production of hidden heavy-flavor mesons as open quantum systems

Pol B Gossiaux, SUBATECH (NANTES)

20th Strangeness in Quark Matter Conference
Busan (Republic of Korea), June 2022

1. **Background and Motivation**
2. **Selected results and facts from OQS**
3. **Quarkonia as OQS**

Just the survival kit due to time constrain; see extra pedagogical material in the back up slides

Subatech and Pays de la Loire
IMT Atlantique Bretagne-Pays de la Loire École Mines-Télécom
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STRONG 2020

Provocative question :

What is a quarkonia ?

Provocative question :

What is a quarkonia ?

… in a (time evolving) QGP

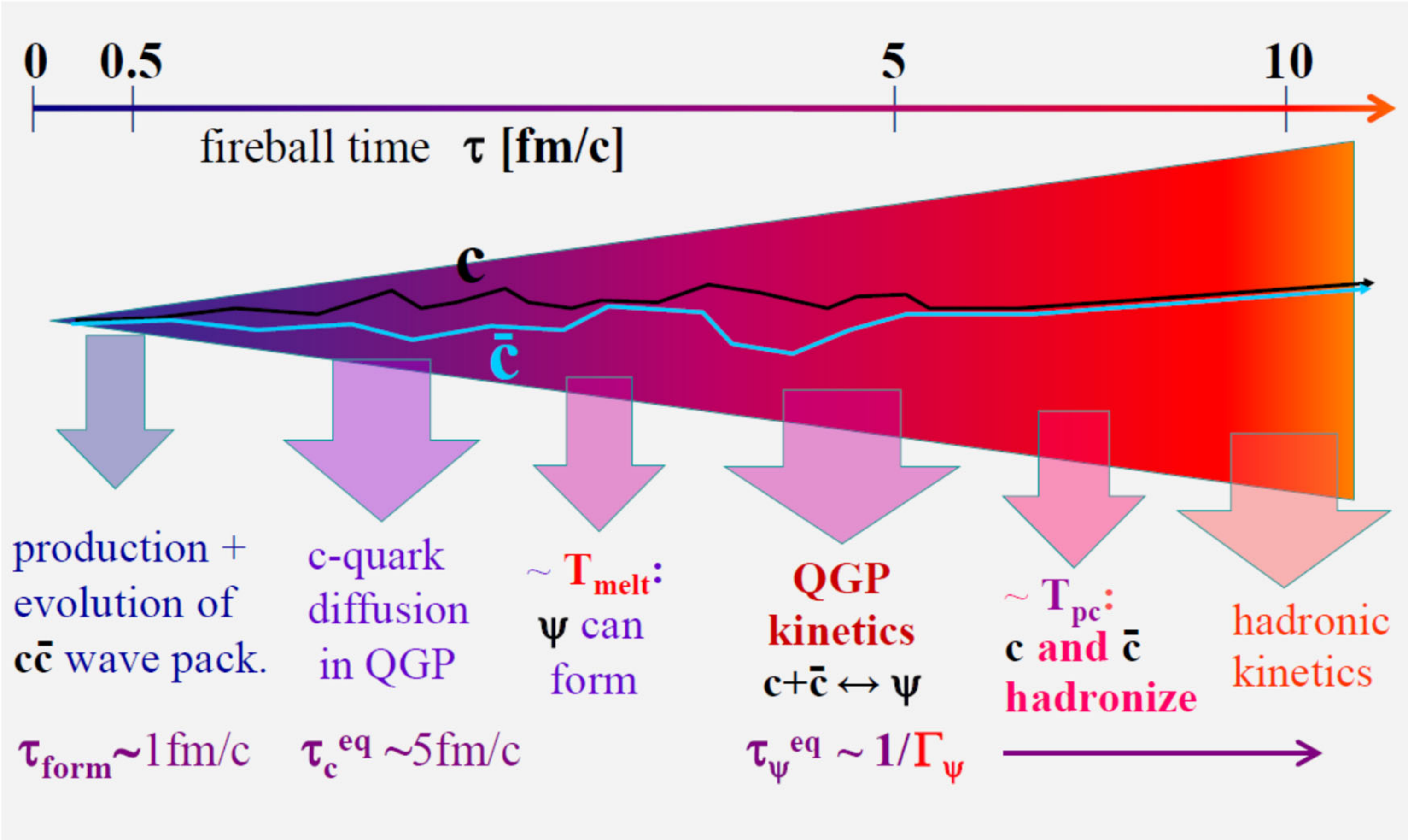
Bonus question:

When is it produced ?

Provocative answer :

If you are able to answer these questions in less then 10’’, you don’t need to listen to this talk !

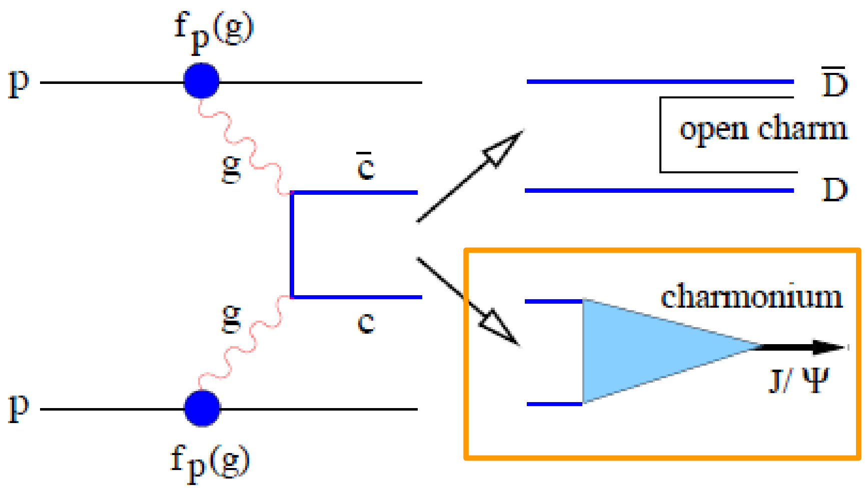
Fate of a c-cbar in the QGP (semi-classical transport viewpoint)



Rapp and Du *Nucl.Phys.A* 967 (2017) 216-224

Quantum coherence

Picture behind transport theory (taken from H. Satz):



Open heavy flavor and quarkonia assumed to be uncorrelated

Formed after some "formation time" $\tau_f \approx 1/\Delta E$ (typically the Heisenberg time), usually assumed to be independent of the surrounding medium

Common belief in the transport community:

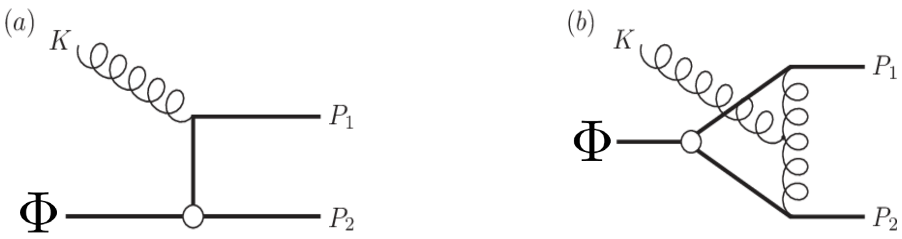
Primordial quarkonia are initially « formed » in QGP and survive with an *individual* survival probability

$$S(t) = e^{-\int_{\tau_f}^{t} \Gamma_\psi(T(t'))dt'}$$

A central quantity: the dissociation rate Γ

Several approaches

pQCD view (Bhanot & Peskin), later on consolidated by NRQCD (Brambilla & Vairo)



$$\Phi + g \to Q + \bar{Q}$$

Dissociation cross section σ

$$\Gamma_\Phi(T) = \langle \sigma n_g \rangle_T$$

Other mechanisms : $x + \Phi \to x + Q + \bar{Q}$

QFT/Lattice QCD

Time correlator

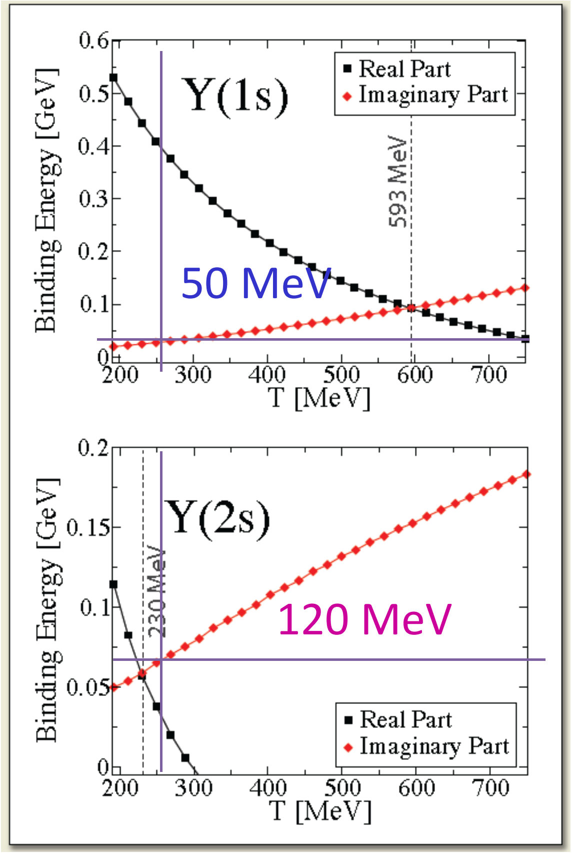
$$\mathcal{C}_>(t,\vec{r}) \approx \langle \psi(t,\tfrac{\vec{r}}{2})\bar{\psi}(t,-\tfrac{\vec{r}}{2})\psi(0,0)\bar{\psi}(0,0)\rangle$$

Satisfies Schroedinger equation with imaginary potential iW . Breakthrough by Laine et al. (2006)

$$\Gamma_\Phi(T) = -2\langle \Phi | W | \Phi \rangle$$

Suppression of the bottomonium « candle »

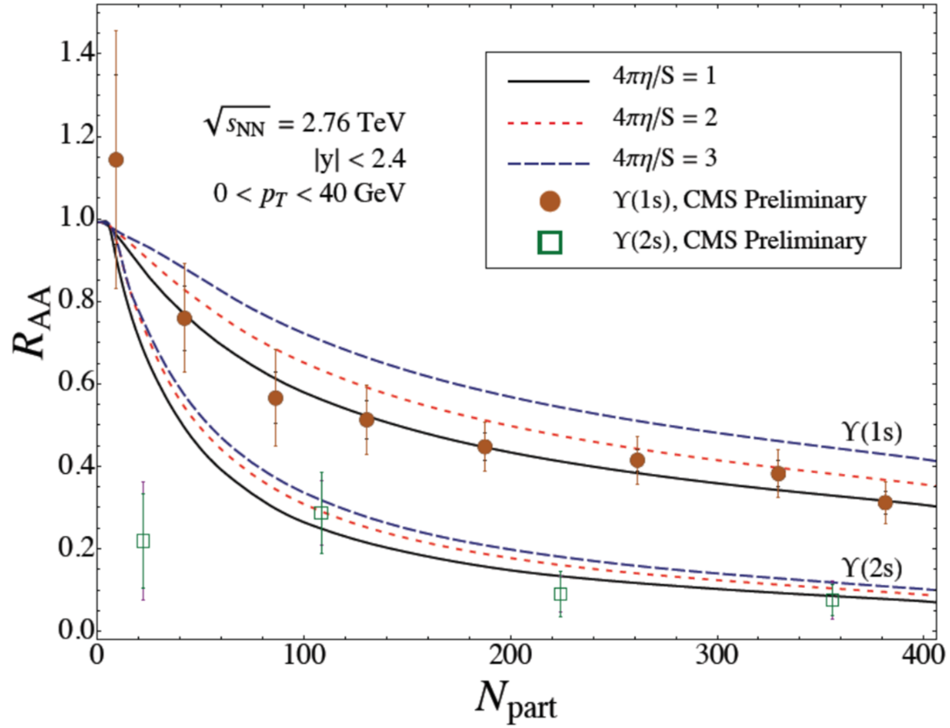
Strickland et al.



Resulting decay rate $\Gamma_T \equiv -2\, \mathrm{Im}[E_{bind}]$

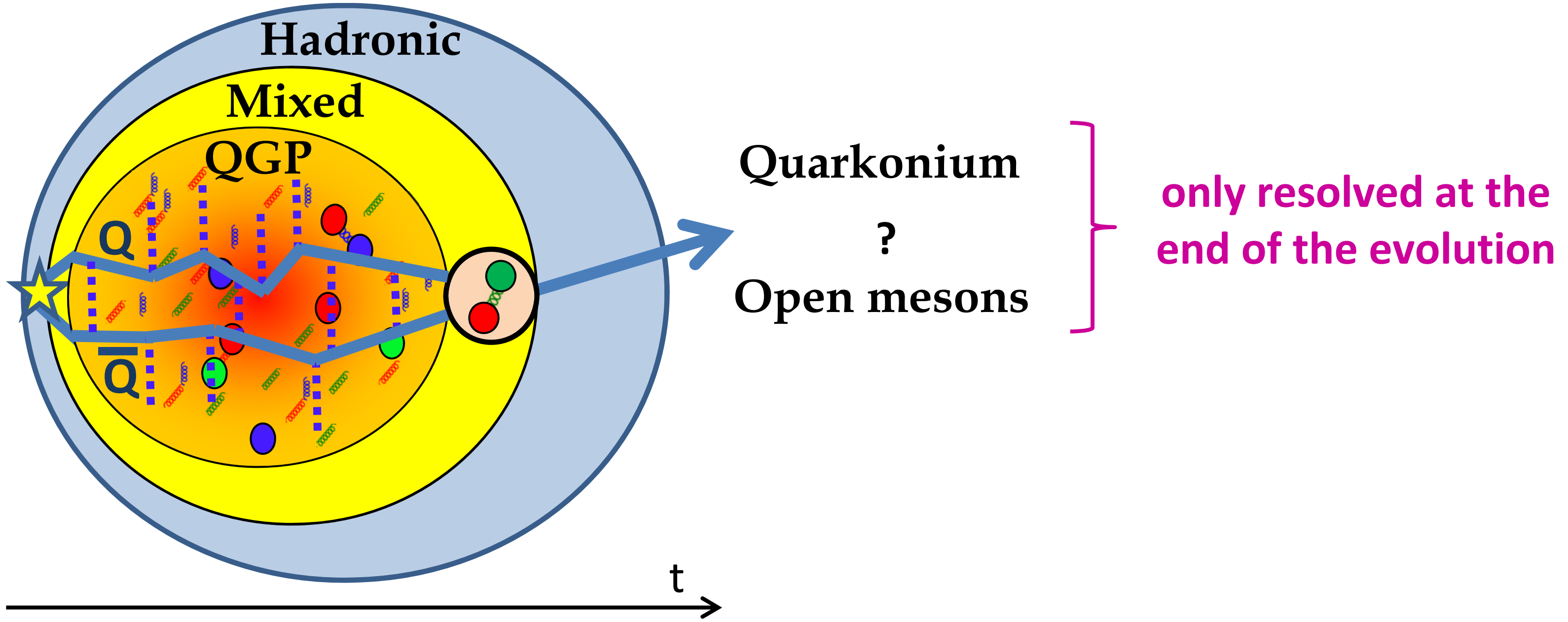
$b\bar{b} \Rightarrow$
- Less shadowing
- Less final state effect

B. Krouppa, R. Ryblewski, and M Stickland, Phys. Rev. C 92, 061901(R) (2015).



… as well as QGP viscosity

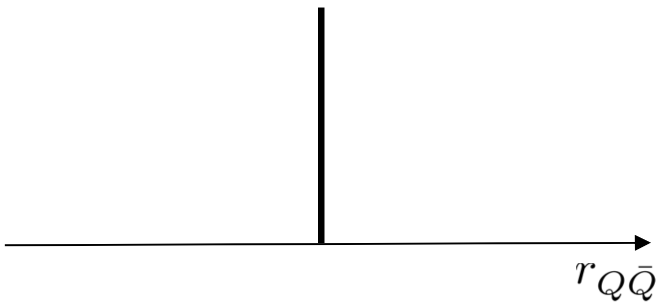
Quantum coherence



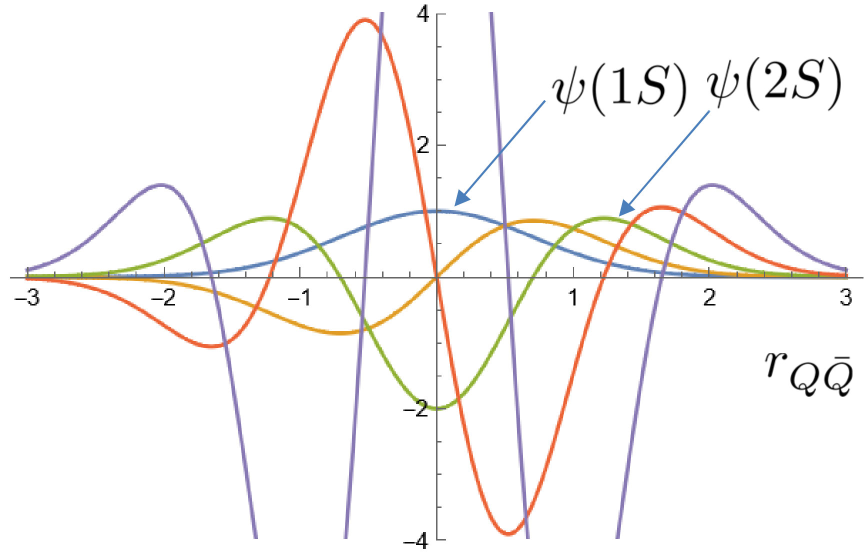
Be aware and beware of quantum coherence, during the whole evolution, but especially at early time…

At, least need for reliable estimates of decoherence time

Quantum coherence at early time



$= \sum$



Initial compact state : $\delta(r_{Q\bar{Q}})$

Dissociation rate: $\Gamma(r_{Q\bar{Q}}) \propto \alpha_S T \times \Phi(m_D r_{Q\bar{Q}}) \sim \alpha_S^2 T^3 \times r_{Q\bar{Q}}^2 \qquad (r m_D \ll 1)$

Coherence

Neglect of coherence

$$\Gamma(r_{Q\bar{Q}}) \approx 0 \propto \sum c_j^\star c_i \langle \psi_j | r^2 | \psi_i \rangle \longrightarrow \Gamma \propto \sum_i |c_i|^2 \langle \psi_i | r^2 | \psi_i \rangle \approx \sum_i |c_i|^2 \Gamma_i \neq 0$$

Correct answer

Crucial to include quantum coherence !

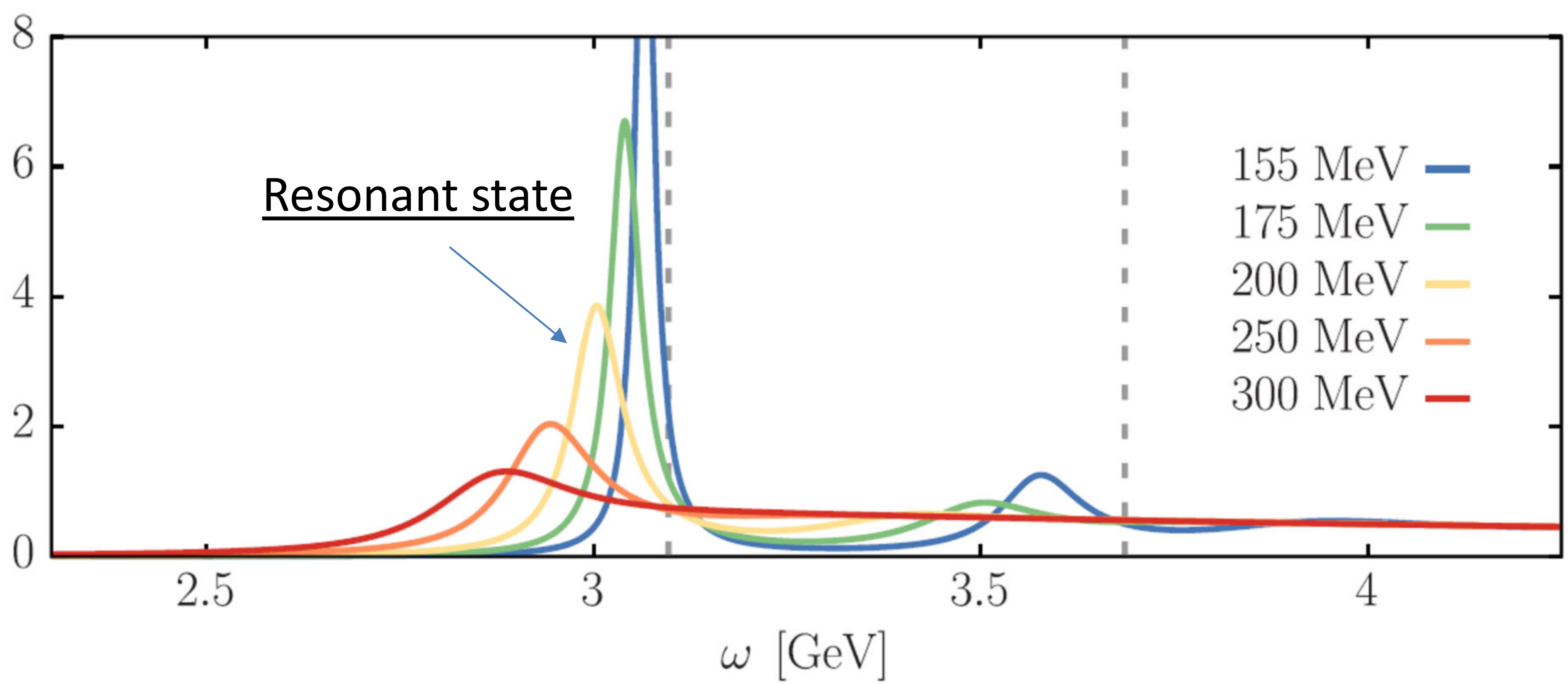
Wrong answer

Comment on formation time then incoherent evolution : role of QGP in decoherence

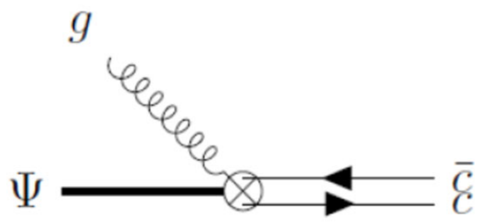
Extra Motivation

Spectral density

David Lafferty, Alexander Rothkopf , Phys. Rev. D 101, 056010 (2020)



How justified is it to deal with the quantum evolution of such « resonant states » with cross sections, meant to describe the reactions of asymptotic states ?



New motto: Q-Qbar real-time dynamics with the Open Quantum System framework

INNER DYNAMICS OF EACH $Q\bar{Q}$ PAIR

A dynamical and continuous quantum picture of the dissociation, recombination, transitions between states, and energy-momentum exchanges with the QGP

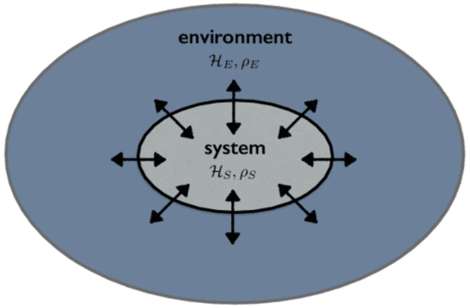
+

EVOLUTION IN A QGP (environment)

Realistic t-dependent background:
Monte-Carlo event generator with initial fluctuations

=> *Quarkonia as QGP continuous thermometers*

Quantum Master Equations



Quite generally, system (Q-Qbar pair) builds correlation with the environment thanks to the Hamiltonian $\hat{H} = \hat{H}^{(0)}_{Q\bar{Q}} + \hat{H}_E + \hat{H}_{\rm int}$

Von Neumann equation for the total density operator ρ

System + environment (QGP)

$\rho(t=0) = \rho_{Q\bar{Q}} \otimes \rho_{QGP}$

$\frac{\mathrm{d}}{\mathrm{d}t}\rho = -i[H, \rho]$ →

Evolution of the total system

$\rho(t) = U(t,0)\left[\rho_{Q\bar{Q}} \otimes \rho_{QGP}\right] U(t,0)^\dagger$

Trace out QGP degrees of freedom =>
Reduced density matrix $\rho_{Q\bar{Q}}$

System ($Q\bar{Q}$ pair)

$\rho_{Q\bar{Q}}(t=0)$

→ $\frac{\mathrm{d}\rho_{Q\bar{Q}}}{\mathrm{d}t} = \mathcal{L}[\rho_{Q\bar{Q}}]$

Evolution of the system

$\rho_{Q\bar{Q}}(t) = Tr_{QGP}\left[U(t,0)\rho(t=0)U(t,0)^\dagger\right]$

However, $\mathcal{L}[\cdot]$ is generically a non local super-operator in time

A special QME: The Lindblad Equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)\right] + \sum_i \boxed{\gamma_i}\left[L_i\rho_{Q\bar{Q}}(t)L_i^\dagger - \frac{1}{2}\left\{L_iL_i^\dagger, \rho_{Q\bar{Q}}(t)\right\}\right]$$

$H_{Q\bar{Q}}$: $\{Q,\bar{Q}\}$ $\underbrace{\text{kinetics + Vacuum potential V}}_{\hat{H}^{(0)}_{Q\bar{Q}}}$ + Lamb shift / screening

L_i : Collapse (or Lindblad) operators, depend on the properties of the medium

3 important conservation properties :

$\rho^\dagger_{Q\bar{Q}} = \rho_{Q\bar{Q}}$ (Hermiticity)

$\mathrm{Tr}[\rho_{Q\bar{Q}}] = 1$ (Norm)

$\langle\varphi|\rho_{Q\bar{Q}}|\varphi\rangle > 0, \forall|\varphi\rangle$ (Positivity)

γ_i Characterize the coupling of the system (Q-Qbar) with the environment

A special QME: The Lindblad Equation

Non unitary / dissipative evolution ≡ decoherence

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)\right] + \sum_i \gamma_i \left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2}\left\{L_i L_i^\dagger, \rho_{Q\bar{Q}}(t)\right\}\right]$$

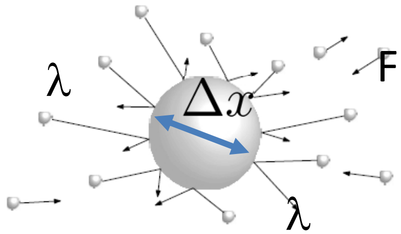
Genuine transitions :

- ✓ Singlet <-> octet
- ✓ Octet <-> octet

Can be reshuffled into non Hermitic effective hamiltonian

$$H_{\text{eff}} = H_{Q\bar{Q}} - i\sum_j \frac{L_j L_j^\dagger}{2}$$ ≡ Dissociation width

For **infinitely massive single Q** and environment wave length λ >> wave packet size Δx:



Fluctuations from env. ⟺ $\frac{\partial \rho_Q(x_Q, x'_Q)}{\partial t} = -F(x_Q - x'_Q)\rho_Q(x_Q, x'_Q)$

Decoherence factor: $F \approx \kappa\,(x_Q - x'_Q)^2$

HQ momentum diffusion coefficient

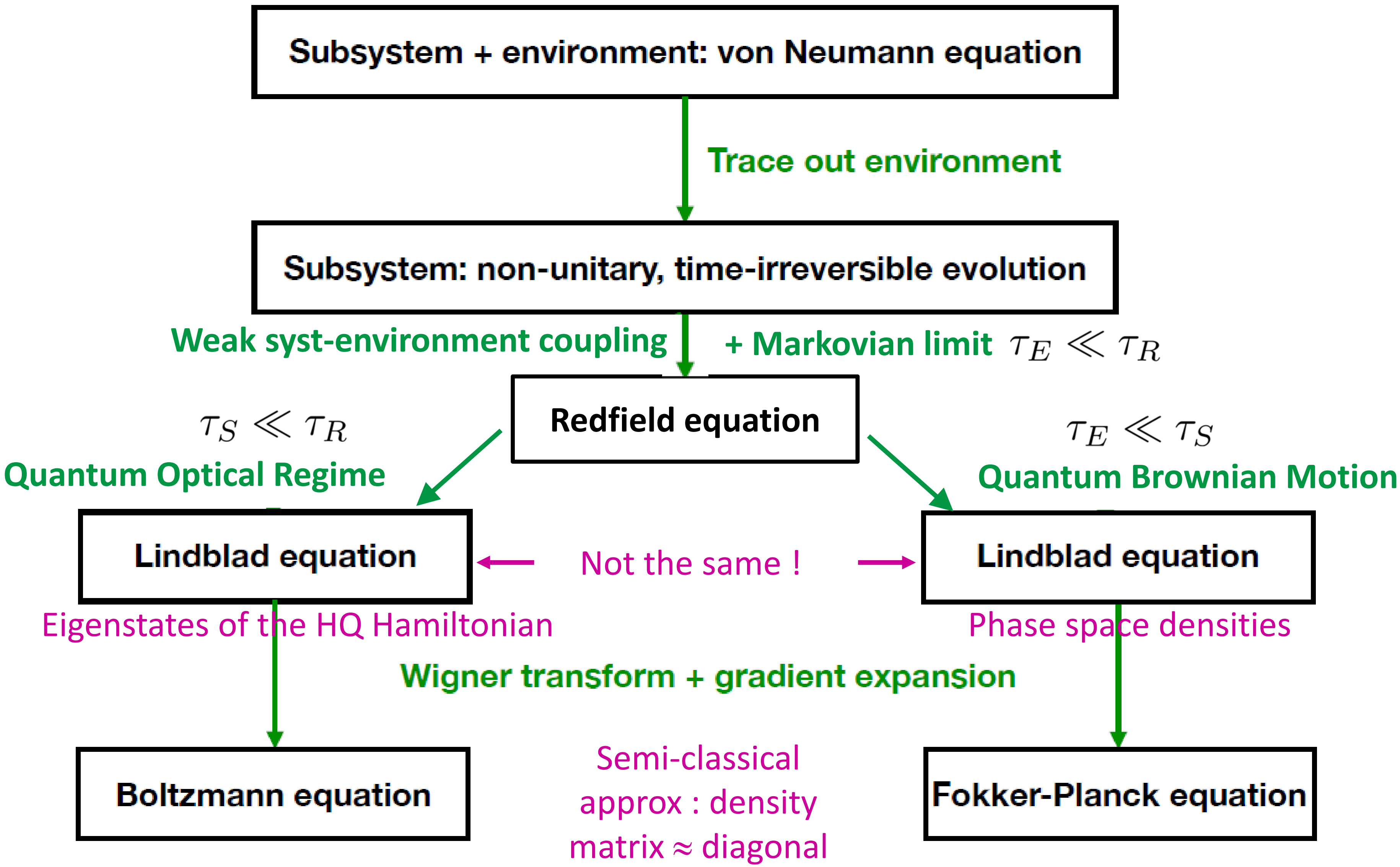
In Q world: smaller objects live longer !

At 1rst order in 1/m_Q : recoil corrections ⟺ friction / dissipation

Pictorial summary

Inspired from Yao Int. J. of Mod. Phys. A, Vol. 36, No. 20, 2130010 (2021)

τ_E: environment autocorrelation time **τ_S: system intrinsic time scale** **τ_R: system relax. time**

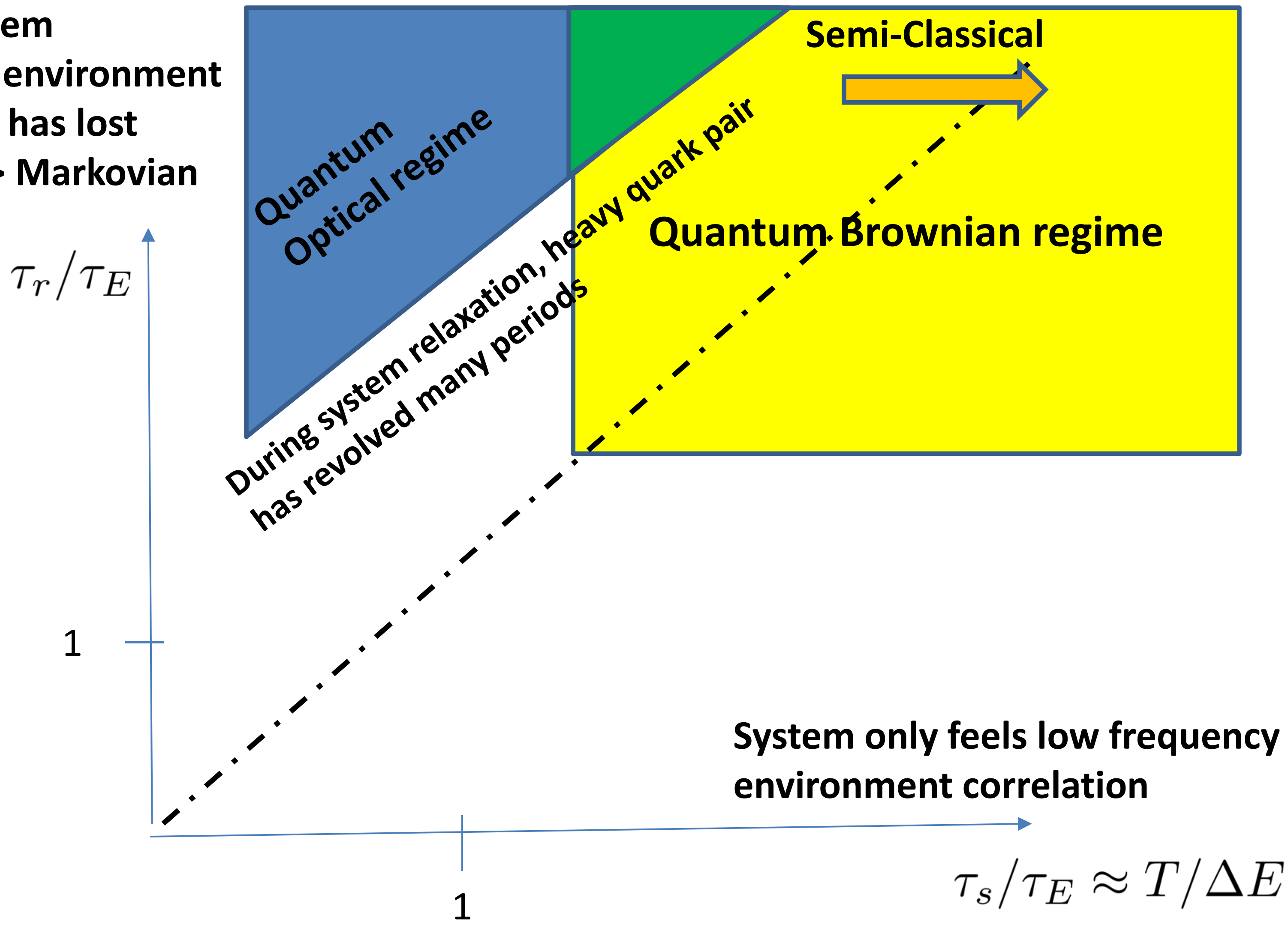


τ_E: environment autocorrelation time **τ_S: system intrinsic time scale** **τ_R: system relax. time**

$$\tau_E \sim \frac{1}{T}$$

$$\tau_S \approx \frac{1}{\Delta E}$$

During system relaxation, environment correlation has lost memory => Markovian process



System only feels low frequency part of environment correlation

Not clear all states goes from one regime to the other at the same T

τ_E: environment autocorrelation time

$$\tau_E \sim \frac{1}{T}$$

τ_S: system intrinsic time scale

$$\tau_S \approx \frac{1}{\Delta E} \approx \frac{1}{m_Q v^2}$$

with $v \approx \alpha_S$

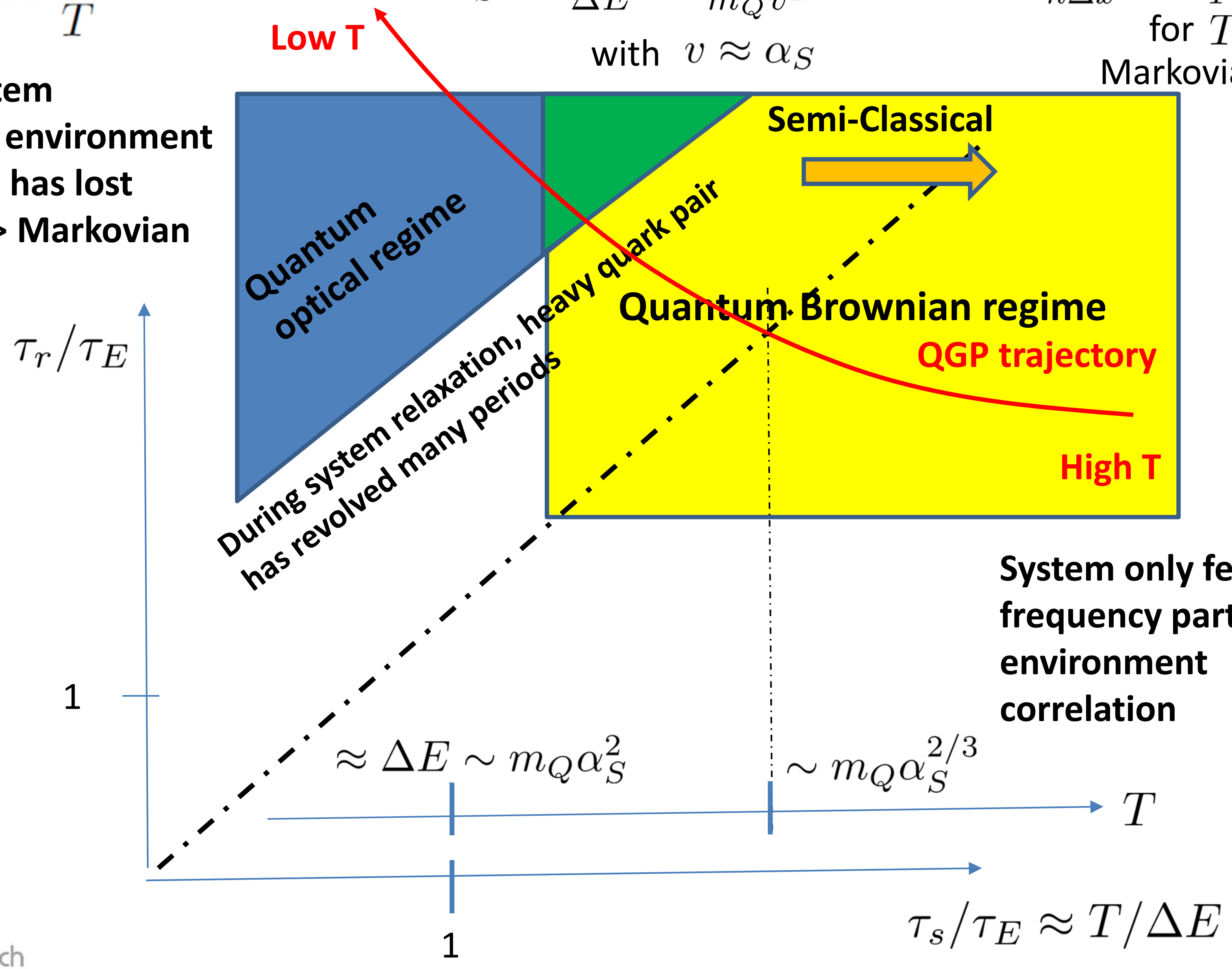
τ_R: system relax. time

$$\tau_R \approx \frac{1}{\kappa \Delta x^2} \approx \frac{m_Q^2}{T^3} \gg \tau_E$$

for $T \ll m_Q$

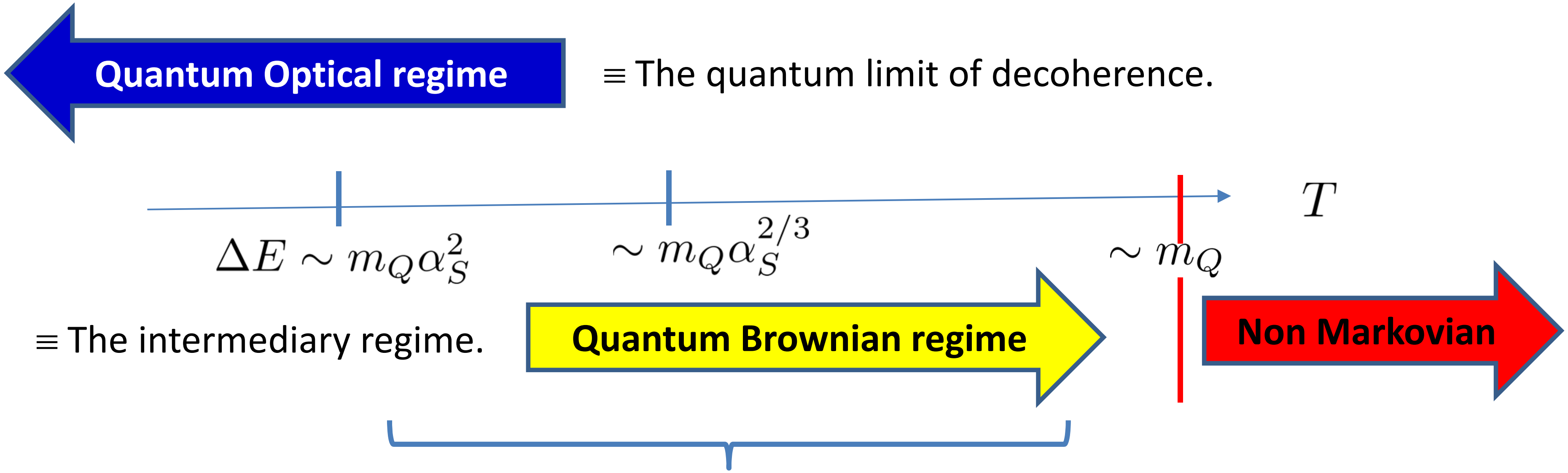
Markovian : OK !

During system relaxation, environment correlation has lost memory => Markovian process



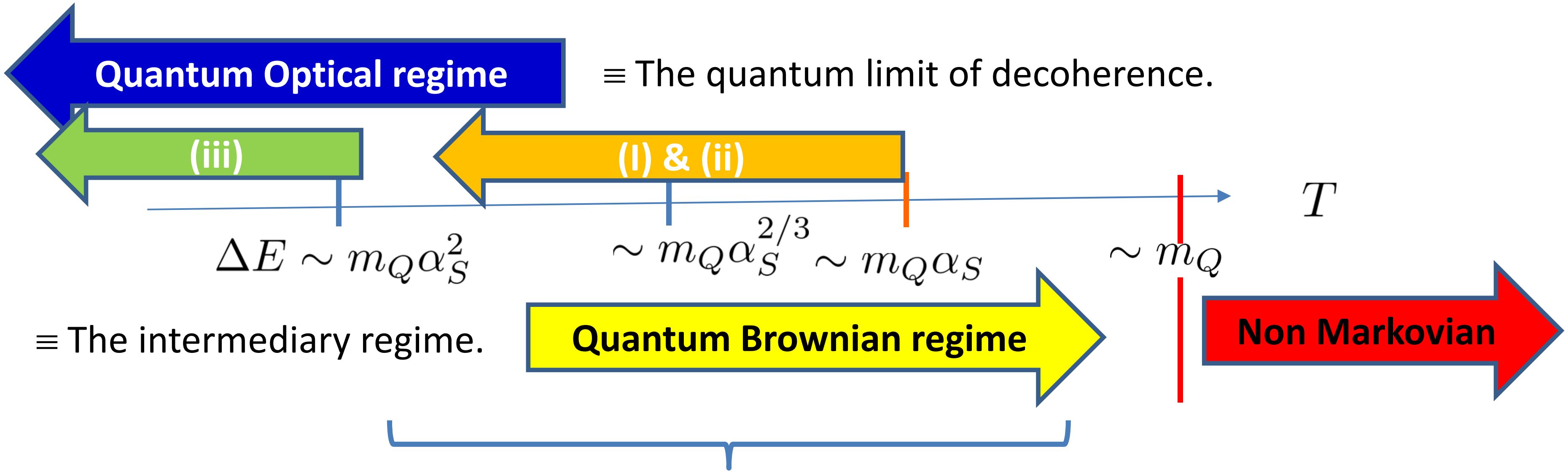
System only feels low frequency part of environment correlation

Subatech



For these « large » temperatures, the Q-Qbar gain enough energy to overwhelm the real binding potential => larger distance => larger decoherence ….

Subatech



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Refined subregimes when playing with the scales of NRQCD / pNRQCD (series of recent papers by N. Brambilla, M.A. Escobdo, A. Vairo, M Strickland et al, Yao and Mehen,…)

NRQCD: $Mv, \Lambda_{\mathrm{QCD}}, T \ll \mu_{\mathrm{NR}} \ll M$: most general scheme for markovian OQS !

pNRQCD:

(Singlet and octet quarkonium fields)

- (i) $1/r \gg T \sim m_D \gg E$: « strongly coupled » QME same as small dipole limit of NRQCD (applies for small time evolution)
- (ii) $1/r \gg T \gg E \gg m_D$: « weakly coupled » : g T << T : essential contribution is gluo – dissociation from hard mode T
- (iii) $1/r \gg T \sim E \gg m_D$: Quantum optical regime ? (ongoing debate due to discrete <-> continuum transitions)

Recent OQS implementations (single $Q\bar{Q}$)

(Year > 2015) Not exhaustive See as well table in 2111.15402v1

regime	SU3 ?	Dissipation ?	3D / 1D	Num method	year	remark	ref
NRQCD ⇔ QBM	No	No	1D	Stoch potential	2018		Kajimotoet al. , Phys. Rev. D 97, 014003 (2018), 1705.03365
	Yes	No	3D	Stoch potential	2020	Small dipole	R. Sharma et al Phys. Rev. D 101, 074004 (2020), 1912.07036
	Yes	No	3D	Stoch potential	2021		Y. Akamatsu, M. Asakawa, S. Kajimoto (2021), 2108.06921
	No	Yes	1D	Quantum state diffusion	2020		T. Miura, Y. Akamatsu et al, Phys. Rev. D 101, 034011 (2020), 1908.06293
	Yes ✓	Yes ✓	1D	Quantum state diffusion	**2021**		Akamatsu & Miura, EPJ Web Conf. 258 (2022) 01006, 2111.15402
	No	Yes	1D	Direct resolution	**2021**		O. Ålund, Y. Akamatsu et al, Comput. Phys. 425, 109917 (2021), 2004.04406
	Yes ✓	Yes ✓	1D	Direct resolution	**2022**		S Delorme et al, https://inspirehep.net /literature/ 2026925
pNRQCD (i)	Yes	No	1D+	Direct resolution	2017	S and P waves	N. Brambilla et al, Phys. Rev. D96, 034021 (2017), 1612.07248
(i) Et (ii)	Yes	No	1D+	Direct resolution	2017	S and P waves	N. Brambilla et al, Phys. Rev. D 97, 074009 (2018), 1711.04515
(i)	Yes	No	Yes	Quantum jump	**2021**	See SQM 2021	N. Brambilla et al. , JHEP 05, 136 (2021), 2012.01240 & *Phys.Rev.D* 104 (2021) 9, 094049, 2107.06222
(i)	Yes ✓	Yes ✓	Yes ✓	Quantum jump	**2022**		N. Brambilla et al. 2205.10289
(iii)	Yes ✓	Yes ✓	Yes ✓	Boltzmann (?)	2019		Yao & Mehen, Phys.Rev.D 99 (2019) 9, 096028, 1811.07027
NRQCD & « pNRQCD »	Yes	Yes	1D	Quantum state diffusion	**2022**		Miura et al. http://arxiv.org/abs/2205.15551v1
Other	No	Yes	1D	Stochastic Langevin Eq.	2016	Quadratic W	Katz and Gossiaux

Bottomonia… the 50 shades of pNRQCD

TUM + Kent State + … : most of the studies performed in the « strongly coupled » pNRQCD hierarchy :

$1/r \gg T \sim m_D \gg E \gg \Lambda_{QCD}$ regime (Strongly coupled ⇔ Quantum Brownian Regime)

Quarkonium fields : S & O + dipole approximation for g-quarkonium coupling (best for bottomonia, but can be questionable for higher states as b and bbar diffuse away)

Bonus from this simplification : only 2 parameters describing the QGP-quarkonium coupling:

κ: HQ momentum diffusion coefficient & γ : ≈ dipole Lamb-shift energy

$$\Sigma_s(t) = \frac{r^2}{2}\left[\kappa(t) + i\gamma(t)\right] ,$$

$$\Sigma_o(t) = \frac{N_c^2-2}{2(N_c^2-1)}\frac{r^2}{2}\left[\kappa(t) + i\gamma(t)\right] ,$$

Dipole self energy (complex => relaxation)

$$\Xi_{so}(\rho_o, t) = \frac{1}{N_c^2-1}\, r^i\, \rho_o\, r^i\, \kappa(t) ,$$

$$\Xi_{os}(\rho_s, t) = r^i\, \rho_s\, r^i\, \kappa(t) ,$$

$$\Xi_{oo}(\rho_o, t) = \frac{N_c^2-4}{2(N_c^2-1)}\, r^i\, \rho_o\, r^i\, \kappa(t) .$$

Transition / « jump » operators in the Lindblad equations

Lattice QCD estimates: $1.8 \lesssim \frac{\kappa}{T^3} \lesssim 3.4$; presently no direct estimate for γ.

TUM + Kent State: Strongly coupled pNRQCD

2017 — 2020 — 2021 **(SQM)** — 2022 — time

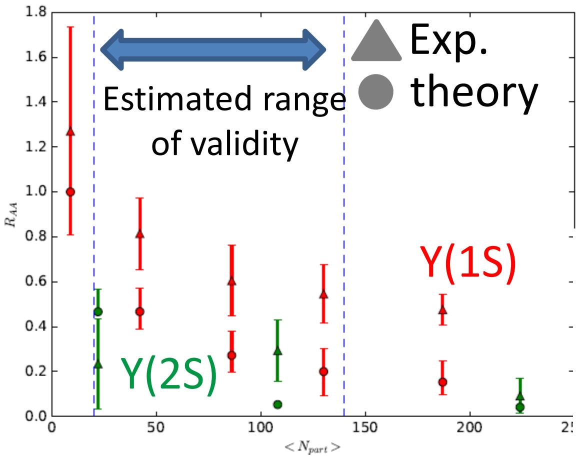
N. Brambilla et al, Phys. Rev. D 97, 074009 (2018), 1711.04515

N. Brambilla et al. , JHEP 05, 136 (2021), 2012.01240 & *Phys.Rev.D* 104 (2021) 9, 094049, 2107.06222

N. Brambilla et al. 2205.10289

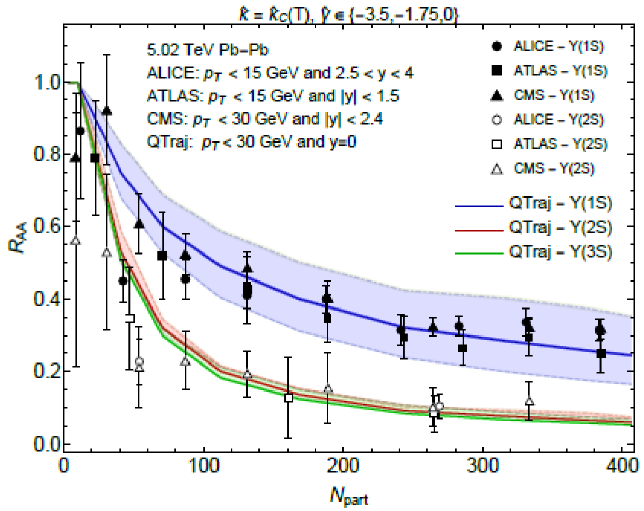
Direct solution, restrained to l=0 & l=1 sph. harmonics

$\rho_{l,l'}$ assumed diagonal



Numerical improvement due to the use of Quantum Jump algorithm (QTRAJ)

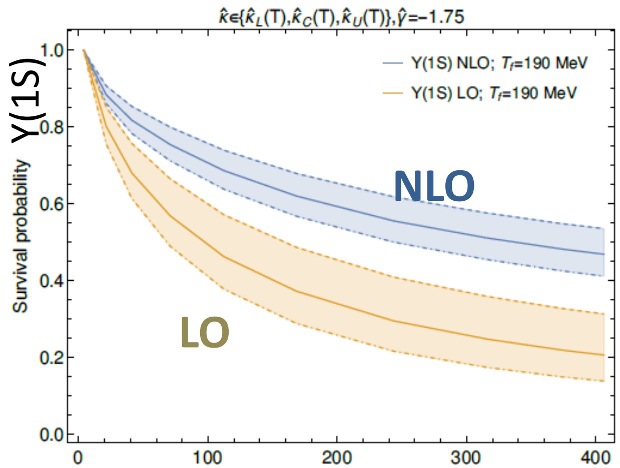
- No limit on ang. harmonics
- More realistic T(t) sampling
- QGP hydro-evolution
- => constrains on the DLS γ



- $R_{AA}(p_T)$: flat

Next To Leading order in T/E:

- ≈ recoil terms / friction
- Allows to go to lower T



- Sizable effect on the survival (friction prevents the $b\bar{b}$ from overheating => less suppr.)
- Complex H_{eff} & parameter uncertainties dominate over Q-jumps => not syst. included

TUM + Kent State: Strongly coupled pNRQCD

2017 | 2020 | 2021 **(SQM)** | 2022 | time

N. Bra… Rev. D 97, 0740…

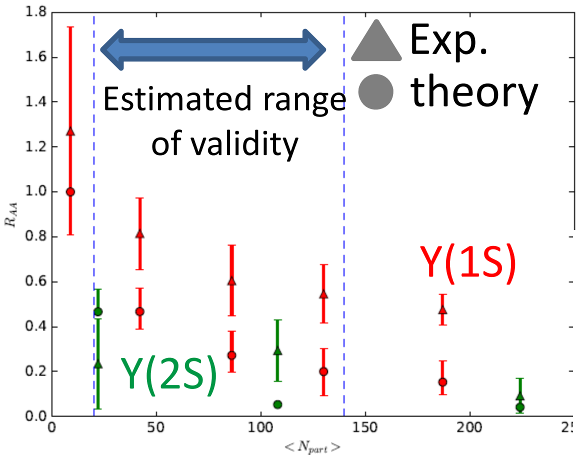
N. Brambilla et al. , JHEP 05, 136 (2021), 2012.01240 & *Phys.Rev.D* 104 (2021) 9, 094049, 2107.06222

N. Brambilla et al. 2205.10289

Program nearly completed , great !

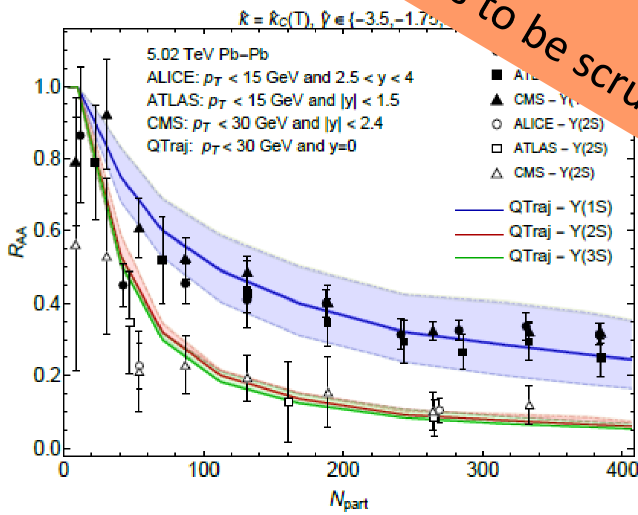
Dipole approximation needs to be scrutinized

Direct solution, r…
l=0 & l=1 sph. harmon…
$\rho_{l,l'}$ assumed diagonal



… improvement due to …tum Jump

- N… …nics
- More … QGP hydro… => constrains on…



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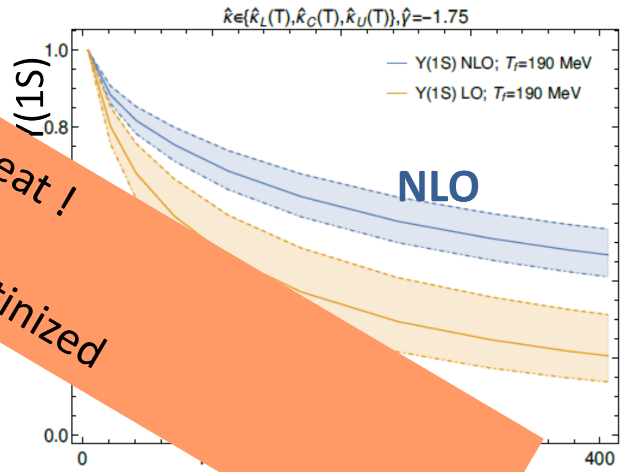

- Sizable effect … …ival (friction prevents t… from overheating => less suppr.)
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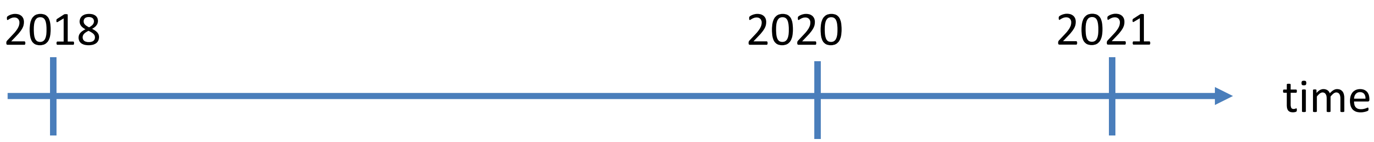
Bottomonia… the 50 shades of pNRQCD

Duke: low T pNRQCD hierarchy (weak coupled small dipole):

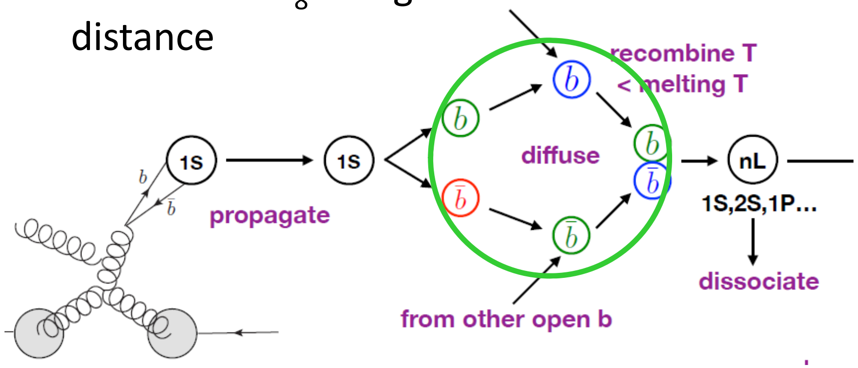
$1/r >> E >> T >> m_D > \Lambda_{QCD}$ regime (treats late T> E as « instantaneous dissociation)



Yao & Mehen, Phys.Rev.D 99 (2019) 9, 096028, 1811.07027

Equivalence between QME and Boltzmann equations:

- For the evolution of the singlet BS
- After the Wigner transform is performed
- Need/justify smooth octet distribution f_8 along relative distance



Correlated regeneration : same $b\bar{b}$ pair

What, there is nothing quantum in the end ?

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2018 — 2020 — 2021 — time

Yao & Mehen, Phys.Rev.D 99 (2019) 9, 096028, 1811.07027

Yao et al, JHEP 01 (2021) 046, 2004.06746

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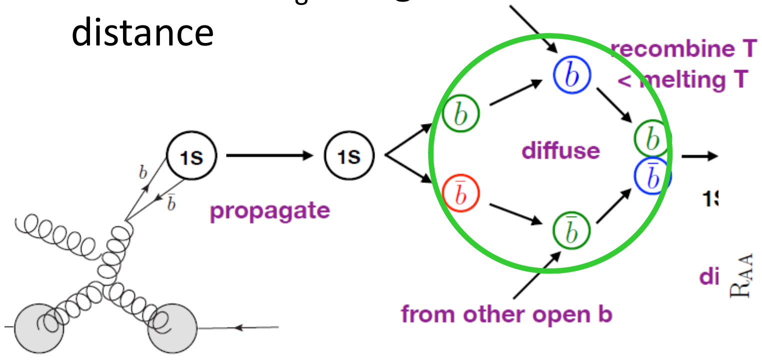
Coupled transp. Eq. in LIDO:

- Initial $b\bar{b}$ state : uncorrelated bottomonia (no open contribution)
- Good agreement with exp. results for α_s**=0.3**
- Crucial role of the correlated regenaration. Could be tested by $R_{AA}(\chi_b(1P))/R_{AA}(Y(2S))$



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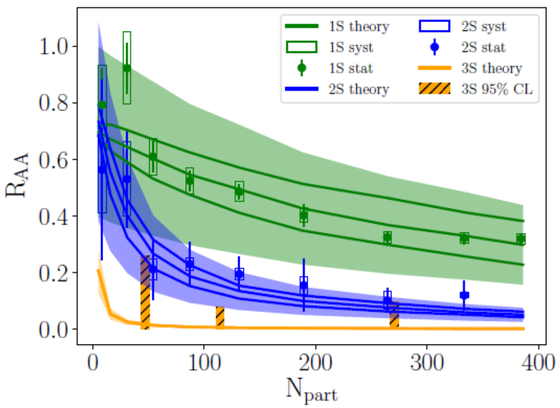

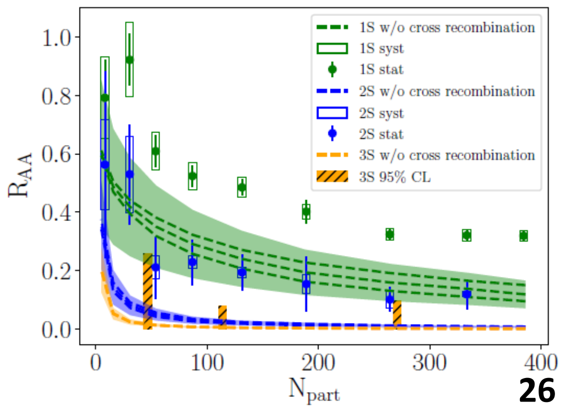

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Equivalence between QME and Boltzmann equations:

- For the evolution of the singlet BS
- After the Wigner transform is performed
- Need/justify smooth octet distribution f_8 along relative distance

Yao et al, JHEP 01 (2021) 046, 2004.06746

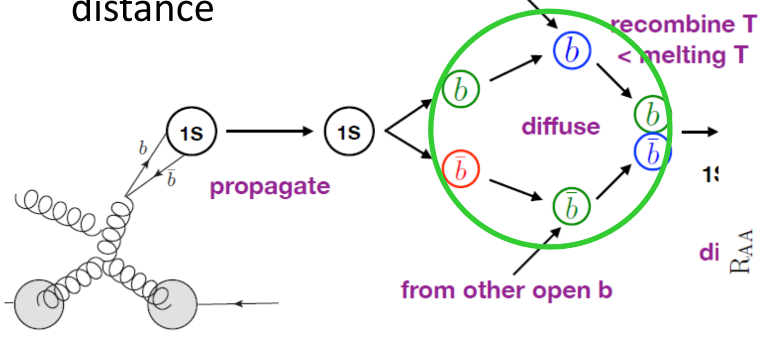
Coupled transp. Eq. in LIDO:

- Initial $b\bar{b}$ state : uncorrelated bottomonia (no open contribution)
- Good agreement with exp. results for α_s**=0.3**
- Crucial role of the correlated regenaration. Could be tested by $R_{AA}(\chi_b(1P))/R_{AA}(Y(2S))$

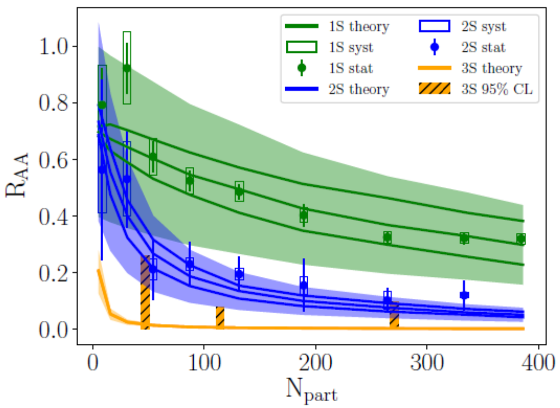
Yao & Mehen al, JHEP 02 (2021), 2009.02408

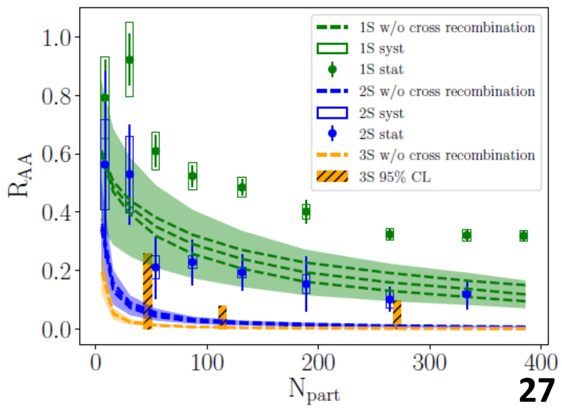
Revisit & extend previous work:

- Boltzmann ⇔ semi-classical gradient expansion of the octet distribution f_8 along relative distance
- Quantum correction derived formally… **Awaits for quantitative estimate**

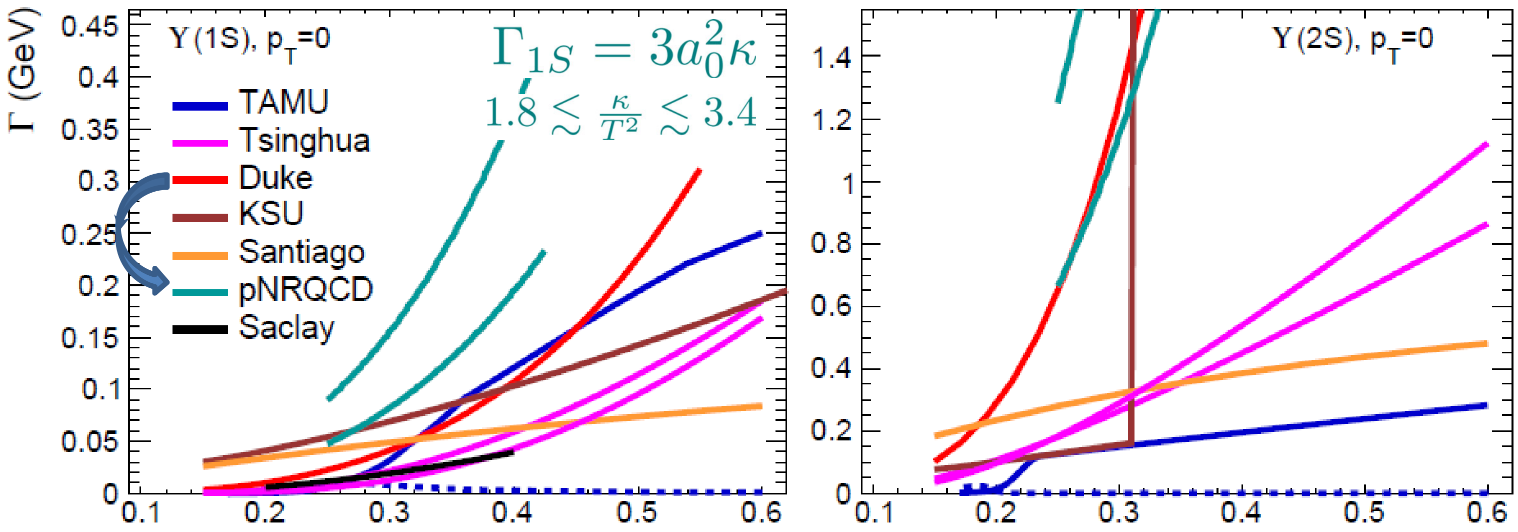


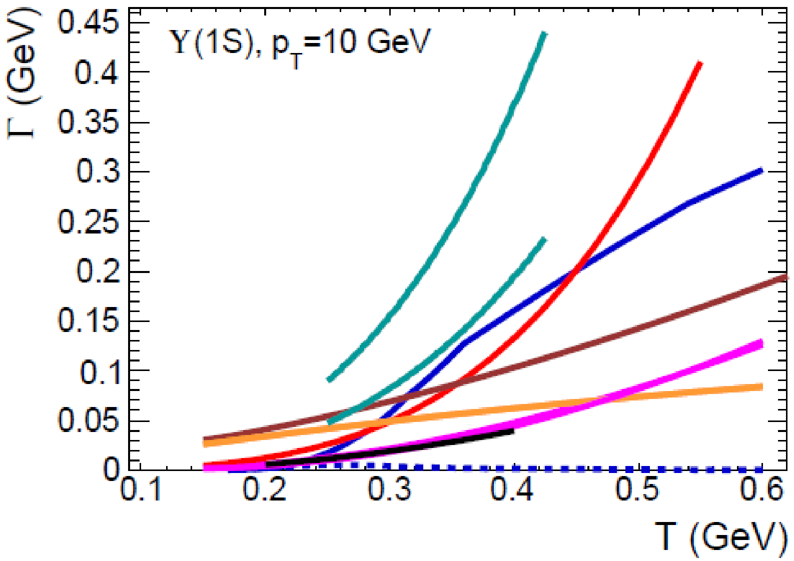
Correlated regeneration : same $b\bar{b}$ pair





Comparison TUM+Kent State vs Duke





- QGP is hot… a relative perspective => Equally good description of the data with 2 different hierarchies of pNRQCD !!!
- Could be the sign that these hierarchies are not so strictly defined or that Duke approach still applies for T << larger scale then E.
- … However, the dissociation Υ(1S) rate from both versions differ significantly
- => Need to gather and discuss / compare further

EMMI RRTF on QUARKONIA (Dec 2019)

See https://indico.gsi.de/event/9314/overview and manuscript in preparation

Fresh News from NRQCD

Akamatsu: (recent)

2020 — 2021 — 2022 → time

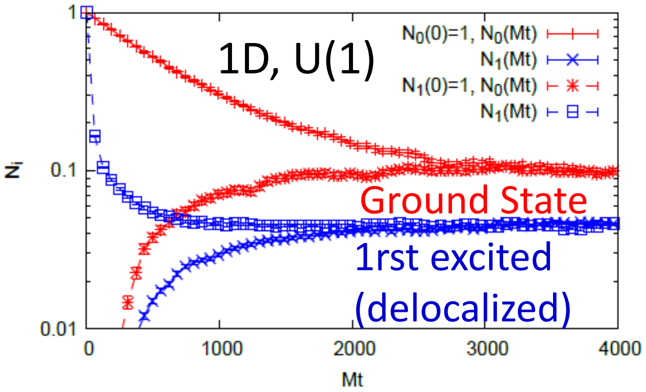
T. Miura, Y. Akamatsu et al, Phys. Rev. D 101, 034011 (2020), 1908.06293

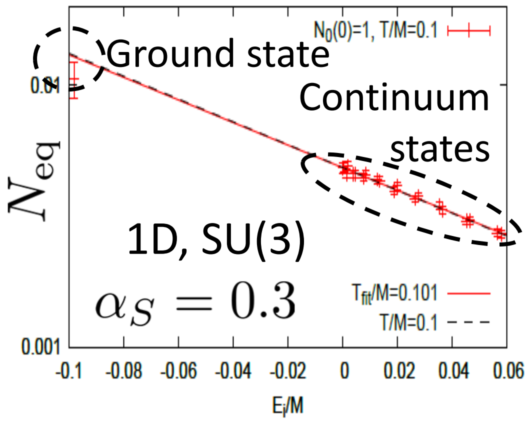
Akamatsu & Miura, EPJ Web Conf. 258 (2022) 01006, 2111.15402

Quantum jump : numerical solution through Quantum State Diffusion Method => friction could be included

Extension -> SU(3) (still 1D)

Approach to equil can be studied





Quantum State -> equilibrium irrelevant of the starting condition -> Boltzmann distribution

See as well discussion in Katz & Gossiaux, Annals Phys. 368 (2016) 267-295, 1504.08087

Fresh News from NRQCD

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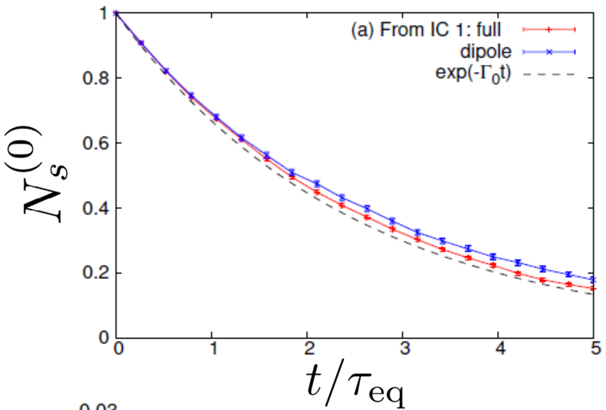
- 2021: Akamatsu & Miura, EPJ Web Conf. 258 (2022) 01006, 2111.15402
- 2022: T. Miura et al, http://arxiv.org/abs/2205.15551v1

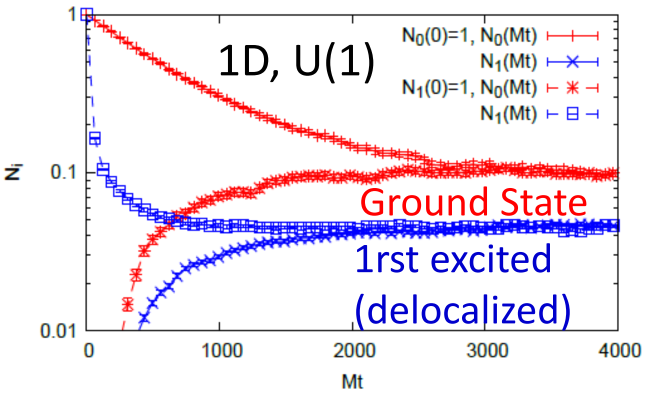
Quantum jump : numerical solution through Quantum State Diffusion Method => friction could be included

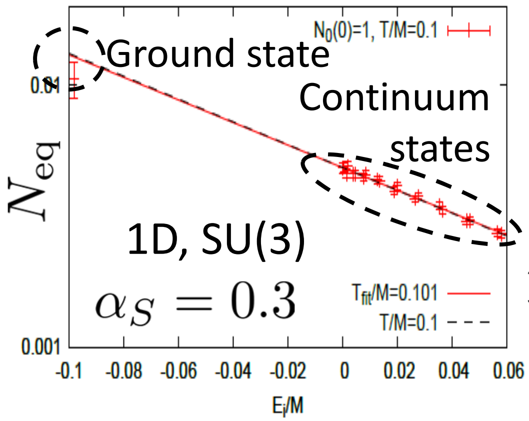
Extension -> SU(3) (still 1D)

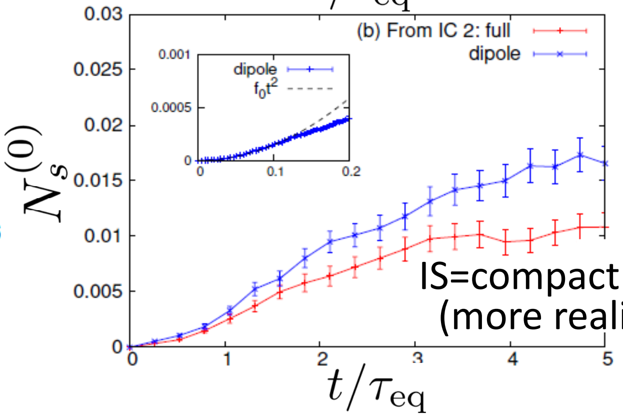
Comparison btwn full scheme & dipole limit (+nice scale analysis)



Approach to equil can be studied







IS=compact octet (more realistic)

Quantum State -> equilibrium irrelevant of the starting condition -> Boltzmann distribution
See as well discussion in Katz & Gossiaux, Annals Phys. 368 (2016) 267-295, 1504.08087

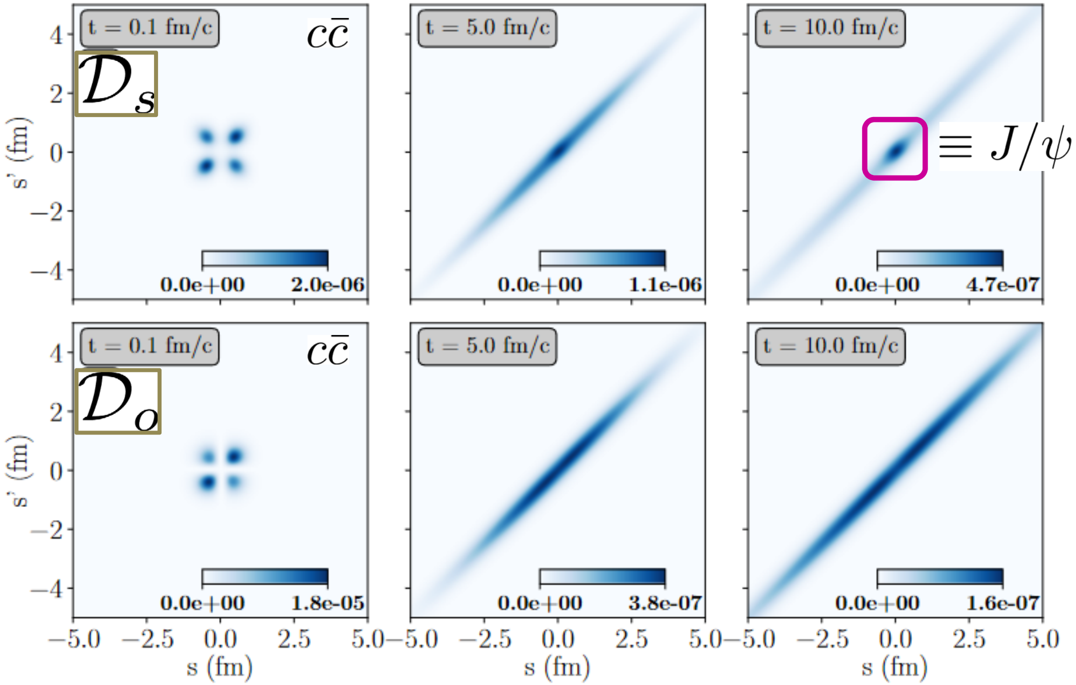
b and $\bar{b}$ are unbound in 8-channel -> density decreases and feeds less the singlet channel.

Fresh News from NRQCD

- S. Delorme (https://inspirehep.net/literature/2026925 and Ph.D. thesis; manuscript coming soon on arxiv), solving BE equations JP Blaizot & MA Escobedo JHEP 06 (2018) 034,1711.10812

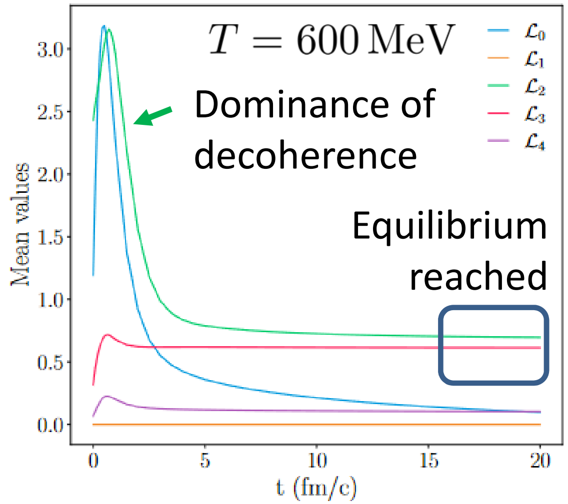
1D potential tuned to 3D : Katz et al. , arxiv2205.05154

Evolution from initial P-like octet state in the T(t) Bjorken scenario :

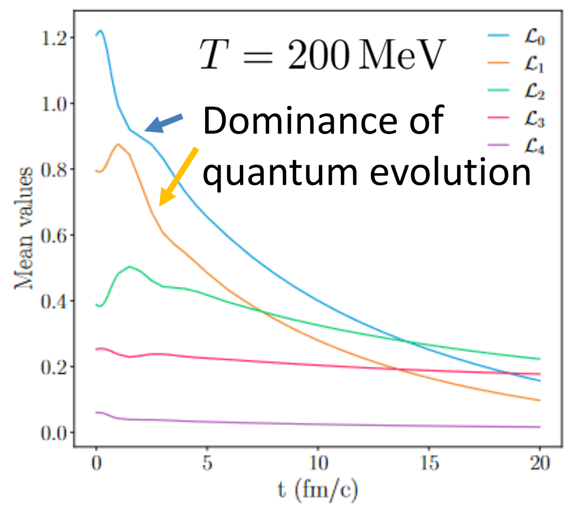


Evolution -> correlation in S-like singlet channel, surviving at the end of the evolution

various $\mathcal{L}$ contributions at fixed T



kinetic
potential
deco /fluct
dissipation
Pos. Preserv.



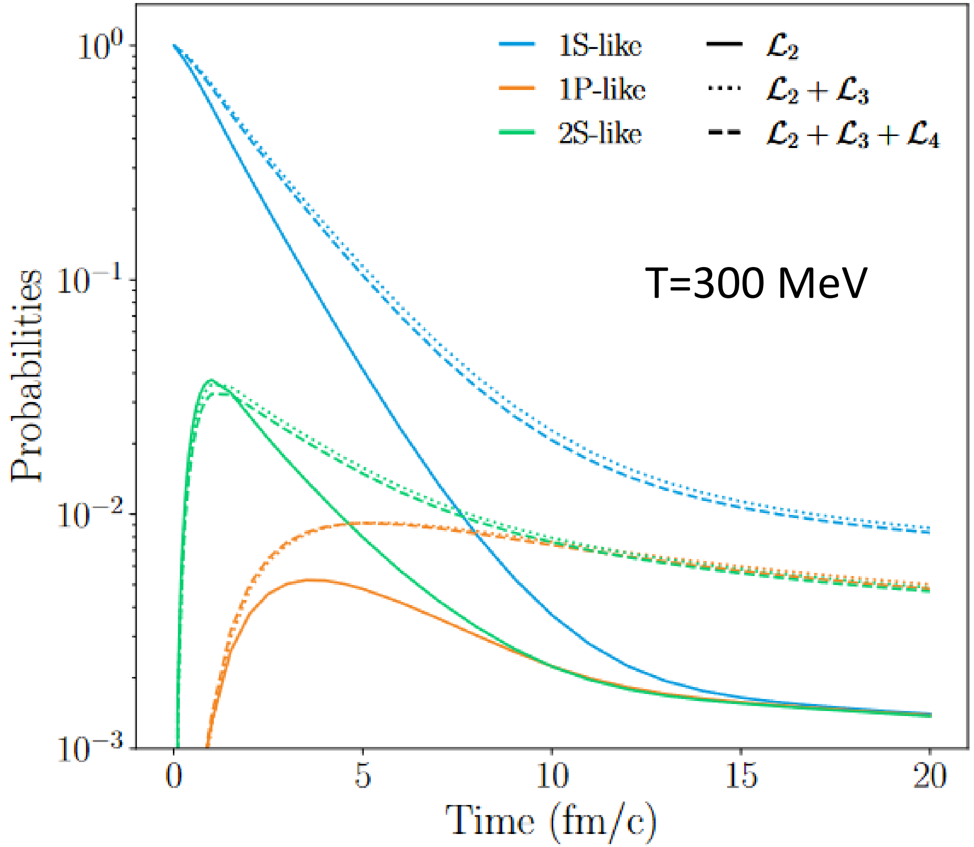
kinetic
potential
deco /fluct
dissipation
Pos. Preserv.

Several subregimes in the Quantum Brownian Motion regime : from classical dominated (high T) -> quantum dominated (« low » T)

Fresh News from NRQCD

- S. Delorme (https://inspirehep.net/literature/2026925 and Ph.D. thesis (http://www.theses.fr/2021IMTA0264 ; manuscript coming soon on arxiv) solving BE equations

Role of friction / dissipation in the probabilities



Fluctuation / decoherence only
Fluctuation + dissipation
Fluctuation + dissipation + positivity preserving

Same trend as the one found for instance in Brambilla et al when including the friction

Subatech

On the charmonium side

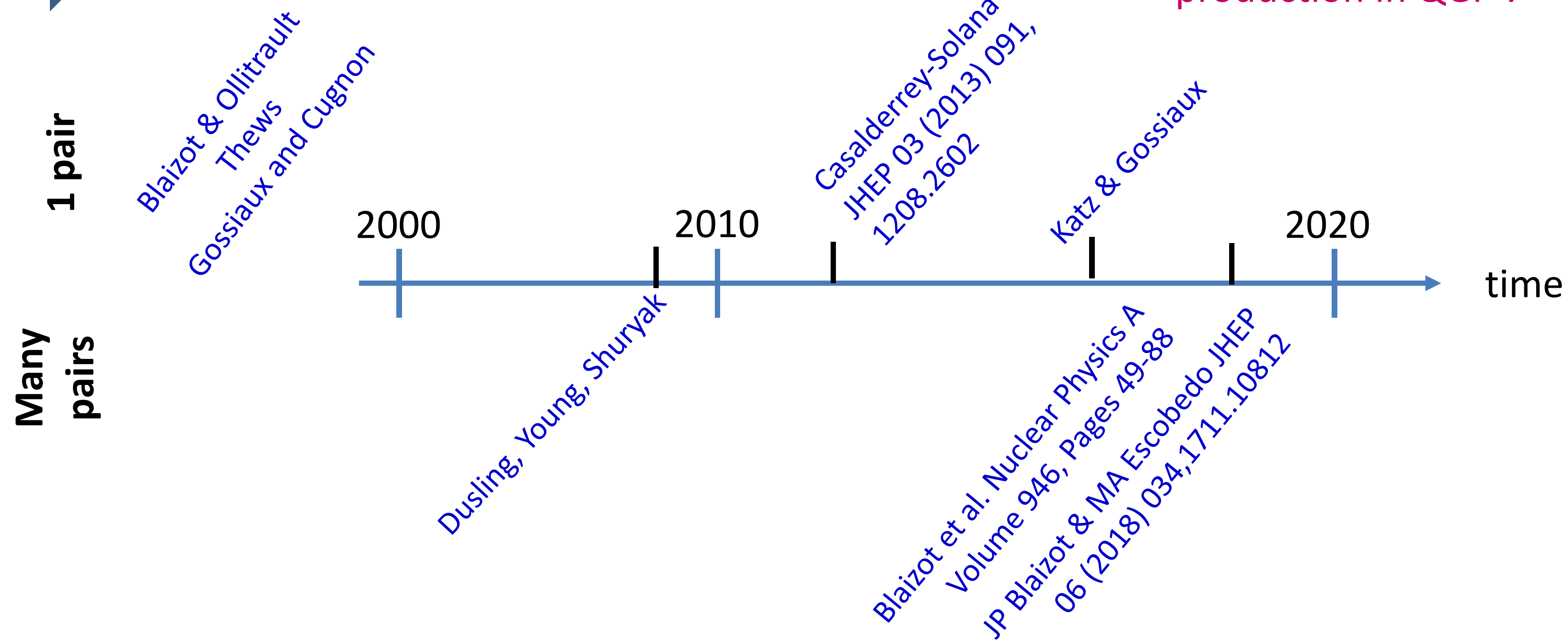
- More on the T >> M v^2 side and even T >> Mv side => pNRQCD is not right theory
- Besides, r* T may be >> 1 => not weak coupling to the QGP either

NRQCD should be privileged over pNRQD… or inspired models

Go microscopic in c and $\bar{c}$

Extra complication: For RHIC and LHC : many pairs !

Probes another aspect of Quarkonium production in QGP !



Pioneering work of **Blaizot and Escobedo** for many c-cbar pairs (NRQCD) => mixed Fokker-Planck + gain/loss rates for color transitions; awaits for implementation in realistic conditions

On the charmonium side

- **Arrebato et al.** (2206.01308) : new microscopic model inspired by OQS principles and Remler method

$$\text{prob}^{\Psi}(t) = \text{Tr}\left[\hat{\rho}^{\Psi}_{Q\bar{Q}}\hat{\rho}_N(t)\right]$$

E.A. Remler, ANNALS OF PHYSICS 136, 293-316 (1981)

Single quarkonia density operator $\hat{\rho}^{\Psi_i}_{Q\bar{Q}} = \sum_i |\Psi^i_{Q\bar{Q}}\rangle\langle\Psi^i_{Q\bar{Q}}|$

N-body density matrix (bulk partons + many c and many cbar)

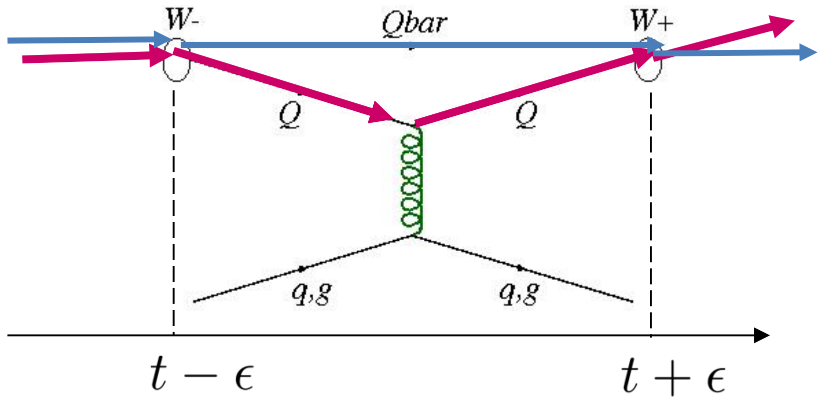
Reformulation : $\text{prob}^{\Psi}(t) = \text{prob}^{\text{prim}}(t_0) + \int_{t_0}^{t} \Gamma^{\Psi}(t')dt'$

Von Neumann eq.

With rate of creation/destruction: $\Gamma^{\psi}(t) = \frac{d\text{prob}^{\Psi}(t)}{dt} = \text{Tr}\left[\hat{\rho}^{\Psi}_{Q\bar{Q}}\frac{d\hat{\rho}_N(t)}{dt}\right]$

Passing to Wigner representation and using semi-classical trajectories for **Q** :

$$\Gamma^{\Psi}(t) = \sum_{i=1,2}\sum_{j\geq 3}\delta(t-t_{ij})\int\frac{d^3p_i d^3x_i}{h^3}W^{\Psi}_{Q\bar{Q}}(p_1,x_1;p_2,x_2)\left[W_N(t+\epsilon)-W_N(t-\epsilon)\right]$$

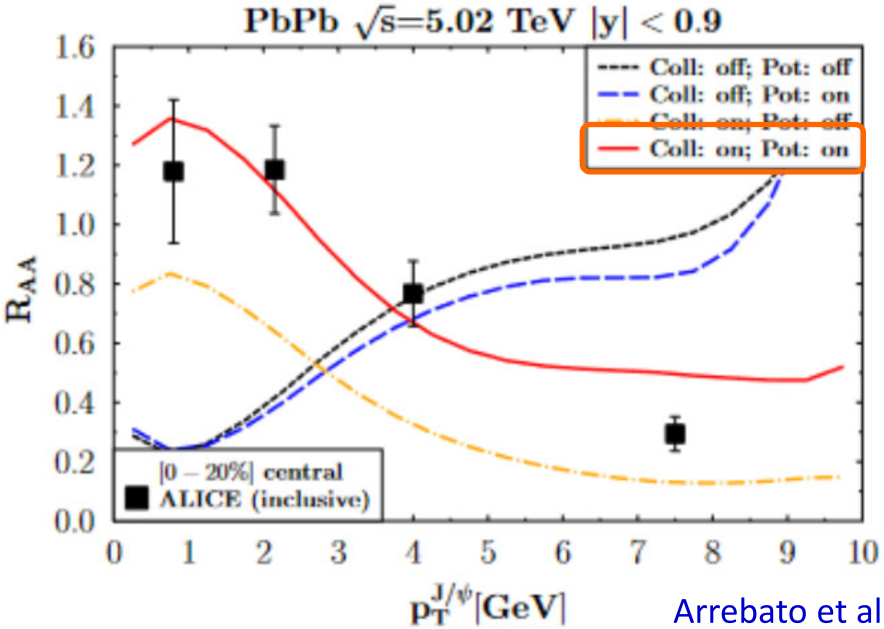


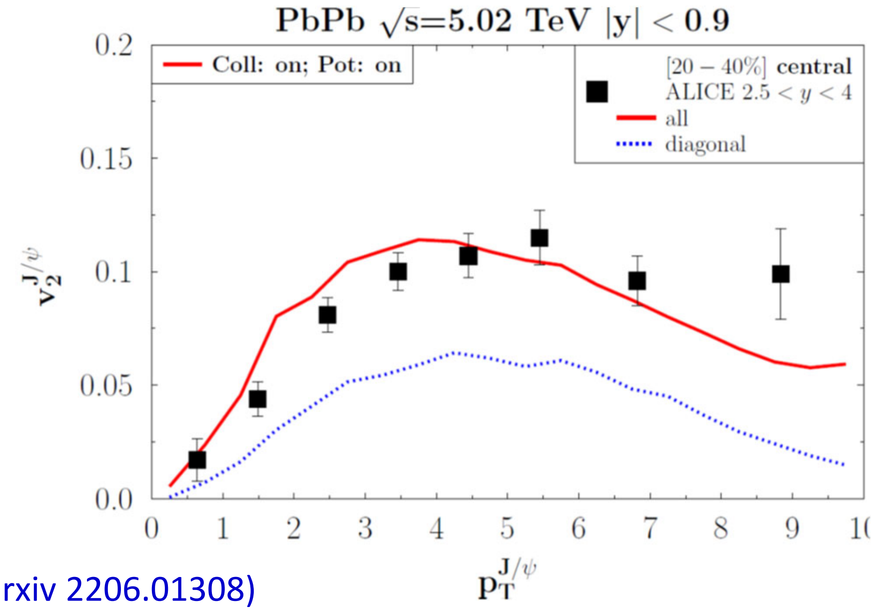
- Quarkonia production in the model is a 3-body process, the c & cbar interact only by collision !
- The "details" of H_{int} between Q and bulk partons are incorporated into the evolution of W_N after each collision / time step (good for the MC simulation)
- Dissociation and recombination treated in the same scheme

+ state of the art c-cbar potential

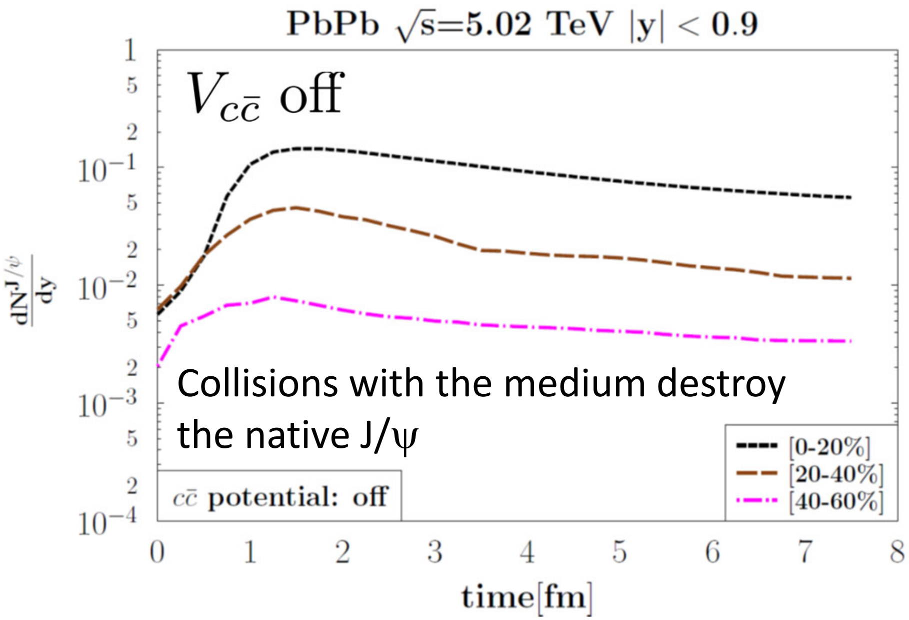
D. Lafferty and A. Rothkopf, PHYS. REV. D 101, 056010 (2020)

Some results from our new microscopic model

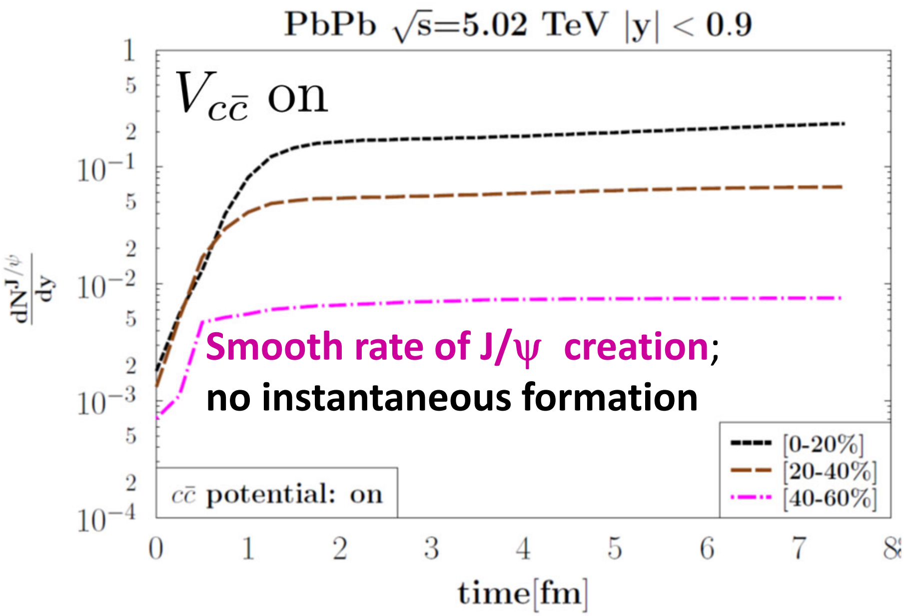




Arrebato et al. (arxiv 2206.01308)



Collisions with the medium destroy the native J/ψ



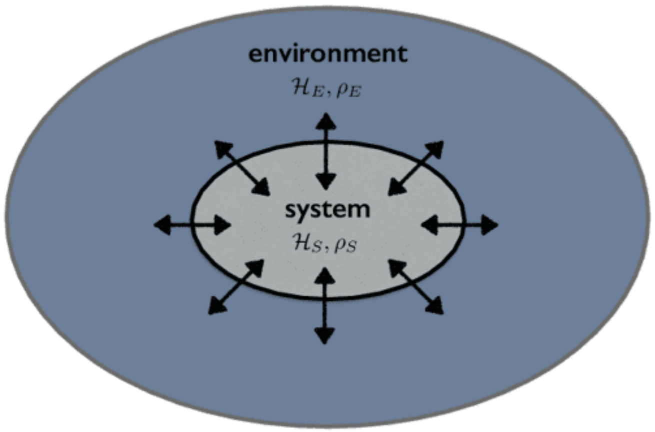
Smooth rate of J/ψ creation; **no instantaneous formation**

Conclusions and Open Questions for OQS

- For b-bar:
  - ✓ pNRQCD is now close to « full OQS » for some relevant hierarchies
  - ✓ Some fundamental QGP quantities can be constrained using bottomonia production in AA collisions
  - ✓ Need to better understand the domain of applicability of various hierarchies and possibly match them
  - ✓ Need to fully quantify the systematic uncertainties stemming from
    - The dipolar approximation (impacts Υ(3S) the most)
    - the « choice » of the initial state
    - the semi-classical approximations if one use them
- For single c-cbar pair:
  - ✓ Full OQS treatment still uncomplete for NRQCD (dissipation + SU(3) ok, but just for 1D) but field is progressing fast
  - ✓ Need to better understand the various regimes and the legitimity of various semi-classical approximations (transport, Fokker Planck)
- For many c-cbar pairs:
  - ✓ Open room for OQS inspired microscopic models properly applied in the relevant regimes
  - ✓ Then, need to make contact with transport theories and SHM.

Back up : more on quantum coherence

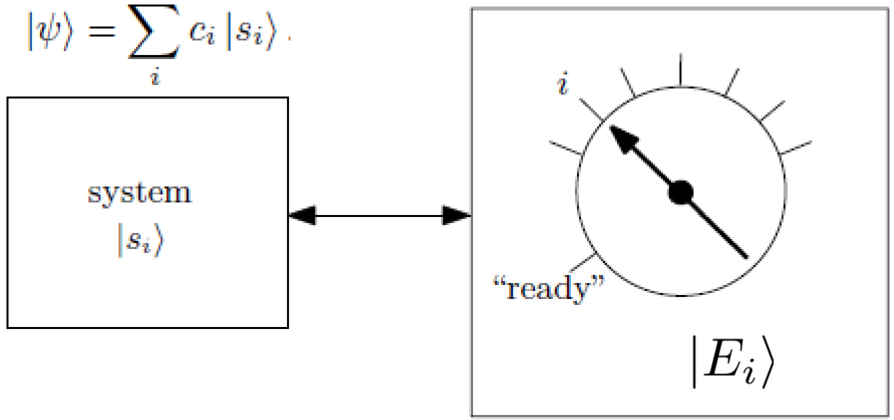
Decoherence, system-environment interaction, OQS



From B. Vacchini

- Since 1970
- Revisited all longstanding problems of quantum mechanics with a new paradigm : quantum systems are naturally coupled to some external environment whose role cannot be ignored.
- Starting from uncorrelated system – detector / environment state, the interaction between the system and the environment will build some global entangled state

$$|\psi\rangle = \sum_i c_i |s_i\rangle .$$



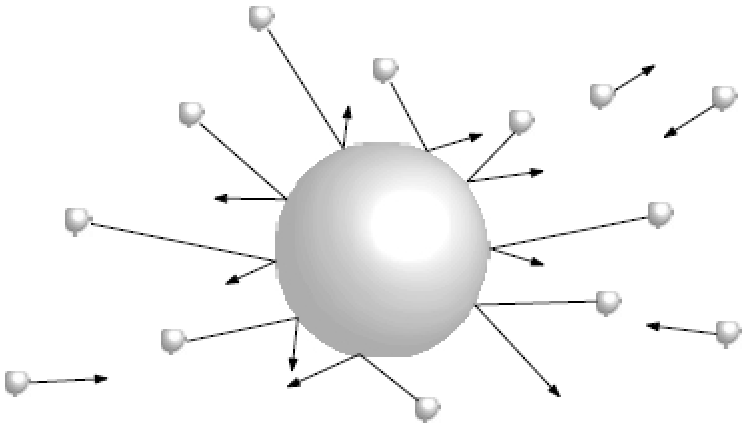
$$\frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)|E_0\rangle \rightarrow \frac{1}{\sqrt{2}}(|\psi_1\rangle|E_1\rangle + |\psi_2\rangle|E_2\rangle)$$

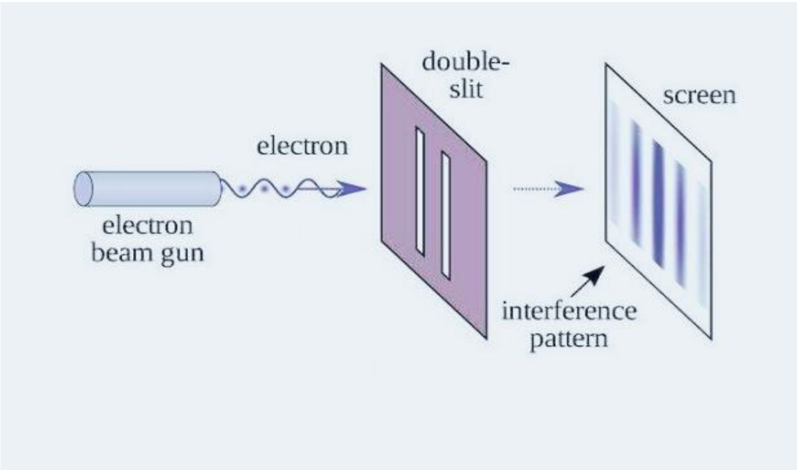
Entangled !

Coherence has been « delocalized » in the larger system … But this is basis « invariant »

Decoherence, system-environment interaction, OQS

- As postulated by Zurek : by interacting with different states of the quantum System, the environment outcome (in $|E_1>$, $|E_2>$,…) get the information about this system which can be considered as a genuine measurement of this system





Analogy with the 2 slits experiment and the « which state » observer

- Environment « measures » the system.
- There exists a preferred ensemble of states (in $|E_1>$, $|E_2>$,…) such that the interactions with the system leads to progressive (but « fast ») orthogonality $\langle E_1|E_2\rangle \sim 0$

Preferred basis or « pointer states » $\langle E_1|E_2\rangle \propto e^{-\frac{t}{t_d}}$ t_d: decoherence time

Decoherence, syst-env. interaction, OQS

$$\frac{1}{\sqrt{2}}\left(|\psi_1\rangle+|\psi_2\rangle\right)|E_0\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|\psi_1\rangle|E_1\rangle+|\psi_2\rangle|E_2\rangle\right)$$

Consequence on the reduced density matrix

$$\hat{\rho}=|\Psi\rangle\langle\Psi| \longrightarrow \hat{\rho}_S=\mathrm{tr}_E\left(|\Psi\rangle\langle\Psi|\right)$$

Tracing out the environment

<u>t=0</u>

$$\hat{\rho}_S=\frac{\langle E_0|E_0\rangle}{2}\left(|\psi_1\rangle\langle\psi_1|+|\psi_2\rangle\langle\psi_2|+|\psi_1\rangle\langle\psi_2|+|\psi_2\rangle\langle\psi_1|\right)$$

$$\hat{\rho}_S=\frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$$ pure state

$\underline{t\gg t_d}$

$$\hat{\rho}_S=\frac{1}{2}\left(|\psi_1\rangle\langle\psi_1|+|\psi_2\rangle\langle\psi_2|+\langle E_2|E_1\rangle|\psi_1\rangle\langle\psi_2|+\langle E_1|E_2\rangle|\psi_2\rangle\langle\psi_1|\right)$$

$$\hat{\rho}_S\approx\frac{1}{2}\begin{pmatrix}1 & \approx 0\\ \approx 0 & 1\end{pmatrix}$$

Appears as quasi classical mixed state

- Coherence still present at the global level (entangled state)
- Not observable at the « local » (system) level
- Appears as a « decohered » / classical density (1/2 for « 1 » state and ½ for « 2 » state)

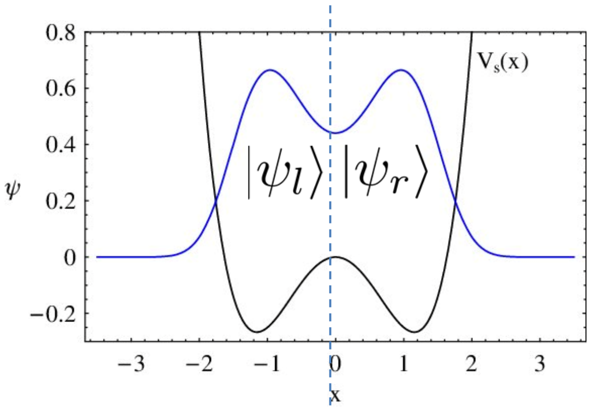
Decoherence, syst-env. interaction, OQS

$t \gg t_d$

- Disappearance of interferences
- Measurements (and dynamics) in terms of classical probabilities
- **… but in the system states that are the best « measured » by the environment the so-called pointer states (preferred basis) !**

(superselection rule)

Link between decoherence and « suppression »



Case A: Ground state of double well potential put in contact with environment…
Assume $|\psi_1\rangle = |\psi_l\rangle$ and $|\psi_2\rangle = |\psi_r\rangle$ are 2 pointer states

$$|GS\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle) \qquad \hat{\rho}_{\rm GS} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$${\rm prob}({\rm GS}, t=0) = 1 \longrightarrow {\rm prob}({\rm GS}, t \gg t_d) = {\rm tr}(\hat{\rho}_S(t)\cdot\hat{\rho}_{\rm GS})$$

$$\approx {\rm tr}\left(\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\cdot\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\right)$$

$$\approx \frac{1}{2}$$

Decoherence ≡ suppression

Decoherence, syst-env. interaction, OQS

$t \gg t_d$

- Disappearance of interferences
- Measurements (and dynamics) in terms of classical probabilities
- **… but in the system states that are the best « measured » by the environment the so-called pointer states (preferred basis) !**

(superselection rule)

Link between decoherence and « suppression »

Case B: compact state put in contact with environment…
Assumes preferred states : {eigenstates of the system Hamiltonian}

$$|\Psi\rangle = \underbrace{\left(\sqrt{p_{\rm GS}}|\psi_{\rm GS}\rangle + \sqrt{p_1}|\psi_1\rangle + \cdots\right)}_{|\psi\rangle}|E_0\rangle$$

$$\text{prob}(\text{GS}, t=0) = p_{\rm GS} \longrightarrow \text{prob}(\text{GS}, t \gg t_d) = \text{tr}(\hat\rho_S(t)\cdot\hat\rho_{\rm GS})$$

$$\approx \text{tr}\left(\begin{pmatrix} p_{\rm GS} & 0 & 0 \\ 0 & p_1 & 0 \\ 0 & 0 & p_2 \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right)$$

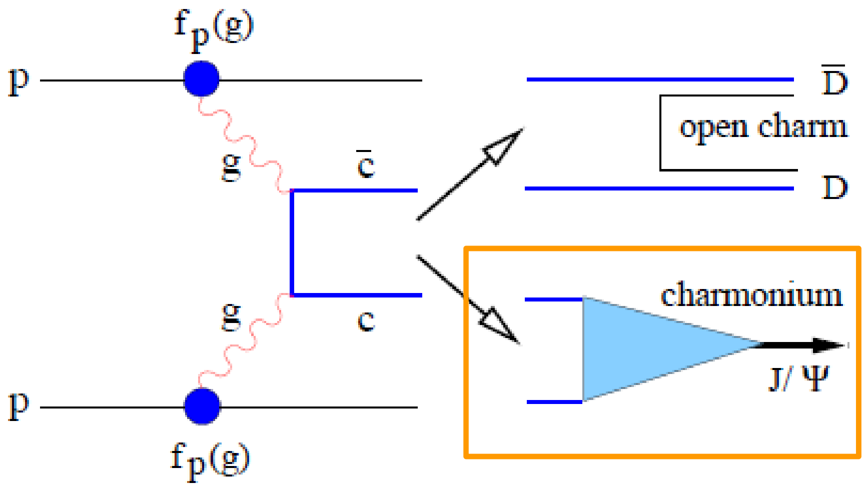
$$\approx p_{\rm GS}$$

Decoherence : ≠ suppression
: just « reveals » the various components of the initial eigenstate

Consequence for Quarkonia :

Iff case B apply :

Picture behind transport theory (taken from H. Satz):



Open heavy flavor and quarkonia assumed to be uncorrelated

~~**Formed after some "formation time" τ_f (typically the Heisenberg time), usually assumed to be independent of the surrounding medium**~~

Formed after some "decoherence time" τ_d, which is a scale depending both on the the initial state and the surrounding medium

And then, classical transport theory rules !

Decoherence, syst-env. interaction, OQS

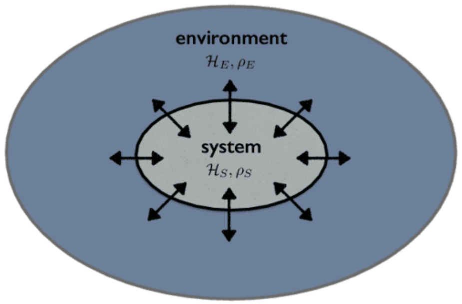
<u>First conclusion from the study of usual decoherence scheme :</u>

- ✓ Essential to characterize the « preferred states » : *maximally robust against decoherence*
- ✓ Need to (semi)-quantify the decoherence time

⇨ Hamiltonian $\hat{H} = \hat{H}_S + \hat{H}_E + \hat{H}_{\rm int}$

Interaction hamiltonian btwn S and E

Usual linear coupling: $\hat{H}_{\rm int} = g \sum_a \hat{S}_a \otimes \hat{E}_a$



<u>Time scales :</u>

τ_S : Time scale associated to the system (indep. Environment)

$\tau_S \sim \frac{1}{\Delta E_S}$ ≡ Heisenberg time (mind if discrete states + continuous)

Energy gap

τ_E : Time scale associated to the environment (indep. System)

$\tau_E \sim \frac{1}{T}$ (hard modes) $\tau_E \sim \frac{1}{m_D}$ (soft modes)

Temperature Debye mass

τ_R : System relaxation time scale resulting from interaction with the env. (in general ≠ t_d !)

Several estimates in the litterature,… in general >> at least τ_E (weak coupling) : Markovian approx

Decoherence, syst-env. interaction, OQS

Standard textbook (M. Schlosshauer) results for OQS:

✓ 3 regimes :

$$\hat{H} = \hat{H}_S + \hat{H}_E + \hat{H}_{\text{int}}$$

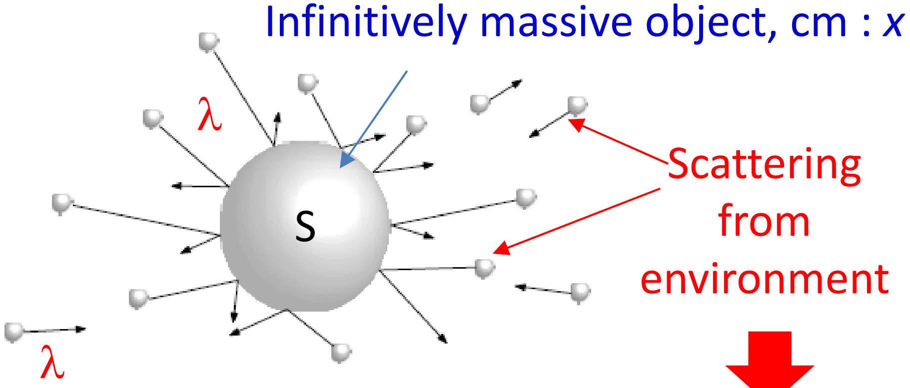
Name	condition	Consequence for Preferred States	Relevance for quarkonia
quantum-measurement limit	Evolution of the system dominated by H_{int}	PS: eigenstates of H_{int} (often : ⇔ positions)	limited
The intermediary regime	evolution of the system governed by H_{int} and H_S in roughly equal strengths	PS: localized in phase space, i.e., in both position and momentum	High Temperature
The quantum limit of decoherence.	The environment is slow and the Hamiltonian H_S dominates the evolution of the system	PS: Eigenstates of H_S	« Low » Temperature (but still > T_{SC})

⇔ Case B presented before

Decoherence, syst-env. interaction, OQS

Standard textbook (M. Schlosshauer) results for OQS:

✓ Decoherence :



$$\hat{H} = \hat{H}_S + \hat{H}_E + \hat{H}_{\text{int}}$$

Reduced density matrix $\rho_S(\mathbf{x}, \mathbf{x}')$

⇕

Wigner density $W_S(\mathbf{X} = \frac{\mathbf{x}+\mathbf{x}'}{2}, p)$

Fourier conjugate of $\mathbf{x} - \mathbf{x}'$

$$\frac{\partial \rho_S(\mathbf{x}, \mathbf{x}', t)}{\partial t} = -F(\mathbf{x} - \mathbf{x}')\rho_S(\mathbf{x}, \mathbf{x}', t)$$

Decoherence factor:

$$F(\mathbf{x} - \mathbf{x}') = \int dq \rho(q) v(q) \int \frac{d\hat{n} d\hat{n}'}{4\pi} \left(1 - e^{iq(\hat{n}-\hat{n}')\cdot(\mathbf{x}-\mathbf{x}')/\hbar}\right) |f(q\hat{n}, q\hat{n}')|$$

Scattering amplitude

Short wave length (λ << Δx)

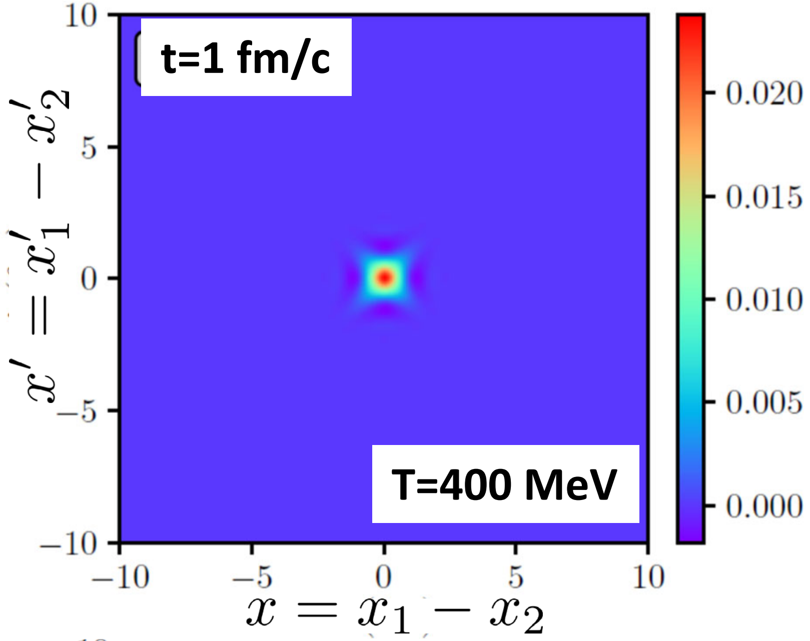
$$F(\mathbf{x} - \mathbf{x}') = \Gamma_{\text{tot}}$$

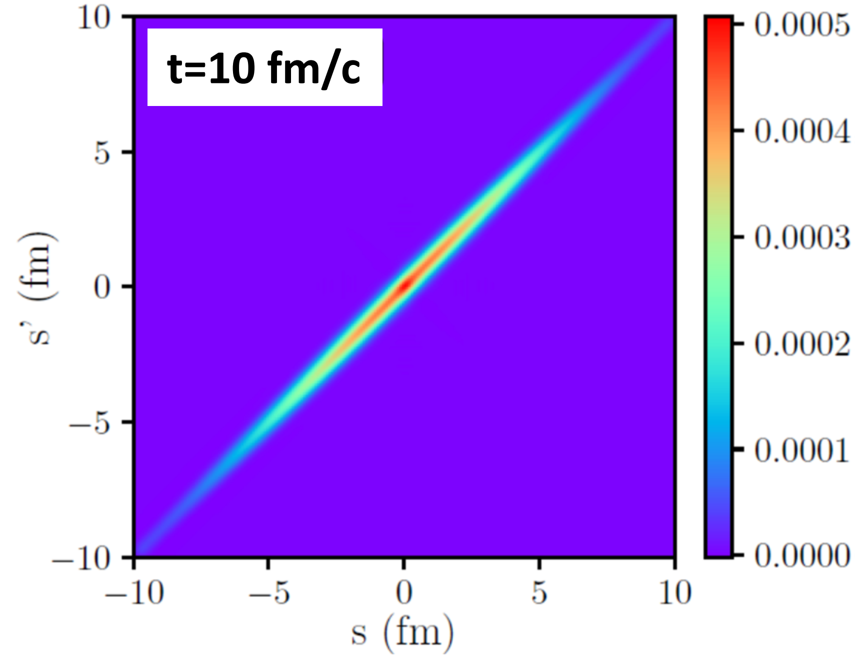
Long wave length (λ >> Δx)

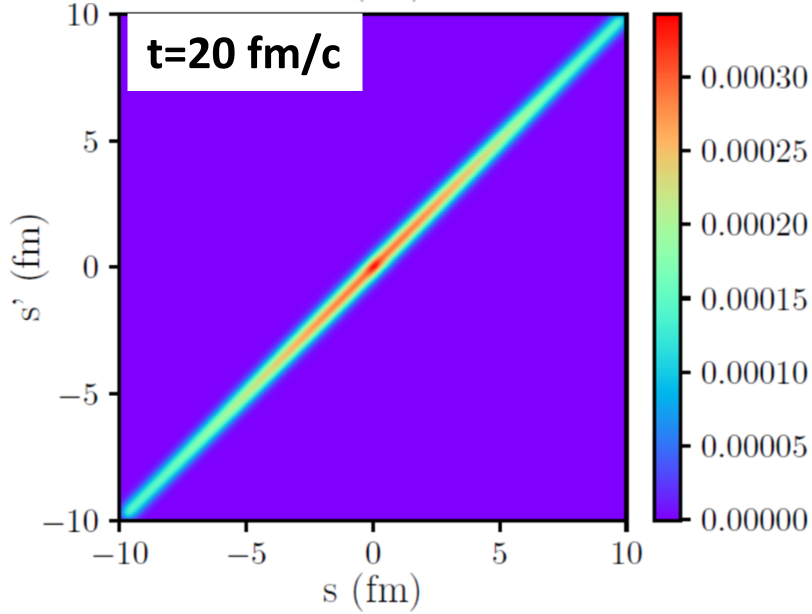
$$F \approx \int dq \rho(q) v(q) q^2 \sigma_{\text{transp}}(q) \times (\mathbf{x} - \mathbf{x}')^2 \approx \kappa(\mathbf{x} - \mathbf{x}')^2$$

Standard textbook (M. Schlosshauer) results for OQS:

✓ Decoherence :







$$\rho_S(x,x',t) \sim \rho_S(x,x',0)e^{-\Lambda(x-x')^2 t}$$

- Compactification along the short diagonal = « classicalization »
- $t_d \sim \frac{1}{\kappa(\Delta x)^2} \sim \frac{1}{TM\eta_D(\Delta x)^2}$

Einstein relation Single part. relaxation rate

$$t_d \sim \frac{\tau_R^{\text{single}}}{\frac{1}{\lambda_{th}^2}(\Delta x)^2} \sim \tau_R^{\text{single}} \times \left(\frac{\lambda_{\text{th}}}{\Delta x}\right)^2$$

Standard textbook (M. Schlosshauer) results for OQS:

✓ Long time dynamics:

Initial state

Decoherence >> dissipation
or
Decoherence << dissipation

$t \gg \{t_d, t_R\}$ Long time dynamics

Equilibration: Decoherence ≈ dissipation

The quantum limit of decoherence.
(« low » T)

Prob of H_S eigenstate $n : p_n \propto e^{-\frac{E_n}{T}}$

Contact with Statistical Hadronization Model

intermediary regime.
(high T)

Gaussian in phase space with

$\Delta p^2 \sim M\,T \qquad \Delta x^2 \sim (M\,T)^{-1} \sim \lambda_{\rm th}^2$

One has in general $p_{n+1} < p_n$, but not

$p_n \propto e^{-\frac{E_n}{T}}$ Exception: $\hat{H}_S = \hat{H}_{HO}$

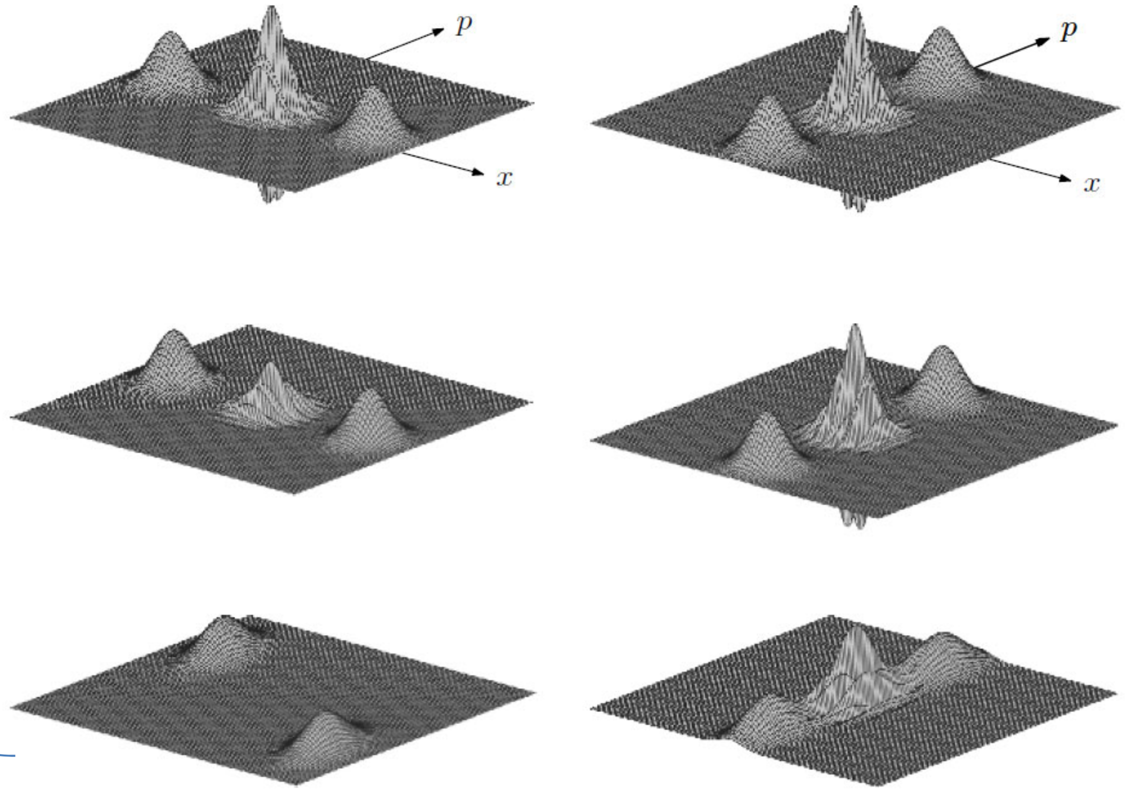
Temperature range has a drastic influence on the physics representation we have for quarkonia evolution in the QGP medium !

Standard textbook (M. Schlosshauer) results for OQS:

- ✓ Decoherence in the intermediary regime: 2 overlapping gaussians

Need to introduce dissipation terms

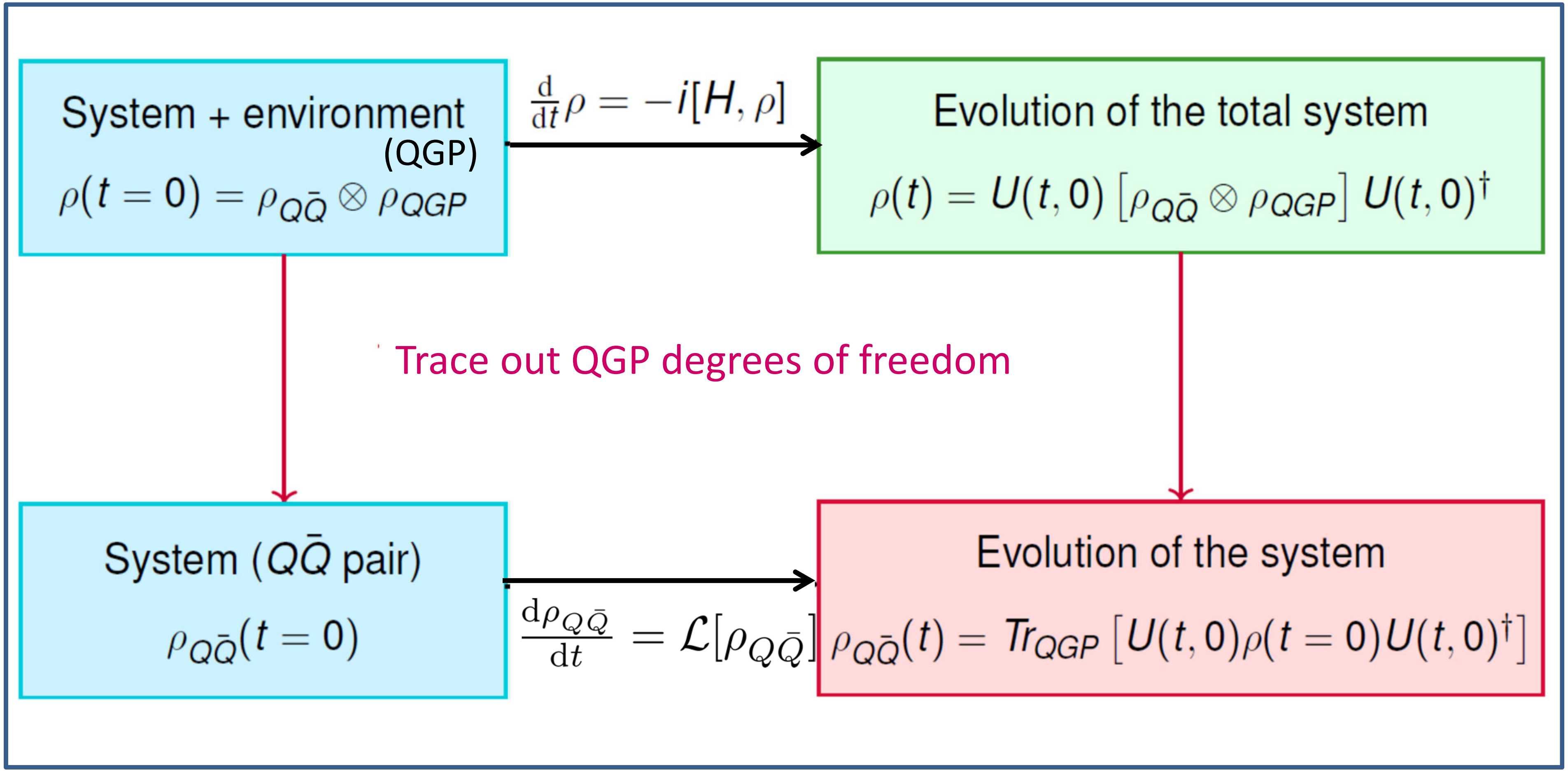
Fast decoherence: the environement can directly monitor the x due to the structure of Hint



Slow decoherence: the environement cannot directly monitor the p => need first that both peaks spearated in p get separated in x to be monitored

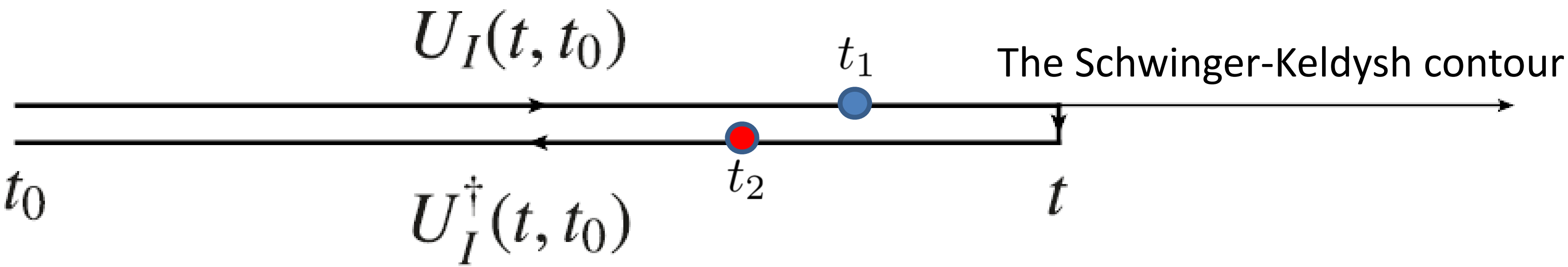
For initial Q-Qbar pair : more the 2[nd] case => long decoherence

Quantum Master Equations



However, $\mathcal{L}[\cdot]$ is generically a non local super-operator in time

Quantum Master Equations



2 times involved in the propagation of ρ (direct & conjugated) !

Von Neumann equation (in the interaction picture) $$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}^{(I)}(t) = -\mathrm{i}\left[\hat{H}_{\mathrm{int}}(t), \hat{\rho}^{(I)}(t)\right]$$

Iterative solution up to the second order + derivative ./. t : $$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}_S^{(I)}(t) = -\int_0^t \mathrm{d}t' \,\mathrm{Tr}_{\mathcal{E}}\left[\hat{H}_{\mathrm{int}}(t), \left[\hat{H}_{\mathrm{int}}(t'), \hat{\rho}^{(I)}(t')\right]\right].$$

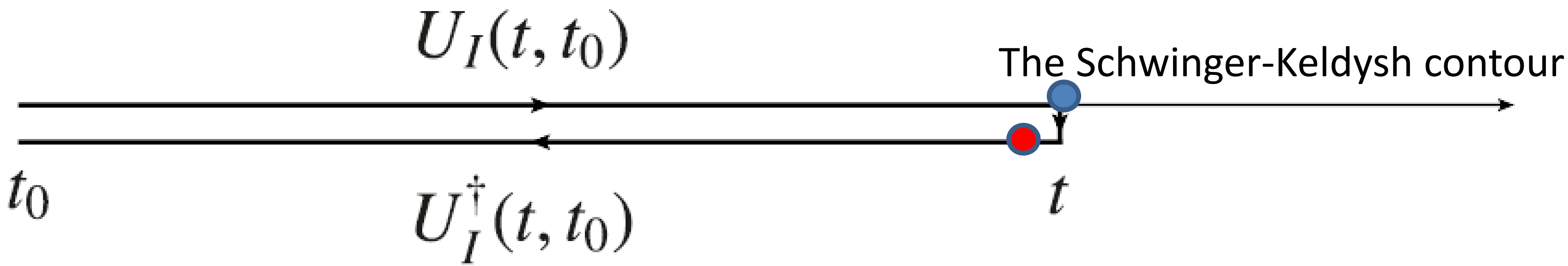
Contains the environment evolution under the influence of the system

First assumption :

- Born: Weak coupling and large environment => environment can be considered as stationnary in the iterated equation : $\hat{\rho}^{(I)}(t') \approx \hat{\rho}_S^{(I)}(t') \otimes \hat{\rho}_{\mathcal{E}}(0) \quad \forall t' \geq 0$

⇒ $$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}_S^{(I)}(t) = -\int_0^t \mathrm{d}t' \,\mathrm{Tr}_{\mathcal{E}}\left[\hat{H}_{\mathrm{int}}(t), \left[\hat{H}_{\mathrm{int}}(t'), \hat{\rho}_S^{(I)}(t') \otimes \hat{\rho}_{\mathcal{E}}\right]\right]$$

Quantum Master Equations



$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}_S^{(I)}(t) = -\int_0^t \mathrm{d}t' \,\mathrm{Tr}_{\mathcal{E}}\left[\hat{H}_{\mathrm{int}}(t), \left[\hat{H}_{\mathrm{int}}(t'), \hat{\rho}_S^{(I)}(t') \otimes \hat{\rho}_{\mathcal{E}}\right]\right]$$

2nd assumption :

- Markov: Autocorrelation time of Environment operators in $\hat{H}_{\mathrm{int}}(t)$ is τ_E and << relaxation time τ_R.

➡ Take t (higher boundary for the integral) as $\tau_E \ll t \ll \tau_R$

➡ $\rho_S(t')$ Does not vary much on [0,t] and can be chosen as $\rho_S(t)$

$$\frac{d}{dt}\hat{\rho}_S^{(I)}(t) = -\int_0^t dt' \mathrm{Tr}_{\mathcal{E}}[\hat{H}_{\mathrm{int}}(t), [\hat{H}_{\mathrm{int}}(t'), \hat{\rho}_S^{(I)}(t) \otimes \hat{\rho}_{\mathcal{E}}]] \equiv \mathcal{L}[\hat{\rho}_S^{(I)}(t)]$$

Quantum Master Equations

Explicit linear form of the H_{int} :

$$\hat{H}_{\text{int}} = \sum_\alpha O_\alpha^{(S)}(t) \otimes O_\alpha^{(E)}(t)$$

System operator (density, current, dipole) Environment operator (potential, field)

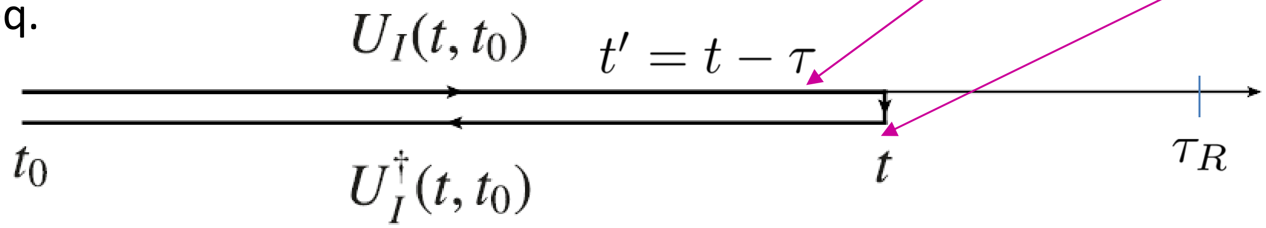
$$\Rightarrow \quad -\int_0^t dt' \mathrm{Tr}_{\mathcal{E}}[\hat{H}_{\text{int}}(t), [\hat{H}_{\text{int}}(t'), \hat{\rho}_S^{(I)}(t) \otimes \hat{\rho}_{\mathcal{E}}]]$$

Only implies environment through 2 points functions $D_{\alpha\beta}(t_1,t_2) = \mathrm{Tr}_E\left(\rho_E\, O_\alpha^{(E)}(t_1) O_\beta^{(E)}(t_2)\right)$ (field / potential thermal correlators)

$$\frac{d}{dt}\hat{\rho}_S^{(I)}(t) = -\int_0^{+\infty} d\tau \sum_{\alpha\beta}(\tau)\left\{D_{\alpha\beta}(\tau)\left[O_\alpha(t)O_\beta(t-\tau)\hat{\rho}_S^{(I)}(t) - O_\beta(t-\tau)\hat{\rho}_S^{(I)}(t)O_\alpha(t)\right]\right.$$
$$\left. + D_{\beta\alpha}(-\tau)\left[\hat{\rho}_S^{(I)}(t)O_\beta(t-\tau)O_\alpha(t) - O_\alpha(t)\hat{\rho}_S^{(I)}(t)O_\beta(t-\tau)\right]\right\}$$

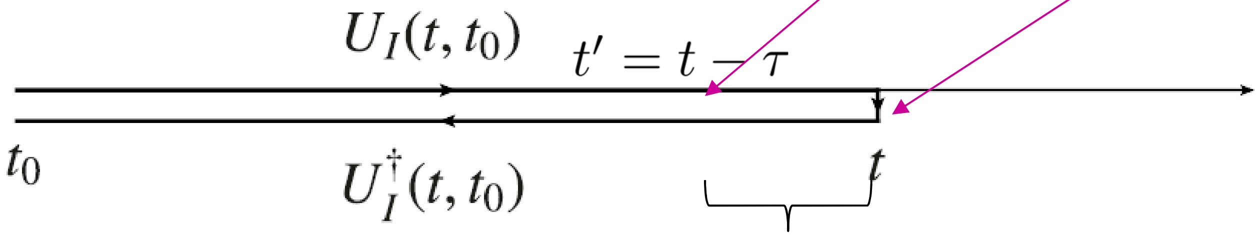
$$\equiv O_\alpha^{(S)}(t)$$

Redfield eq.



Going further:

$$\frac{d}{dt}\hat{\rho}_S^{(I)}(t) = -\int_0^{+\infty} d\tau \sum_{\alpha\beta}(\tau)\left\{D_{\alpha\beta}(\tau)\left[O_\alpha(t)O_\beta(t-\tau)\hat{\rho}_S^{(I)}(t) - O_\beta(t-\tau)\hat{\rho}_S^{(I)}(t)O_\alpha(t)\right] + \cdot\right.$$

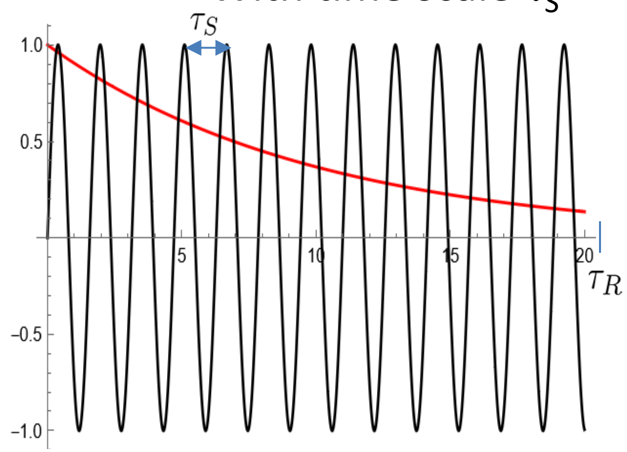


A) $\tau_R \gg \tau_S$ (low T)

Oscillations of $O_\alpha^{(I)}(t) \cdots O_\beta^{(I)}(t-\tau)$

With time scale τ_S

The numerous oscillations within τ_R allow to better reformulate the time integral in the ω Fourier space (**Rotating Wave Approx**)



$$\sum_{k,l,m,n} \int d\omega D_{\alpha\beta}(\omega)\delta(\omega - \Delta E_{k,l})\delta_{\Delta E_{k,l} - \Delta E_{m,n}} \cdots$$

$$\langle k|O_\alpha^{(S)}|l\rangle\langle n|O_\beta^{(S)}|m\rangle$$

Discrete energy jumps

Coupled differential equations for density matrix elements expressed in the basis of system eigenstates

$$\frac{d\rho_S(t)}{dt} = -i\left[H_S + \sum_{n,k}\sigma_{nk}|n\rangle\langle k|, \rho_S(t)\right]$$ Yao 21

$$+ \sum_{n,m,k,l}\gamma_{nm,kl}\left(|n\rangle\langle m|\rho_S(t)|k\rangle\langle l| - \frac{1}{2}\{|k\rangle\langle l|n\rangle\langle m|, \rho_S(t)\}\right)$$

Quantum Optical Regime

$$\sigma_{nk} \,\&\, \gamma_{nm,kl} : \mathcal{F}(D_{\alpha\beta}(\omega))$$

Lindblad Equation

Born Markov + Quantum Optical Regime => Lindblad equation (local in time)

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)\right] + \sum_i \boxed{\gamma_i}\left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2}\left\{L_i L_i^\dagger, \rho_{Q\bar{Q}}(t)\right\}\right]$$

$H_{Q\bar{Q}}$: $\{Q, \bar{Q}\}$ kinetics + Vacuum potential V + Lamb shift

L_i : Collapse (or Lindblad) operators, depend on the properties of the medium

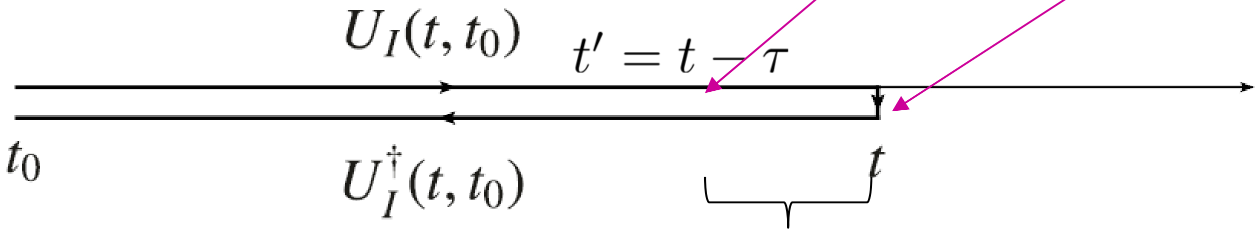
3 important conservation properties :

$\rho^\dagger_{Q\bar{Q}} = \rho_{Q\bar{Q}}$	$\mathrm{Tr}[\rho_{Q\bar{Q}}] = 1$	$\langle\varphi\|\rho_{Q\bar{Q}}\|\varphi\rangle > 0, \forall\|\varphi\rangle$
(Hermiticity)	(Unitarity)	(Positivity)

γ_i Characterize the coupling of the system with the environment

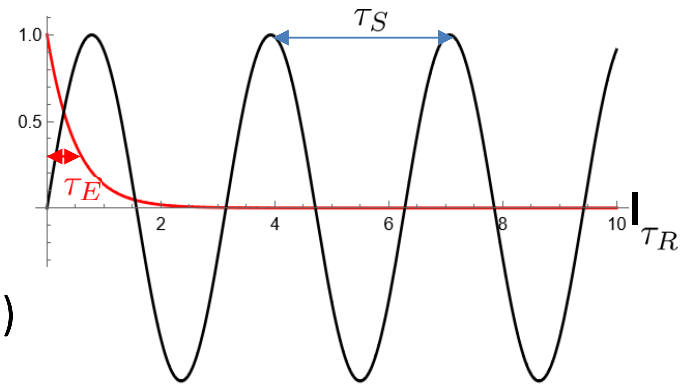
Going further:

$$\frac{d}{dt}\hat{\rho}_S^{(I)}(t) = -\int_0^{+\infty} d\tau \sum_{\alpha\beta} \left\{ D_{\alpha\beta}(\tau) \left[O_\alpha(t) O_\beta(t-\tau)\hat{\rho}_S^{(I)}(t) - O_\beta(t-\tau)\hat{\rho}_S^{(I)}(t) O_\alpha(t) \right] + \cdots \right.$$



Oscillations of $O_\alpha^{(I)}(t) \cdots O_\beta^{(I)}(t-\tau)$

B) $\tau_E \ll \tau_S$ (high T)



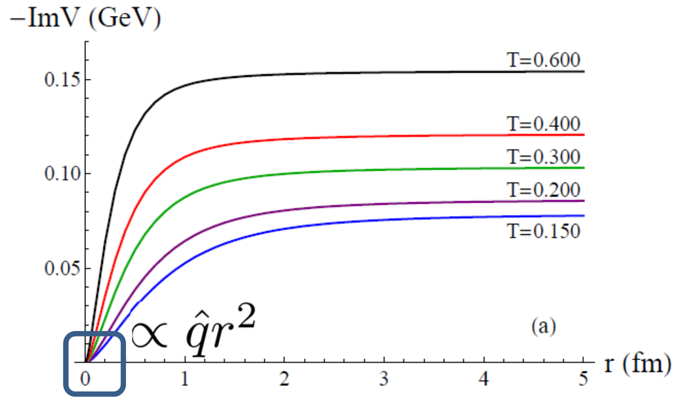
The ultra small τ_E allows for expansion wrt τ of O(t-τ)(=> in τ_E/τ_S) around origin **(Gradient Expansion)**

- Implies $O^{(S)}(t)$ (≡ positions) and $\dot{O}^{(S)}(t)$(≡ velocities)
  => formulation in phase space
- Implies correlators D through $\int_0^{+\infty} d\tau D_{\alpha\beta}(\tau)$ and $\int_0^{+\infty} d\tau\, \tau D_{\alpha\beta}(\tau)$
  => behaviour of $D_{\alpha\beta}(\omega)$ and $\frac{\partial D_{\alpha\beta}(\omega)}{\partial\omega}$ at $\omega \approx 0$
  => potentials (real and imaginary) !

Quantum Brownian Motion (Caldeira Leggett Model)

B) $\tau_E \ll \tau_S$ (high T)

Case of a single $Q\bar{Q}$ pair:



Screening / Lamb shift

Imaginary potential

$$\frac{d}{dt}\rho_S(t) = -i\left[H_S + \Delta H_S, \rho_S\right] + \int_{x,y} \gamma(\vec{x}-\vec{y})\left[\tilde{V}^a_{S,\Delta}(\vec{y})\rho_S \tilde{V}^{a\dagger}_{S,\Delta}(\vec{x}) - \frac{1}{2}\left\{\tilde{V}^{a\dagger}_{S,\Delta}(\vec{x})\tilde{V}^a_{S,\Delta}(\vec{y}), \rho_S\right\}\right]$$

$$= -i\left[H_S + \Delta H_S, \rho_S\right] + \int_k \gamma(\vec{k})\left[\tilde{V}^a_{S,\Delta}(\vec{k})\rho_S \tilde{V}^{a\dagger}_{S,\Delta}(\vec{k}) - \frac{1}{2}\left\{\tilde{V}^{a\dagger}_{S,\Delta}(\vec{k})\tilde{V}^a_{S,\Delta}(\vec{k}), \rho_S\right\}\right],$$

$$\Delta H_S = \left[-2S(\vec{x}_Q - \vec{x}_{Q_c}) - \frac{1}{8MT}\left\{\vec{p}_Q - \vec{p}_{Q_c}, \vec{\nabla}\gamma(\vec{x}_Q - \vec{x}_{Q_c})\right\}\right] t^a_Q t^{a*}_{Q_c},$$

$$\tilde{V}^a_{S,\Delta}(\vec{k}) = e^{i\vec{k}\cdot\vec{x}_Q/2}\left(\boxed{1} - \frac{\vec{k}\cdot\vec{p}_Q}{4MT}\right)e^{i\vec{k}\cdot\vec{x}_Q/2}t^a_Q - e^{i\vec{k}\cdot\vec{x}_{Q_c}/2}\left(1 - \frac{\vec{k}\cdot\vec{p}_{Q_c}}{4MT}\right)e^{i\vec{k}\cdot\vec{x}_{Q_c}/2}t^{a*}_{Q_c}$$

$$+ \frac{S(\vec{x}_Q - \vec{x}_{Q_c})}{2T}\left(e^{i\vec{k}\cdot\vec{x}_Q} - e^{i\vec{k}\cdot\vec{x}_{Q_c}}\right) i f^{abc} t^b_Q t^{c*}_{Q_c}.$$

Akamatsu 20
See as well Blaizot & Escobedo 18

Recoiless limit
Infinitively massive object

1rst recoil correction for energy conservation
Essential for dissipation

N.B.: in QBM regime, QME not doomed to be positivity-preserving => need to add extra term to insure this property