Theory Summary Lecture
Strangeness in Quark Matter 2022

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SQM2022

Pusan (Korea)

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Theory Overview (Kapusta)

A large array of topics and open questions.

- What's most important?
- What are the ultimate goals?
- Which new insights did SQM2022 bring?
- What are the most promising directions for research?

Disclaimer:

This Summary will not be comprehensive

My sincere apologies to all whose beautiful work is not mentioned

Equation of State

- Is there a critical point in the phase diagram? Many models suggest so. Where is the critical curve?
- If a critical point exists it would be in the same universality class as the 3D Ising model and liquid-gas transitions. How would it be related to a chiral symmetry restoring phase transition, which is in a different universality class, if the up and down quarks were massless?
- To describe the matter created at RHIC beam energies and below, knowledge is required of the equation of state as a function of T, μ_B, μ_Q, and μ_S to conserve energy, baryon number, electric charge, and strangeness. This is nontrivial when there is critical behavior in the phase diagram.
- Such an equation of state is also needed for modeling neutron star mergers and closely related to the cold dense matter comprising neutron stars.
- Will hard working lattice practioners ever be able to provide the answers?

Simulations are the Key to Comparison between Theory and Experiment

- Analytic models were very useful at the beginning of the field. Numerical simulations are now essential to make contact between theory and experiment.
- Fluctuations are typically computed in coordinate space but measured experimentally in momentum space. Dynamical simulations are necessary to compare them.
- Photon and dilepton emission are computed theoretically as functions of the thermodynamic variables. They must be folded with the space-time evolution of the matter.
- Jet and charm evolution are computed theoretically as functions of the thermodynamic variables. They must be folded with the space-time evolution of the matter. Feedback on the matter should be taken into account.

Hydrodynamics and Transport Theory

- Perfect fluid dynamics is not realistic. System is out of equilibrium. How to define flow velocity? Via energy flow, baryon flow, electric charge flow, or something else? Not an issue for transport models.
- Is the standard model of viscous hydrodynamics coupled with an hadronic afterburner the correct one for lower beam energies?
- What is the initial condition as it depends on beam energy?
- Transport theory with hadrons and mean fields is undoubtedly a better description at lower beam energies. Better able to handle longer nuclear transit times.
- How and when to transition from transport theory to hydrodynamics as a function of beam energy, impact parameter, or even at a fixed impact parameter? When does the assumption of (approximate) local equilibrium become reasonable?

Strangeness

- How does strangeness first appear? As strange quarks and anti-quarks or as strange mesons and baryons? How does it depend on beam energy or impact parameter or even as a function of transverse position in a given collision?
- The differences between Λ and $\bar{\Lambda}$ and between K^+ and K^- are very small at the LHC but become increasingly large as the energy decreases. What interesting physics can we infer?
- The connection between polarized Λ and $\bar{\Lambda}$ baryons, aligned K^{*0} mesons, and vorticity of the matter is extremely interesting. Current models assume their spins are equilibrized but is that correct?
- How does strangeness evolve? Is the strange current directly proportional to the baryon current? If the quarks are polarized, how do they pass that on to the strange mesons and baryons?

Charm

- Charm production in p+p and p+Pb at the LHC is not well reproduced by fragmentation in PYTHIA, which is based on e^+e^- collisions. Some physics is missing.
- The charm diffusion coefficient is calculated as a function of T on the lattice and detailed modeling can connect it to experimental data. What about the chemical potential dependence at lower beam energies?
- Many new exotic charmed baryons and mesons have been detected at the LHC. Are they tetraquark and pentaquark states or are they molecular states of two hadrons?
- Do charmed nuclei exist?
- K^{*0} vector mesons in Pb+Pb collisions at the LHC are aligned but J/ψ mesons are not. What dynamics is involved?

Fluctuations

- Initial state fluctuations from randomly chosen locations of nucleons.
- Initial state fluctuations from randomly chosen locations of quarks within nucleons.
- Initial state fluctuations of color gluon field at very high energy.
- Hydrodynamic fluctuations are intimately connected to transport coefficients due to the fluctuation-dissipation theorem. Inclusion of viscosity and heat conductivity requires them for consistency.
- Hydrodynamic fluctuations in the evolving matter is challenging to implement numerically in 3D.

Correlations

- Hanbury-Brown and Twiss intensity interferometry provides information on the space-time evolution of the matter. Correlations are readily inferred from experiment, but a rigorous derivation for heavy ion collisions is subtle and lacking.
- Charge balance functions can provide information on the charge diffusivity, but extraction from data requires detailed knowledge and modeling of the space-time evolution of the matter.
- How can we understand theoretically the transition in dynamics from small to medium to large collision systems as defined by size of the beam and target or by multiplicity?
- Can Disoriented Chiral Condensates (DCC) be revived at the LHC?

Critical Point and First Order Phase Transition Dynamics

- One may need to be within 10^{-2} or 10^{-3} of T_c or even less to see the true critical exponents. The distance is not universal. Beyond that one typically measures mean field exponents. Quite a challenge for heavy ion collisions.
- Crossing a line of first order phase transition in a heavy ion collision ought to be more dramatic than passing near a critical point.
- It is uncertain how hydrodynamic models can deal with a first order phase transition. Nucleation is the statistical formation of a critical size bubble or droplet of the other phase in the metastable region, but the critical size may be too large for relevance in a heavy ion collision or may not happen fast enough.
- It is uncertain how hydrodynamic models can deal with a first order phase transition. Spinodal decomposition occurs when the system crosses into the unstable region where $c_T^2 < 0$. Subsequent evolution depends on pre-existing fluctuations and inhomogeneities.
- Transport theories can use mean fields to deal with a two phase system dynamically. But, fundamentally, does one use hadron or quark and gluon degrees of freedom? No matter whether a smooth crossover or a critical point, the issue remains.

A Sampling of Publicly Available Codes

- Initial state: Glauber, LEXUS, IP-Glasma, Trento
- Relativistic hydrodynamics: MUSIC, iEBE-MUSIC, iEBE-VISHNU, SONIC & superSONIC, SPheRIO, CLVISC, vHLLE, ECHO-QGP
- Transport theory: UrQMD, ZPC, AMPT, SMASH
- Particlization of hydrodynamics: iSS, Microcanonical Cooper-Frye

The Big Picture

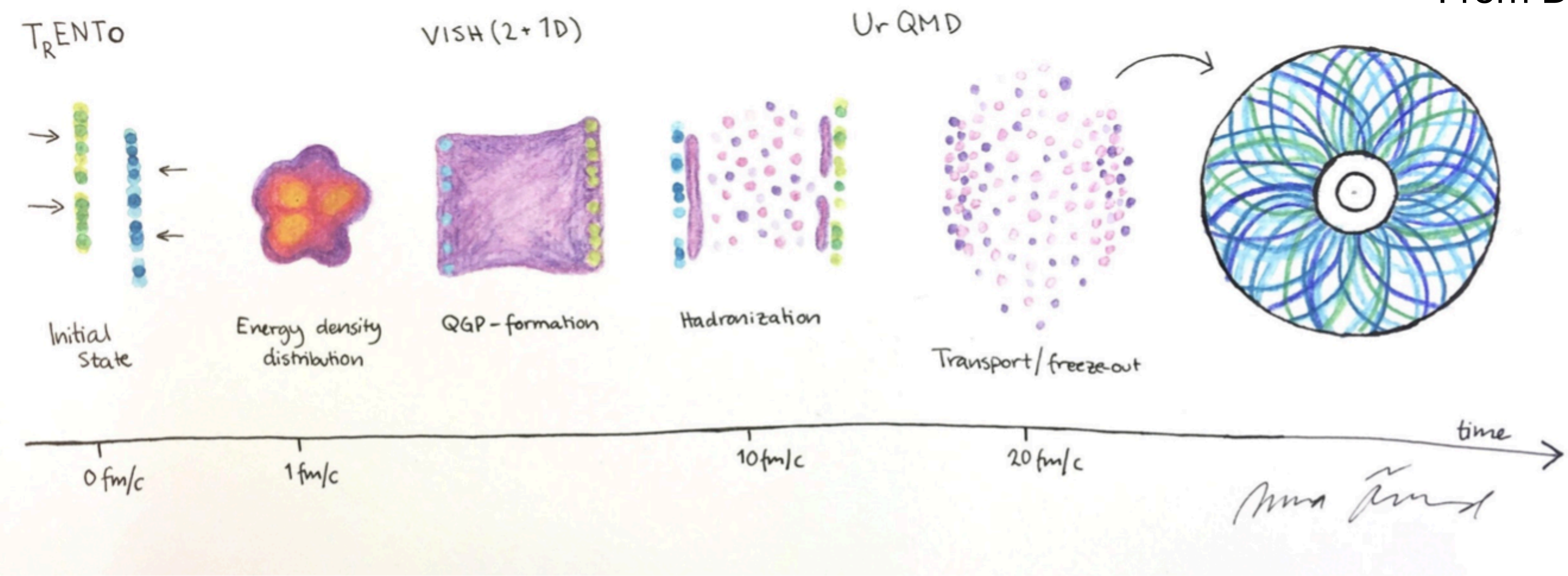
- Remarkable progress continues to be made
- Unexpected new insights continue to be generated
- The topics of “SQM” have expanded beyond what was imaginable 40 years ago
- Huge amount of data of a wide variety is both, a benefit and a curse
- The community needs to stay focused on its core goals
- The top-level picture:
  - We have an accepted “standard model” for the central rapidity region in collisions at $\sqrt{s_{\mathrm{NN}}} > 100$ GeV
  - We **don’t** have a widely accepted and data tested model for collisions at $\sqrt{s_{\mathrm{NN}}} < 20$ GeV
  - Theoretical tools for the baryon-rich region remain imprecise
  - Hadronization / Particle emission remains poorly understood
  - Probes of chiral symmetry restoration remain at best tenuous

Outline

- Flaws in the “standard model”
- Core-corona models
- BES energies
- Critical fluctuations
- Anomalous chiral phenomena
- Hadron production
- Quarkonium
- Strangeness
- Bayesian analysis

The Standard Model

From D.J. Kim's talk



Credits to Anna Onnerstad

$$T^{\mu\nu} = eu^{\mu}u^{\nu} - (P + \Pi)\Delta_{\mu\nu} + \pi^{\mu\nu}, \qquad \delta_{\mu}T^{\mu\nu} = 0$$

Model framework relies on initial energy deposition to be treatable like a sudden "quench"

This requires that both nuclei are Lorentz contracted to << 1 fm.

New challenges for the SM

Plumberg: 2^{nd} order “causal” viscous hydrodynamics models are **not causal**, even in Pb+Pb

→ Causality implies $0 \leq v^2 \leq c^2$, so evolution equations **must**:

(i) be hyperbolic $(v^2 \geq 0)$

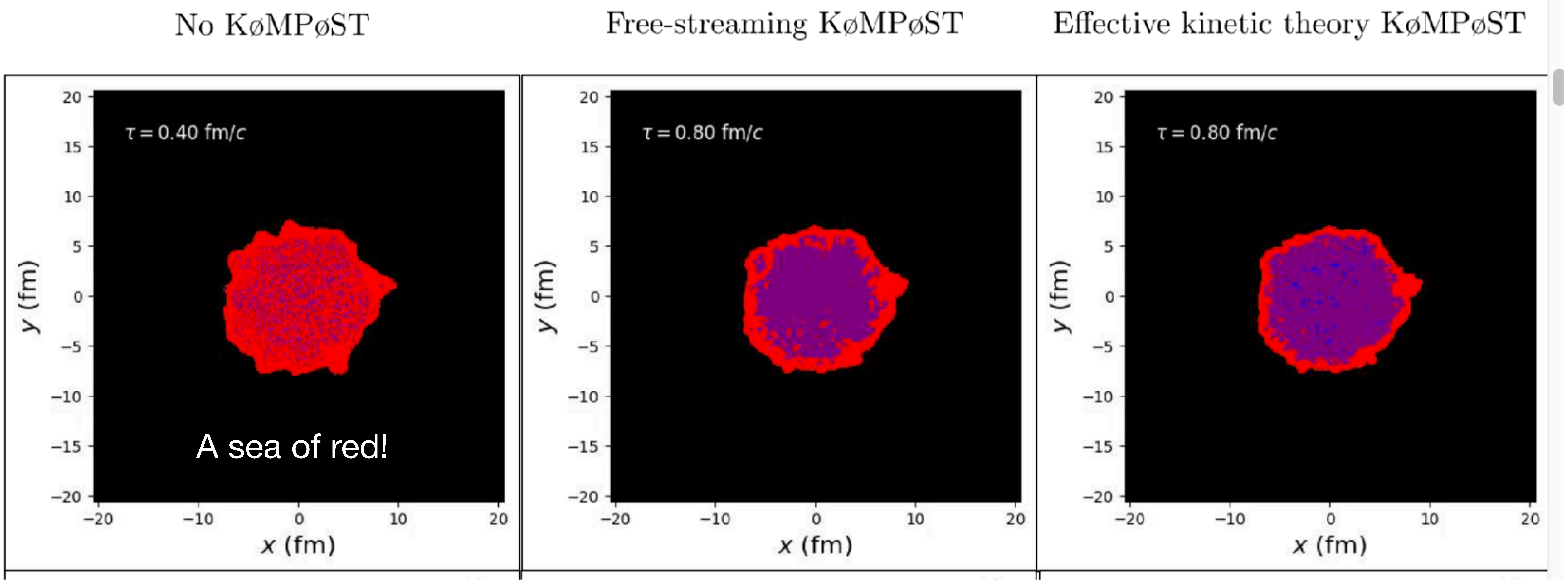
(ii) have no superluminal propagation $(v^2 \leq c^2)$

→ Causality conditions (6 necessary and 8 sufficient) can be checked **for each fluid cell**

1. **Blue** - cells where all sufficient conditions are met (definitely causal)
2. **Purple** - cells where not all sufficient conditions are met but all necessary conditions are met (maybe causal or acausal)
3. **Red** - cells where one or more necessary conditions are violated (definitely acausal)

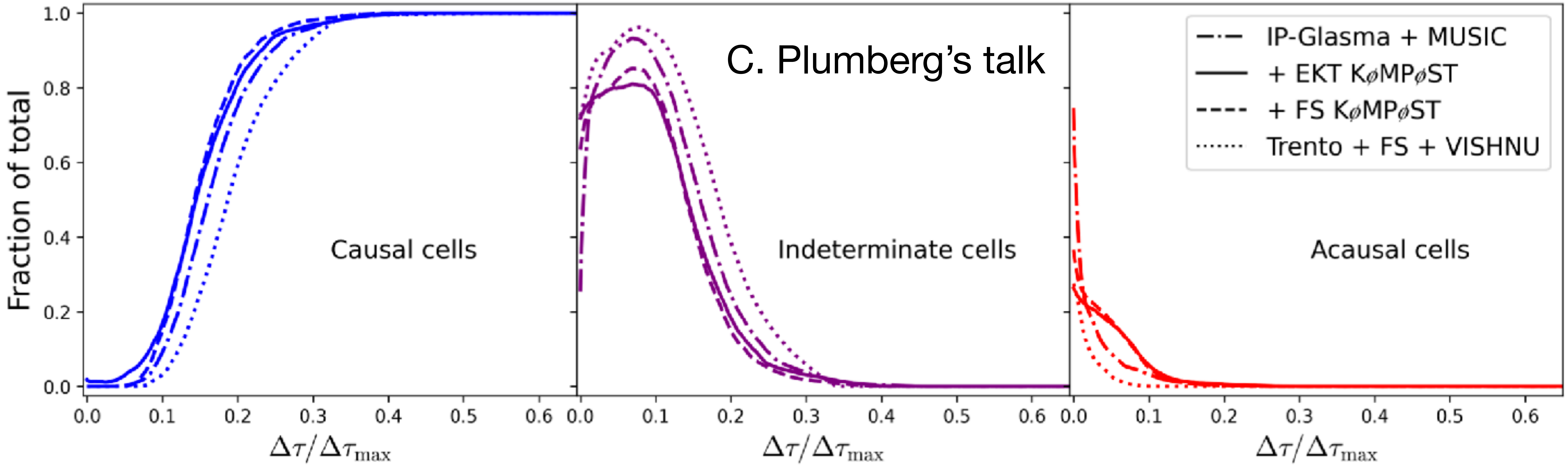
Results for Pb+Pb at 2.76 TeV

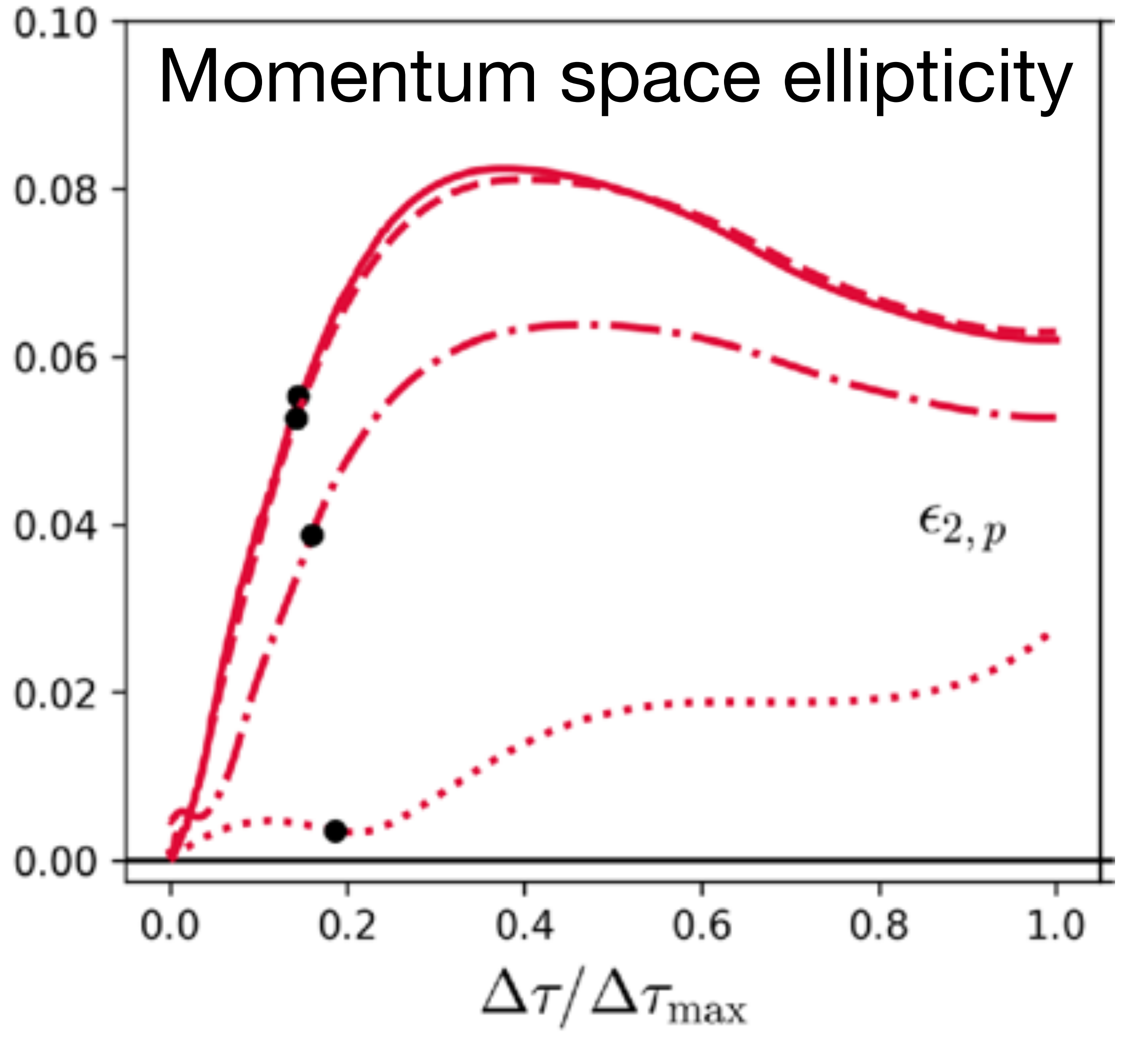
Model implementation: IP-Glasma + MUSIC with and w/o quasiparticle evolution before hydrodynamics



Origin of causality violation: Reynolds number is too small = fluid gradients (in some d.o.f.) are too large

How bad is it?





A large fraction of our most valued observables is generated when the hydro evolution contains cells with questionable causality

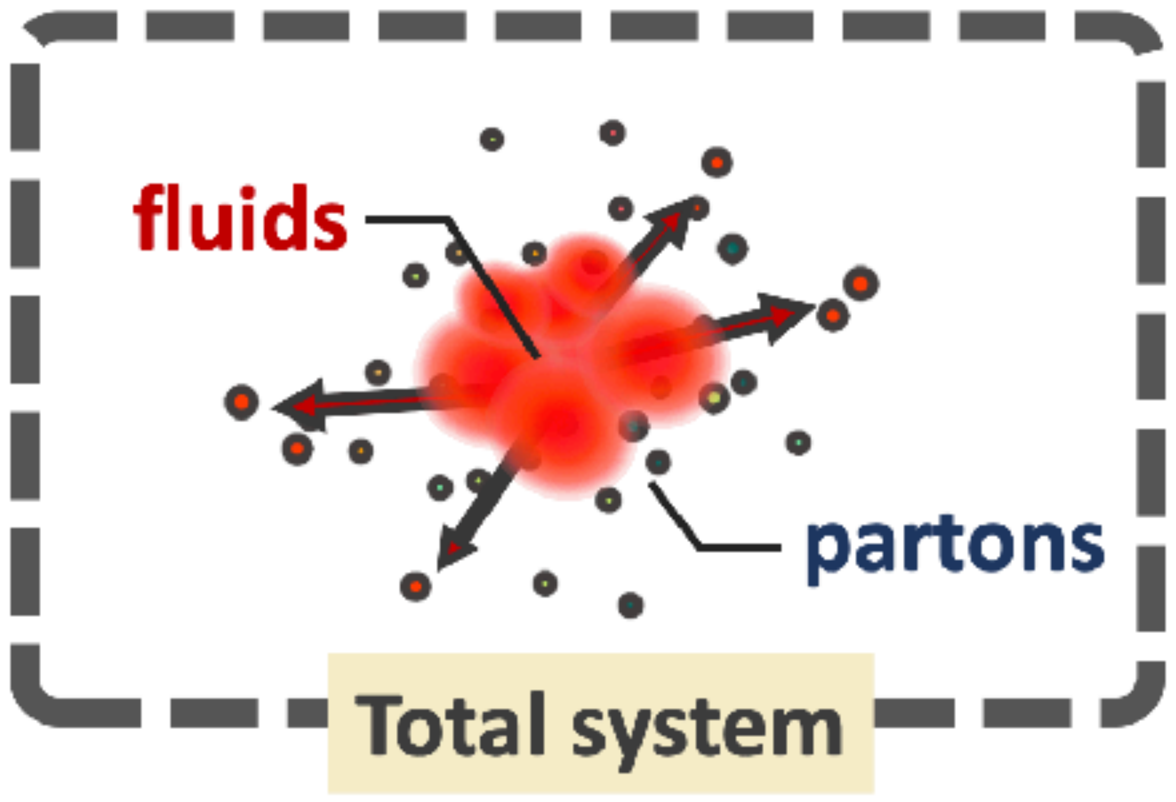
Is there a remedy that retains the indispensable features of the fluctuating nature of the initial state?

E.g., can we work with moments of the initial density fluctuations instead of the full fluctuating density distribution?

Core-corona models

- Causality violations in hydro are especially serious at the edge of the fireball
- This suggests that core/corona models are a possible solution
- Talk by Yuuka Kanakubo on implementation of the Dynamical Core-Corona Initialization framework (DCCI)

Two-component picture
➔ fluids and partons



Continuum eq. for fluid+parton

$$\partial_\mu \left(T^{\mu\nu}_{\mathrm{fluid}} + T^{\mu\nu}_{\mathrm{parton}}\right) = 0$$

Hydrodynamic eq. with source term

$$\partial_\mu T^{\mu\nu}_{\mathrm{fluid}} = J^\nu$$

Gaussian profile G and straight trajectory for a parton

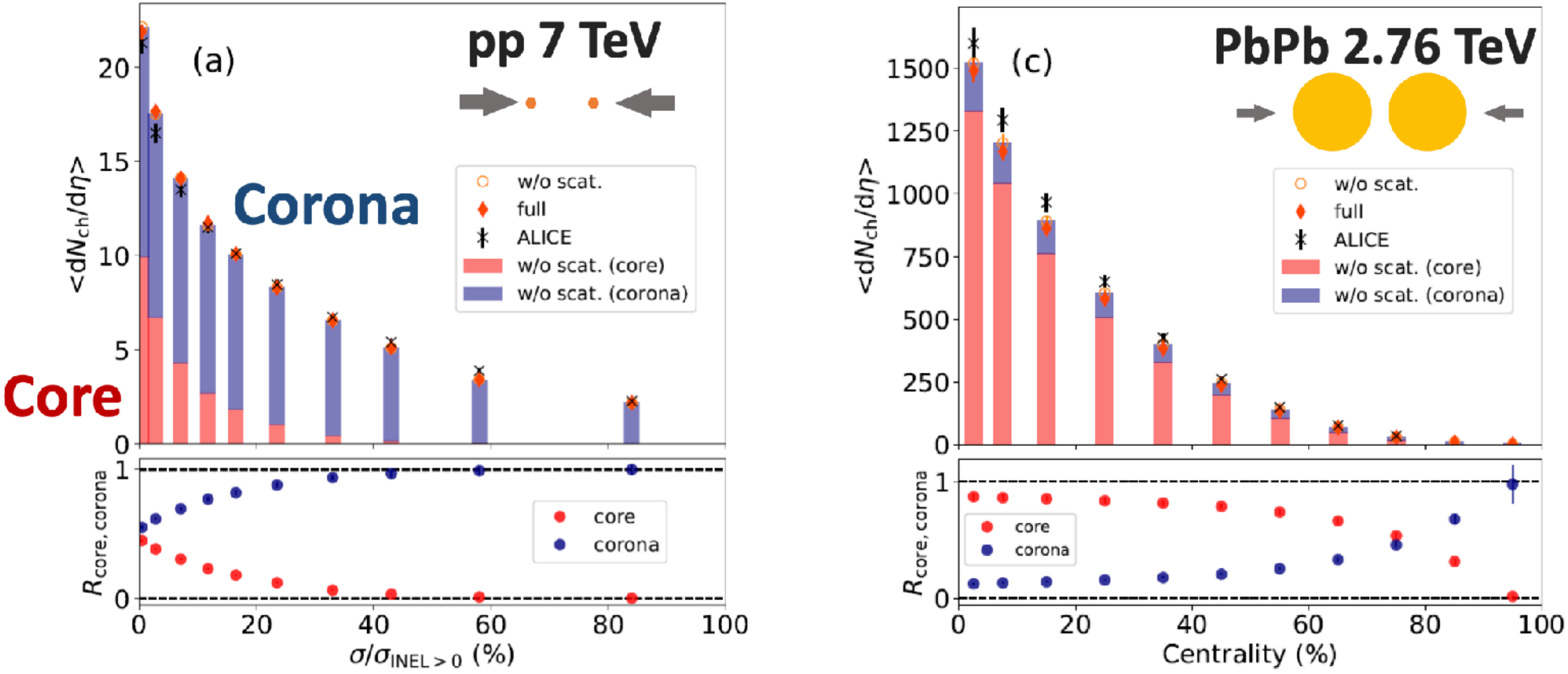
$$J^\nu \to -\sum_i \frac{dp_i{}^\nu(t)}{dt} G(x - x_i(t))$$

"Sources of fluids"
= "Four-momentum deposition from partons"

$$\frac{dp_i{}^\mu}{d\tau} = -\sum_j^{N_{\mathrm{scat}}} \rho_{i,j}\sigma_{i,j}|v_{\mathrm{rel},i,j}|\, p_i{}^\mu$$

Partons rescatter and are added to the fluid when their momenta are thermal
↩

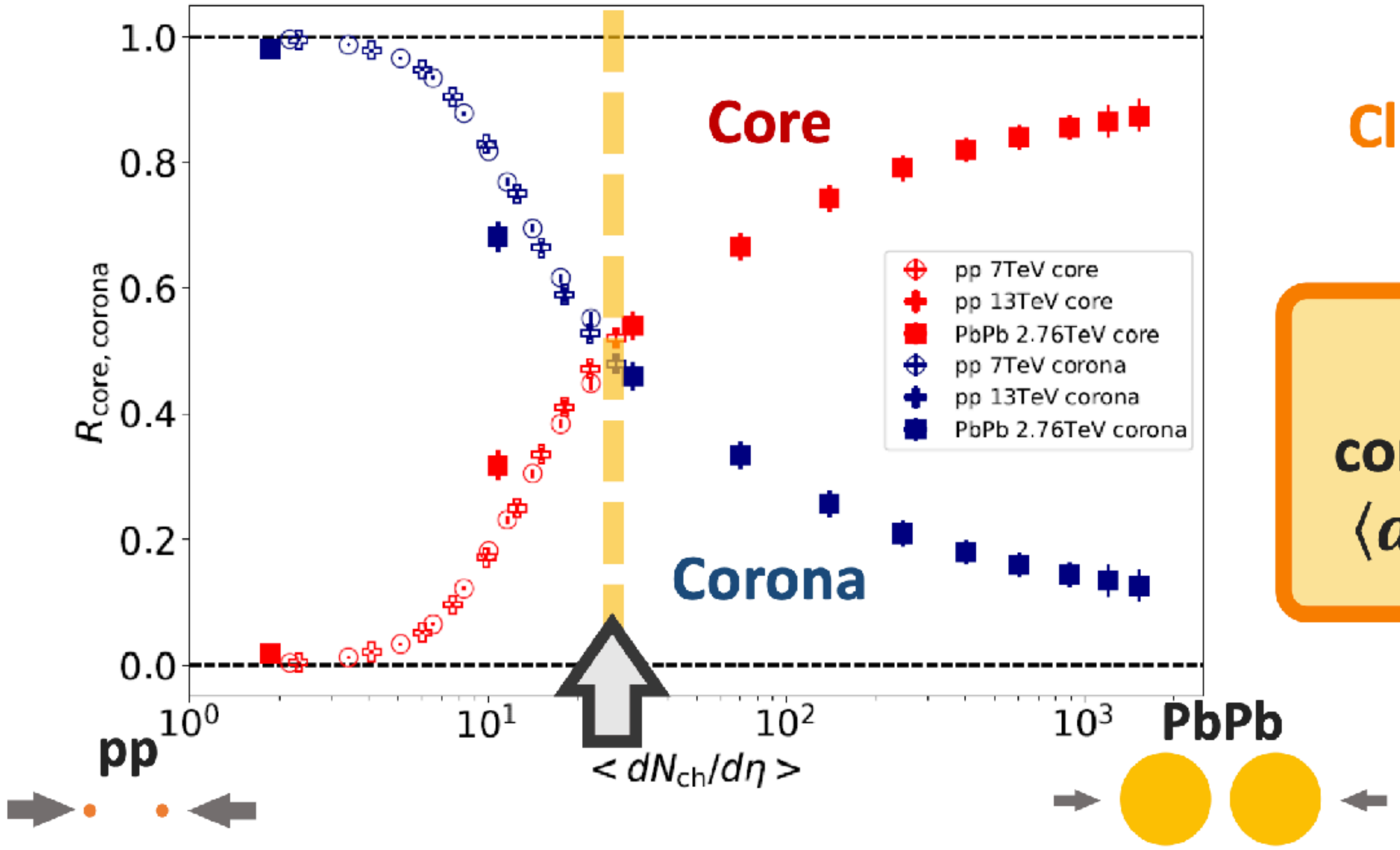
p+p vs. Pb+Pb



Need for equilibrated **and** non-equilibrated matter in both, p+p and Pb+Pb

Core vs corona (dN/dy)

Consistent with observation that p+p and p+Pb collisions with dN_{ch}/dy = 30 exhibit collective behavior

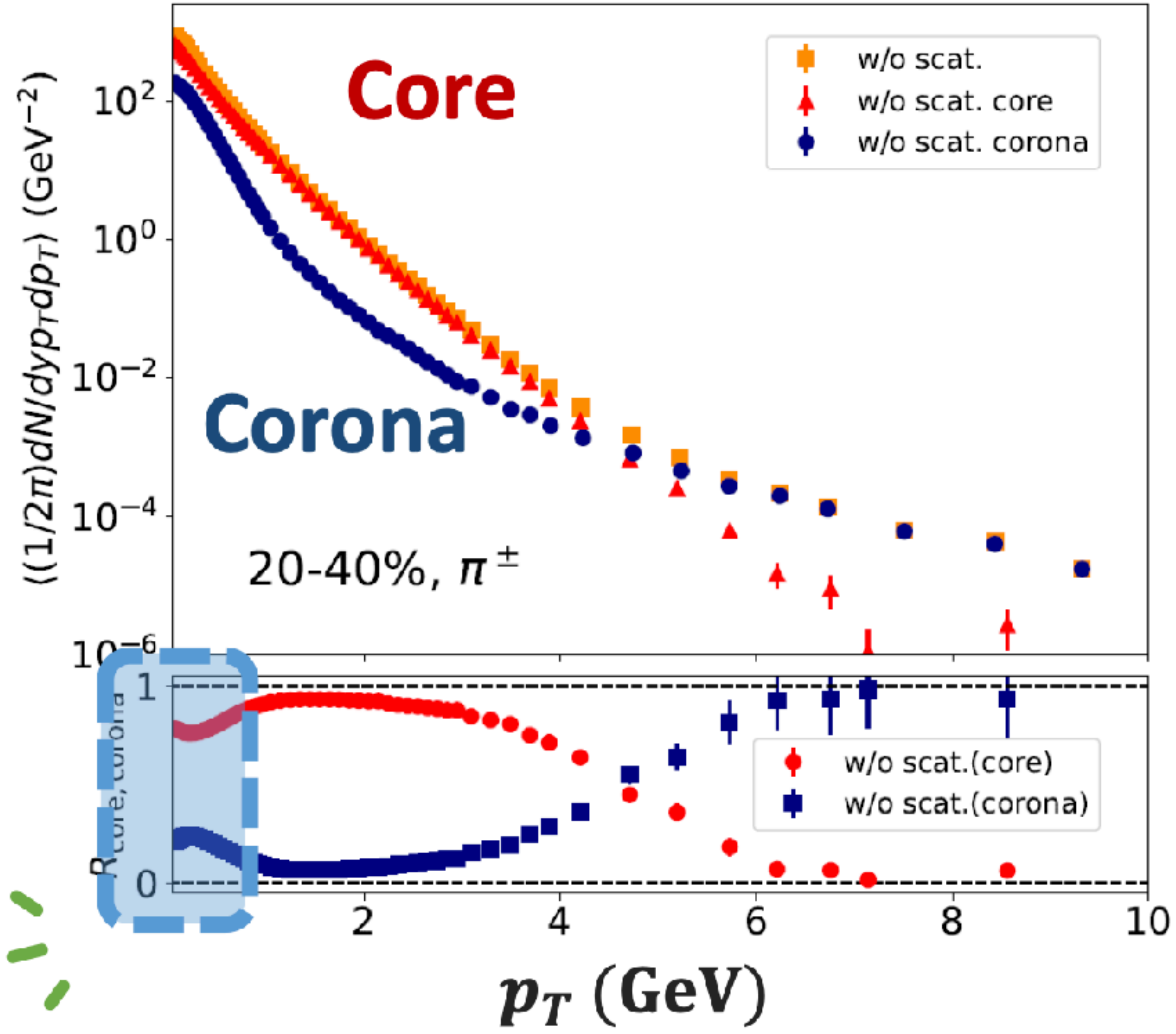


Clear scaling with multiplicity

Onset of core dominance at $\langle dN_{ch}/d\eta \rangle \sim 20$

Core vs. corona (p_T)

$\pi^{\pm}$ in 2.76 TeV Pb+Pb 20-40% centrality



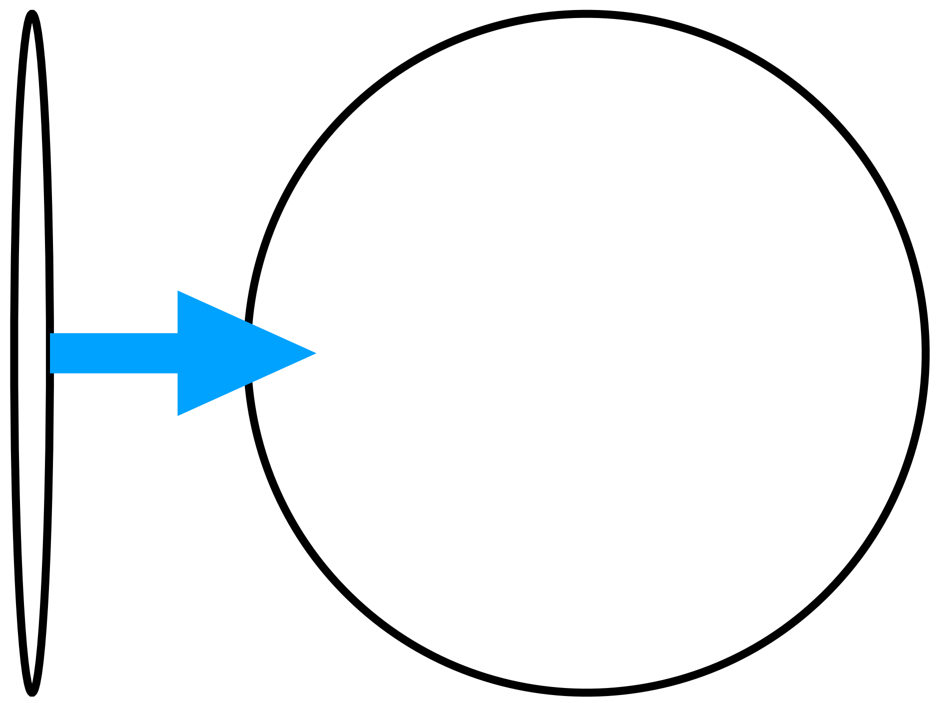
Core-corona models highlight some of the deficiencies of the “standard model” for heavy ion collisions. **Not everything is thermal.**

Can a core-corona model of this kind for central y be formulated as a rigorous kinetic theory for QCD similar to the kinetic equilibration scenarios?

What are the limits for such a formulation?

Would a kinetic theory avoid causality violations **and** provide a more realistic description of the entire mid-rapidity fireball?

BES energies

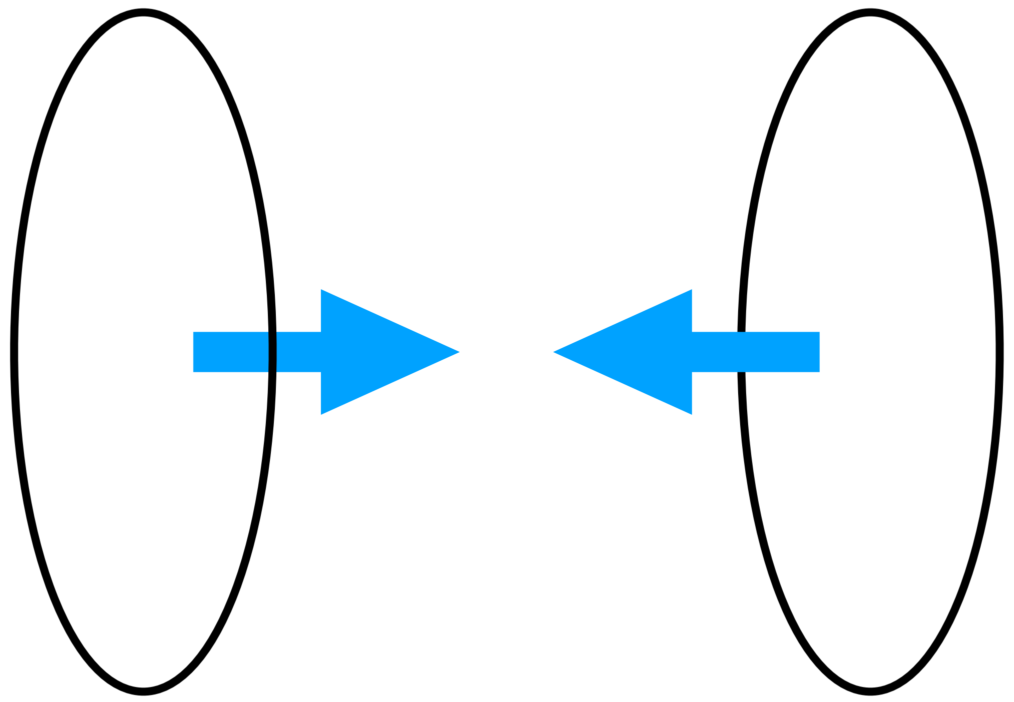


Target rest frame (for fragmentation region):

Can be studied in the fixed target portion of the RHIC BES-II

Collision is a **time-staggered local quench**, and space-like Cauchy hyper-surface for hydrodynamics initial conditions $t_i(z)$ can be defined

Does this allow to develop a "standard model" (Anishetty et al, 1980)?



Center-of-mass frame: Both nuclei are mildly Lorentz contracted $2R/\gamma_{cm}$

Collision **cannot** be treated as a quench; energy and baryon number deposition occurs over extended period of time.

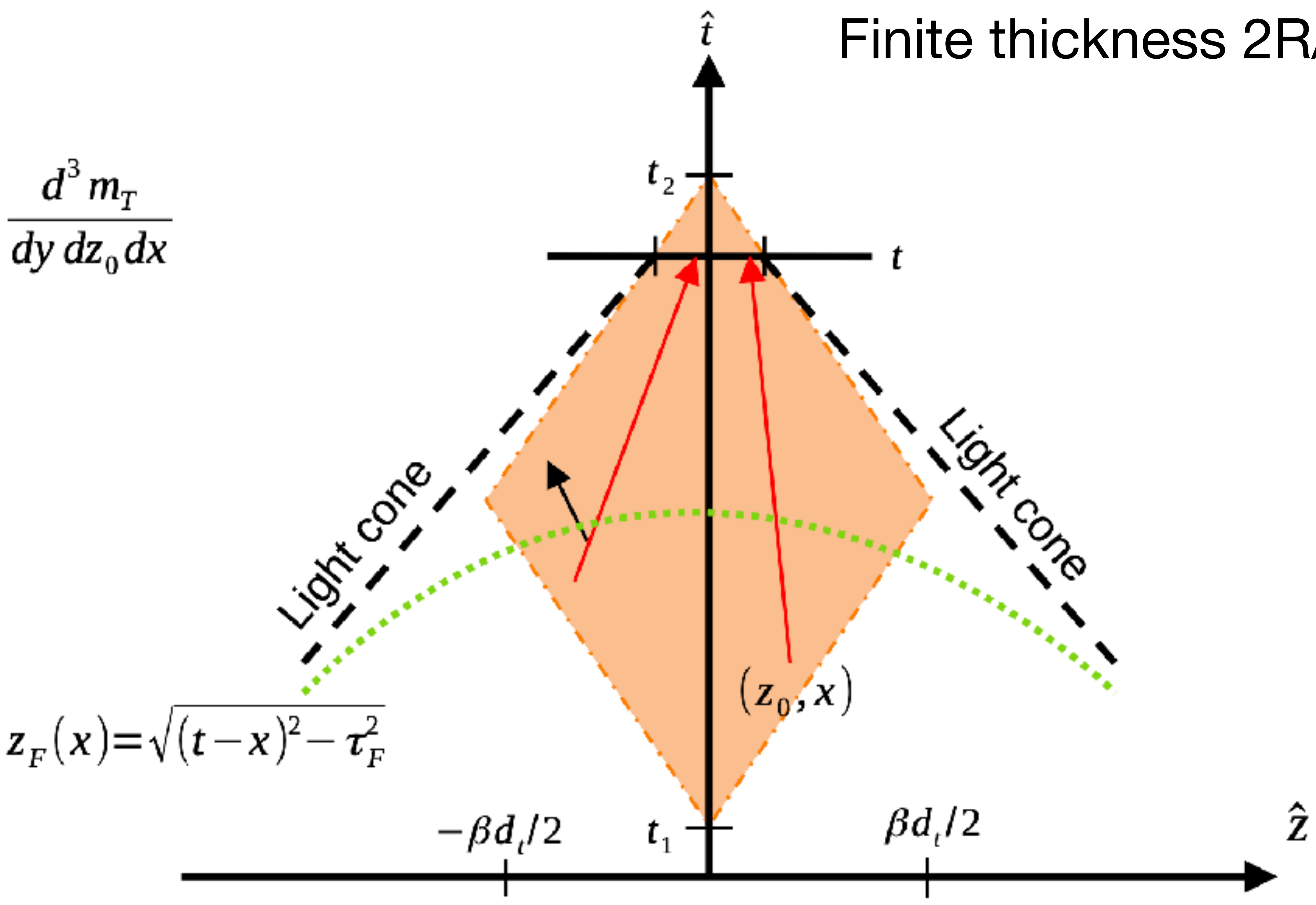
This limits the reachable maximum energy and baryon number density.

Schematic model

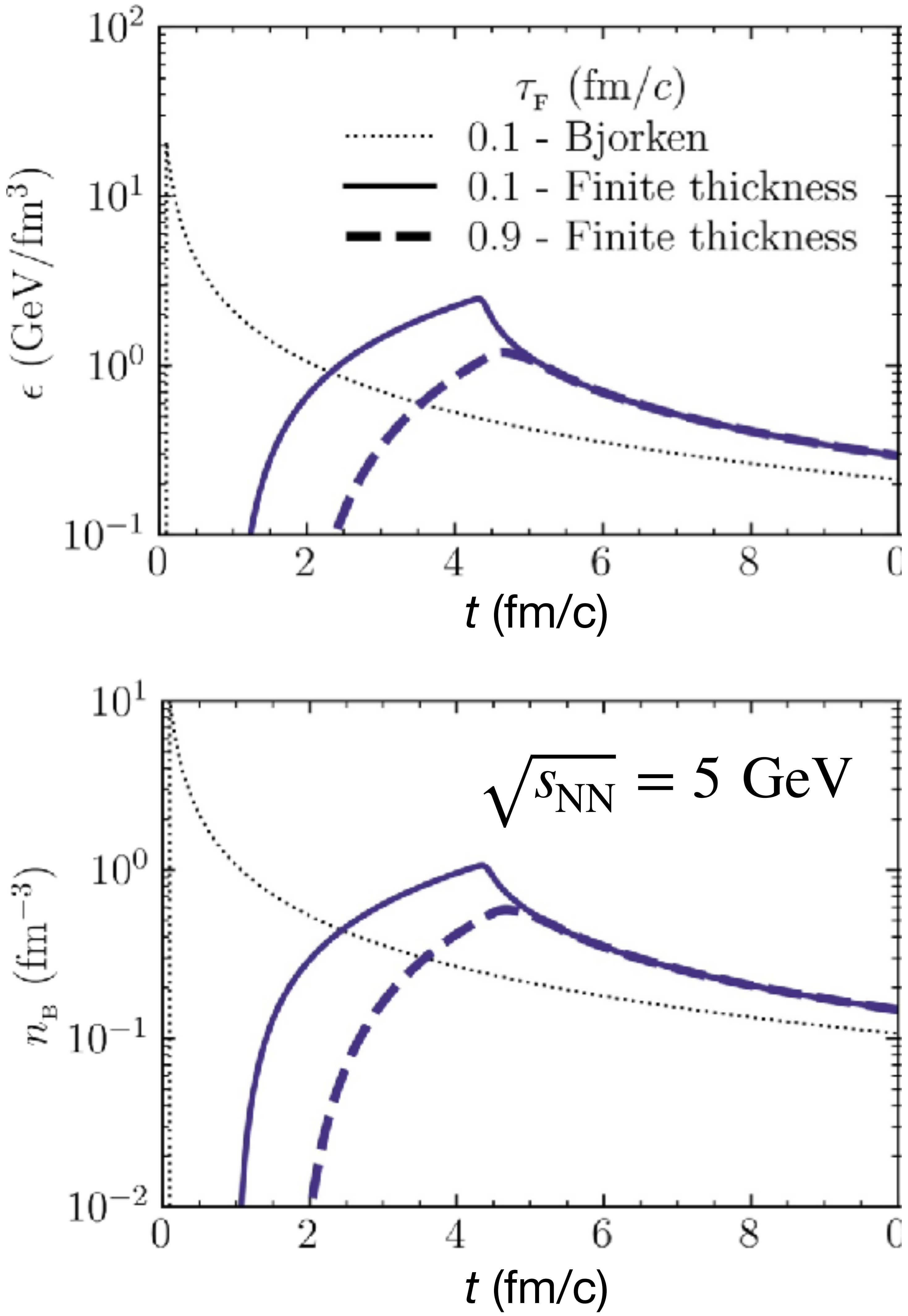
Todd Mendenhall

Formation time τ_f

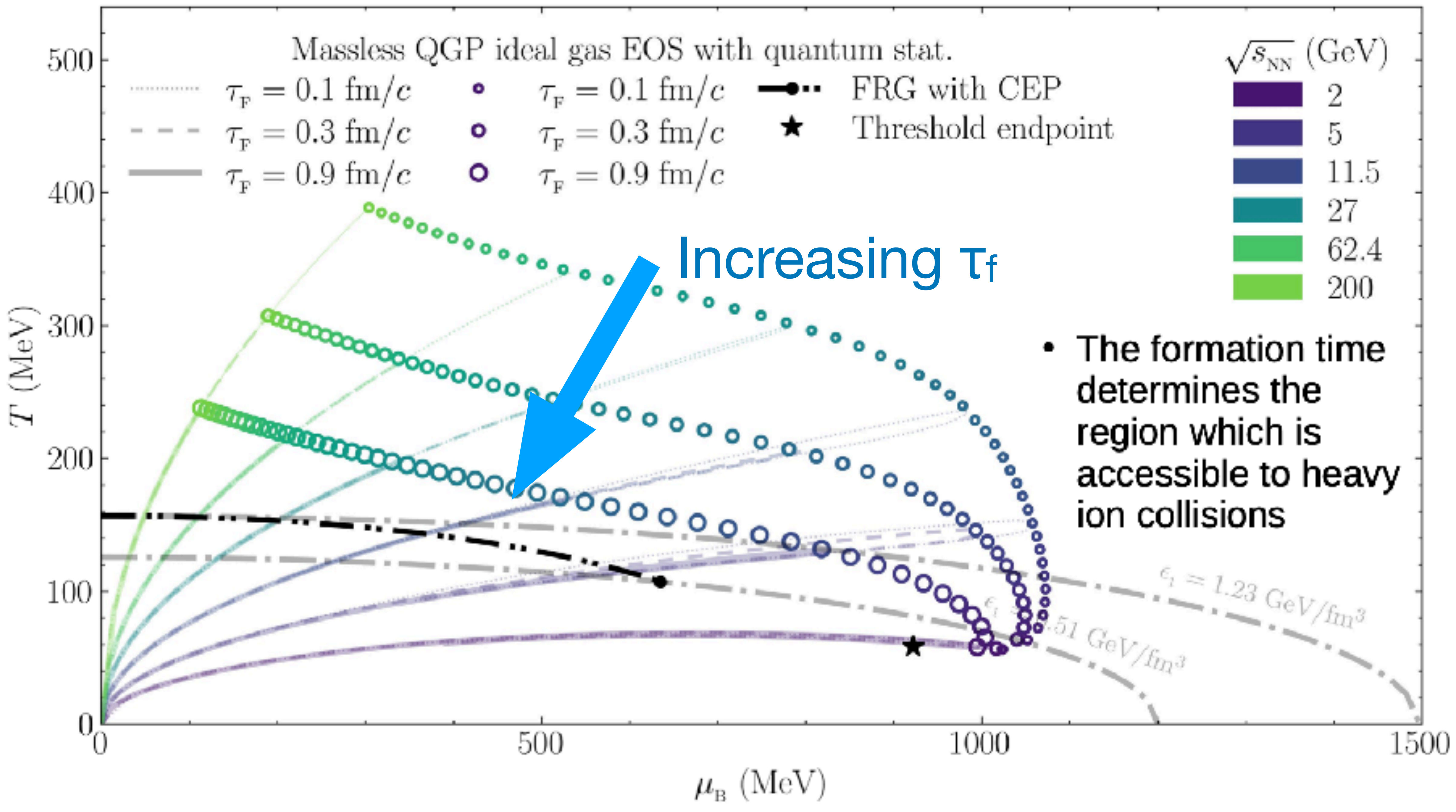
Finite thickness $2R/\gamma$



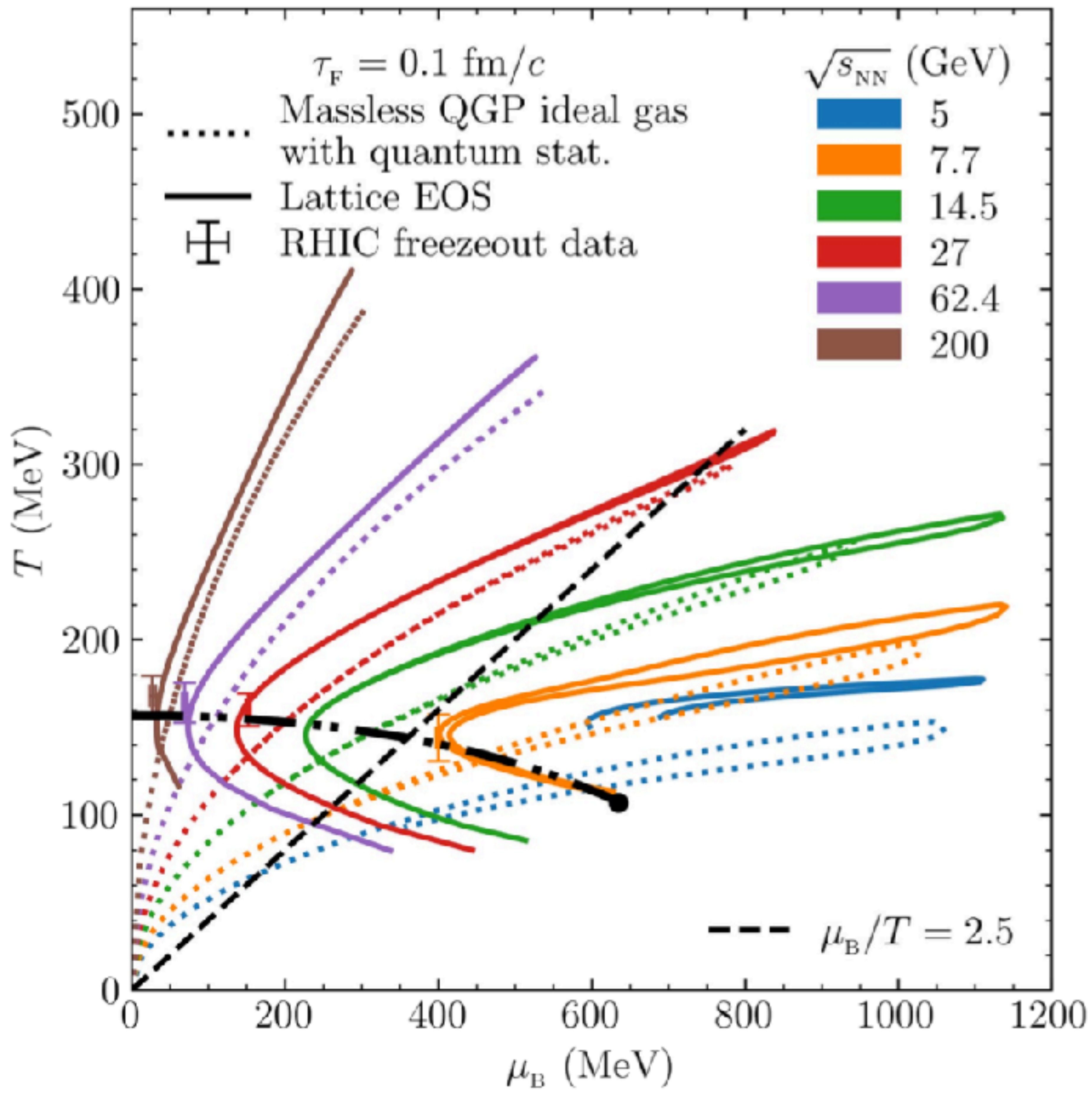
$$\epsilon(t) = \frac{1}{A_T} \iint_{S(t)} \frac{dz_0\, dx}{t-x} \frac{d^3 m_T}{dy\, dz_0\, dx} \cosh^3 y$$



T-μ_B trajectories



- The formation time determines the region which is accessible to heavy ion collisions



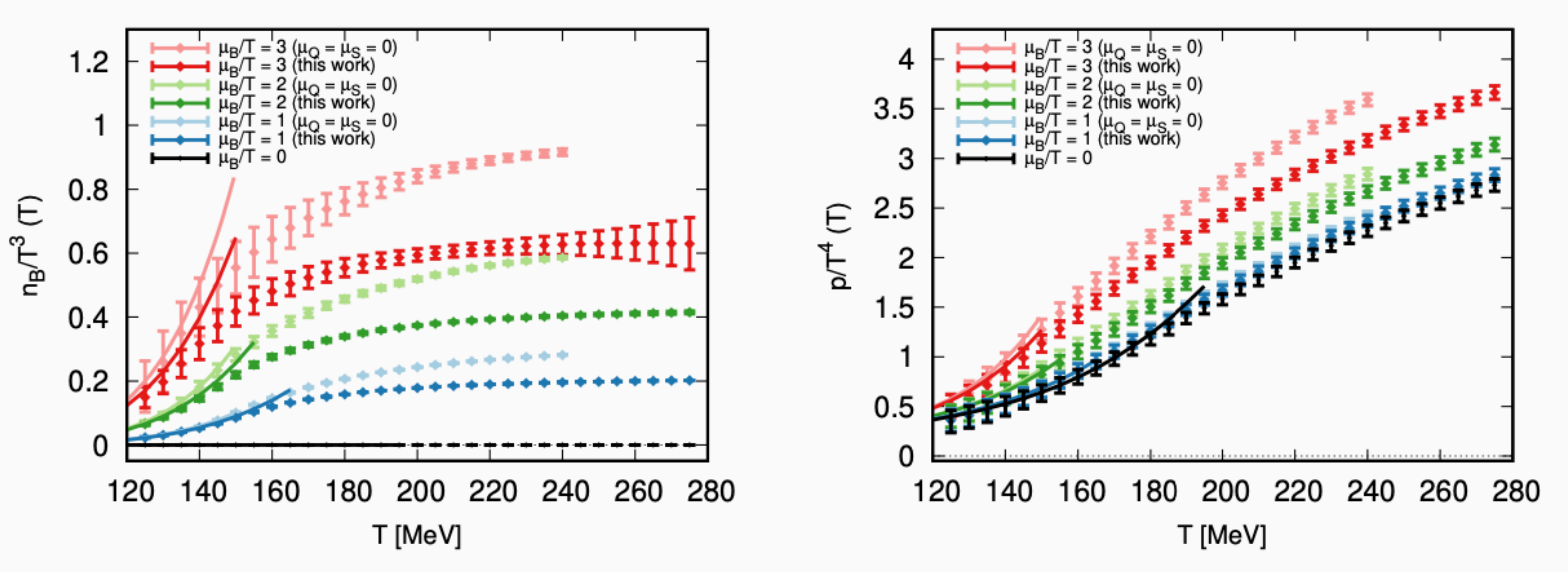
Lattice EOS at $\mu_B \neq 0$

New method to extrapolate to $\mu_B \neq 0$ by reorganizing the Taylor series expansion ($\hat{\mu}_B = \mu_B/T$):

$$\frac{\chi_1^B(T,\,\hat{\mu}_B)}{\hat{\mu}_B} = \chi_2^B(T',0)\ , \quad T' = T\left(1+\kappa_2(T)\,\hat{\mu}_B^2 + \kappa_4(T)\,\hat{\mu}_B^4 + \mathcal{O}(\hat{\mu}_B^6)\right)$$

Allows to reach $\mu_B/T = 3.5$ with reasonable uncertainties with or w/o strangeness neutrality:

(Talk by P. Parotto)



Critical fluctuations

"Hydro+" treats slowest mode, which is the source of critical fluctuations, separately:

(Talk by M. Pradeep)

Fluctuations of $\hat{s} = s/n_B$

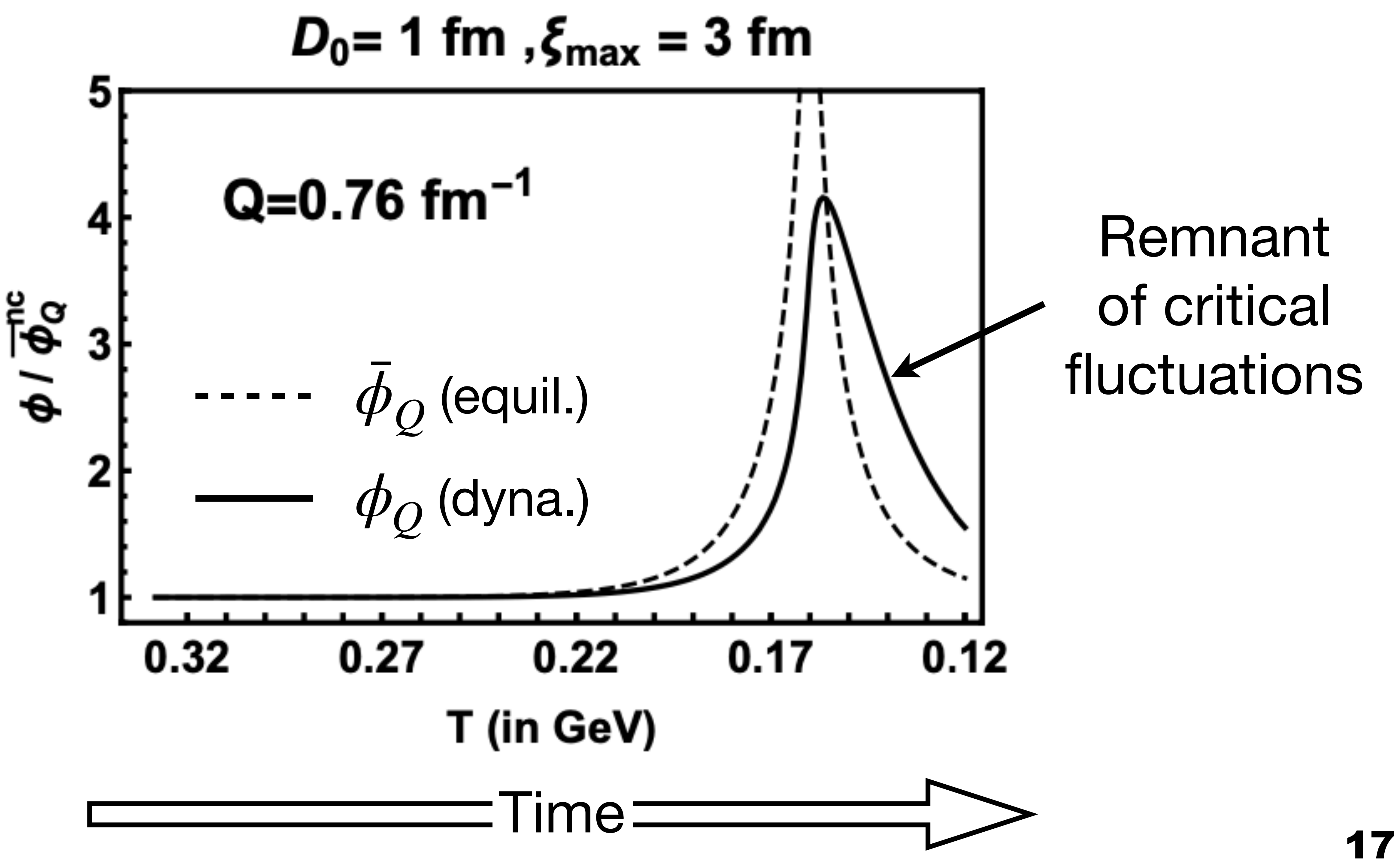
$$\phi_{\mathbf{Q}} = \int_{\Delta\mathbf{x}} e^{-i\,\mathbf{Q}\cdot\Delta\mathbf{x}} \left\langle \delta\hat{s}(x_+)\,\delta\hat{s}(x_-) \right\rangle$$

The mode does not propagate; it relaxes to its instantaneous equilibrium value $\bar{\phi}_Q \propto \xi^2$

$$u\cdot\partial\phi_{\mathbf{Q}} = -\Gamma(\mathbf{Q})\left(\phi_{\mathbf{Q}} - \bar{\phi}_{\mathbf{Q}}\right)$$

Damping rate vanishes at the critical point (critical slowing down): $\Gamma(Q) \propto \xi^{-3}$

Low-Q modes are suppressed due to conservation laws and slower relaxation

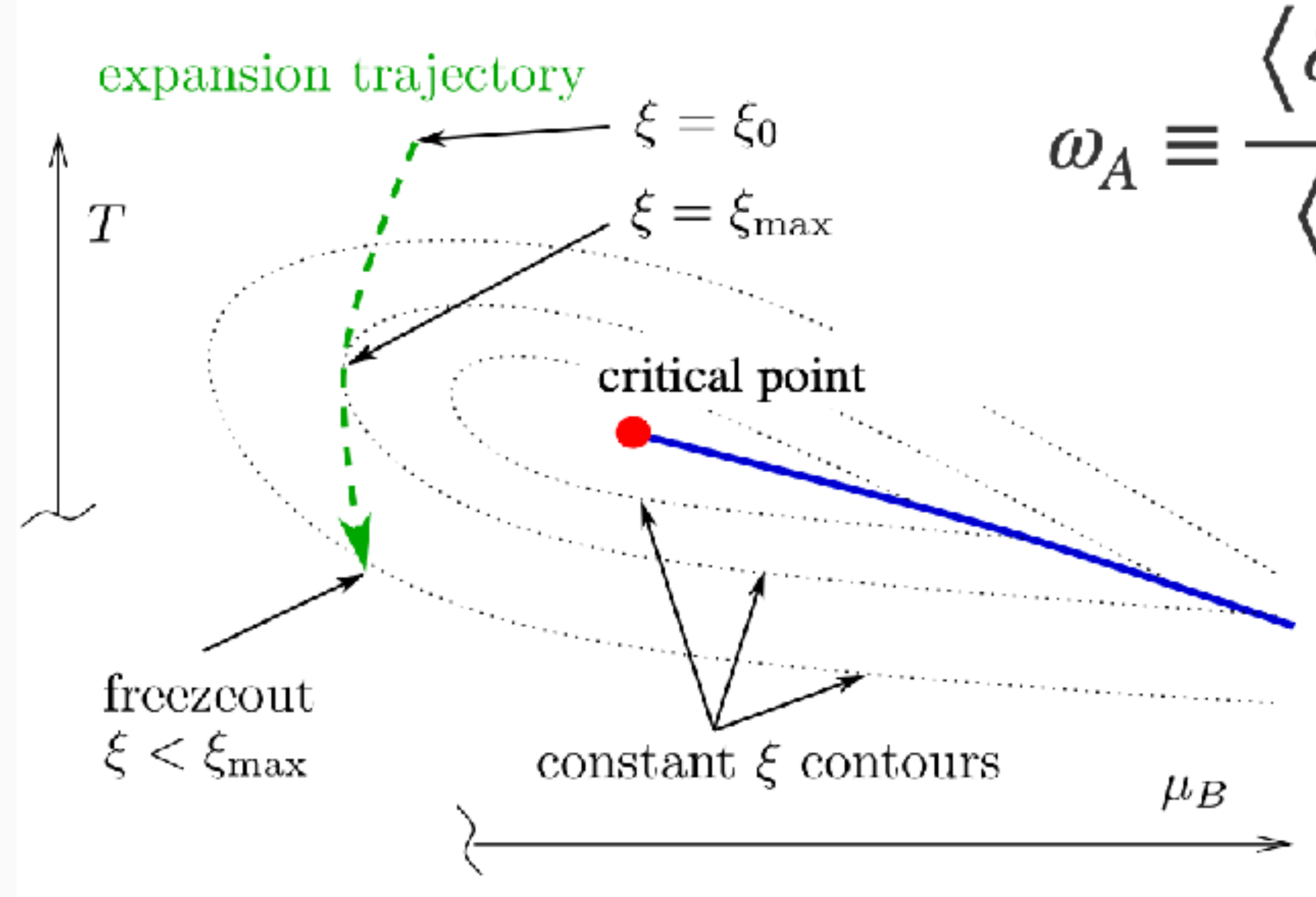


Critical fluctuations II

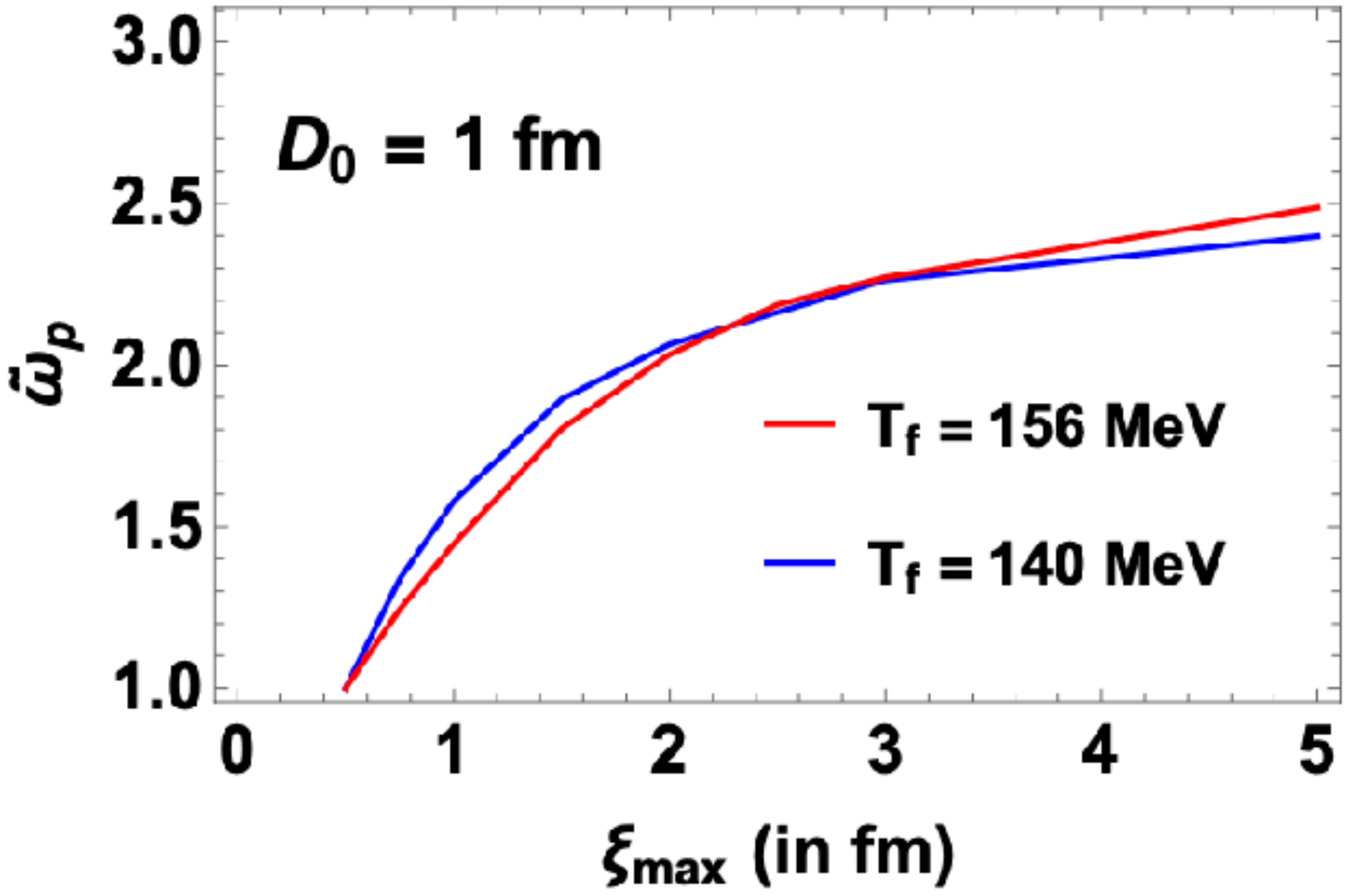
Fluctuations of critical mode couple to baryon masses at freezeout via collective σ field:

$$f_A = \langle f_A \rangle + g_A \frac{\partial \langle f_A \rangle}{\partial m_A} \sigma \qquad \langle \sigma(x_+)\sigma(x_-) \rangle \approx Z^{-1} \langle \delta\hat{s}(x_+)\delta\hat{s}(x_-) \rangle$$

This is an additional contribution (beyond Poisson) to baryon number fluctuations: $\langle \delta N_A^2 \rangle_\sigma$

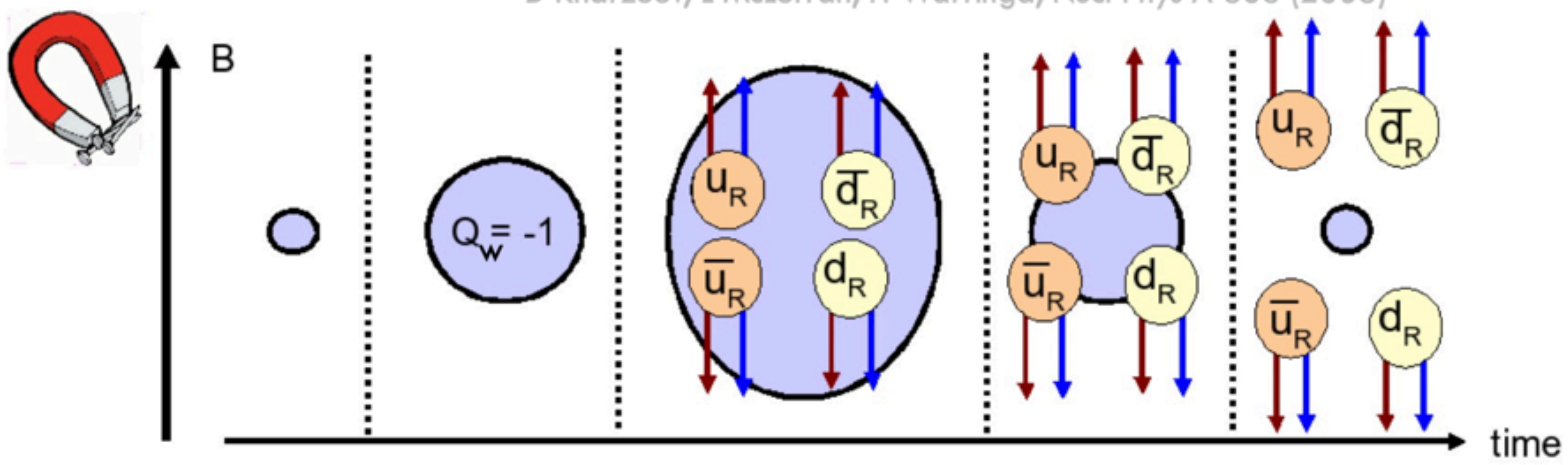


$$\omega_A \equiv \frac{\langle \delta N_A^2 \rangle_\sigma}{\langle N_A \rangle}$$



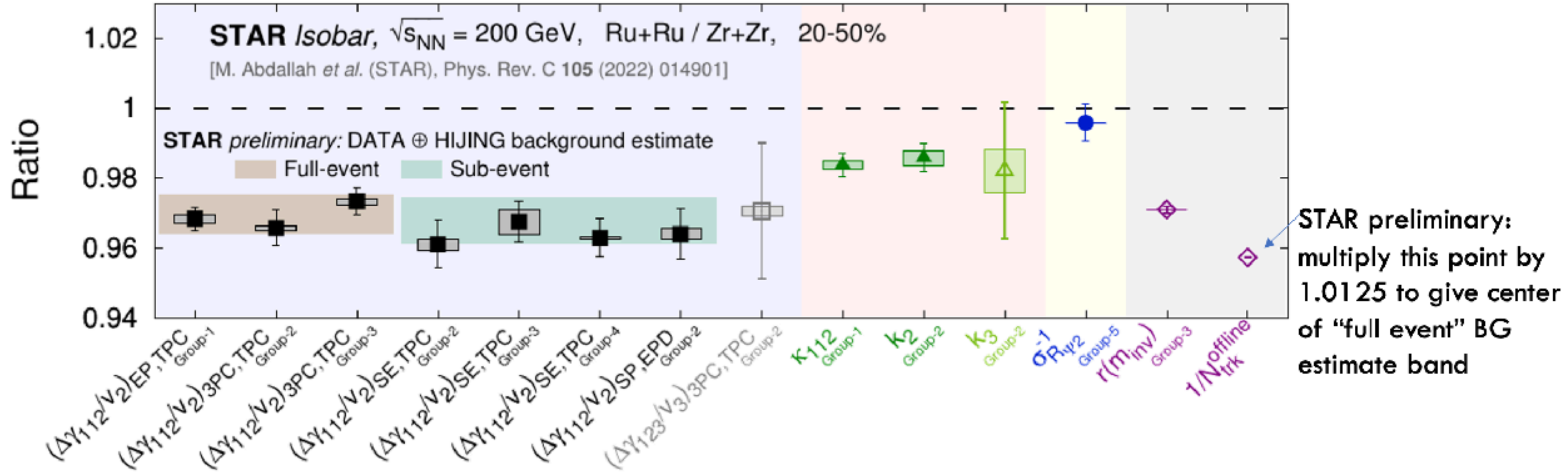
Anomalous chiral phenomena

- Anomalous chiral current phenomena in the presence of strong magnetic fields, such as the CME and CMW, are an integral property of QCD under conditions of chiral symmetry restoration.



- The isobar comparison run provided the most stringent test yet for anomalous chiral current fluctuations in heavy ion collisions.

Evan Finch's overview talk



Charge separation

- Searches for other manifestations of AChP will continue.
- But it's time for theorists to seriously explore what the highly improved experimental limits tell us.
- How can this be done?

Anomalous current: $\mathbf{J} = \sum_f \frac{(Q_f e)^2}{2\pi^2} \mu_5 \mathbf{B} \equiv C\mu_5 \mathbf{B}$ with $\mu_5 = \frac{3n_5}{T^2}$ (n_5 = axial number density)

In QCD, n_5 is determined by the winding number density of the gluon field: $n_5 = -\int dt \frac{g^2}{8\pi^2} \mathbf{E}^a \cdot \mathbf{B}^a$

Current conservation $\partial\rho/\partial t = -\nabla \cdot \mathbf{J}$ tells us that the separated charge ΔQ is

$$\Delta Q = \frac{3Cg^2}{8\pi^2 T^2} \int d^3x \int dt\, \mathbf{B} \cdot \nabla \int dt'\, \mathbf{E}^a \cdot \mathbf{B}^a$$

A more rigorous description including charge transport is provided by anomalous hydrodynamics.

Analysis of CME Limits I

$$\Delta Q = \frac{3Cg^2}{8\pi^2 T^2}\int d^3x \int dt\,\mathbf{B}\cdot\nabla\int dt'\,\mathbf{E}^a\cdot\mathbf{B}^a$$

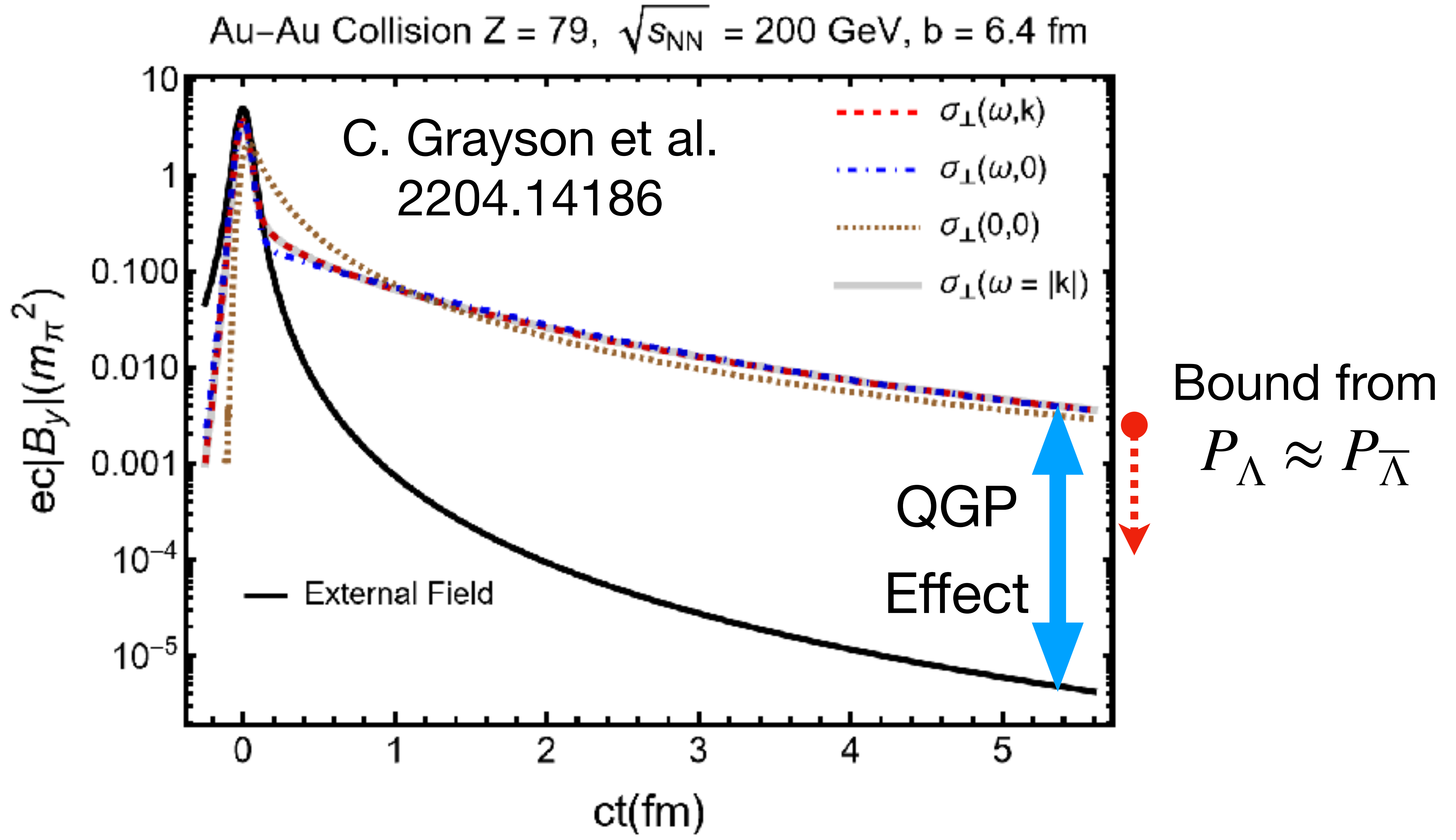
ΔQ is determined by two factors: $\int dt\,\mathbf{B}$, and the space-time dependent fluctuations of $\mathbf{E}^a\cdot\mathbf{B}^a$.

Induced magnetic field in QGP due to Eddy currents (Tuchin 2013):

$$|B_{\rm ind}(t)| \approx \frac{Ze\beta b\sigma}{8\pi t^2} \rightarrow \int dt\,|B_{\rm ind}| \approx \frac{Ze\beta b\sigma}{8\pi\tau_i}$$

Vacuum field contribution: $\int dt\,|B_{\rm vac}| \approx \frac{Ze\beta b}{4\pi R^2}$

For $\sigma \approx 5$ MeV, $\tau_i = 0.5$ fm/c both contributions are approximately equal.



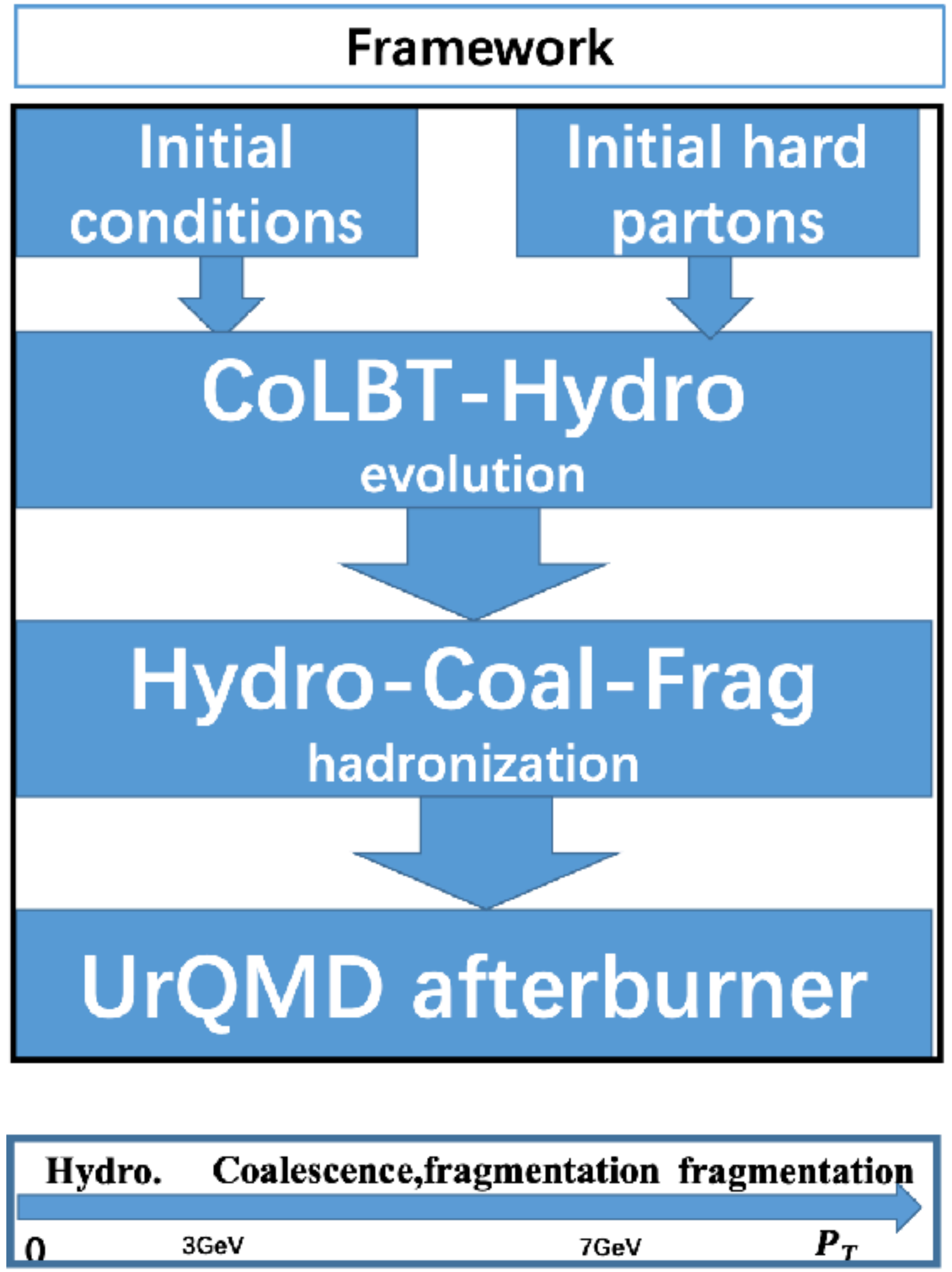
Analysis of CME Limits II

$$\Delta Q = \frac{3Cg^2}{8\pi^2 T^2} \int d^3x \int dt\, \mathbf{B} \cdot \nabla \int dt'\, \mathbf{E}^a \cdot \mathbf{B}^a$$

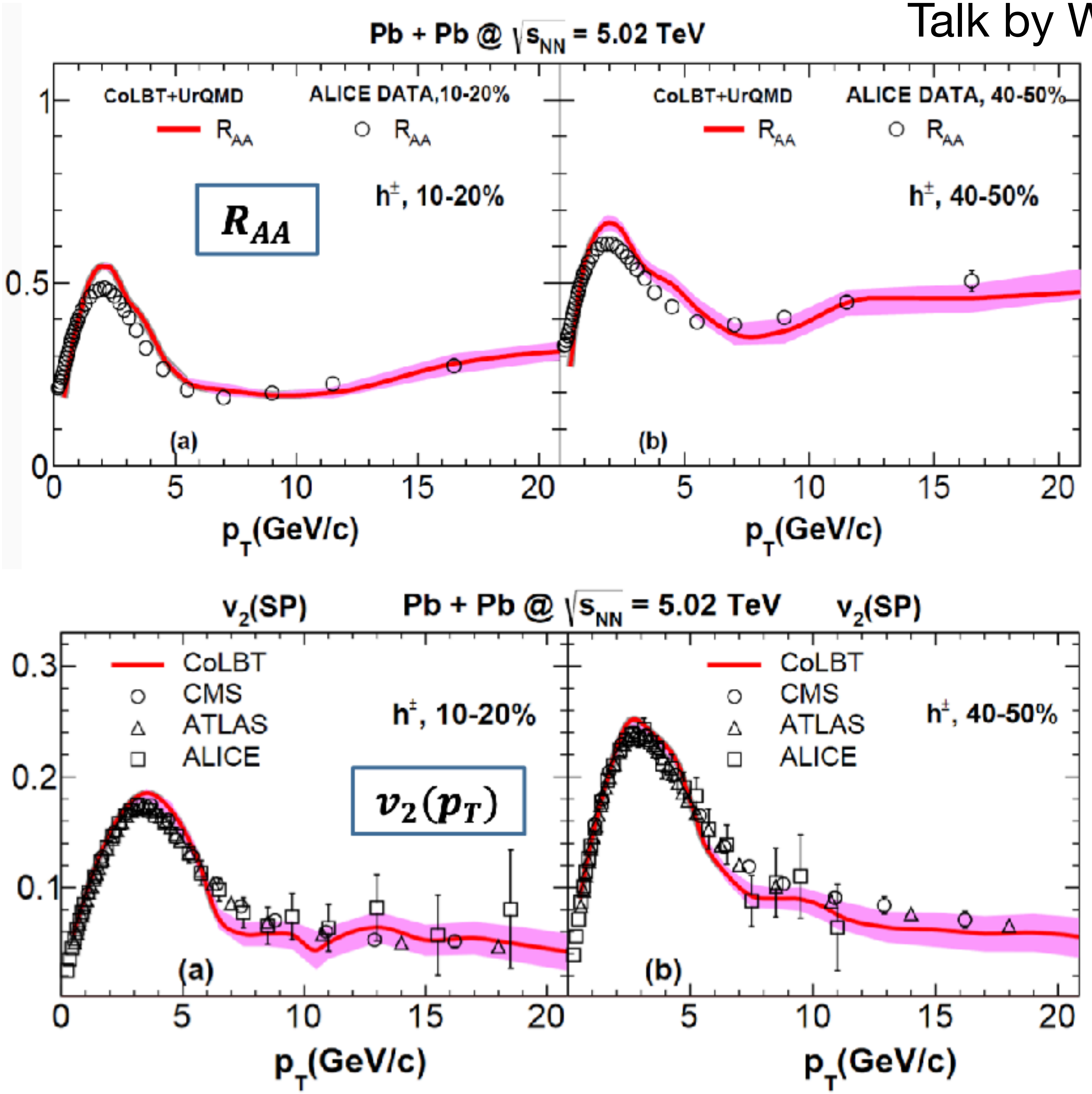
- With reasonable effort, reliable estimates of the time-dependent magnetic field including fully dynamical response of the QGP, can be obtained.
- Experimental limits on CME can then set an upper bound on winding number fluctuations over the course of the heavy ion collision.
- How valuable would this information be?
- Certainly the tremendous effort invested in the experimental measurement justifies a commensurate theoretical effort.

- In the meantime, the experimental search will for AChP continue. A detected signal would be better than an upper limit.
- But the latest predictions of B(t) also suggest that a detection via $\Lambda, \overline{\Lambda}$ polarization may be within reach.

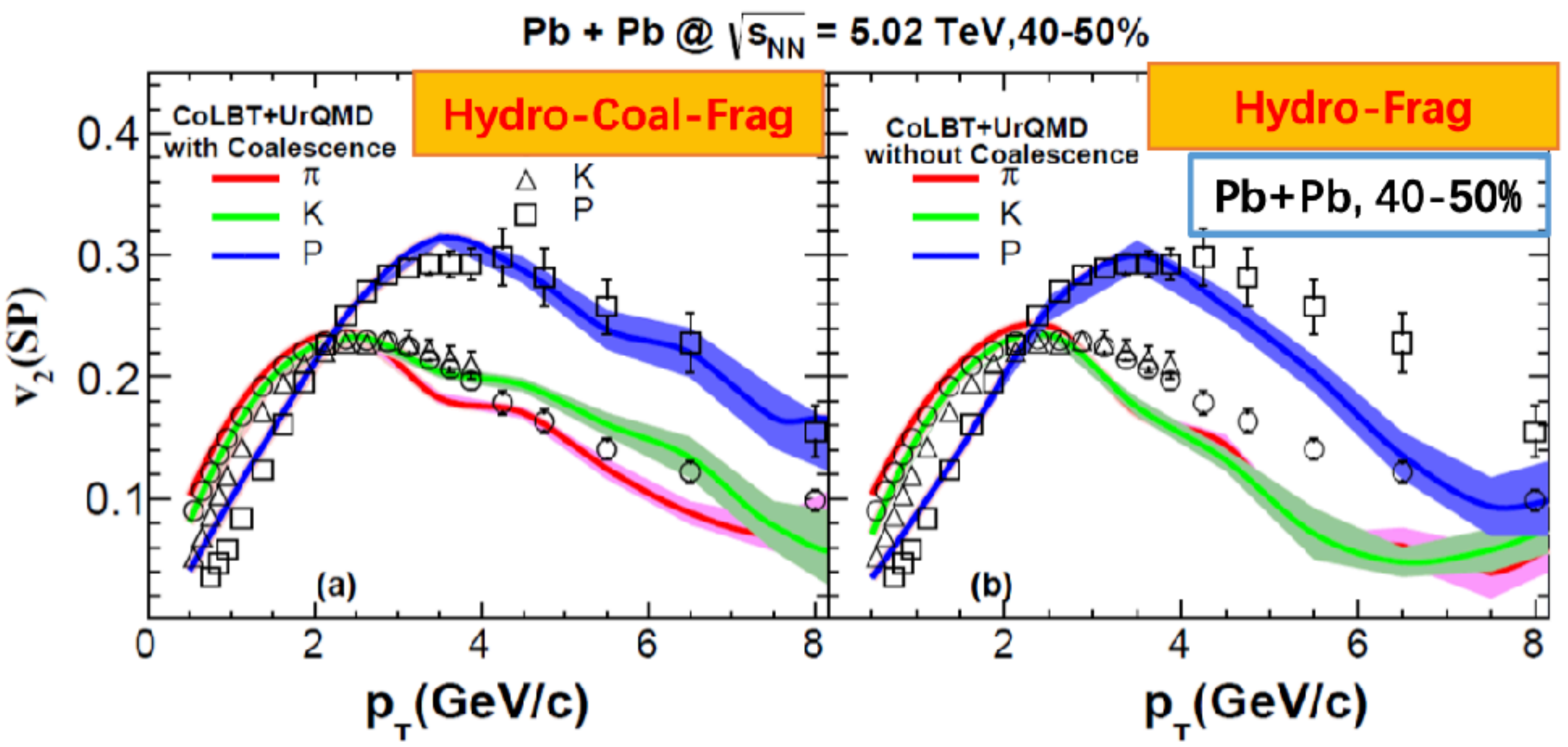
Dynamical hadronization

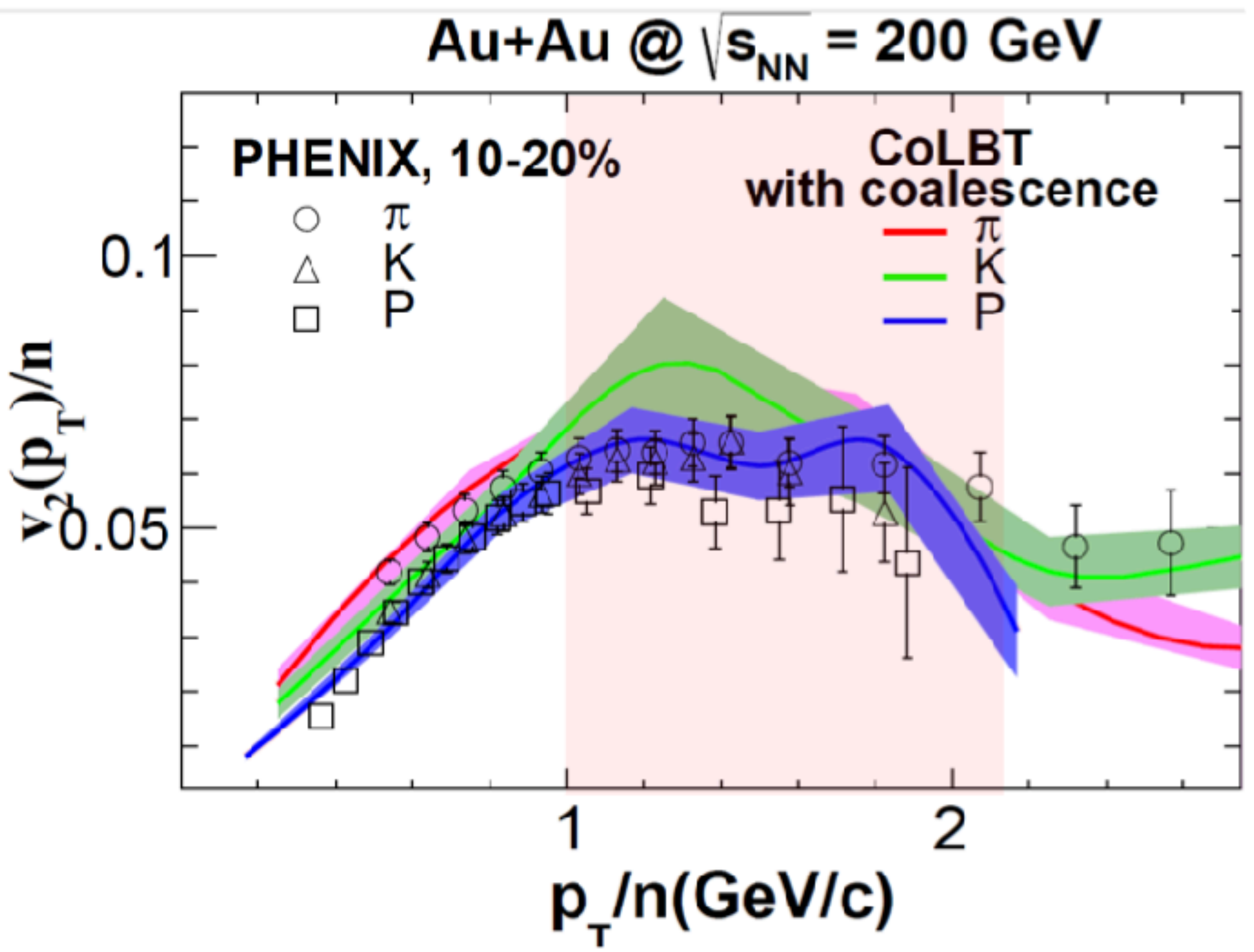


Talk by Wenbin Zhao



Recombination/fragmentation works



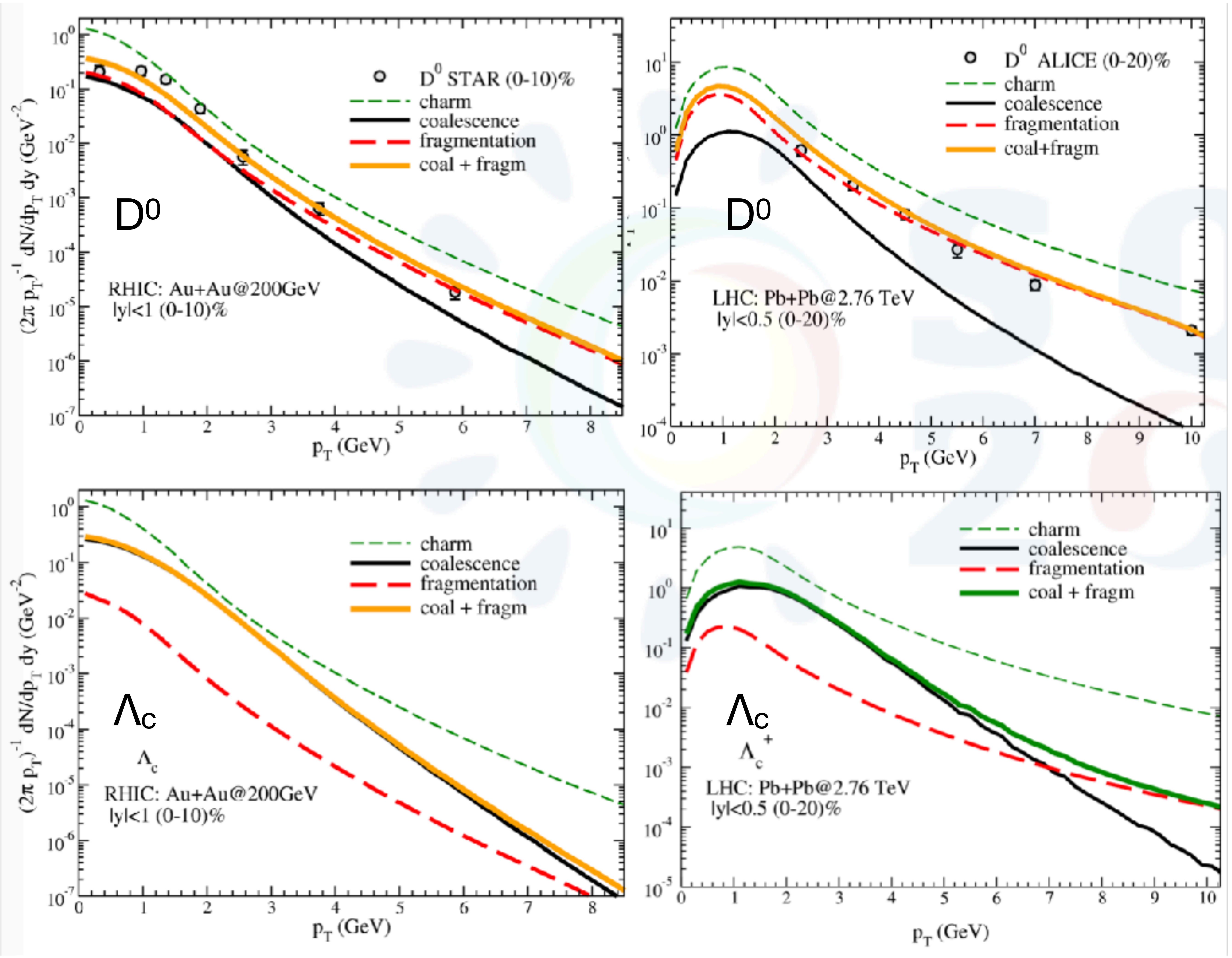


Coalescence required to describe flow of identified hadrons in intermediate momentum range (3 GeV < p_T < 7 GeV)

- n_Q scaling reproduced
- Extrapolation to RHIC successful

Reco/frag for charm

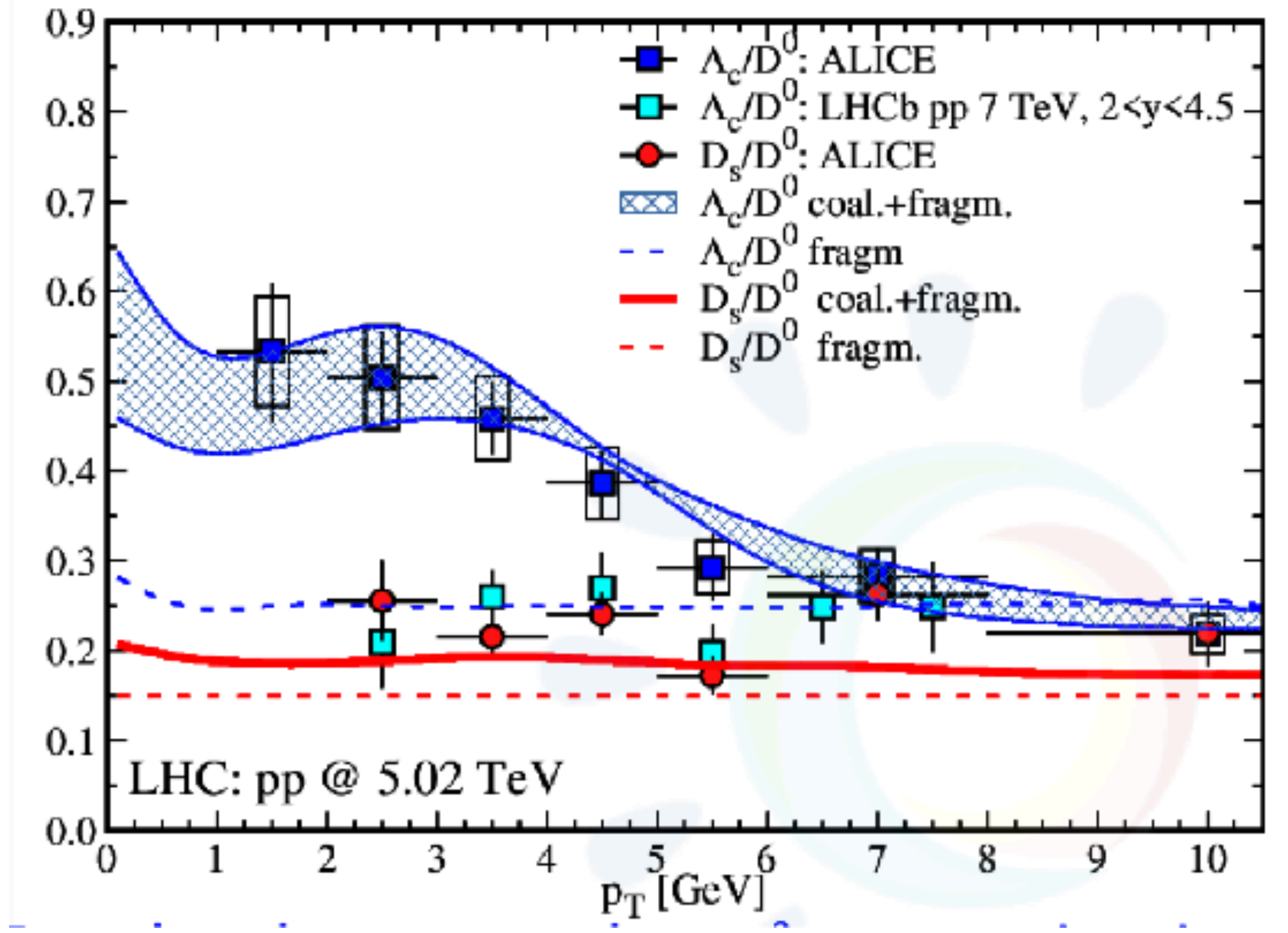
Catania coalescence+fragmentation model (V. Minissale)



Coalescence generally more important at RHIC than at LHC

Coalescence even important for Λ_c for p_T < 5 GeV/c in p+p at LHC

V. Minissale, S. Plumari, V. Greco, Physics Letters B 821 (2021) 13662

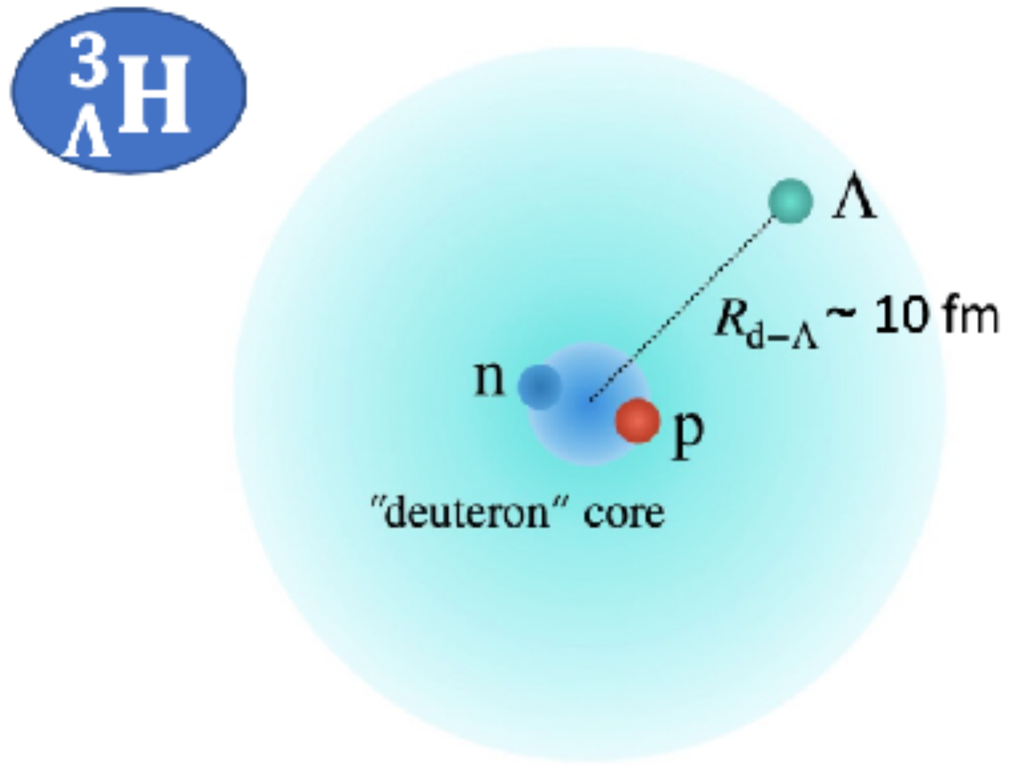


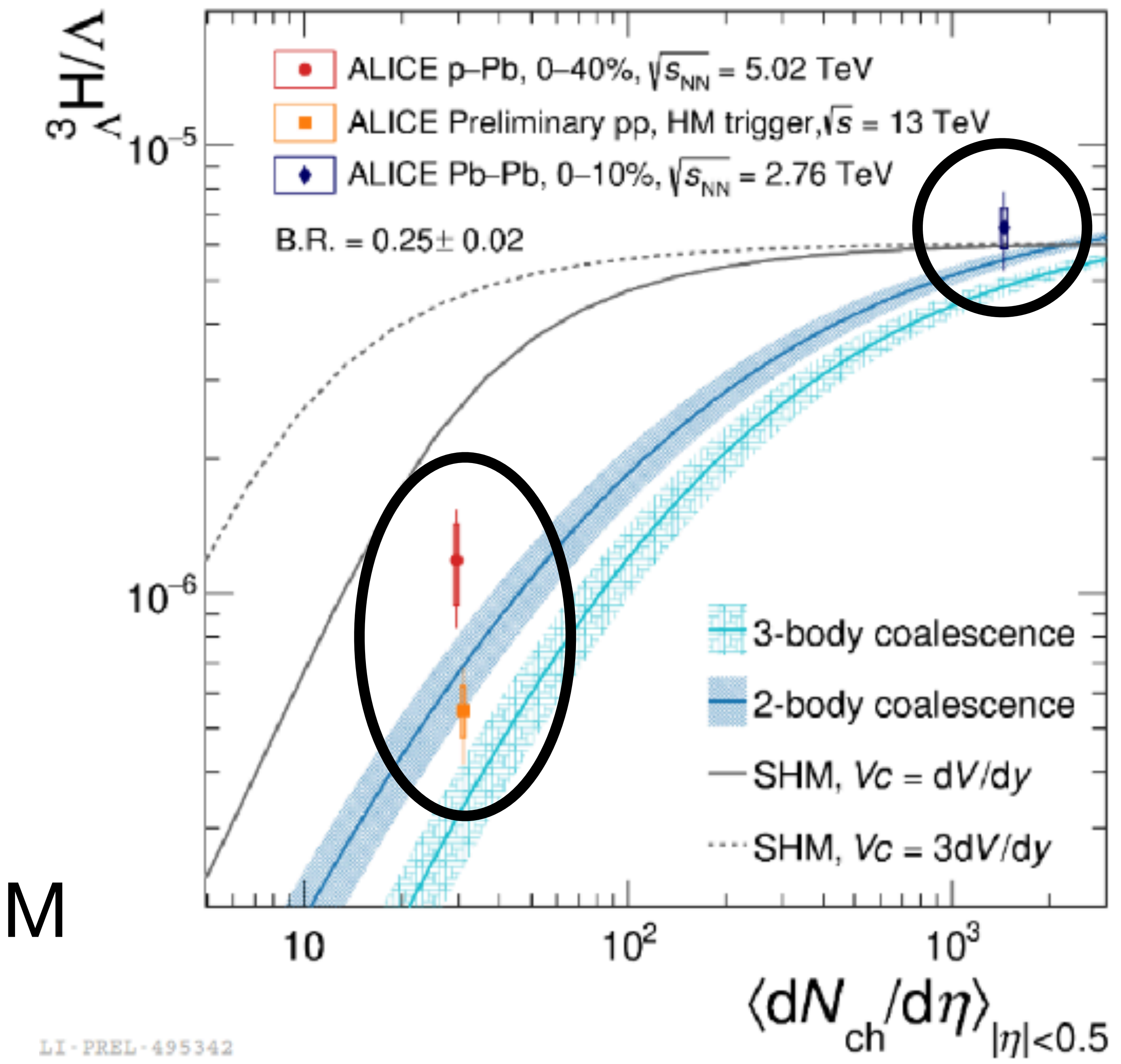
Statistical Hadronization

The statistical hadronization model (SHM) for particle yields works astoundingly well. Why?

This includes light nuclear fragments, which have binding energies $E_B \ll T_H$.

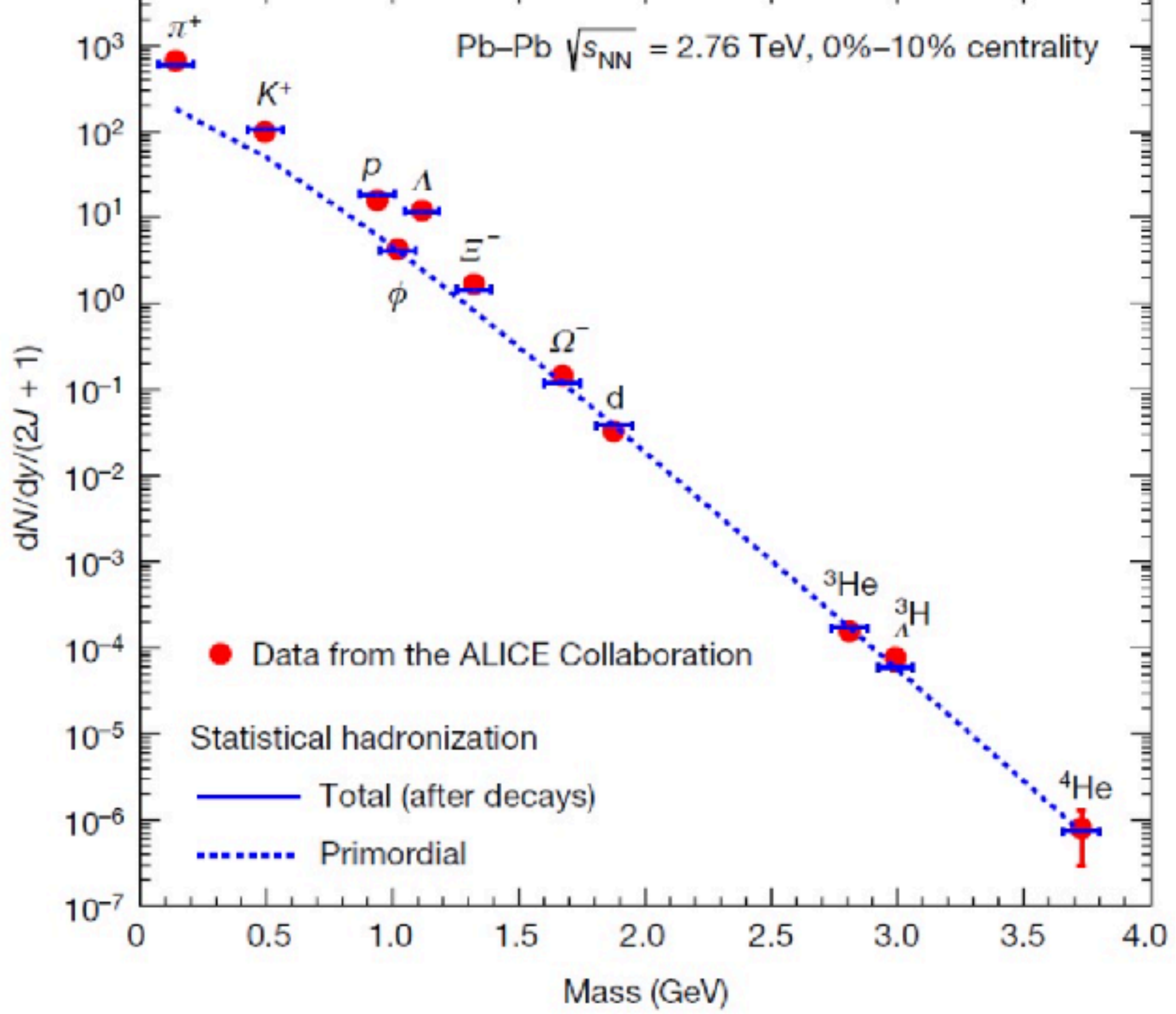
The hyper-triton ${}^3_\Lambda H$ with a rms radius > 5 fm provides an extreme test case.





Conclusion: Data favor coalescence model over SHM

C. Pinto's talk



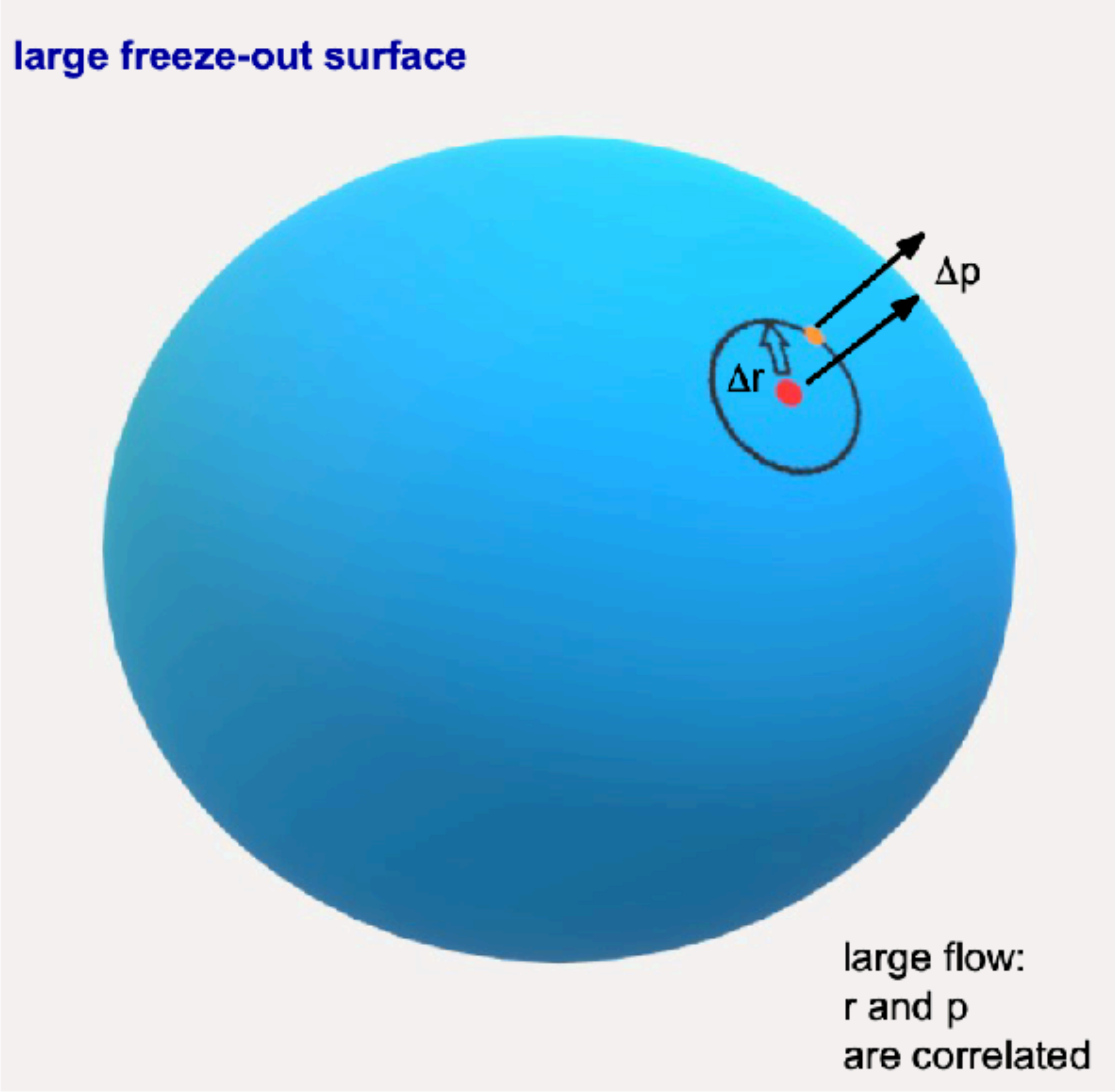
Particle yields of light-flavour hadrons described over 9 orders of magnitude with a **common $T_{chem} \approx 156$ MeV**

(Hyper-) Nuclei

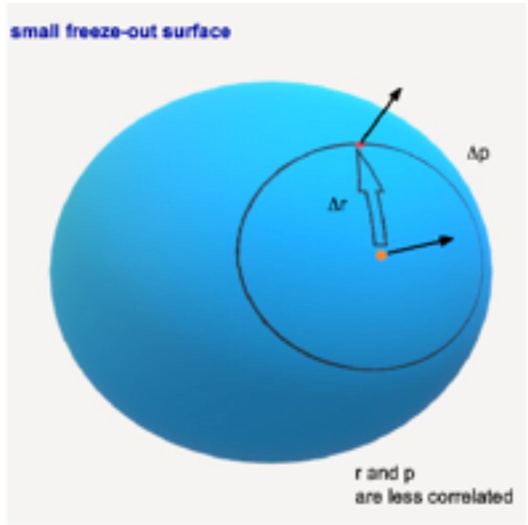
Size of bound state and size of source affect coalescence probability (Talk by T. Reichert)

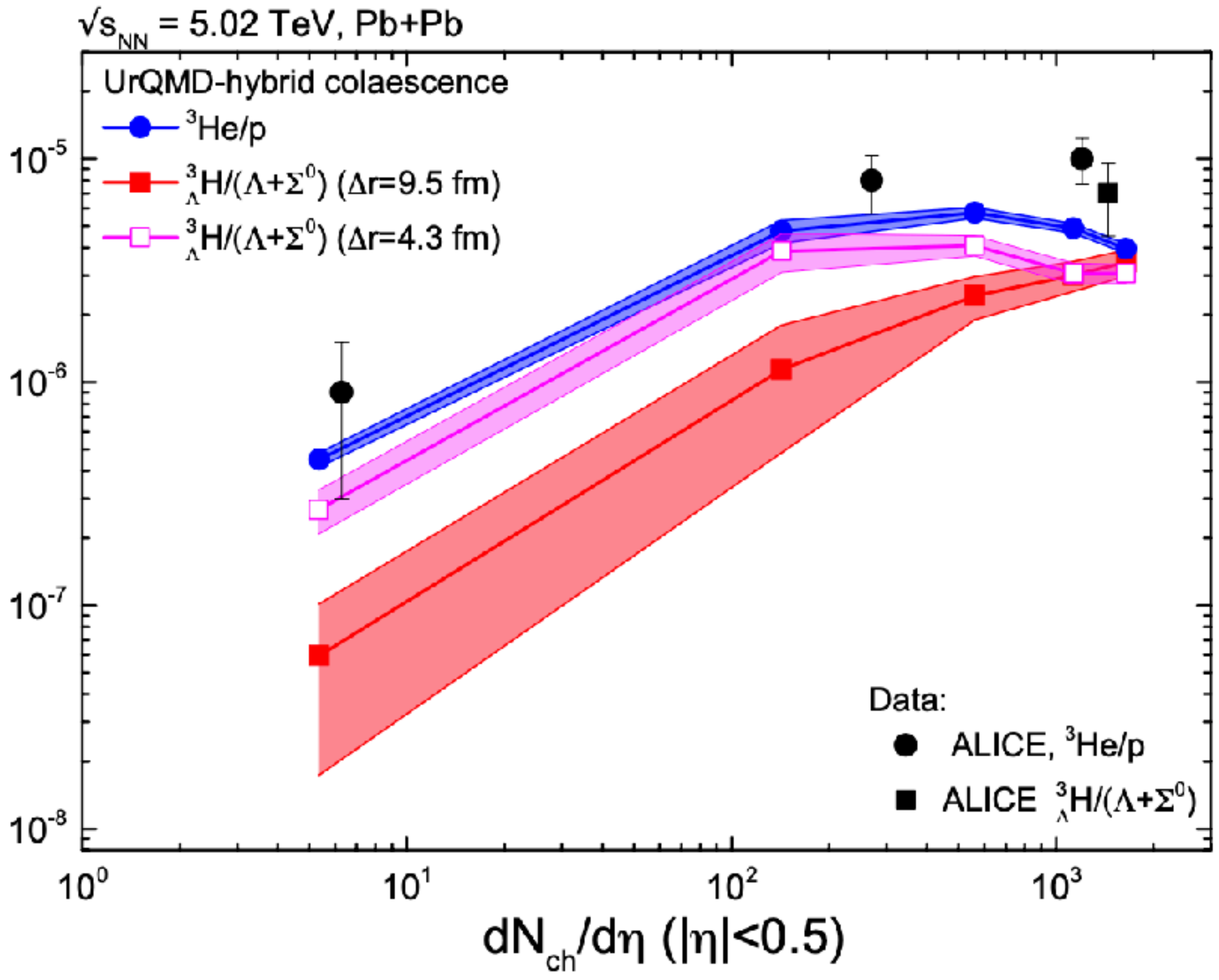
Large ratio $R_{\text{bound state}}/R_{\text{source}}$ result in large divergence of flow field and lead to reduced coalescence

Large source

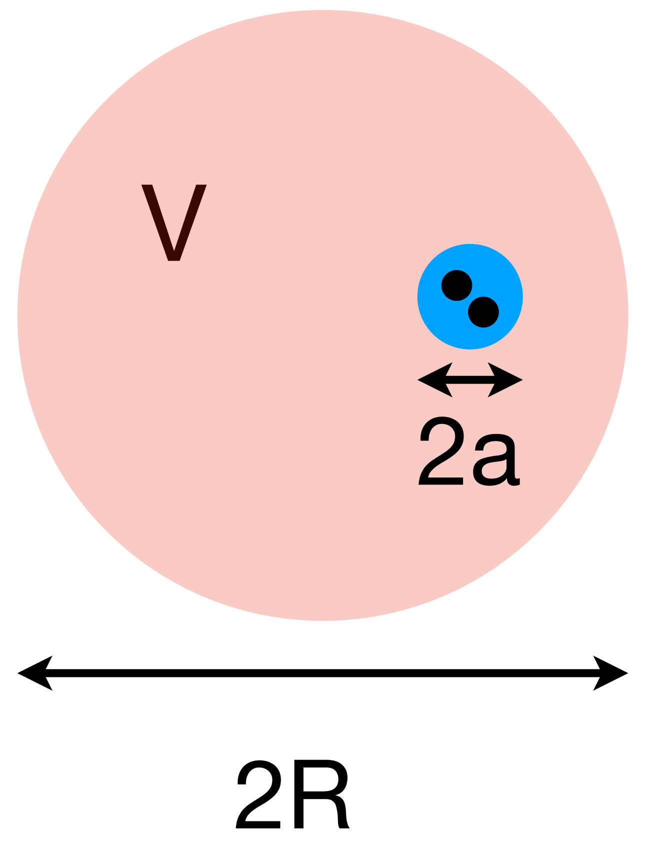


Small source





SHM *does* care about hadron size

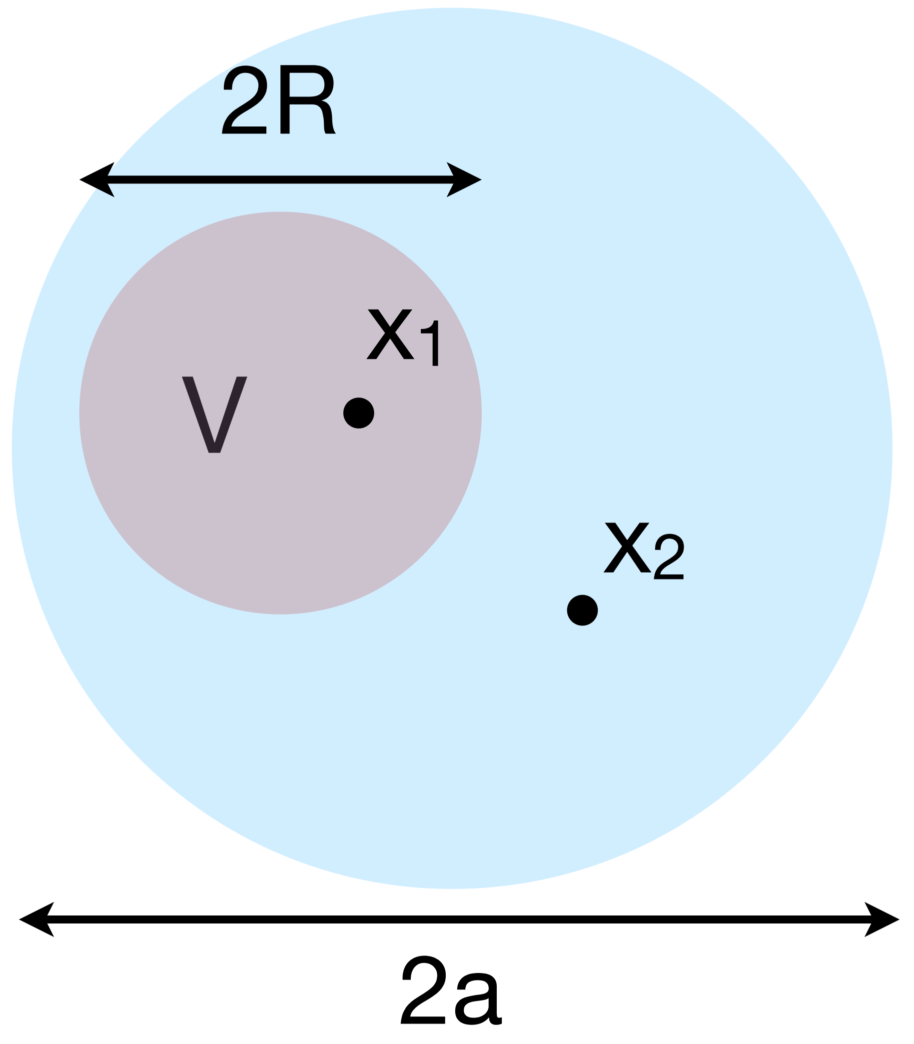


$$P_H = \mathrm{Tr}[e^{-H/T}\theta(V_H)] = \int d^3p\, \langle \psi_p | e^{-H/T} \theta_V | \psi_p \rangle$$

$$P_H = \int d^3p\, e^{-E_p/T} \int d^3x_1\, d^3x_2\, |\psi_p(x_1 - x_2)|^2 \theta_V(x_1)\theta_V(x_2)$$

When $a \ll R$, integrate out (x_1–x_2):

$$P_H \approx \int d^3p\, e^{-E_p/T} \int d^3X\, \theta_V(X) = V_H \int d^3p\, e^{-E_p/T}$$

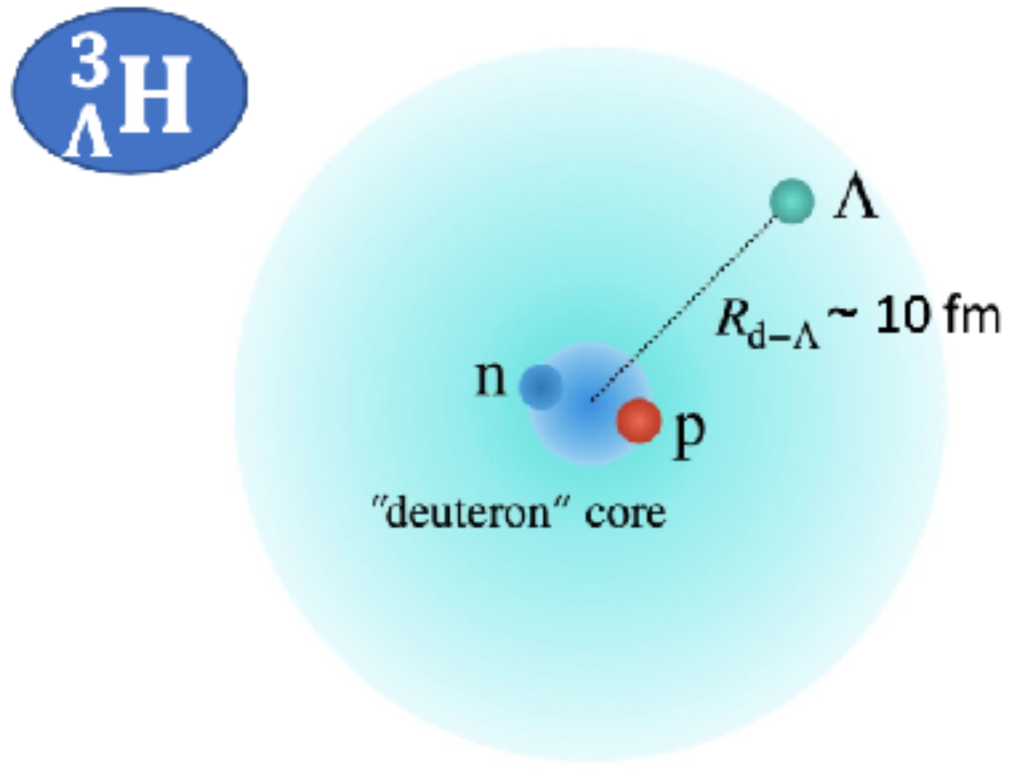


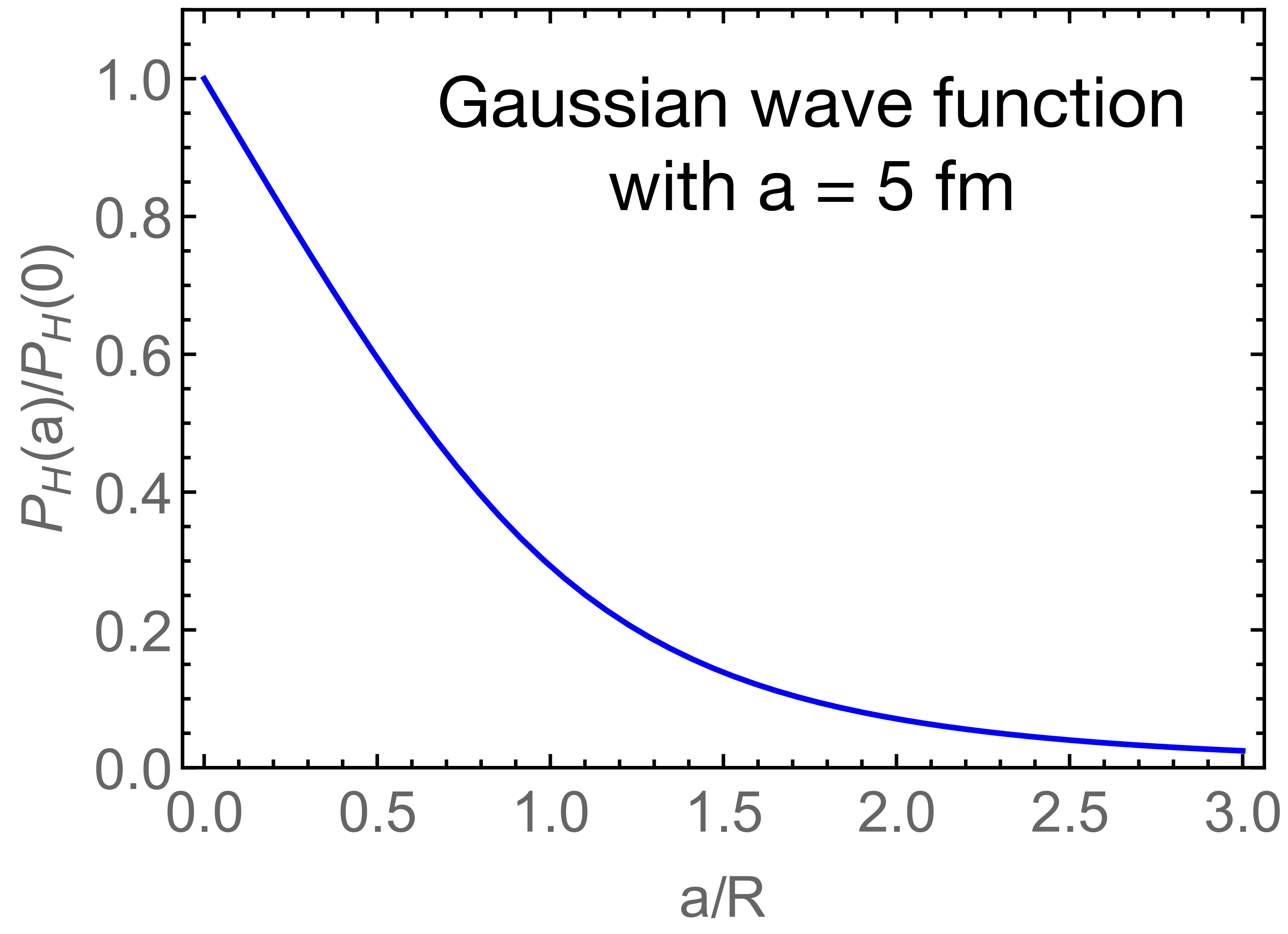
When $a \gg R$, requires $|x_1 - x_2| < R$:

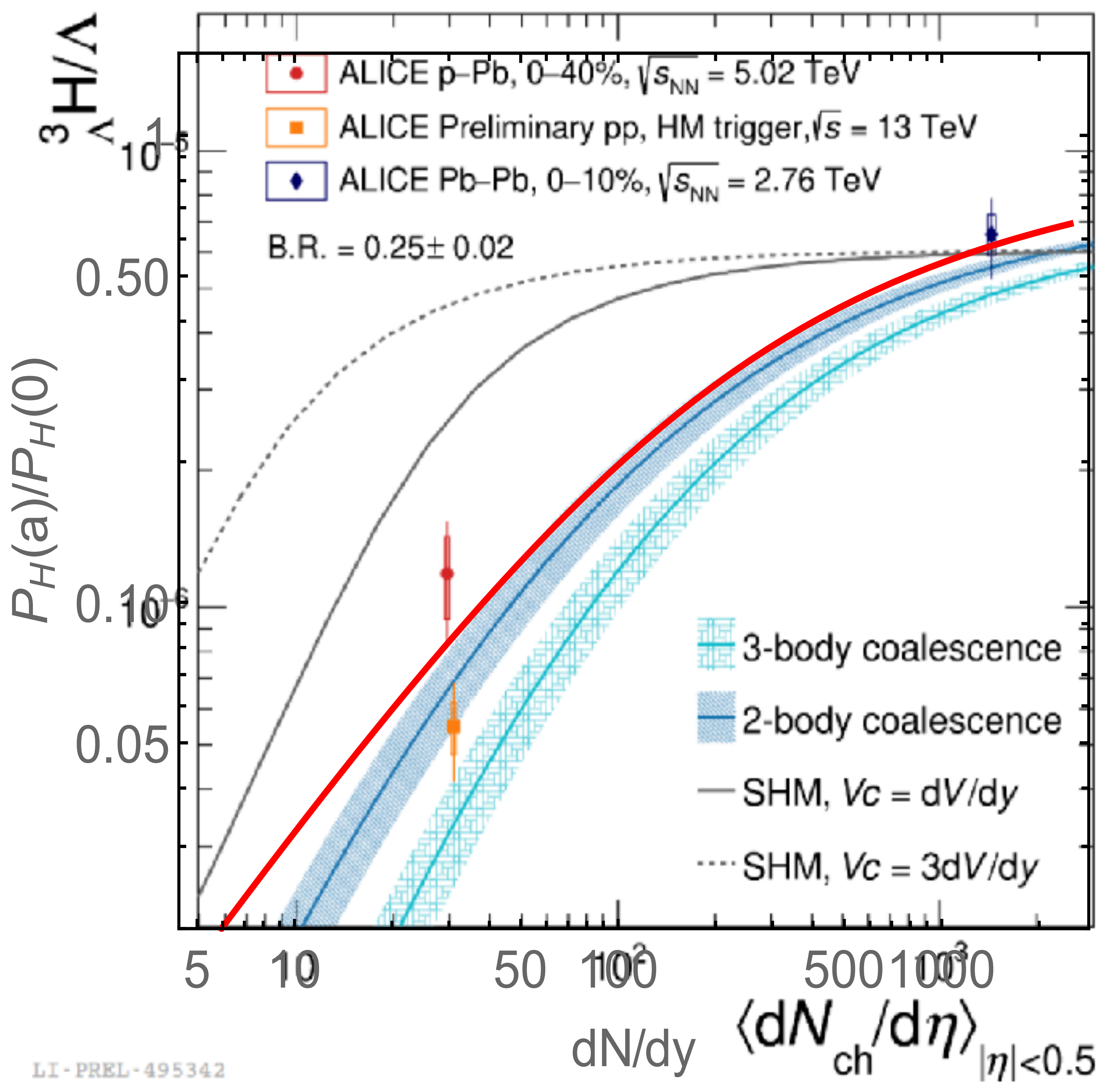
$$P_H \approx \int d^3p\, e^{-E_p/T} \int d^3x_1\, d^3x_2\, |\psi_p(0)|^2 \theta_V(x_1)\theta_V(x_2)$$

$$P_H \approx V_H^2 \int d^3p\, e^{-E_p/T} |\psi_p(0)|^2 \propto \frac{V_H^2}{a^3} \int d^3p\, e^{-E_p/T}$$

Particle size matters







Particle size effect similar to 2-body coalescence.

(n-p) distance is small compared to fireball radius.

Quarkonium

Take-aways from B. Gossiaux's overview talk:

- Theoretical description of Upsilon suppression is reaching a mature state with general agreement about the formal framework (open quantum systems + NRQCD)
- What is now needed are high statistics data - these will be taken in the next 3-4 years
- Can the effect of color screening (Debye mass) be isolated?
- Upsilon suppression at p_T > few GeV requires more theoretical work (maybe + SCET?)

- Charmonium suppression is extraordinarily complex. Many mechanisms contribute:
  - Nuclear shadowing (nPDFs) and other cold nuclear matter effects
  - Color screening and thermal ionization
  - Regeneration, especially at LHC, influenced by charm thermalization
  - Gluon fragmentation dominates at high p_T, influenced by parton energy loss
- Comprehensive, rigorous theoretical framework does not yet exist (m_c not large enough)

J/ψ regeneration

Statistical hadronization model seems to work quite well for J/ψ and ψ(2s) in Pb+Pb at LHC.
This raises many questions:

- Does it prove that regeneration occurs at the QGP-HRG phase boundary?
- If not, why doesn't J/ψ decouple earlier and exhibit a higher temperature?
- Is there regeneration during the HRG phase?
- How is J/ψ polarized? Does regeneration at T_{chem} fully explain its elliptic flow?

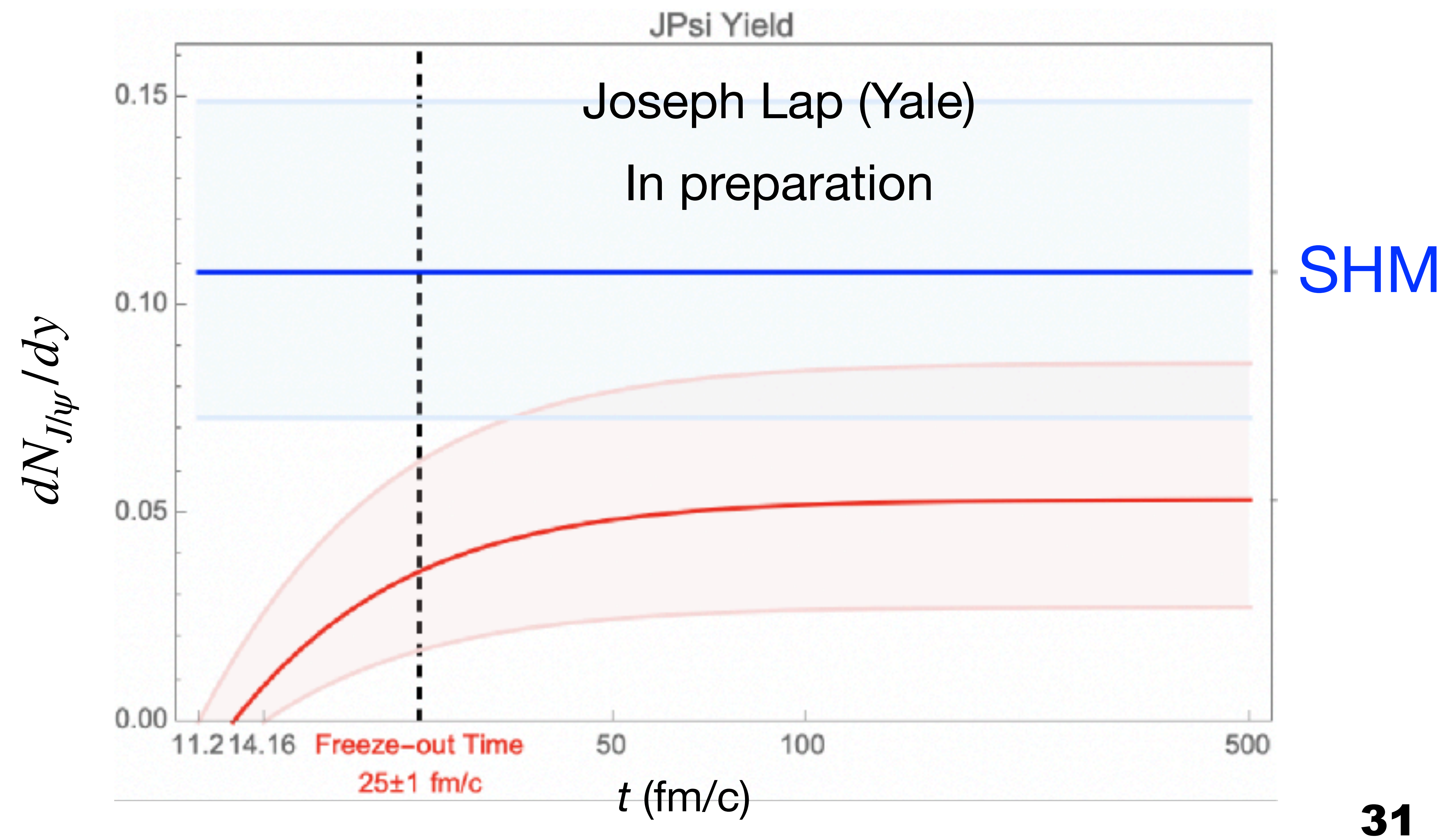
Hadronic reactions, e.g.

$D^* + \bar{D} \to J/\psi + \pi,\ D_s^* + \bar{D} \to J/\psi + K$

are exothermic and have 1-3 mb cross sections
(Abreu et al., PRC 97 (2017) 044902).

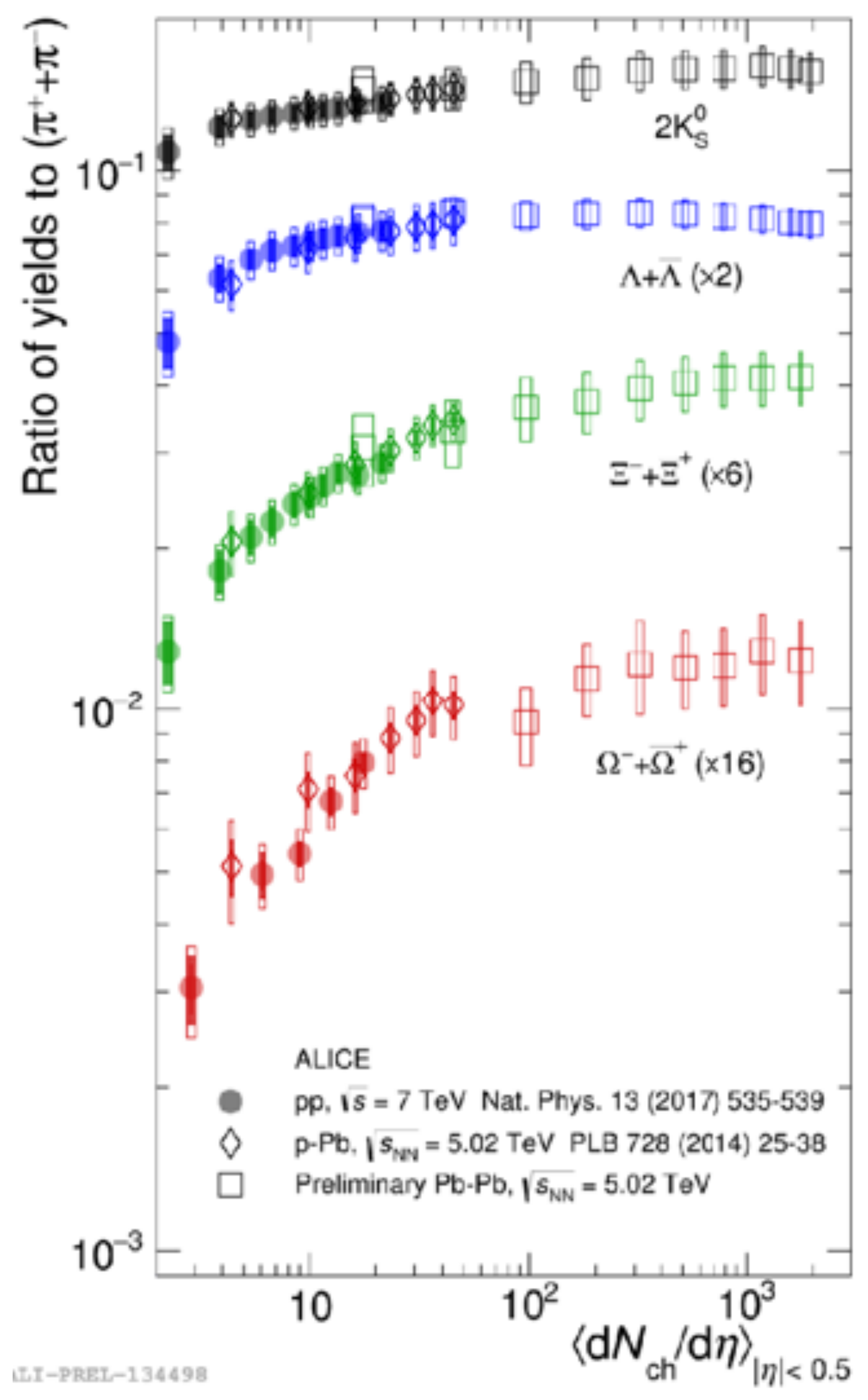
For $\lambda_c \sim 30$ they lead to substantial J/ψ regeneration.

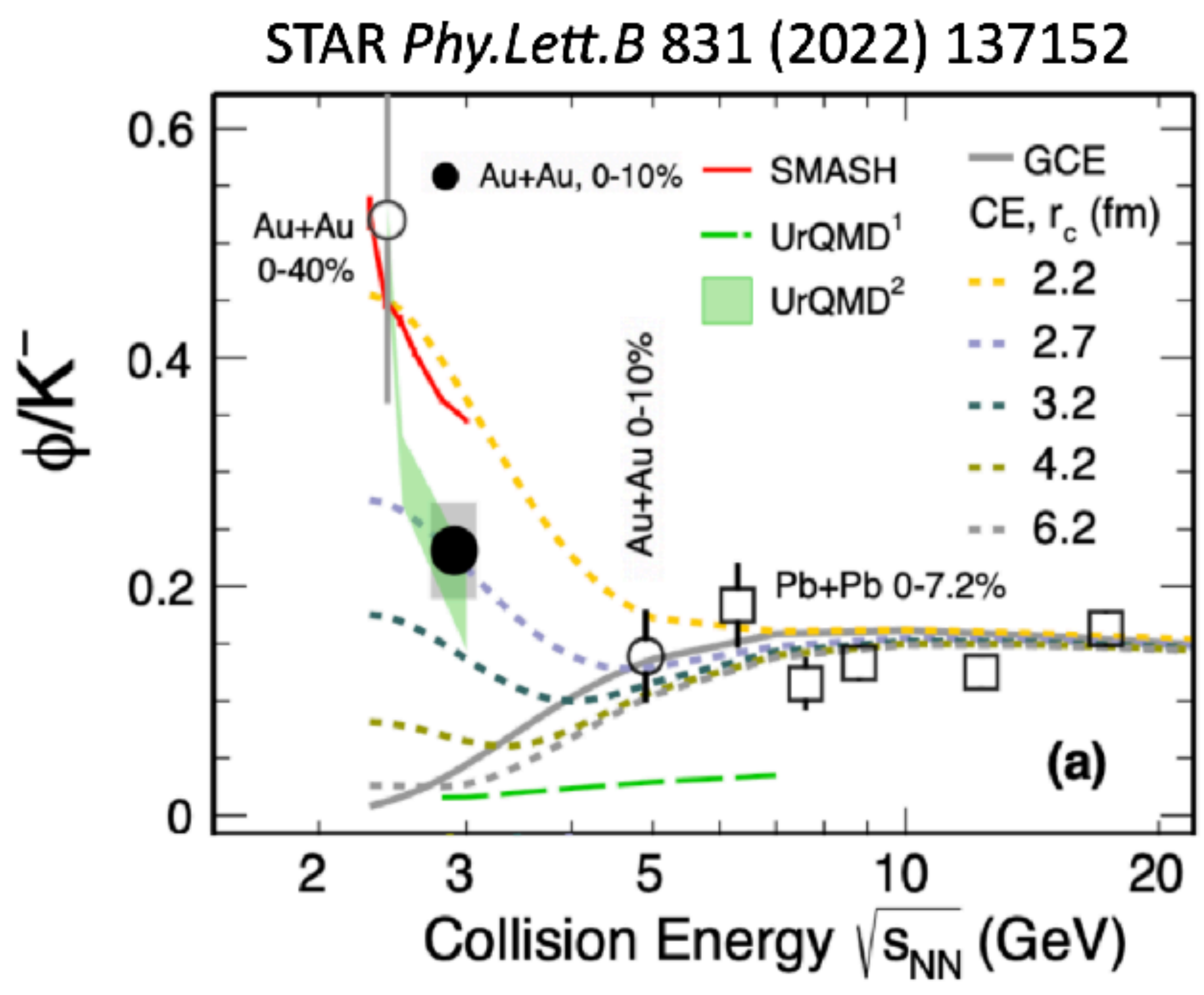
$$\sum_{ij,h} \langle \sigma_{D_i+D_j \to J/\Psi+h} \rangle \frac{dN_{D_i}}{dy} \frac{dN_{D_j}}{dy} = (27.4 \pm 14.1)\ \mathrm{fm}^2 .$$



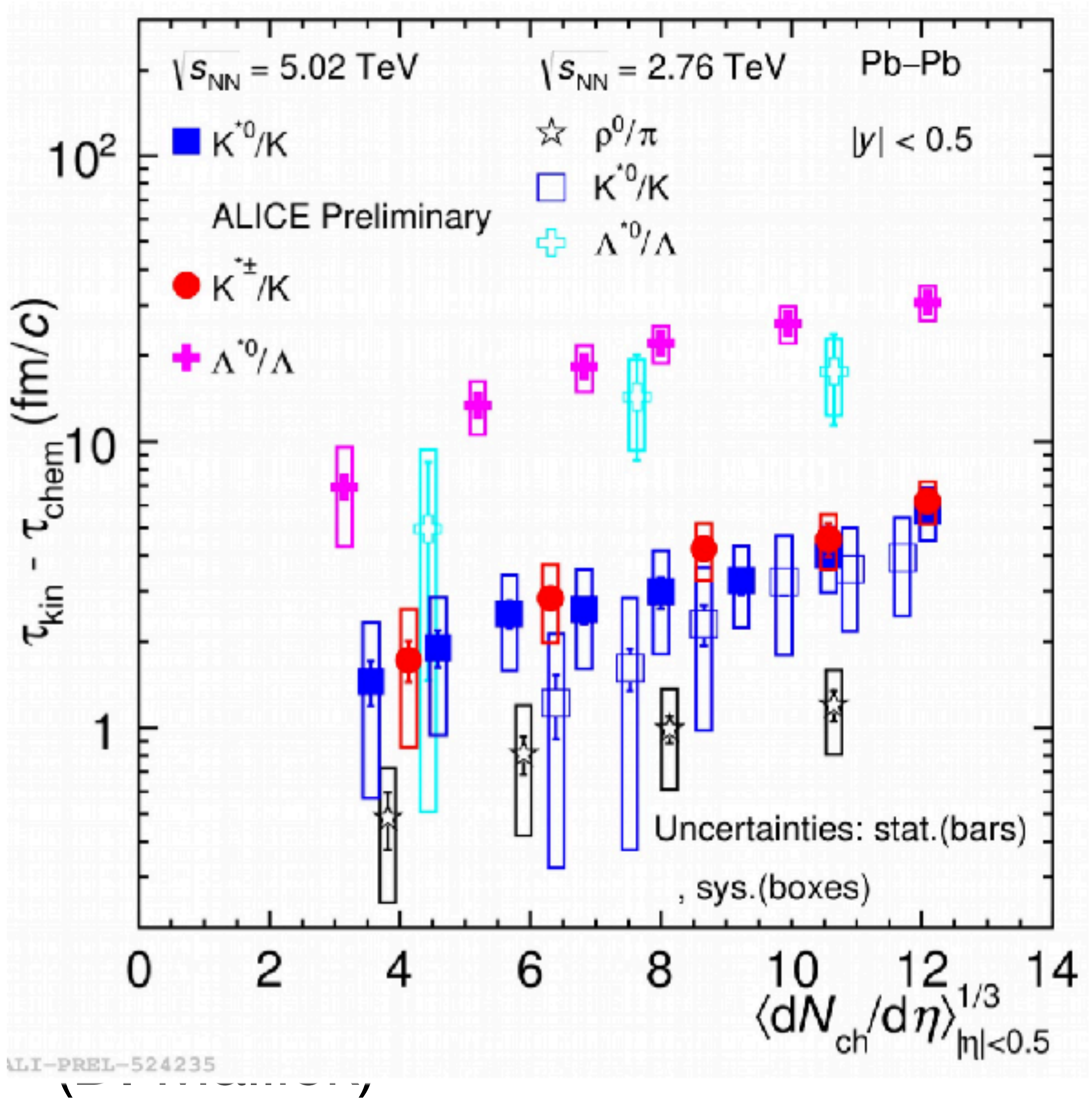
Strangeness!

- There are plenty of data for identified strange hadrons, including ϕ, for many sytems
- Theoretical activity has lately focused on semi-microscopic phenomenological models that are applicable to small systems, including p+p
- See Livio Bianchi's overview talk this morning





Life-time of hadronic phase



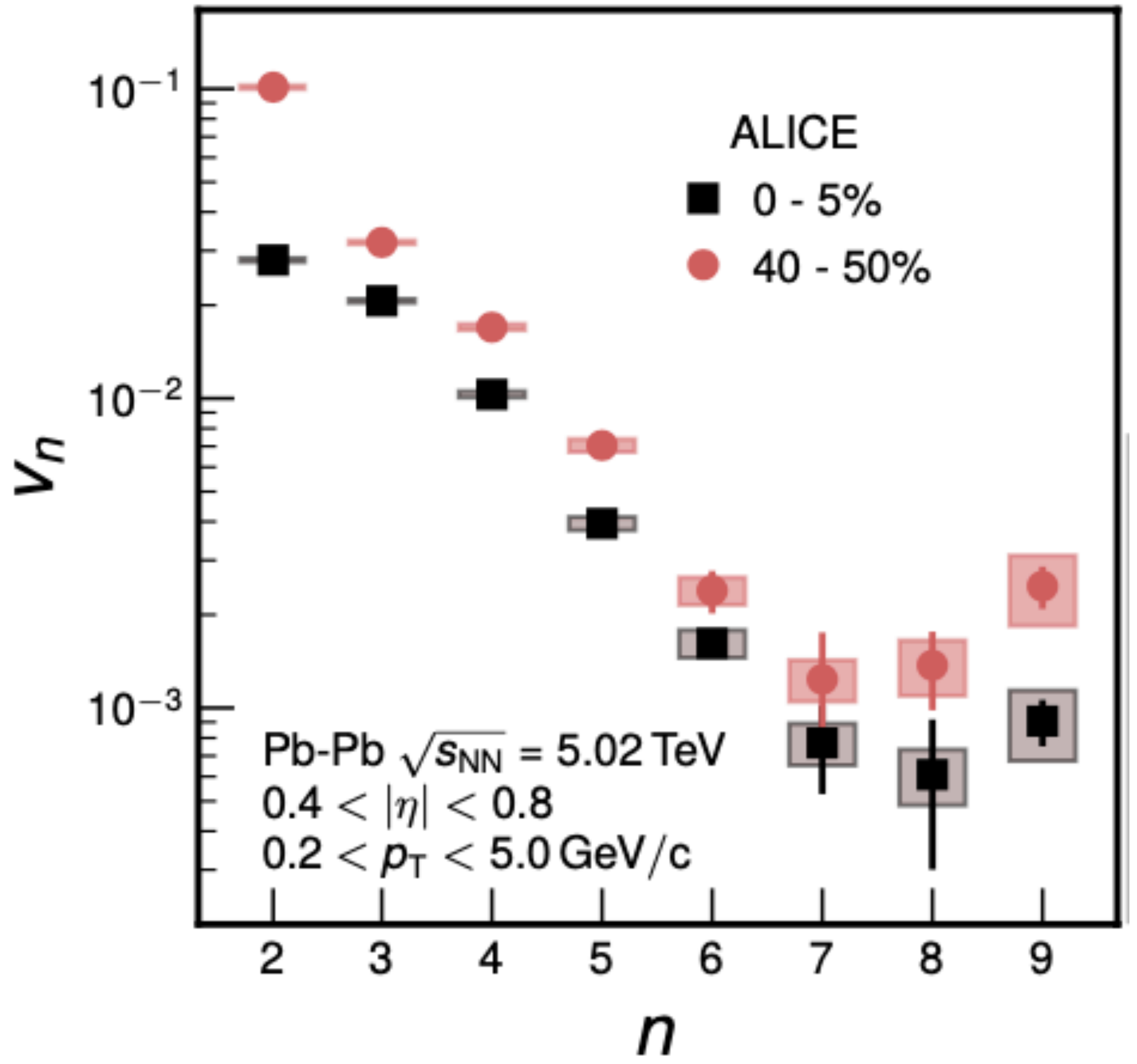
Bayesian analysis

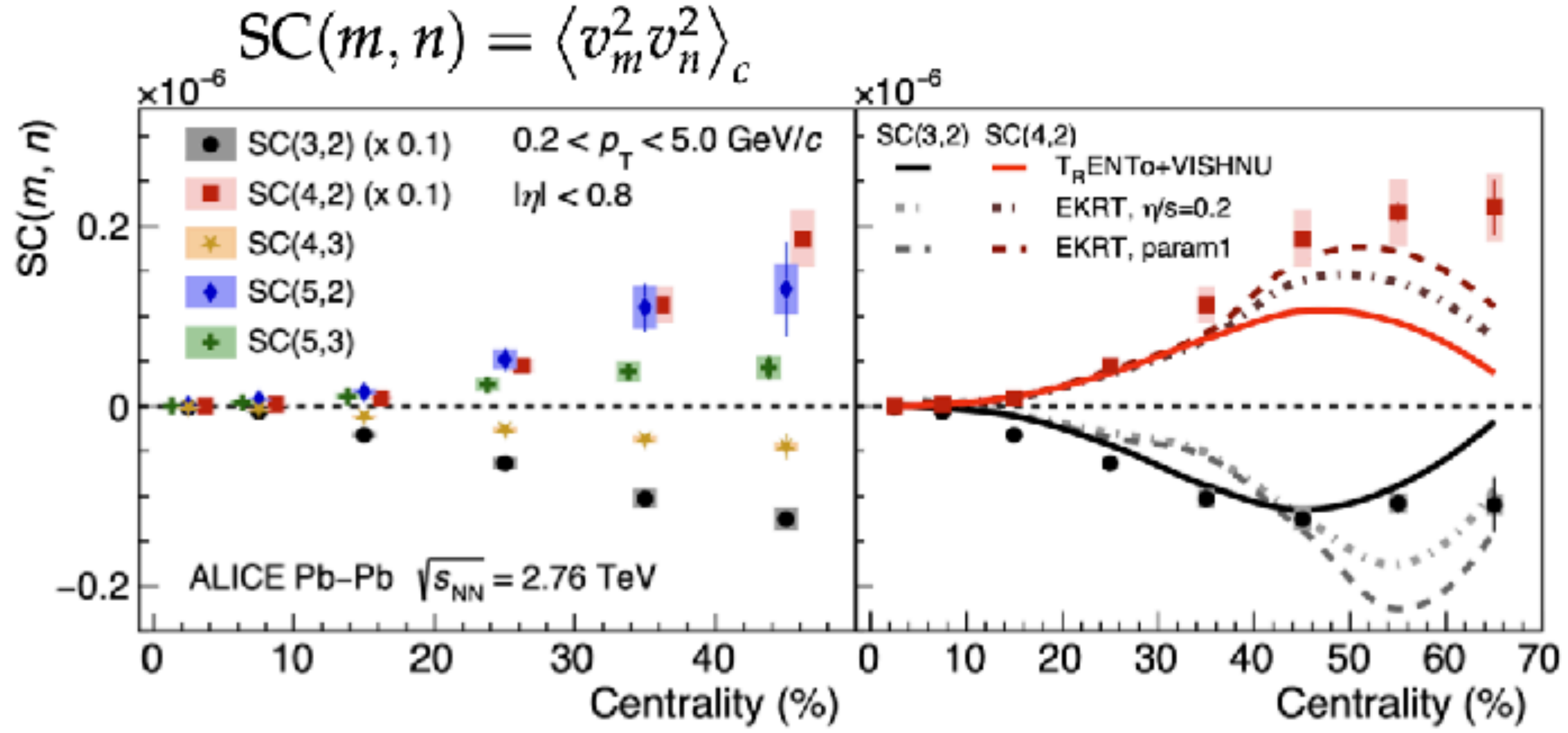
- Bayesian model-to-data comparison is becoming ever more ambitious (DJ Kim's talk)
- Requires substantial computing resources, e.g., O(10^5) CPU-years for 8 parameters
- New Jyväskylä analysis uses an much expanded set of observables

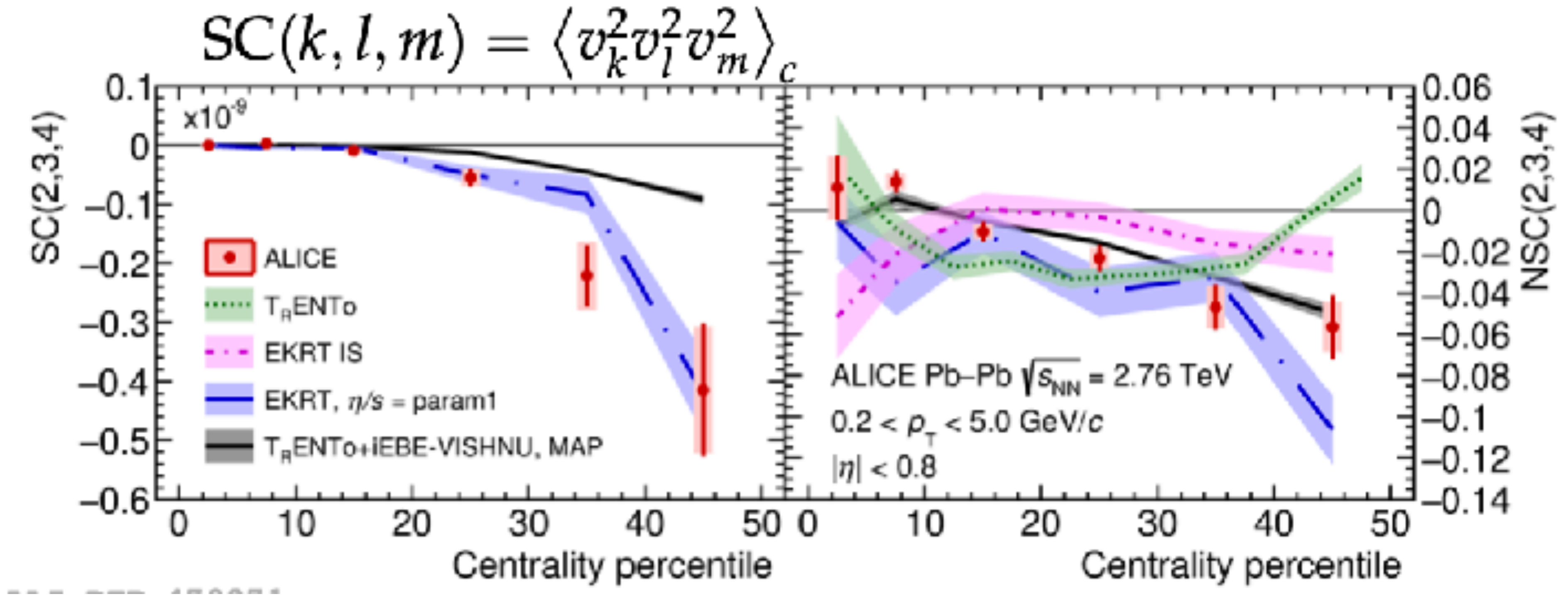
[2] Jyvaskyla (2022)

2.76 TeV
- N_{ch}
- NSC(3,2), NSC(4,2)
- NSC(2,3,4), NSC(2,3,5)
- PID[1] $\langle p_T \rangle$
- v_2 to v_4
- $\chi_{4,22}$ to $\chi_{6,mk}$
- $\rho_{4,22}$ to $\rho_{6,mk}$

5.02 TeV
- PID[2] mult. and N_{ch}
- NSC(3,2) to NSC(4,3)
- PID $\langle p_T \rangle$
- v_2 to v_7
- $\chi_{4,22}$ to $\chi_{6,mk}$
- $\rho_{4,22}$ to $\rho_{6,mk}$







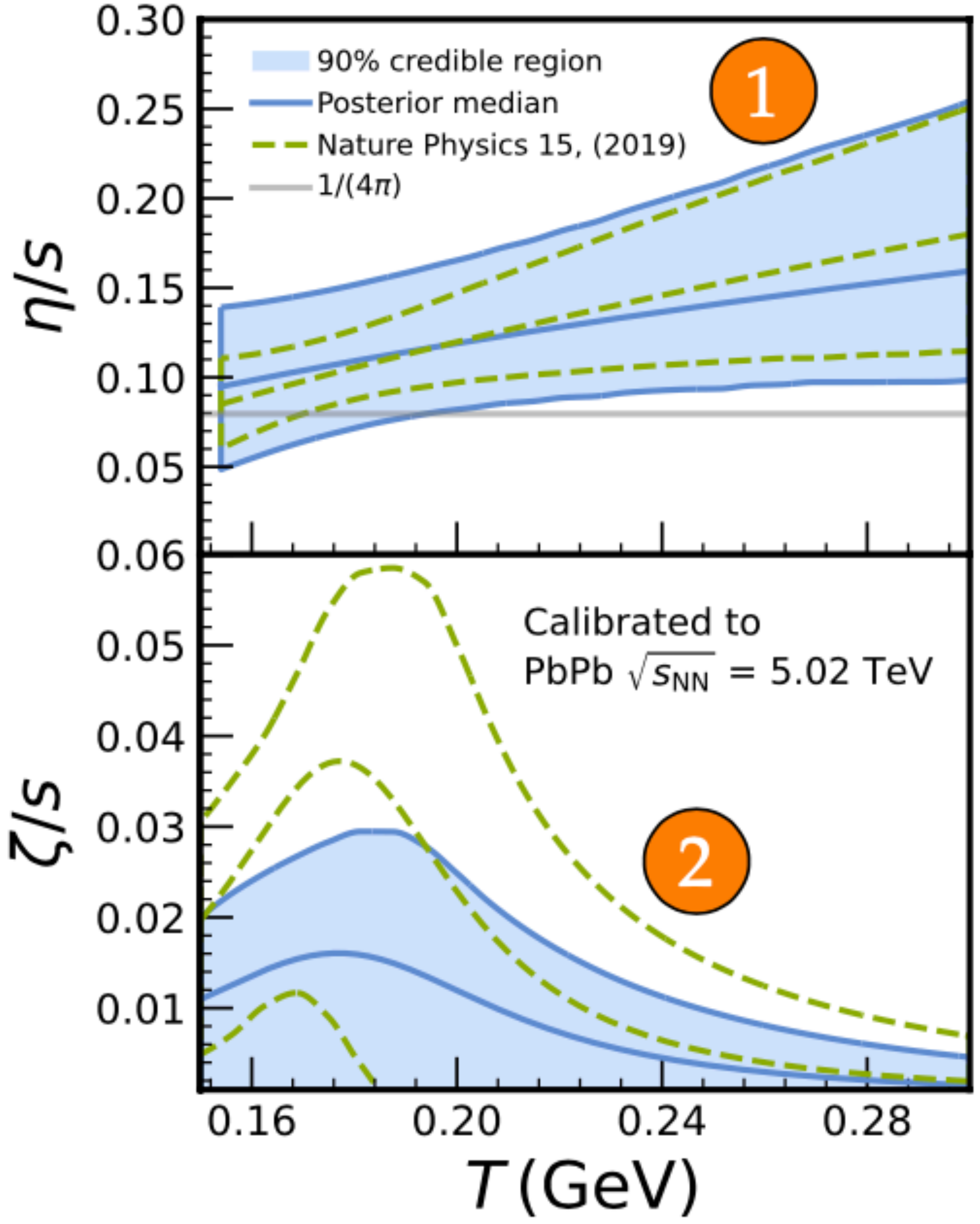
Observables sensitivity

- Together various flow observables cover the sensitivity for all components of transport properties.

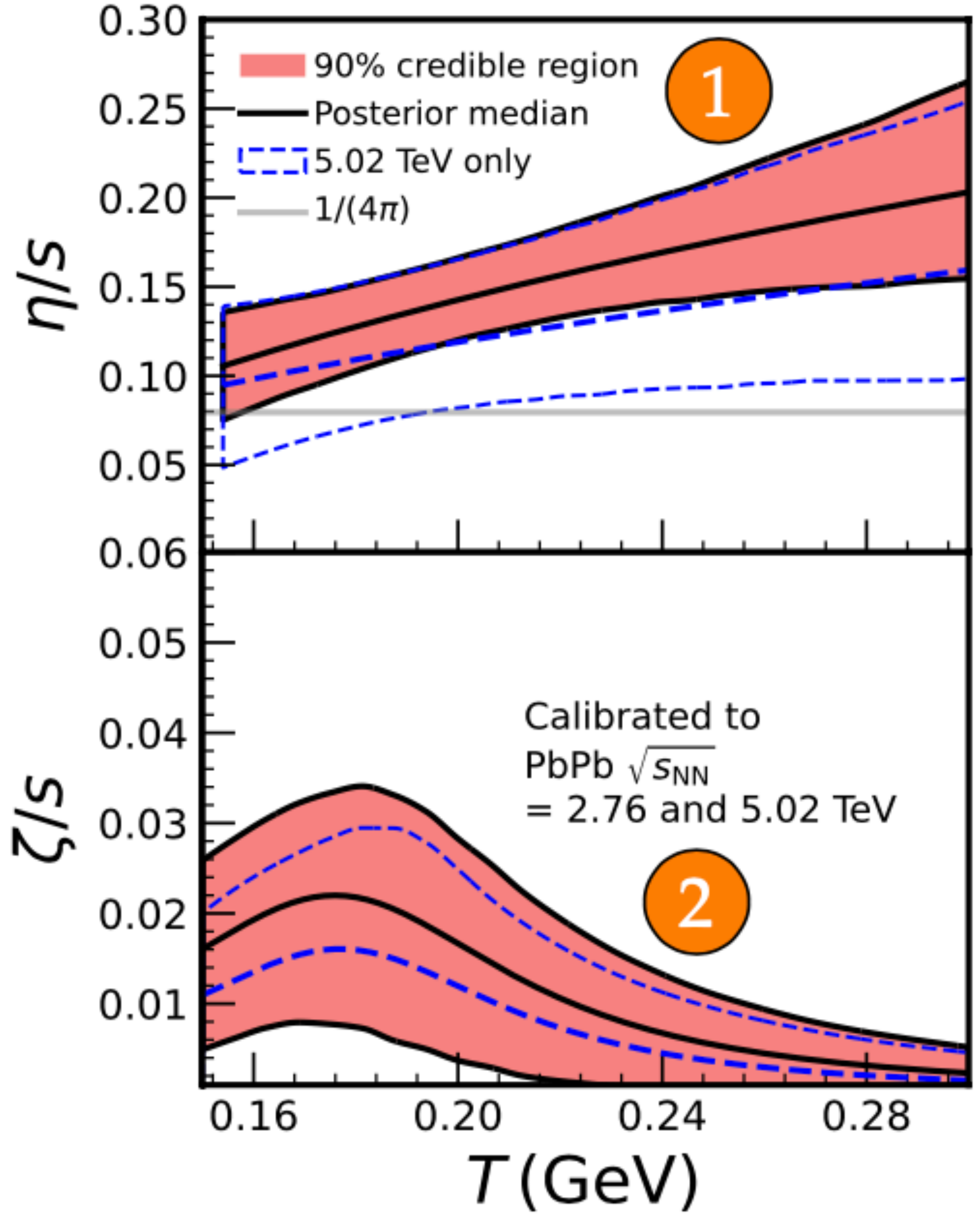
Name	Symbol	Measure	Sensitivity-stochastic approach
Flow coefficients	v_n	System expansion and anisotropy of the flow	Average $\langle \eta/s \rangle$ and $\zeta/s(T)$ peak
(Normalized) Symmetric cumulants	(N)SC(k, l, m)	Correlation between magnitudes of flow harmonics	$\eta/s(T)$ temperature dependence
Linear and non-linear contributions	$v_{n,\mathrm{L}}$, $v_{n,mk}$	Magnitude of the linear and non-linear contributions	$\eta/s(T)$ and initial conditions, not used
Non-linear flow mode coefficients	$\chi_{n,mk}$	Quantification of the non-linear response	$\eta/s(T)$ at the freeze-out
Symmetry-plane correlations	$\rho_{n,mk}$	Correlations between the directions of flow harmonics	$\eta/s(T)$

Improved analysis

2021 analysis - 5.02 TeV



2022 analysis - 2.76 + 5.02 TeV



- More data reduce uncertainties
- Higher v_n and nonlinear flow observables give stronger constraints
- Sensitivities of different observables quantified
- v_2 for 5.02 TeV too high
- 10-20% discrepancies for PID yields
- Improvements needed:
  - Initial state model
  - Hadron transport?

Instead of Conclusions:

Many thanks and congratulations to the organizers for conducting an inspiring conference

It was not easy by any stretch, and we all are grateful for their very hard work

I hope you found this survey of theory presentations useful