Two and three-body interactions among kaons and nucleons tested at the LHC

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Kaon (K) nucleon (N) and anti-Kaon $\bar{K}$N interactions are fundamental for the study of low-energy QCD

- $K = K^+(u,\bar{s}), K^0(d,\bar{s}); \bar{K} = K^-(s,\bar{u}), \bar{K}^0(s,\bar{d})$
- Traditionally, these interactions are studied by scattering experiments ($K^+(p,n,d)$ and $K^-(p,n,d)$) at low energies
  - Few experimental measurements with big uncertainties and not at low-energy $p_{\text{lab}} < 50$ MeV/c

$$\sigma(K^+_{\text{p,n,d}} \rightarrow X) \text{ (mB)}$$
$$\sigma(K^-_{\text{p,n,d}} \rightarrow X) \text{ (mB)}$$

**K⁺p interaction**

- **K⁺p interaction**
  - Repulsive (due to Coulomb and strong interactions)
  - No coupled channels
  - No resonances
  - Well known [1]

K⁻p interaction

- K⁻p interaction
  - deeply attractive
  - several resonances
  - several coupled-channels ($\bar{K^0}n$, $\pi^+\Sigma^-$, $\pi^0\Sigma^0$, $\pi^-\Sigma^+$, $\pi^0\Lambda$, $\Lambda^*$, $\Sigma^*$, ...)

- Systems close to the K⁻p threshold and with the same quantum numbers
**K⁻p interaction**

- **K⁻p interaction**
  - deeply attractive
  - several **resonances**
  - several **coupled-channels** \((\bar{K}^0 n, \pi^+\Sigma^-, \pi^0\Sigma^0, \pi^-\Sigma^+, \pi^0\Lambda)\)

  - systems close to the K⁻p threshold and with the same quantum numbers
  - \(\bar{K}N \leftrightarrow \pi\Sigma\) dynamics leads to the formation of the **Λ(1405)**, ~27 MeV below K⁻p threshold

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<table>
<thead>
<tr>
<th>E(MeV/c)</th>
<th>K⁻p elastic</th>
<th>K⁻p total</th>
<th>K⁻p (\leftrightarrow) (\bar{K}^0 n)</th>
<th>(\pi\Lambda)</th>
<th>(\pi\Sigma)</th>
<th>(\Lambda(1405))</th>
<th>(\bar{K}^0 n)</th>
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</thead>
<tbody>
<tr>
<td>-177</td>
<td>-100</td>
<td>-27</td>
<td>+58</td>
<td>-100</td>
<td>3</td>
<td>-100</td>
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</tbody>
</table>
K^-p interaction and Λ(1405)

- Nature of Λ(1405): dynamically generated resonance
  - Models based on below-threshold extrapolations
    - Pole positions is model dependent
      (relative contributions not measured experimentally)

Y. Kamiya et al. NPA 954 (2016) 41-57
T. Hyodo et al. PPNP 67 (2012)
U. Meißner and Tetsuo Hyodo: PDG review (2020) (Section 83)
**K^-p interaction and Λ(1405)**

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    - State-of-the-art chiral models (χEFT) are in agreement above threshold
    - Large discrepancies in the region below threshold

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A. Cieplý et al, arxiv:2001.08621
- MI, MII Z. H. Guo, et al. PRC 87 (2013) 035202
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    - Constraint at threshold by SIDDARTHA measurement [1] of kaonic hydrogen 1s level shift and width

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[1] SIDDHARTA collaboration PLB704 (2011) 113
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      - Scattering length

[1] SIDDHARTA collaboration PLB704 (2011) 113
KN and $\overline{K}N$ interactions: the game changer

Two particle momentum correlation measured with ALICE at the LHC

- KN and $\overline{K}N$ interaction
  - ALICE collaboration PRL 124 (2020) 9, 092301
  - ALICE collaboration PLB 822 (2021) 136708
  - ALICE collaboration arXiv: 2205.15176

- and other interactions:
  - $pp, p\Lambda, \Lambda\Lambda$: ALICE collaboration PRC 99(2019)
  - $\Lambda\Lambda$: ALICE collaboration PLB 797 (2019) 134822
  - $p\Xi$: ALICE collaboration PRL 123 (2019) 134822
  - $p\Sigma^0$: ALICE collaboration PLB 805 (2020) 135419
  - $p\Omega$: ALICE collaboration Nature 588 (2020) 232-238
  - $p\phi$: ALICE collaboration PRL 127 (2021) 172301
  - $B-B$: ALICE collaboration PLB B 829 (2022) 137060
  - $p\Lambda$: ALICE collaboration arXiv:2104.04427
  - $pD$: ALICE collaboration arXiv:2201.05352
  - $\Lambda\Xi$: ALICE collaboration arXiv:2204.10258
  - $ppp$ and $pp\Lambda$: ALICE collaboration arXiv:2206.03344
Two particle momentum correlation...

\[ k^* = \frac{|\vec{p}_1 - \vec{p}_2|}{2} \]

\[
C(k^*) = \int S(\vec{r}^*) \left| \psi(\vec{k}^*, \vec{r}^*) \right|^2 d^3 \vec{r}^* = N(k^*) \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}
\]
Two particle momentum correlation...

\[ k^* = \frac{|\vec{p}_1 - \vec{p}_2|}{2} \]

Emission source \( S(\vec{r}^* ) \)

\[
C(k^*) = \int S(\vec{r}^*) \left| \psi(\vec{k}^*, \vec{r}^*) \right|^2 d^3 \vec{r}^* = \mathcal{N}(k^*) \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}
\]
Two particle momentum correlation...

Emission source $S(\vec{r}^*)$

$$k^* = \frac{|\vec{p}_1 - \vec{p}_2|}{2}$$

Potentials

- Repulsive
- Attractive

Correlation functions

$$C(k^*) = \int S(\vec{r}^*) \left| \psi(\vec{k}^*, \vec{r}^*) \right|^2 d^3 \vec{r}^* = N(k^*) \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}$$
Two particle momentum correlation...

\[ k^* = \frac{|\vec{p}_1 - \vec{p}_2|}{2} \]

**Emission source** \( S(\vec{r}^*) \)

**Potentials**
- Repulsive
- Attractive

**Correlation functions**
- Attractive
- Repulsive

**Two-particle wave function**

\[
C(k^*) = \int S(\vec{r}^*) \left| \psi(\vec{k}^*, \vec{r}^*) \right|^2 d^3 \vec{r}^* = \frac{N(k^*)}{N_{\text{same}}(k^*)} \frac{N_{\text{mixed}}(k^*)}{N_{\text{mixed}}(k^*)}
\]

Measure \( C(k^*) \) \( \rightarrow \) fixing the source \( S(r^*) \), study the interaction
... from small to large systems

- By changing the colliding system it is possible to probe interaction distances ranging from ~1 fm up to ~10 fm.
KN and $\bar{K}N$ interactions: the game changer

Two particle momentum correlation measured with ALICE at the LHC

- The Coulomb-only potential is not able to describe $K^+p$ interaction and the introduction of the strong potential is needed to fit the data:
  - CFs are sensitive to the strong interaction

ALICE collaboration PRL 124 (2020) 9, 092301
Fit: CATS D. L. Mihaylov et al. EPJ C78 (2018) 5, 394
Two particle momentum correlation measured with ALICE at the LHC

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Two particle momentum correlation measured with ALICE at the LHC

ALICE pp $\sqrt{s}=13$ TeV

$r_0 = 1.18 \pm 0.01 \pm 0.12$ fm

$\lambda = 0.61 \pm 0.06$

$\uparrow$ K$^+p \ominus K^-p$

Coulomb

Coulomb+Strong (Kyoto Model)

Coulomb+Strong (Jülich Model)

$0.7 < S_T < 1$

$K^+p$

$K^-p$

$\Lambda(1520)$

$\pi\Lambda$  $\pi\Sigma$  $\Lambda(1405)$  $K^-p$  $K^0n$  $\Lambda(1520)$

$-177$  $-100$  $-27$  $+58$  $+244$

ALICE collaboration  PRL 124 (2020) 9, 092301
Fit: CATS D. L. Mihaylov et al. EPJ C78 (2018) 5, 394)
**Two particle momentum correlation** measured with ALICE at the LHC

ALICE pp $\sqrt{s} = 13$ TeV

- $r_0 = 1.18 \pm 0.01 \pm 0.12$ fm
- $\lambda = 0.64 \pm 0.06$

$K^+p \oplus K^0n$

ALICE collaboration PRL 124 (2020) 9, 092301

Fit: CATS D. L. Mihaylov et al. EPJ C78 (2018) 5, 394
Two particle momentum correlation measured with ALICE at the LHC

- First experimental evidence for the opening of the $K^0n$ channel
- New constraints for low-energy QCD chiral models
Two particle momentum correlation measured with ALICE at the LHC

ALICE pp $\sqrt{s} = 13$ TeV
$r_0 = 1.18 \pm 0.01 \pm 0.12$ fm
$\lambda = 0.64 \pm 0.06$

$K^+p$ and $K^-p$

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ALICE collaboration PLB 822 (2021) 136708

KN and $\bar{K}N$ interactions: the game changer

17/06/2022
K\(^{-}\)p interaction: improved chiral model

Koonin-Prat formula for coupled channels (CC)

\[ C_{K^{-}p}(\vec{k}^{*}) = \int d^{3}\vec{r}^{*} S_{K^{-}p}(\vec{r}^{*}) \left| \psi_{K^{-}p}(\vec{k}^{*}, \vec{r}^{*}) \right|^{2} + \sum_{j} \omega_{j} \int d^{3}\vec{r}^{*} S_{j}(\vec{r}^{*}) \left| \psi_{j}(\vec{k}^{*}, \vec{r}^{*}) \right|^{2} \]

- Coupled-channel are short-range features of the strong interaction
  - the shape and strength of the correlation function are modified at small distances
- Improved Kyoto chiral model to describe CC potential \( V_{j} \)
- Conversion weights \( (\omega_{j}) \)
  - control CC contribution
  - depend on primary yield and kinematics

K\(^-\)p interaction: improved chiral model

Koonin-Prat formula for coupled channels (CC)

\[
C_{K^-p}(\vec{k}^*) = \int d^3\vec{r}^* S_{K^-p}(\vec{r}^*) \left| \psi_{K^-p}(\vec{k}^*, \vec{r}^*) \right|^2 + \sum_j \omega_j \int d^3\vec{r}^* S_j(\vec{r}^*) \left| \psi_j(\vec{k}^*, \vec{r}^*) \right|^2 
\]

\(j = \bar{K}^0n, \pi^0\Sigma^0, \pi^+\Sigma^-, \pi^-\Sigma^+, \pi^0\Lambda\)

- System size survey
  - For large radii contribution from CC gets negligible → elastic scattering

The emitting source in small colliding systems

- Data-driven analysis on p–p and p–Λ pairs
  - Possible presence of collective effects $\rightarrow m_T$ scaling of the core radius
  - Contribution of strongly decaying resonances with $c\tau \sim 1$ fm (*)
- Common universal core source for baryons

ALICE Collaboration PLB 811 (2020) 135849
17/06/2022

The emitting source in small colliding systems

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  - Possible presence of collective effects → $m_T$ scaling of the core radius
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- Common universal core source for baryons

- What about meson-baryon pairs?
  - $K^+p$ interaction is well known → extract $r_{\text{core}}$ for Kp pairs
    - For small systems:
      - build effective sources for Kp($K^0n$) and one for $\pi\Sigma$ ($\pi\Lambda$) pairs using different resonances

ALICE Collaboration arXiv: 2205.15176
**K⁻p in large systems**

\[ R_{kp} = 8.1 \pm 0.26^{+0.30}_{-1.09} \text{stat} \pm 0.30 \text{syst} \text{ fm} \]

- **K⁺p used to extract source size**
  - Gaussian source
  - Lednický-Lyuboshitz (LL) fit to extract \( r_{\text{eff}} \)

ALICE collaboration PLB 822 (2021) 136708

\[ \text{ALICE} \]

\[ \text{Pb–Pb } \sqrt{s_{\text{NN}}} = 5.02 \text{ TeV} \]

\[ \text{K}^+p \oplus \text{K}^-\bar{p} \]

\[ \text{L-L fit} 5-10\% \]
**K⁻p in large systems**

- **K⁺p** used to extract source size
  - Gaussian source
  - Lednický-Lyuboshitz (LL) fit to extract $r_{\text{eff}}$
- Large system: no coupled channels (as in Kyoto model)
- Use Lednický-Lyuboshitz (LL) fit to extract $\Re f_0$ and $\Im f_0$
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- Large system: no coupled channels (as in Kyoto model)
- Use Lednický-Lyuboshitz (LL) fit to extract $\Re f_0$ and $\Im f_0$
- $\Re f_0$ and $\Im f_0$ in agreement with available data and calculations
  - Alternative to exotic atoms and scattering experiments!

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ALICE collaboration PLB 822 (2021) 136708

ALICE-PUB-500325

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K⁻p from small to large systems

\[ C_{K^-p}(k^*) = \int d^3 r^* S_{K^-p}(r^*) \left| \psi_{K^-p}(\vec{k}^*, \vec{r}^*) \right|^2 + \sum_j \omega_j \int d^3 r^* S_j(r^*) \left| \psi_j(\vec{k}^*, \vec{r}^*) \right|^2 \]

Each coupled channels is accounted in the \( \omega_j \) weights

- primary production yields fixed from thermal model (Thermal-FIST) [1]
- estimate amount of pairs in FSI sensitive kinematic region
- distribute particles according to blast-wave model [2,3,4]
- normalize to expected yield of K⁻p

\[ C_{K^-p}(k^*) = \int d^3 \vec{r}^* S_{K^-p}(\vec{r}^*) \left| \psi_{K^-p}(\vec{k}^*, \vec{r}^*) \right|^2 + \sum_j \omega_j \int d^3 \vec{r}^* S_j(\vec{r}^*) \left| \psi_j(\vec{k}^*, \vec{r}^*) \right|^2 \]

State-of-the-art Kyoto Model is not able to describe the data from small to large source size.
$C_{K^-p}(k^*) = \int d^3\vec{r}^* S_{K^-p}(\vec{r}^*) \left| \psi_{K^-p}(\vec{k}^*, \vec{r}^*) \right|^2 + \sum_j \omega_j^{prod} \cdot \alpha_j \int d^3\vec{r}^* S_j(\vec{r}^*) \left| \psi_j(\vec{k}^*, \vec{r}^*) \right|^2$
K^-p from small to large systems

\[ C_{K^-p}(k^*) = \int d^3 \vec{r}^* S_{K^-p}(\vec{r}^*) \left| \psi_{K^-p}(\vec{k}^*, \vec{r}^*) \right|^2 + \sum_j \omega_j^{\text{prod}} \cdot \alpha_j \int d^3 \vec{r}^* S_j(\vec{r}^*) \left| \psi_j(\vec{k}^*, \vec{r}^*) \right|^2 \]

A correction factor \( \alpha_j \) is introduced to quantify the model-to-data deviation.
Unique constraint and direct access to $K^-p \leftrightarrow \bar{K}^0 n$ and $K^-p \leftrightarrow \pi \Sigma$ dynamics

$\alpha_{K^0-n}$ deviates from unity:
- $K^-p \leftrightarrow \bar{K}^0 n$ currently implemented in Kyoto
  - $\chi$EFT is too weak
- fine tuning of Kyoto $\chi$EFT is needed and data from hadron-hadron collisions have to be taken into account
What about many-body systems?

- KNN: exotic bound states of anti-kaon with nucleons predicted more than 30 years ago [1,2] due to the strongly attractive KN interaction in I = 0 channel

- First positive experimental evidence of the p-p-K⁻ bound state by the E15 Collaboration [3]

- Next experimental challenge: genuine three-body interaction measurements

Three-particle correlation function

\[ C(p_1, p_2, p_3) = \frac{P(p_1, p_2, p_3)}{P(p_1)P(p_2)P(p_3)} = \mathcal{N} \frac{N_{\text{same}}(Q_3)}{N_{\text{mixed}}(Q_3)} \]

The Lorentz invariant \( Q_3 \) is defined as:

\[ q_{ij}^\mu = (p_i - p_j)^\mu - \frac{(p_i - p_j) P_{ij}}{p_{ij}^2} P_{ij}^\mu \]

\[ p_{ij} = p_i + p_j \]

ALICE collaboration arXiv: 2206.03344

\[
c_3(Q_3) = [C'(Q_3)] - \left[ C_{12}(Q_3) - C_{23}(Q_3) - C_{31}(Q_3) + 2 \right]
\]

Cumulant: genuine 3-body correlations

Measured triplets: include two- and three-particle correlations

Lower-order correlations

**Figure: ALICE Preliminary**

- **pp $\sqrt{s} = 13$ TeV**
- **High Mult. (0–0.17% INEL)**

**p–p–K$^+$**

**p–p–K$^-$**

**17/06/2022**

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Genuine 3-body correlations

- **Statistical significance:**
  - \( n_{\sigma} = 2.3 \) for \( Q_3 < 0.4 \text{ GeV}/c \)

- **Conclusions:**
  - The measured cumulant is compatible with zero within the uncertainties
  - Above 180 MeV/c, the genuine three-body effects do not contribute significantly in the dynamics of the \( p-p-K^+ \) system
**Genuine 3-body correlations**

- **Statistical significance:**
  - $n_0 = 0.5$ for $Q_3 < 0.4 \text{ GeV/c}$

- **Conclusion:**
  - The measured cumulant is consistent with zero within uncertainties

- **Genuine three-body effects are not significant in p-p-K\(^-\) systems**
  - The measurement confirms that three-body strong interaction is not relevant in the formation of the exotic kaonic bound states
Conclusions

- Momentum correlation technique applied to data collected at the LHC in different collision systems
  - Unique way to access KN and ČN interaction: New constraints for low-energy QCD chiral models
    - First experimental access to coupled channels dynamics (K⁻p ↔ K⁰n, K⁻p ↔ πΣ, K⁻p ↔ πΛ)
    - Data-model tension in description of K⁻p interaction:
      - K⁻p ↔ K⁰n currently implemented in state-of-the-art Kyoto χEFT is too weak
  - First direct measurement of p-p-K⁺ and p-p-K⁻ interaction
    - Cumulants compatible with 0, no evidence of a genuine three-body force
    - Kaonic bound state formation driven by two-body forces

- More precision studies within reach with large statistics in LHC Run 3 & 4
  - Unique way to access coupled-channels dynamics in the meson-baryon sector: open a new era in the charm sector!
Momentum correlation technique applied to data collected at the LHC in different collision systems

- Unique way to access KN and ŦN interaction: New constraints for low-energy QCD chiral models
  - First experimental access to coupled channels dynamics ($K^-p \leftrightarrow \bar{K}^0n$, $K^-p \leftrightarrow \pi\Sigma$, $K^-p \leftrightarrow \pi\Lambda$)
  - Data-model tension in description of $K^-p$ interaction:
    - $K^-p \leftrightarrow \bar{K}^0n$ currently implemented in state-of-the-art Kyoto $\chi$EFT is too weak
  - Direct access to $K^0_s p$ ($Kp + \bar{K}p$) interaction:
    - state-of-the-art theory Kyoto $\chi$EFT well describes the experimental data

- First direct measurement of $p-p-K^+$ and $p-p-K^-$ interaction
  - cumulants compatible with 0, no evidence of a genuine three-body force
    - kaonic bound state formation driven by two-body forces

- More precision studies within reach with large statistics in Run 3 & 4!
Accessing KN and $\bar{K}N$ interaction with $K^0$

- $K^0_s - p$ system is a combination of strong eigenstates

$$|K^0_s p\rangle = \frac{1}{\sqrt{2}} \left( |K^0 p\rangle - |\bar{K}^0 p\rangle \right) \Rightarrow C_{K^0_s p} = \frac{1}{2} [C_{K^0 p} + C_{\bar{K}^0 p}]$$

- Weak strong repulsion
- 1 CC below threshold: $K^+ n$
  - predicted to be a weak coupling
- Calculations from Aoki-Jido $\chi$EFT model for KN\[1]\n
Accessing KN and $\bar{K}N$ interaction with $K^0$

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\[ |K^0_s p\rangle = \frac{1}{\sqrt{2}} \left[ |K^0 p\rangle - |\bar{K}^0 p\rangle \right] \Rightarrow C_{K^0 sp} = \frac{1}{2} \left[ C_{K^0 p} + C_{\bar{K}^0 p} \right] \]

- Moderate attraction
- 3 CC below threshold: $\pi^0 \Sigma^+, \pi^+ \Sigma^0, \pi^+ \Lambda$
  - large $\pi \Sigma$ coupling (as in $K^-p$)
- Calculations from Kyoto $\chi$EFT model for $K\bar{N}$ used for $K^-p$ [1,2]

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- Calculations from Kyoto $\chi$EFT model for $K\bar{N}$ used for $K^-p$ [1,2]
**K⁰_S – p interaction**

- Gaussian source function with $r=1.18\pm0.12$ fm [1]
- $K^0 p(\bar{p})$ and $\bar{K}^0 (\bar{p}) \psi$ with CC provided by Kyoto $\chi^2_{\text{EFT}}$
- Conversions weights $\omega = 1$ for $K^0 p$, $K^+ n$, and $\pi^+ \Lambda$; $\omega_{\Sigma\pi} = 2.95$ [2]

- **Model describes data within 2σ between 0 and 300 MeV/c**
  - State-of-the-art theory well describes the experimental data
  - Small caveat: source not (yet) studied in details

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Best fit of $K^-p$ observables: cross section data

Deser-type relation connects shift $\varepsilon_{1s}$ and width $\Gamma_{1s}$ to the real and imaginary part of $\alpha_{K^-p}$ and $\alpha_{K^-d}$:

\[
\frac{i\Gamma}{2} = 2\alpha^3 \mu^2 a_{K^-p} = 412 \text{ GeV} \frac{\text{fm}}{a_{K^-p}} \\
\frac{i\Gamma}{2} = 2\alpha^3 \mu^2 a_{K^-d} = 601 \text{ GeV} \frac{\text{fm}}{a_{K^-d}}
\]

- done by SIDDHARTA
- aim of SIDDHARTA-2

one can obtain the isospin dependent antikaon-nucleon scattering lengths

\[
a_{K^-p} = \frac{a_0(I=0) + a_1(I=1)}{2} \\
a_{K^-d} = \frac{\frac{1}{2} m_N + \frac{m_K}{2}}{2 m_N + \frac{m_K}{2}} (3a_1 + a_0) + C
\]

- Fundamental inputs of low-energy QCD effective field theories
For modeling the source every resonance with a $c\tau > 8$ fm is taken out and the yields properly renormalized. These resonance are used to determine the decay-kinematics with EPOS.

<table>
<thead>
<tr>
<th>Primordial fraction</th>
<th>Resonance fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c\tau &lt; 1$ fm</td>
</tr>
<tr>
<td>28 %</td>
<td>15 %</td>
</tr>
</tbody>
</table>

$<m(\pi)> = 1124$ MeV/$c^2$

$<c\tau(\pi)> = 1.5$ fm

Only resonances which contribute more than 2% to total yield are shown.

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Yield (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^0$</td>
<td>9.01</td>
</tr>
<tr>
<td>$\rho^+$</td>
<td>8.71</td>
</tr>
<tr>
<td>$\omega$</td>
<td>7.67</td>
</tr>
<tr>
<td>$K(892)^{\ast\ast}$</td>
<td>2.29</td>
</tr>
</tbody>
</table>
Resonances used for $\pi\Sigma(\Lambda)$ source ($\Sigma\Lambda$)

- For modeling the source every resonance with a $c\tau > 8$ fm is taken out and the yields properly renormalized. These resonance are used to determine the decay-kinematics with EPOS.

<table>
<thead>
<tr>
<th>Primordial fraction</th>
<th>$c\tau &lt; 1$ fm</th>
<th>$1 &lt; c\tau &lt; 2$ fm</th>
<th>$2 &lt; c\tau &lt; 5$ fm</th>
<th>$c\tau &gt; 5$ fm</th>
</tr>
</thead>
<tbody>
<tr>
<td>26 %</td>
<td>0 %</td>
<td>5 %</td>
<td>5 %</td>
<td>64 %</td>
</tr>
</tbody>
</table>

$<m(\Sigma)> = 1463$ MeV/c²

$<c\tau(\Sigma)> = 4.7$ fm

Only resonances which contribute more than 2% to total yield are shown

<table>
<thead>
<tr>
<th>Resonance</th>
<th>$\Sigma^0$</th>
<th>$\Sigma^{*0}$</th>
<th>$\Sigma^{*+}$</th>
<th>$\Sigma^{*-}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield (in %)</td>
<td>27</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>
Contributions to the experimental correlation function

- Fit of the \( C(k^*) = \frac{C_{\text{data}}(k^*)}{C_{\text{baseline}}(k^*)} \) to obtain the parameters of the strong interaction between \( K^0_S \) and \( p(\bar{p}) \) is performed with the function:

\[
C(k^*) = \left[ 1 + \lambda_{\text{genuine}} (C_{\text{FSI}}(k^*) - 1) + \sum_{i,j} \lambda_{ij} (C_{ij}(k^*) - 1) \right] \cdot \text{Norm}
\]

- Fraction of identified and primary particles, used as \( C_{\text{FSI}}(k^*) \) weight
- Final-state interactions contribution
- Contribution linked to the presence of misidentified particles
- Normalization

\[
\sum_{i,j} \lambda_{ij} (C_{ij}(k^*) - 1) = \lambda_K (C_K(k^*) - 1) + \lambda_{\bar{p}(\bar{p})} (C_{\bar{p}(\bar{p})}(k^*) - 1)
\]
$K^0_S$–p correlation function fit with Lednický-Lyuboshitz

$$C_{FSI}(k^*) = \sum S \rho_S \left[ \frac{1}{2} \left| f(k^*) \right|^2 \left( 1 - \frac{d_0}{2\sqrt{\pi}R} \right) + \frac{2\Re f(k^*)}{\sqrt{\pi}R} F_1(2k^*R) - \frac{\Im f(k^*)}{R} F_2(2k^*R) \right]$$

$$C_{Lednicky}(k^*) = 1 + C_{FSI}(k^*)$$

Scattering amplitude:

$$f(k^*) = \left( \frac{1}{f_0} + \frac{1}{2}d_0k^{*2} - ik^* \right)^{-1}$$

- $f_0$ scattering length, $d_0$ effective range of interaction
  - $\Re f_0$, $\Im f_0$ estimated parameters
  - $\Re f_0 > 0$ : attractive interaction
  - $\Im f_0 \neq 0$ : presence of annihilation processes

ALICE Preliminary
pp $\sqrt{s}$ = 13 TeV

$r = 1.18 \pm 0.12$ fm
$\lambda = 0.74 \pm 0.15$

$\Re(f_0) = 0.29^{+0.04}_{-0.04}^{(stat.)} +0.07_{-0.06}^{(syst.)}$ fm
$\Im(f_0) = 0.26^{+0.05}_{-0.05}^{(stat.)} +0.06_{-0.06}^{(syst.)}$ fm
$\chi^2$/ndf = 9/12
Lednický-Lyuboshitz model

\[
C(k^*) = \frac{\int S(r^*, k^*) |\psi(r^*, k^*)|^2 \, d^4 r^*}{\int S(r^*, k^*) \, d^4 r^*}
\]

\[
|\psi(r^*, k^*)| = \sqrt{A_C(\eta)} \left[ \exp(-i k^* r^*) F(-i \eta, 1, i \xi) + f_c(k^*) \frac{G(r^*)}{r^*} \right]
\]

\[
f_c(k^*) = \left( \frac{1}{f_0} + \frac{d_0 \cdot k^{*2}}{2} - \frac{-2h(k^{*} a_c)}{s_c} - i k^{*} A_C(k^{*}) \right)^{-1}
\]

- Numerically solvable (strong+Coulomb)
- **3 parameters**: \( \Re f_0, \Im f_0 \) and source r define the correlation function.
- \( d_0 = 0 \) (zero effective range approx.)
Kaon-proton in Pb−Pb

- No $K^0 n$ structure
- Simultaneous description (and fit) of the correlation functions for 6 centralities (0-50%) with two parameters and 6 radii
- Radii constrained from $K^+ p$
Kubo’s cumulant expansion method

- Genuine three-particle correlations isolated using the Kubo’s cumulant expansion method:


- In terms of correlation functions:

\[ c_3(Q_3) = C(Q_3) - C_{12}(Q_3) - C_{23}(Q_3) - C_{31}(Q_3) + 2 \]
Lower-order contributions evaluation

Data-driven approach

- Using the **same** and **mixed-events** distributions:

\[
C ([p_1, p_2] , p_3) = \frac{N_2 (p_1, p_2) N_1 (p_3)}{N_1 (p_1) N_1 (p_2) N_1 (p_3)}
\]

- The scalar \(Q_3\) is calculated from the measured single-particle momenta

\[
(p_1, p_2, p_3) \rightarrow Q_3
\]

Projector method

- Using the two-body correlation function of the pair (1,2).

- A kinematic transformation from

\[
k_{12}^{*}\text{(pair)} \rightarrow Q_3\text{(triplet)}
\]

\[
C_2(k_{12}^{*}) \rightarrow C_3(Q_3)
\]

is performed.

- For the pair \((i, j)\) we have

\[
C_3^{ij}(Q_3) = \int C_2(k_{ij}^{*}) W_{ij}(k_{ij}^{*}, Q_3) dk_{ij}^{*}
\]

\[\text{two-body correlation function} \hspace{1cm} \text{projector}\]
p–p–p correlation function

Lower-order correlations

More details in Bawani Talk
$p-p-p$ cumulant

**cumulant** (Flat feed-down from the resonances is considered)

- **Statistical significance:**
  - $\sigma_n = 6.7$ for $Q_3 < 0.4$ GeV/c

- **Conclusion:**
  - Presence of a genuine three-body effect in $p-p-p$

- **Possible interpretations:**
  - Pauli blocking at the three-particle level
  - long-range Coulomb interaction effects
  - three-body strong interaction
    - Collaboration with A. Kievsky, L. Marcucci and M. Viviani (Pisa University - INFN) for the theoretical interpretation.

$p$–$p$–$K^+$ correlation function

Lower-order correlations
Small Sources: Collective Effects and Strong Resonances

Elliptic flow
- Anisotropic pressure gradients within the source

Radial flow
- Expanding source with constant velocity
- Different effect on different masses

Strong decays of broad resonances
- Resonances with $c\tau \sim r_0 \sim 1$ fm ($\Delta^*$, $N^*$, $\Sigma^*$) introduce an exponential tail to the source
- Different for each particle species

Core Radius

Strong decays of specific resonances