamma^{\mu},\gamma^{\nu}]\right\}\Psi$$

- de Groot-van Leeuwen-van Weert (GLW)

S. R. De Groot, Relativistic Kinetic Theory. Principles and Applications, 1980
W. Florkowski, A. Kumar, R. Ryblewski, Prog. Part. Nucl. Phys. 108 (2019) 103709

$$\widehat{\mathcal{S}}_{\mathrm{GLW}}^{\lambda,\mu\nu} = \frac{\mathrm{i}}{4m}\left(\bar{\Psi}\sigma^{\mu\nu}\overleftrightarrow{\partial}^{\lambda}\Psi - \partial_{\rho}\epsilon^{\mu\nu\lambda\rho}\bar{\Psi}\gamma^{5}\Psi\right)$$

- Hilgevoord-Wouthuysen (HW)

J. Hilgevoord, S.A. Wouthuysen, Nuclear Physics 40, 1 (1963)
E. Speranza and N. Weickgenannt, Eur. Phys. J. A 57, 155 (2021)

$$\widehat{\mathcal{S}}_{\mathrm{HW}}^{\lambda,\mu\nu} = \frac{\mathrm{i}}{4m}\bar{\Psi}\sigma^{\mu\nu}\overleftrightarrow{\partial}^{\lambda}\Psi$$

Spin polarization in other pseudo-gauges

$$\widehat{\rho}^{\Phi}_{\rm LTE} = \frac{1}{\mathcal{Z}} \exp\left\{ -\int {\rm d}\Sigma_\mu \left[\widehat{T}^{\mu\nu}_{\rm B} \beta_\nu - \frac{1}{2}\left(\varpi_{\lambda\nu} - \Omega_{\lambda\nu}\right) \widehat{\Phi}^{\mu,\lambda\nu} + \xi_{\lambda\nu} \widehat{\Phi}^{\lambda,\mu\nu} - \frac{1}{2}\Omega_{\lambda\nu} \nabla_\rho \widehat{Z}^{\lambda\nu,\mu\rho} - \widehat{j}^\mu \zeta \right] \right\}$$

- Belinfante

$$S^\mu_{\rm B}(k) \simeq S^\mu_\varpi(k) + S^\mu_\xi(k)$$

$$S^\mu_\varpi(k) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau} k_\tau \frac{\int_\Sigma {\rm d}\Sigma \cdot k\, n_{\rm F}\,(1-n_{\rm F})\,\varpi_{\rho\sigma}}{\int_\Sigma {\rm d}\Sigma \cdot k\, n_{\rm F}}$$

$$S^\mu_\xi(k) = -\frac{1}{4m}\epsilon^{\mu\lambda\sigma\tau} \frac{k_\tau k^\rho}{\varepsilon_k} \frac{\int_\Sigma {\rm d}\Sigma \cdot k\, n_{\rm F}\,(1-n_{\rm F})\,\hat{t}_\lambda \xi_{\rho\sigma}}{\int_\Sigma {\rm d}\Sigma \cdot k\, n_{\rm F}}$$

- Canonical spin tensor

$$S^\mu_{\rm C}(k) \simeq S^\mu_{\rm B}(k) + \Delta^{\rm C}_\Theta S^\mu(k)$$

$$\Delta^{\rm C}_\Theta S^\mu(k) = \frac{\epsilon^{\lambda\rho\sigma\tau}\hat{t}_\lambda (k^\mu k_\tau - \eta^\mu_\tau m^2)}{m\varepsilon_k} \frac{\int_\Sigma {\rm d}\Sigma(x) \cdot k\, n_{\rm F}\,(1-n_{\rm F})\,(\varpi_{\rho\sigma} - \Omega_{\rho\sigma})}{\int_\Sigma {\rm d}\Sigma \cdot k\, n_{\rm F}}$$

- GLW and HW

$$S^\mu_{\rm GLW,HW}(k) \simeq S^\mu_{\rm C}(k) + \Delta^{\rm GLW,HW}_\Theta S^\mu(k) + \Delta^{\rm GLW,HW}_\xi S^\mu(k)$$

$$\Delta^{\rm GLW,HW}_\xi S^\mu(x) = -S^\mu_\xi(k) \qquad \Delta^{\rm GLW,HW}_\Theta S^\mu(k) = -\frac{1}{8m}\epsilon^{\mu\lambda\rho\tau}\hat{t}_\lambda \frac{k_\tau k^\sigma}{\varepsilon_k} \frac{\int_\Sigma {\rm d}\Sigma \cdot k\, n_{\rm F}\,(1-n_{\rm F})\,(\varpi_{\rho\sigma} - \Omega_{\rho\sigma})}{\int_\Sigma {\rm d}\Sigma \cdot k\, n_{\rm F}}$$

No shear induced polarization here!

- Arbitrary

$$\widehat{\Phi}^{\lambda,\mu\nu}_{\partial\Sigma} = \frac{\rm i}{m} K \bar{\Psi} \overset{\leftrightarrow}{\partial}{}^\lambda \sigma^{\mu\nu}\Psi \qquad \Delta^{\partial\Sigma} S^\mu_\xi(k) = \mp 2K \frac{1}{4m}\epsilon^{\mu\lambda\sigma\tau} \frac{k_\tau k^\rho}{\varepsilon_k} \frac{\int_\Sigma {\rm d}\Sigma \cdot k\, n_{\rm F}\,(1-n_{\rm F})\,\hat{t}_\lambda \xi_{\rho\sigma}}{\int_\Sigma {\rm d}\Sigma \cdot k\, n_{\rm F}}$$