Chiral Magnetic Effect and Relativistic Heavy-Ion Collisions

Lecture in the student day

Strangeness in Quark Matter 2022

Busan, South Korea

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Ho-Ung Yee
(University of Illinois, Chicago)

Suggested Reviews

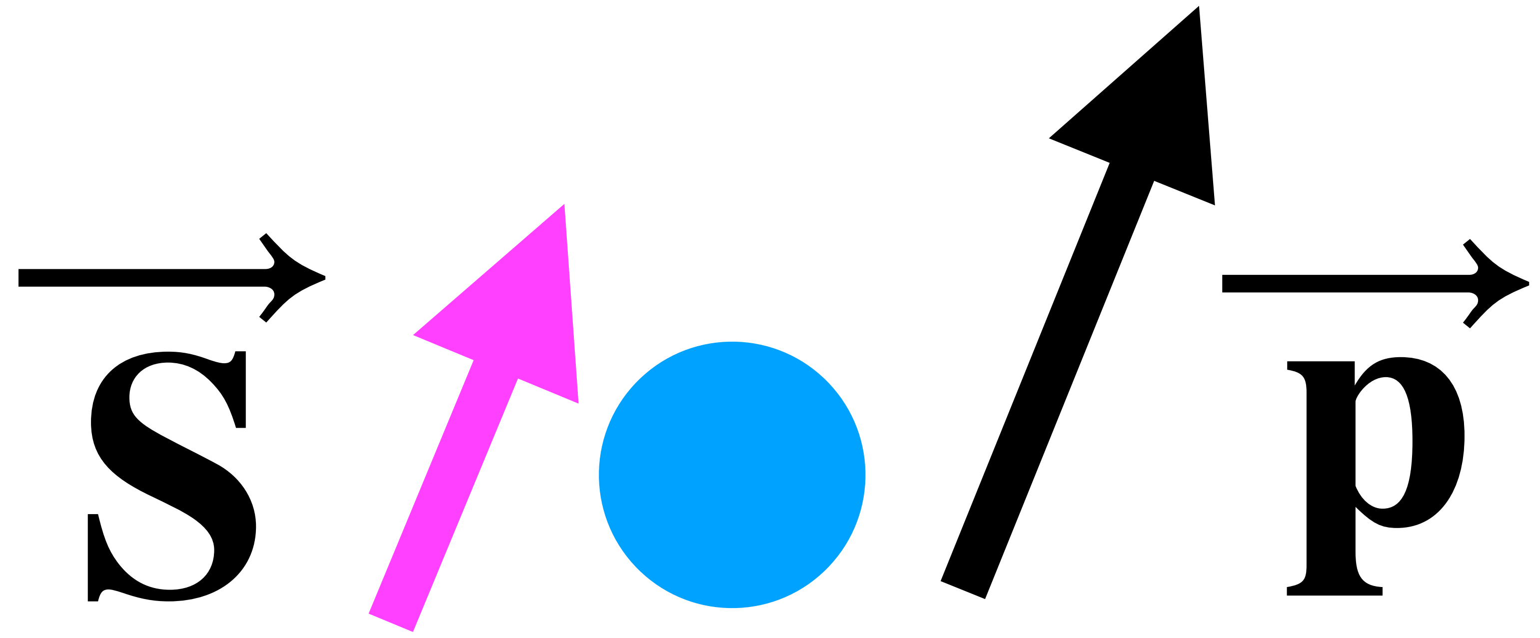
- Kharzeev -1312.3348 - introduction and history of CME
- Fukushima - 1209.5064 - early anecdotes and conceptual issues
- Kharzeev-Liao-Voloshin-Wang -1511.04050 - comprehensive
- CME Task Force Report -1608.00982 - concise summary
- Kharzeev-Liao -2102.06623 - review in Nature Physics
- Hattori-Huang -1609.00747 - broad topics
- Li-Wang - 2002.10397 - review of experiments

The lecture will be focused on the basics,
aiming to motivate you to search deeper in literature

Plan

- Chiral Symmetry of QCD and Chiral Anomaly
- Chiral Magnetic Effect (CME) - Theory
- CME in Heavy-Ion Collisions - Experiments
- Chiral Magnetic Wave (CMW)
- Questions and Discussions

Relativistic Massless Fermions: Helicity

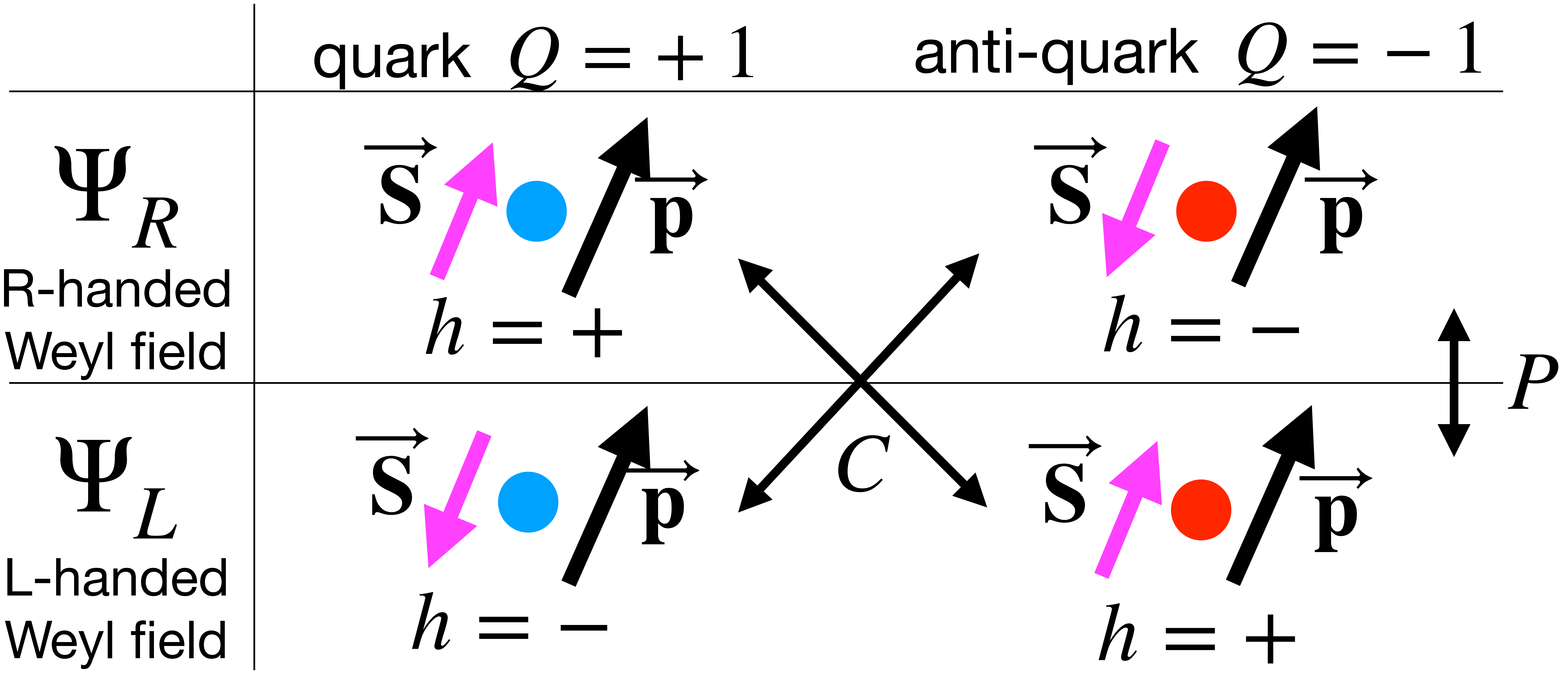


- Helicity: $h = \overrightarrow{\mathbf{S}} \cdot \hat{\mathbf{p}} = \pm \dfrac{\hbar}{2}$: $h = +\hbar/2$ (Right-Handed), $h = -\hbar/2$ (Left-Handed)

- Under the parity $\overrightarrow{x} \rightarrow -\overrightarrow{x}$ transformation (P), $\overrightarrow{\mathbf{S}} \rightarrow \overrightarrow{\mathbf{S}}$ and $\overrightarrow{\mathbf{p}} \rightarrow -\overrightarrow{\mathbf{p}}$, and h is P-odd

- Any observable that correlates $\overrightarrow{\mathbf{S}}$ and $\overrightarrow{\mathbf{p}}$ breaks P

QCD Quark Field : $\Psi = (\Psi_R, \Psi_L)$ (Dirac)



Chiral Symmetry of QCD

$$U(1)_R \times U(1)_L$$

$\Psi_R \to \Psi_R e^{i\alpha}$

$\Psi_L \to \Psi_L e^{i\alpha}$

$N_R = N_R^+ - N_R^-$ conservation

$N_L = N_L^+ - N_L^-$ conservation

$$U(1)_V = U(1)_R + U(1)_L, \quad U(1)_A = U(1)_R - U(1)_L$$

Vector charge $N_V = N_R + N_L$: Net quark number

Axial $U(1)_A$ Symmetry

$$N_A = N_R - N_L = N_R^+ + N_L^- - (N_L^+ + N_R^-)$$

Total number of Helicity $h = +$ fermions

Total number of Helicity $h = -$ fermions

Axial charge is Parity-odd, but C-even (CP-odd)
(it doesn't care whether quarks or anti-quarks)
It is simply the Net Helicity

For N_F flavors, we have an extended symmetry

$$U(N_F)_R \times U(N_F)_L$$

We usually extract $U(1)_V \times U(1)_A$ part :

$$SU(N_F)_R \times SU(N_F)_L \times U(1)_V \times U(1)_A$$

since $U(1)_A$ part is violated quantum mechanically, called Chiral Anomaly (we will come back to this later)

Quark Mass Breaks Chiral Symmetry

$$m_q \bar{\Psi}\Psi = m_q(\bar{\Psi}_L \Psi_R + \mathrm{h.c.})$$

This is invariant only under $U(N_F)_R = U(N_F)_L$

$$U(N_F)_R \times U(N_F)_L \longrightarrow U(N_F)_V$$

QCD Chiral Symmetry is an Approximate Symmetry
since $m_q \approx 5\,\mathrm{MeV} \ll \Lambda_{QCD} \sim 1\,\mathrm{GeV}$

Spontaneous Breaking of Chiral Symmetry

Even in the absence of quark mass and chiral anomaly, the QCD vacuum breaks chiral symmetry dynamically,

$$\langle \bar{\Psi}\Psi \rangle = \langle \bar{\Psi}_L \Psi_R \rangle + \mathrm{h.c.} \approx (1\,\mathrm{GeV})^3$$

(Chiral Condensate)

$$U(N_F)_R \times U(N_F)_L \longrightarrow U(N_F)_V$$

Why does this have to be non-perturbative?

Let's look at the operator more explicitly in $\mathbf{p}$-space

$$\Psi_R = \int_{\mathbf{p}} u_R(\mathbf{p})a_R(\mathbf{p})e^{i\mathbf{p}\cdot\mathbf{x}} + v_L(\mathbf{p})b_L^\dagger(\mathbf{p})e^{-i\mathbf{p}\cdot\mathbf{x}}$$

$$\Psi_L = \int_{\mathbf{p}} u_L(\mathbf{p})a_L(\mathbf{p})e^{i\mathbf{p}\cdot\mathbf{x}} + v_R(\mathbf{p})b_R^\dagger(\mathbf{p})e^{-i\mathbf{p}\cdot\mathbf{x}}$$

$u_{R/L}, v_{R/L}$ = Spinor Wave Functions

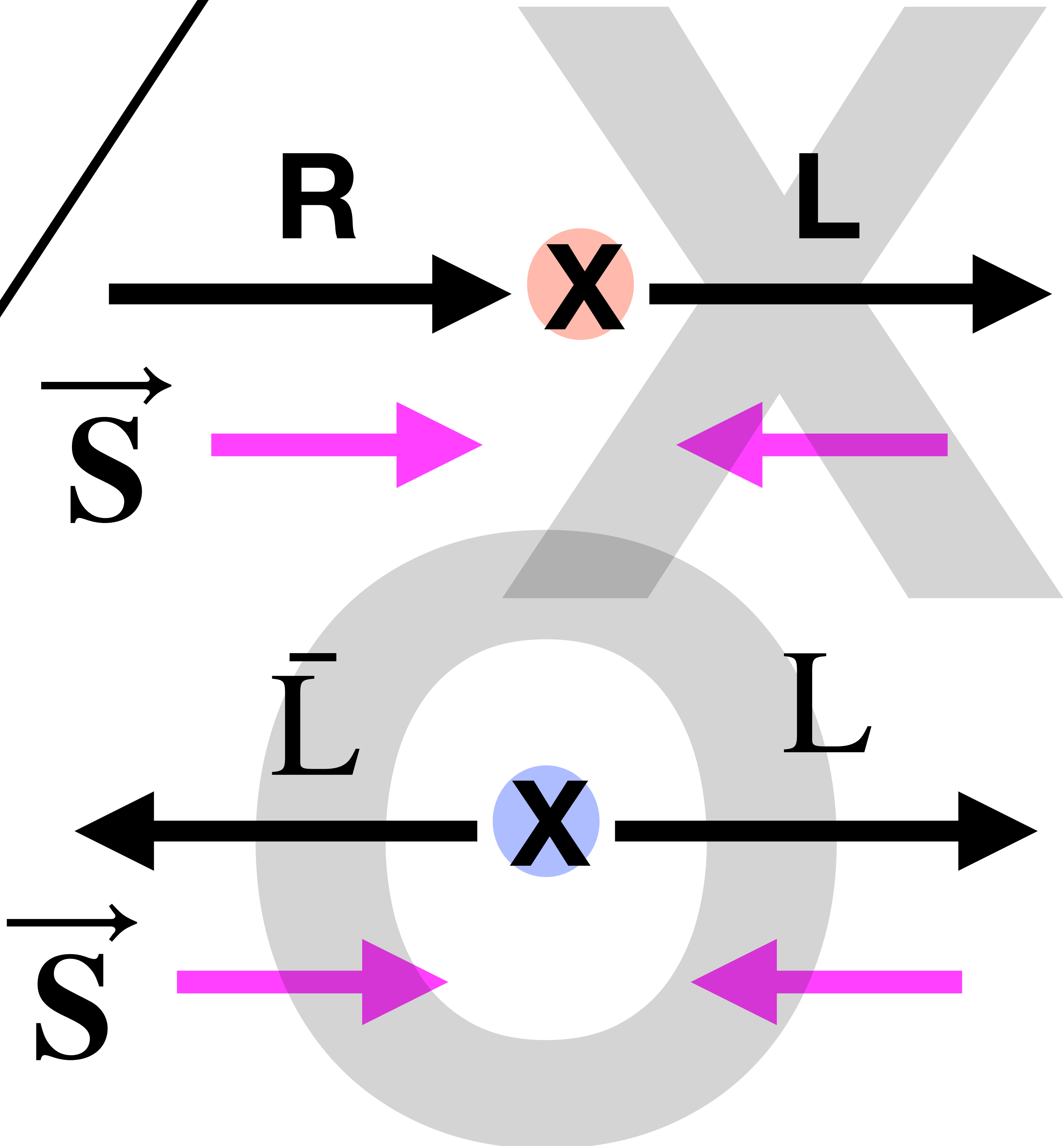
$a^\dagger_{L/R}, b^\dagger_{L/R}$ = Quark, Anti-Quark Creation Operators

$$\bar{\Psi}_L\Psi_R = \int_{\mathbf{p}} \bar{u}_L(\mathbf{p})u_R(\mathbf{p})a_L^\dagger(\mathbf{p})a_R(\mathbf{p}) + \bar{v}_R(\mathbf{p})v_L(\mathbf{p})b_R^\dagger(\mathbf{p})b_L(\mathbf{p})$$

$$+\bar{u}_L(\mathbf{p})v_L(-\mathbf{p})a_L^\dagger(\mathbf{p})b_L^\dagger(-\mathbf{p}) + \bar{v}_R(\mathbf{p})u_R(-\mathbf{p})b_R(\mathbf{p})a_R(-\mathbf{p})$$

$$\bar{u}_L(\mathbf{p})u_R(\mathbf{p}) = 0$$

$$\bar{u}_L(\mathbf{p})v_L(-\mathbf{p}) \neq 0$$

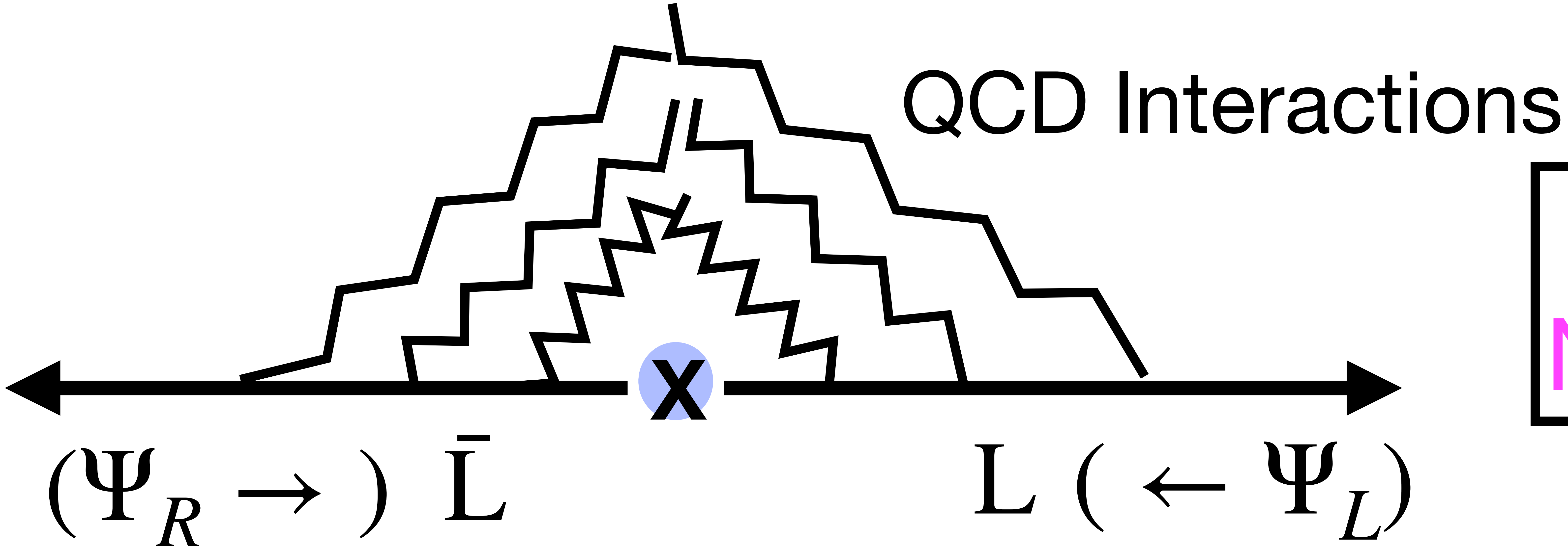


Spin flip is forbidden by angular momentum conservation

Creation of L-handed fermion and anti-fermion pair back-to-back

$$\langle \bar{\Psi}_L \Psi_R \rangle = \langle a_L^\dagger(\mathbf{p}) b_L^\dagger(-\mathbf{p}) \rangle + \langle a_R(\mathbf{p}) b_R(-\mathbf{p}) \rangle$$

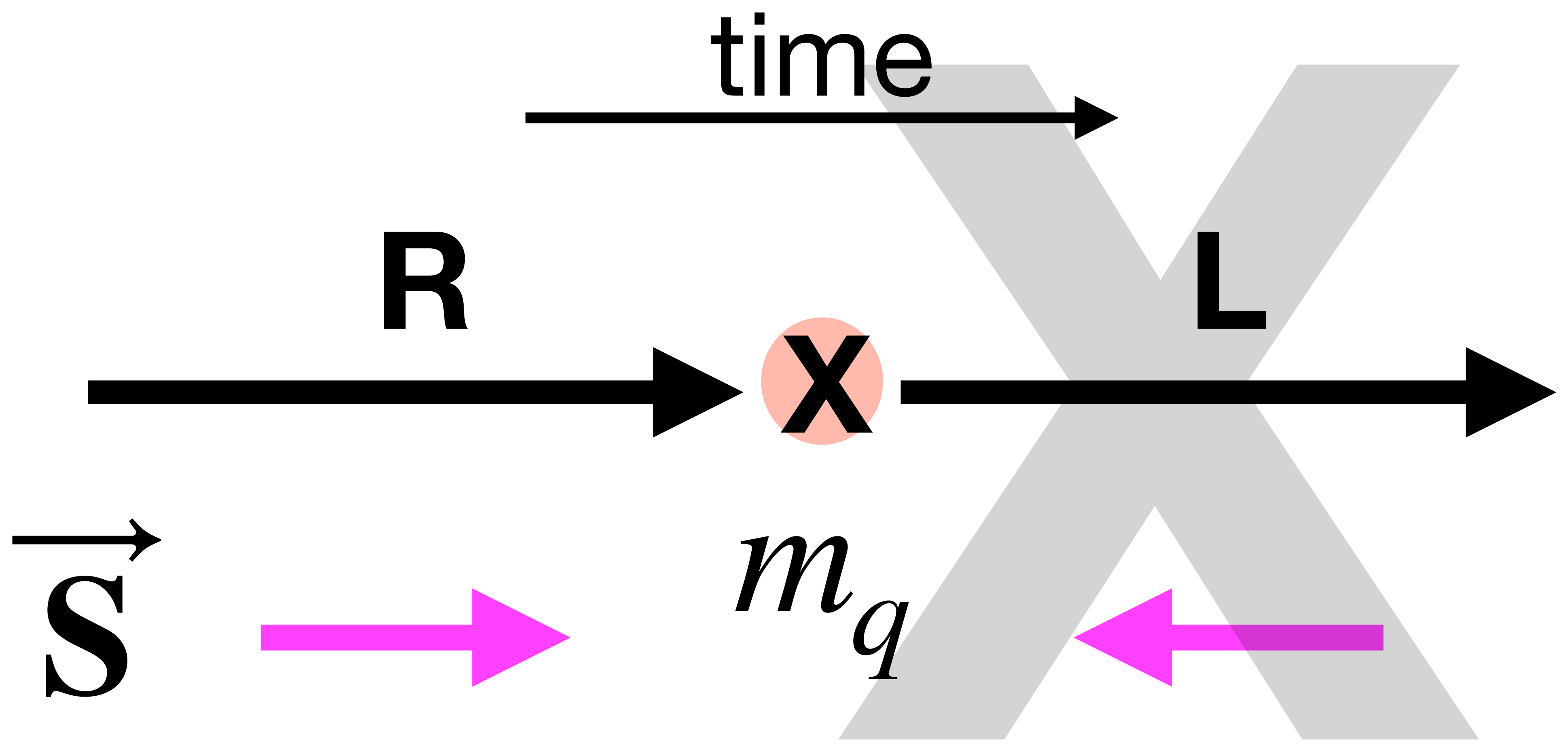
Similar to Cooper Pair $\Delta = \langle a^\dagger a^\dagger \rangle$ in Superconductivity, but with Quark-Antiquark Pair



Possible only Non-Perturbatively

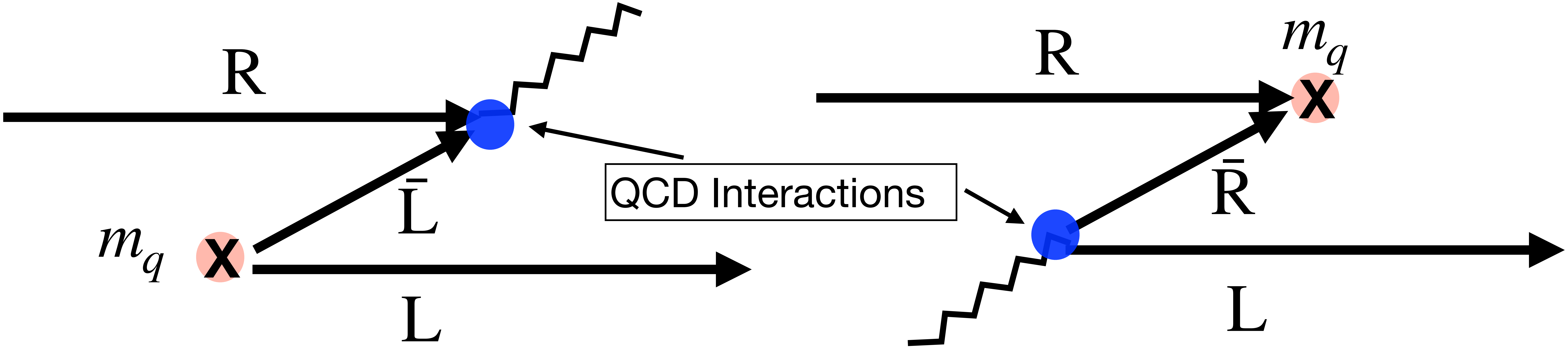
Chirality Flipping by Quark Mass

(Grabowska-Kaplan-Reddy, 1409.3602)

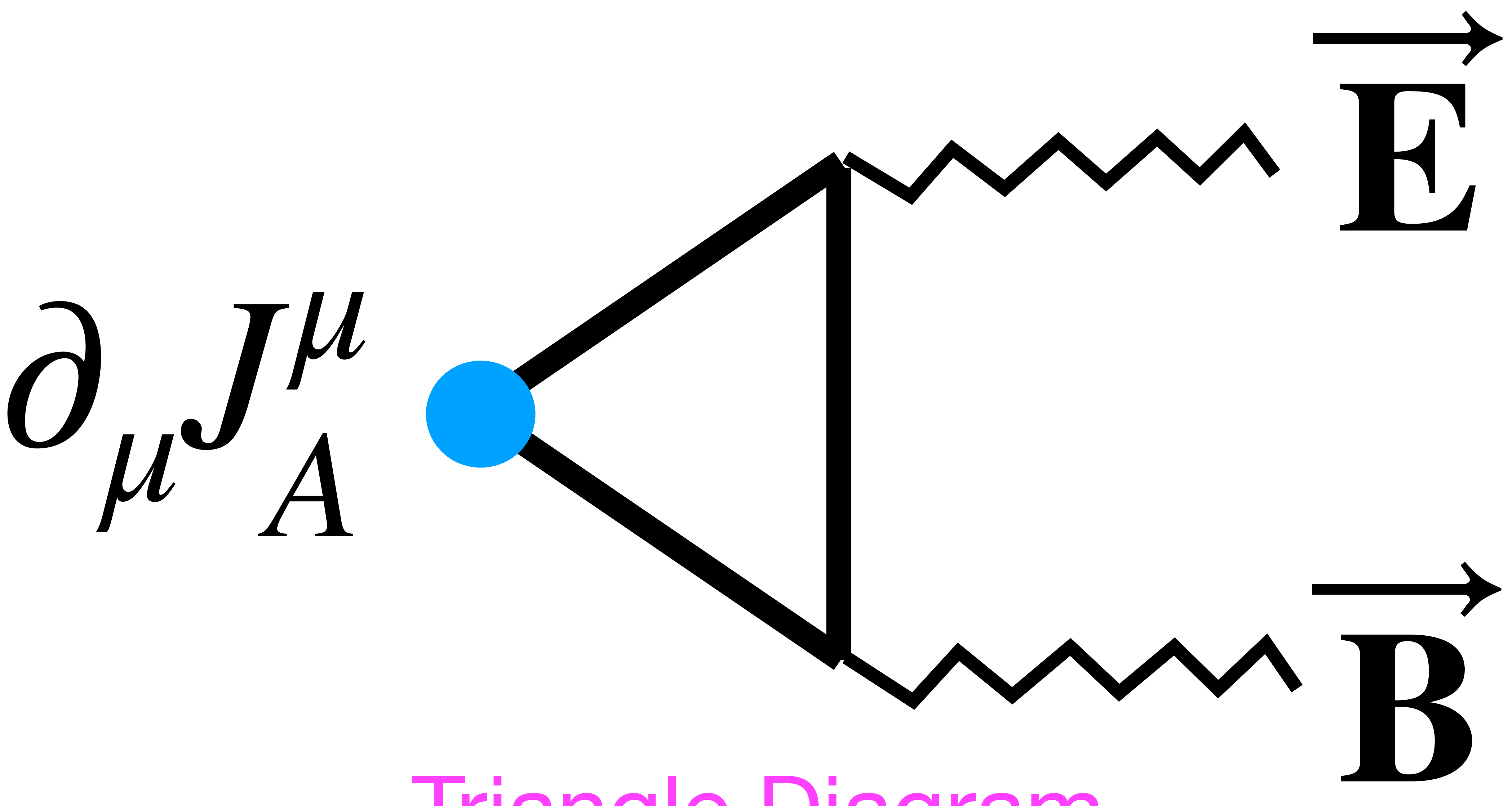


A simple ``mass insertion" cannot flip helicity, due to angular momentum conservation

We need QCD interactions and m_q to flip chirality



Chirality Anomaly of $U(1)_A$



Triangle Diagram

QCD Gluons
Topological Fluctuations, e.g.
Instantons in vacuum
Sphalerons in high T

$$\frac{dN_A}{dt} = \frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B} + \frac{g^2}{4\pi^2}\mathbf{E}_g\cdot\mathbf{B}_g$$

$U(1)_A$ is strongly broken in QCD vacuum
$m_{\eta'} \approx 950\,\mathrm{MeV},\ m_{\pi^0,\pi^\pm} \approx 140\,\mathrm{MeV}$

Note
$m_u \approx 2\,\mathrm{MeV},\ m_d \approx 5\,\mathrm{MeV}$
Why $m_{\pi^0} \approx m_{\pi^\pm}$?

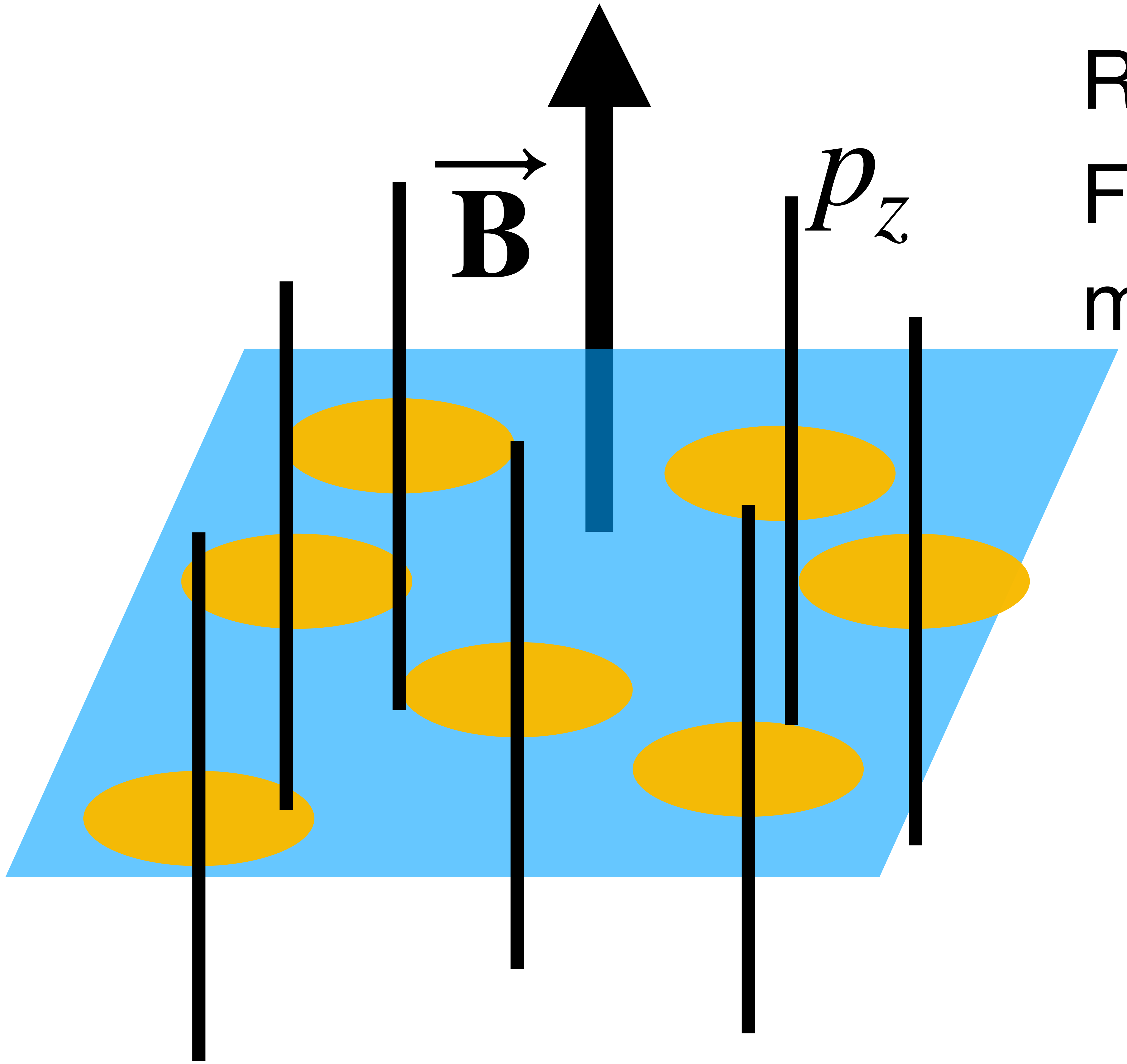
Subtle Aspects of Chiral Anomaly

Topological

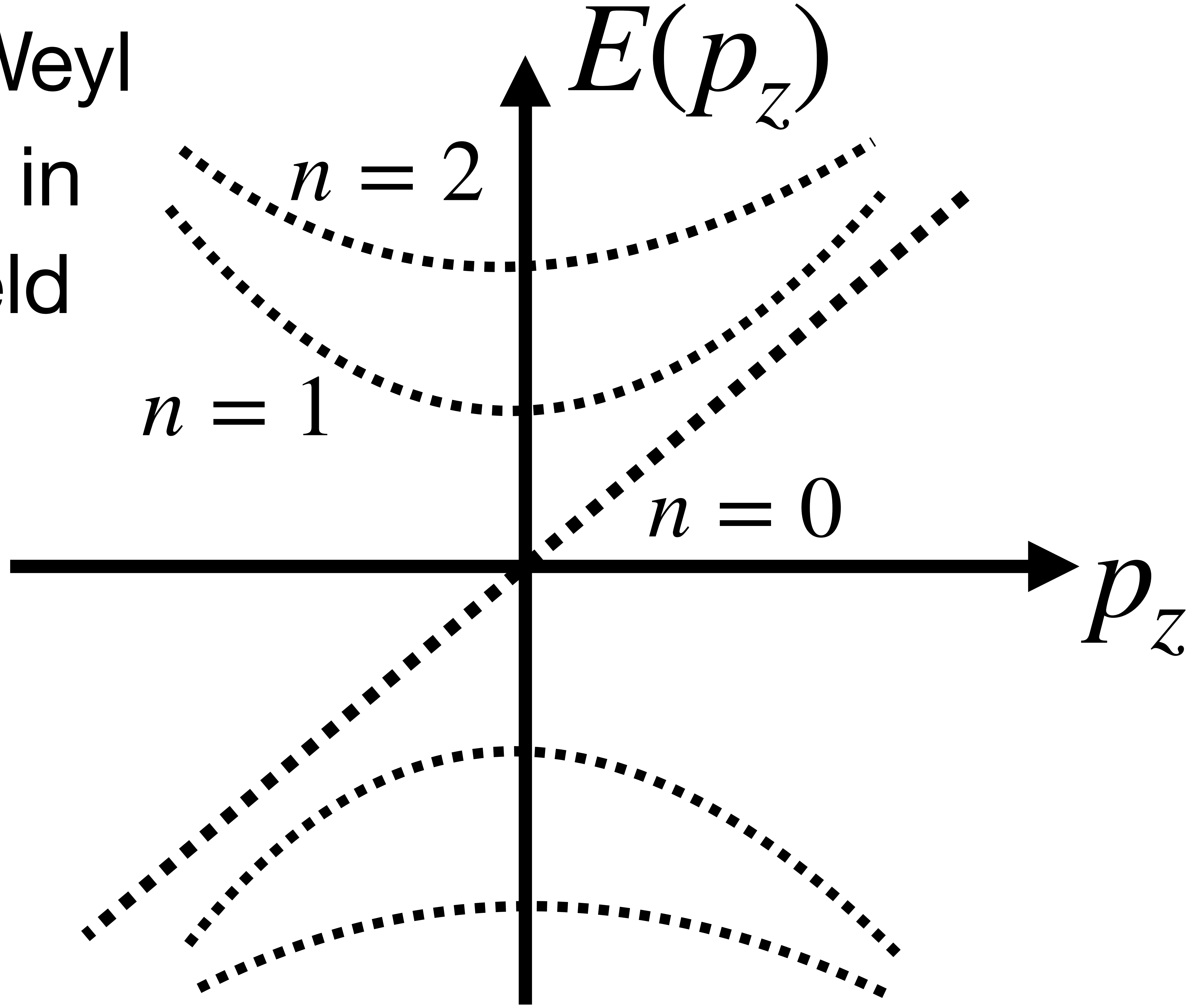
$$\frac{1}{4\pi^2}\int d^4x\, \mathbf{E}_g \cdot \mathbf{B}_g = \text{Integer}$$

UV-IR Connection

Gribov's Picture of Anomaly



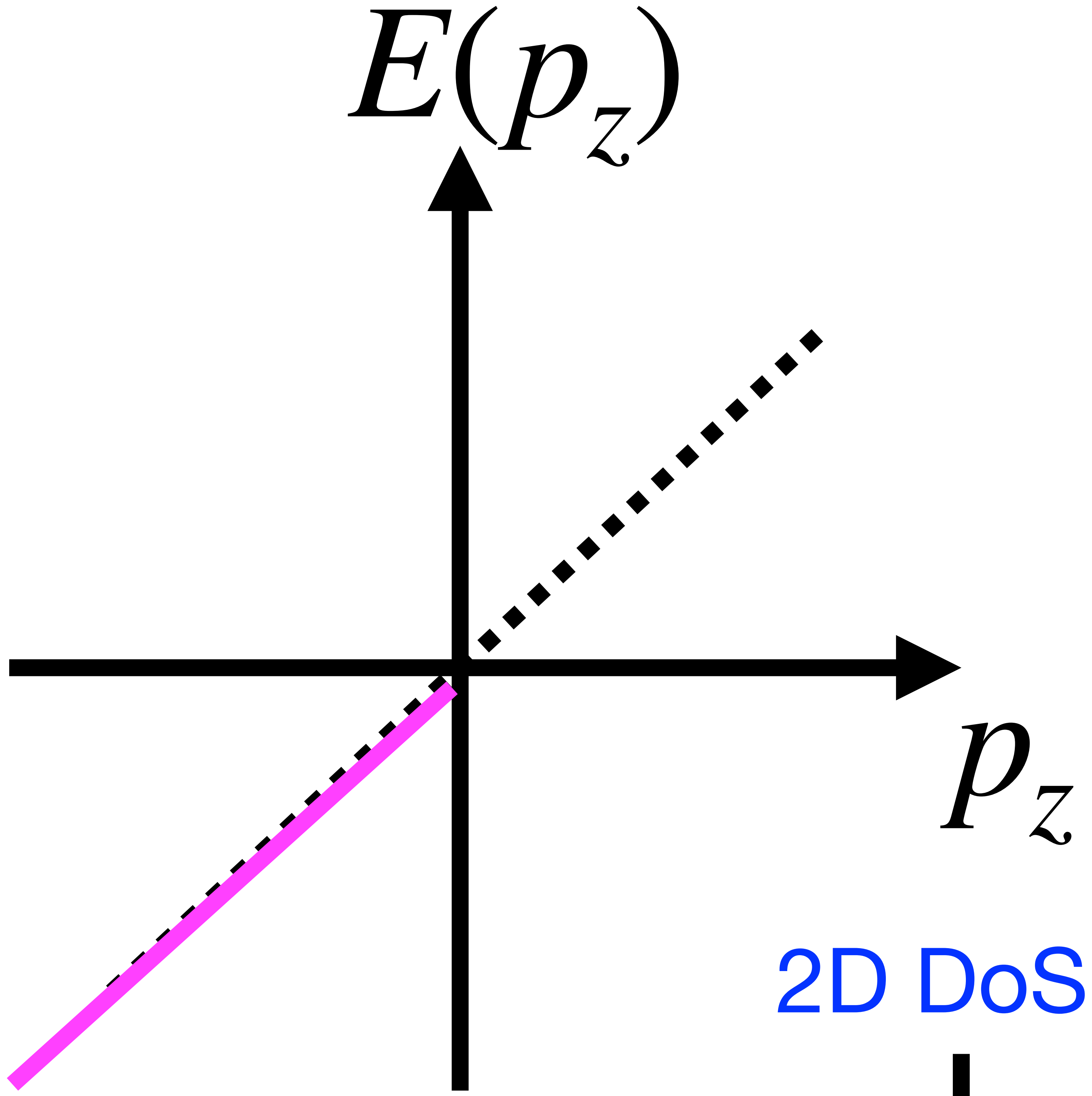
R-Handed Weyl Fermion Ψ_R in magnetic field



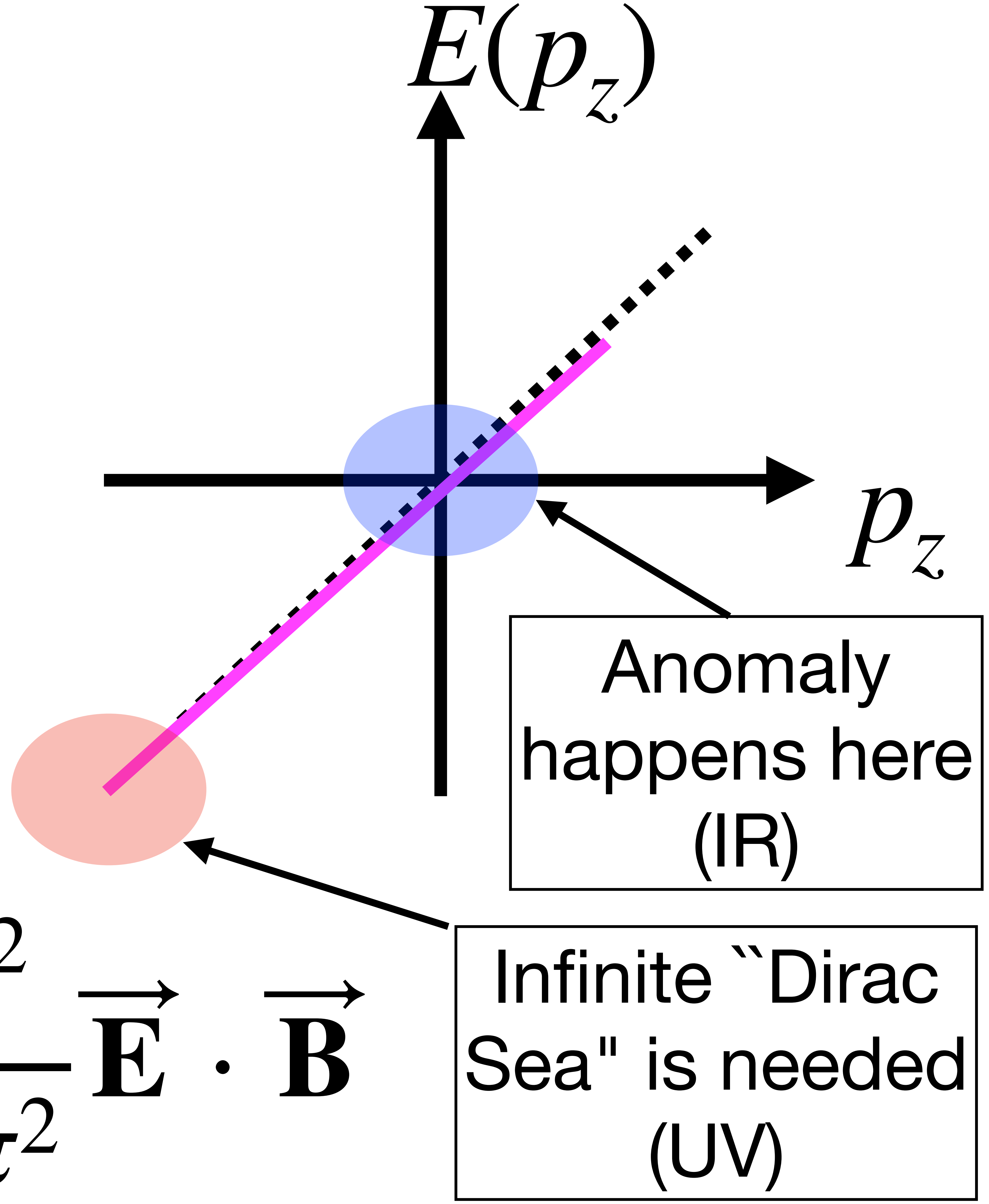
Landau Levels with 2D density of states $\frac{eB}{2\pi}$

$n = 0$ Chiral Zero Mode

UV-IR Connection



$$\frac{dp_z}{dt} = eE$$



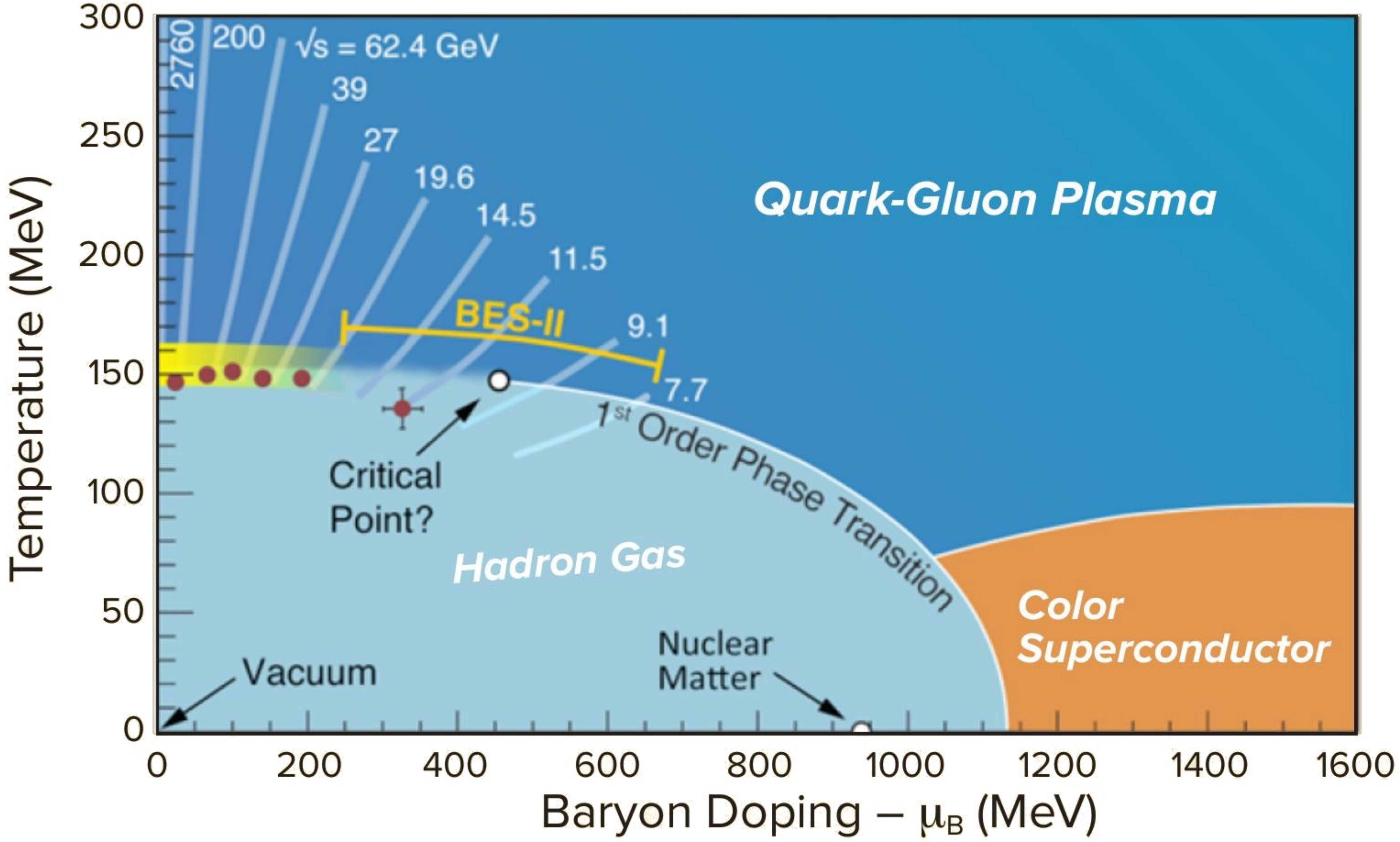
2D DoS　　1D DoS

$$\frac{dn_R}{dt} = \left(\frac{eB}{2\pi}\right)\frac{1}{2\pi}\frac{dp_z}{dt} = \frac{e^2}{4\pi^2}\vec{\mathbf{E}}\cdot\vec{\mathbf{B}}$$

Axial Chemical Potential

In high T deconfined quark-gluon plasma,
chiral symmetry $U(1)_A$ is approximately restored

(We will come back to this later)



N_R and N_L
approximately conserved

↓

Chiral chemical potentials
μ_R and μ_L
or, axial $\mu_A = (\mu_R - \mu_L)/2$

Chiral Magnetic Effect

(Fukushima-Kharzeev-Warranga, 0808.3382, Vilenkin '79)

$$\vec{\mathbf{J}} = \frac{e^2}{2\pi^2} \mu_A \vec{\mathbf{B}}$$

P-odd
C-odd
T-odd

Chiral Anomaly Coefficient

P-odd
C-even
T-even

P-even
C-odd
T-odd

Axial Charge

Non-Dissipative

Chiral Version of CME

There is also the Chiral Separation Effect (CSE)

$$\vec{\mathbf{J}}_A = \frac{e^2}{2\pi^2}\mu_V \vec{\mathbf{B}} \quad \text{where } \mu_V = (\mu_R + \mu_L)/2$$

Spin Polarization $\vec{\mathbf{S}}$
for Dirac fermions
(Xu-Guang Huang's Lecture)

CME + CSE →

$$\vec{\mathbf{J}}_R = \frac{e^2}{4\pi^2}\mu_R \vec{\mathbf{B}}$$

$$\vec{\mathbf{J}}_L = -\frac{e^2}{4\pi^2}\mu_L \vec{\mathbf{B}}$$

Derivation of CME (I) - Nielsen-Ninomiya

Poynting's Theorem : $$\frac{\partial u}{\partial t} + \nabla \cdot \mathbf{S} = -\mathbf{E} \cdot \mathbf{J}$$

Power to the chiral matter :

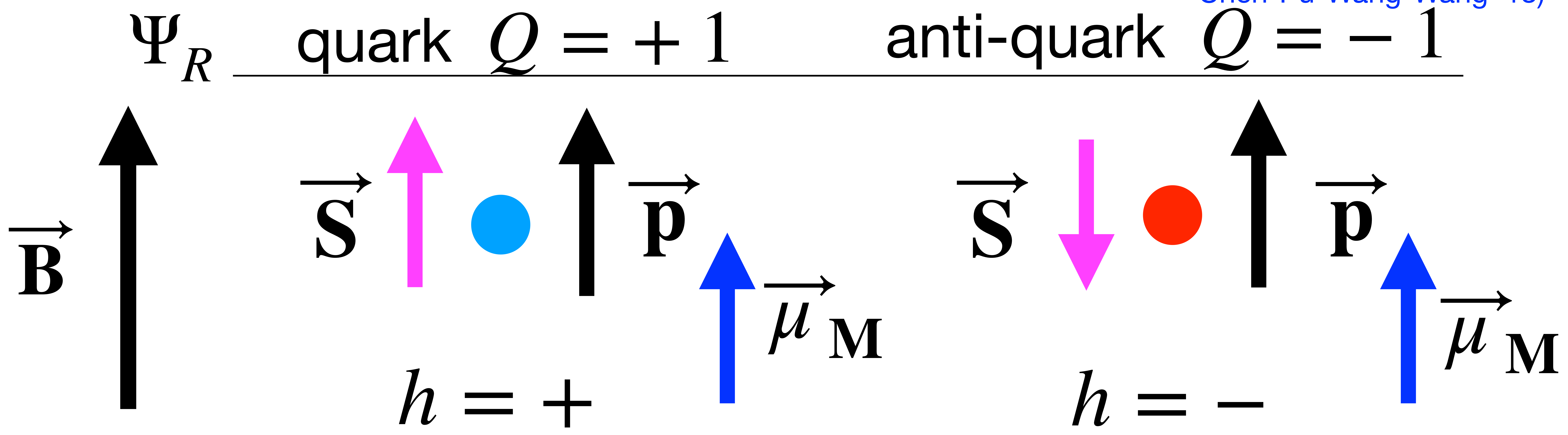
$$P = \overrightarrow{\mathbf{E}} \cdot \overrightarrow{\mathbf{J}} = \frac{dn_A}{dt}\mu_A = \frac{e^2}{2\pi^2}\overrightarrow{\mathbf{E}} \cdot \overrightarrow{\mathbf{B}}\mu_A$$

$$\Longrightarrow \overrightarrow{\mathbf{J}} = \frac{e^2}{2\pi^2}\mu_A \overrightarrow{\mathbf{B}}$$

Same Coefficients

Derivation of CME (II) - Chiral Kinetic Theory

(Son-Yamamoto '12,
Stephanov-Yin '12,
Chen-Pu-Wang-Wang '13)



Magnetic Moment Interaction

$$\Delta H = -\vec{\mu}_{\mathbf{M}} \cdot \vec{\mathbf{B}} = -\frac{eQ}{|\vec{\mathbf{p}}|}\vec{\mathbf{S}} \cdot \vec{\mathbf{B}} = -\frac{e\hbar}{2|\vec{\mathbf{p}}|^2}\vec{\mathbf{p}} \cdot \vec{\mathbf{B}}$$

$$\overrightarrow{\mathbf{J}}_R = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(\overrightarrow{\mathbf{v}}_+ f_+(\mathbf{p}) - \overrightarrow{\mathbf{v}}_- f_-(\mathbf{p}) \right)$$

Classical Velocity : $\overrightarrow{\mathbf{v}}_+ = \overrightarrow{\mathbf{v}}_- = \dfrac{\mathbf{p}}{|\mathbf{p}|} \equiv \hat{\mathbf{p}}$

Equibrium : $f_\pm^{\mathrm{eq}} = \dfrac{1}{e^{\beta(|\mathbf{p}| \mp \mu_R)} + 1}$, $(\beta = 1/kT)$

Both of them are modified at $\mathcal{O}(\hbar)$ with $\overrightarrow{\mathbf{B}}$

Magnetic moment interaction

$$\Delta H = -\frac{e\hbar}{2|\vec{\mathbf{p}}|^2}\vec{\mathbf{p}}\cdot\vec{\mathbf{B}}$$

$$f_\pm = f_\pm^{\mathrm{eq}}(|\mathbf{p}| + \Delta H) = f_\pm^{\mathrm{eq}}(|\mathbf{p}|) + \frac{\beta e\hbar(\mathbf{p}\cdot\mathbf{B})}{2|\mathbf{p}|^2} f_\pm^{\mathrm{eq}}(1 - f_\pm^{\mathrm{eq}})$$

$\mathcal{O}(\hbar)$ - correction

This accounts for 1/3 of total CME

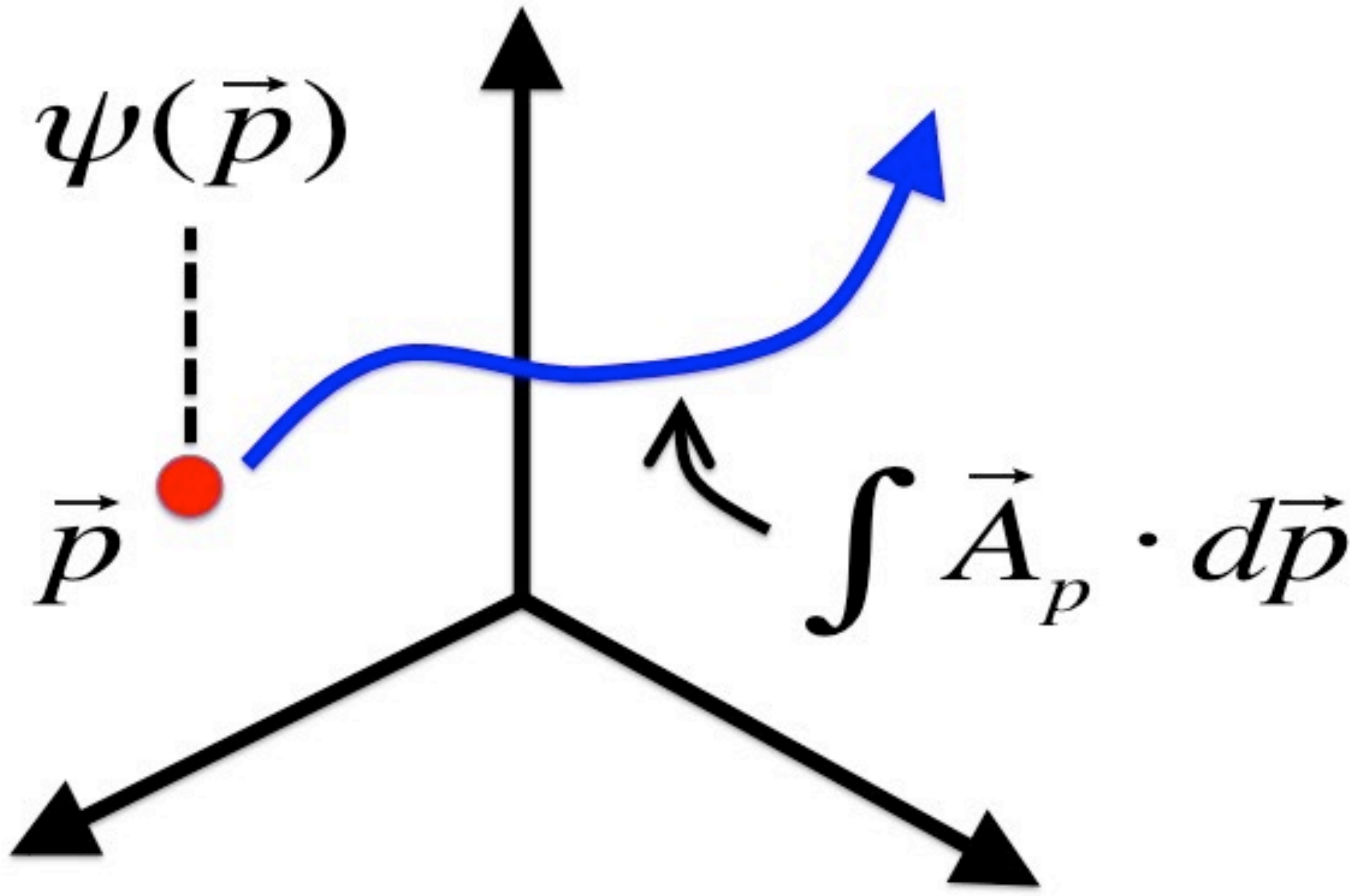
$$\Delta \mathbf{J}_R = \frac{\beta\hbar(e\mathbf{B})}{6}\int_{\mathbf{p}} \frac{1}{|\mathbf{p}|}\left(f_+^{\mathrm{eq}}(1 - f_+^{\mathrm{eq}}) - f_-^{\mathrm{eq}}(1 - f_-^{\mathrm{eq}})\right) = \frac{1}{3}\cdot\frac{\mu_R}{4\pi^2}(e\mathbf{B})$$

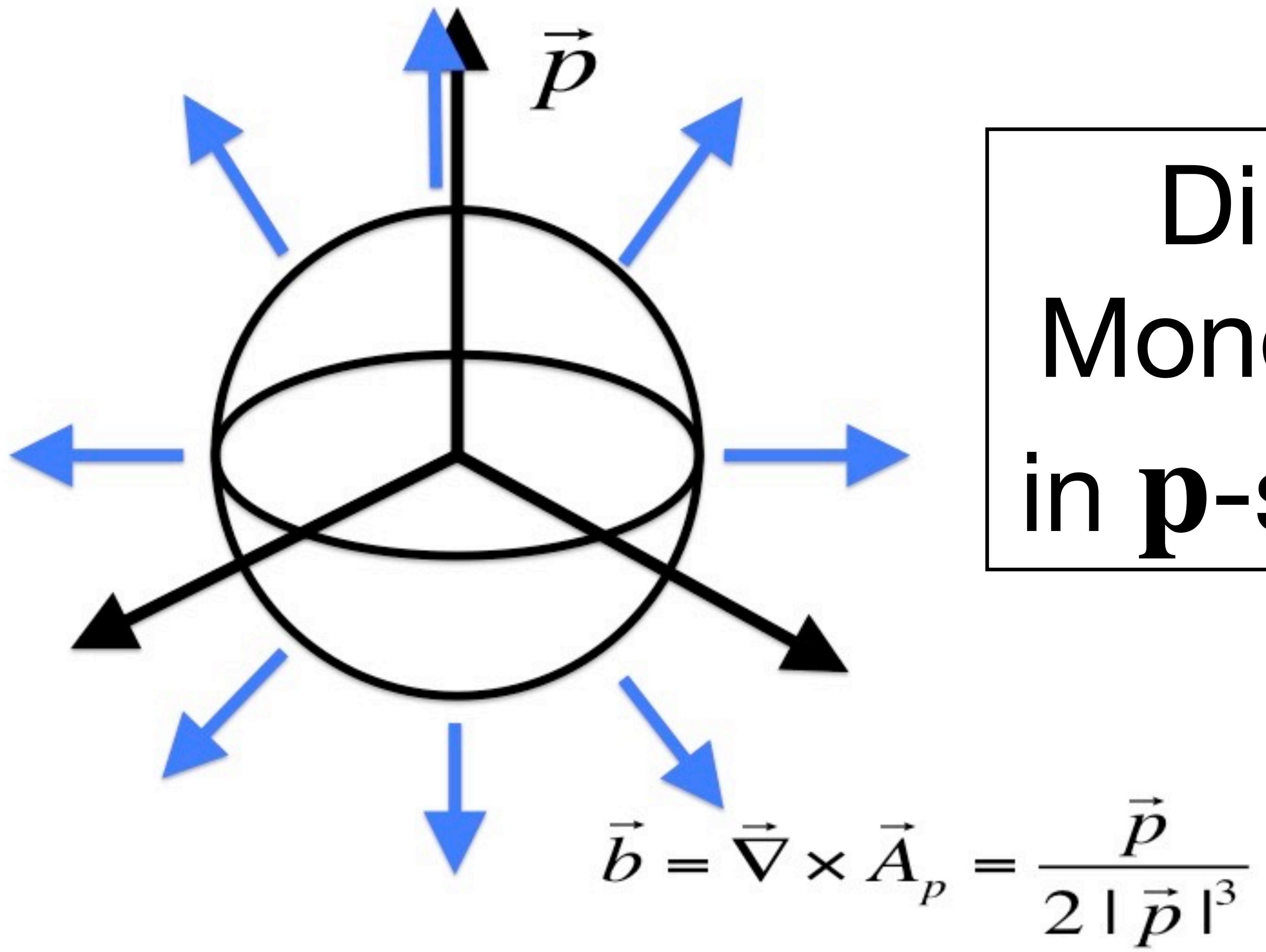
The rest 2/3 comes from quantum correction to the classical velocity $\Delta\vec{\mathbf{v}}_{\pm}$, due to

the Berry's Phase of spinor wave functions in $\mathbf{p}$-space

$$\mathscr{A}_p = (i\hbar)u_R^\dagger(\mathbf{p})\,\nabla_{\mathbf{p}}u_R(\mathbf{p})$$

(Son-Yamamoto '12,
Stephanov-Yin '12,
Chen-Pu-Wang-Wang '13)





Dirac Monopole in $\mathbf{p}$-space

Quantum correction to velocity

$$\Delta \mathbf{v}_{\pm} = \frac{\hbar \hat{\mathbf{p}}(\hat{\mathbf{p}} \cdot (e\mathbf{B}))}{|\mathbf{p}|^2}$$

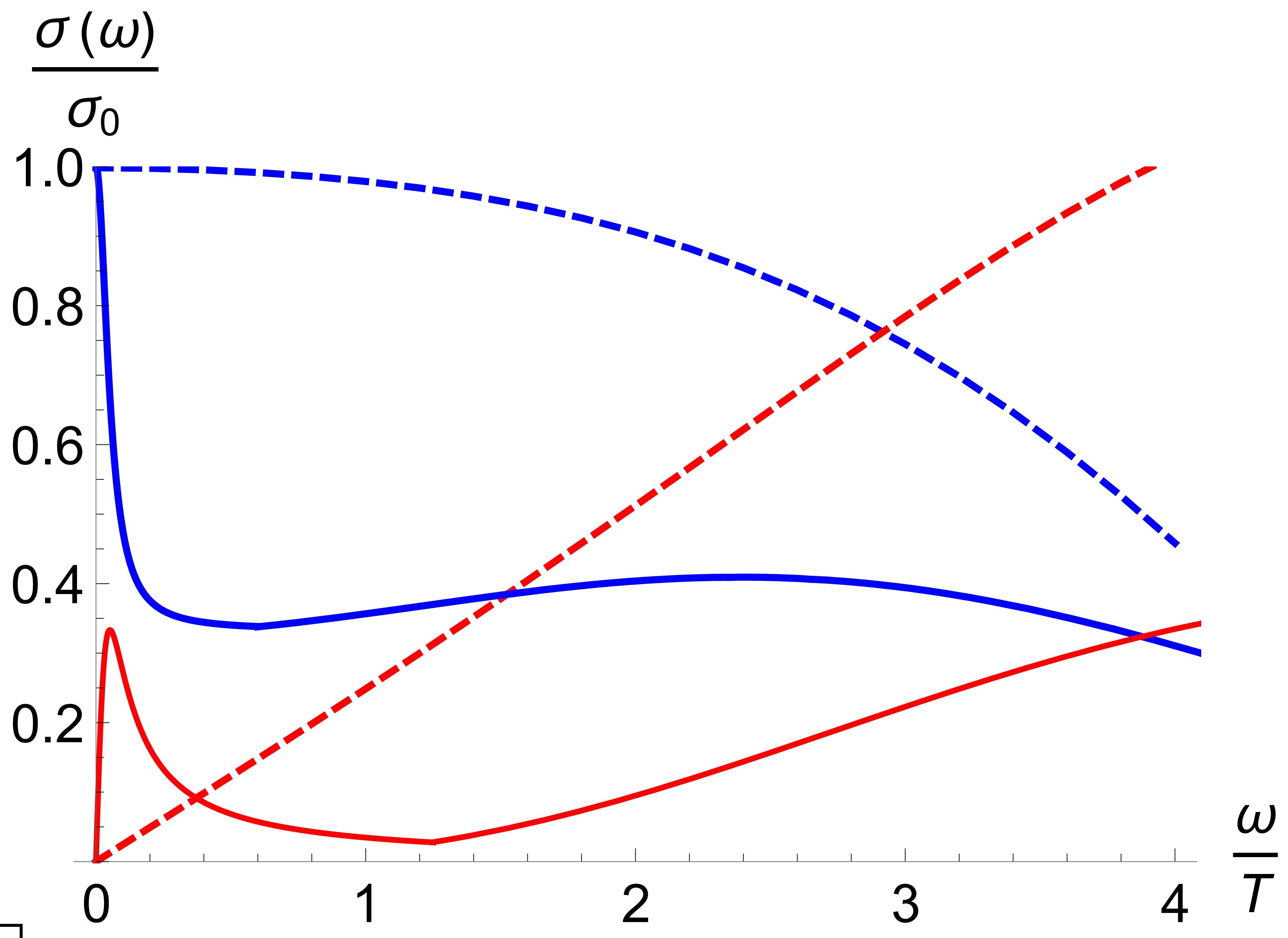
This accounts for 2/3 of total CME

$$\Delta \mathbf{J}_R = \frac{\hbar(e\mathbf{B})}{3} \int_{\mathbf{p}} \frac{1}{|\mathbf{p}|^2} \left(f_+^{\mathrm{eq}} - f_-^{\mathrm{eq}}\right) = \frac{2}{3} \cdot \frac{\mu_R}{4\pi^2}(e\mathbf{B})$$

$\Delta f_{\pm}$	$\frac{1}{3}$
$\Delta \mathbf{v}_{\pm}$	$\frac{2}{3}$
Total	1

Out-of Equilibrium CME : $\mathbf{J}(\omega) = \sigma_5(\omega)\mathbf{B}(\omega)$

	$\omega \ll \tau_R^{-1}$	$\omega \gg \tau_R^{-1}$
$\Delta f_\pm$	$\frac{1}{3}$	0
$\Delta \mathbf{v}_\pm$	$\frac{2}{3}$	$\frac{2}{3}$
$\mathbf{J_M}$	0	$-\frac{1}{3}$
Total	1	$\frac{1}{3}$



Solid: Perturbative QCD, Dashed: AdS/CFT
Blue: Real part, Red: Imaginary part

Magnetization Current $\mathbf{J_M} = \nabla \times \mathbf{M}$,

$$\mathbf{M} = \int_{\mathbf{p}} \frac{e\hbar}{2|\overrightarrow{\mathbf{p}}|^2}\overrightarrow{\mathbf{p}}\left(f_+(\mathbf{p}) + f_-(\mathbf{p})\right)$$

(Kharzeev-Stephanov-Yee, 1612.01674)

Chiral Vortical Effect (CVE)

$$\vec{\mathbf{J}} = \frac{e^2}{2\pi^2}\mu\mu_A\vec{\omega}$$

$$\omega = \nabla \times \mathbf{v}$$

P-odd
C-odd
T-odd

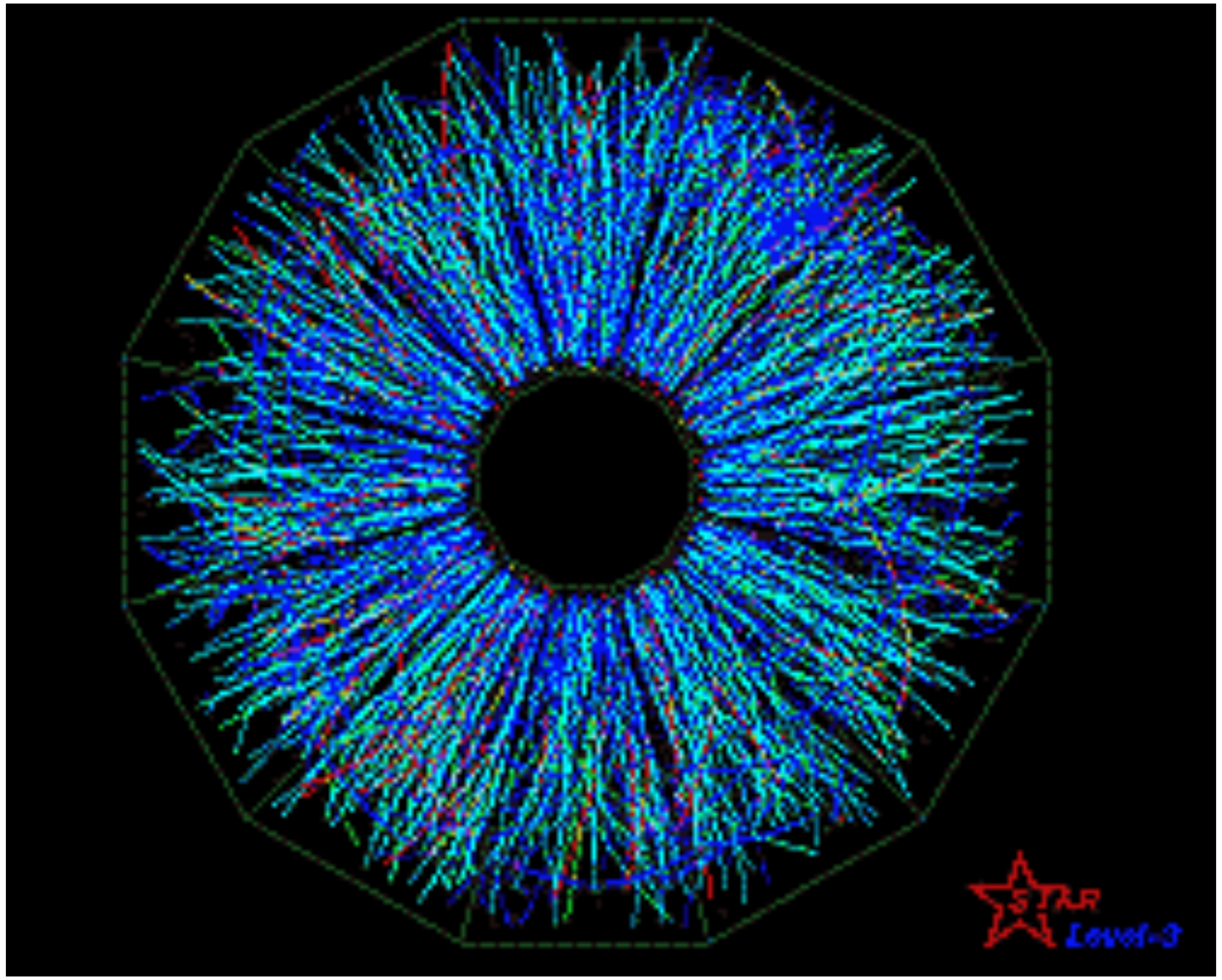
P-odd
C-odd
T-even

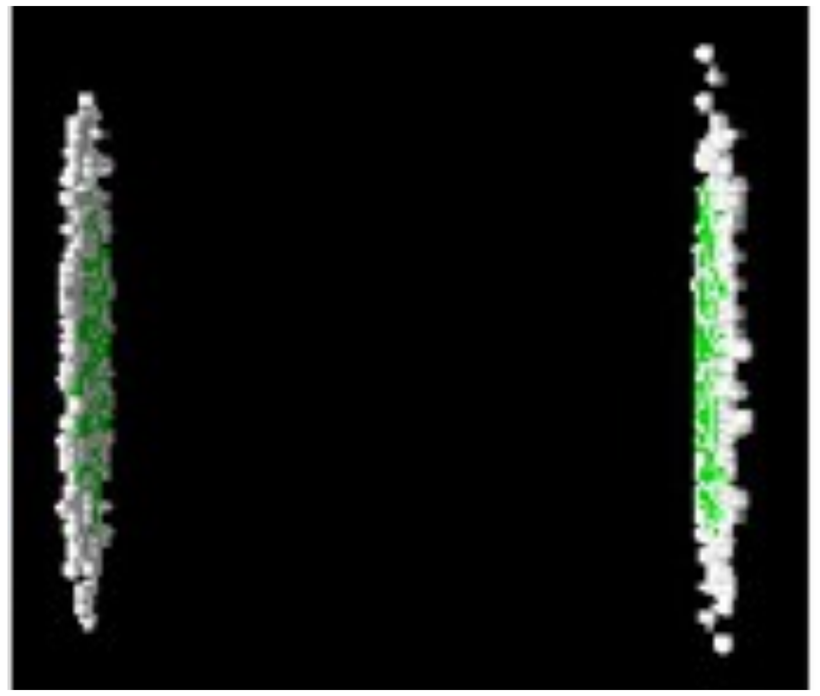
P-even
C-even
T-odd

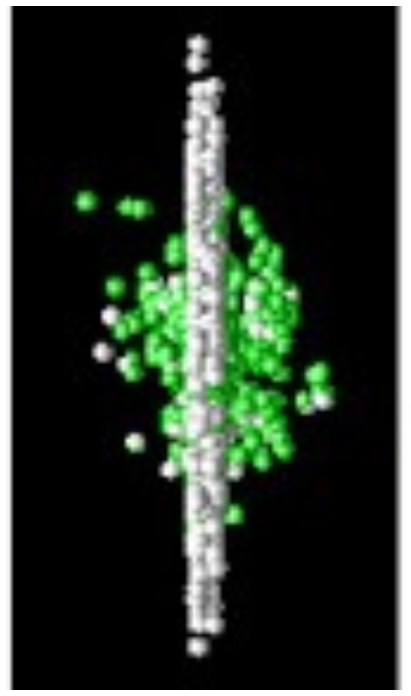
Axial Charge

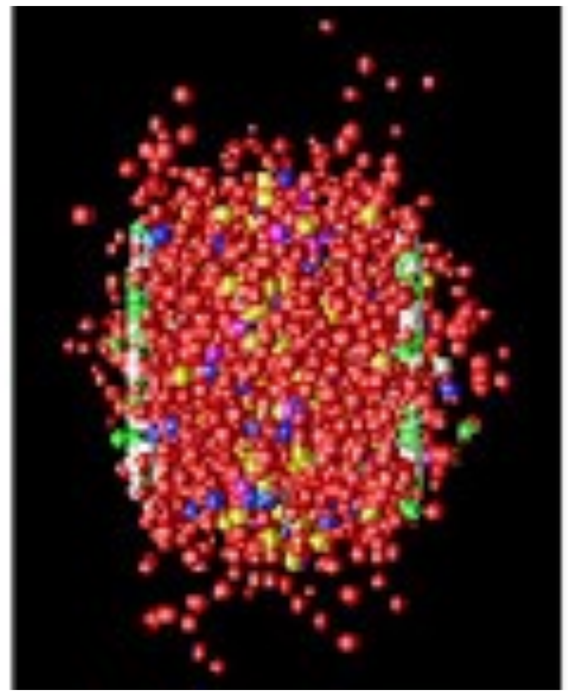
Non-Dissipative

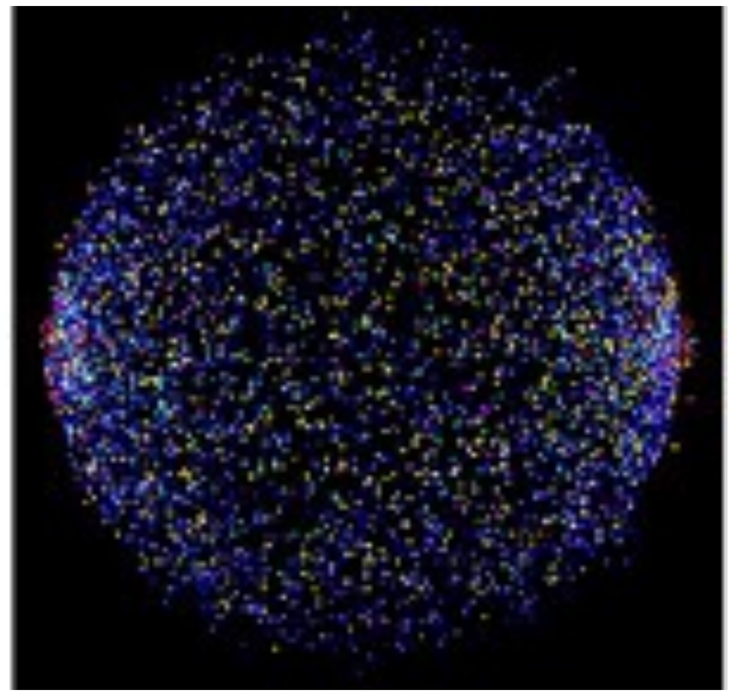
Relativistic Heavy-Ion Collisions (RHIC)



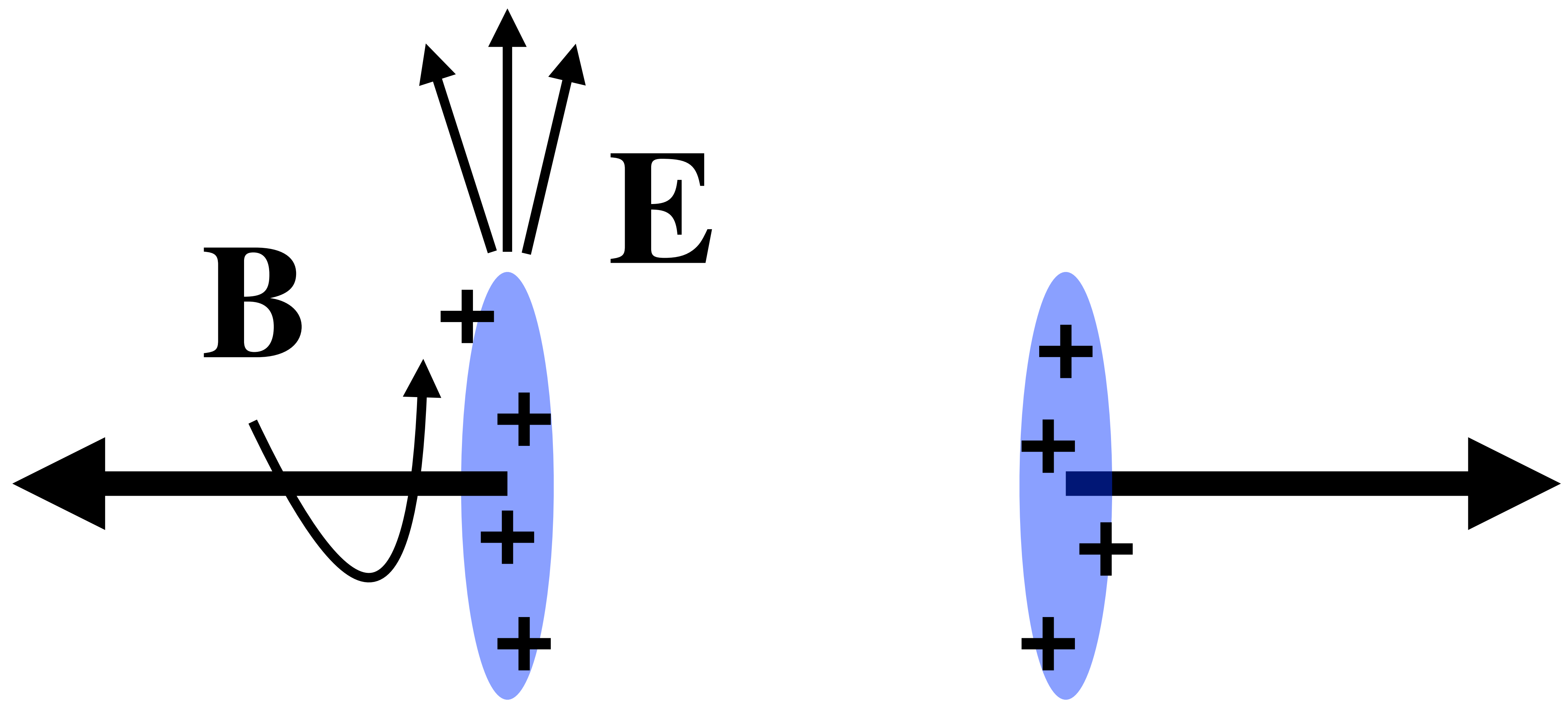


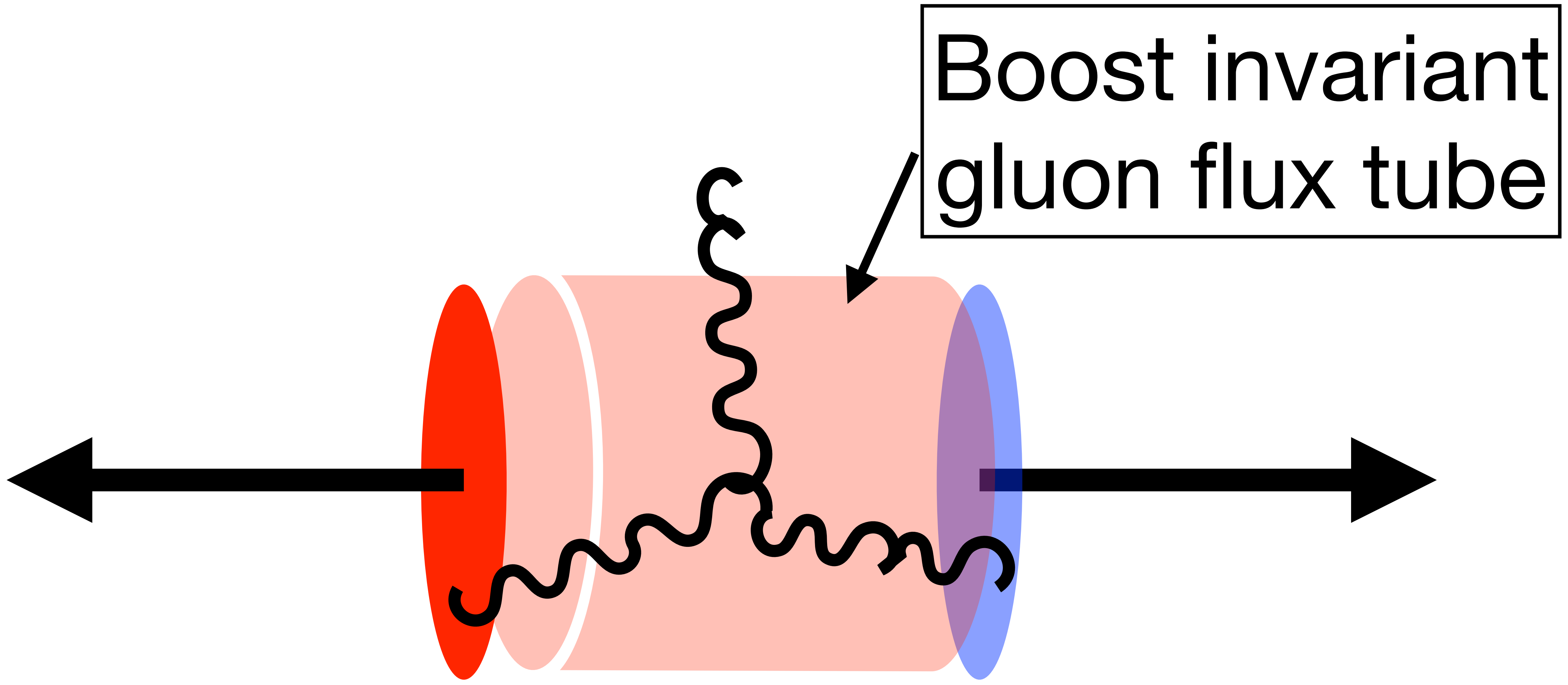






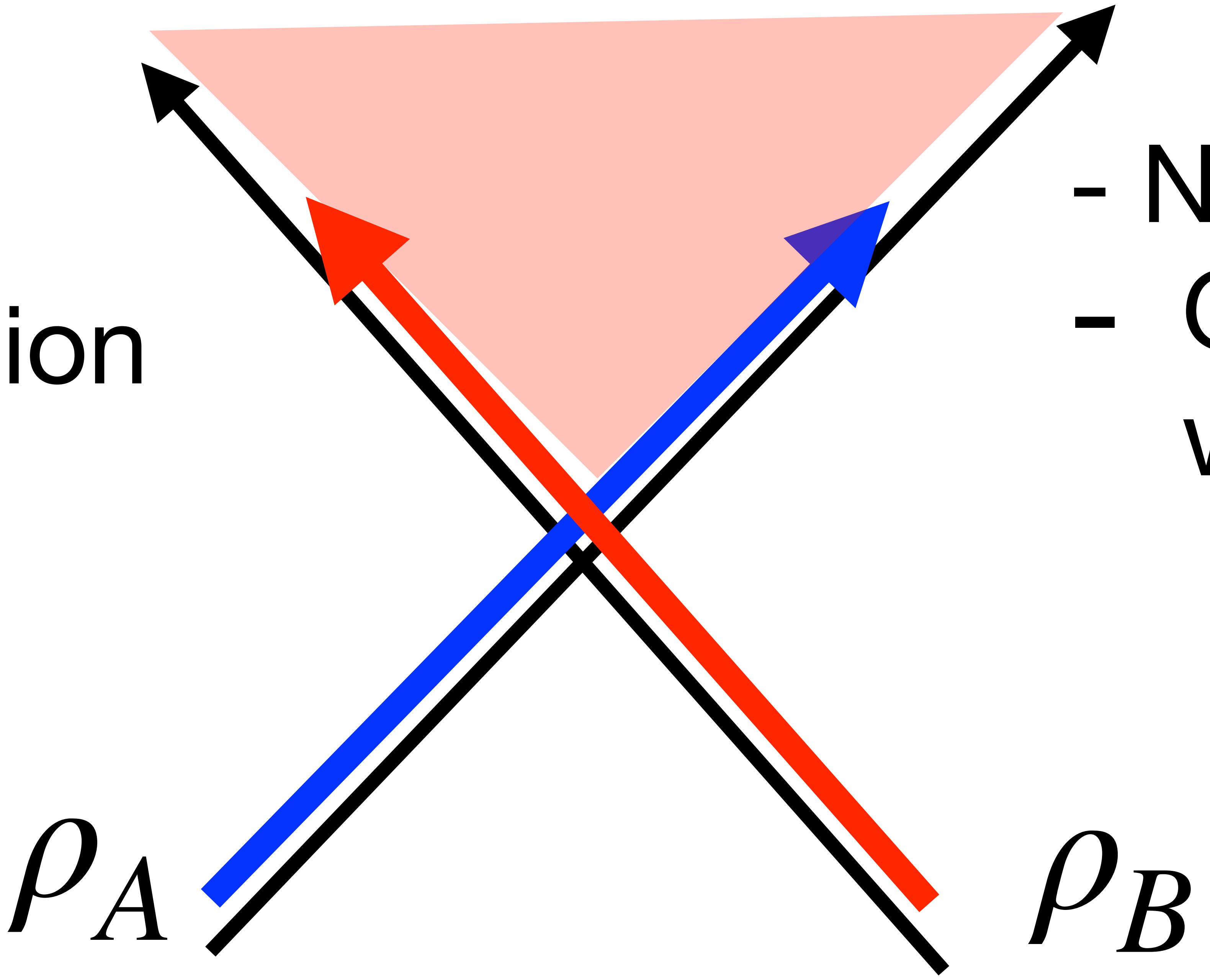
Heavy-Ion Collisions : Basics





QED

- Linear superposition
- Collinear forward radiation



QCD

- Non-linear YM eqns
- Gluon production in wide rapidity

(Gunion-Bertsch '82)

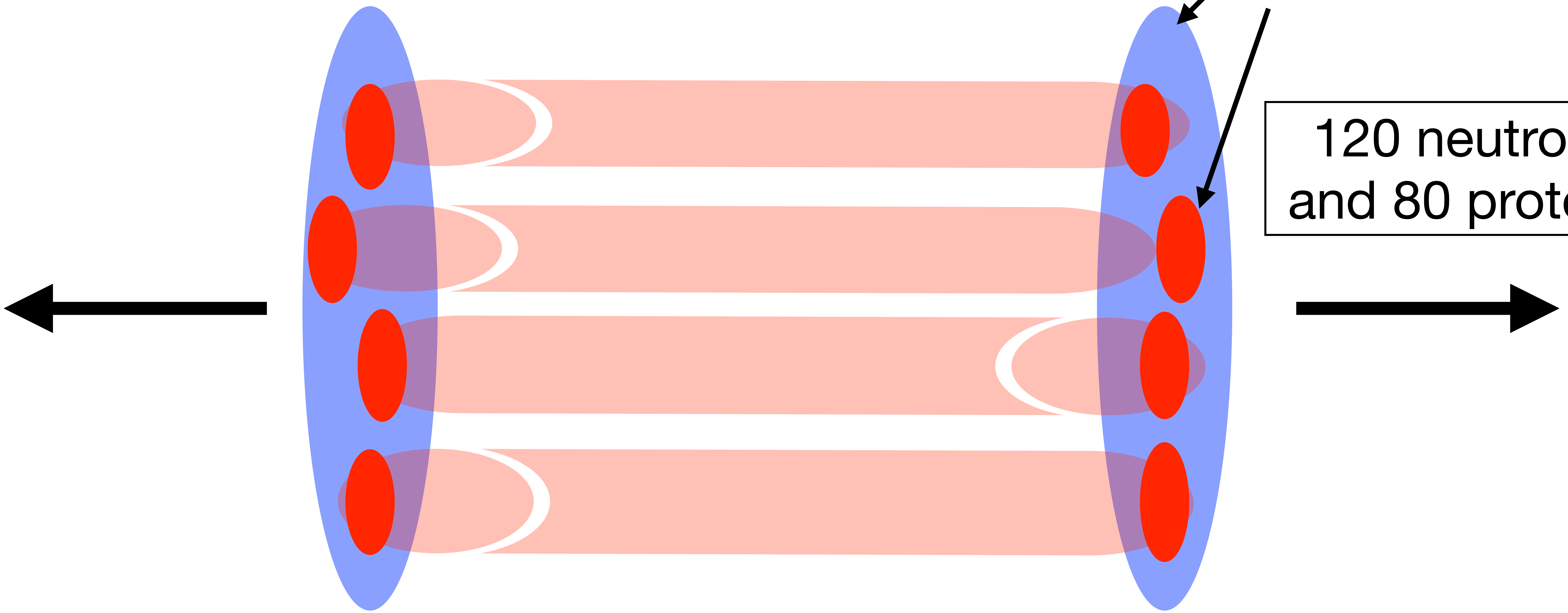
Au-Au $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$ (RHIC)

Pb-Pb $\sqrt{s_{NN}} = 2.76\,\mathrm{TeV}$ (LHC)

Fluctuating Color Charges
(Color Glass Condensate)

(McLerran-Venugopalan '93)

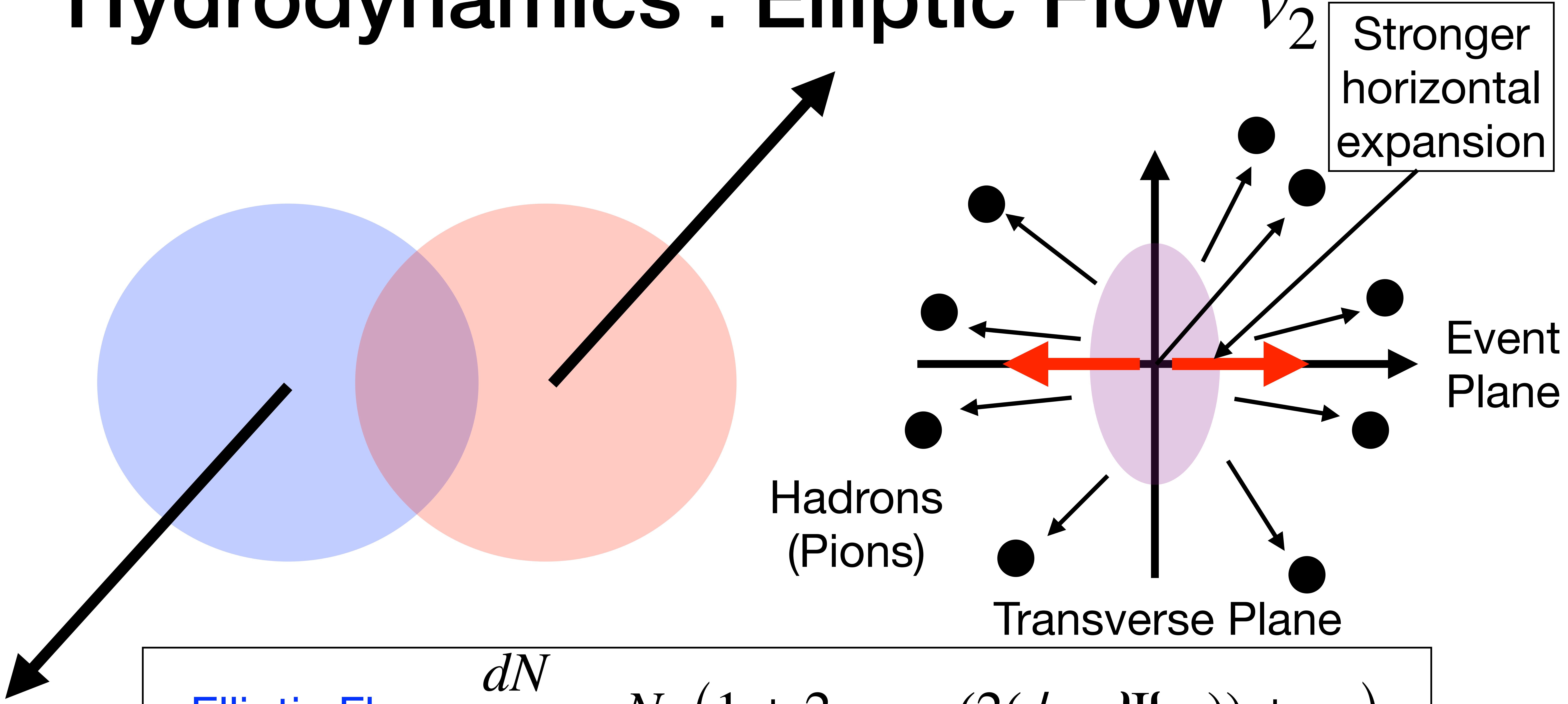
120 neutrons
and 80 protons



Gluon fields → Quark-Gluon Plasma (QGP)

Lifetime $\sim 10\,\mathrm{fm/c}$, Initial temperature $T \sim 300 - 400\,\mathrm{MeV}$

Hydrodynamics : Elliptic Flow v_2



Elliptic Flow : $\frac{dN}{d\phi} = N_0 \left(1 + 2v_2 \cos(2(\phi - \Psi_{EP})) + \cdots\right)$

Typical value $v_2 \sim 0.01 - 0.1$

CME in Heavy-Ion Collisions (I) : Axial Charge

The initial gluon fields have random topological fluctuations of $\mathbf{E}^g \cdot \mathbf{B}^g \neq 0$
(Kharzeev-Krasnitz-Venugopalan, hep-ph/0109253)

→ Event-by-event fluctuations of μ_A

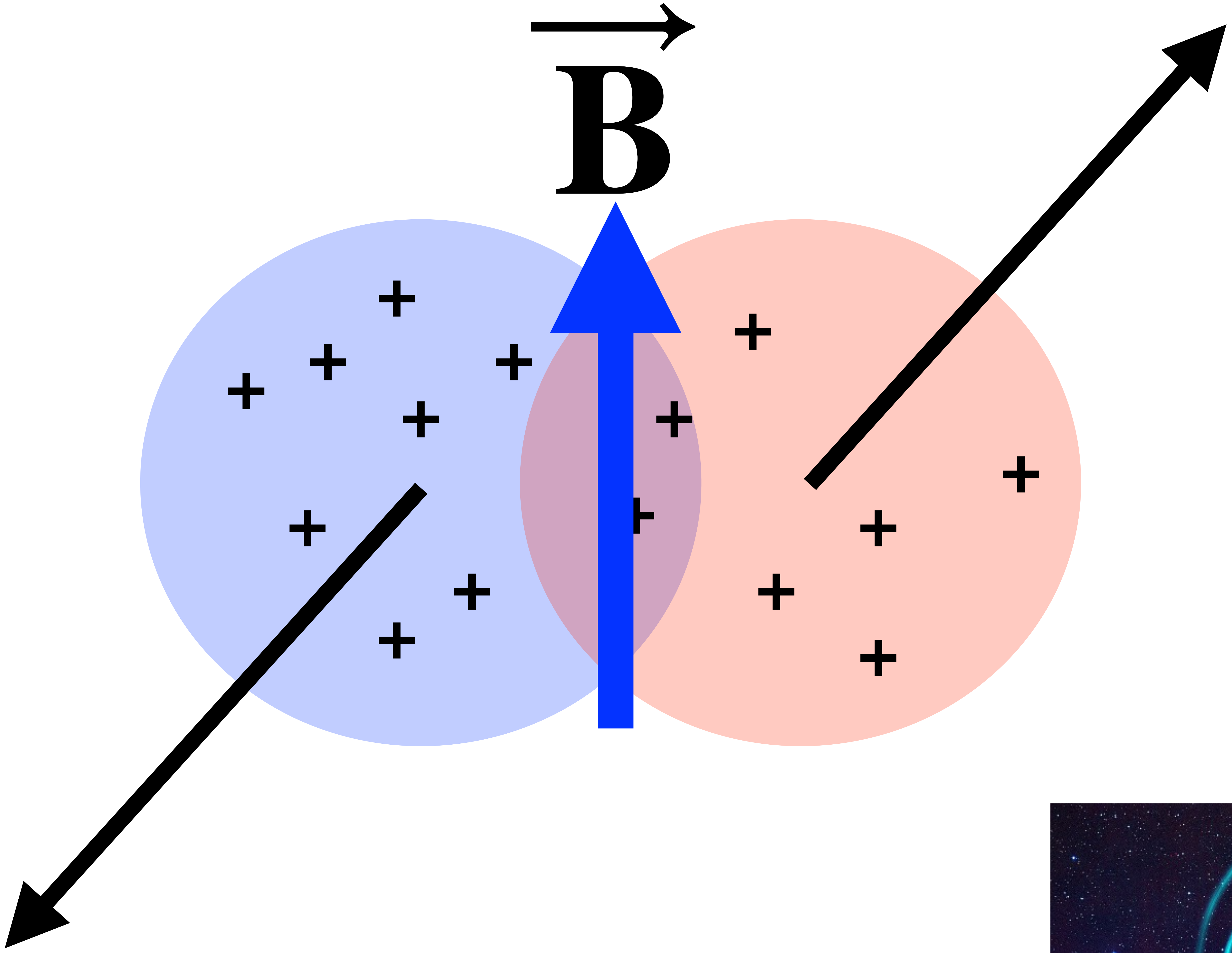
How long does μ_A last in QGP ?

- Relaxation rate due to Sphaleron transitions $\sim \alpha_s^5 T$
- Relaxation rate due to quark mass $\sim m_q^2 \alpha_s^2 / T$

μ_A lasts up to $\sim 10\,\mathrm{fm/c}$

(Kapusta-Rrapaj-Rudaz, 2012.13784, Lin-Yee, 1305.3949)

CME in Heavy-Ion Collisions (II) : Magnetic Field



Initial Magnetic Field

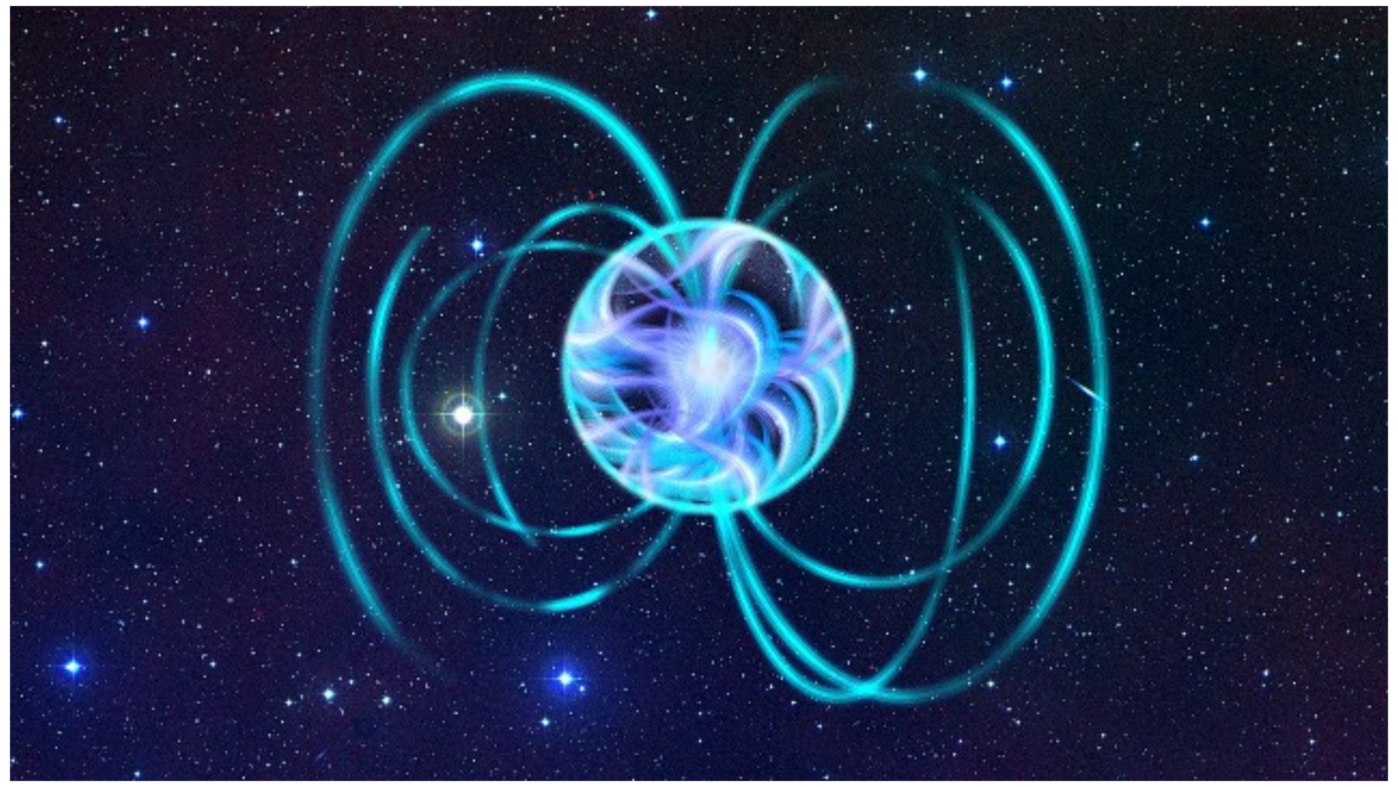
$$e\mathbf{B} \sim \frac{\alpha_{\rm EM} Z \gamma}{b^2} \sim (100\,\mathrm{MeV})^2$$

(Kharzeev-McLerran-Warringa, 0711.0950)

$$Z \sim 100\,,\ \gamma \sim 100\,,\ b \sim 10\,\mathrm{fm}$$

$$eB \sim (100\,\mathrm{MeV})^2 \sim 10^{18}\,\mathrm{G}$$

is comparable to initial T



Magnetars

$eB \sim 10^{16}\,\mathrm{G}$

Magnetic Field Diffusion

How long does B last in QGP ?

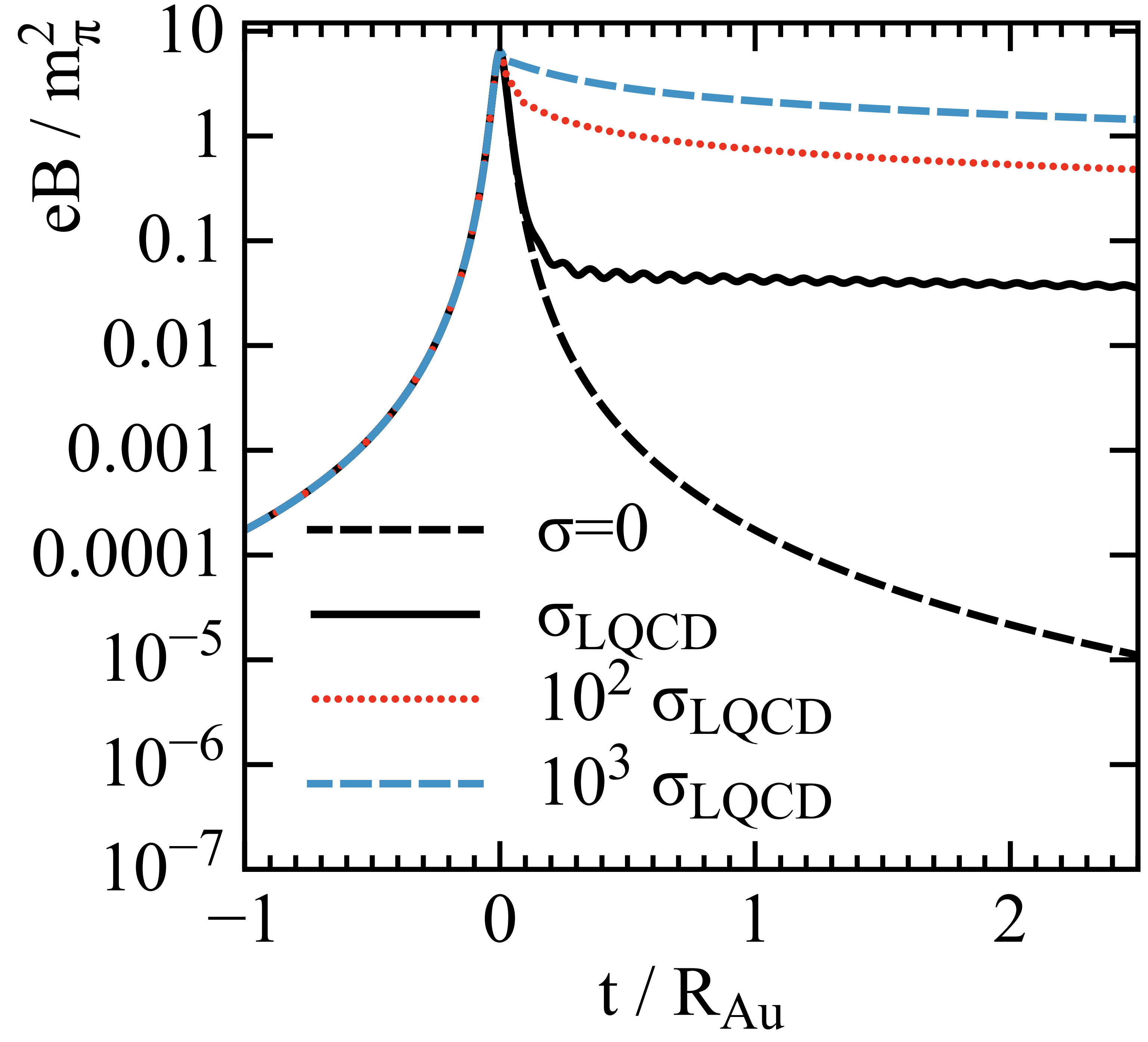
Maxwell's eqns in a conducting plasma

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t} = \sigma \mathbf{E} + \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times (\nabla \times \mathbf{B}) = -\nabla^2 \mathbf{B} = -\sigma \frac{\partial \mathbf{B}}{\partial t} - \frac{\partial^2 \mathbf{B}}{\partial t^2} \approx -\sigma \frac{\partial \mathbf{B}}{\partial t}$$

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\sigma} \nabla^2 \mathbf{B} = D_B \nabla^2 \mathbf{B}}$$

(K. Tuchin, 1006.3051)



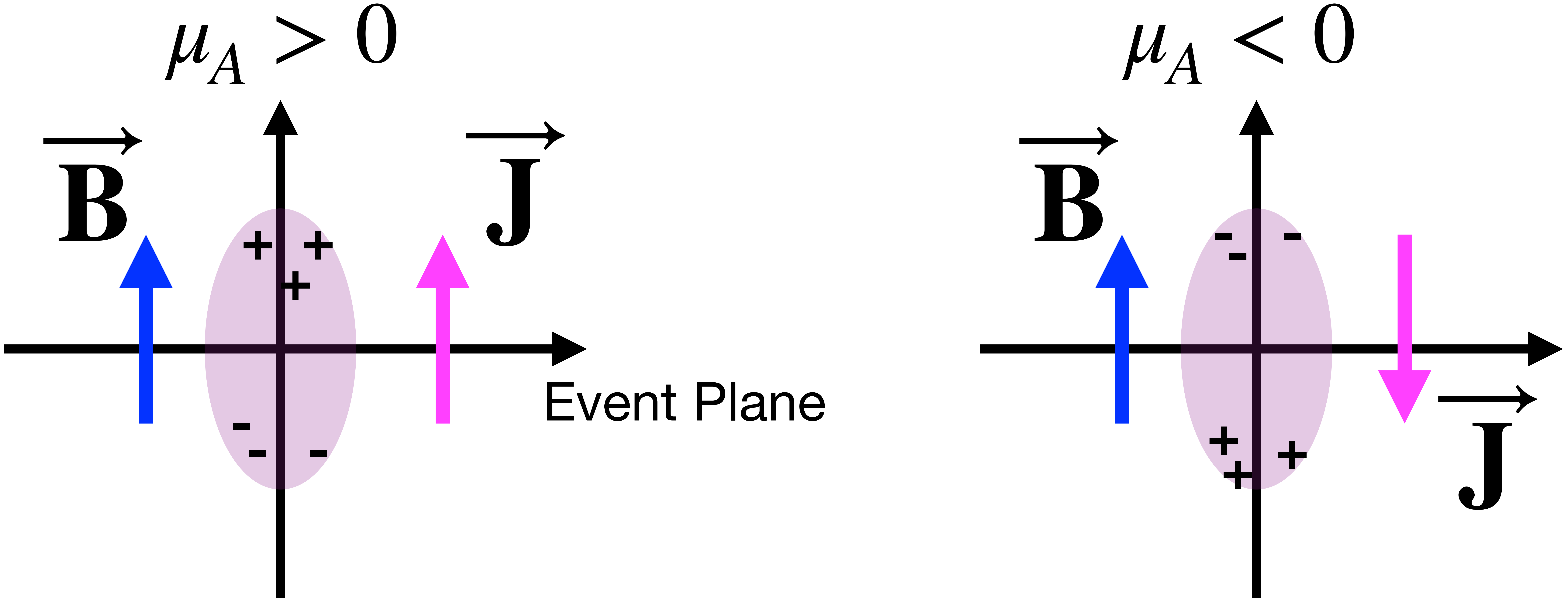
(McLerran-Skokov,1305.0774)

Conductivity from Lattice QCD : $\sigma_{\mathrm{Lattice}} \approx 0.1 e^2 T \sim 0.01/\mathrm{fm}$

Magnetic field diffusion time $\tau_D = \sigma L^2 \approx 1\ \mathrm{fm/c}$: Not long nor too short

Experimental Observables of CME

(D. Kharzeev, hep-ph/0406125)



Event-by-Event Fluctuating Charge Dipoles

$$\frac{dN_\pm}{d\phi} = N_0 \left(1 + 2v_2 \cos(2(\phi - \Psi_{EP})) \pm 2a_1 \sin(\phi - \Psi_{EP}) + \cdots\right)$$

$$\langle a_1^{\text{CME}} \rangle_{\text{events}} = 0 \ \rightarrow \langle (a_1)^2 \rangle \neq 0,$$ but it is now P-even

(We will come back to this later)

Event-by-Event $dN/d\phi$ is a large N approximation

Each event gives $(\phi_1, \phi_2, \cdots, \phi_N)$

Averaging over events gives Probability Dist. : $P(\phi_1, \phi_2, \cdots, \phi_N)$

Measured in experiments

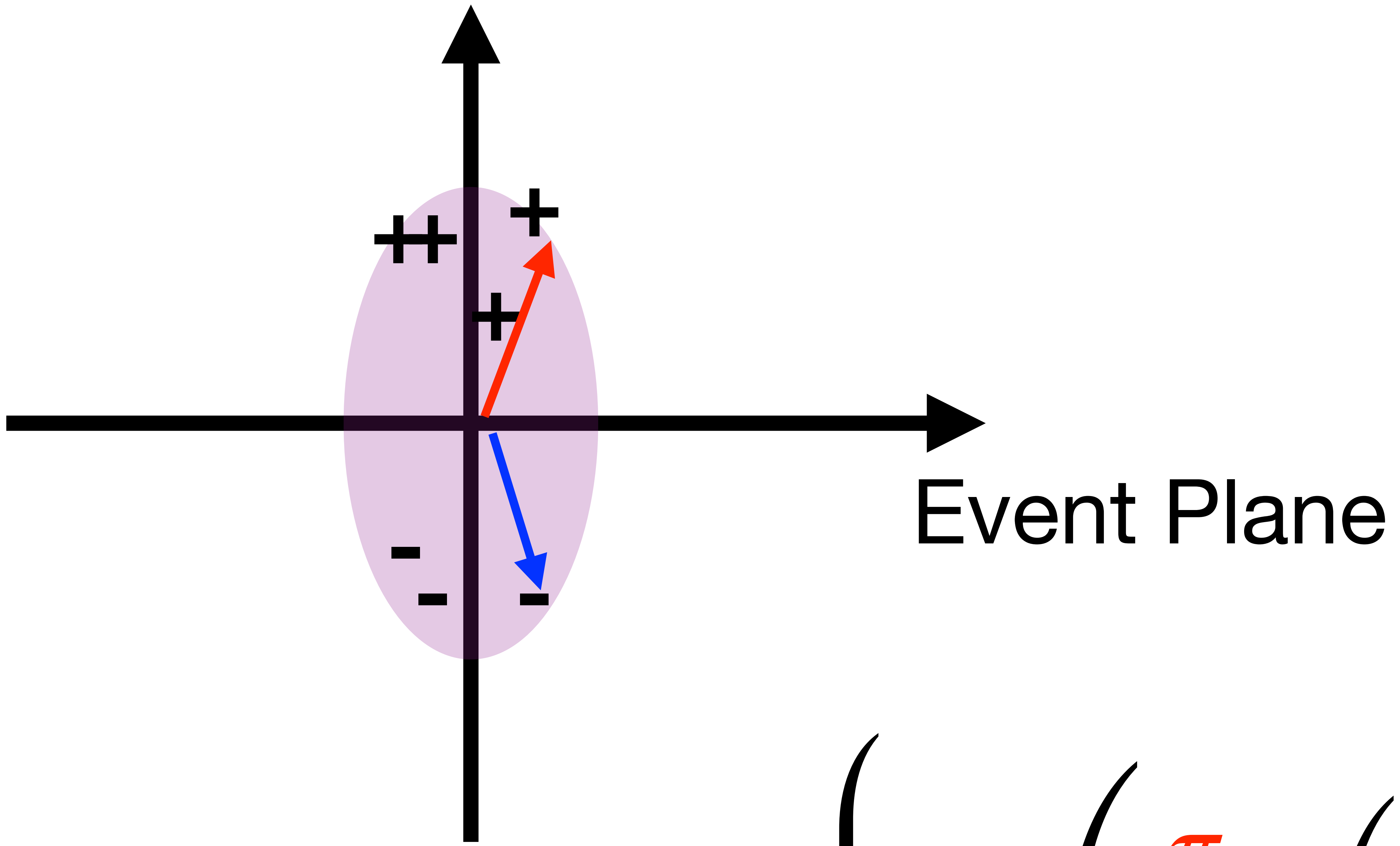
E.g., 2-Particle Distribution

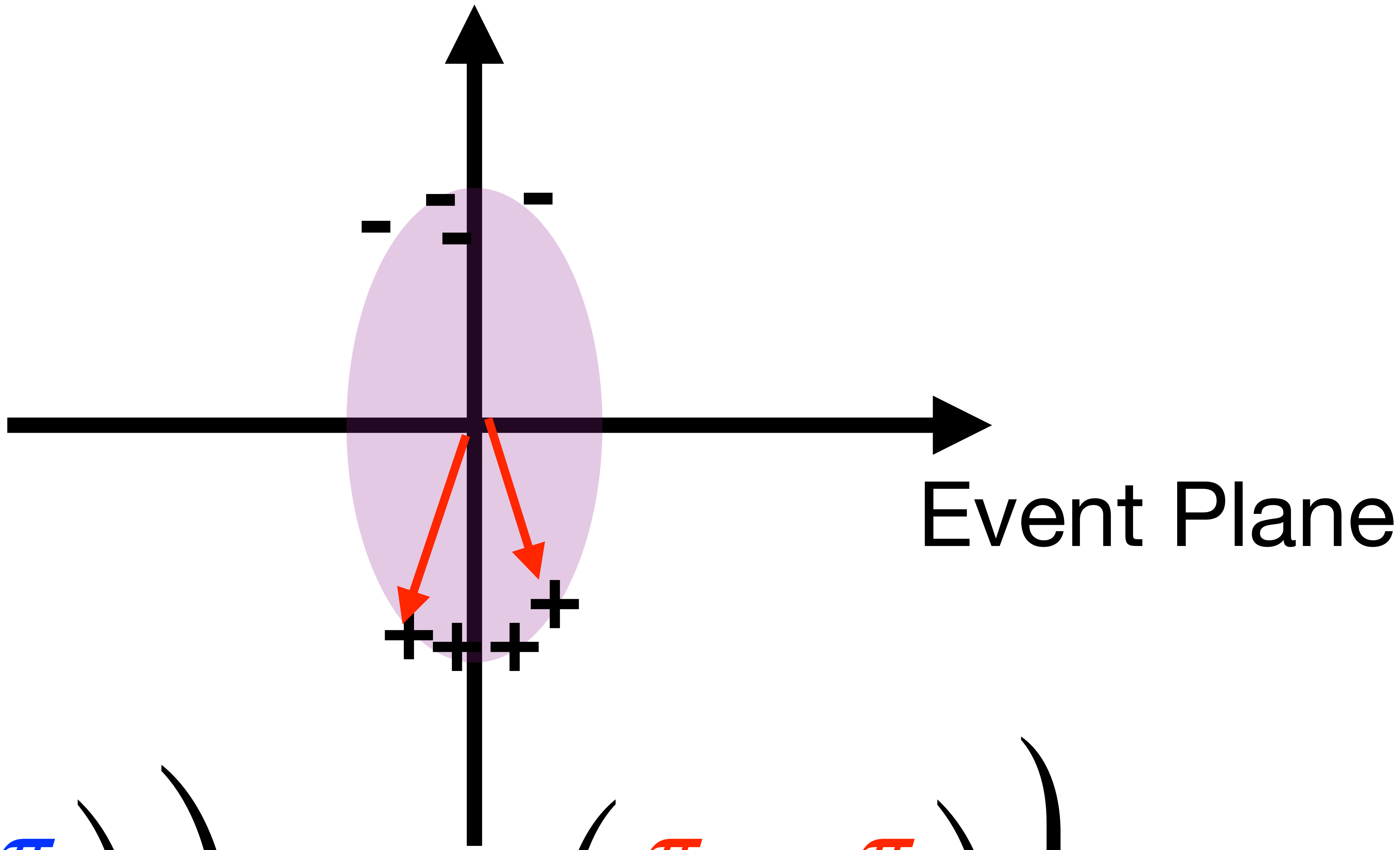
$$\frac{d^2N}{d\phi_1 d\phi_2} = \mathcal{N} \int_{\phi_3,\phi_4,\cdots,\phi_N} P(\phi_1, \phi_2, \phi_3, \cdots, \phi_N)$$

γ-Correlator

(S. Voloshin, hep-ph/0406311)

$$\gamma = \langle \cos(\phi_1 + \phi_2 - 2\Psi_{EP}) \rangle = \frac{1}{\mathcal{N}} \int_{\phi_1, \phi_2} \cos(\phi_1 + \phi_2 - 2\Psi_{EP}) \frac{d^2N}{d\phi_1 d\phi_2}$$





$$\Delta\gamma = \gamma_{+-} - \gamma_{++} \approx a_1^2 \left(\cos\left(\frac{\pi}{2} + \left(-\frac{\pi}{2}\right)\right) - \cos\left(\frac{\pi}{2} + \frac{\pi}{2}\right) \right) = 2\langle a_1^2 \rangle$$

$$\gamma_{++/+-} = \langle \cos\phi_1 \cos\phi_2 \rangle - \langle \sin\phi_1 \sin\phi_2 \rangle \approx v_1^2 \mp a_1^2 + (B_{\rm IN} - B_{\rm OUT})$$

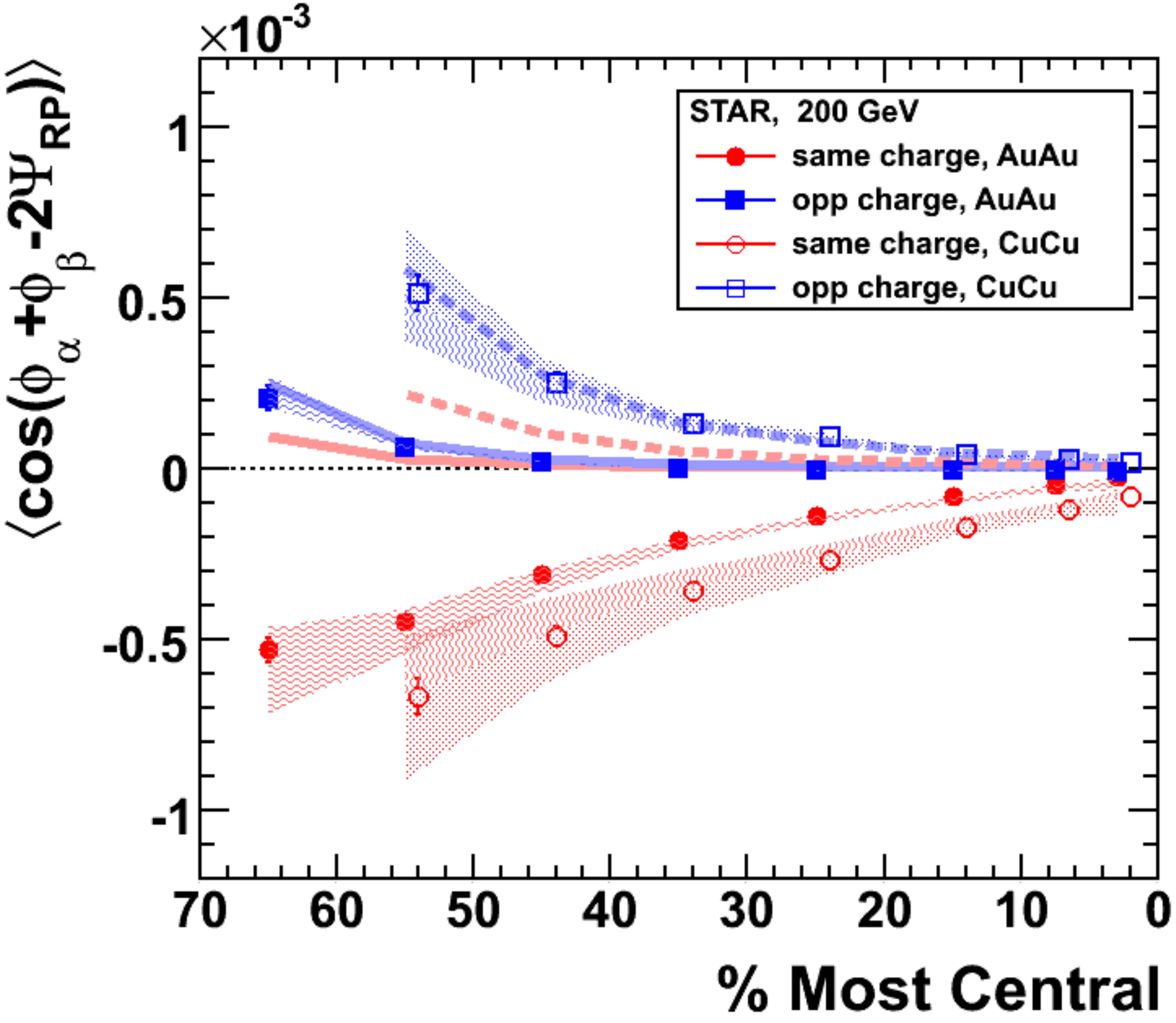
CME Effects

The key idea is the cancellation of angle-independent backgrounds, e.g., resonance decays : $B_{\rm IN} - B_{\rm OUT}$

(Directed flow v_1 can be eliminated experimentally)

This reduces the backgrounds to only those correlated with global azimuthal asymmetry, i.e., the elliptic flow :

$$B_{\rm IN} - B_{\rm OUT} \sim v_2/N \sim \mathcal{O}(10^{-3})$$



STAR '09 (0909.1739)

$\mathrm{Au} + \mathrm{Au}\,,\mathrm{Cu} + \mathrm{Cu}$ at $\sqrt{s} = 200$ and $62.4\,\mathrm{GeV}$

$\Delta\gamma = \gamma_{\mathrm{OS}} - \gamma_{\mathrm{SS}} > 0$
and $\gamma \to 0$ for $|\Delta\eta| > 1$

But, $\gamma_{\mathrm{OS}} \approx 0$
and $\Delta\gamma \sim \mathcal{O}(10^{-3})$

δ -Correlator

$$\delta = \langle \cos(\phi_1 - \phi_2) \rangle = \langle \cos\phi_1 \cos\phi_2 \rangle + \langle \sin\phi_1 \sin\phi_2 \rangle$$

CME : $\Delta\delta = \delta_{\rm OS} - \delta_{\rm SS} \approx -2\langle (a_1)^2 \rangle < 0$

Experiments : $\Delta\delta^{\rm exp} > 0$

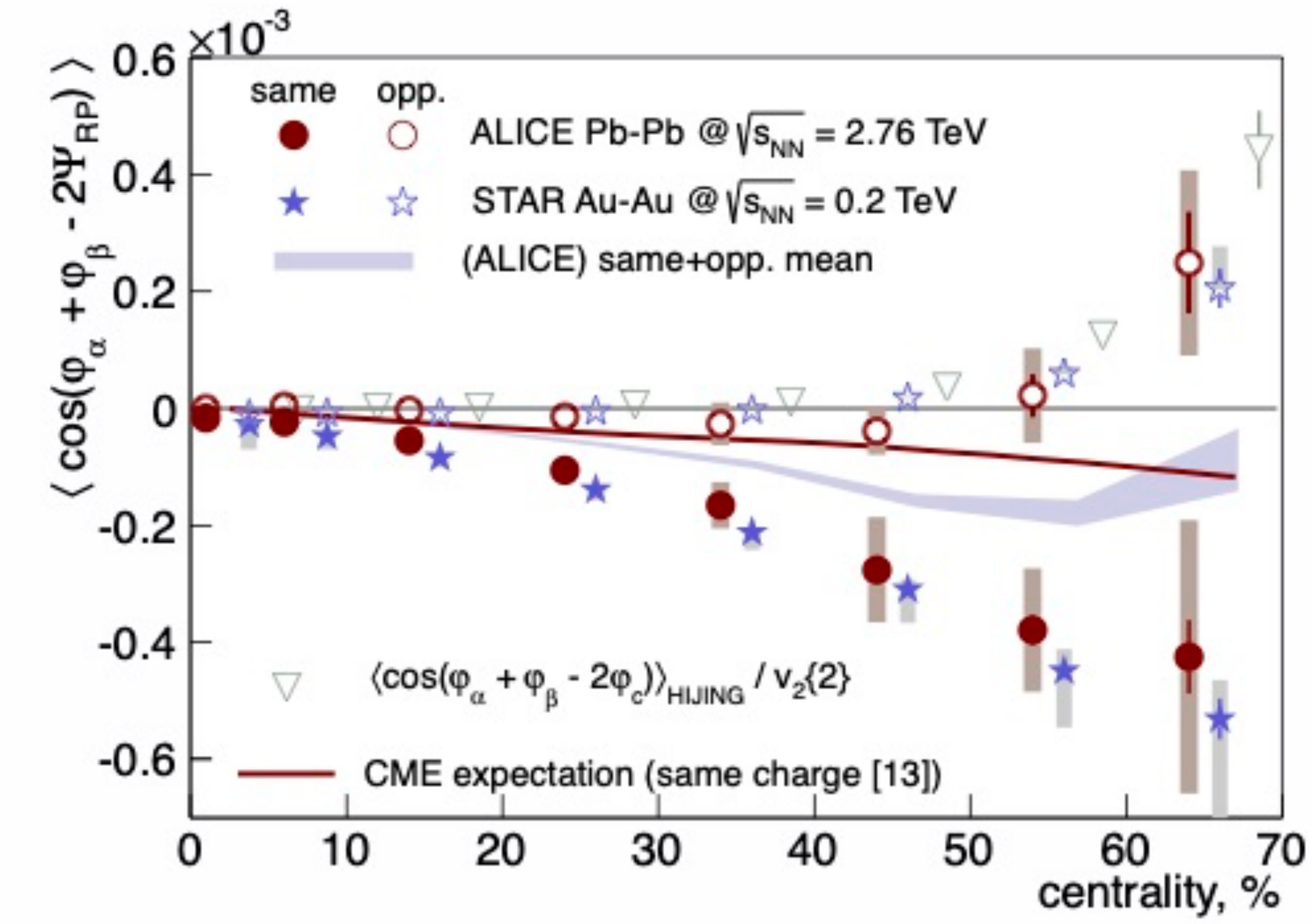
In-Plane and Out-Plane can be separated

$$\langle \cos\phi_1 \cos\phi_2 \rangle = \frac{1}{2}(\gamma + \delta)\,, \quad \langle \sin\phi_1 \sin\phi_2 \rangle = \frac{1}{2}(-\gamma + \delta)$$

(Bzdak-Koch-Liao, 0912.5050)

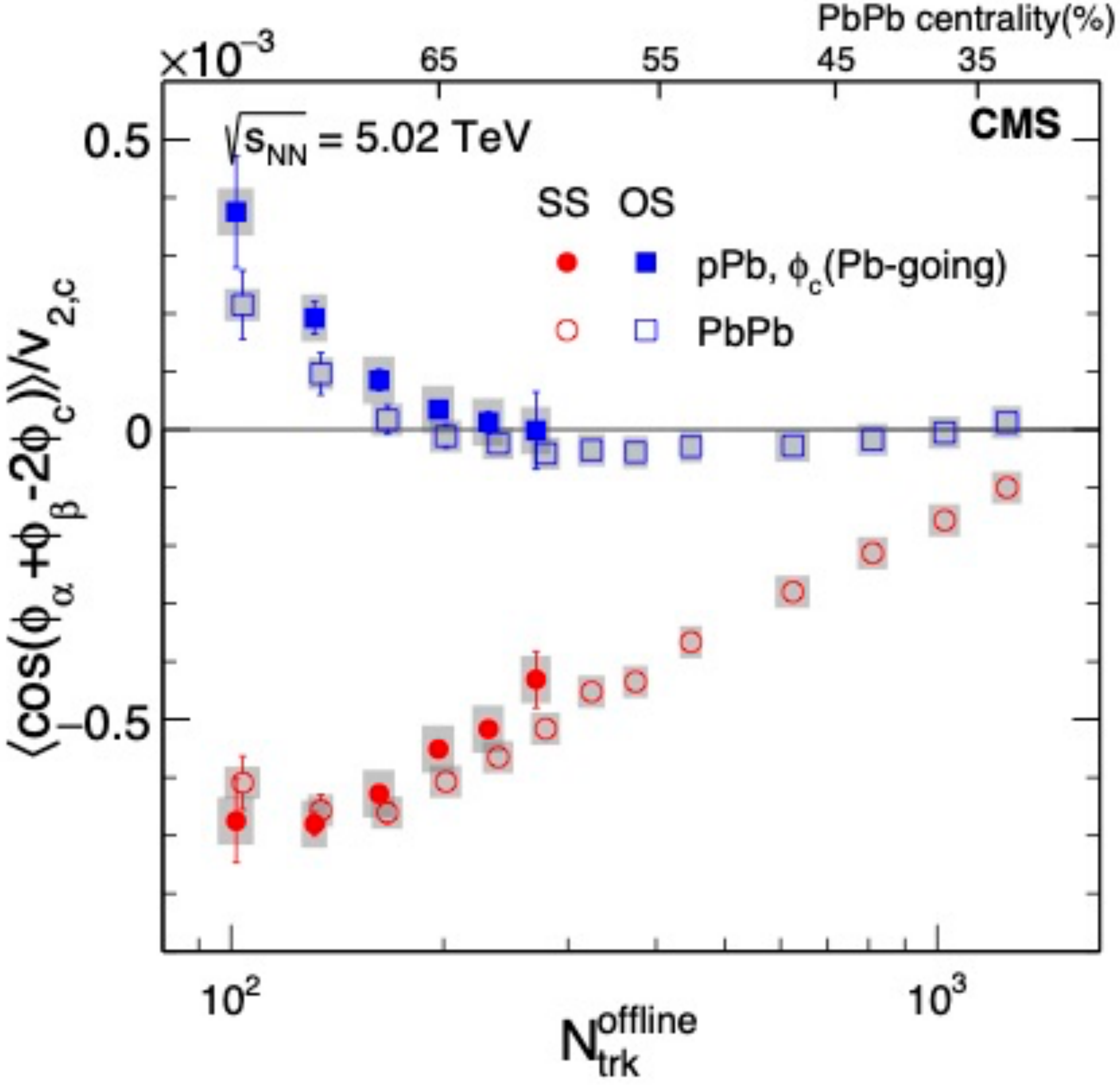
Very Tricky and Interesting to Explain Theoretically

LHC '12 (1207.0900) Pb + Pb at $\sqrt{s} = 2.76\,\mathrm{TeV}$



Similar Pattern is Observed

LHC '16 (1610.00263) $\mathrm{P} + \mathrm{Pb}$ at $\sqrt{s} = 5.02\,\mathrm{TeV}$



Magnetic Field is Much Smaller in $\mathrm{P} + \mathrm{Pb}$ than in $\mathrm{Pb} + \mathrm{Pb}$

Non-CME Backgrounds is large

Backgrounds

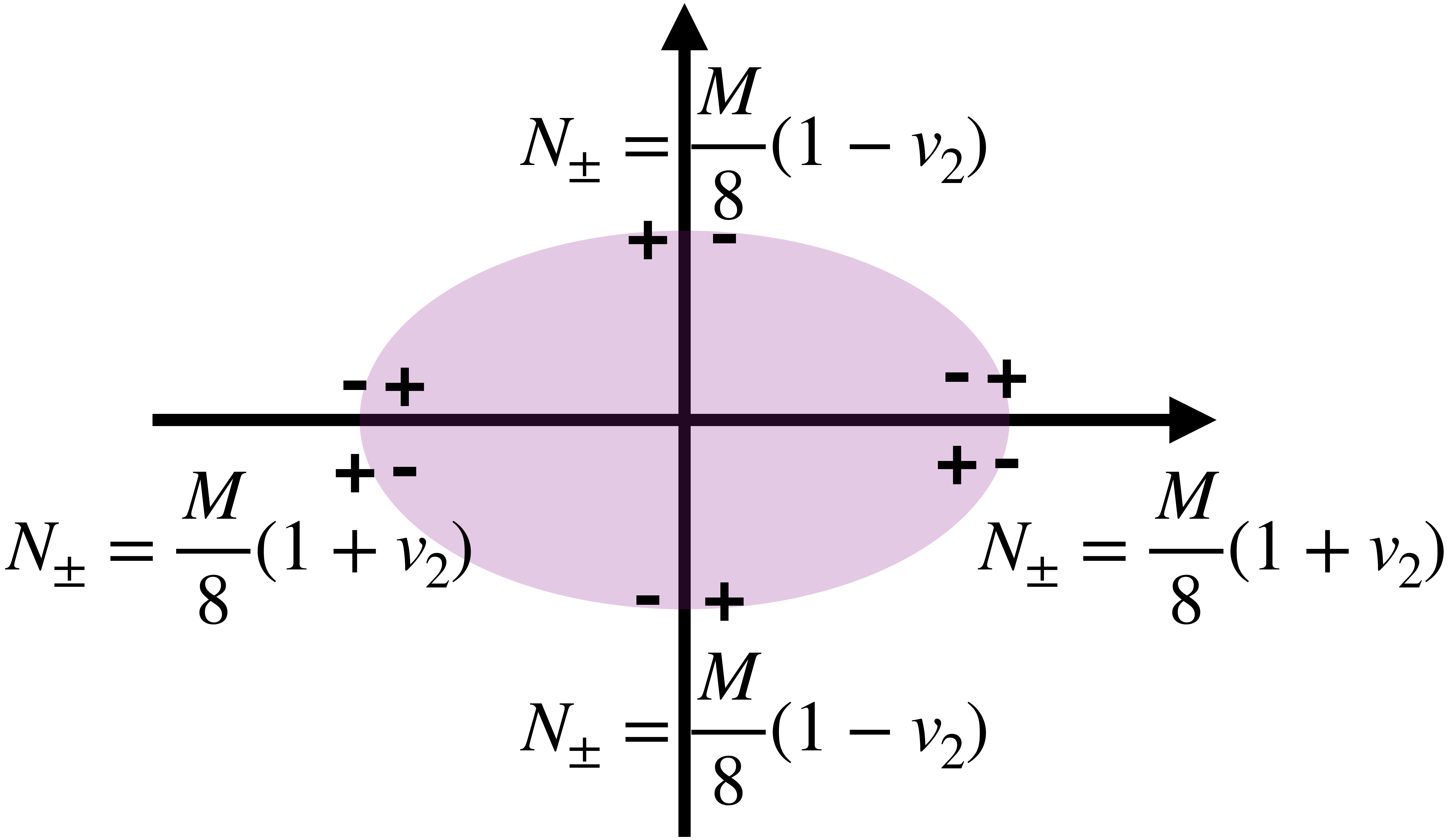
A finite multiplicity of particles, $M = N_+ + N_- = 2N$,
have self-correlations of $1/N$

If one particle is selected, it is no longer available in the second selection experimentally, which affects $\dfrac{d^2N}{d\phi_1 d\phi_2}$

In addition, $\dfrac{d^2N}{d\phi_1 d\phi_2}$ in general has genuine
charge-dependent 2-particle correlations

These effects combined is described by the Balance Function of ``Local Charge Conservation" (Pratt-Schlichting, 1005.5341)

A Toy Example of Local Charge Conservation



Neutral pairs of $\pi^\pm$ at
$\phi = 0, \pi/2, \pi, 3\pi/2$
with elliptic flow v_2

$$\langle\cos(2\phi)\rangle = M/4(1+v_2)/(M/2) - M/4(1-v_2)/(M/2) = v_2$$

$$\gamma_{++} = M/4(1+v_2)/(M/2)(-1/(M/2-1)) + M/4(1-v_2)/(M/2)(1/(M/2-1)) = -2v_2/M$$

$$\gamma_{+-} = 0$$

$$\Delta\gamma = \frac{2v_2}{M}$$

Sum Rules

From Self-Correlations

$$\Delta\gamma = \frac{2v_2}{M} + \frac{1}{2}\langle d_y^2 - d_x^2 \rangle, \quad \Delta\delta = \frac{2}{M} - \frac{1}{2}\langle d_y^2 + d_x^2 \rangle$$

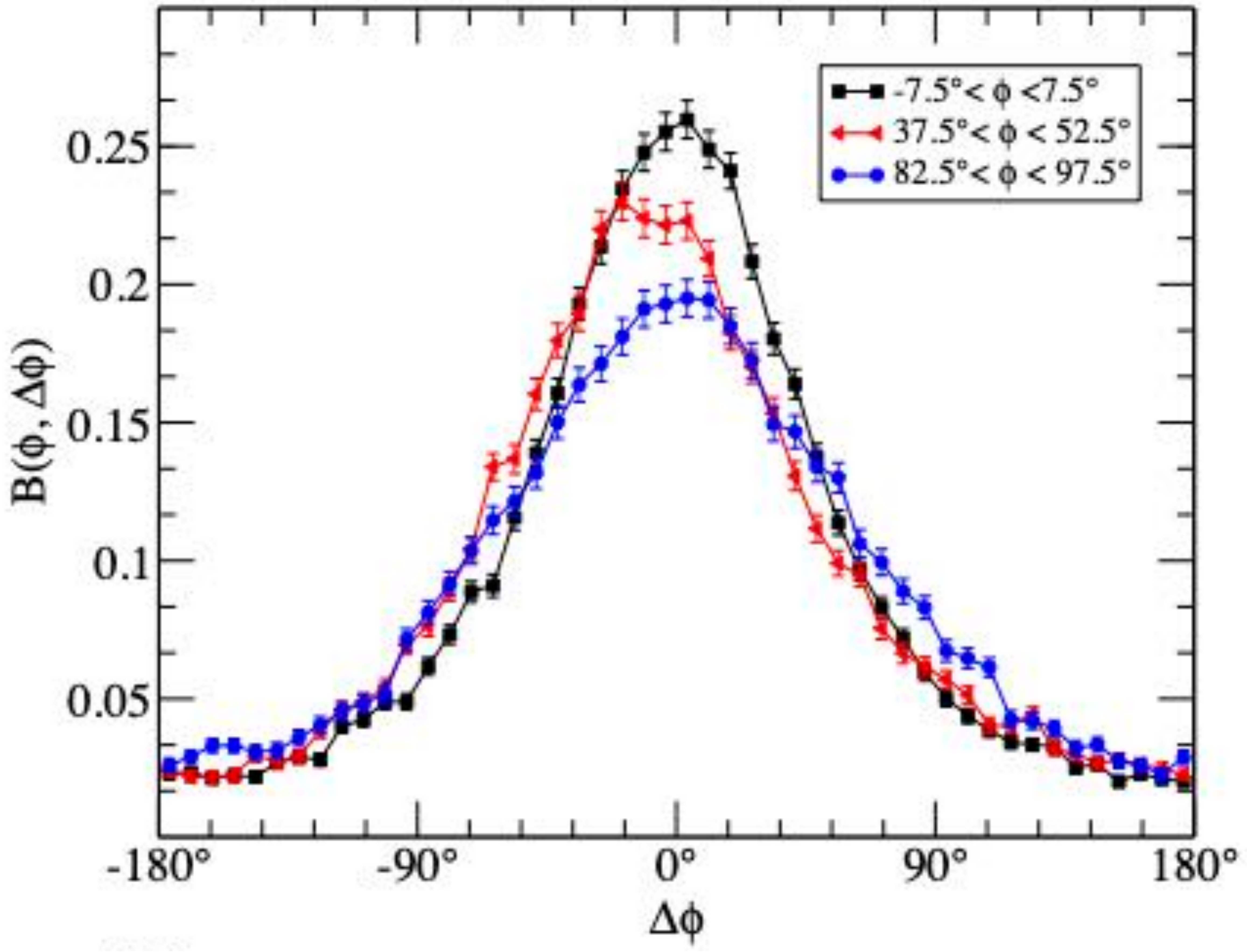
From CME and Finite size of the 2-particle Balance Function

$$d_y = \frac{1}{N}\left(\sum_{i=1}^{N} \sin\phi_i^+ - \sum_{i=1}^{N} \sin\phi_i^- \right), \quad d_x = \frac{1}{N}\left(\sum_{i=1}^{N} \cos\phi_i^+ - \sum_{i=1}^{N} \cos\phi_i^- \right)$$

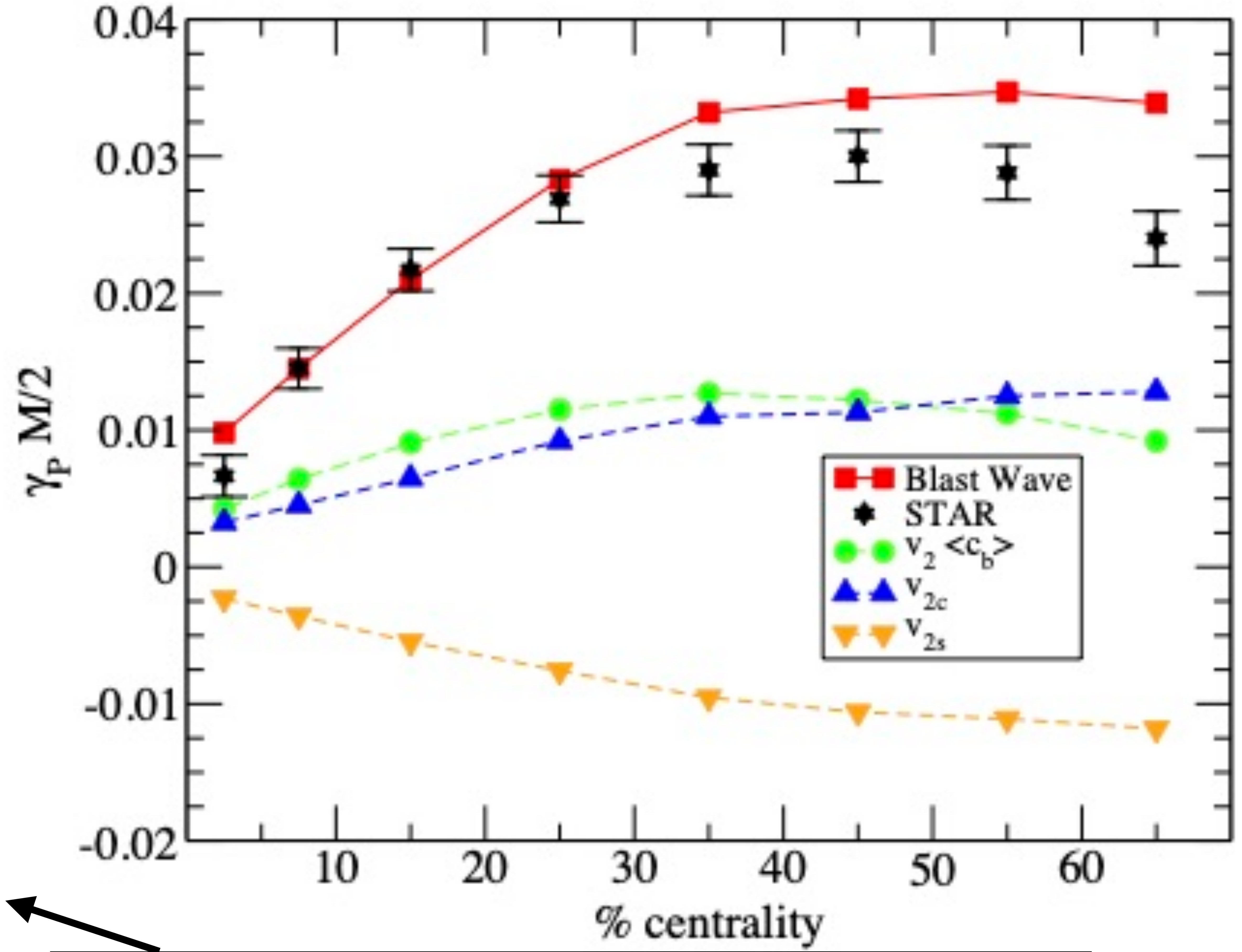
Event-by-Event Mean Charge Dipole

The Balance Function

(Pratt-Schlichting,1005.5341)



$\Delta\gamma^{\text{exp}}$ may be explained by Local Charge Conservation



$$\frac{d^2N}{d\phi^+ d\phi^-} - \frac{d^2N}{d\phi^+ d\phi^+} = \frac{dN}{d\phi} \cdot B(\phi, \Delta\phi)$$

The Charge Balance Function

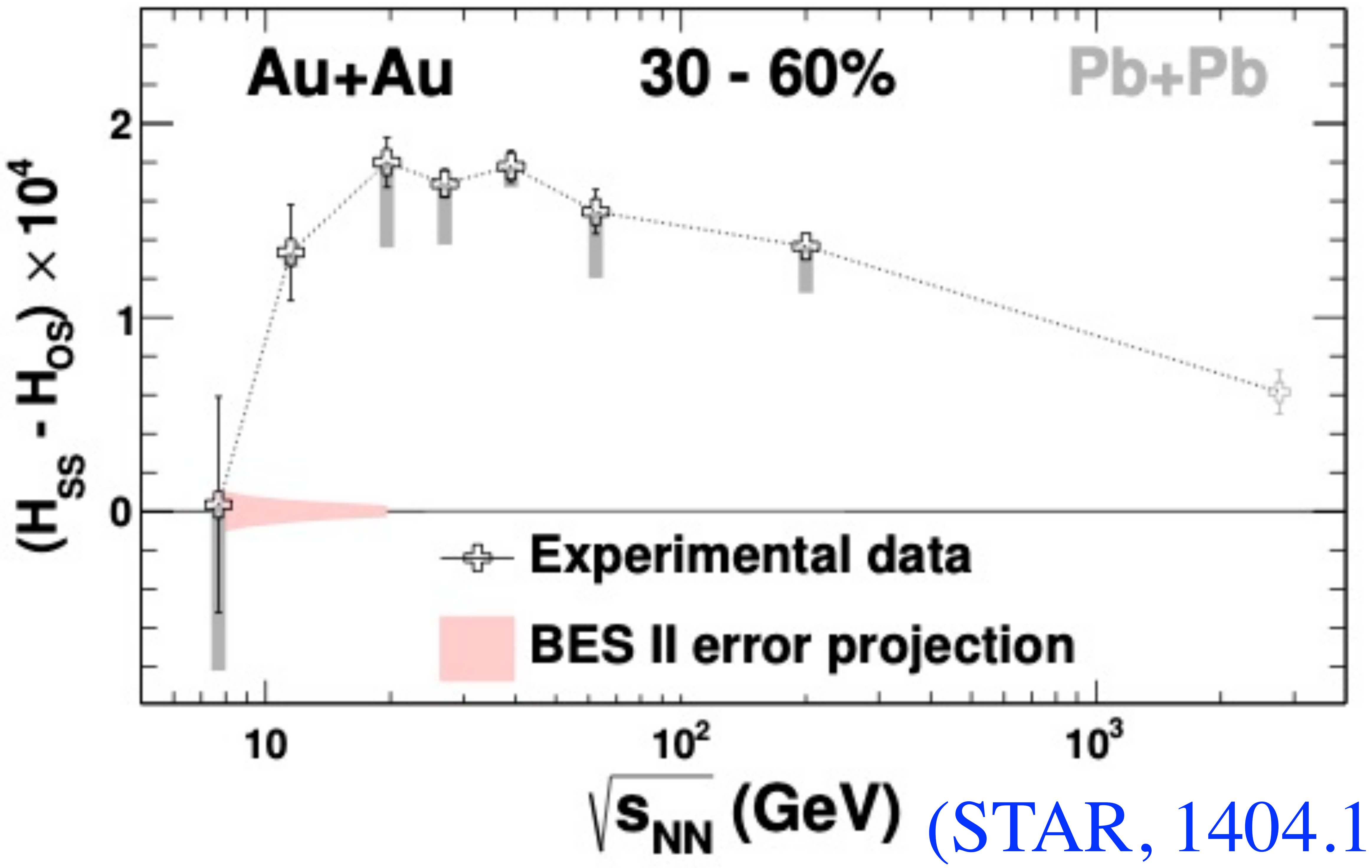
The H - Observable

(Bzdak-Koch-Liao, 1207.7327)

This motivates $\Delta\gamma = \kappa v_2 B - H$, $\Delta\delta = B + H$,
with the backgrounds $B \sim 1/N$

$$H = \frac{\kappa v_2 \Delta\delta - \Delta\gamma}{1 + \kappa v_2}$$

$$\kappa \sim 1$$



(STAR, 1404.1433)

Experimental Efforts

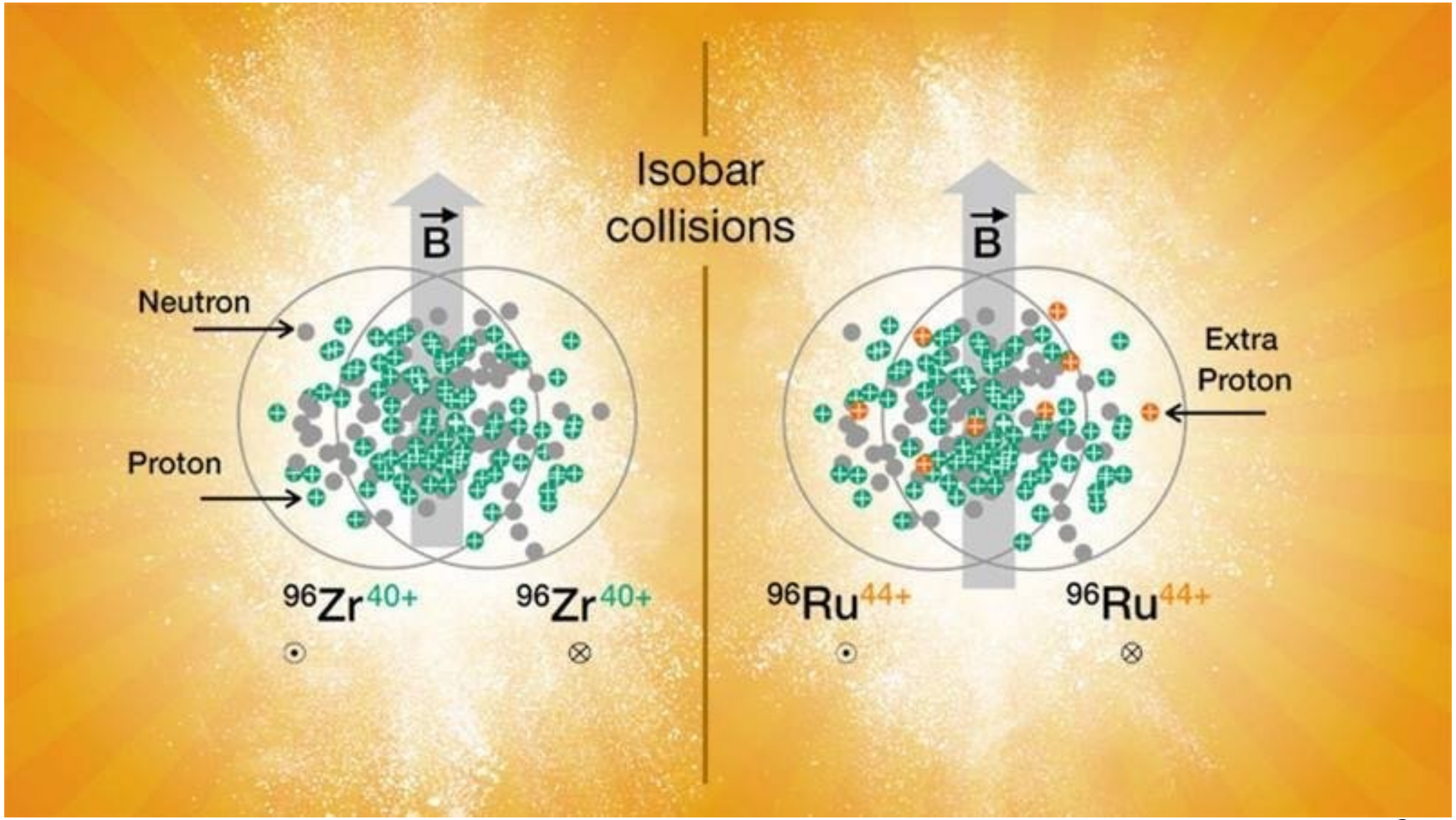
Beam Energy Scan in RHIC - S. Voloshin, G. Wang, 0907.2213, 1210.5498
Event Shape Eng. U+U in RHIC - Chatterjee-P. Tribedy, 1412.5103
Pb+Pb in LHC - 1207.0900, 2005.14640
pA vs AA in LHC - W. Li, 1610.00263
New Observables, e.g., Pair-Invariant Mass - F. Wang *et al.*, 1705.05410
R-correlator - R. Lacey, *et al.* 1710.01717
Signed Balance Function - A. Tang, 1903.04622
Isobar collisions of Zr and Ru in RHIC (2018) (New Result !) - 2109.00131

Theoretical Efforts

3D Viscous Anomalous Hydrodynamics - J. Liao, S. Shi, *et al.*
Chiral Magneto-Hydrodynamics - Y. Yin, Gursoy-Kharzeev-Rajagopal
Beam Energy Scan Theory Collaboration (BEST) - 2108.13867

Isobar Collisions : $^{96}_{40}\mathrm{Zr}$ and $^{96}_{44}\mathrm{Ru}$

(S. Voloshin, 0907.2213)

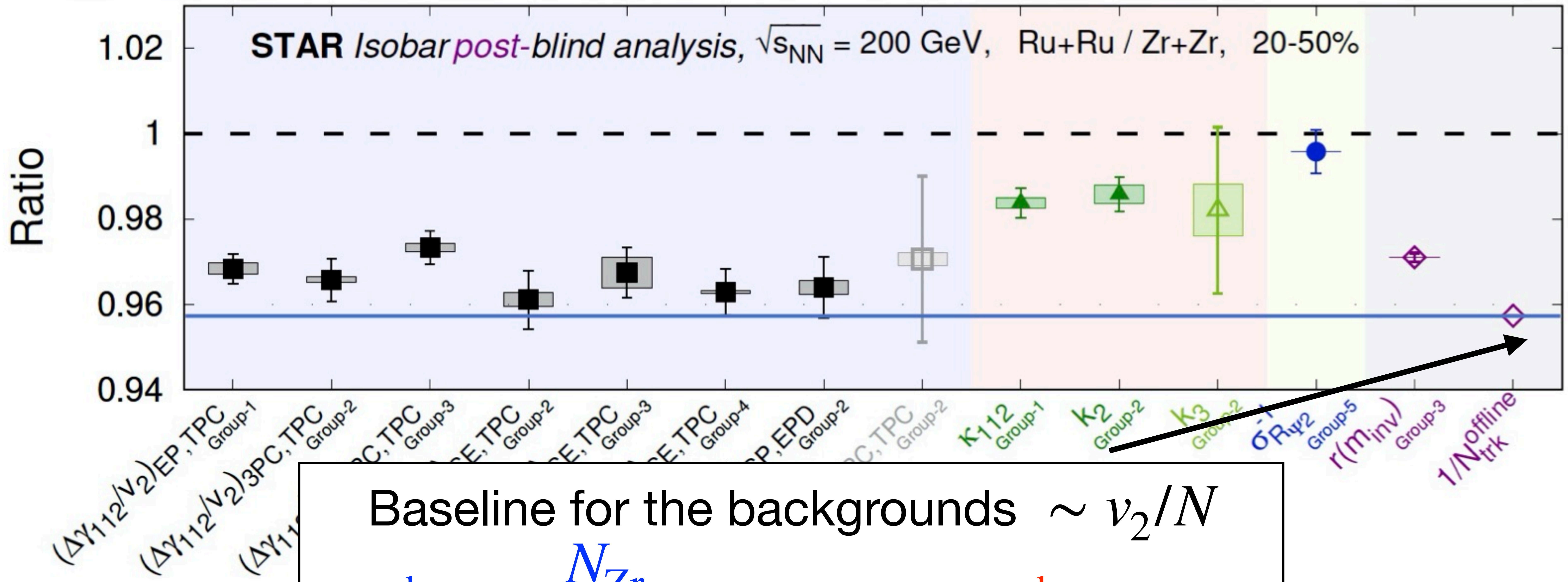


Picture from BNL

CME signal scales with $(eB)^2$: $R = \dfrac{\mathrm{CME(Ru)}}{\mathrm{CME(Zr)}} \sim \left(\dfrac{44}{40}\right)^2 \approx 1.2$

STAR '21 Result : Predefined Observables

Predefined observables, assuming identical backgrounds for Zr and Ru
$R^{\mathrm{exp}} < 1$



STAR '21 Result
(Run in 2018)
2109.00131

Baseline for the backgrounds $\sim v_2/N$

$R^{\mathrm{base}} = \dfrac{N_{\mathrm{Zr}}}{N_{\mathrm{Ru}}} < 1$ and $R^{\mathrm{exp}}/R^{\mathrm{base}} > 1$

CME signal may exist !

(Kharzeev-Liao-Shi, 2205.00120)

Chiral Magnetic Wave (CMW)

(Kharzeev-Yee, 1012.6026)

$$\partial_t n_{R/L} + \vec{\nabla} \cdot \vec{\mathbf{J}}_{R/L} = \partial_t n_{R/L} \pm \frac{e}{4\pi^2\chi} \vec{\mathbf{B}} \cdot \vec{\nabla} n_{R/L} = (\partial_t + \vec{\mathbf{v}}_\chi \cdot \vec{\nabla}) n_{R/L} = 0$$

$$\vec{\mathbf{J}}_{R/L} = \pm \frac{e^2}{4\pi^2} \mu_{R/L} \vec{\mathbf{B}} \approx \pm \frac{e}{4\pi^2\chi} n_{R/L} \vec{\mathbf{B}}$$

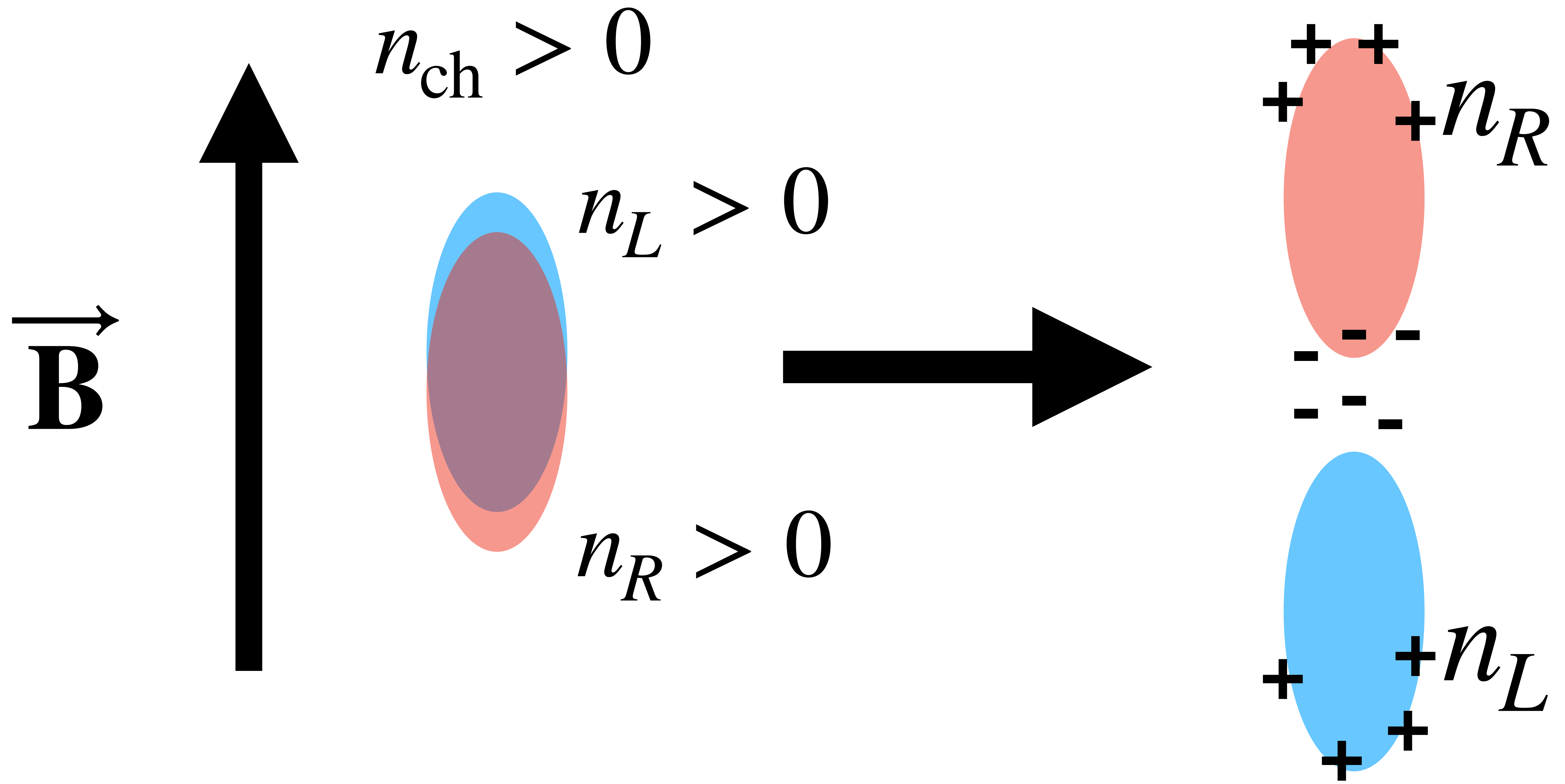
$\chi =$ charge susceptibility

Hydrodynamic propagating modes of chiral charges with velocity

$$\vec{\mathbf{v}}_\chi = \pm \frac{1}{4\pi^2\chi} \vec{\mathbf{B}}$$

Similar to sound waves

Experimental Signature of CMW



Quadrupole Moment of Charges

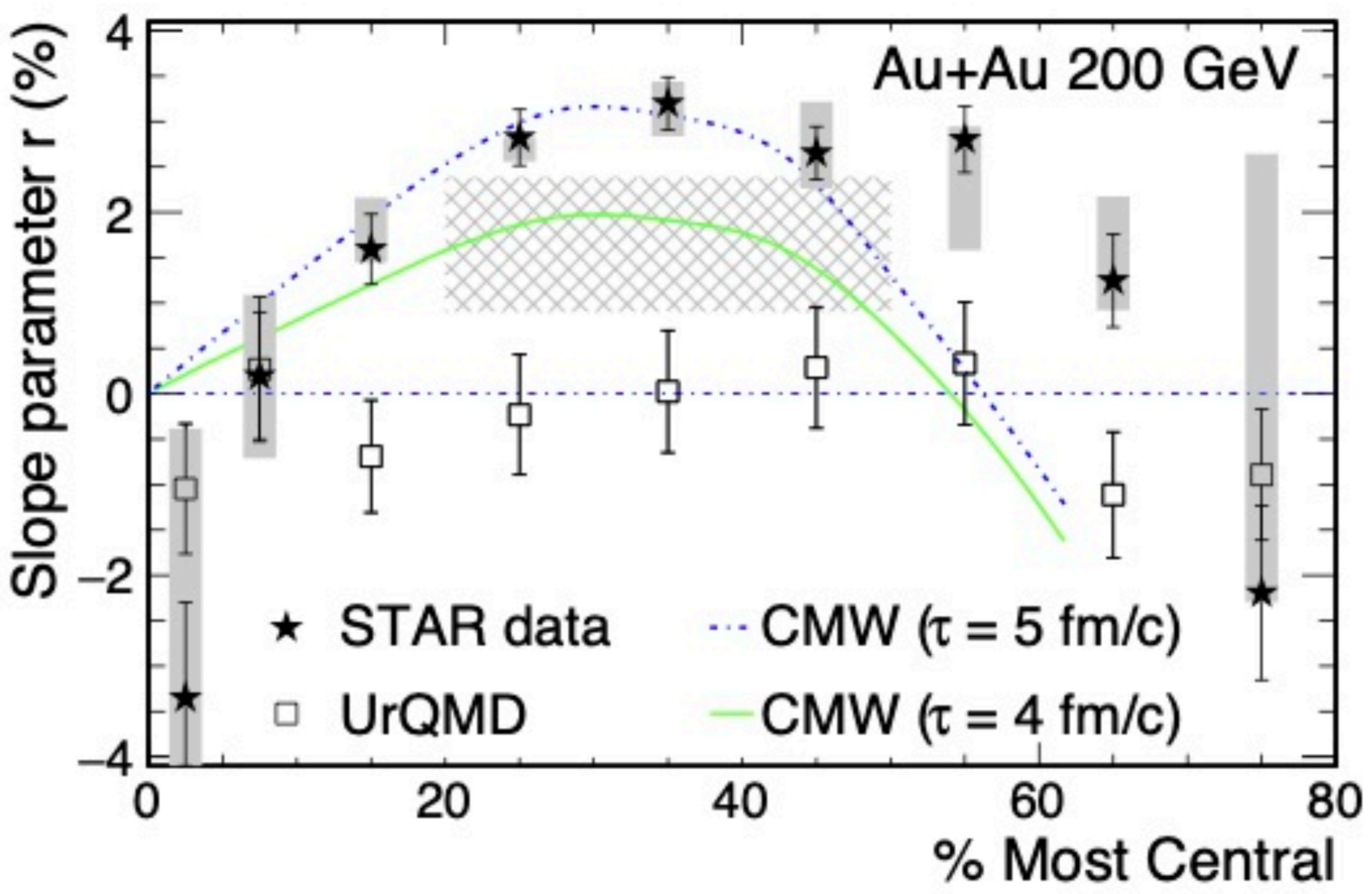
(Gorbar-Miransky-Shovkovy, 1101.4954
Burnier-Kharzeev-Liao-Yee, 1103.1307)

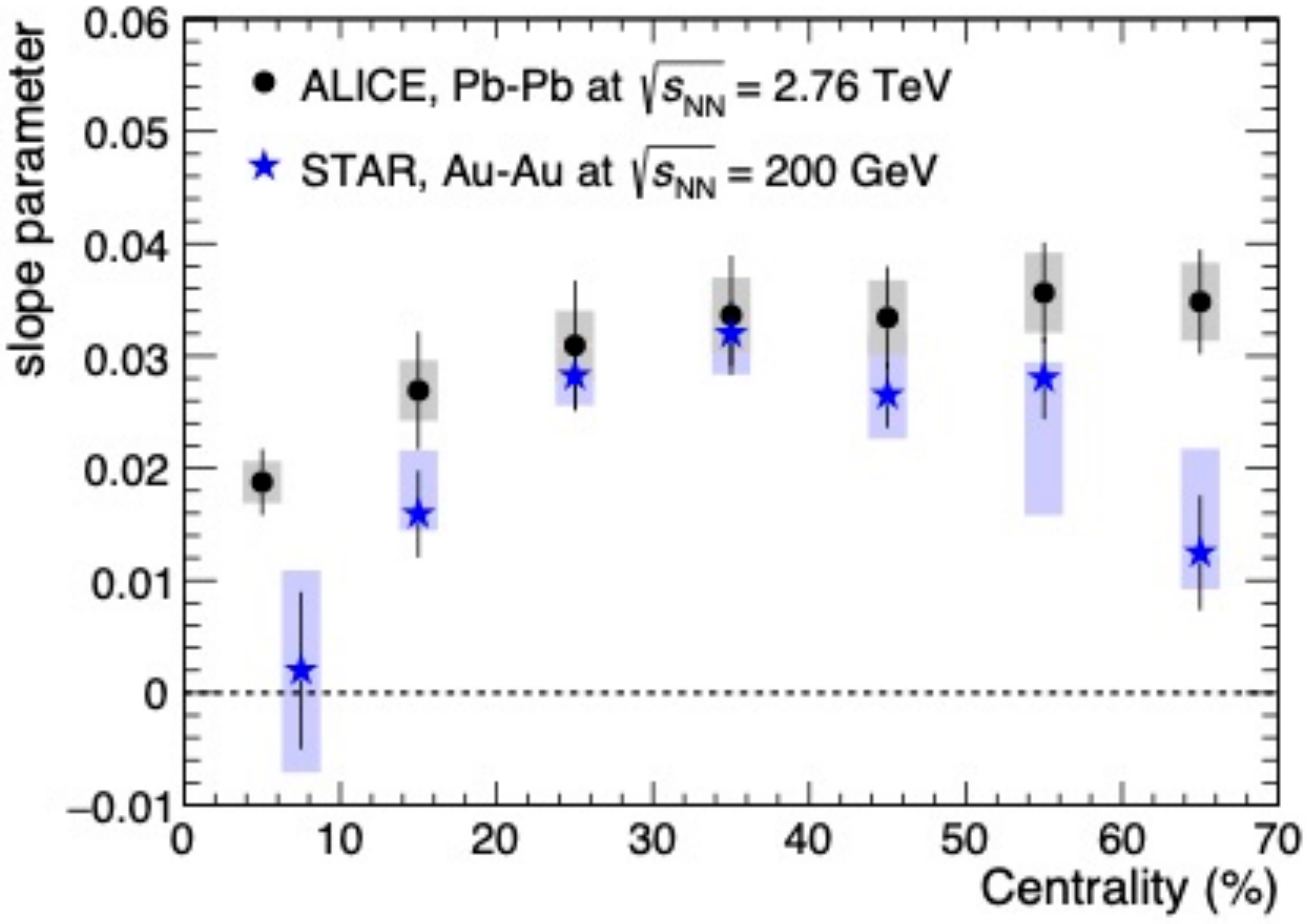
Charge Dependent Elliptic Flows

$$v_2(\pi^-) - v_2(\pi^+) = r\,A_{\rm ch}\,, \quad A_{\rm ch} \equiv \left(\frac{N_{\pi^+} - N_{\pi^-}}{N_{\pi^+} + N_{\pi^-}} \right)$$

Slope Parameter $r > 0$

Slope Parameter in RHIC and LHC





STAR, 1504.02175

LHC, 1512.05739

Agrees with the CMW Predictions
(but, there are backgrounds effects, too)

What we did not discuss

- Chiral Hydrodynamics - D. Son-P. Surowka, 0906.5044
- Collective Modes - I. Shovkovy, 1807.07608, 2111.11416
- Chiral Plasma Instability and Chiral Turbulence - N. Yamamoto, 1302.2125, 1603.08864
- CME in Dirac/Weyl semi-metals - D.Kharzeev, Q. Li, et al., 1412.6543, K. Landsteiner, 1306.4932, 1610.04413

Related Presentations

Tuesday, June 14

- Parallel 1, 4:30 pm, by Wenya Wu
- Parallel 4, 4:00 pm, by Roy Lacey
- Parallel 4, 4:20 pm, by Yicheng Feng

Thursday, June 16

- Plenary, 9:05 am, by Evan Finch