

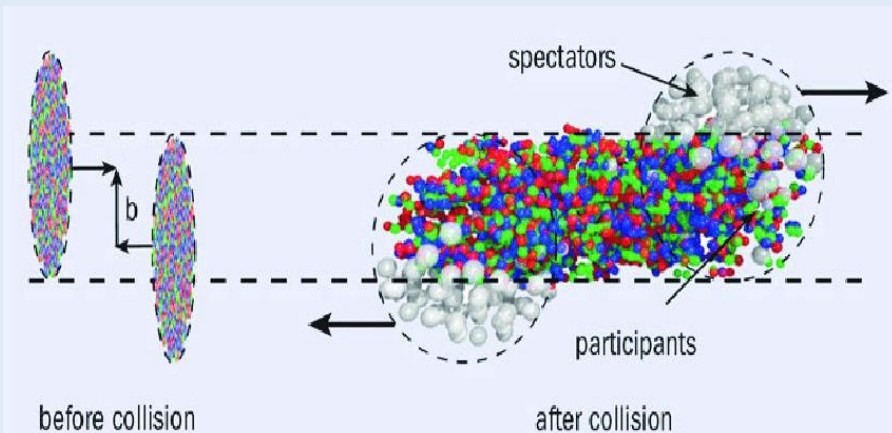
Causal second-order magnetohydrodynamics from kinetic theory using RTA approximation



BY : ANKIT KUMAR PANDA , NISER (INDIA)

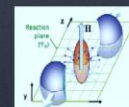
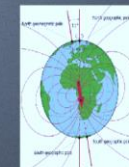
COLLABORATORS : Dr. Ashutosh dash , Dr. Victor ROY , Rajesh Biswas

QGP AND MAGNETIC FIELD PRODUCTION

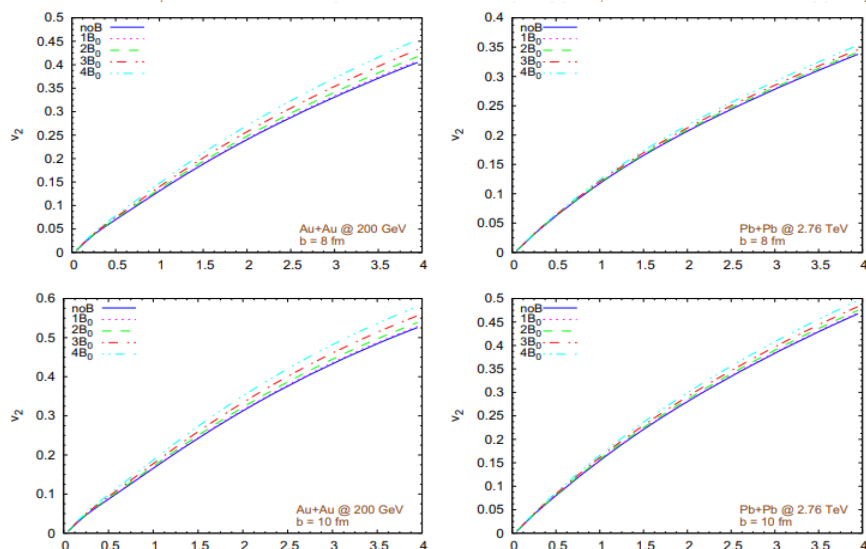


MAGNETIC FIELD MAGNITUDE COMPARISON

- Earth's magnetic field $\sim 5 \times 10^{-5} \text{ T}$
- MRI $\sim 1 \text{ T}$
- Strongest laboratory fields $\sim 1.2 \times 10^3 \text{ T}$
- Magnetar (static) $\sim 10^{11} \text{ T}$
- Heavy Ion Collisions (transient) $\sim 10^{14} \text{ T}$

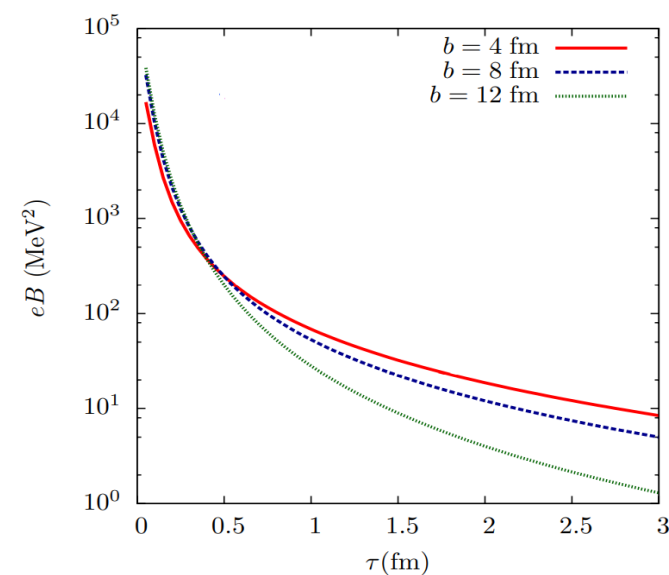


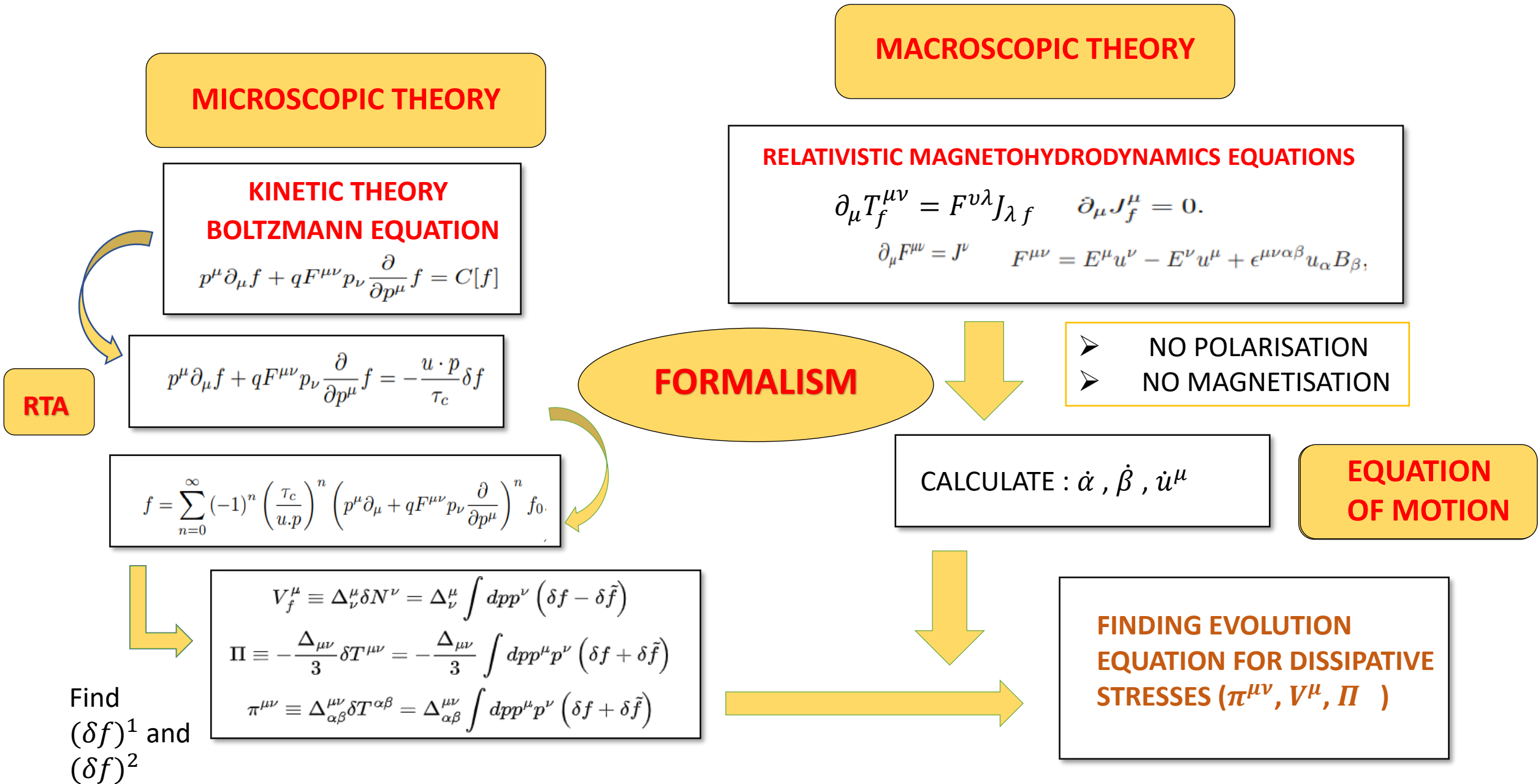
v_2 VARIATION WITH MAGNETIC-FIELD AND b



MOTIVATION

MAGNETIC-FIELD VARIATION WITH b





**ANISOTROPIC
CO-EFFICIENT**

DIFFUSION COEFFICIENTS

$$\begin{aligned} \kappa_{\parallel} &= \beta_V \tau_c, \\ \kappa_{\perp} &= \frac{\beta_V \tau_c}{1 + (qB\tau_c \delta_{VB})^2}, \\ \kappa_{\chi} &= \frac{\beta_V q B \tau_c^2 \delta_{VB}}{1 + (qB\tau_c \delta_{VB})^2} = \kappa_{\perp} q B \tau_c \delta_{VB}. \end{aligned}$$

CONDUCTIVITY COEFFICIENTS

$$\begin{aligned} \sigma_E^{\parallel} &= q^2 \tau_c \beta \beta_V, \\ \sigma_E^{\perp} &= \frac{q^2 \tau_c \beta \beta_V}{1 + (qB\tau_c \delta_{VB})^2}, \\ \sigma_E^{\times} &= \frac{q^3 B \tau_c^2 \beta \beta_V \delta_{VB}}{1 + (qB\tau_c \delta_{VB})^2}. \end{aligned}$$

SHEAR COEFFICIENTS

$$\begin{aligned} \eta_0 &= 2\beta_{\pi} \tau_c, \\ \eta_1 &= \frac{2\beta_{\pi} \tau_c}{1 + (2qB\tau_c \delta_{\pi B})^2}, \\ \eta_2 &= \frac{4\beta_{\pi} q B \tau_c^2 \delta_{\pi B}}{1 + (2qB\tau_c \delta_{\pi B})^2} = 2\eta_1 q B \tau_c \delta_{\pi B}, \\ \eta_3 &= \frac{2\beta_{\pi} \tau_c}{1 + (qB\tau_c \delta_{\pi B})^2}, \\ \eta_4 &= \frac{2\beta_{\pi} q B \tau_c^2 \delta_{\pi B}}{1 + (qB\tau_c \delta_{\pi B})^2} = \eta_3 q B \tau_c \delta_{\pi B}. \end{aligned}$$

IDEAL-MHD EVOLUTION EQUATION

$$\frac{\Pi}{\tau_c} = -\dot{\Pi} - \delta_{\Pi\Pi} \Pi \theta + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu} - \tau_{\Pi V} V \cdot \dot{u} - \lambda_{\Pi V} V \cdot \nabla \alpha - l_{\Pi V} \partial \cdot V - \beta_{\Pi} \theta$$

$$+ \tau_c \tau_{\Pi V B} \dot{u}_{\alpha} q B b^{\alpha\beta} V_{\beta} - \tau_c q \delta_{\Pi V B} \nabla_{\mu} (B b^{\mu\beta} V_{\beta}) - \tau_c q B \lambda_{\Pi V B} b^{\mu\beta} V_{\beta} \nabla_{\mu} \alpha$$

$$\frac{V^{\mu}}{\tau_c} = -\dot{V}^{(\mu} - V_{\nu} \omega^{\nu\mu} - \lambda_{VV} V^{\nu} \sigma_{\nu}^{\mu} - \delta_{VV} V^{\mu} \theta + \lambda_{V\Pi} \Pi \nabla^{\mu} \alpha - \lambda_{V\pi} \pi^{\mu\nu} \nabla_{\nu} \alpha - \tau_{V\pi} \pi_{\nu}^{\mu} \dot{u}^{\nu}$$

$$+ \tau_{V\Pi} \Pi \dot{u}^{\mu} + l_{V\pi} \Delta^{\mu\nu} \partial_{\gamma} \pi_{\nu}^{\gamma} - l_{V\Pi} \nabla^{\mu} \Pi + \beta_V \nabla^{\mu} \alpha - q B \delta_{VB} b^{\mu\gamma} V_{\gamma} + \tau_c q B l_{V\pi B} b^{\sigma\mu} \partial^{\kappa} \pi_{\kappa\sigma}$$

$$+ \tau_c q B \tau_{V\Pi B} b^{\gamma\mu} \Pi \dot{u}_{\gamma} - \tau_c q B l_{V\Pi B} b^{\gamma\mu} \nabla_{\gamma} \Pi - q \tau_c \delta_{VV B} B b^{\mu\nu} V_{\nu} \theta - q \tau_c \lambda_{VV B} B b^{\gamma\nu} V_{\nu} \sigma_{\gamma}^{\mu}$$

$$- q \tau_c \rho_{VV B} B b^{\gamma\nu} V_{\nu} \omega_{\gamma}^{\mu} - \tau_c q \tau_{VV B} \Delta_{\gamma}^{\mu} D (B b^{\gamma\nu} V_{\nu})$$

$$\frac{\pi^{\mu\nu}}{\tau_c} = -\dot{\pi}^{\mu\nu} + 2\beta_{\pi} \sigma^{\mu\nu} + 2\pi_{\gamma}^{(\mu} \omega^{\nu)\gamma} - \tau_{\pi\pi} \pi_{\gamma}^{(\mu} \sigma^{\nu)\gamma} - \delta_{\pi\pi} \pi^{\mu\nu} \theta + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu} - \tau_{\pi V} V^{(\mu} \dot{u}^{\nu)}$$

$$+ \lambda_{\pi V} V^{(\mu} \nabla^{\nu)} \alpha + l_{\pi V} \nabla^{(\mu} V^{\nu)} + \delta_{\pi B} \Delta_{\eta\delta}^{\mu\nu} q B b^{\eta\gamma} g^{\beta\rho} \pi_{\gamma\rho} - \tau_c q B \tau_{\pi V B} \dot{u}^{(\mu} b^{\nu)\sigma} V_{\sigma}$$

$$- \tau_c q B \lambda_{\pi V B} V_{\gamma} b^{\gamma(\mu} \nabla^{\nu)} \alpha - q \tau_c \delta_{\pi V B} \nabla^{(\mu} (B^{\nu)\gamma} V_{\gamma})$$

RESULTS

$$\delta f^{(l)} = \frac{f_0 \bar{f}_0 \tau_c}{u \cdot p} (A \Pi + B^{\beta} V_{\beta} + C^{\rho} \pi_{\rho}),$$

Cooper-Frye Formalism

NUMERICAL CODES

RESISTIVE-MHD EVOLUTION EQUATIONS

$$\frac{\Pi}{\tau_c} = -\dot{\Pi} - \delta_{\Pi\Pi} \Pi \theta + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu} - \tau_{\Pi V} V \cdot \dot{u} - \lambda_{\Pi V} V \cdot \nabla \alpha - l_{\Pi V} \partial \cdot V - \beta_{\Pi} \theta - q B \lambda_{\Pi V B} b^{\mu\beta} V_{\beta} V_{\mu}$$

$$+ \tau_c \tau_{\Pi V B} \dot{u}_{\alpha} q B b^{\alpha\beta} V_{\beta} - q \delta_{\Pi V B} \nabla_{\mu} (\tau_c B b^{\mu\beta} V_{\beta}) - q^2 \tau_c \chi_{\Pi E E} E^{\mu} E_{\mu}.$$

$$\frac{V^{\mu}}{\tau_c} = -\dot{V}^{(\mu} - V_{\nu} \omega^{\nu\mu} + \lambda_{VV} V^{\nu} \sigma_{\nu}^{\mu} - \delta_{VV} V^{\mu} \theta + \lambda_{V\Pi} \Pi \nabla^{\mu} \alpha - \lambda_{V\pi} \pi^{\mu\nu} \nabla_{\nu} \alpha - \tau_{V\pi} \pi_{\nu}^{\mu} \dot{u}^{\nu} - q B \delta_{VB} b^{\mu\gamma} V_{\gamma}$$

$$+ \tau_{V\Pi} \Pi \dot{u}^{\mu} + l_{V\pi} \Delta^{\mu\nu} \partial_{\gamma} \pi_{\nu}^{\gamma} - l_{V\Pi} \nabla^{\mu} \Pi + \beta_V \nabla^{\mu} \alpha + \tau_c q B l_{V\pi B} b^{\sigma\mu} \partial^{\kappa} \pi_{\kappa\sigma} - q \tau_c \lambda_{VV B} B b^{\gamma\nu} V_{\nu} \sigma_{\gamma}^{\mu}$$

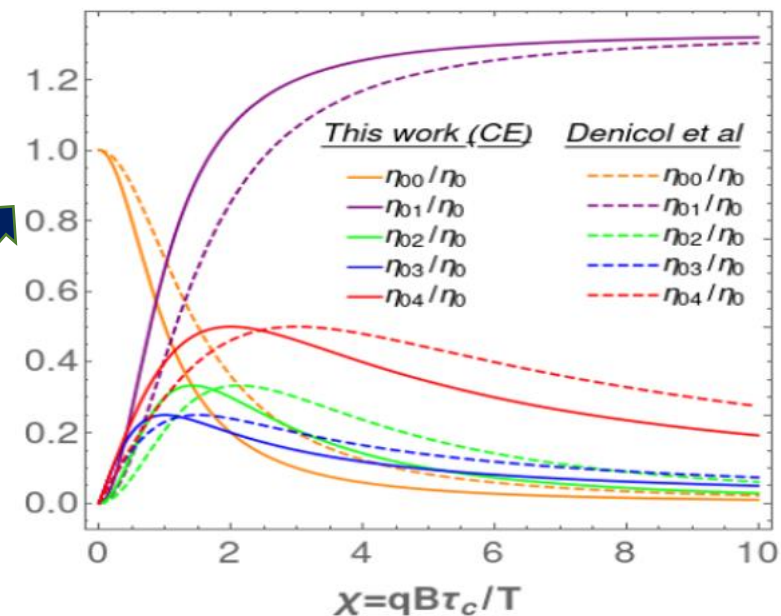
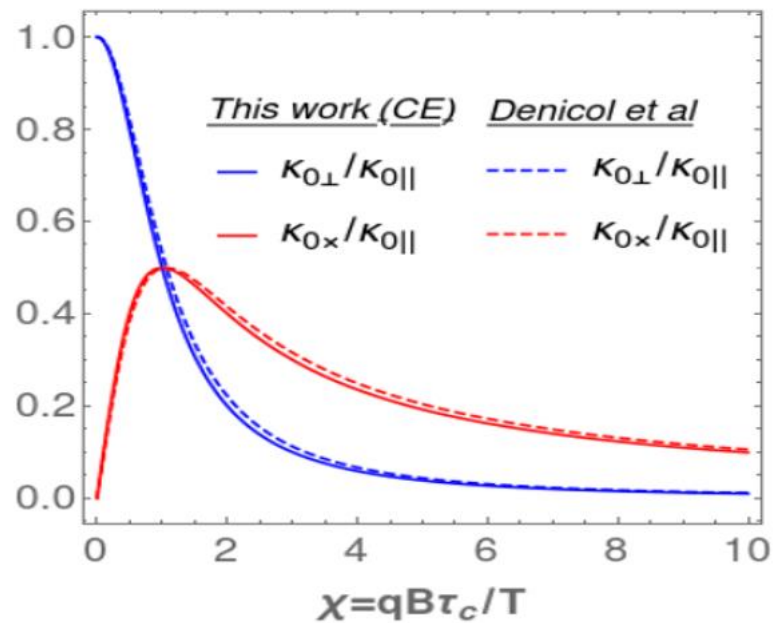
$$+ \tau_c q B \tau_{V\Pi B} b^{\gamma\mu} \Pi \dot{u}_{\gamma} - \tau_c q B l_{V\Pi B} b^{\gamma\mu} \nabla_{\gamma} \Pi - q \tau_c \delta_{VV B} B b^{\mu\nu} V_{\nu} \theta - q \tau_c \rho_{VV B} B b^{\gamma\nu} V_{\nu} \omega_{\gamma}^{\mu}$$

$$+ \chi_{VE} q E^{\mu} + q \Delta_{\alpha}^{\mu} \chi_{VE} D (\tau_c E^{\alpha}) - q \tau_c \rho_{VE} E^{\mu} \theta - q \tau_{VV B} \Delta_{\gamma}^{\mu} D (\tau_c B b^{\gamma\nu} V_{\nu}).$$

$$\frac{\pi^{\mu\nu}}{\tau_c} = -\dot{\pi}^{(\mu\nu)} + 2\beta_{\pi} \sigma^{\mu\nu} + 2\pi_{\gamma}^{(\mu} \omega^{\nu)\gamma} - \tau_{\pi\pi} \pi_{\gamma}^{(\mu} \sigma^{\nu)\gamma} - \delta_{\pi\pi} \pi^{\mu\nu} \theta + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu} - \tau_{\pi V} V^{(\mu} \dot{u}^{\nu)} - \tau_c q B \tau_{\pi V B} \dot{u}^{(\mu} b^{\nu)\sigma} V_{\sigma}$$

$$+ \lambda_{\pi V} V^{(\mu} \nabla^{\nu)} \alpha - l_{\pi V} \nabla^{(\mu} V^{\nu)} + \delta_{\pi B} \Delta_{\eta\delta}^{\mu\nu} q B b^{\eta\gamma} g^{\beta\rho} \pi_{\gamma\rho} - q B \lambda_{\pi V B} V_{\gamma} b^{\gamma(\mu} V^{\nu)} - q \delta_{\pi V B} \nabla^{(\mu} (\tau_c B^{\nu)\gamma} V_{\gamma})$$

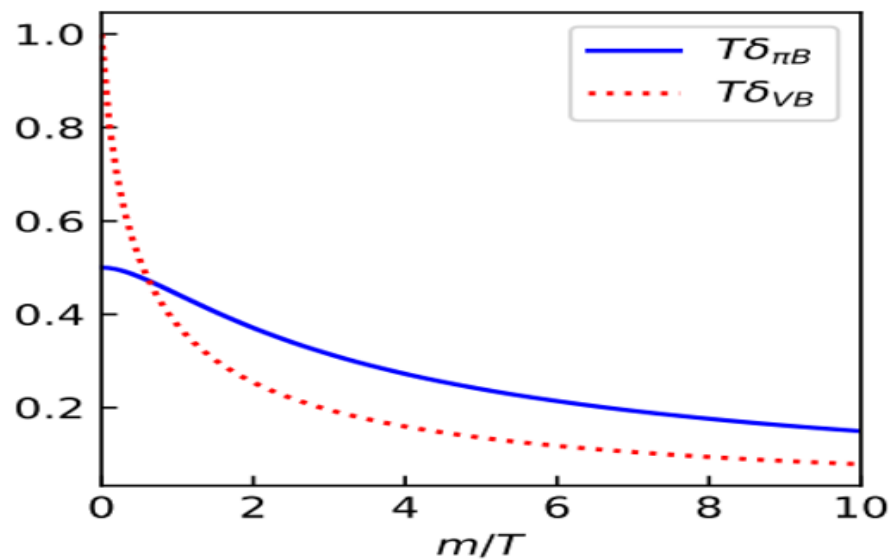
$$+ q^2 \tau_c \chi_{\pi E E} \Delta_{\sigma\rho}^{\mu\nu} E^{\sigma} E^{\rho}.$$



OTHER RESULTS

Phys. Rev. D 98 (2018) 076009 [1804.05210].

DOI : "10.1007/JHEP03(2021)216"

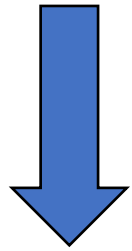


SUMMARY

- ❑ We Found out the 2nd-ORDER evolution equations of viscous stresses .
- ❑ All the Transport coefficients pertaining to this study have been evaluated .
- ❑ The anisotropic transport coefficients of shear , diffusion stresses and for the electrical conductivities have also been evaluated in the presence of external electromagnetic field.
- ❑ From kinetic theory perspective we have found out the small corrections to equilibrium distribution function which can be readily used in the cooper-frey formula to find out different flow harmonics.

THANK YOU

$$\pi^{\mu\nu} = \eta\sigma^{\mu\nu} \quad \Pi = -\zeta\nabla_\lambda u^\lambda \quad V_f^\mu = \kappa\nabla^\mu\alpha \quad \longrightarrow \quad \text{FIRST ORDER CONSTITUTIVE RELATIONS}$$



$$v_g \propto k$$

WHERE

$$v_g = \frac{d\omega}{dk}$$

LEADING TO ACAUSALITY



2ND ORDER

Truncation upto 2nd order .

Find $(\delta f)^1$ and $(\delta f)^2$.

Smallness parameter $Kn = \tau_c T$, $\chi = \frac{qB\tau_c}{T}$ and $\xi = \frac{qE\tau_c}{T}$.