The phases of cold dense nuclear matter

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SQM2020

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Plan of the talk:

1. Dense QCD: From NJL model EoS to synthetic parametrization
2. Mass-Radius diagram and astrophysical constraints
3. Tidal deformabilities and the GW170817 event
4. Universal relations and an example of application

In collaboration with:

Mark Alford (Washington University, St. Louis, USA)

Jia-Jie Li (Goethe-University → South Western University, China)

also for universalities: V. Paschalidis, K. Yagi, D. Blaschke, Alvarez-Castillo, Largani, Fischer, Raduta, Oertel, Khadkikar

Dense QCD: From NJL model EoS to synthetic parametrization

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Dense QCD

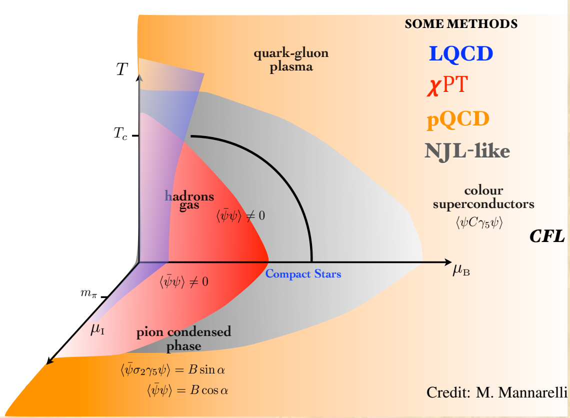
Mass-Radius diagram

Universalities relations

The QCD Lagrangian is written for $\psi_q = (\psi_{qR}, \psi_{qG}, \psi_{qB})^T$ as

$$\mathcal{L}_{QCD} = \underbrace{\bar{\psi}^i_q (i\gamma^\mu)(D_\mu)_{ij}\psi^j_q - m_q \bar{\psi}^i_q \psi_{qi}}_{\text{quarks}} - \underbrace{\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}}_{\text{gluons (Yang-Mills)}} ,$$

where $\underbrace{(D_\mu)_{ij} = \delta_{ij}\partial_\mu - ig_s t^a_{ij} A^a_\mu}_{\text{covarinat derivative}}$, and $\underbrace{F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - 2q(A^\mu \times A^\nu)}_{\text{gluonic field (Yang-Mills) field tensor}}$



Credit: M. Mannarelli

NJL model description of quark matter

$$\mathcal{L}_{NJL} = \underbrace{\bar\psi(i\gamma^\mu\partial_\mu - \hat m)\psi}_{\text{quarks}} + \underbrace{G_V(\bar\psi i\gamma^\mu\psi)^2}_{\text{vector}} + \underbrace{G_S\sum_{a=0}^{8}[(\bar\psi\lambda_a\psi)^2 + (\bar\psi i\gamma_5\lambda_a\psi)^2]}_{\text{scalar}-\text{pseudoscalar}}$$

$$+ \underbrace{G_D\sum_{\gamma,c}[\bar\psi^a_\alpha i\gamma_5\epsilon^{\alpha\beta\gamma}\epsilon_{abc}(\psi_C)^b_\beta][(\bar\psi_C)^r_\rho i\gamma_5\epsilon^{\rho\sigma\gamma}\epsilon_{rsc}\psi^s_\sigma]}_{\text{pairing}}$$

$$- \underbrace{K\left\{\det{}_f[\bar\psi(1+\gamma_5)\psi] + \det{}_f[\bar\psi(1-\gamma_5)\psi]\right\}}_{\text{t}'\text{Hooft interaction}},$$

- quarks: ψ^a_α, color $a = r, g, b$, flavor ($\alpha = u, d, s$); mass matrix: $\hat m = \mathrm{diag}_f(m_u, m_d, m_s)$;
- other notations: λ_a, $a = 1, ..., 8$, $\psi_C = C\bar\psi^T$ and $\bar\psi_C = \psi^T C$, $C = i\gamma^2\gamma^0$.

Parameters of the model:

- G_S the scalar coupling and cut-off Λ are fixed from vacuum physics
- G_D is the di-quark coupling $\simeq 0.75G_S$ (via Fierz) but free to change
- G_V and $\rho_{\rm tr}$ are treated as free parameters

QCD interactions pairing interactions and gaps

- Symmetric in space wave function (isotropic interaction) $\langle 0|\psi^a_{\alpha\sigma}\psi^b_{\beta\tau}|0\rangle$
- Antisymmetry in colors a, b for attraction
- Antisymmetry in spins σ, τ (Cooper pairs as spin-0 objects)
- Antisymmetry in flavors α, β

2SC phase:

Low densities, large m_s (strange quark decoupled)

$$\Delta(2SCs) \propto \Delta\epsilon^{ab3}\epsilon_{\alpha\beta} \qquad \delta\mu \ll \Delta,$$

Crystalline or gapless phases:

Intermediate densities, large m_s (strange quark decoupled)

$$\Delta(\text{cryst.}) \propto \epsilon_{\alpha\beta}\Delta_0\, e^{i\vec{Q}\cdot\vec{r}} \qquad \delta\mu \geq \Delta,$$

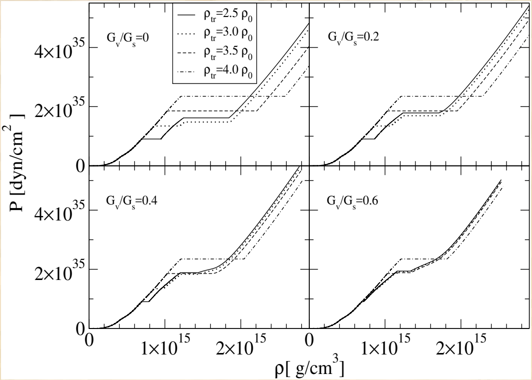
CFL phase:

High densities nearly massless u, d, s quarks

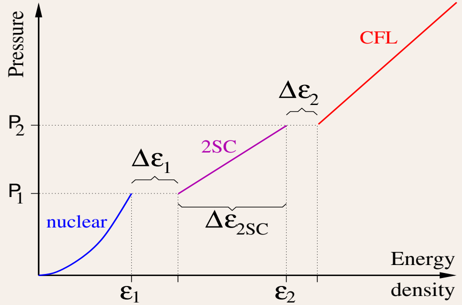
$$\Delta(CFL) \propto \langle 0|\psi^a_{\alpha L}\psi^b_{\beta L}|0\rangle = -\langle 0|\psi^a_{\alpha R}\psi^b_{\beta R}|0\rangle = \Delta\epsilon^{abC}\Delta\epsilon_{\alpha\beta C}.$$

EOS hadronic matter + Q1 (2SC) and Q2 (CFL) phases of matter

- Maxwell: large surface tension –> sharp jump: $P_N(\mu_B) = P_Q(\mu_B)$
- Glendenning: low surface –> smooth transition



Synthetic equations of state with constant speed of sound



Parameters of the models:

$$(\epsilon_1, P_1) \quad \Delta\varepsilon_1, \quad \Delta\varepsilon_{2SC} \quad (\varepsilon_2, P_2) \quad \Delta\varepsilon_2$$

Note that there are five independent parameters.

Mass-Radius diagram

Quarks and new equilibria of compact objects

150 The Equilibrium and Stability of Fluid Configurations

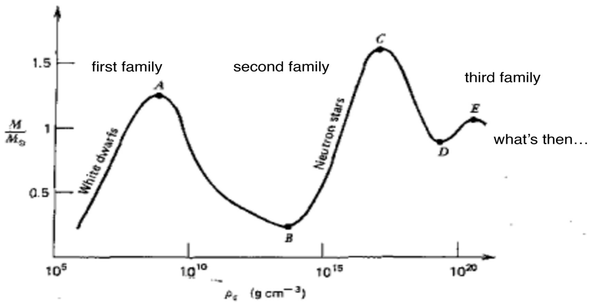
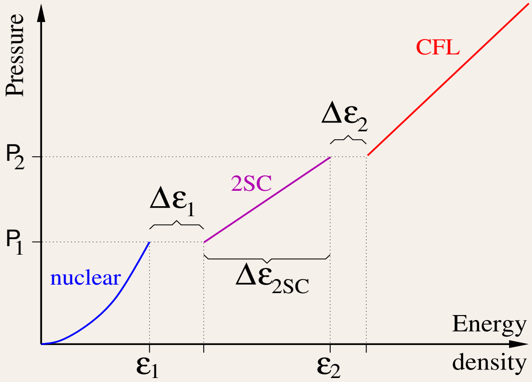


Figure 6.2 Schematic diagram showing the turning points in the mass versus central density diagram for equilibrium configurations of cold matter.

S. Shapiro, S. Teukolsky, "Black holes, White dwarfs and Neutron Stars"

-*White dwarfs* -first family, $M \leq 1.5 M_\odot$, [S. Chandrasekhar, L. Landau (1930-32)]
-*Neutron Stars* - second family, $M \leq 2 M_\odot$, [Oppenhimer-Volkoff (1939)]
-*Hybrid Stars* - third family, $M \leq 2 M_\odot$, [Gerlach (1968), Glendenning-Kettner (2000)]
- *Fourth Family* - see Phys. Rev. Lett. 119, 161104 (2017).

EoS with sequential phase transitions



Need to specify:

The scheme extends the EoS with constant speed of sound (CSS) of M. G. Alford, S. Han, M. Prakash, Phys. Rev. D 88, 083013 (2013) to double phase transitions: Phys. Rev. Lett. 119, 161104 (2017).

EoS in analytical form

$$P(\varepsilon) = \begin{cases} P_1, & \varepsilon_1 < \varepsilon < \varepsilon_1 + \Delta\varepsilon_1 \\ P_1 + s_1\left[\varepsilon - (\varepsilon_1 + \Delta\varepsilon_1)\right], & \varepsilon_1 + \Delta\varepsilon_1 < \varepsilon < \varepsilon_2 \\ P_2, & \varepsilon_2 < \varepsilon < \varepsilon_2 + \Delta\varepsilon_2 \\ P_2 + s_2\left[\varepsilon - (\varepsilon_2 + \Delta\varepsilon_2)\right], & \varepsilon > \varepsilon_2 + \Delta\varepsilon_2 \,. \end{cases}$$

Need to specify:

- the two speeds of sounds: s_1 and s_2
- the point of transition from NM to QM ε_1, P_1
- the magnitude of the first jump $\Delta\varepsilon_1$
- the size of the 2SC phase, i.e, the second transition point ε_2, P_2
- the size of the second jump $\Delta\varepsilon_2$

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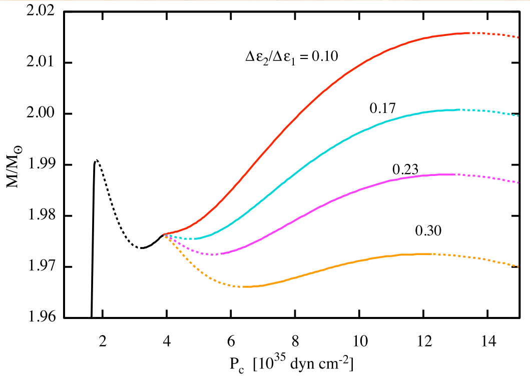
Dense QCD

Mass-Radius diagram

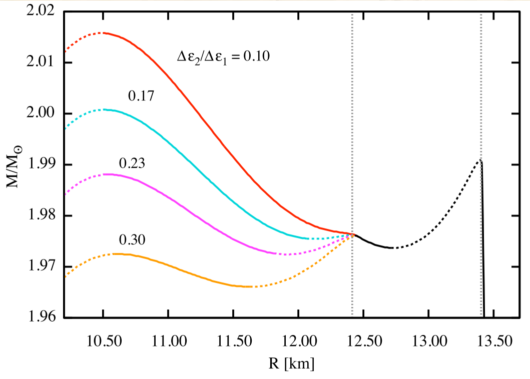
Universalities relations

Mass-central pressure relation



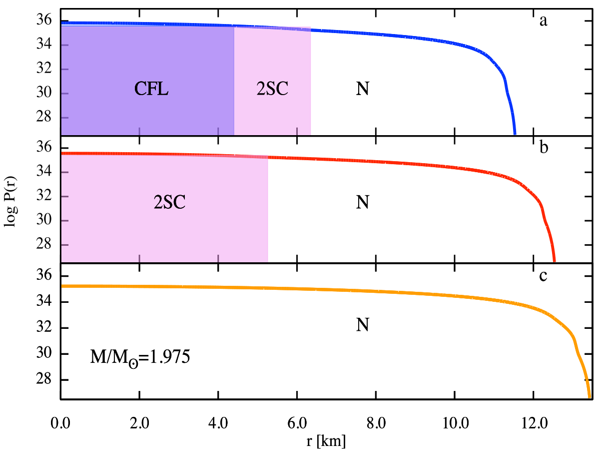
- Phase Q1: $P_1 = 1.7 \times 10^{35}$ dyn cm^{-2}, $\Delta\varepsilon_{2SC}/\varepsilon_1 = 0.27$, $\Delta\varepsilon_1/\varepsilon_1 = 0.6$.
  Phase Q2: 4 different values of $\Delta\varepsilon_2$.
- Speeds of sound $s_1 = 0.7$ and $s_2 = 1$.
- Stable branches –> solid lines, unstable branches –> dashed lines.
- Triplets emerge for $\Delta\varepsilon_2 = 0.23$

Mass-radius relation



Same as previous slide but the $M - R$ relation.
Emergence of twins and triplets –> strong first order phase transition in quark matter.

Internal structure of triplet stars



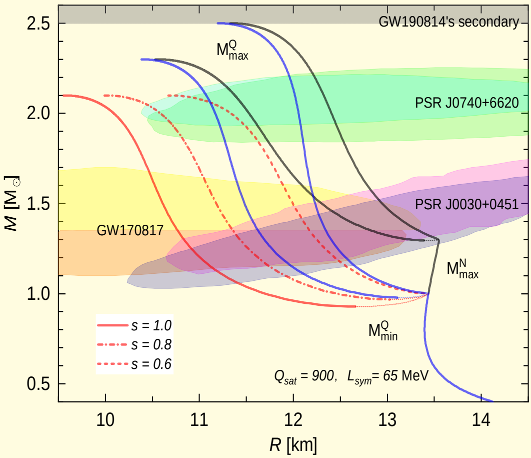
- Internal profiles of triplets with $M = 1.975\ \mathrm{M}_\odot$ and $\Delta\varepsilon_2/\Delta\varepsilon_1 = 0.23$.
- "*N*" –> nuclear only, 2SC —> single phase, CFL,2SC –> two phases.

Stability "matrix" for different magnitudes of jumps

$\Delta\varepsilon_2/\Delta\varepsilon_1$	$\Delta\varepsilon_1/\varepsilon_1$ 0.4	0.5	0.6	0.7
0.1	s, s	s, s	$\underbrace{us, s}_{\text{N-2SC}}$	$\underbrace{u, us}_{\text{N-CFL}}$
0.2	s, s	s, s	$\underbrace{us, us}_{\text{triplet}}$	$\underbrace{u, us}_{\text{N-CFL}}$
0.3	s, s	s, s	$\underbrace{us, us}_{\text{N-2SC;N-CFL}}$	$\underbrace{u, us}_{\text{N-CFL}}$
0.4	s, s	$\underbrace{s, us}_{\text{2SC-CFL}}$	$\underbrace{us, u}_{\text{N-2SC}}$	u, u
0.5	s, s	$\underbrace{s, us}_{\text{2SC-CFL}}$	$\underbrace{us, u}_{\text{N-2SC}}$	u, u

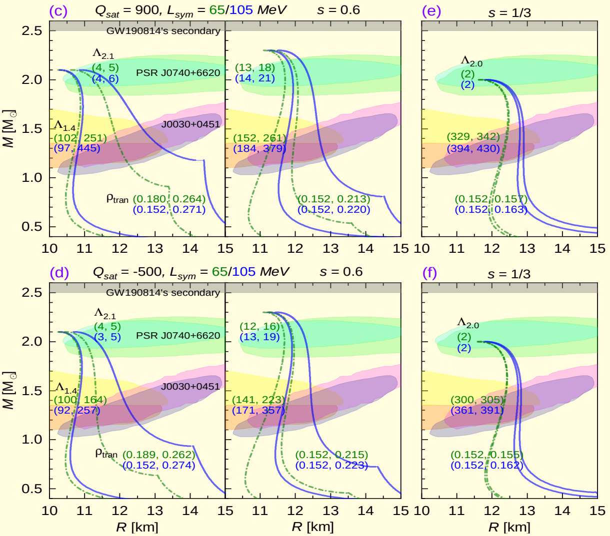
- Stable/unstable branches are referred by s/u, the Q1 and Q2 phases.
- Increasing the jumps –> instability

$M - R$ for twin stars



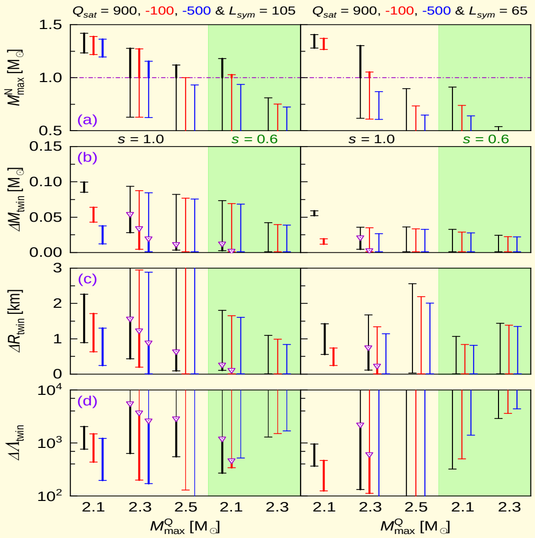
Mass-radius relation for hybrid stars with a single QCD phase translation for different values of hadronic maximum mass.

Early QCD phase transition scenario



Examples of low-density QCD phase transition on the *M*-*R* phase diagram.

Ranges for twin stars



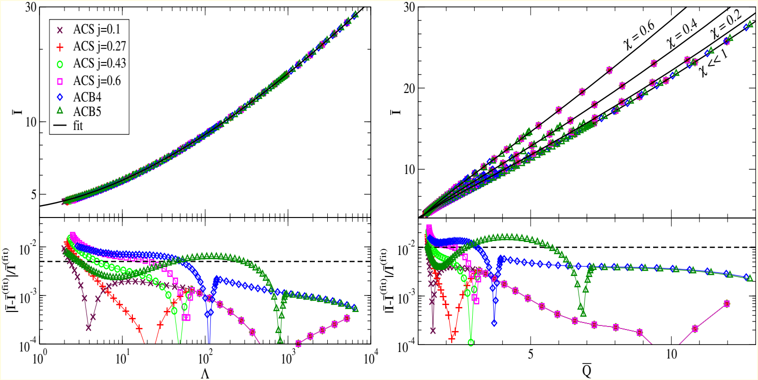
The mass, radius and deformability ranges for twin stars.

Universal relations

Universalities of TOV solutions:

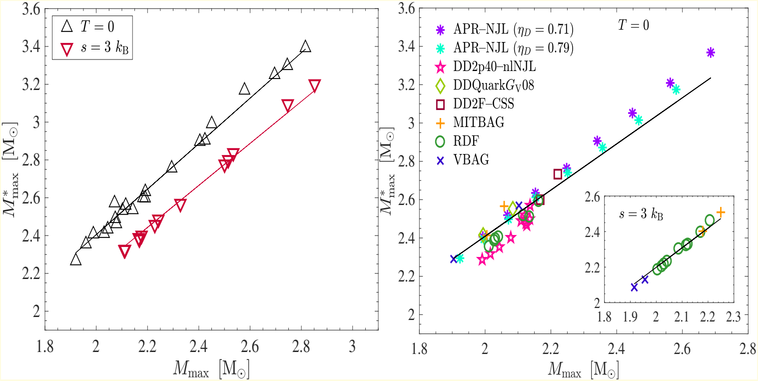
- Universal (independent of the underlying EoS) relations among the global properties of compact stars - $I - L - Q$ relations. (Yagi and Yunes 2013a; Maselli et al. 2013; Breu and Rezzolla 2016; Yagi and Yunes 2017)
  Well established for:
  (a) zero temperature slowly rotating stars
  (b) rapidly rotating cold star
  (c) magnetized cold star
- Recently Khosravi et al. considered a large number of non-rotating and rapidly rotating hybrid stars (arXiv:2112.10439)
- See also the earlier work by Paschalidis et al. (arXiv:1712.00451)

Examples of universalities



- Moment of inertia vs tidal deformability and moment of inertia vs quadrupole moment

Examples of universalities



- Maximum masses of Keplerian vs static hadronic (left) and hybrid (right) stars.