Fluctuations of conserved charges and correlations

Anar Rustamov

GSI

a.rustamov@gsi.de

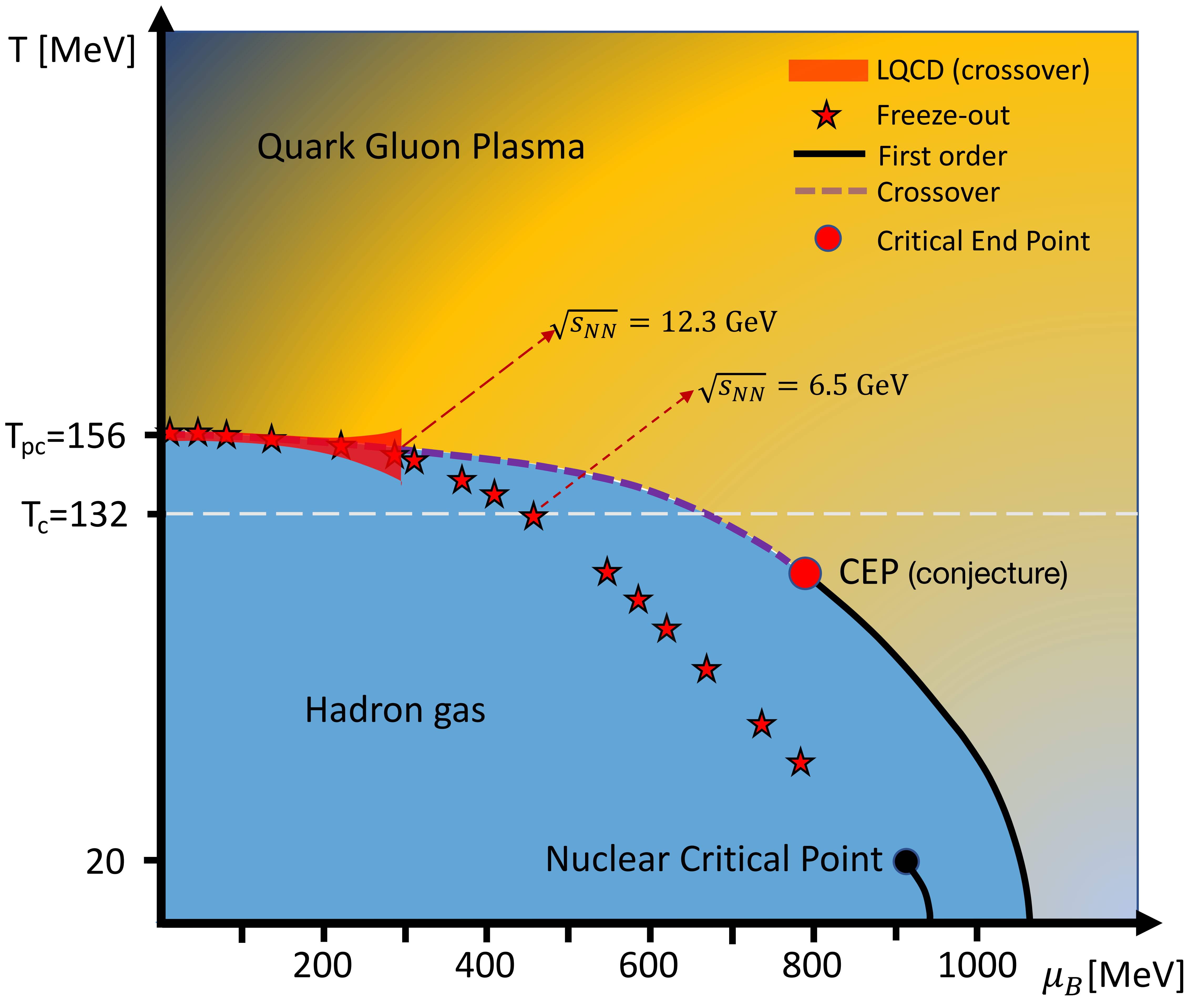
SQM2022

The 20th International Conference on Strangeness in Quark Matter
13-17 June 2022
Busan, Republic of Korea

Outline

- Phase diagram and fluctuations
  - Importance of baselines
    - Correlations in rapidity space
- Experimental results and their interpretations
  - The search for critical phenomena
  - The quest for proton clusters
  - Other ideas
- Conclusions

Phase diagram and fluctuations



A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nature 561, 321–330 (2018)
H. T. Ding et al [HotQCD], arXiv:1903.04801, A. Bazavov et al [HotQCD], arXiv:1812.08235

E-by-E fluctuations are predicted within Grand Canonical Ensemble

direct link to EoS

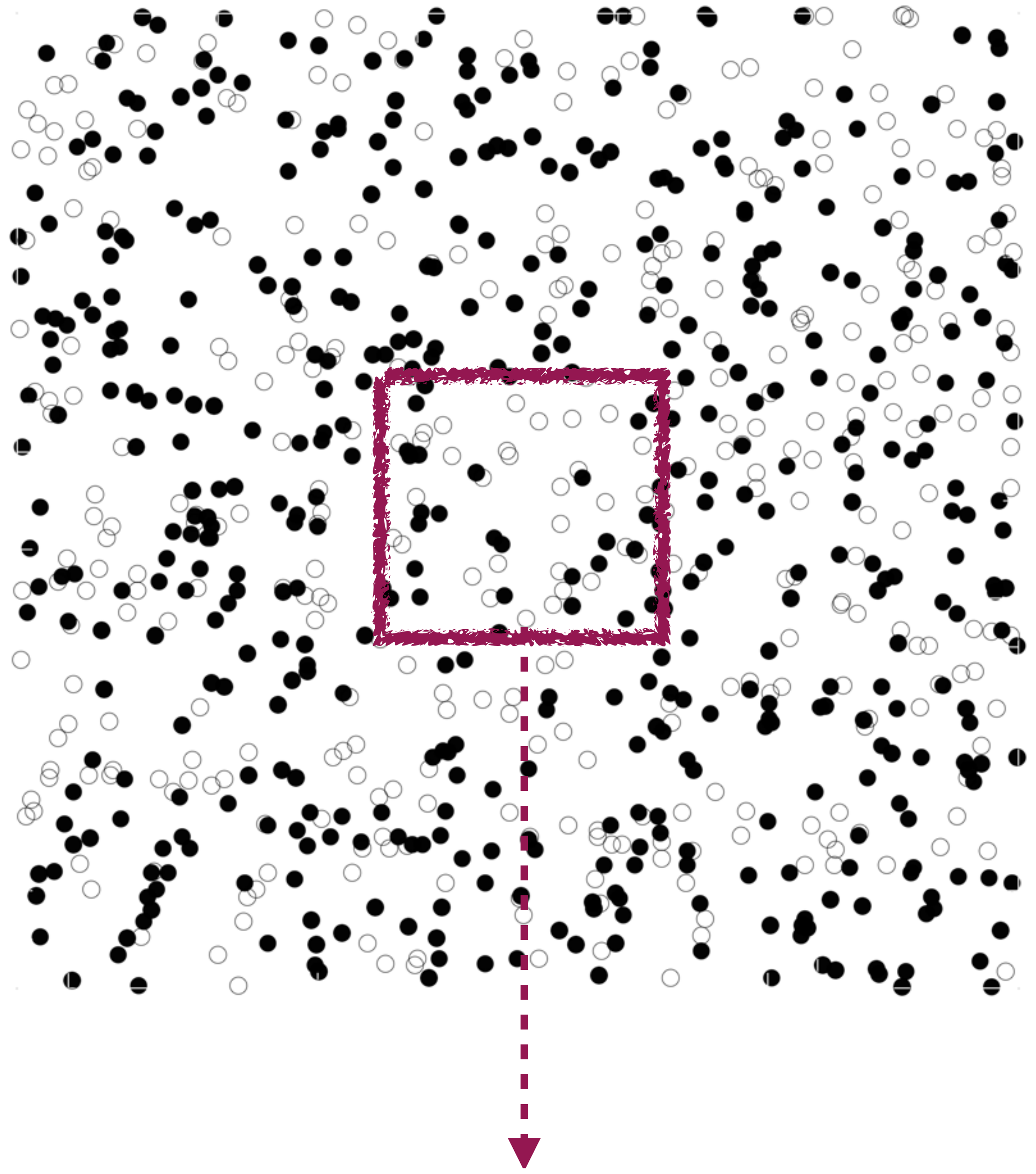
$$\frac{\kappa_n(N_B - N_{\bar{B}})}{VT^3} = \frac{1}{VT^3}\frac{\partial^n lnZ(V,T,\mu_B)}{\partial\left(\mu_B/T\right)^n} \equiv \hat{\chi}_n^B$$

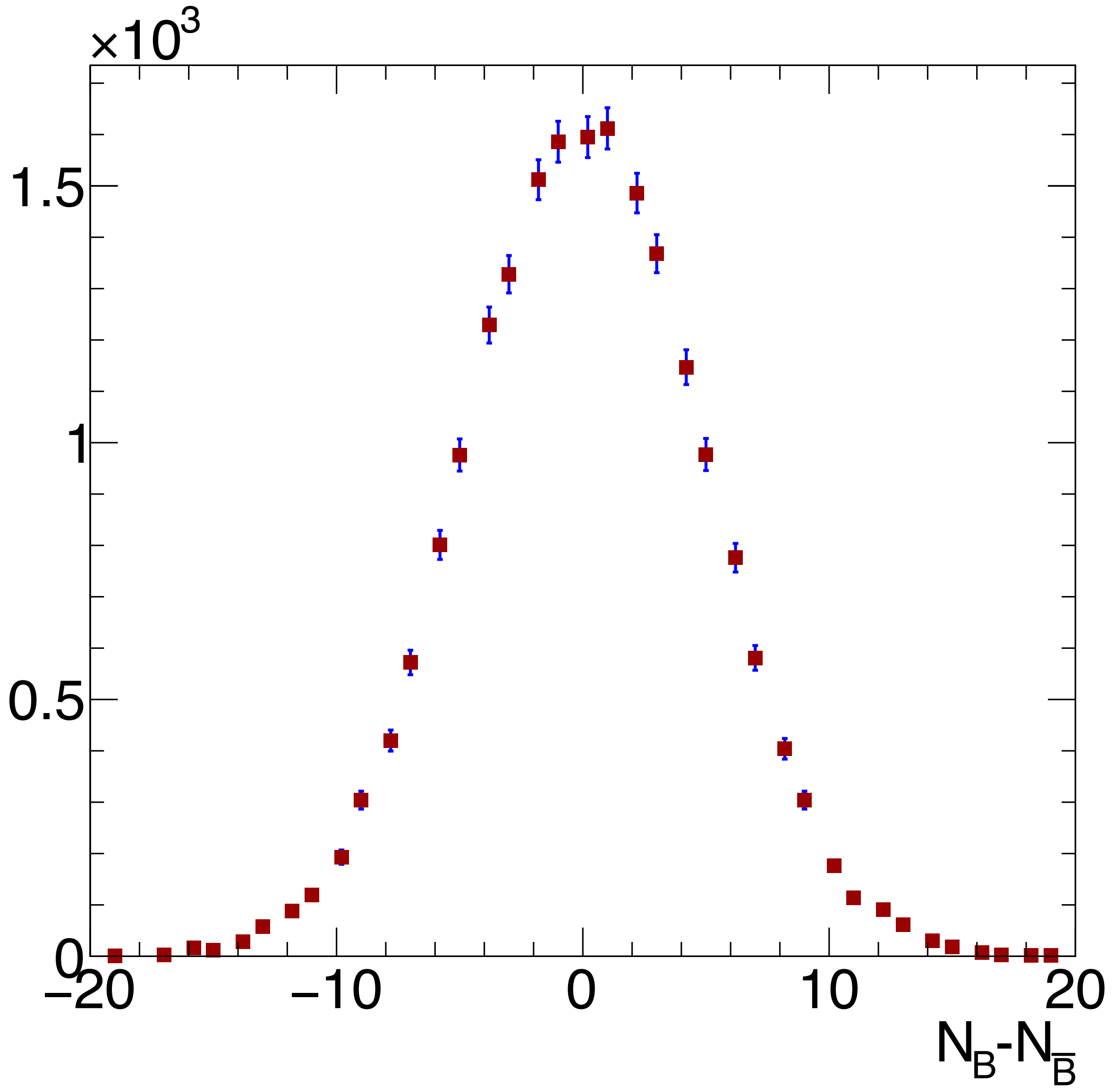
κ_n - cumulants (measurable in experiment)

$\hat{\chi}_n^B$ - susceptibilities (e.g. from lQCD)

decoding the phase structure of matter with E-by-E fluctuations

Cumulants and minimal baseline





- $\Delta N = N_B - N_{\bar{B}}$ occurs with probability $p(\Delta N)$ (measured)
- r^{th} order central moment: $\mu_r = \sum_{\Delta N} (\Delta N - \langle \Delta N \rangle)^r p(\Delta N)$
- first 4 cumulants: $\kappa_1 = \langle \Delta N \rangle,\ \kappa_2 = \mu_2,\ \ \kappa_3 = \mu_3,\ \kappa_4 = \mu_4 - 3\mu_2^2$

- **advantage:** sensitive to small (critical) signals
- **disadvantage:** sensitive to any non-critical contributions

$$\frac{\kappa_n(N_B - N_{\bar{B}})}{VT^3} = \frac{1}{VT^3}\frac{\partial^n lnZ(V,T,\mu_B)}{\partial\left(\mu_B/T\right)^n} \equiv \hat{\chi}_n^B$$

minimal baseline: Ideal Gas EoS + GCE

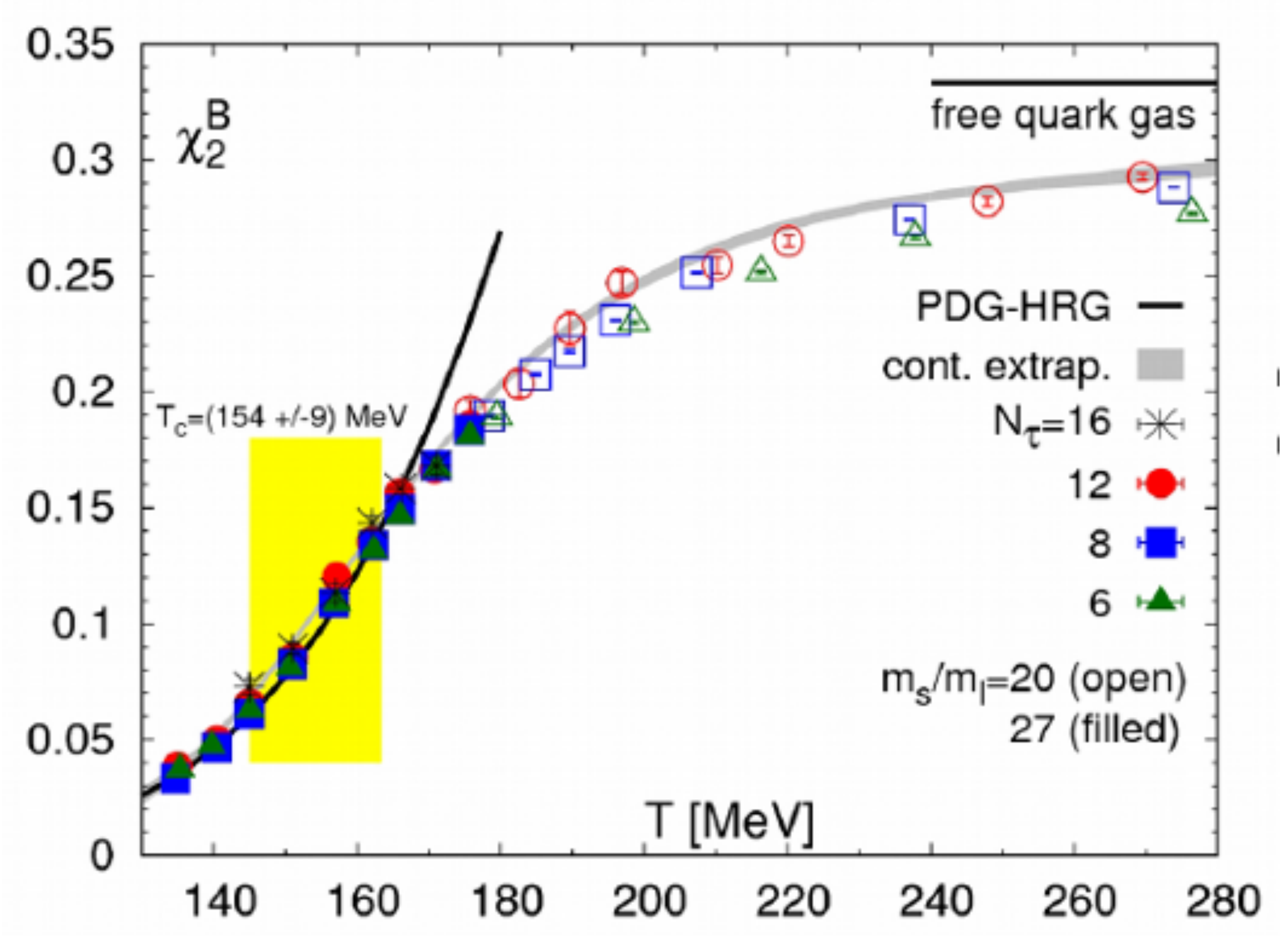
particles (Poisson)

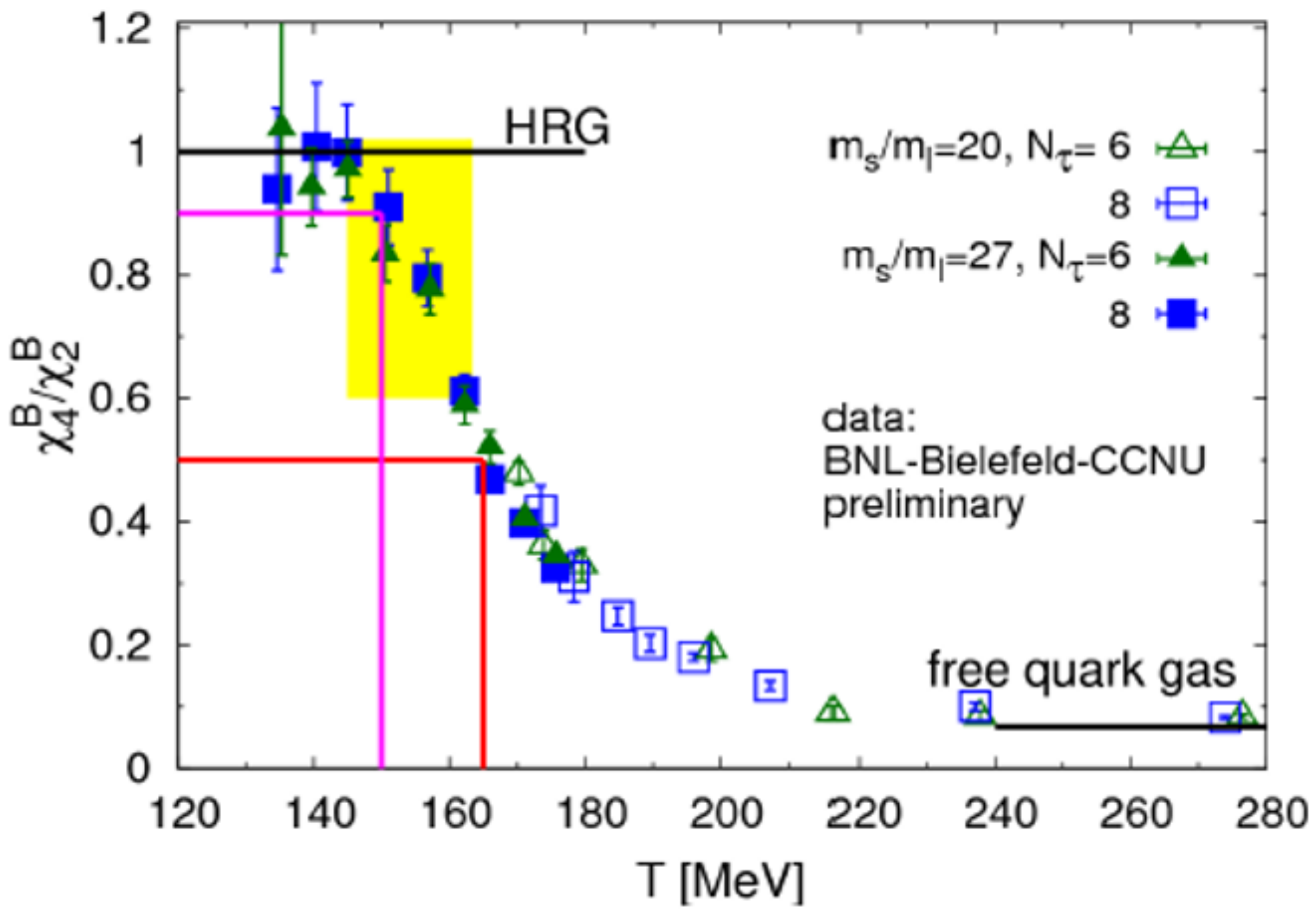
$$\frac{\kappa_m}{\kappa_n} = 1$$

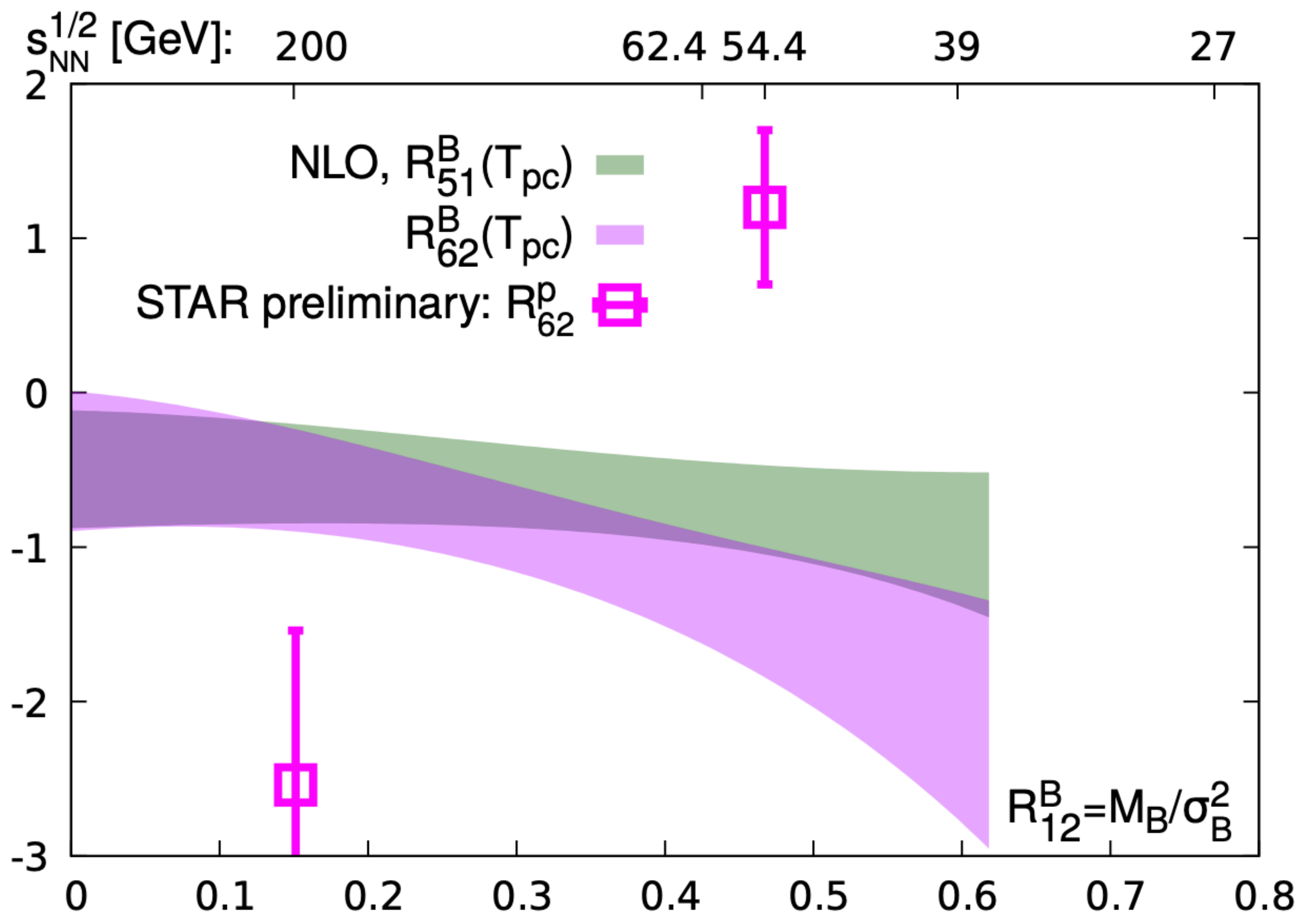
net-particles (Skellam)

$$\frac{\kappa_{2m}}{\kappa_{2n}} = 1, \quad \frac{\kappa_{2m}}{\kappa_{2n+1}} = \frac{\langle N \rangle + \langle \bar{N} \rangle}{\langle N \rangle - \langle \bar{N} \rangle}$$

Results from LQCD







A. Bazavov et al [HotQCD], PRD 101 (2020) 074502
A. Bazavov et al., Phys.Rev. D85 (2012) 054503

- χ_2^B : agreement with the HRG in GCE (for T < 165 MeV)
- χ_4^B/χ_2^B : significant reduction compared to HRG in GCE (for T > 150 MeV)
- $\chi_{5(6)}^B/\chi_{1(2)}^B$: (progressively) negative sign towards lower energies (probe of crossover)

In LQCD calculations

- volume is fixed
- charge conservations are imposed on the averages

In experiments

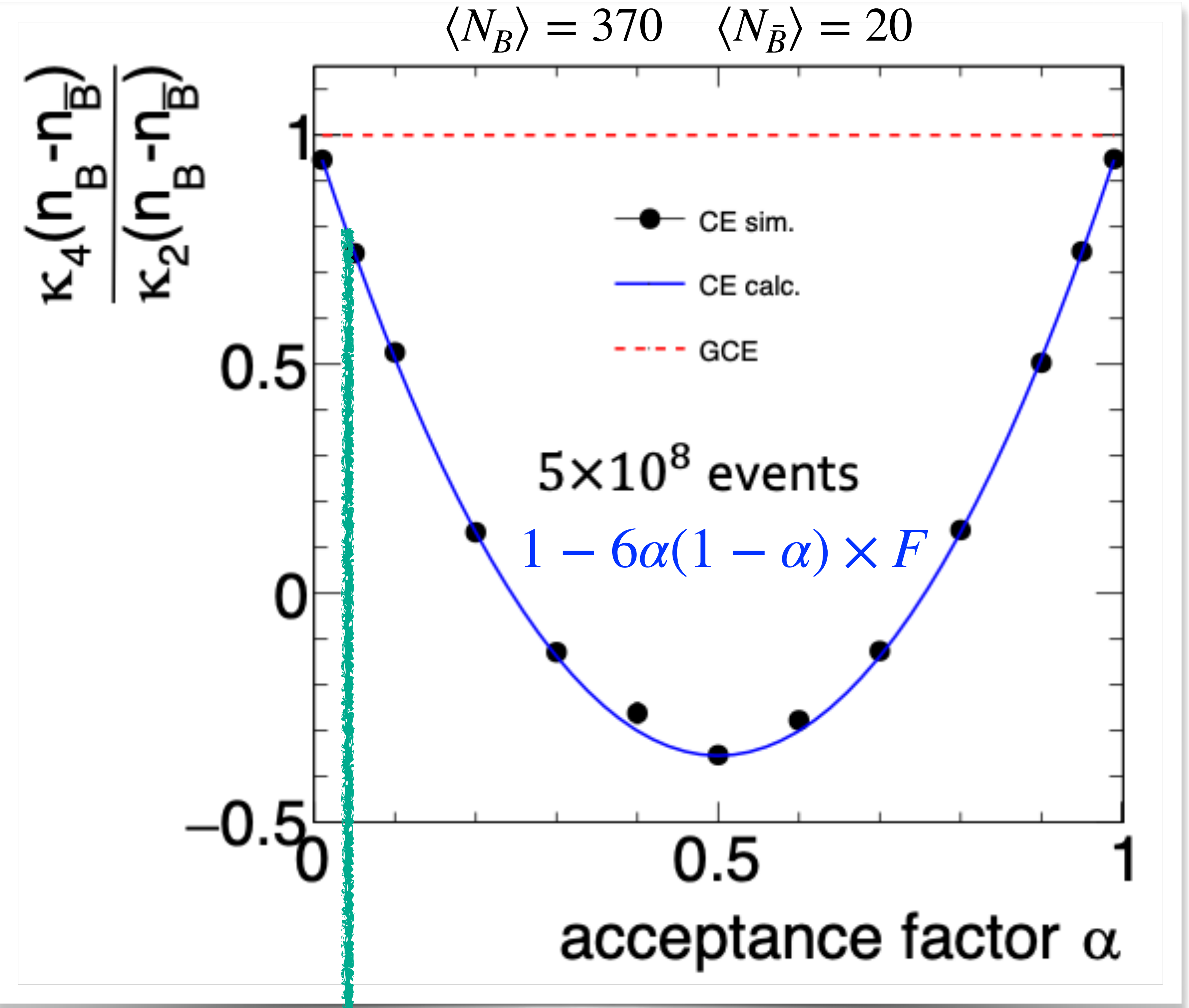
- volume fluctuates from event to event
- charge is conserved in each event

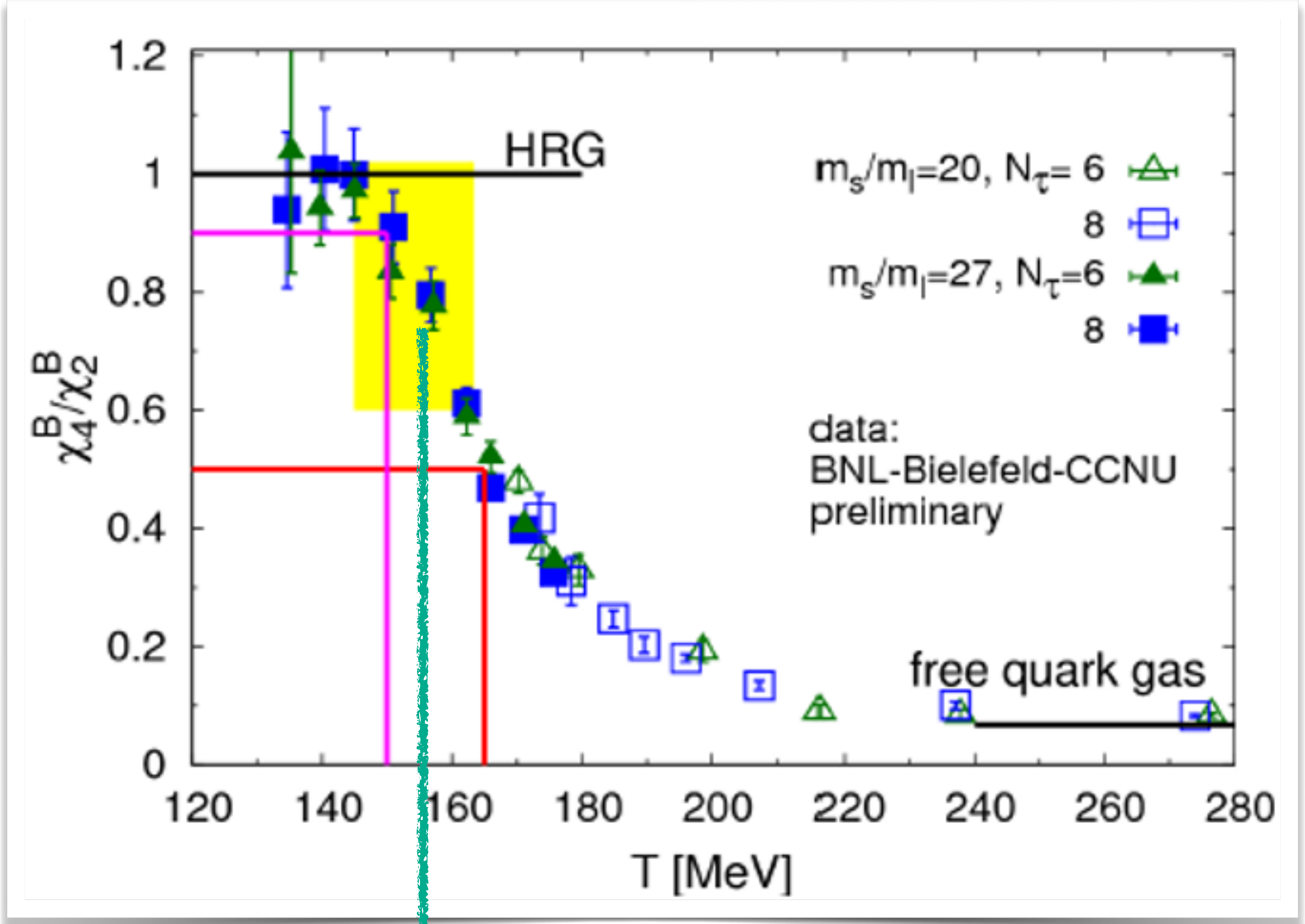
P. Braun-Munzinger, AR., J. Stachel, NPA 960 (2017) 114
V. Skokov, B. Friman, and K. Redlich, Phys.Rev. C88 (2013) 034911

Prerequisite for improved baseline

ideal gas + baryon number conservation in 4π

LQCD





P. Braun-Munzinger, AR, J. Stachel, arXiv:1807.08927
A. Bzdak, V. Koch, V. Skokov, Phys.Rev.C 87 (2013) 1, 014901

about 20% deviation from unity, however, induced by different mechanisms

construction of proper baseline is essential for extracting the true signal

Details of implementation

$$Z_B(V,T) = \sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty} \frac{(\lambda_B z_B)^{N_B}}{N_B!}\frac{(\lambda_{\bar{B}} z_{\bar{B}})^{N_{\bar{B}}}}{N_{\bar{B}}!}\delta(N_B - N_{\bar{B}} - B) = \left(\frac{\lambda_B z_B}{\lambda_{\bar{B}} z_{\bar{B}}}\right)^{\frac{B}{2}} I_B(2z\sqrt{\lambda_B \lambda_{\bar{B}}})$$

B net baryon number, conserved in each event

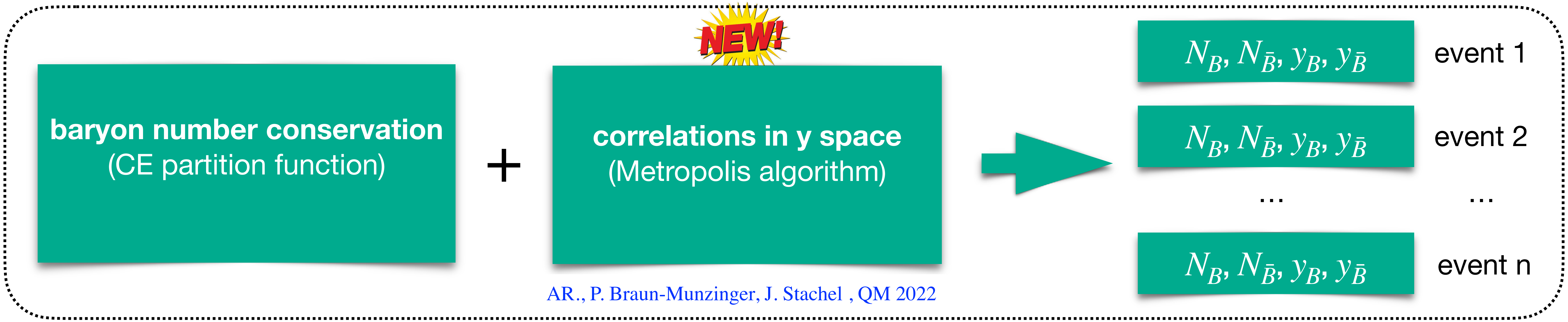
I_B modified Bessel function of the first kind

$z_B, z_{\bar{B}}$ single particle partition functions for baryons, anti baryons

$\lambda_B, \lambda_{\bar{B}}$ auxiliary parameters for calculating cumulants of baryons, anti baryons

P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel , NPA 1008 (2021) 122141

A. Bzdak, V. Koch, V. Skokov, Phys.Rev.C 87 (2013) 1, 014901



Input from experiments

- baryon rapidity distributions
- measured (canonical) $\langle N_B \rangle$, $\langle N_{\bar{B}} \rangle$

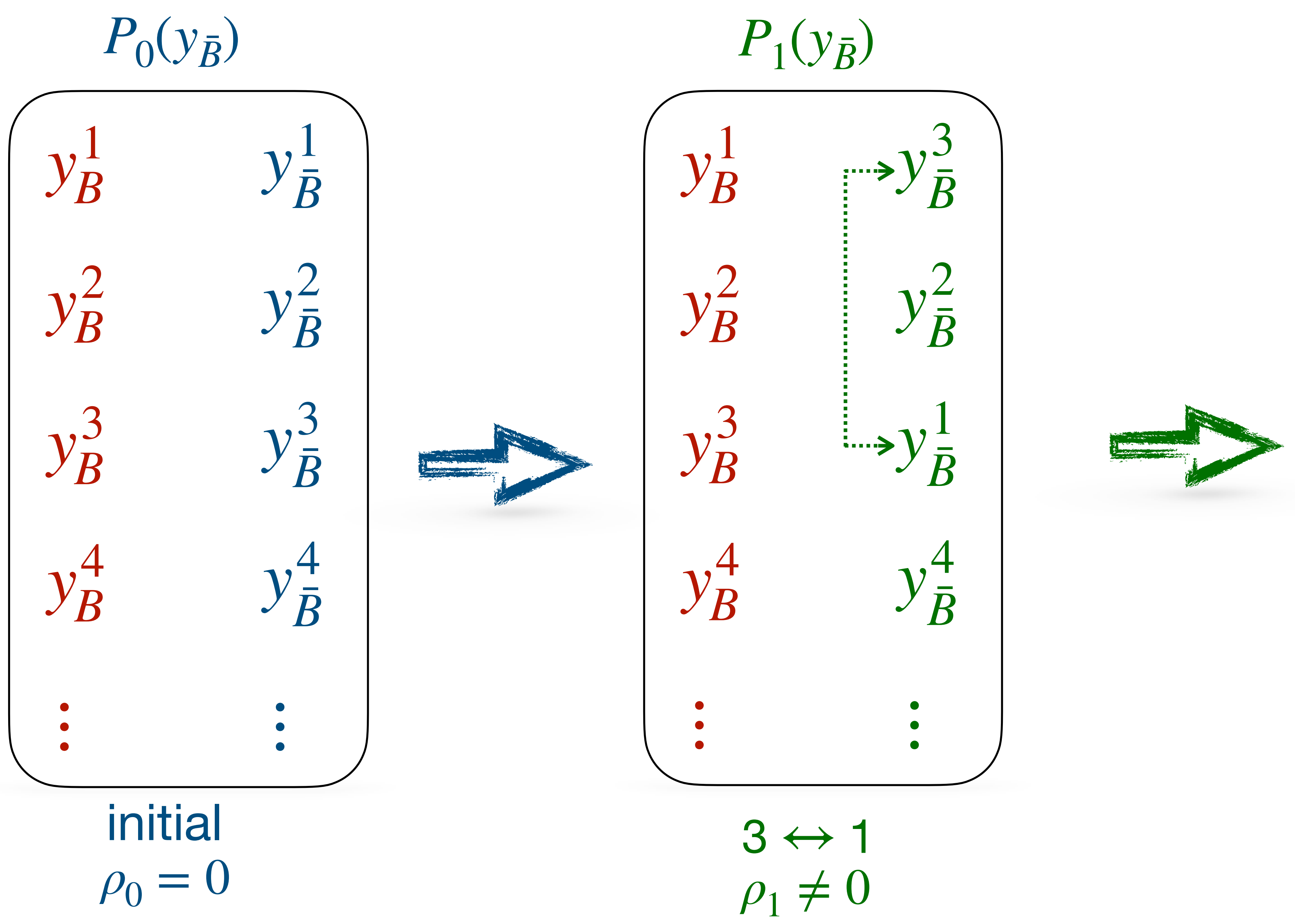
$z = \sqrt{z_B z_{\bar{B}}}$ **is calculated by solving**

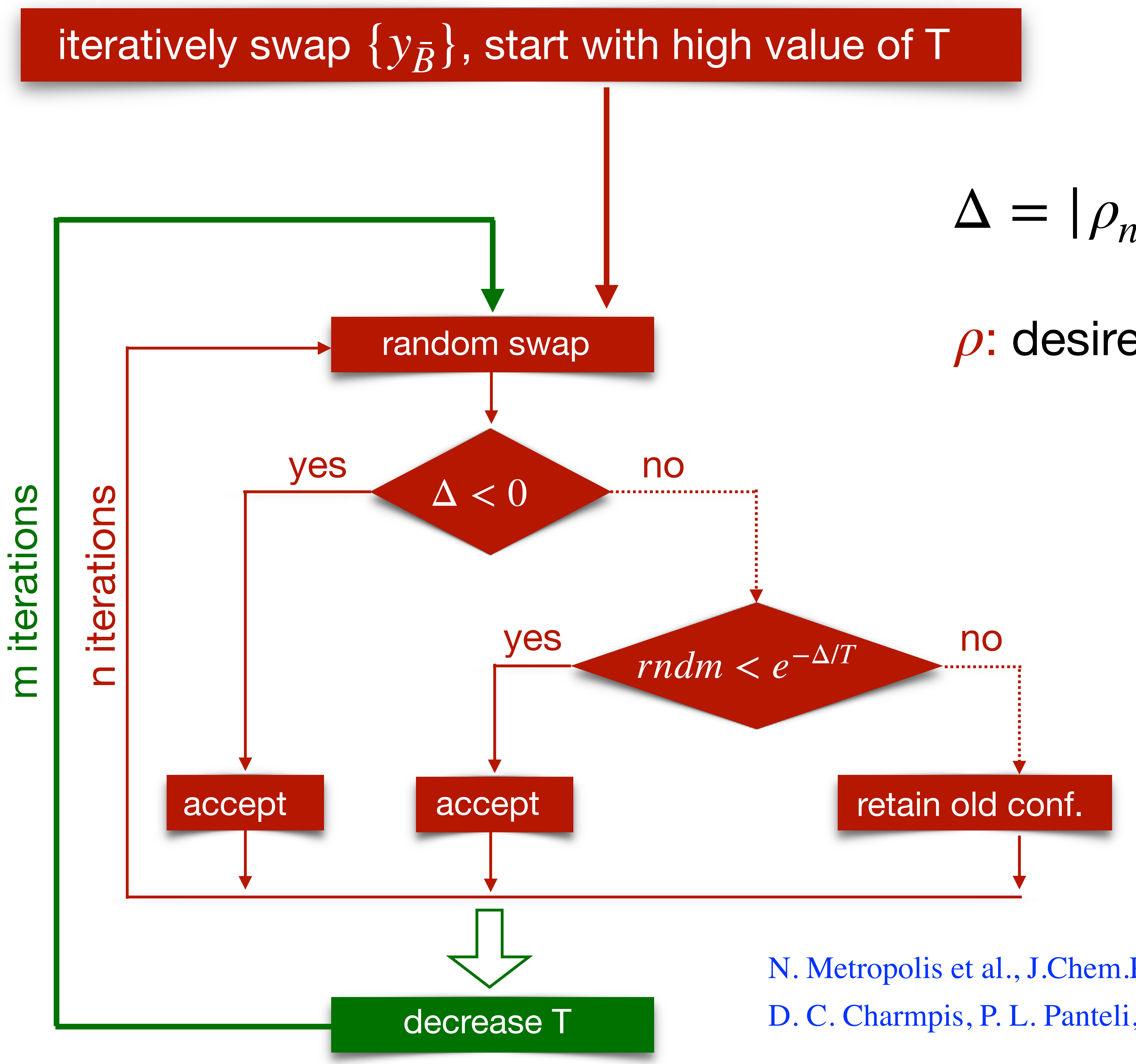
$$\langle N_B \rangle = \lambda_B \frac{\partial \ln Z_B}{\partial \lambda_B}\bigg|_{\lambda_B, \lambda_{\bar{B}} = 1} = z\frac{I_{B-1}(2z)}{I_B(2z)}$$

Metropolis algorithm (Simulated annealing)

AR., P. Braun-Munzinger, J. Stachel , QM 2022

start with uncorrelated $\{y_B\}$, $\{y_{\bar{B}}\}$





stop if accepted rate is too small (< 1%)

$$\Delta = |\rho_n - \rho| - |\rho_{n-1} - \rho|$$

ρ: desired corr. coefficient

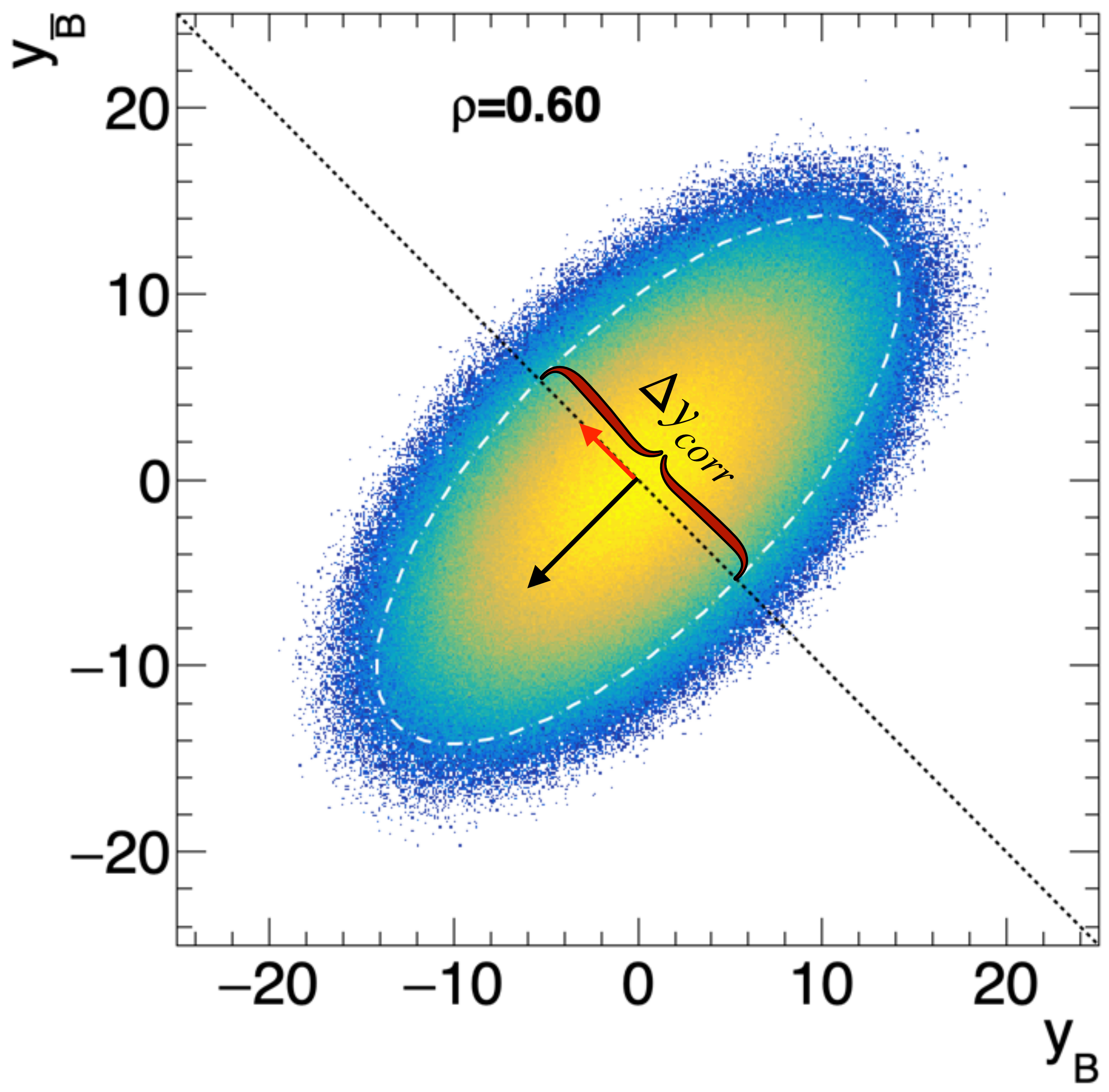
$$\rho_n = \frac{cov[y_B, P_n(y_{\bar{B}})]}{\sigma_{y_B}\sigma_{y_{\bar{B}}}}$$

N. Metropolis et al., J.Chem.Phys. 21 (1953) 1087-1092

D. C. Charmpis, P. L. Panteli, Comp. Stat. 19 (2004) 283-300

works for arbitrary distributions

Improved baseline(s)



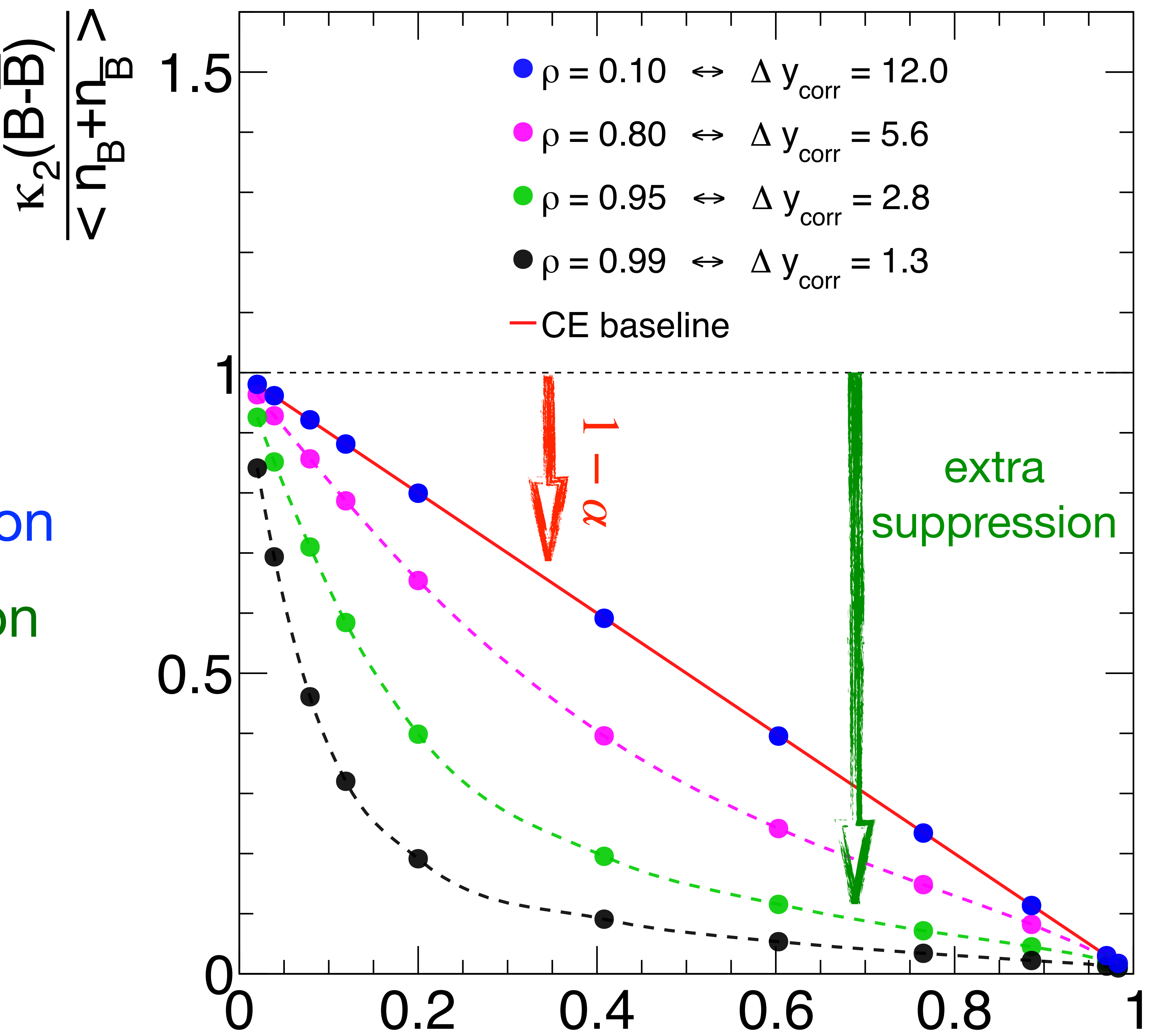
$$\rho = \frac{\langle y_B y_{\bar{B}} \rangle - \langle y_B \rangle \langle y_{\bar{B}} \rangle}{\sigma_{y_B} \sigma_{y_{\bar{B}}}}$$

ρ - correlation coefficient

Δy_{corr} - correlation length

$\rho = 0$: Global baryon number conservation

$\rho \neq 0$: Local baryon number conservation



$$\frac{\kappa_2(B-\bar{B})}{\langle n_B + n_{\bar{B}} \rangle} = 1 - \frac{\alpha_B \langle n_B \rangle + \alpha_{\bar{B}} \langle n_{\bar{B}} \rangle}{\langle n_B + n_{\bar{B}} \rangle} + \left(z^2 - \langle N_B \rangle \langle N_{\bar{B}} \rangle\right) \frac{\left(\alpha_B - \alpha_{\bar{B}}\right)^2}{\langle n_B + n_{\bar{B}} \rangle}$$

$\alpha_B = \langle n_B \rangle / \langle N_B \rangle$, $\alpha_{\bar{B}} = \langle n_{\bar{B}} \rangle / \langle N_{\bar{B}} \rangle$, for $\alpha = \alpha_B = \alpha_{\bar{B}}$;

$$\frac{\kappa_2(B-\bar{B})}{\langle n_B + n_{\bar{B}} \rangle} = 1 - \alpha$$

P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel, NPA 1008 (2021) 122141

$\langle n_B \rangle$ - inside acceptance

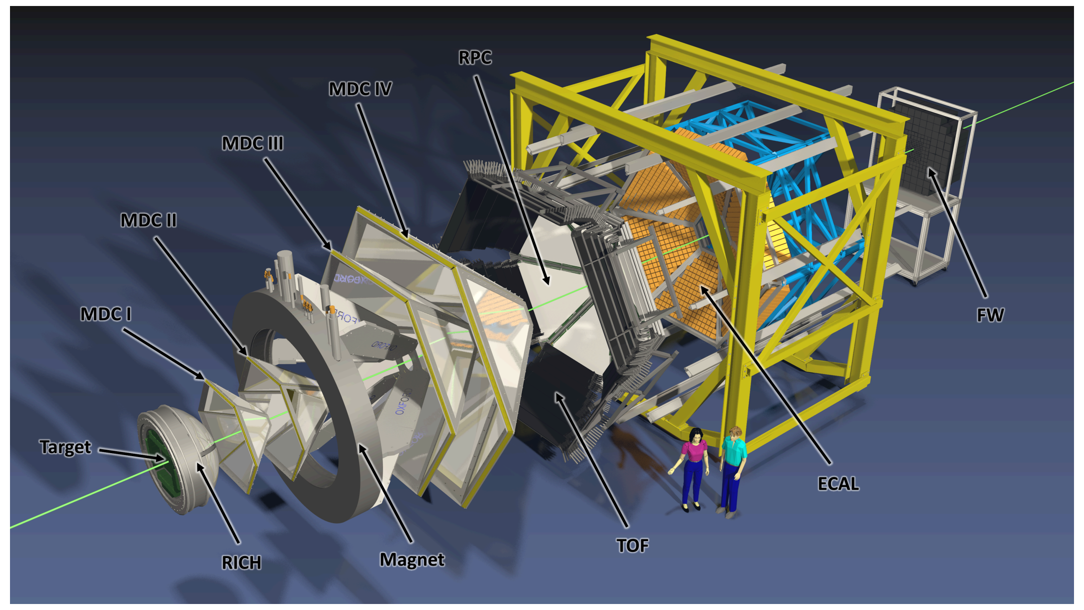
$\langle N_B \rangle$ - in 4π

extra suppression due to local baryon number conservation

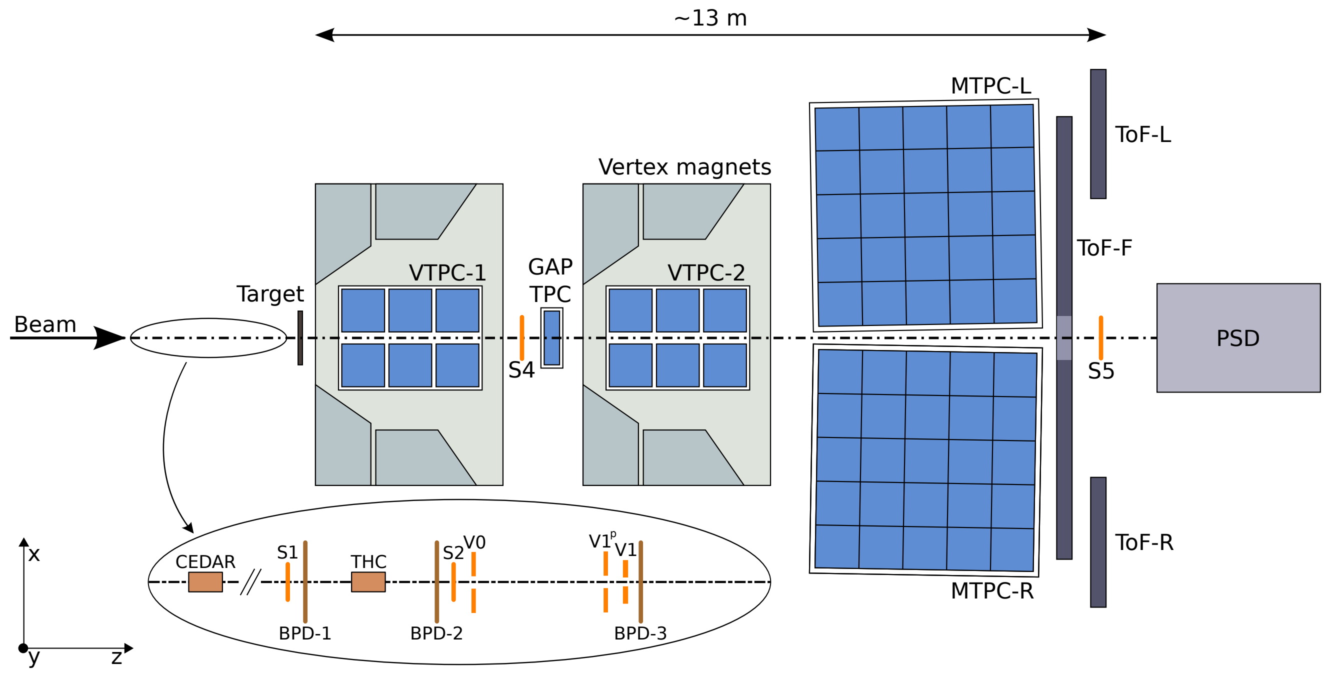
introduced via Choleski decomposition and Metropolis algorithm

AR., P. Braun-Munzinger, J. Stachel , QM 2022, previous version: arXiv:1907.03032

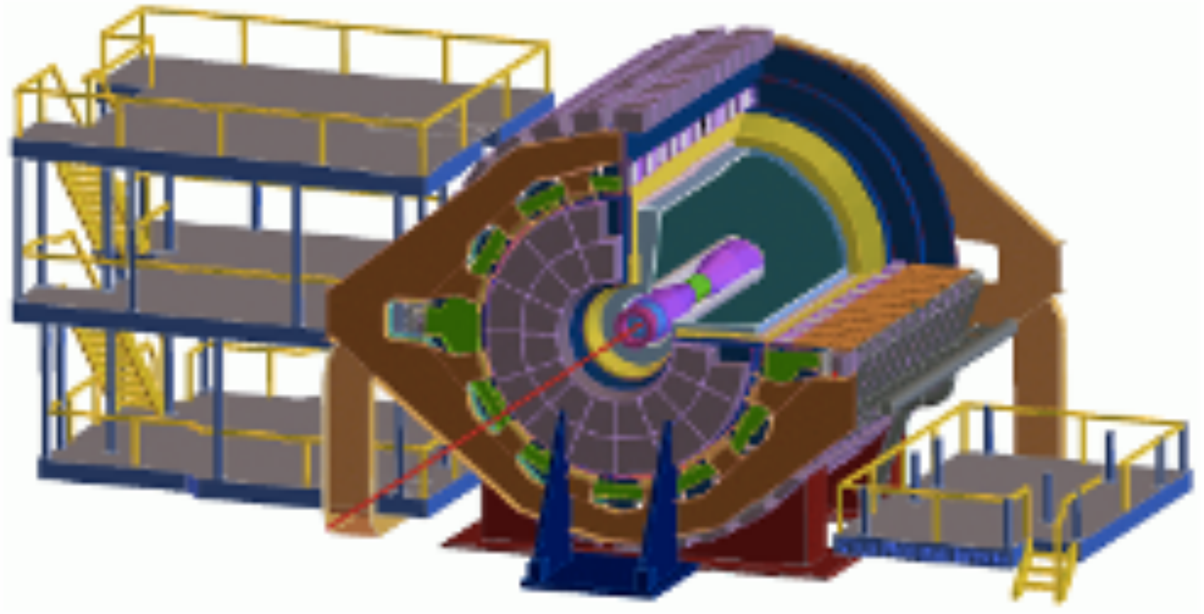
experimental results



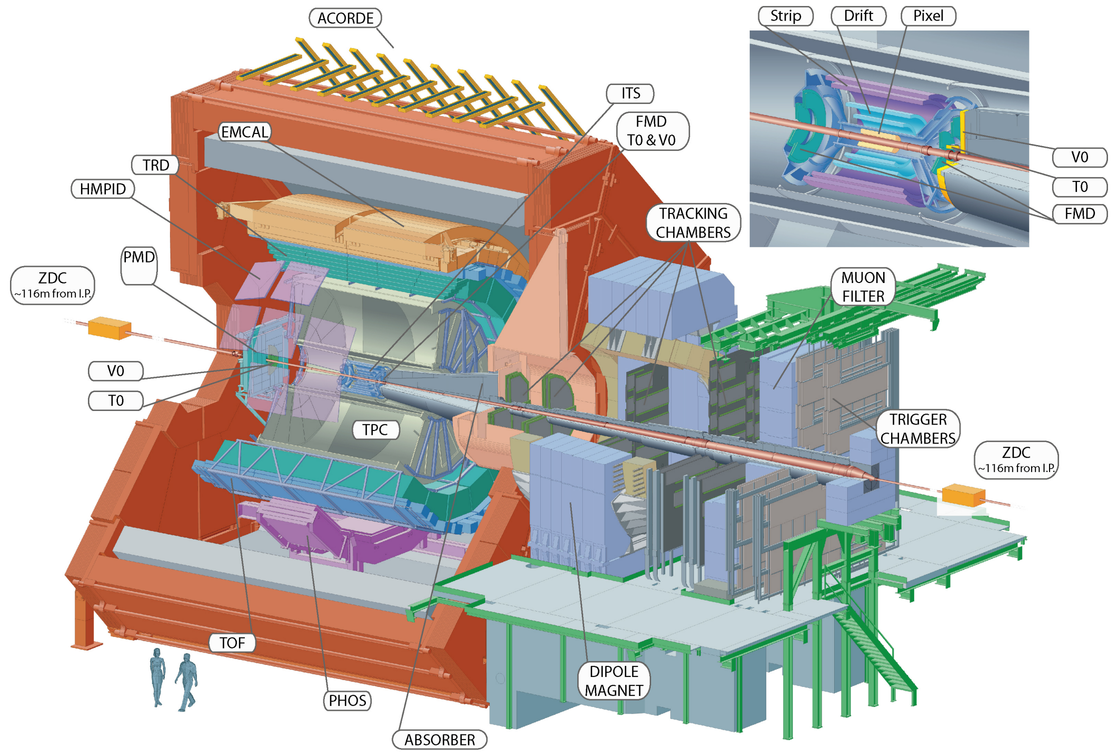
HADES@GSI SIS18 (few GeV)



NA61/SHINE@CERN SPS (5-17 GeV)



STAR@BNL RHIC (3 - 200 GeV)



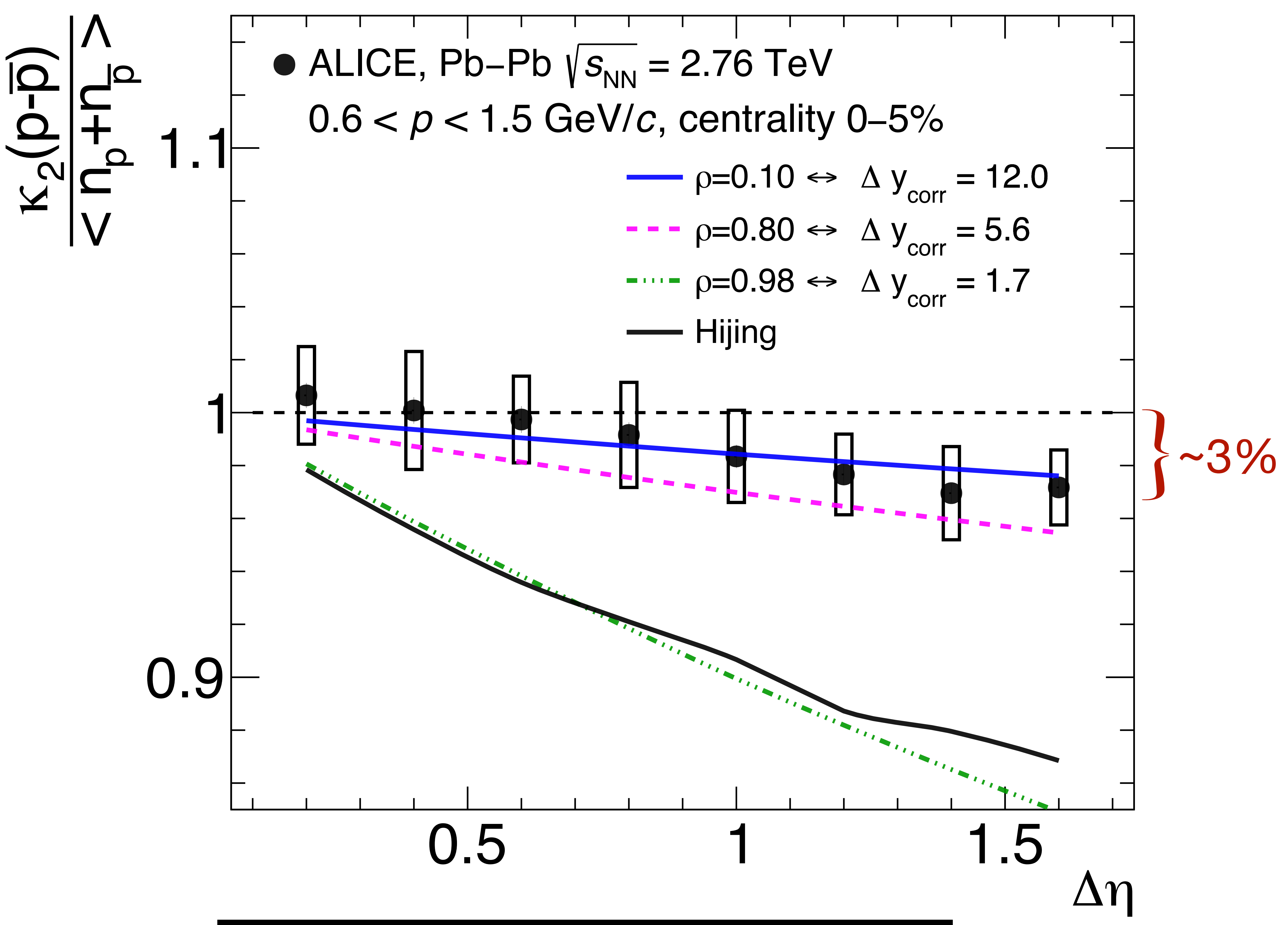
ALICE@ CERN LHC (few TeV)

probing the matter produced at energy scales from several GeV to several TeV

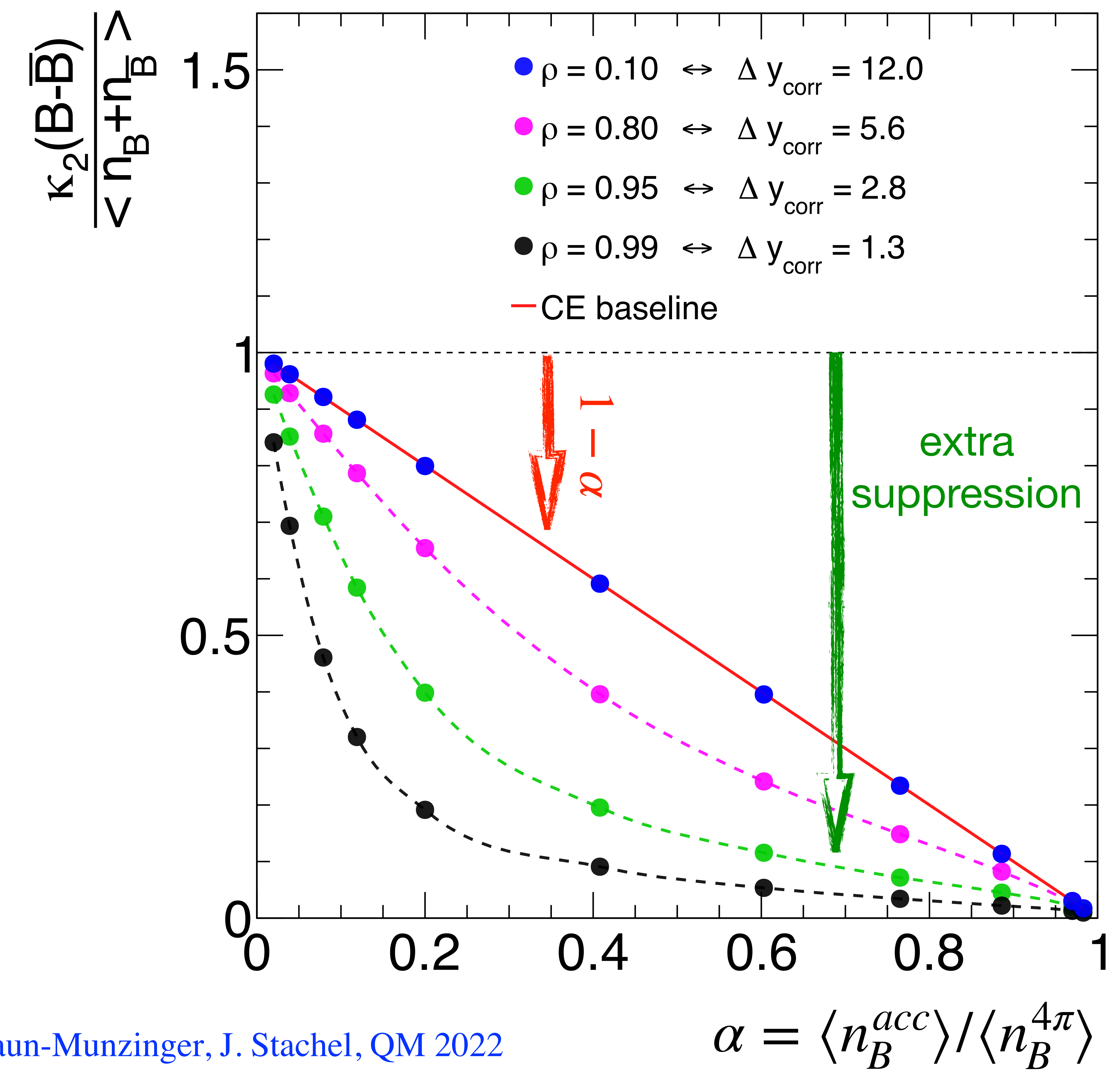
Results at LHC energies (ALICE)

AR., NPA 967 (2017) 453-456 **ALICE:** Phys. Lett. B 807 (2020) 135564

CE baseline: P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel, NPA 1008 (2021) 122141



15 June, 11:30, Mesut Arslandok



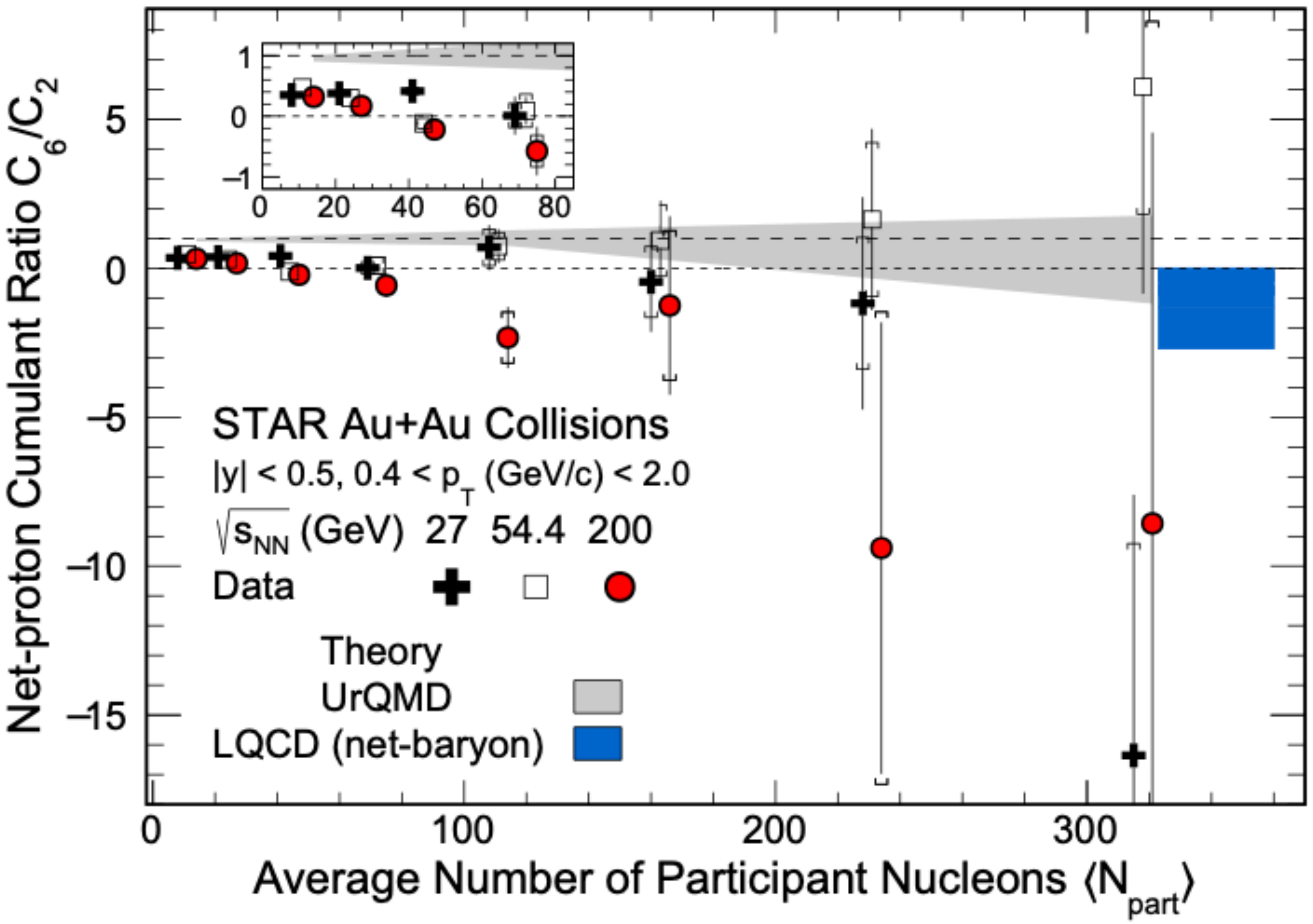
AR., P. Braun-Munzinger, J. Stachel, QM 2022

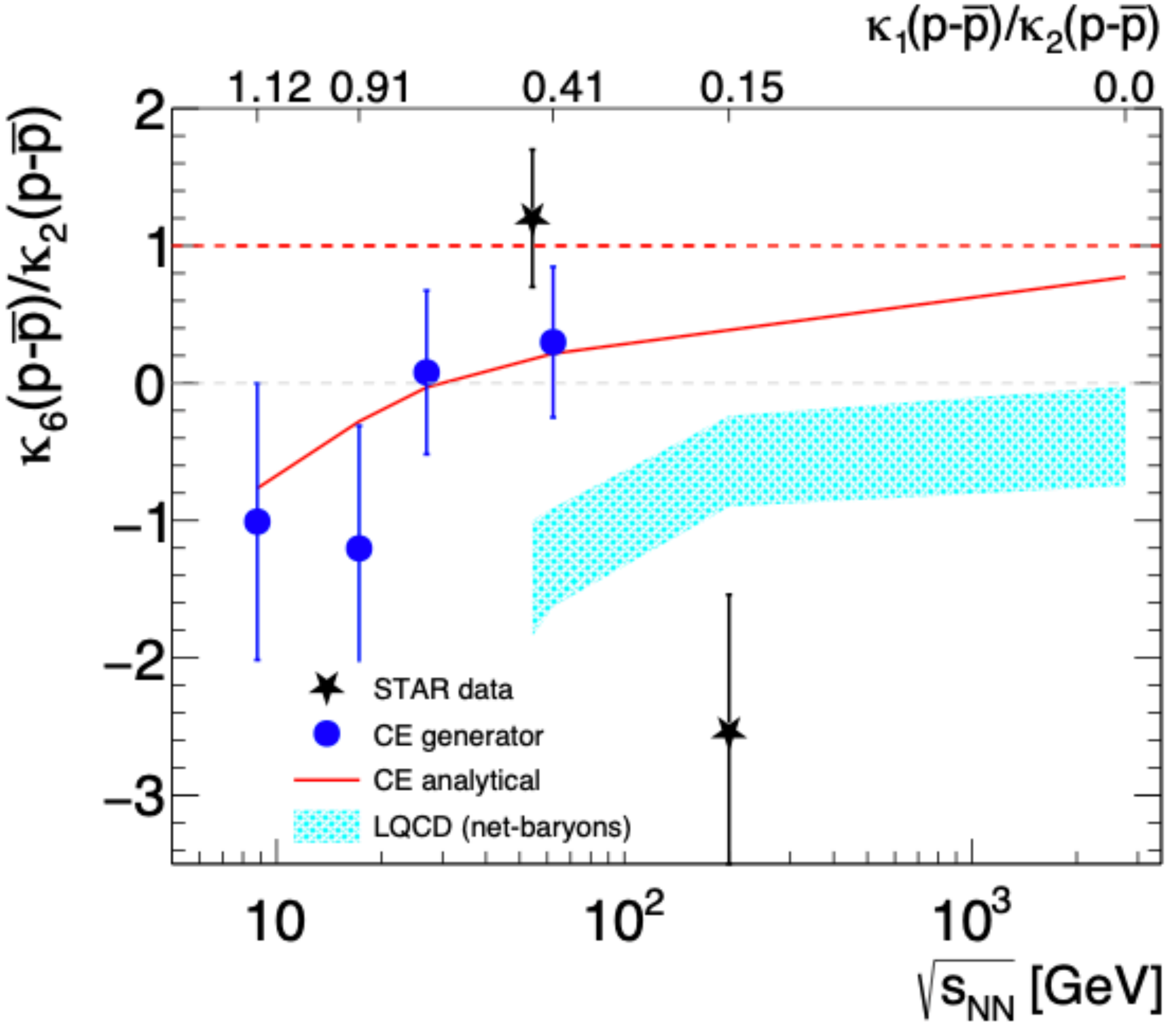
- Alice data: best description with $\rho = 0.1$ ($\Delta y_{corr} = 12$) ↔ Global baryon number conservation
- Hjing (Lund String Fragmentation) results are in conflict with the ALICE data

The experimental search for crossover transition

STAR: Phys.Rev.Lett. 127 (2021) 26, 262301

P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel, NPA 1008 (2021) 122141





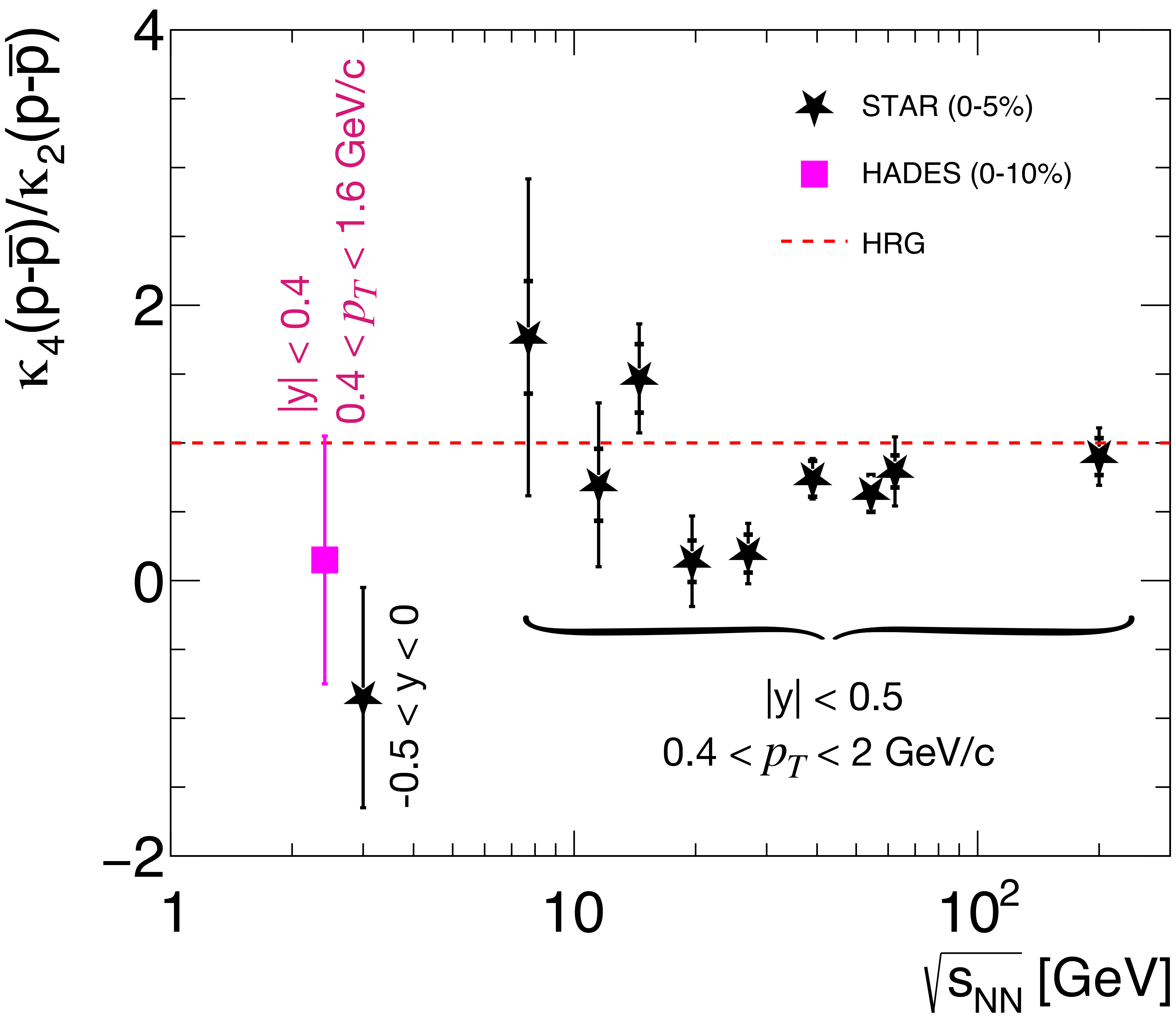
- first try to measure the cross-over signals via fluctuations of net-protons (LQCD - net-baryons)
  - no consistent trend between 54.4 and 200 GeV data (both are negative in LQCD)
  - baryon number conservation also leads to negative values for κ_6/κ_2
- calls for higher statistics for firm conclusions

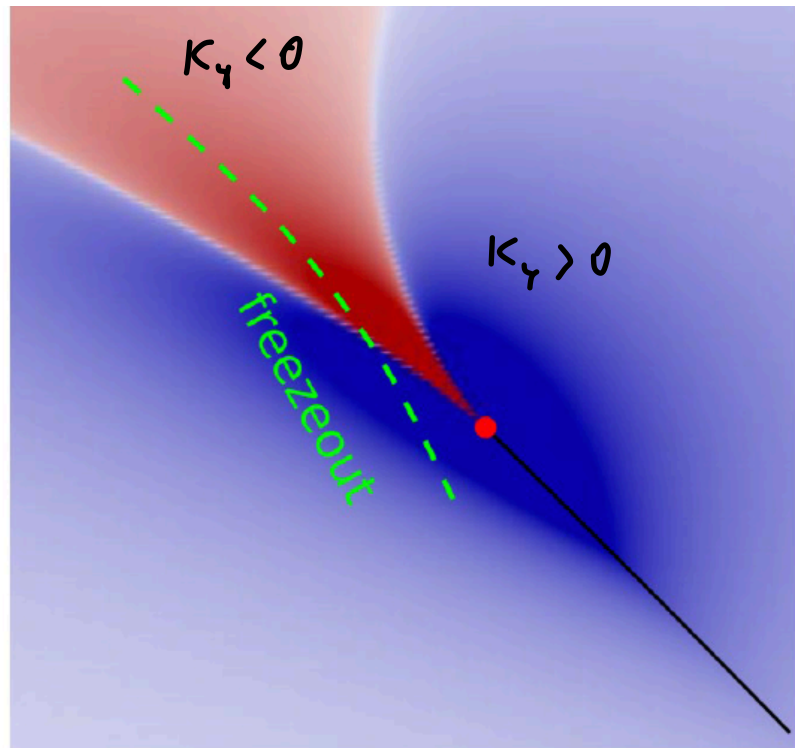
14 June, 14:00, Ashish Pandav

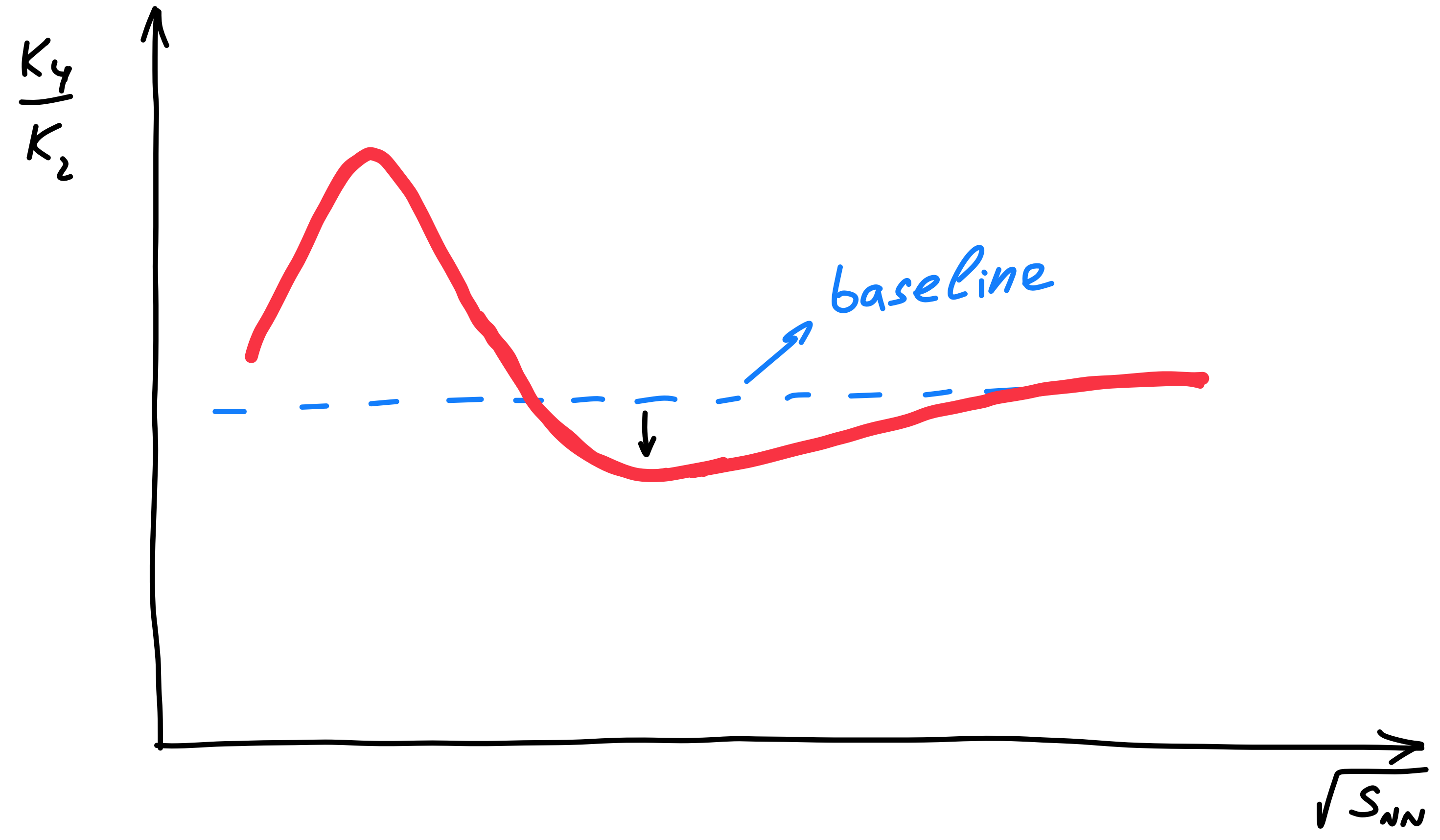
15 June, 14:30, Ho San Ko

Energy excitation function of κ_4/κ_2 in central Au-Au collisions

HADES: Phys.Rev.C 102 (2020) 2, 024914
STAR: Phys.Rev.Lett. 126 (2021) 9, 092301







a dip in the excitation function is generic

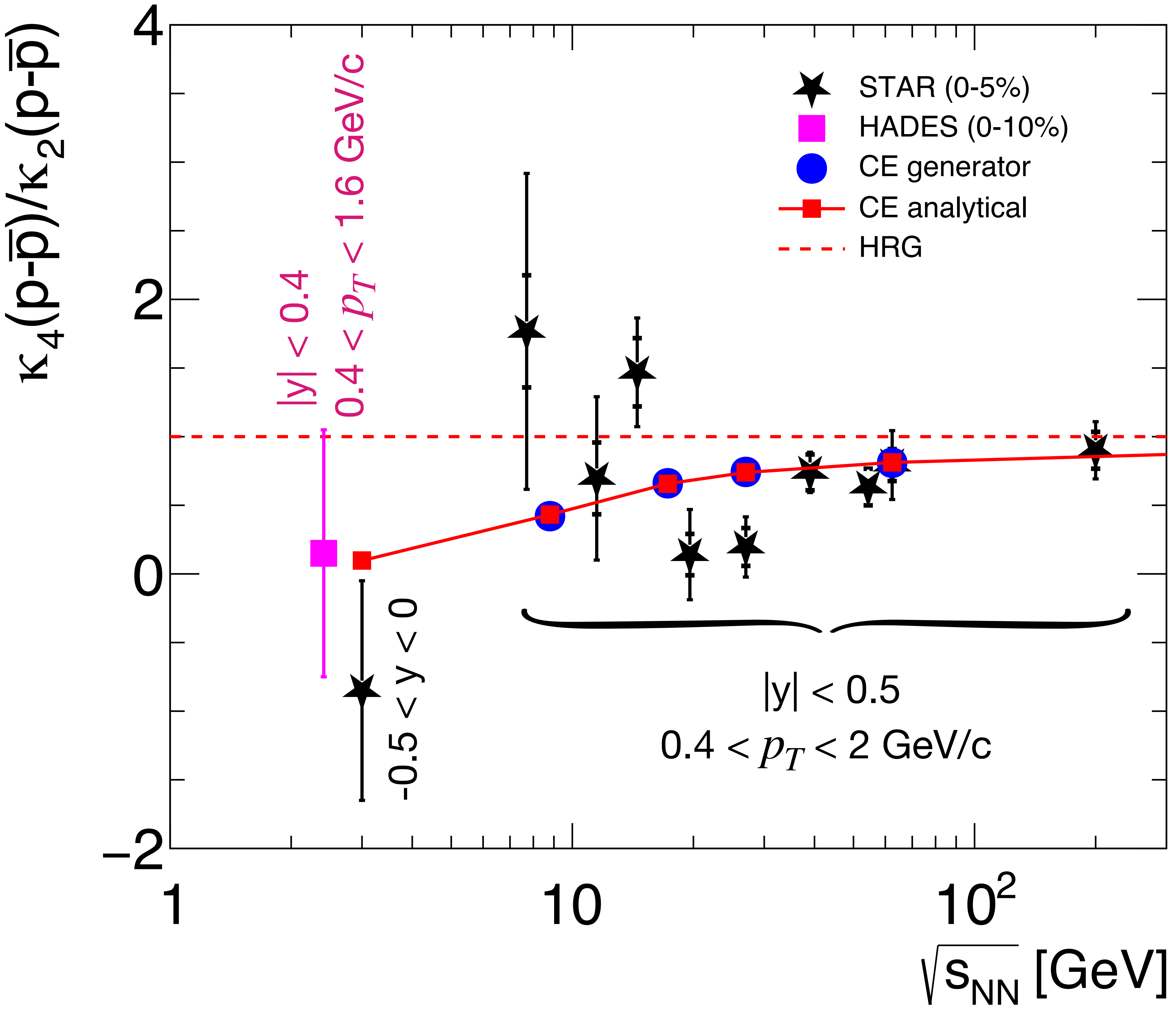
M. Stephanov, PRL102.032301(2009), PRL107.052301(2011)
M.Cheng et al, PRD79.074505(2009)

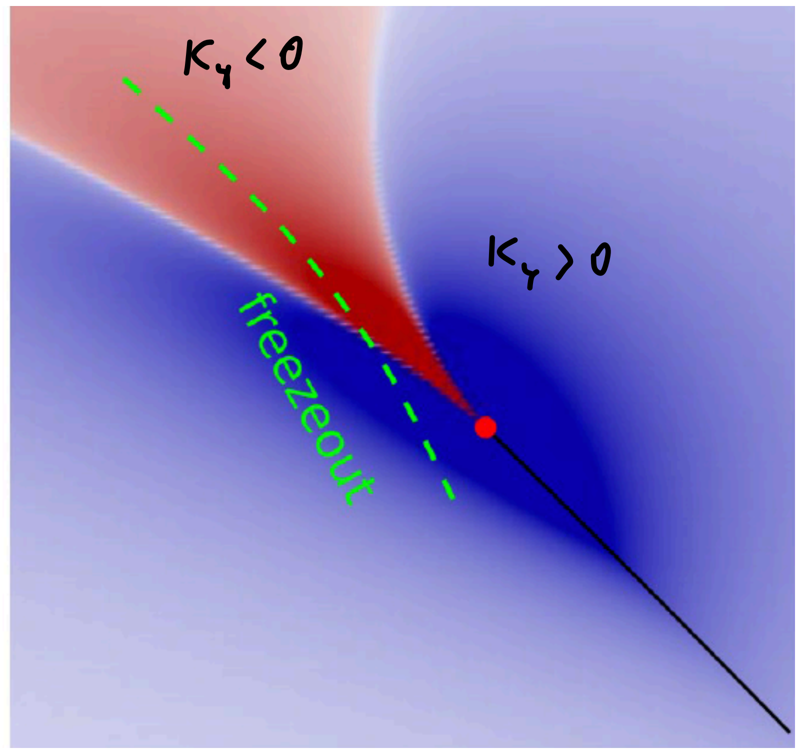
STAR: Phys.Rev.Lett. 126 (2021) 9, 092301

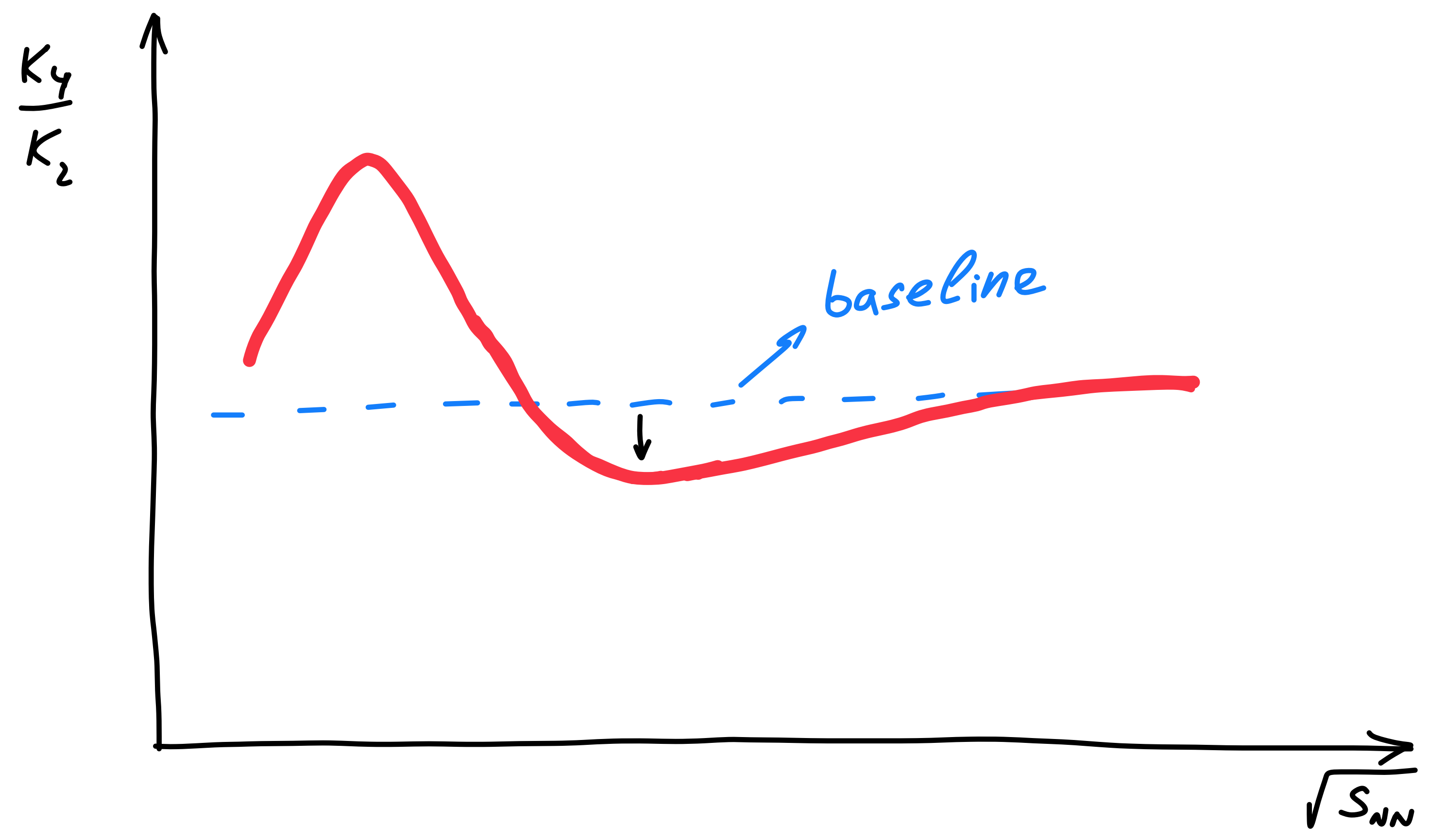
non-monotonic behaviour with a significance of 3.1σ relative to Skellam expectation

Energy excitation function of κ_4/κ_2 in central Au-Au collisions

HADES: Phys.Rev.C 102 (2020) 2, 024914
STAR: Phys.Rev.Lett. 126 (2021) 9, 092301







a dip in the excitation function is generic

M. Stephanov, PRL102.032301(2009), PRL107.052301(2011)
M.Cheng et al, PRD79.074505(2009)

STAR: Phys.Rev.Lett. 126 (2021) 9, 092301

non-monotonic behaviour with a significance of 3.1σ relative to Skellam expectation

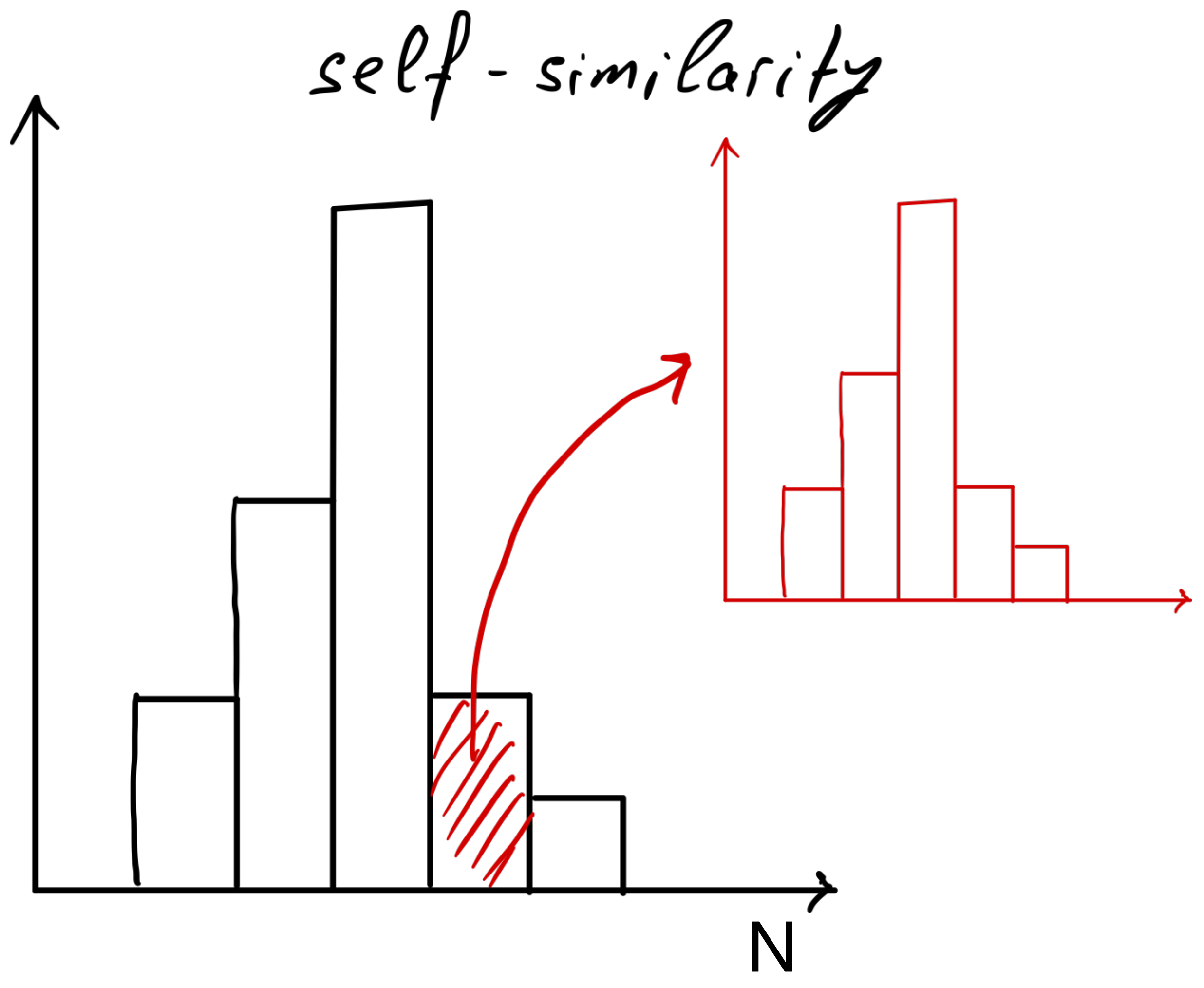
CE Baseline: P. Braun-Munzinger, B. Friman, K. Redlich, AR, J. Stachel, NPA 1008 (2021) 122141

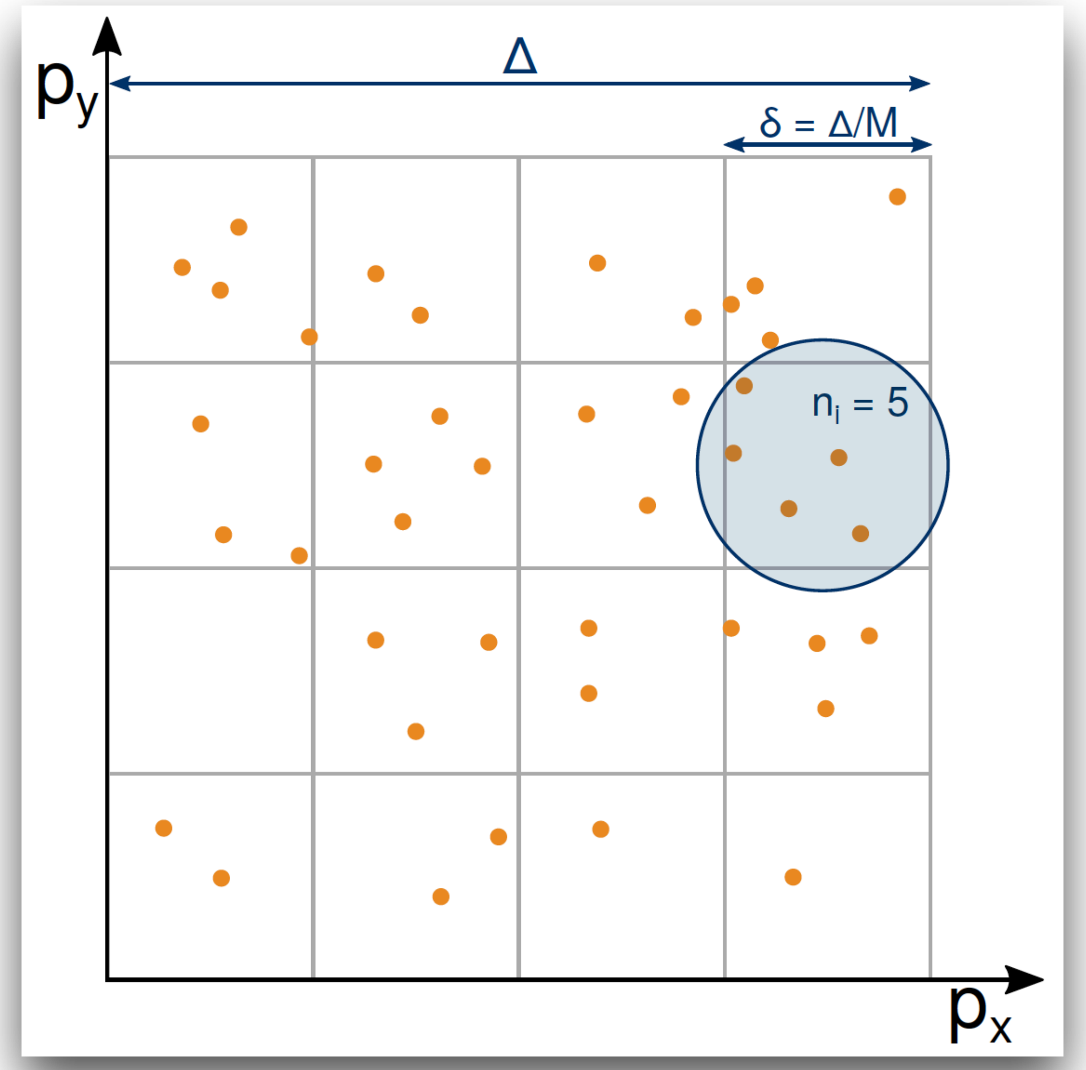
no statistically significant difference between the data and the canonical baseline (KS test: 1.2σ, χ^2 test: 1.5σ)

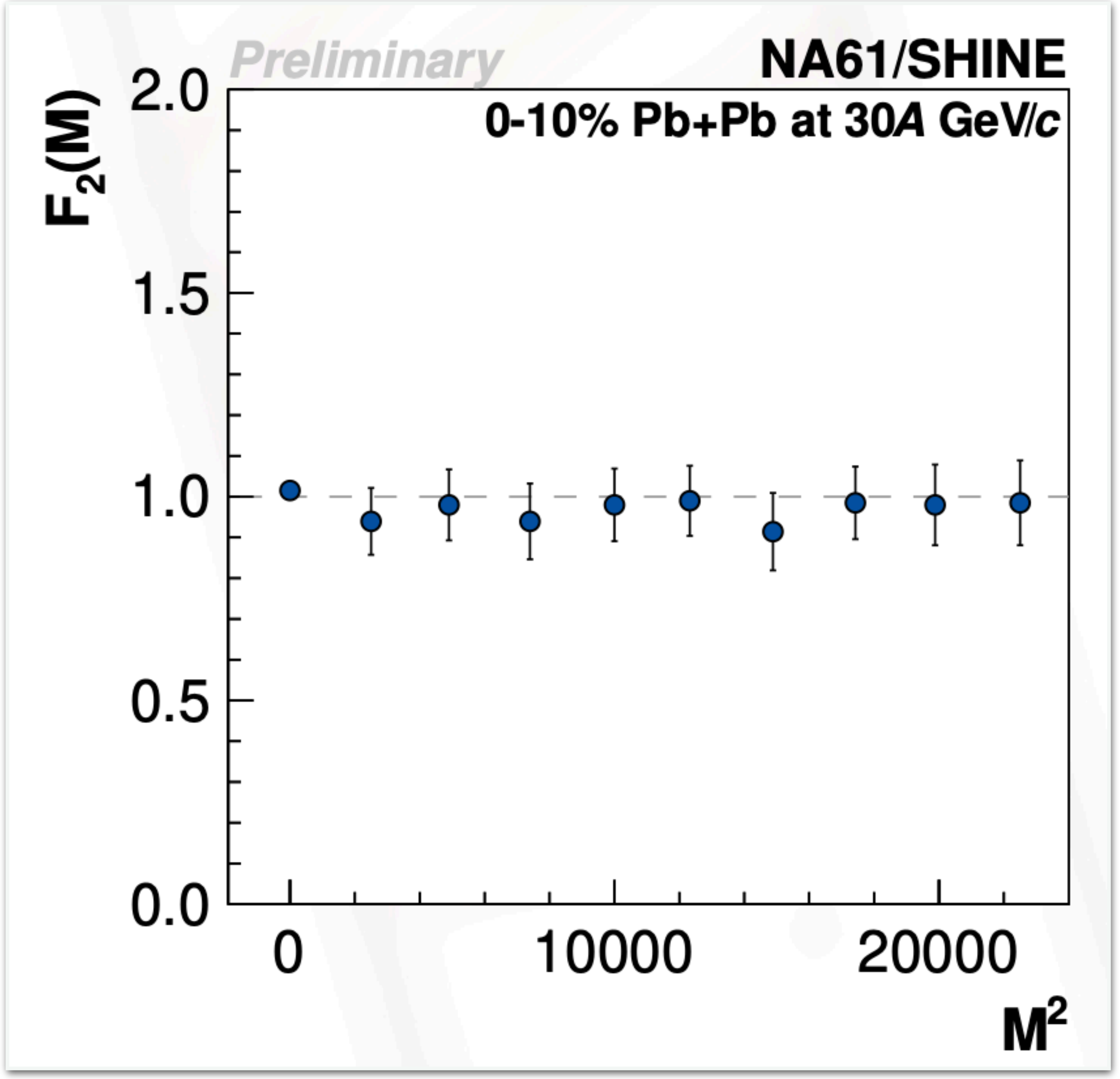
see also: V. Vovchenko, V. Koch, Ch. Shen, Phys.Rev.C 105 (2022) 1, 014904

higher statistics is needed for unambiguous conclusions

Search for self-similarity







$$F_2(M) = \frac{\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i(n_i - 1) \rangle}{\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \rangle^2}$$

δ - width of the i^{th} bin

n_i - number of particles in i^{th} bin

near the QCD critical point (assuming 3D Ising universality class)

$$F_2(M) \sim (M^2)^{\phi_2}, \quad \phi_2 = 5/6$$

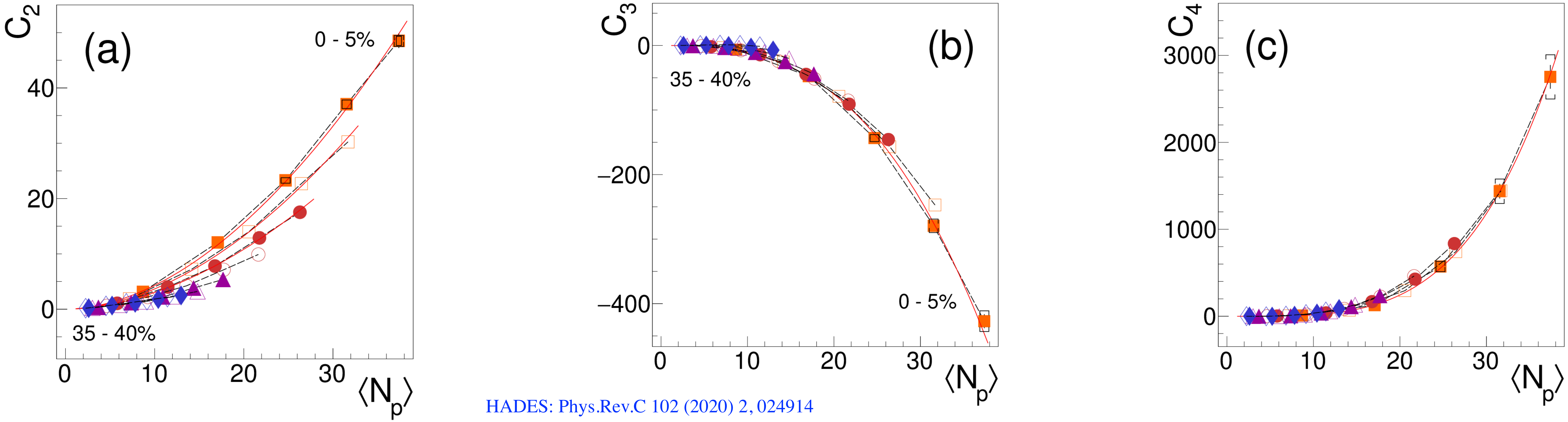
no indication of power-low behaviour

14 June, poster session, Tobiasz Czopowicz

15 June, 09:40, Nikolaos Davis

the quest for proton clusters

Results from HADES, Au-Au $\sqrt{s_{NN}}$=2.4 GeV



HADES: Phys.Rev.C 102 (2020) 2, 024914

$\kappa_2 = \kappa_1 + C_2$

$\kappa_3 = \kappa_1 + 3C_2 + C_3$

$\kappa_4 = \kappa_1 + 7C_2 + 6C_3 + C_4$

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + C_2(y_1, y_2)$$

$$\langle n^2 \rangle - \langle n \rangle^2 = \langle n \rangle + \int C_2(y_1 y_2) dy_1 dy_2$$

integrated correlation function: C_n

B. Ling, M. Stephanov, PRC 93 (2016) 034915

A. Bzdak, V. Koch, N. Strodthoff, PRC 95 (2017) 054906

- $\langle N_p \rangle$ - mean number of protons in selected $y_0 \pm \Delta y$
- Δy = 0.1, 0.2, 0.3, 0.4, 0,5

large values for integrated correlation functions

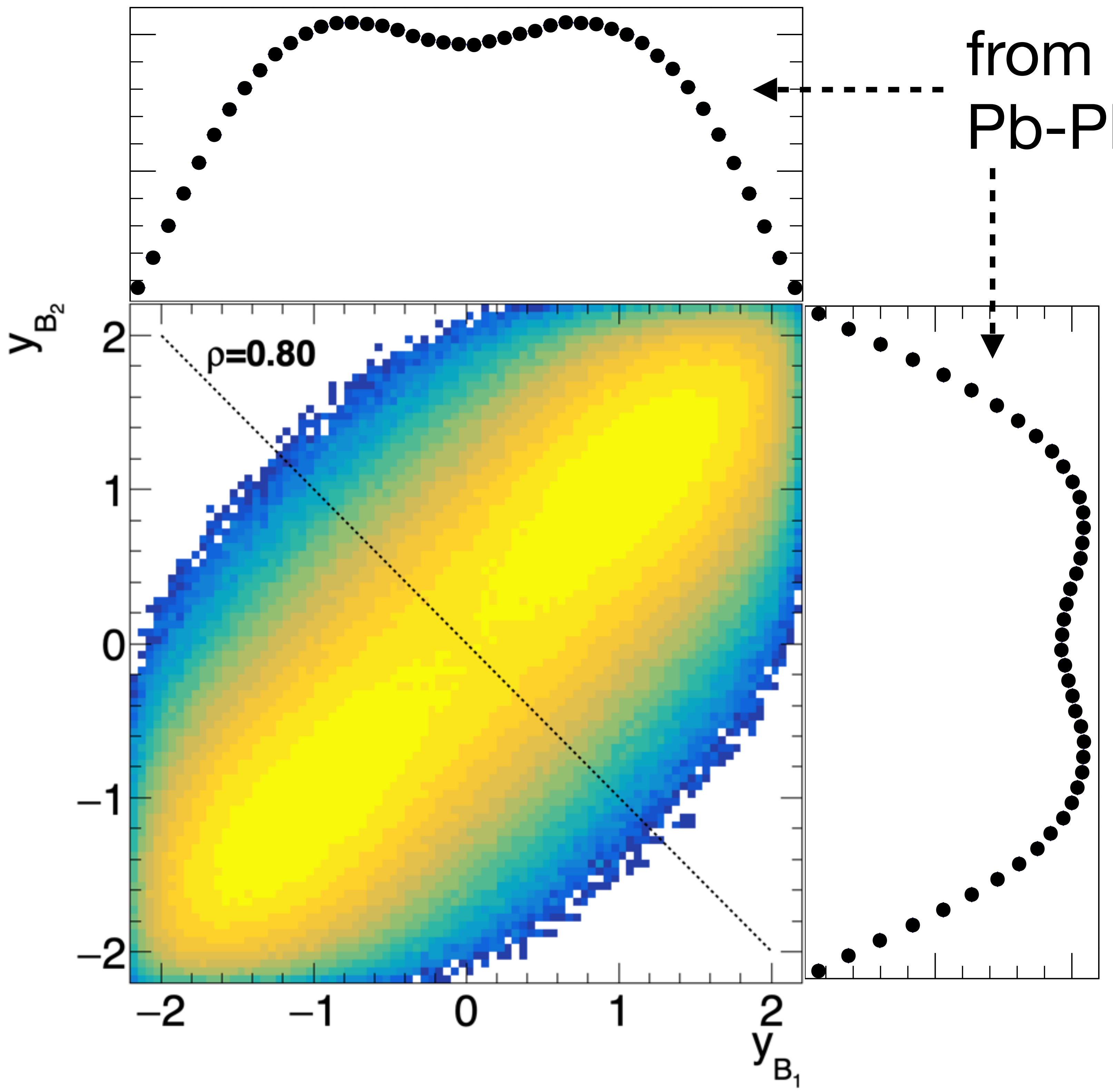
- do data imply multi-cluster formation?
- what is the mechanism behind?

The quest for proton clusters

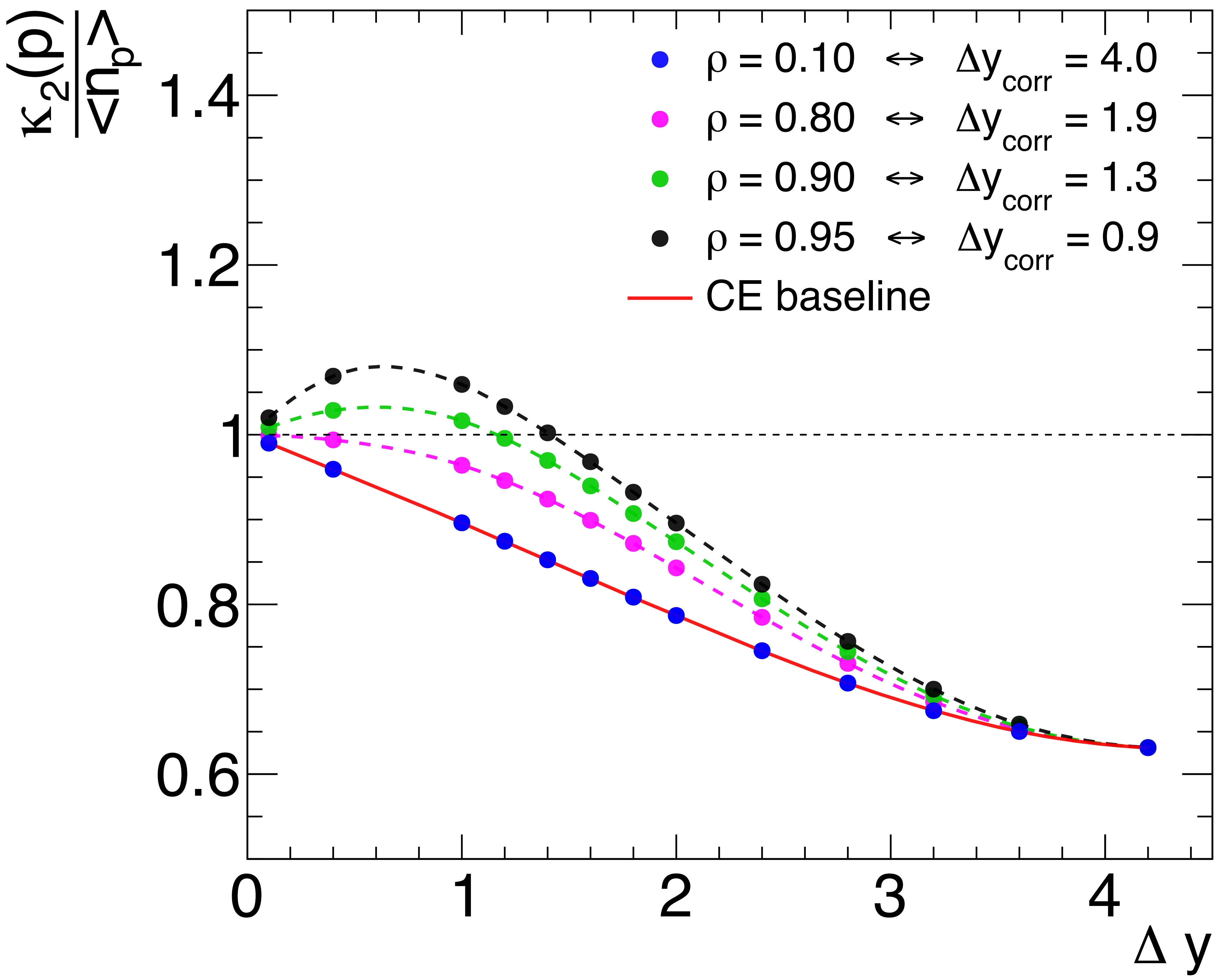
proton clusters and cumulants A. Bzdak, V. Koch, V. Skokov, Eur.Phys.J.C 77 (2017) 5, 288

canonical Ensemble + Metropolis algorithm AR., P. Braun-Munzinger, J. Stachel, QM 2022

introducing correlations between baryons



predictions for $\kappa_2(p)/\langle n_p \rangle$ at $\sqrt{s_{NN}}$ = 8.8 GeV



CE baseline: P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel, NPA 1008 (2021) 122141

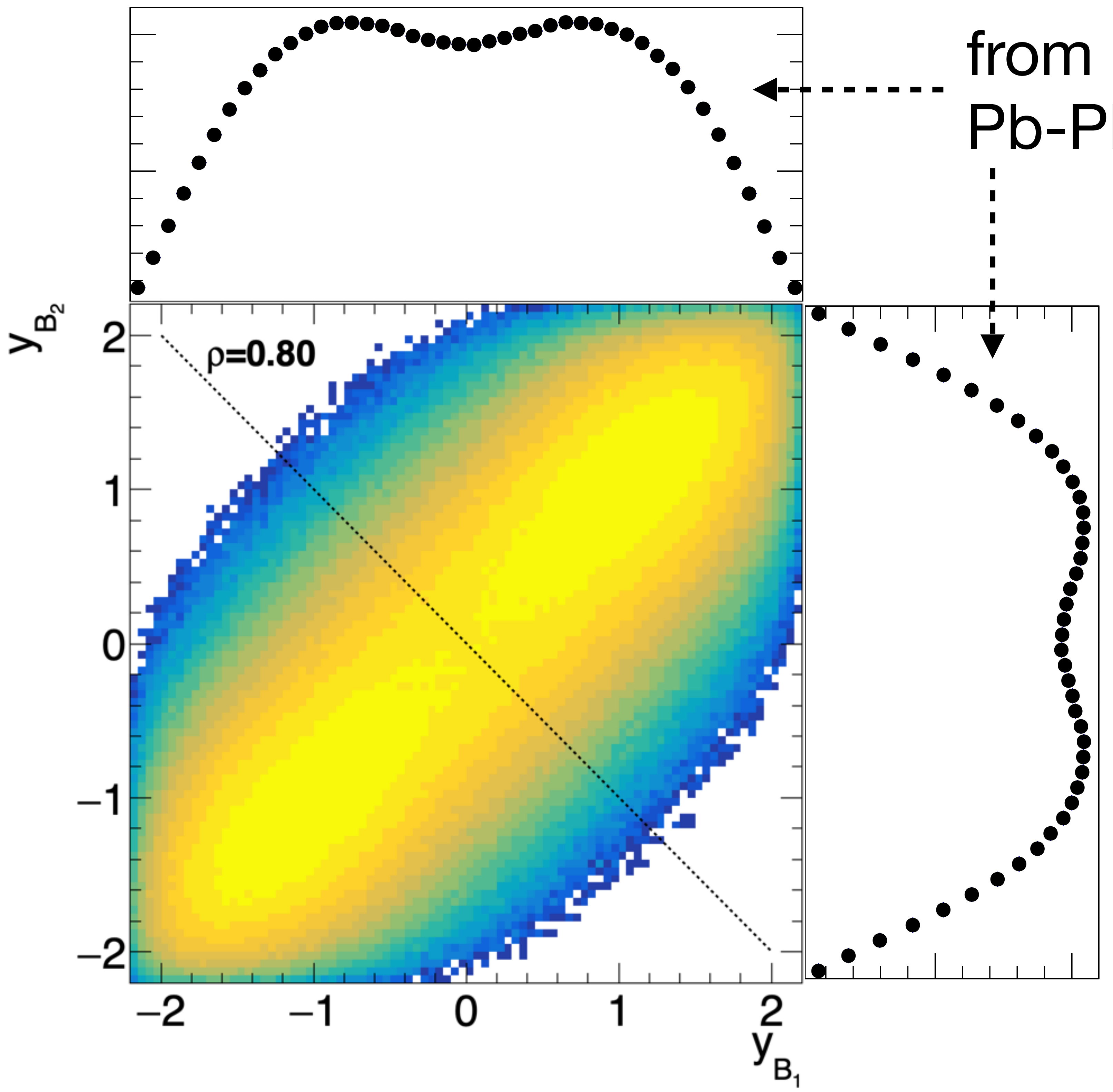
- for large values of ρ and small values of Δy it is more probable to treat protons **in pairs**
- this process increases the finally measured proton number fluctuations

The quest for proton clusters

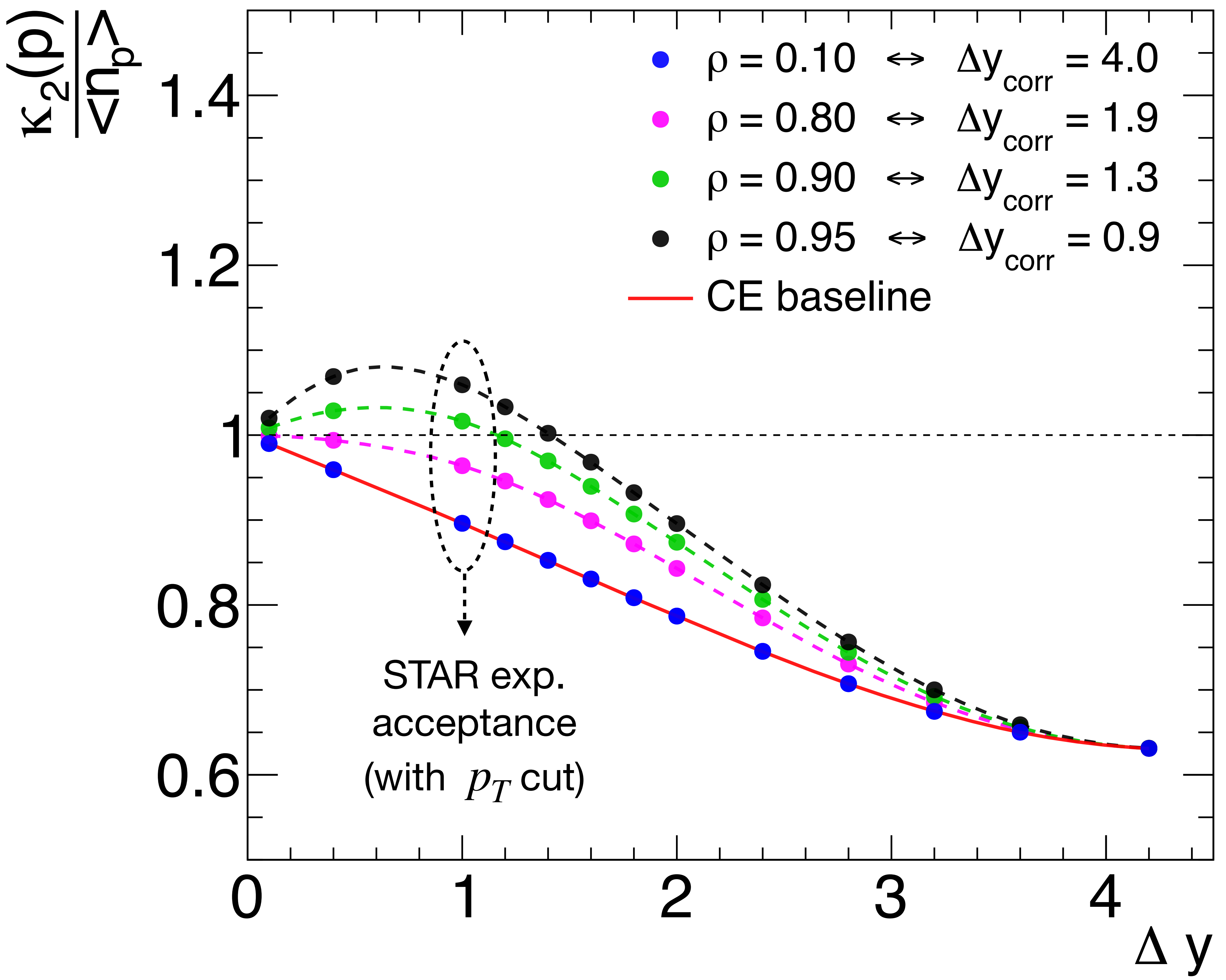
proton clusters and cumulants A. Bzdak, V. Koch, V. Skokov, Eur.Phys.J.C 77 (2017) 5, 288

canonical Ensemble + Metropolis algorithm AR., P. Braun-Munzinger, J. Stachel, QM 2022

introducing correlations between baryons



predictions for $\kappa_2(p)/\langle n_p \rangle$ at $\sqrt{s_{NN}}$ = 8.8 GeV

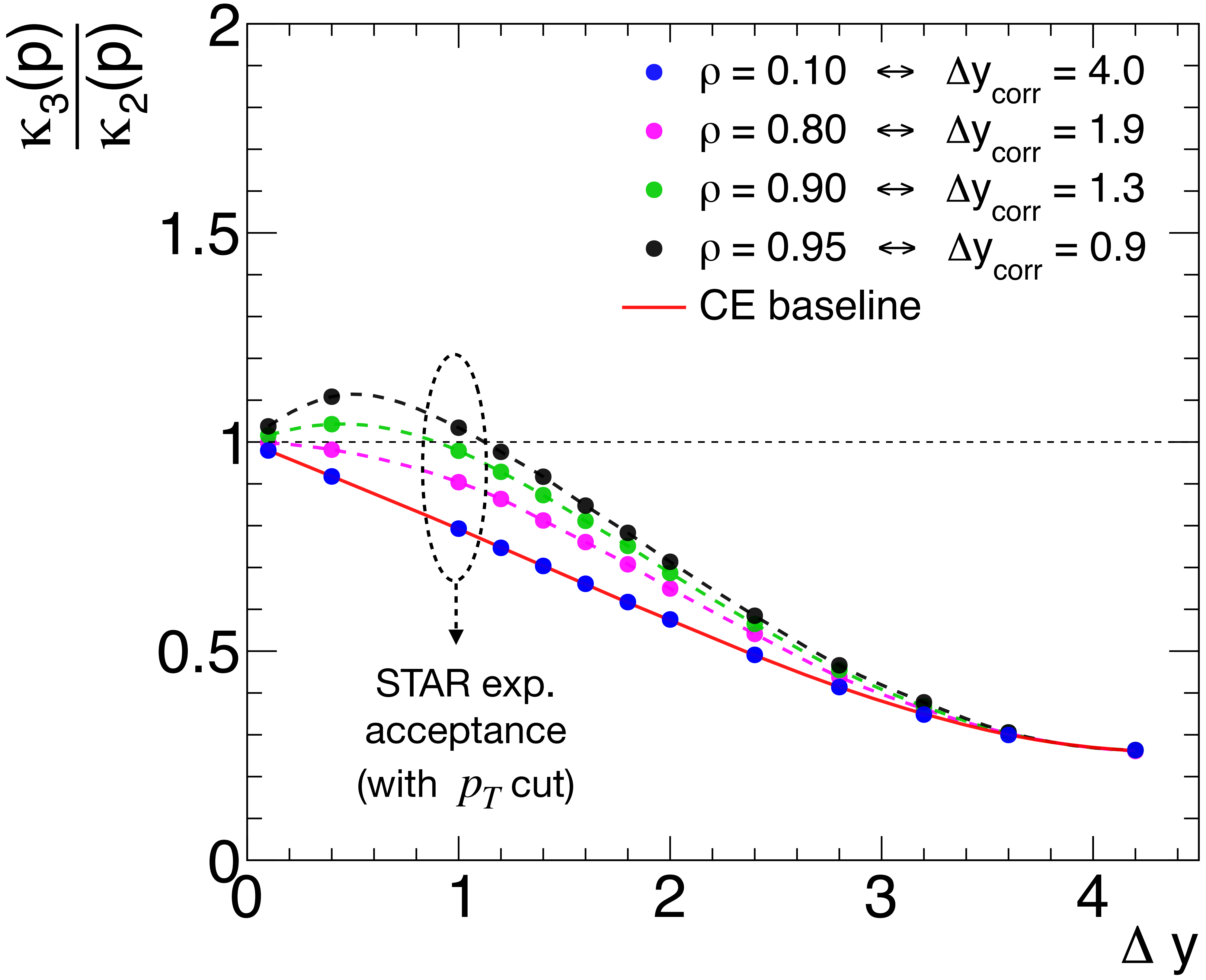


CE baseline: P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel, NPA 1008 (2021) 122141

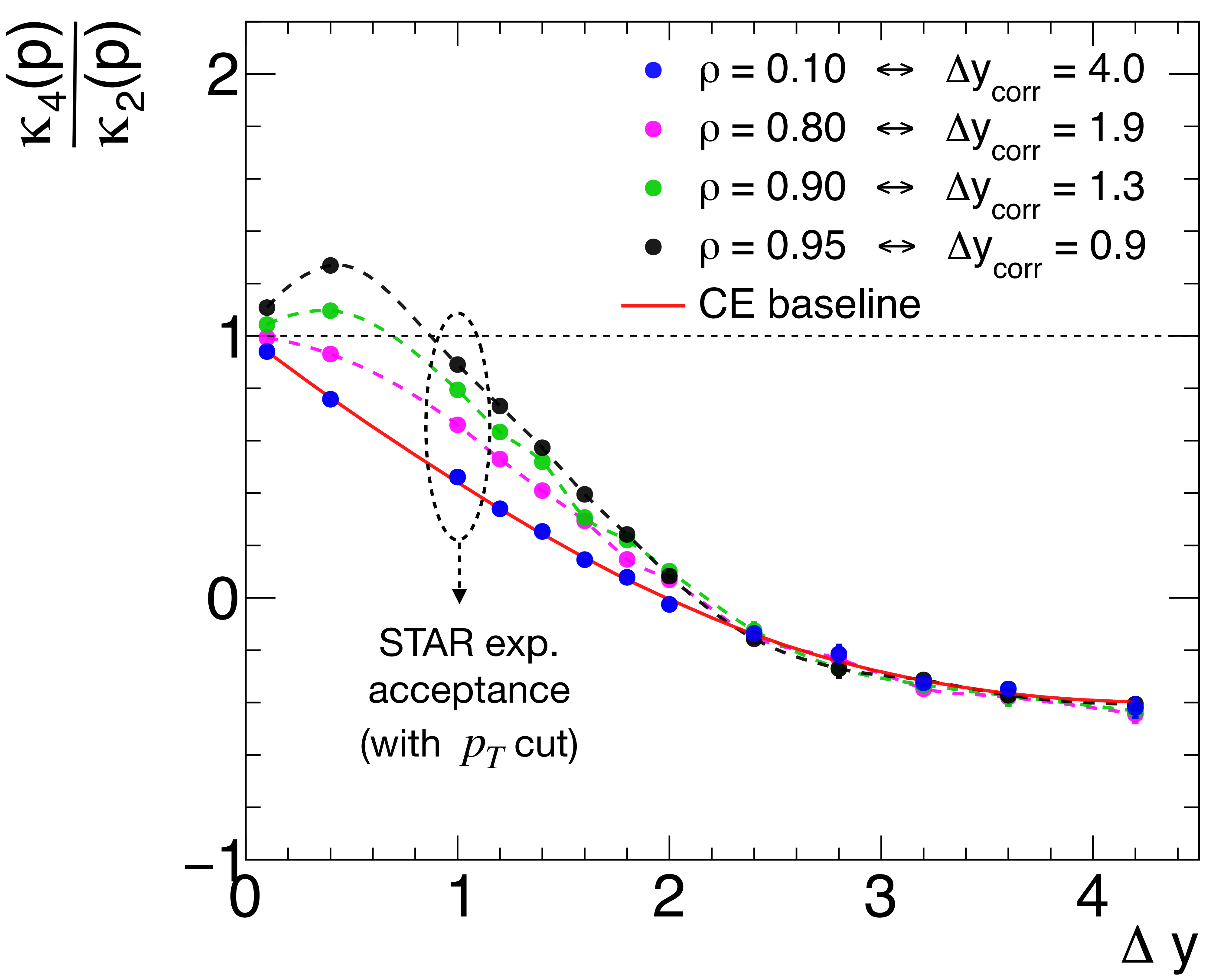
- for large values of ρ and small values of Δy it is more probable to treat protons **in pairs**
- this process increases the finally measured proton number fluctuations

The quest for proton clusters

predictions for $\kappa_3(p)/\kappa_2(p)$ at $\sqrt{s_{NN}}$ = 8.8 GeV



predictions for $\kappa_4(p)/\kappa_2(p)$ at $\sqrt{s_{NN}}$ = 8.8 GeV

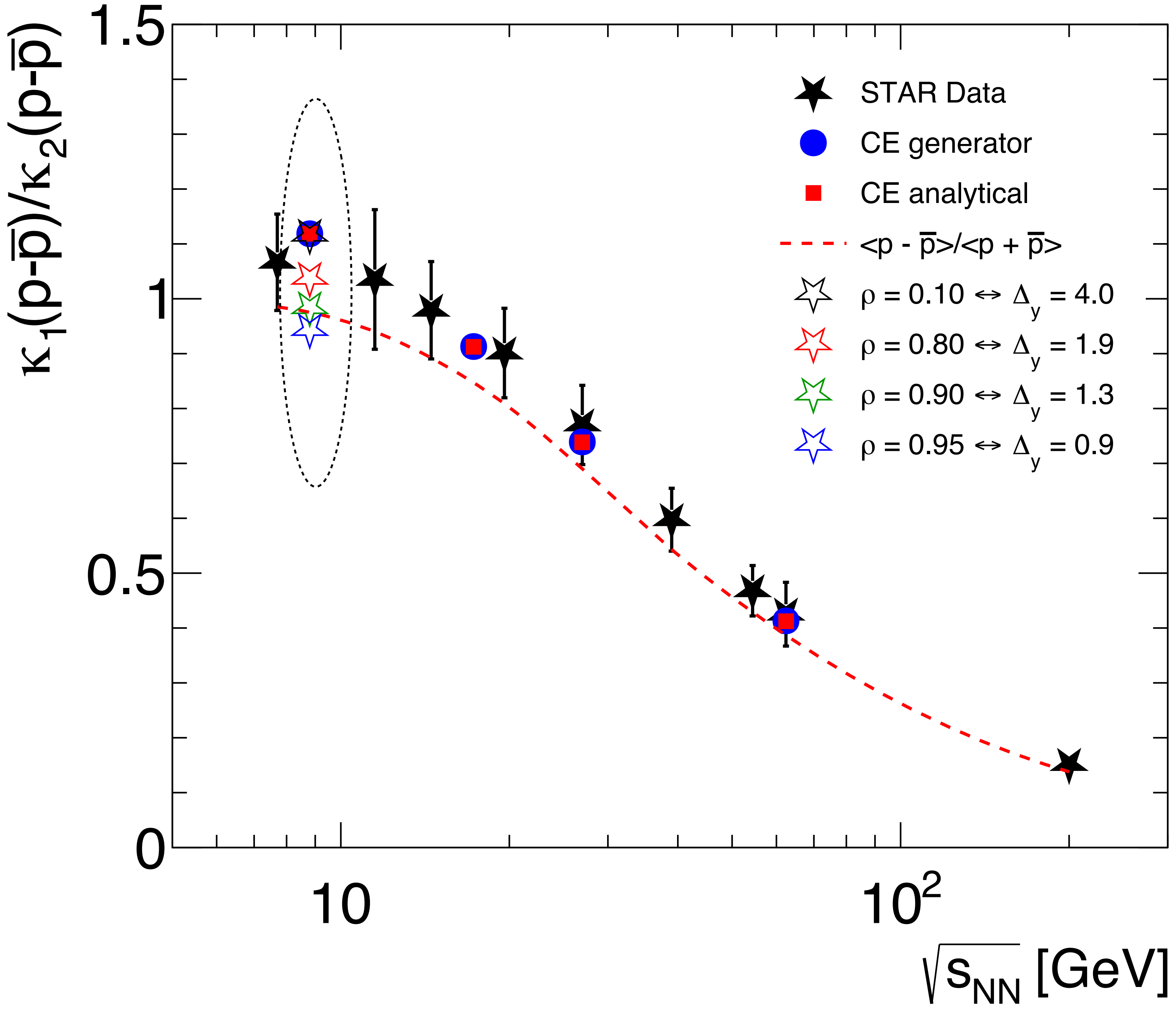


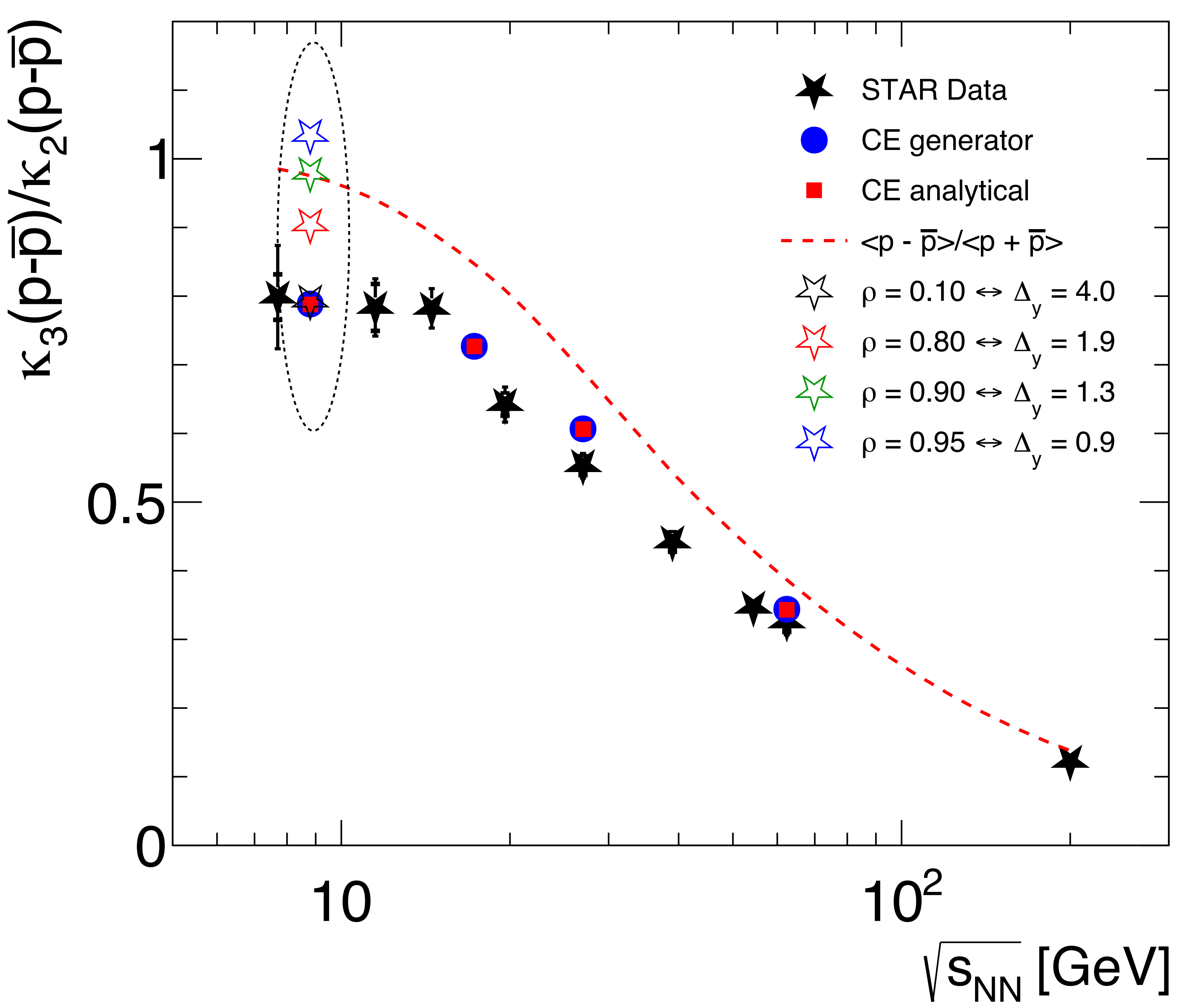
AR., P. Braun-Munzinger, J. Stachel , QM 2022

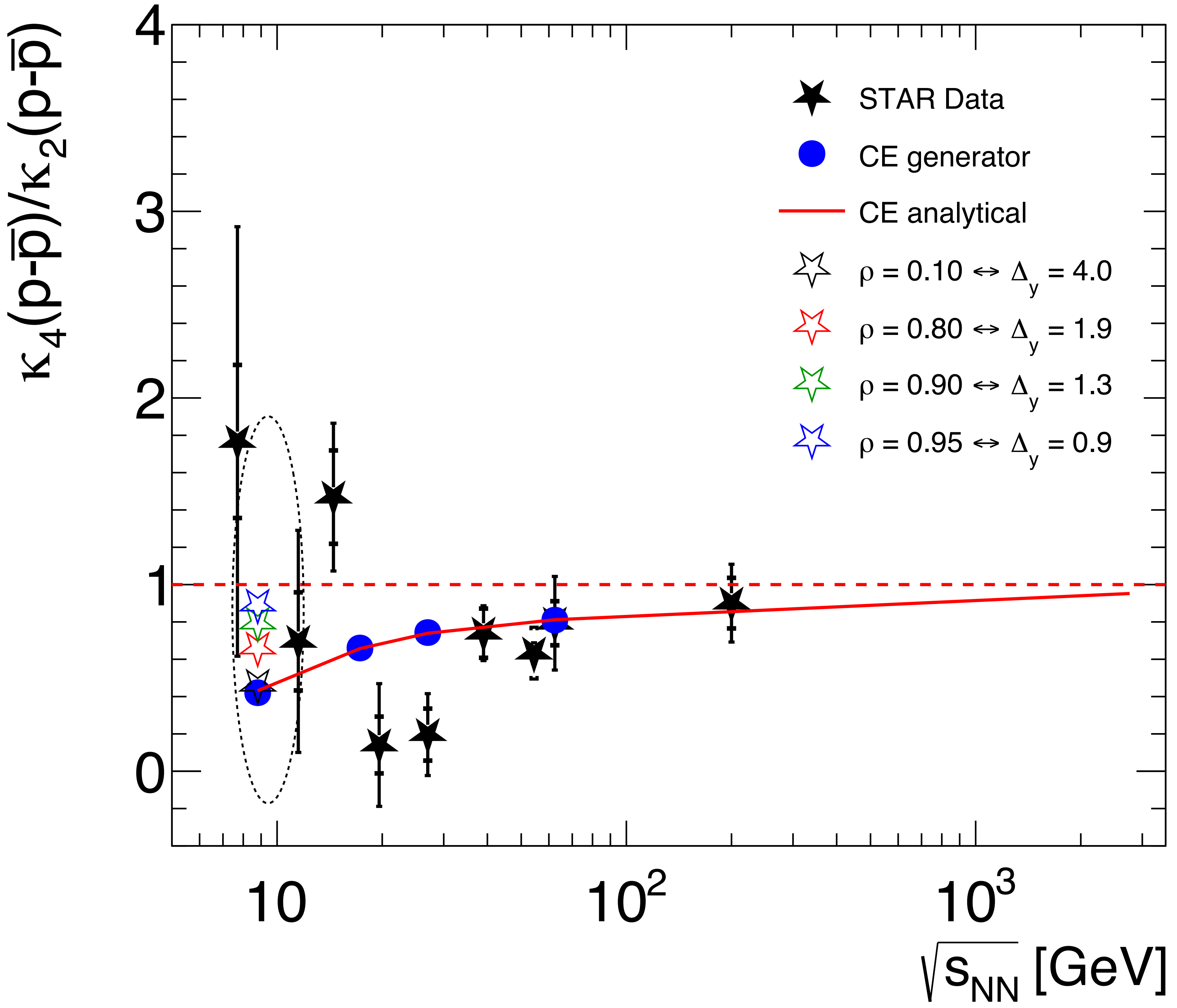
CE baseline: P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel , NPA 1008 (2021) 122141

correlated proton production enhances $\kappa_3(p)/\kappa_2(p)$ and $\kappa_4(p)/\kappa_2(p)$ wrt CE baseline

Comparison to STAR data





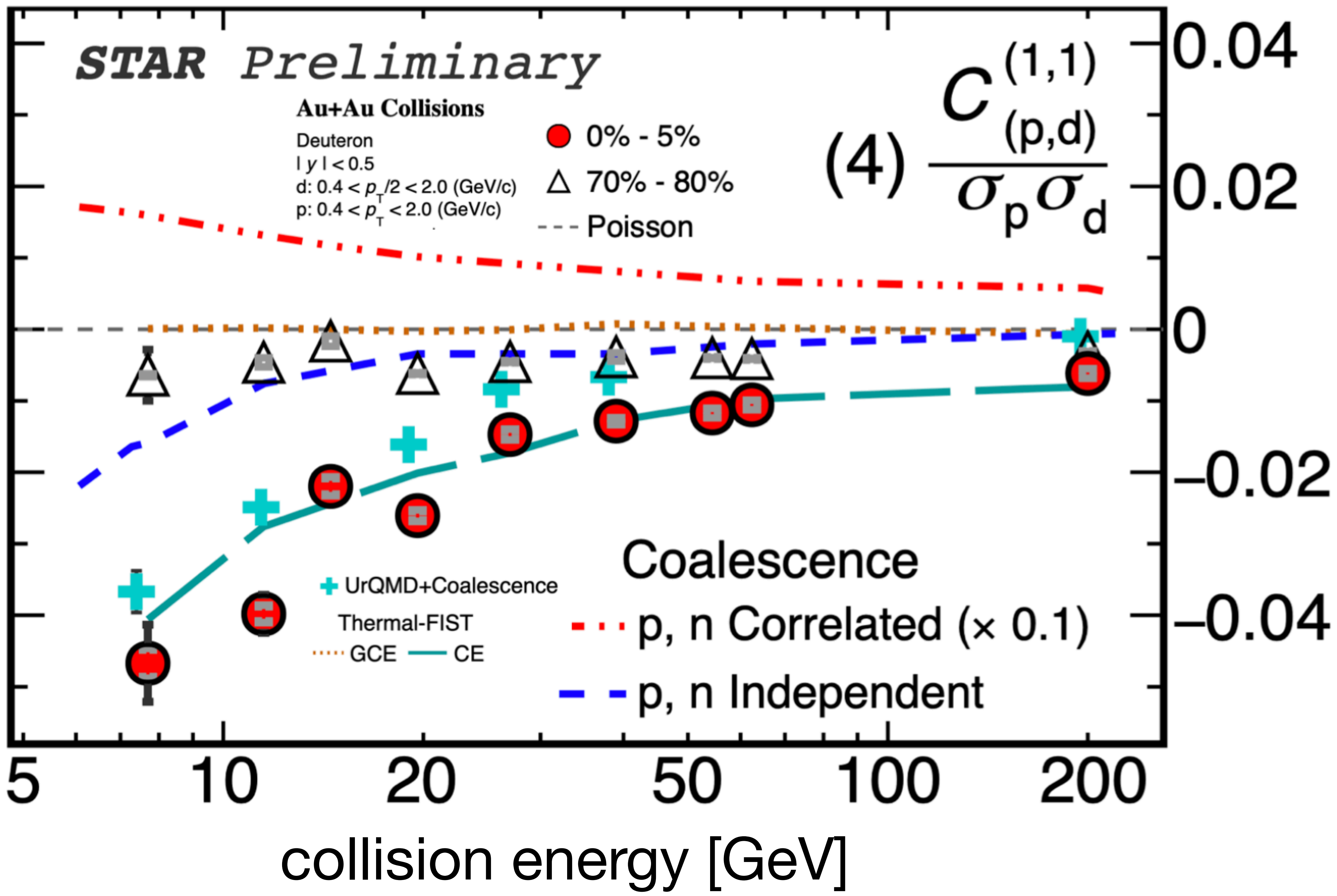


STAR: Phys.Rev.Lett. 126 (2021) 9, 092301
AR., P. Braun-Munzinger, J. Stachel , QM 2022
CE Baseline: P. Braun-Munzinger, B. Friman, K. Redlich, AR, J. Stachel, NPA 1008 (2021) 122141

- the STAR data is best described with the long range correlations ($\rho = 0.1$) (no clustering)
- the precision of the data however, does not exclude the scenario with $\rho = 0.8$
- at the current precision of the data there is no evidence for critical behaviour!

additional ideas

(anti)proton-(anti)deuteron correlations



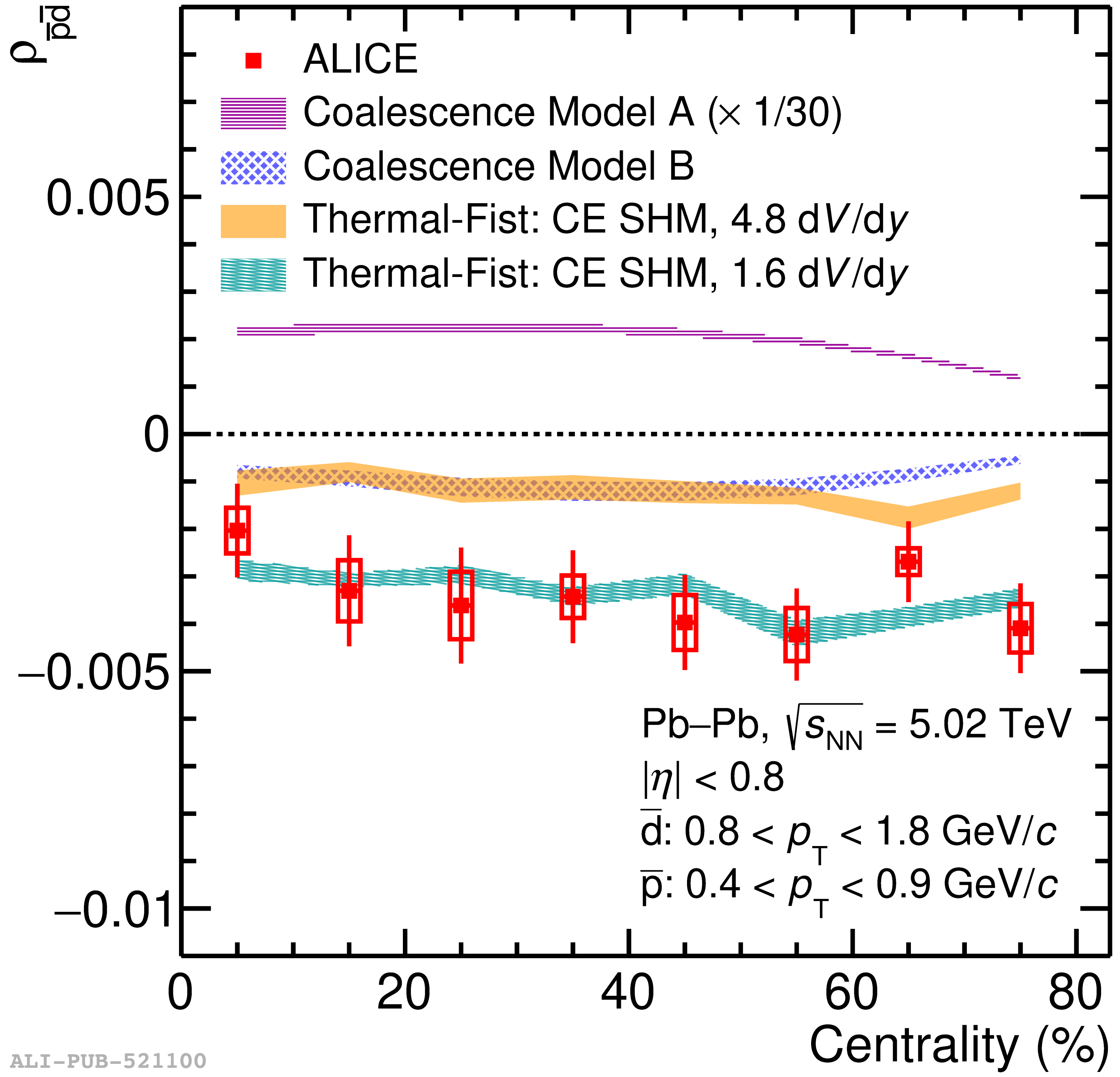
Debasish Mallick, QM22

$$\frac{\langle (n_a - \langle n_a \rangle) \rangle \langle (n_b - \langle n_b \rangle) \rangle}{\sigma_a \sigma_b}$$

Toy Coalescence **Model A:**
statistically from correlated p, n

Toy Coalescence **Model B:**
statistically from uncorrected n, p

Zuzana. Feckovà et al., Phys.Rev.C 93 (2016) 5, 054906



ALICE: arXiv:2204.10166

STAR

- Baryon number conservation reproduces the data
- Both, model A and model B, fail to follow the data
- UrQMD+phase space coalescence model is consistent with the data

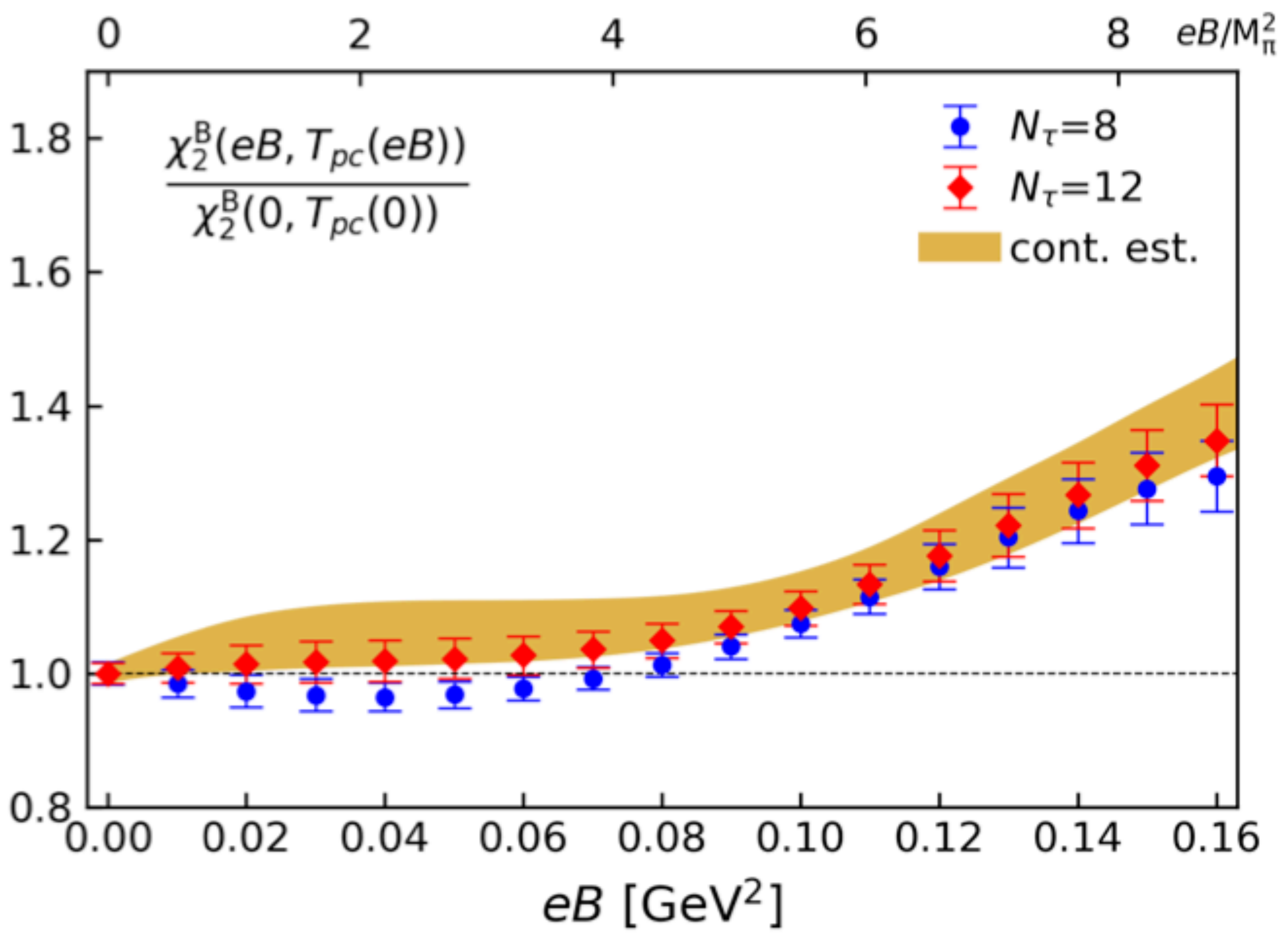
14 June, poster session, Debasish Mallick

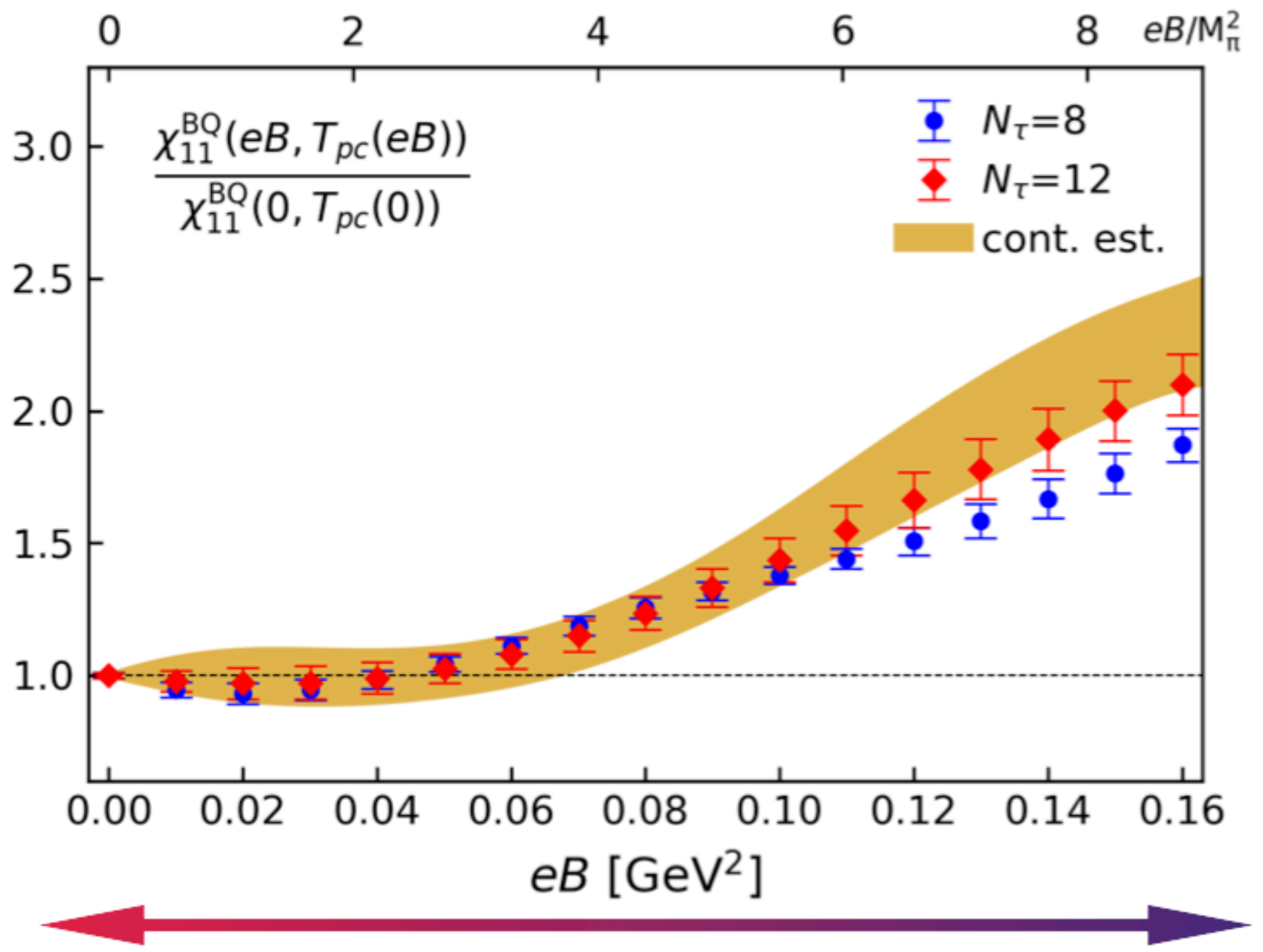
ALICE

- Baryon number conservation reproduces the data
- Model A does not fit the data

14 June, 10:50, Mario Ciacco

Cumulants as probes of magnetic field and missing states





$$\hat{\chi}_2^{X=B,Q,S} = \frac{1}{VT^3}\frac{\partial^2 lnZ(V,T,\mu_{B,Q,S})}{\partial\left(\mu_X/T\right)^2}$$

$$\langle\left(N_X - \langle N_X\rangle\right)^2\rangle$$

$$\hat{\chi}_1^{XY} = \frac{1}{VT^3}\frac{\partial^2 lnZ(V,T,\mu_{B,Q,S})}{\partial\left(\mu_X/T\right)\partial\left(\mu_Y/T\right)}$$

$$\langle\left(N_X - \langle N_X\rangle\right)\left(N_Y - \langle N_Y\rangle\right)\rangle$$

- second order fluctuations/correlations of conserved charges are strongly deteriorated by the presence of magnetic field
- experimentally measurable to probe the magnetic field in nuclear collisions

H.-T. Ding, S.-T. Li, Q. Shi, X.-D. Wang, arXiv: 2104.06843
Jun-Hong Liu, QM2022

14 June, poster session, Jun-Hong Liu

- cumulants, involving strangeness and charm, are sensitive probes of missing strange/charmed hadrons

A. Bazavov et al., Phys. Lett. B 737 (2014) 210–215, arXiv:1404.4043 [hep-lat]
J. Goswami et al., Acta Phys. Polon. Supp. 14 (2021) 251, arXiv:2011.02812 [hep-lat]

Conclusions

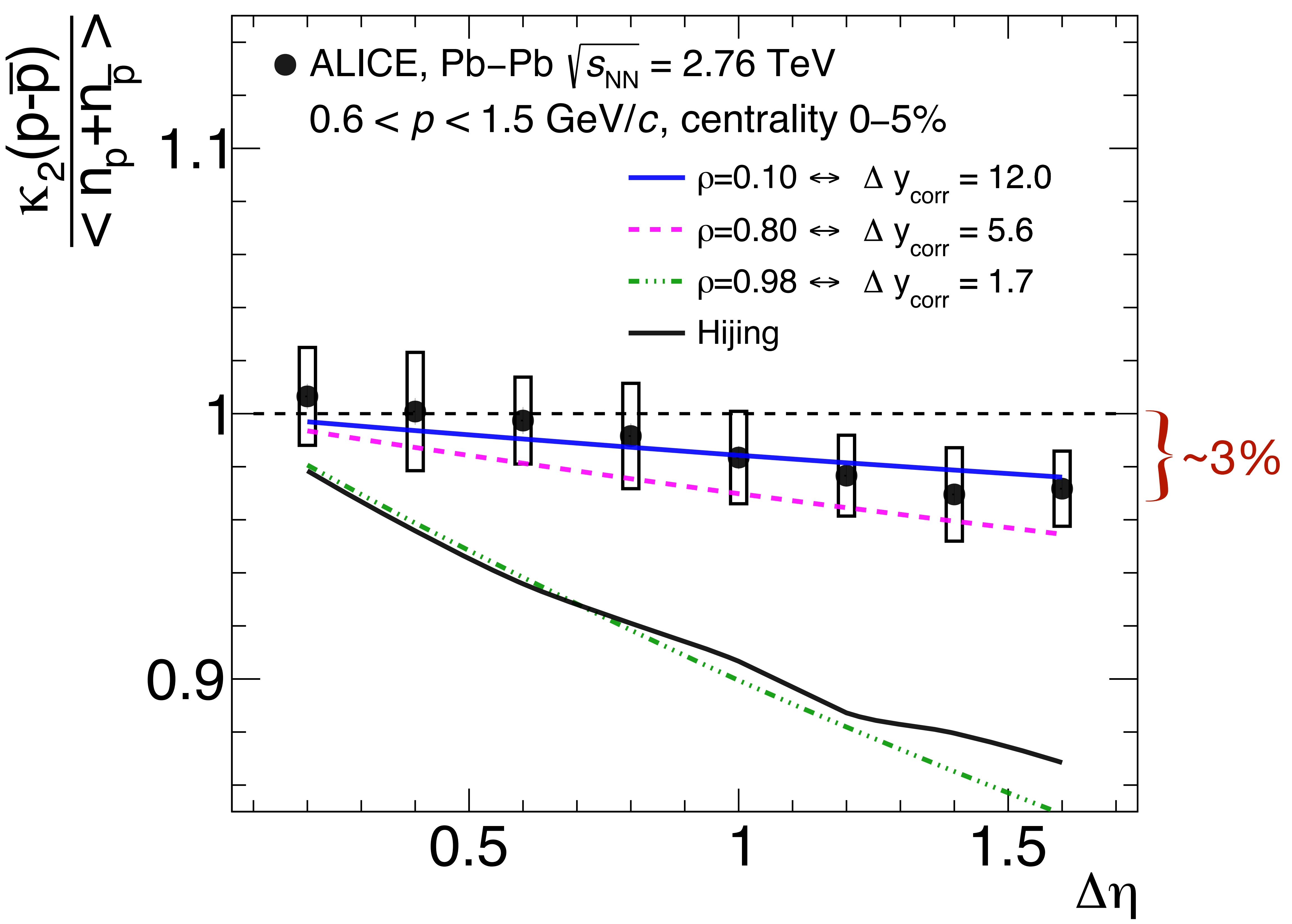
- The ALICE data exclude short range $B\bar{B}$ correlations
    - The data are best described with the correlation coefficient ρ = 0.1 $\leftrightarrow$ Δy_{corr} = 12
    - This behaviour is at odds with the Lund String Fragmentation model for baryon production
- The HADES data on integrated correlation functions show non-linear dependences on acceptance
    - Indication of multi-proton clustering?
    - The STAR data at 7.7 GeV show no strong evidence for clustering
- The energy excitation function of κ_4/κ_2 of (net-)protons, within the experimental uncertainties, is consistent with the non-critical baseline (ideal gas + canonical Ensemble)
- The measured trend of κ_6/κ_2 from STAR at 200 GeV is consistent with the LQCD predictions for cross-over, however this is not the case for 54.4 GeV data
- The intermittency analysis from NA61/SHINE does not exhibit the searched for power-law behaviour for critical point

the current and near future data from ALICE, STAR, HADES/CBM and NA61/SHINE will be a game changer


THANKS FOR YOUR
ATTENTION

Baryon production in string models

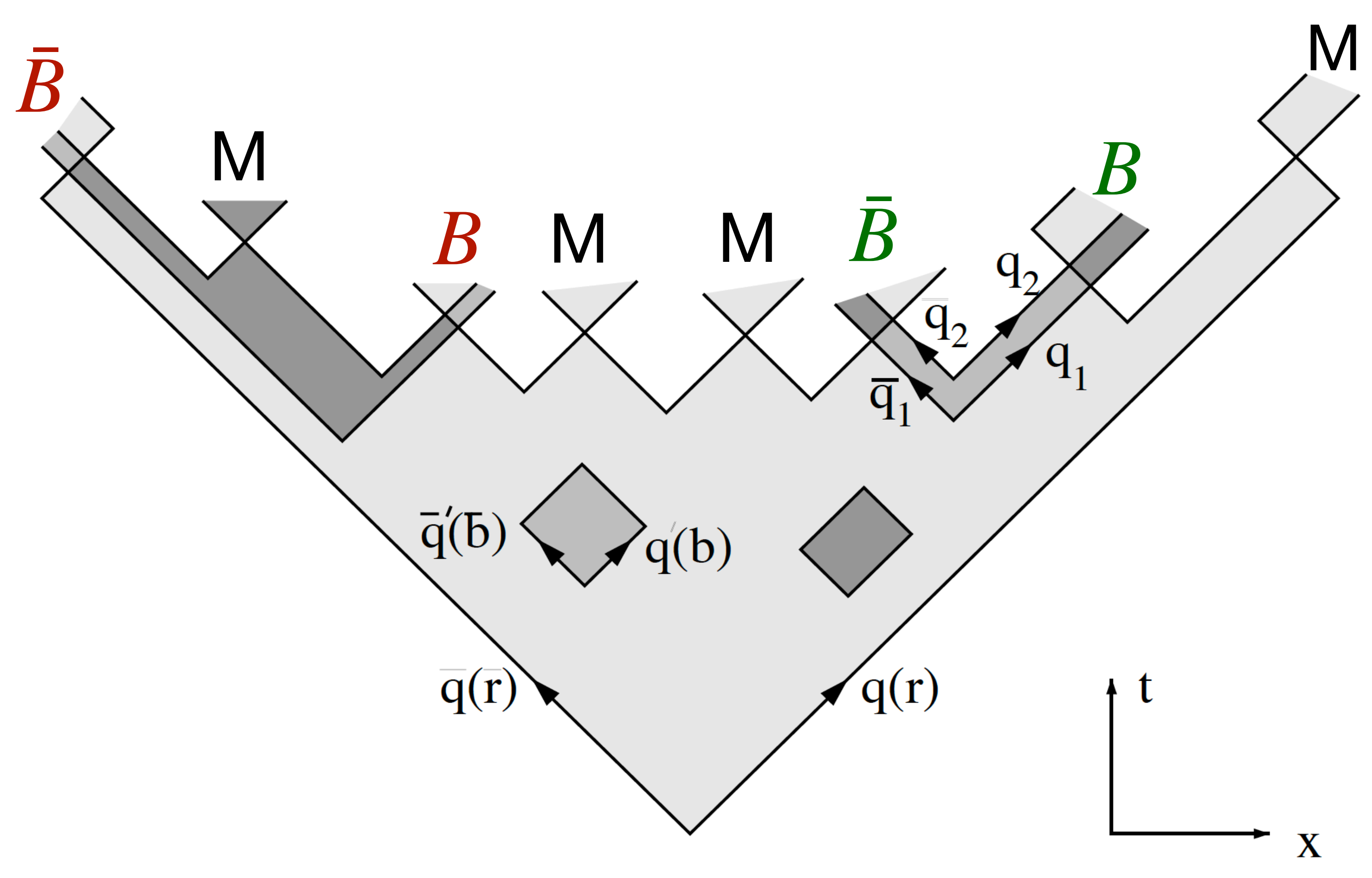
AR., NPA 967 (2017) 453-456   **ALICE:** Phys. Lett. B 807 (2020) 135564



Hjing (Lund String Fragmentation) results are in conflict with the ALICE data

Lund String Fragmentation

baryon production
$q\bar{q}$ pair is replaced by qq-$\bar{q}\bar{q}$ pair



- diquark-antidiquark
- popcorn mechanism

induces short range correlations in rapidity space

B. Andersson, G. Gustafson, G. Ingelman, T. Sjostrand Phys.Rept. 97 (1983) 31-145

$\kappa_2(p - \bar{p})$ measurements are essential to constrain baryon production mechanisms