Spin Polarization Phenomena

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- Local spin polarization
  - Experimental results
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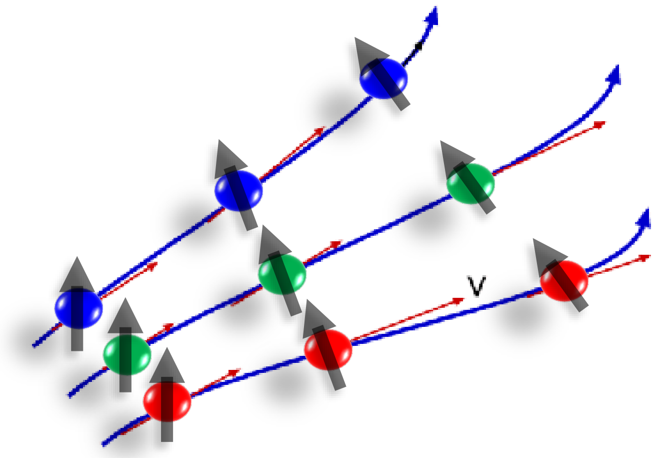
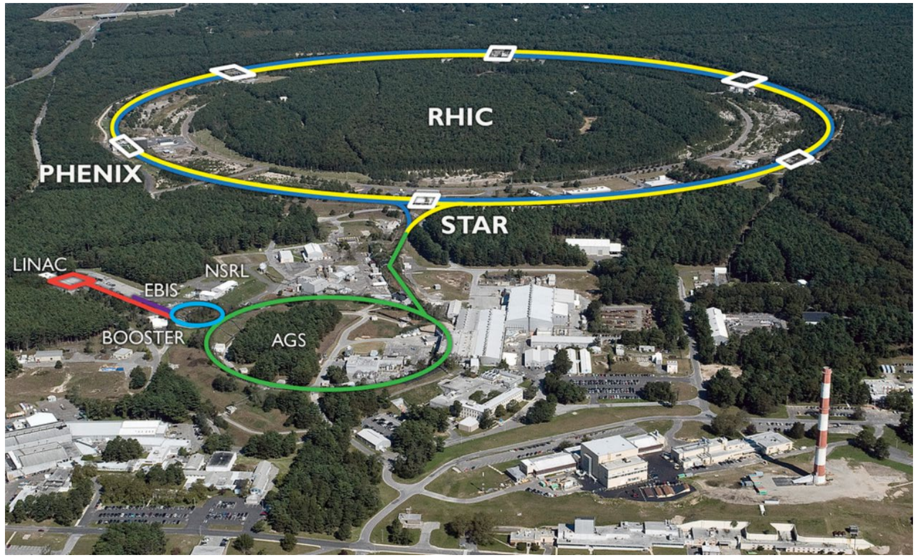
- Spin alignment of vector mesons
- Summary

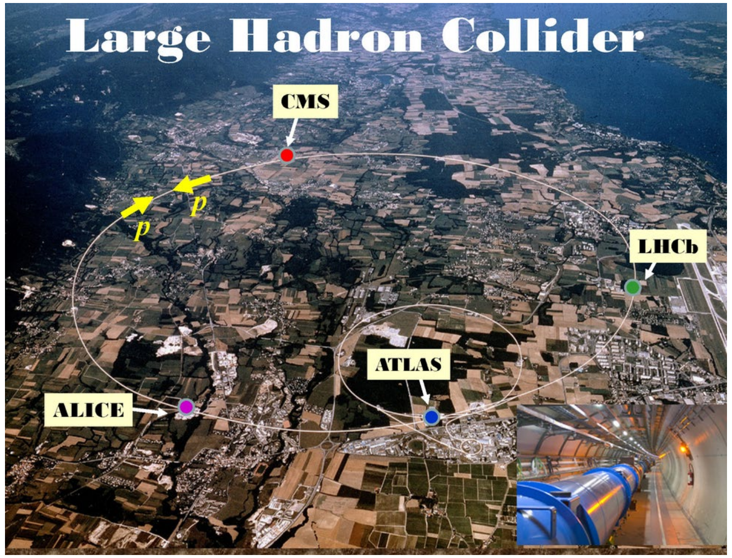


Introduction

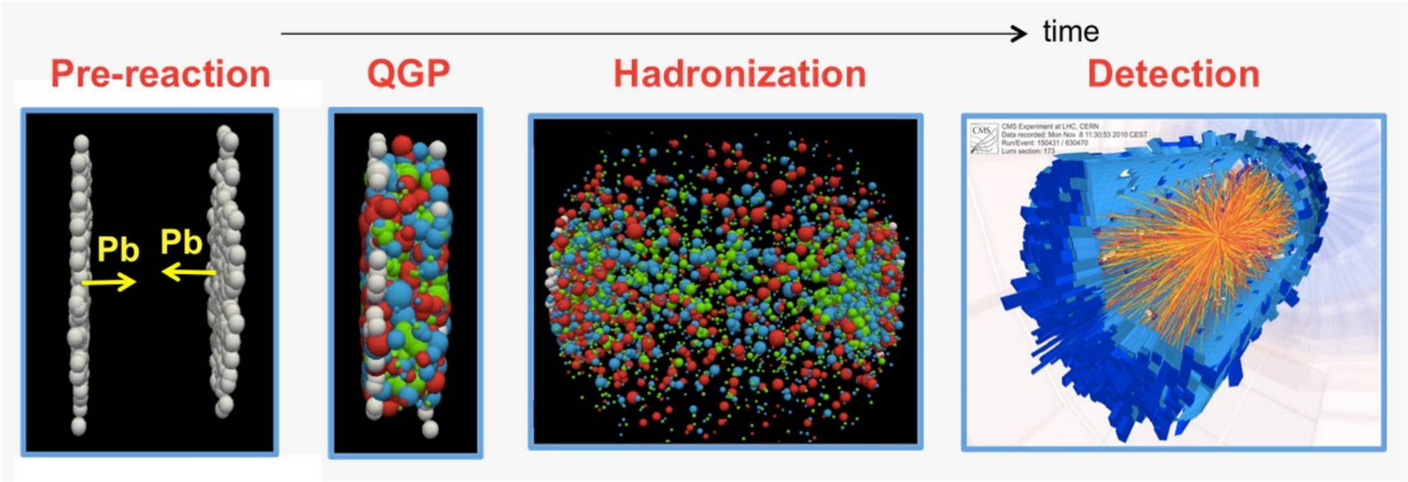
Heavy ion collisions and quark gluon plasma

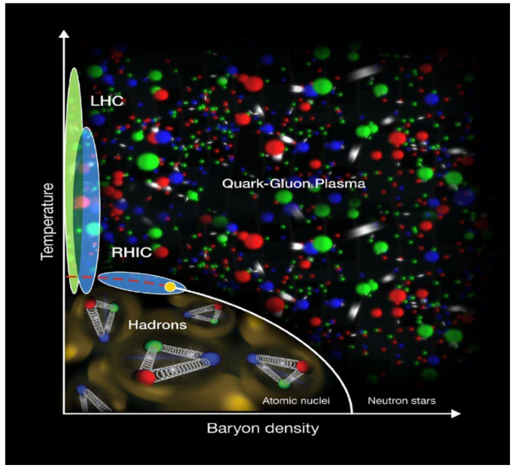


RHIC@BNL



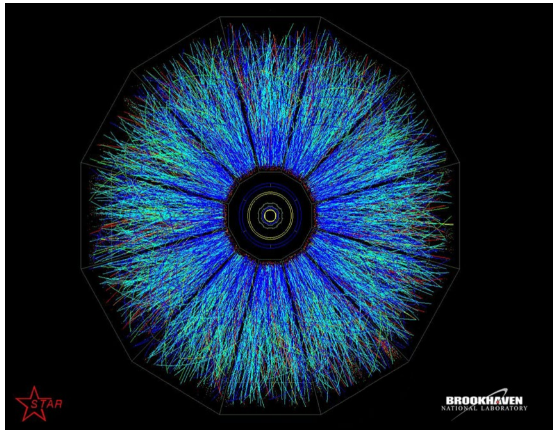
LHC@CERN

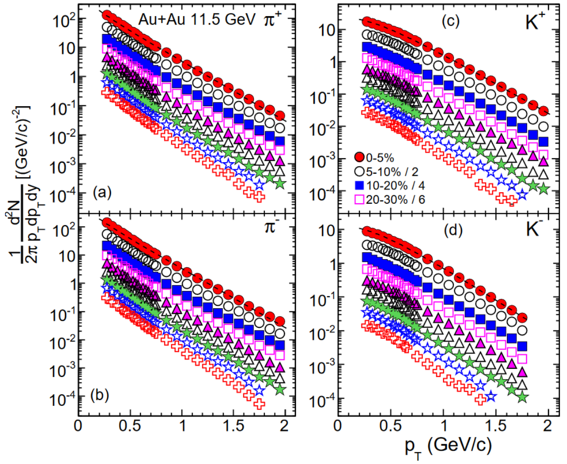


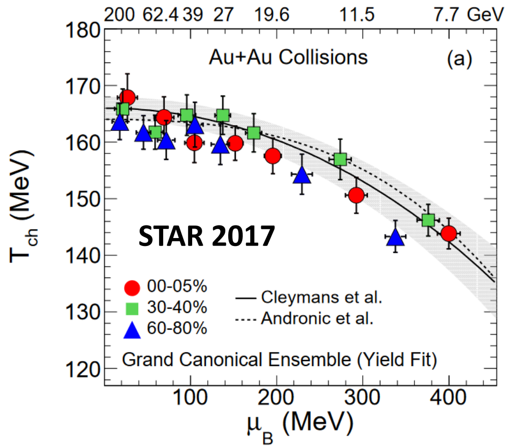


Probes of the quark gluon plasma

- Electric or flavor probes of quark gluon plasma (QGP)
- For example: Multiplicity of produced hadrons
  Thermodynamics, chemical freeze-out, … …



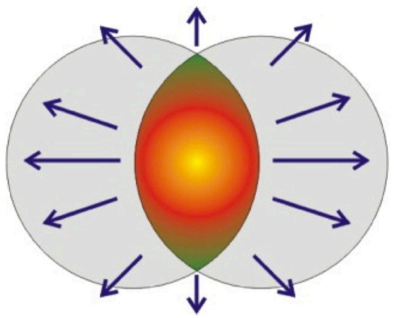




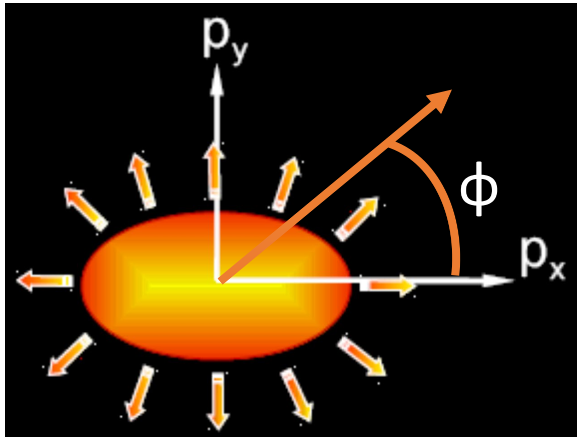
See lecture by A. Andronic

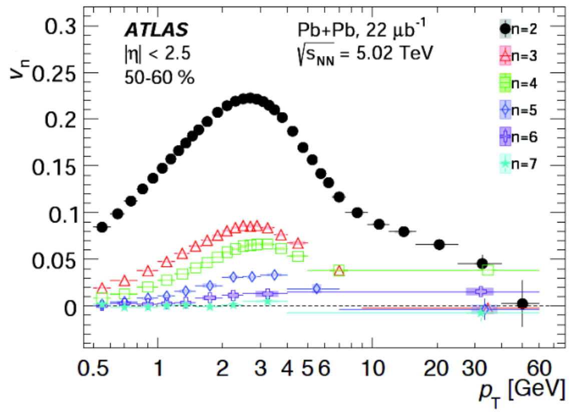
Probes of the quark gluon plasma

- Electric or flavor probes of quark gluon plasma (QGP)
- For example: Anisotropy in charged-hadron spectra
  harmonic flow coefficients -> equation of state, transport properties



⊙ z direction





$$\frac{dN_{\rm ch}}{d\phi} \propto 1 + 2v_1 \cos(\phi - \Psi_1) + 2v_2 \cos[2(\phi - \Psi_2)] + \cdots$$

See lecture by Y. J. Lee

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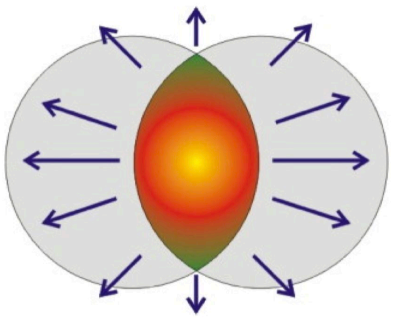
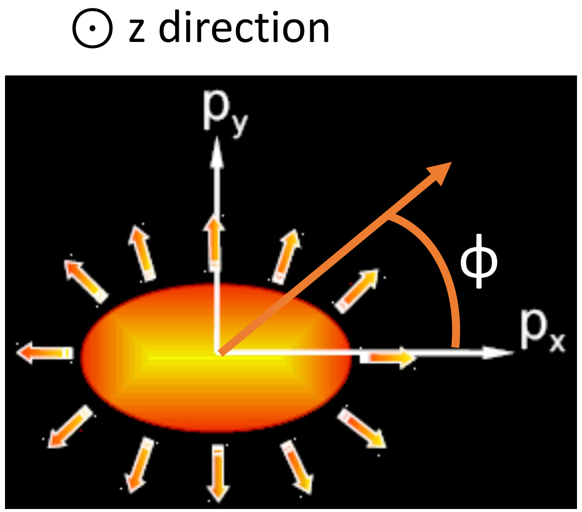
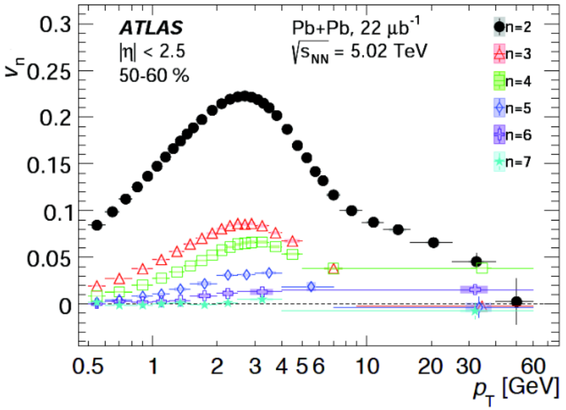






$$\frac{dN_{\rm ch}}{d\phi} \propto 1 + 2v_1 \cos(\phi - \Psi_1) + 2v_2 \cos[2(\phi - \Psi_2)] + \cdots$$

See lecture by Y. J. Lee

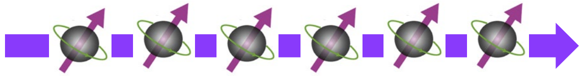
- These are the "electronics (flavortronics)" of QGP

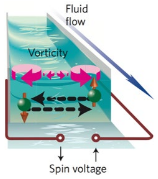
Probes of the quark gluon plasma

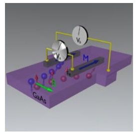
- Electronics vs. spintronics in condensed matter physics (and industry)

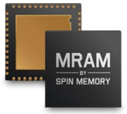
Electronics

Spintronics







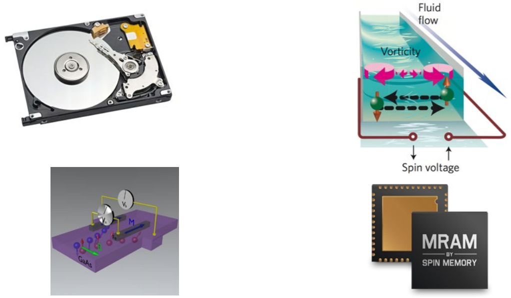


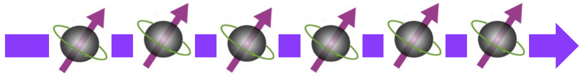
Probes of the quark gluon plasma

- Electronics vs. spintronics in condensed matter physics (and industry)

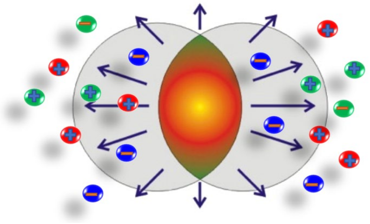
Electronics

Spintronics

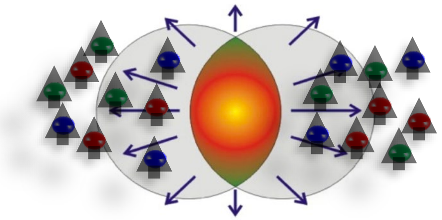




- "Electronics" vs. "spintronics" in heavy-ion collisions?
  - Charged hadrons multiplicity N_{ch}
  - Harmonic flows of charges $v_1, v_2, \ldots$
  - … …



  - Hyperon spin polarization $P_{y,x,z}$
  - Harmonic flows of spin $f_{2;x,y,z}, \ldots$
  - … …



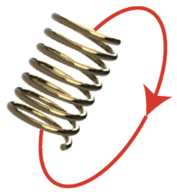
Spintronics

- How to polarize spin?

Magnetic moment

$$H_{\mathrm{Zeeman}} = -\gamma \boldsymbol{S} \cdot \boldsymbol{B}$$

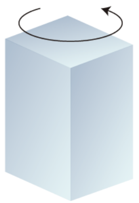
Magnetic field



Angular momentum

$$H_{\text{Spin-rotation}} = -S \cdot \Omega$$

Rotation field



Spin orbit coupling

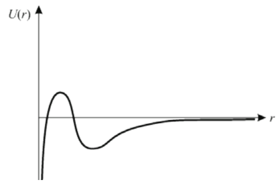
$$H_{\mathrm{SOC-E}} = -\lambda \boldsymbol{S} \cdot (\boldsymbol{p} \times \boldsymbol{E})$$

Electric field

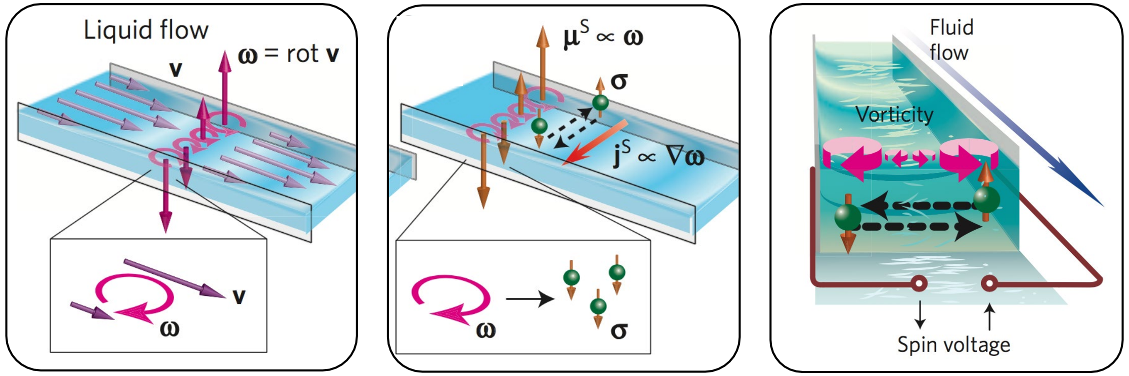
Spin orbit coupling

$$H_{\mathrm{SOC-U}} = \eta \frac{1}{r} \frac{\partial U}{\partial r} \boldsymbol{S} \cdot \boldsymbol{L}$$

External potential



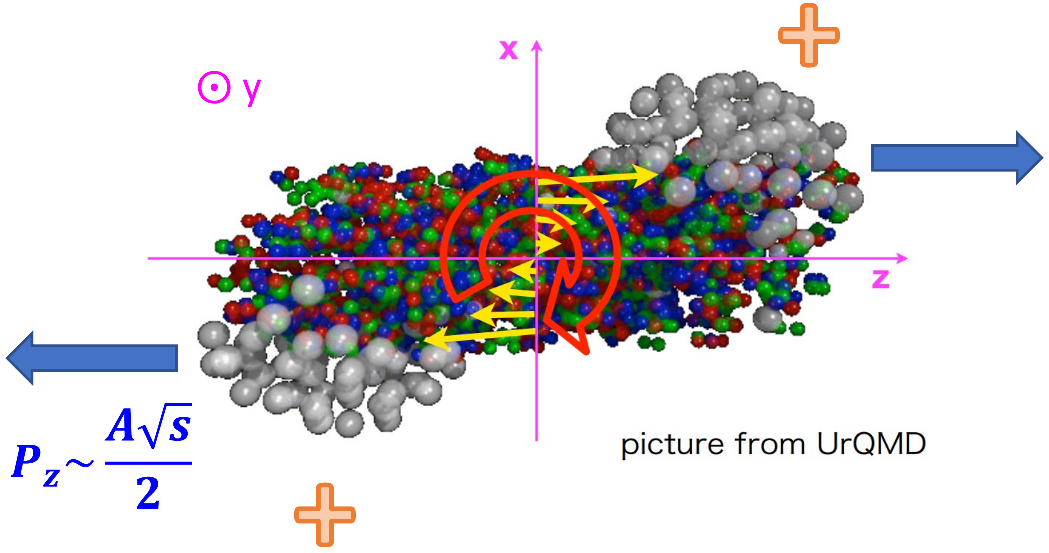
- An interesting example



Takahashi et al. 2016

- To realize spin probes of quark gluon plasma (QGP): Rotation, Magnetic field, ……?

Global angular momentum and magnetic fields



$$J_0 \sim \frac{Ab\sqrt{s}}{2} \sim 10^6 \hbar$$

Global angular momentum

$$eB \sim \gamma \alpha_{\mathrm{EM}} \frac{Z}{b^2} \sim 10^{18}\ \mathrm{G}$$

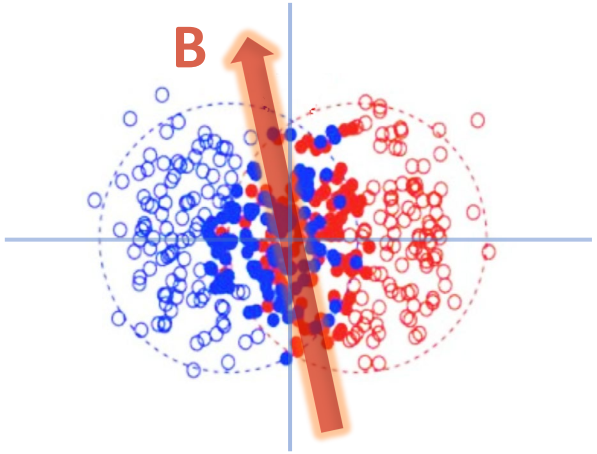
Strong magnetic field

(RHIC Au+Au 200 GeV, b=10 fm)

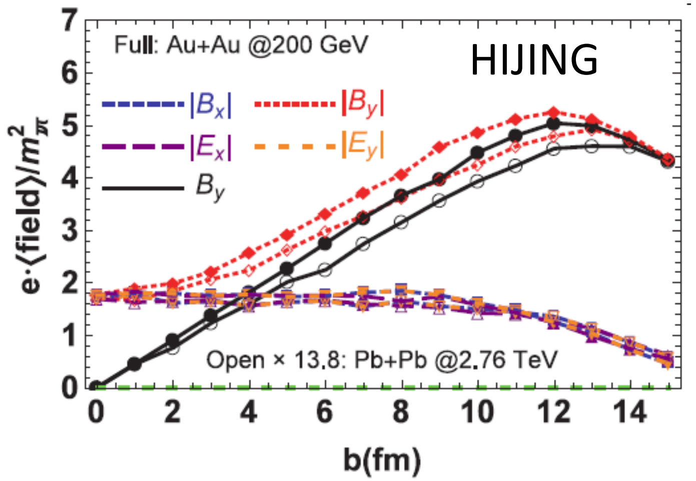
Magnetic fields

- Initial B and E fields:

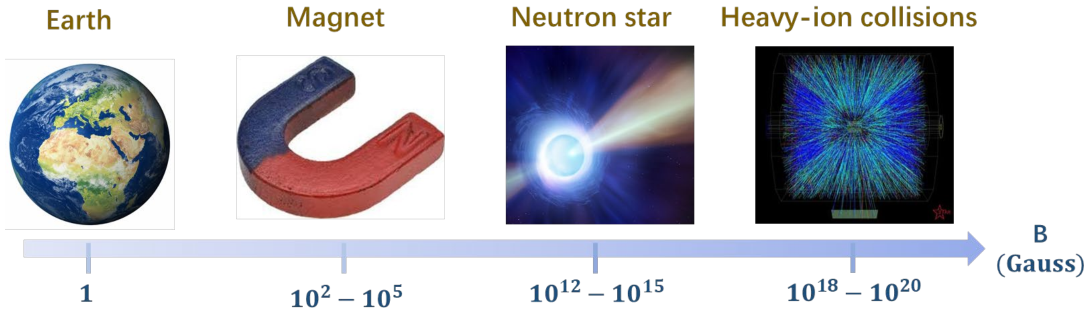
Fluctuations



Event average



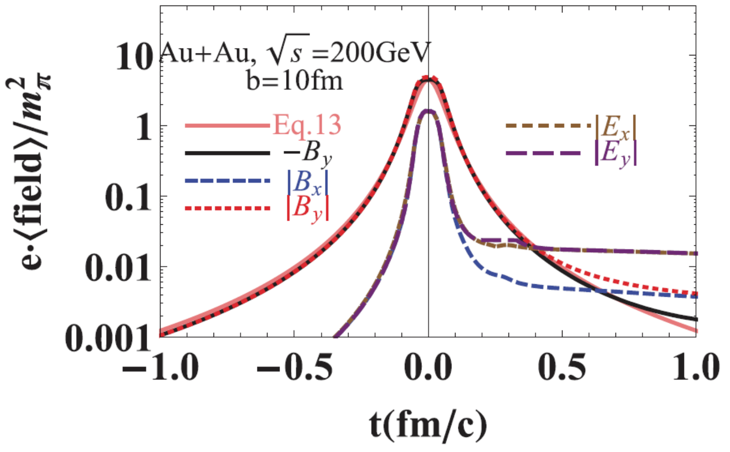
(Deng-XGH 2012)



Many similar simulations.
Sorry that I cannot list all the references

Magnetic fields

- If quark-gluon matter is insulating:



(XGH 2015)

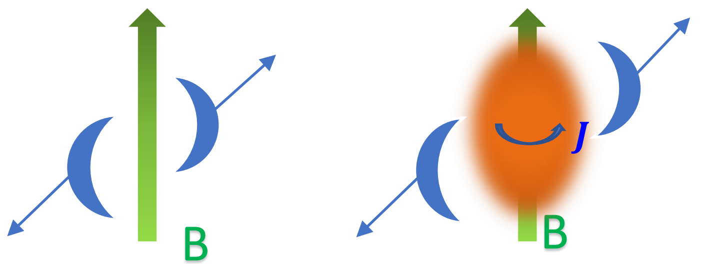
Well fitted by

$$\langle eB_y(t)\rangle \approx \frac{\langle eB_y(0)\rangle}{(1+t^2/t_B^2)^{3/2}}$$

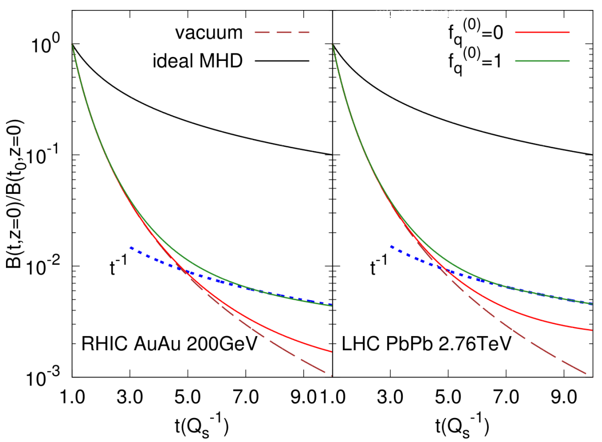
Life time of B field

$$t_B \approx R_A/(\gamma v_z) \approx \frac{2m_N}{\sqrt{s}} R_A$$

- More realistic evolution of B fields:

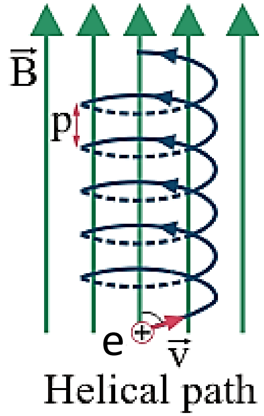


(Yan-XGH 2021)



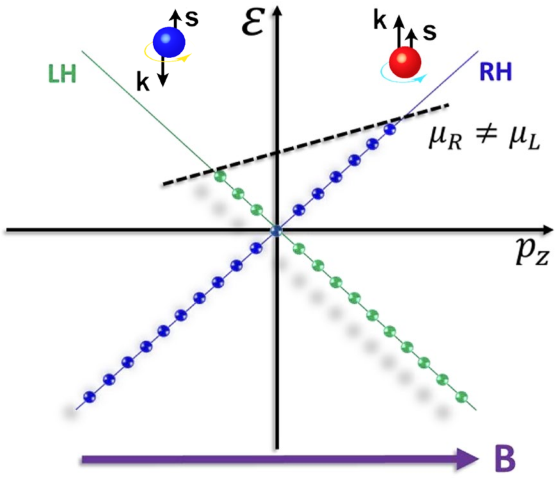
Magnetic fields

- Landau spin polarization of charged particle in a chirality-imbalanced medium:



$$E_n^2 = p_z^2 + 2neB$$

$$n = 0$$



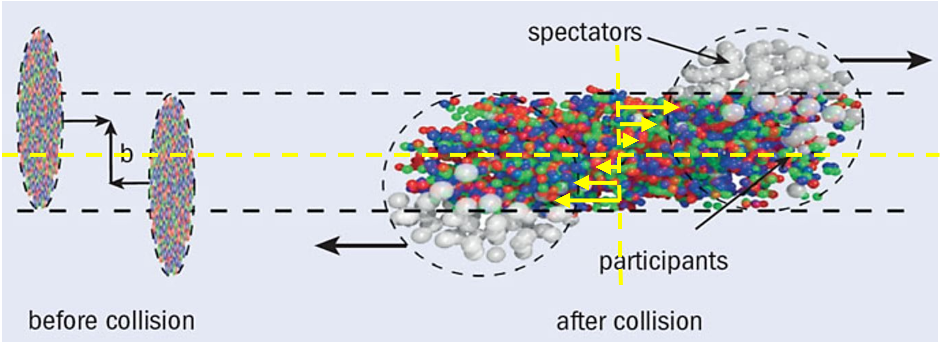
$$\boldsymbol{j}_V = \boldsymbol{j}_R + \boldsymbol{j}_L = \frac{e\boldsymbol{B}}{2\pi^2}\mu_A$$

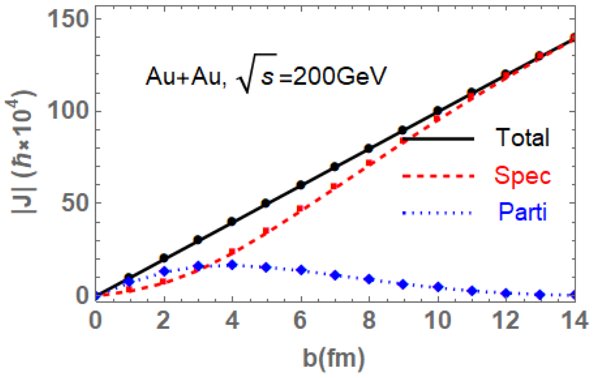
Chiral magnetic effect (CME)

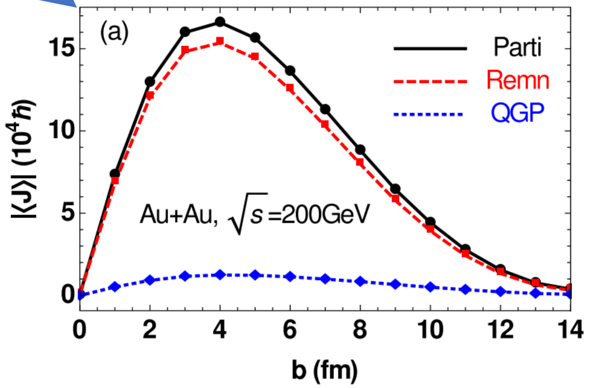
See lecture by H. –U. Yee

Vorticity by global angular momentum

- Angular momentum conservation







Vorticity by global angular momentum

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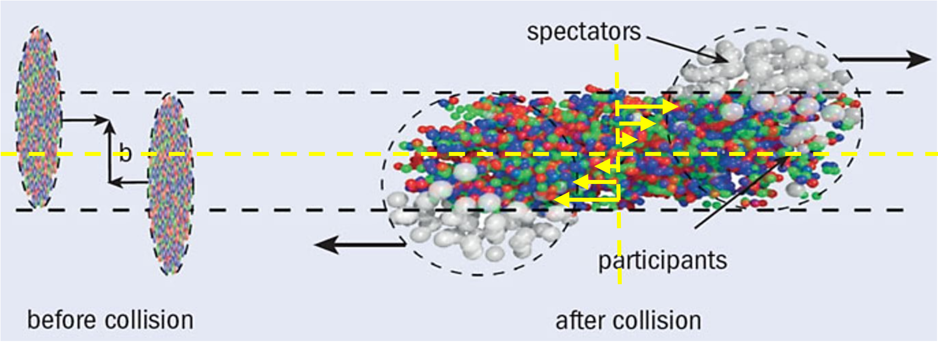
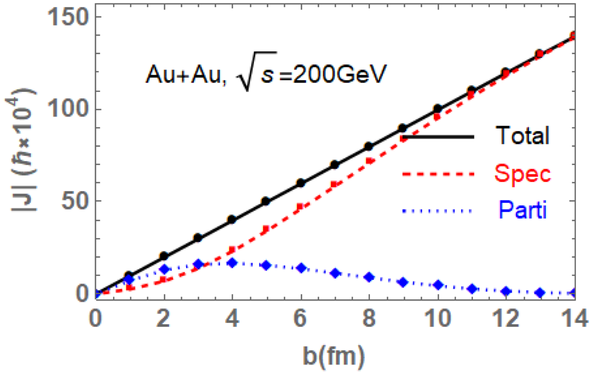
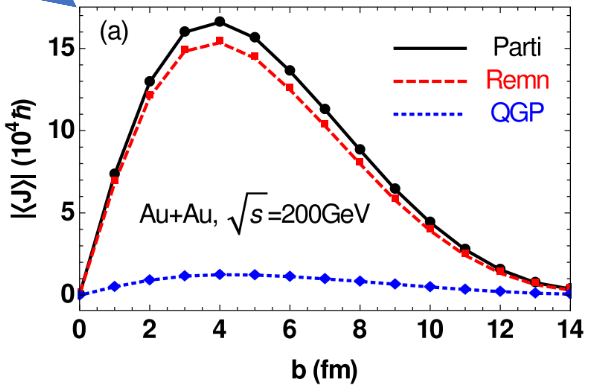


No rigid rotation*, but local fluid vorticity

$$\boldsymbol{\omega} = \frac{1}{2}\nabla \times \boldsymbol{v}$$

(Angular velocity of fluid cell)





* At low energy, there is a possibility that two colliding nuclei fuse into a compound high-spin nucleus

Vorticity by global angular momentum

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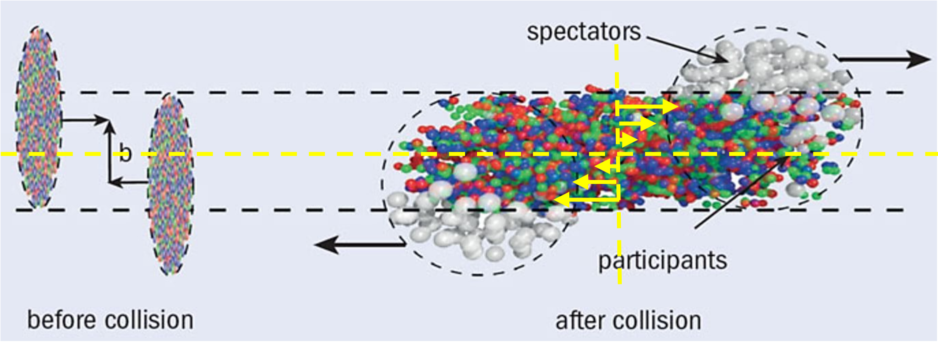


No rigid rotation*, but local fluid vorticity

$$\boldsymbol{\omega} = \frac{1}{2}\nabla \times \boldsymbol{v}$$

(Angular velocity of fluid cell)

- Estimation at low energy $\sqrt{s} \gtrsim 2m_N$

part of $J_0 \sim Ab(\sqrt{s} - 2m_N)$ **retained in the produced matter**:

$J = \int d^3\boldsymbol{x}\, I(\boldsymbol{x})\omega(\boldsymbol{x}) \approx \int d^3\boldsymbol{x}\, \varepsilon(\boldsymbol{x}) x_\perp^2 \varpi \sim 2m_N\, A\, R_A^2\, \varpi$ **for** $b < 2R_A$

$$\varpi \sim \frac{b}{R_A^2}\frac{\sqrt{s} - 2m_N}{2m_N} \sim 10^{22} s^{-1}$$

$(b = R_A, \sqrt{s} = 3\ \mathrm{GeV})$

- Estimation at high energy $\sqrt{s} \gg 2m_N$

part of $J_0 \sim Ab\sqrt{s}$ **retained in the produced matter**:

$J \approx \int d^3\boldsymbol{x}\, \gamma^2(\boldsymbol{x})\varepsilon(\boldsymbol{x}) x_\perp^2 \varpi \sim s\, A\sqrt{s}\, R_A^2\, \varpi/(2m_N)^2$ **for** $b < 2R_A$

$$\varpi \sim \frac{b}{R_A^2}\left(\frac{2m_N}{\sqrt{s}}\right)^2 \sim 10^{19} s^{-1}$$

$(b = R_A, \sqrt{s} = 200\ \mathrm{GeV})$

* At low energy, there is a possibility that two colliding nuclei fuse into a compound high-spin nucleus

Vorticity by global angular momentum

- Relativistic vorticities

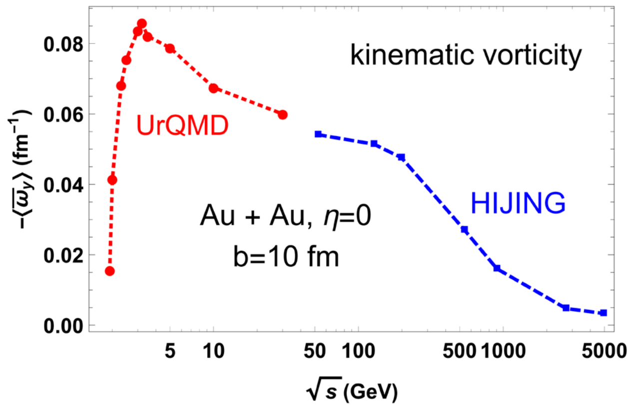
Kinematic vorticity $\omega_{\mu\nu}^{\mathrm{K}} = -\frac{1}{2}(\partial_\mu u_\nu - \partial_\nu u_\mu) \quad \Rightarrow \quad \omega_{\mathrm{K}}^{\mu} = -(1/2)\epsilon^{\mu\nu\rho\sigma} u_\nu \omega_{\rho\sigma}^{\mathrm{K}}$

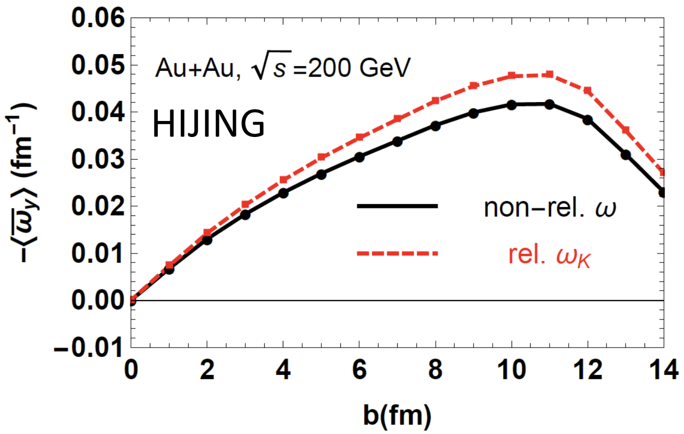
Temperature vorticity $\omega_{\mu\nu}^{\mathrm{T}} = -\frac{1}{2}[\partial_\mu(T u_\nu) - \partial_\nu(T u_\mu)]$

Thermal vorticity $\omega_{\mu\nu}^{\beta} = -\frac{1}{2}[\partial_\mu(\beta u_\nu) - \partial_\nu(\beta u_\mu)]$

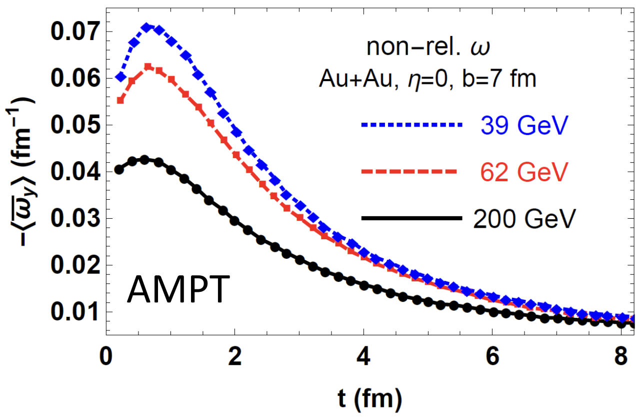
… …

- Numerical results for various vorticities





(Deng-XGH 2016; Deng-XGH-Ma-Zhang 2020)



(Jiang-Lin-Liao 2016)

Vorticity by global angular momentum

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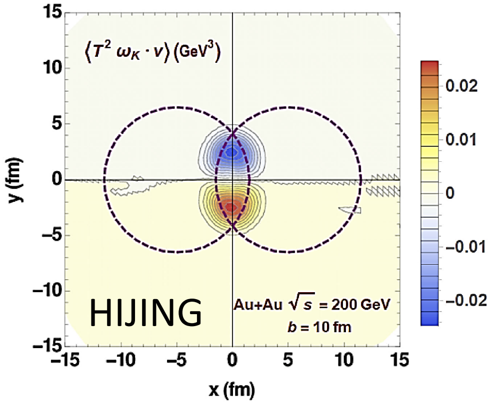
Temperature vorticity $$\omega^{\mathrm{T}}_{\mu\nu} = -\frac{1}{2}[\partial_\mu(Tu_\nu) - \partial_\nu(Tu_\mu)]$$

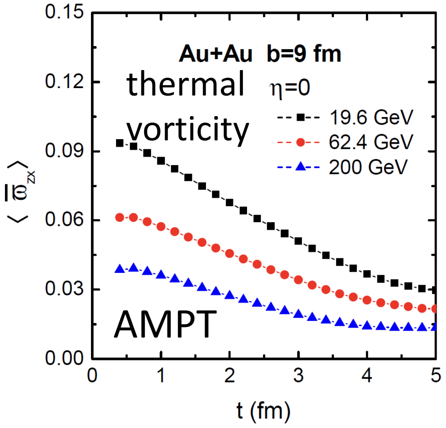
Thermal vorticity $$\omega^{\beta}_{\mu\nu} = -\frac{1}{2}[\partial_\mu(\beta u_\nu) - \partial_\nu(\beta u_\mu)]$$

… …

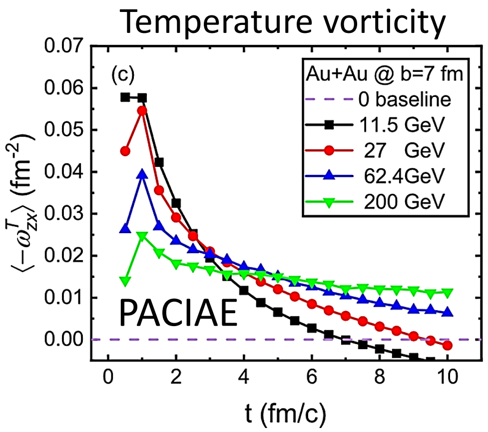
- Numerical results for various vorticities



(Deng-XGH 2016)



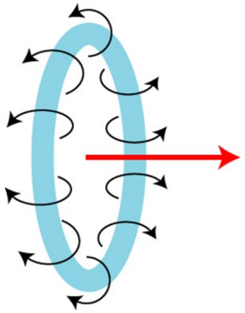
(Wei-Deng-XGH 2018)

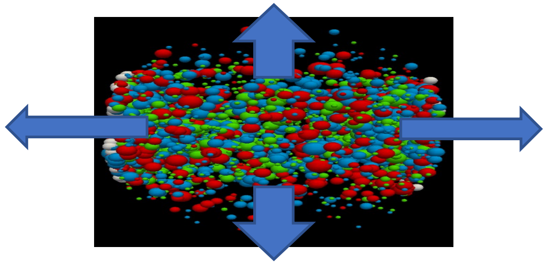


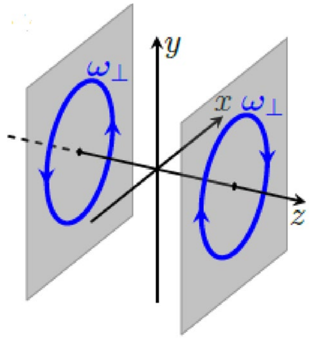
(Lei etal 2021)

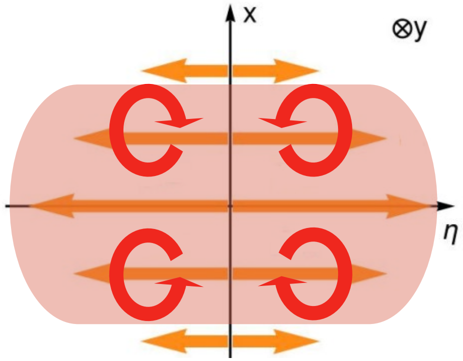
(See also: Becattini-Karpenko etal 2015,2016; Xie-Csernai etal 2014,2016,2019; Pang-Petersen-Wang-Wang 2016; Xia-Li-Wang 2017,2018; Sun-Ko 2017; Ivanov etal 2017-2020; … …)

Vorticity by inhomogeneous expansion

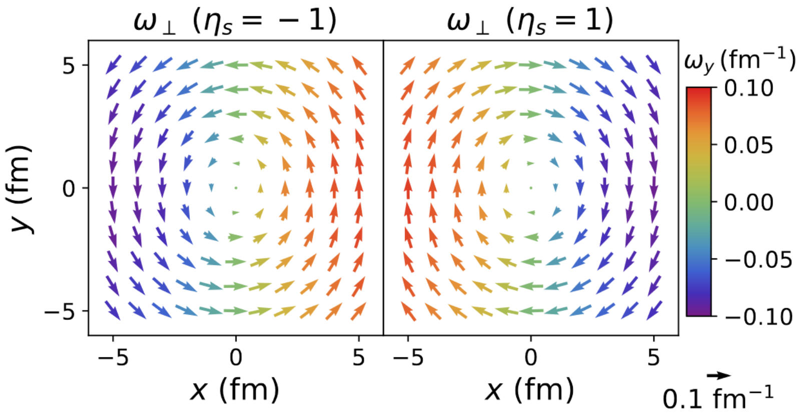




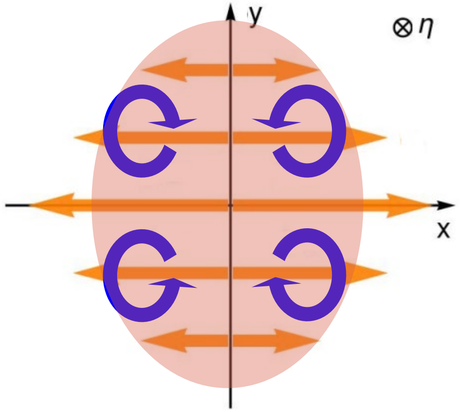




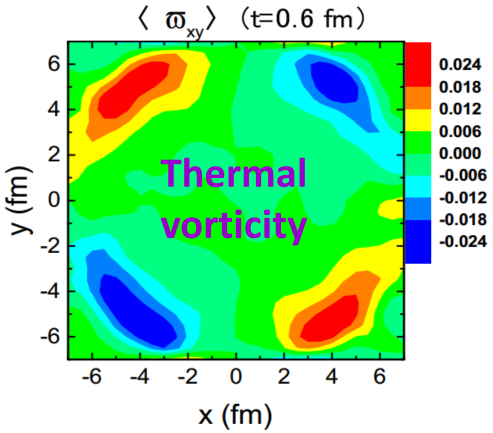
Transverse



(Xia-Li-Wang 2017)



Longitudinal

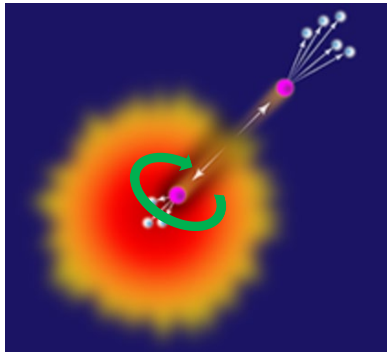


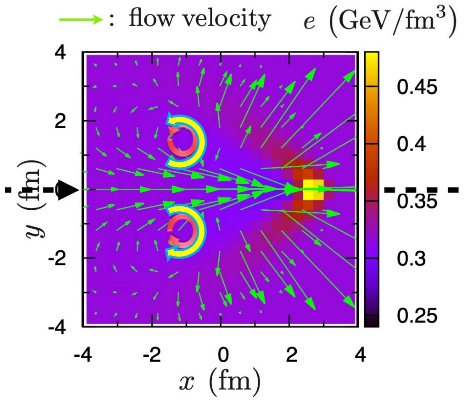
(Wei-Deng-XGH 2019)

(See also: Karpenko-Becattini 2017; Csernai etal 2014; Teryaev-Usubov 2015; Ivanov-Soldatov 2018; Fu etal 2020; Lei etal 2021; … …)

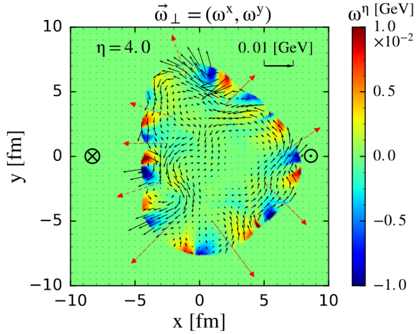
Other sources of vorticity

1) Jet

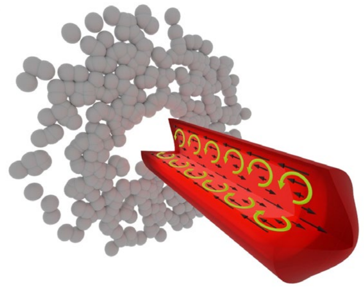




(Betz-Gyulassy-Torrieri 2007)

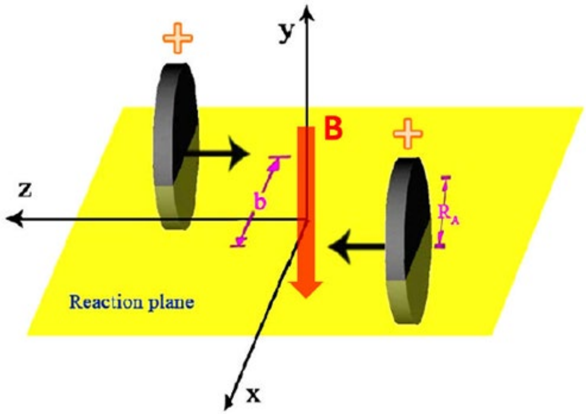


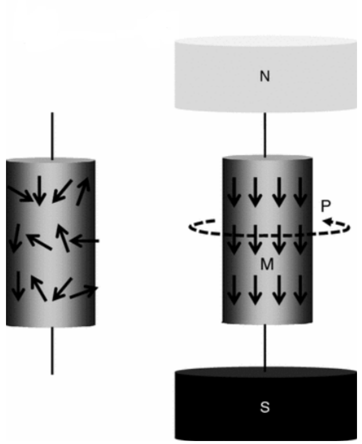
(Pang-Peterson-Wang-Wang 2016)



(Voloshin 2018; Lisa etal 2021)

2) Magnetic field

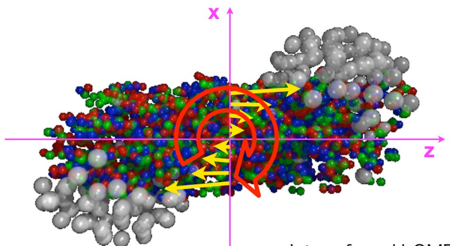




Einstein-de-Haas effect

Main message of this part

1. Global AM induces strong vorticity in HICs

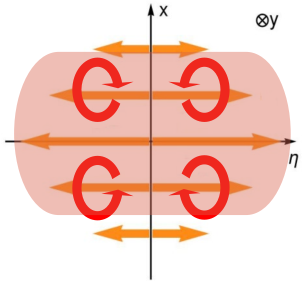


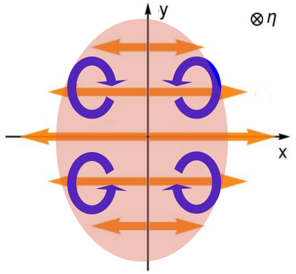
picture from UrQMD

: $\boldsymbol{\omega \approx 10^{19} - 10^{22}\ s^{-1}}$

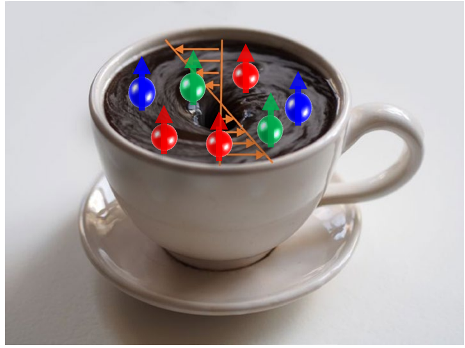
(QGP: The most vortical fluid)

2. Inhomogeneous expansion: quadrupoles in both xy and xz planes





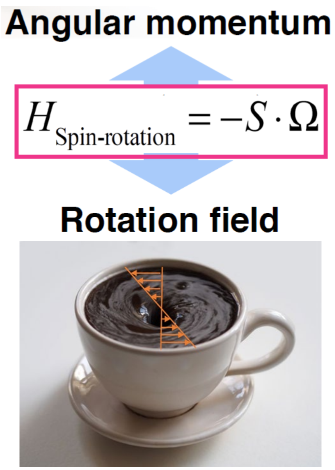
3. Such vorticity (and also magnetic, electric fields) can polarize spin



Global spin polarization

Global spin polarization

- Spin can thus be polarized by vorticity

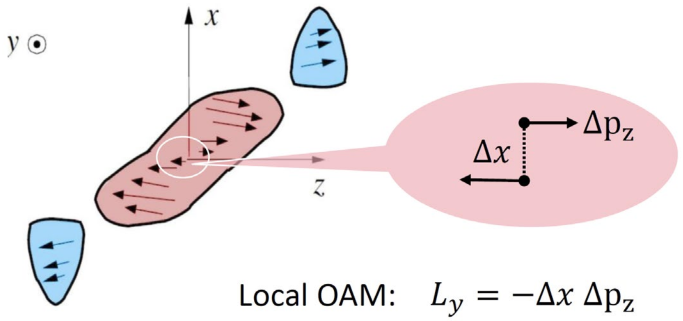


(at thermal equilibrium)

$$\frac{dN_s}{d\boldsymbol{p}} \sim e^{-(H_0 - \boldsymbol{\omega} \cdot \boldsymbol{S})/T}$$

$$\mathrm{P} = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \sim \frac{\omega}{2T}$$

- The original idea was proposed by Liang and Wang



(Figure by J. H. Gao)

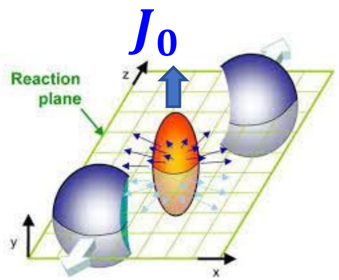
Quark polarization:

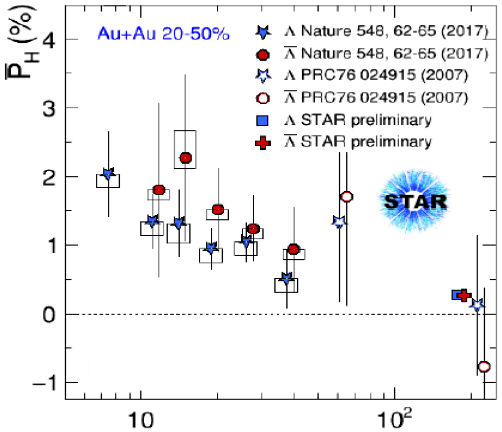
$$\mathrm{P} = -\frac{\pi \mu p}{2E(E+m)}$$

(Liang-Wang 2004)

Global spin polarization: Experiments

- First measurement of Λ global polarization (in rest frame) by STAR@RHIC





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parity-violating decay of hyperons

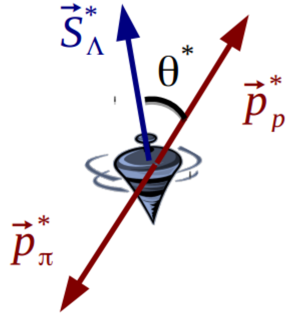
In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P_\Lambda} \cdot \hat{\mathbf{p}}_\mathbf{p}^*)$$

α: Λ decay parameter (α_Λ=0.732)

P_Λ: Λ polarization

p_p^*: proton momentum in Λ rest frame

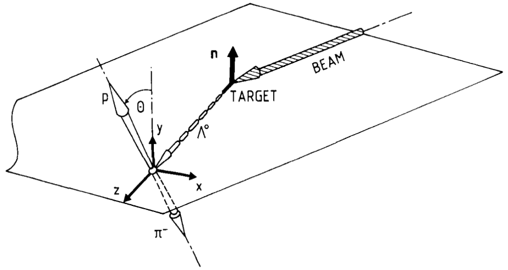


$\Lambda \to p + \pi^-$

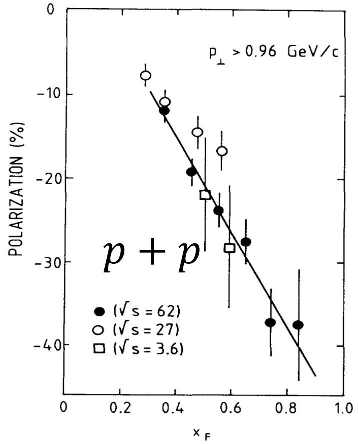
(BR: 63.9%, c τ ~7.9 cm)

- Using Λ to study spin physics in p+p, e+p, e+e collisions has a long history

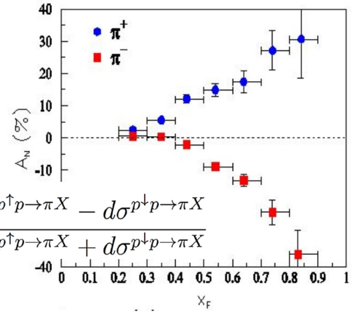
Λ polarization w.r.t production plane



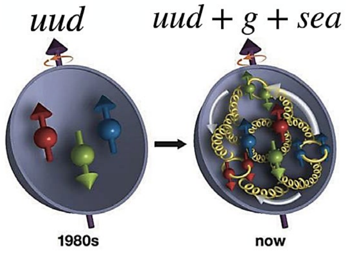
(Lesnik etal 1975; Bunce etal 1976; see e.g. review: Panagiotou 1990)



Useful for understanding e.g. single spin asymmetry, proton spin puzzle, …



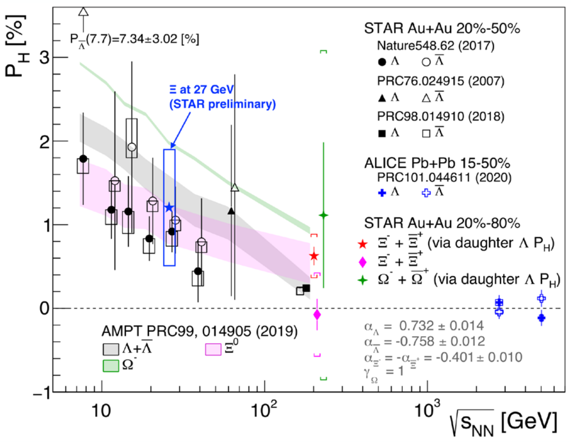
$$A_N = \frac{d\sigma^{p^\uparrow p \to \pi X} - d\sigma^{p^\downarrow p \to \pi X}}{d\sigma^{p^\uparrow p \to \pi X} + d\sigma^{p^\downarrow p \to \pi X}}$$



Global spin polarization: Experiments

- Ξ^-, Ω^- global polarization by STAR@RHIC, Λ global polarization by ALICE@LHC

STAR, PRL126, 162301 (2021)



⬅ Theory curves: Wei-Deng-XGH 2019

Exp. method 1:

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_H \mathbf{P}_H^* \cdot \hat{\boldsymbol{p}}_B^*)$$

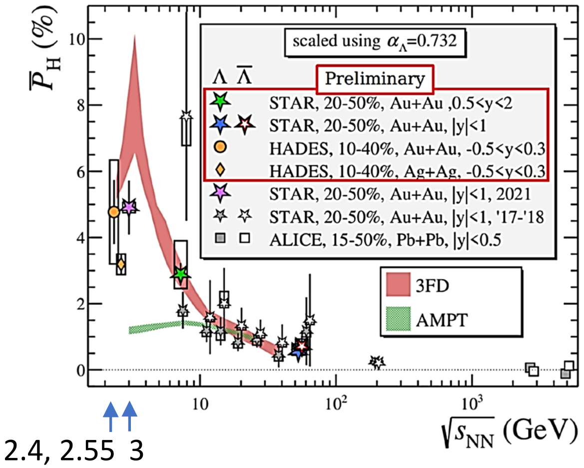
Exp. method 2:

$$\mathbf{P}_\Lambda^* = C_{\Omega^- \Lambda} \mathbf{P}_\Omega^* \qquad \mathbf{P}_\Lambda^* = C_{\Xi^- \Lambda} \mathbf{P}_\Xi^*$$

hyperon	decay mode	α_H	magnetic moment μ_H	spin
Λ (uds)	$\Lambda \to p\pi^-$ (BR: 63.9%)	0.732	-0.613	1/2
Ξ^- (dss)	$\Xi^- \to \Lambda\pi^-$ (BR: 99.9%)	-0.401	-0.6507	1/2
Ω^- (sss)	$\Omega^- \to \Lambda K^-$ (BR: 67.8%)	0.0157	-2.02	3/2

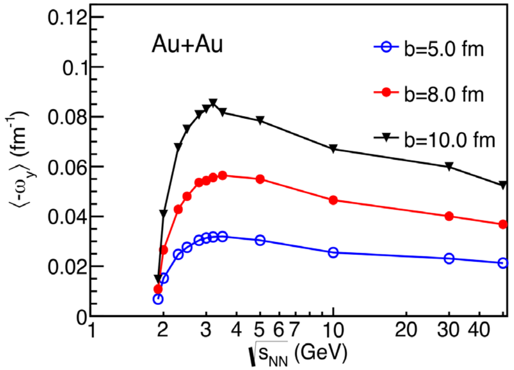
Useful to understand B-field, but still big uncertainties

- Global polarization at low energy by STAR@RHIC 2021, HADES@GSI 2021

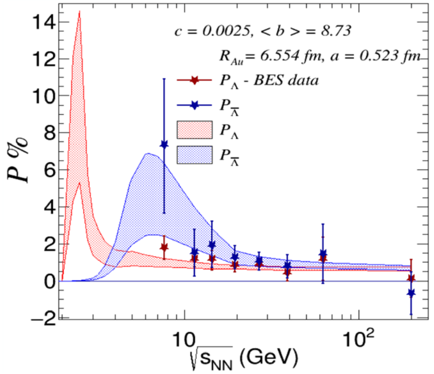


Theories suggest peak at low energy ➡

⬅ Experiments not see peak till 2.X GeV



(Deng-XGH-Ma-Zhang 2020)



(Ayala etal 2021)

A spin polarization formula

- Global polarization is (mainly) due to global angular momentum (AM)
- Vorticity: a bridge connecting initial AM and final global polarization

An estimate for static spin: $$\boldsymbol{P} = \frac{\langle \boldsymbol{s} \rangle}{s} = \frac{1}{sZ}\mathrm{Tr}\left(\boldsymbol{s}e^{-\beta H + \beta \boldsymbol{s}\cdot\boldsymbol{\omega}}\right) \approx \frac{s+1}{3}\frac{\boldsymbol{\omega}}{T}$$

Covariant extension to moving spin-1/2: (Becattini etal 2013, Fang-Pang-Wang-Wang 2016, Liu-Mameda-XGH 2020)

$$P^\mu(p) = -\frac{1}{8E_p}\epsilon^{\mu\nu\rho\sigma}p_\nu \frac{\int d\Sigma_\lambda p^\lambda f'(x,p)\varpi_{\rho\sigma}(x)}{\int d\Sigma_\lambda p^\lambda f(x,p)} + O(\varpi^2)$$

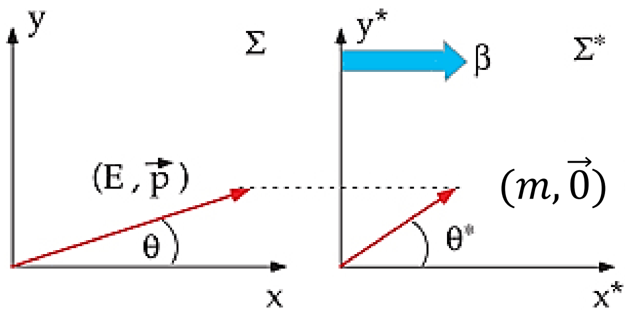
- Valid at global equilibrium in lab frame. $f(x,p)$ is Fermi-Dirac distribution
- Thermal vorticity $\varpi_{\rho\sigma} = \left(\frac{1}{2}\right)(\partial_\sigma \beta_\rho - \partial_\rho \beta_\sigma), \beta_\mu = u_\mu/T$
- Spin polarization is enslaved to thermal vorticity, not dynamical
- Friendly for numerical simulation (A Cooper-Frye type freeze-out formula)
- When magnetic field is present: $\boldsymbol{\omega} \Rightarrow \boldsymbol{\omega} + s^{-1}\mu_H \boldsymbol{B}$ and $\varpi^\perp_{\rho\sigma} \Rightarrow \varpi^\perp_{\rho\sigma} - 2\beta\mu_H F^\perp_{\rho\sigma}$

A spin polarization formula

- Boost into the rest frame of the particle

$$P_0^* = 0$$

$$\boldsymbol{P}^* = \boldsymbol{P} - \frac{\mathbf{p} \cdot \boldsymbol{P}}{E_p(E_p + m)} \boldsymbol{p}$$



- For the global polarization: project onto global AM direction

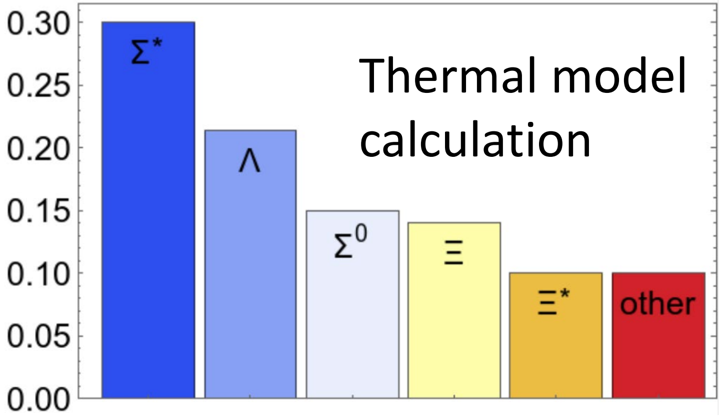
$$P_y^* = \boldsymbol{P}^* \cdot \hat{\boldsymbol{y}}$$

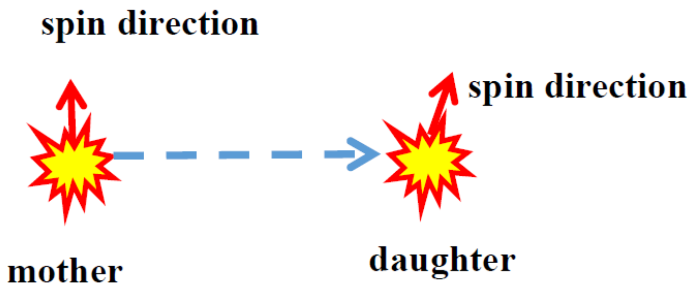
- With these equipments, one can calculate primary spin polarization by calculating temperature, fluid velocity, magnetic fields, and so on.

- But not all the Λs are primary, feed-down decay of heavy hadrons produce many Λ

The feed-down effects

- About 80% of final Λ's are from decays of higher-lying particles





- Spin polarization transfer (Xia-Li-XGH-Huang 2019, Becattini-Cao-Speranza 2019)

	spin and parity	$(1/N)dN/d\Omega^*$	$\mathbf{P}_D$	$\langle \mathbf{P}_D \rangle / \mathbf{P}_P$
strong decay	$1/2^+ \to 1/2^+ 0^-$	$1/(4\pi)$	$2(\mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\hat{\mathbf{p}}^* - \mathbf{P}_P$	-1/3
strong decay	$1/2^- \to 1/2^+ 0^-$	$1/(4\pi)$	$\mathbf{P}_P$	1
strong decay	$3/2^+ \to 1/2^+ 0^-$	$3\left[1 - 2\Delta/3 - (1-2\Delta)\cos^2\theta^*\right]/(8\pi)$	Too long to be shown; see ref.	1
strong decay	$3/2^- \to 1/2^+ 0^-$	$3\left[1 - 2\Delta/3 - (1-2\Delta)\cos^2\theta^*\right]/(8\pi)$		-3/5
weak decay	$1/2 \to 1/2\ 0$	$(1 + \alpha P_P \cos\theta^*)/(4\pi)$		$(2\gamma + 1)/3$
EM decay	$1/2^+ \to 1/2^+ 1^-$	$1/(4\pi)$	$-(\mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\hat{\mathbf{p}}^*$	-1/3

The feed-down effects

- Some decay channels can lead to spin-polarization flip, e.g., EM decay

Angular momentum conservation requires that the daughter Λ to be polarized opposite

$$\Sigma^0 \to \Lambda + \gamma \qquad \left(\frac{1}{2}\right)^+ \to \left(\frac{1}{2}\right)^+ 1^-$$

- For Λ:

	N_i/N_Λ	Spin and parity	Decay channel
Λ	1	$1/2^+$	-
$\Lambda(1405)$	0.236	$1/2^-$	$\Sigma^0\pi$
$\Lambda(1520)$	0.265	$3/2^-$	$\Sigma^0\pi$
$\Lambda(1600)$	0.098	$1/2^+$	$\Sigma^0\pi$
$\Lambda(1670)$	0.061	$1/2^-$	$\Sigma^0\pi$, $\Lambda\eta$
$\Lambda(1690)$	0.112	$3/2^-$	$\Sigma^0\pi$
Σ^0	0.686	$1/2^+$	$\Lambda\gamma$
Σ^{*0}	0.533	$3/2^+$	$\Lambda\pi$
Σ^{*+}	0.535	$3/2^+$	$\Lambda\pi$, $\Sigma^0\pi$
Σ^{*-}	0.524	$3/2^+$	$\Lambda\pi$, $\Sigma^0\pi$
$\Sigma(1660)$	0.068	$1/2^+$	$\Lambda\pi$, $\Sigma^0\pi$
$\Sigma(1670)$	0.125	$3/2^-$	$\Lambda\pi$, $\Sigma^0\pi$
Ξ^0	0.343	$1/2^+$	$\Lambda\pi$
Ξ^-	0.332	$1/2^+$	$\Lambda\pi$
Ξ^{*0}	0.228	$3/2^+$	$\Xi\pi$
Ξ^{*-}	0.224	$3/2^+$	$\Xi\pi$

➡ Feed-down decays suppress primary P_Λ by about **10%**

- For Ξ^-, Ω^-: (Xia-Li-XGH-Huang 2021)

Feed-down contribution for Ω is negligible.

Feed-down contribution for Ξ is mainly:

Spin-3/2

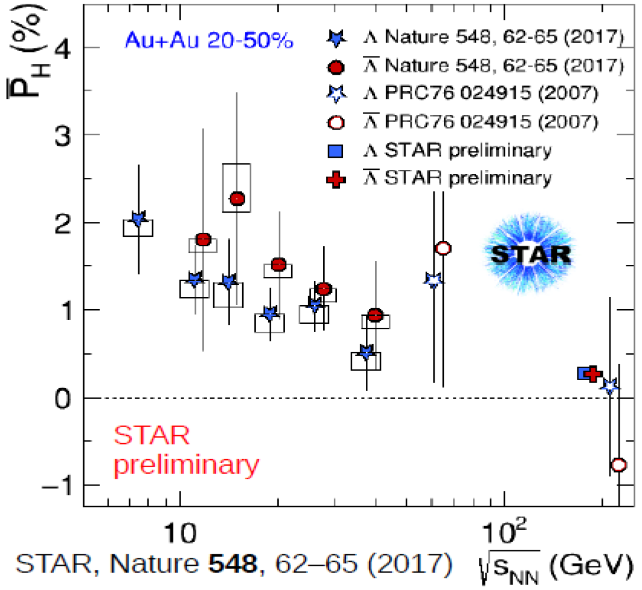
$$P_{\Xi^-}(\text{primary+feed-down}) = \frac{N_{\Xi^-} + \frac{5}{3}N_{\Xi(1530)\to\Xi^-}}{N_{\Xi^-} + N_{\Xi(1530)\to\Xi^-}} P_{\Xi^-}(\text{primary})$$

$$\approx 1.25 P_{\Xi^-}(\text{primary})$$

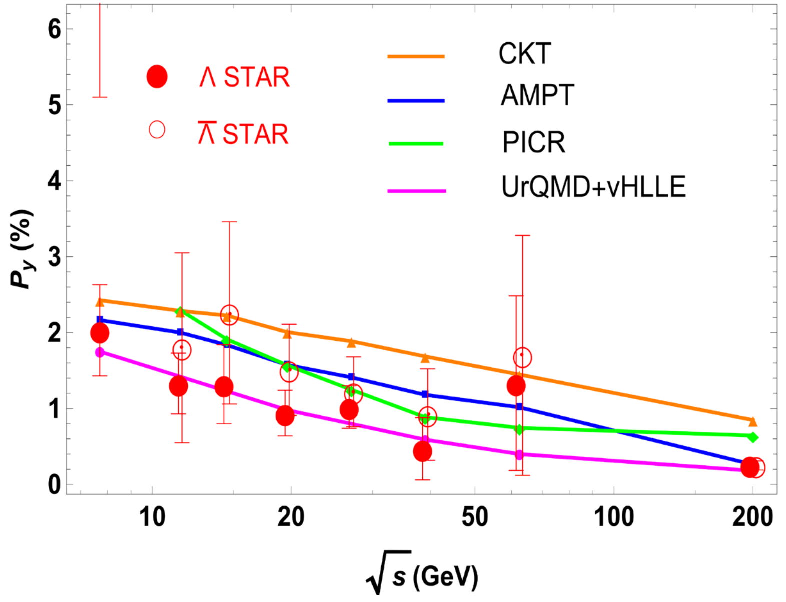
➡ Feed-down decays enhance primary P_Λ by about **25%**

Global spin polarization: Vorticity

Λ hyperons: Experiment = Theory



$$\langle\omega\rangle = \langle T(P_\Lambda + P_{\bar{\Lambda}})\rangle_{\sqrt{s}=7-200\mathrm{GeV}}$$
$$\approx (9 \pm 1) \times 10^{21}\mathrm{s}^{-1}$$



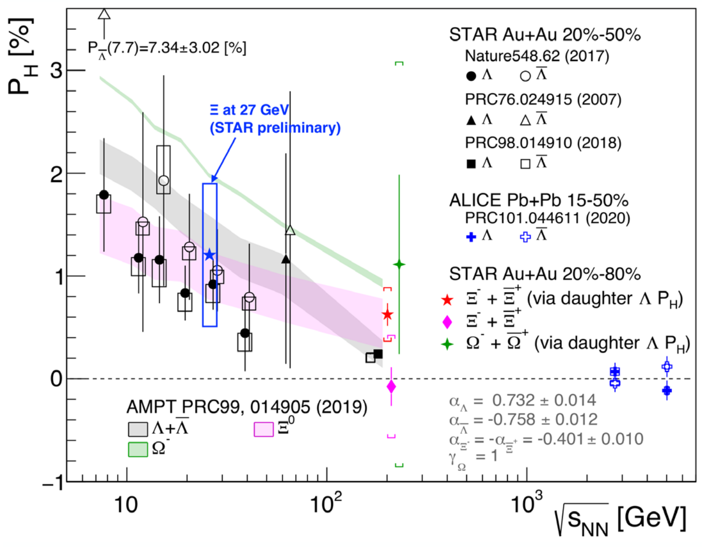
(Li-Pang-Wang-Xia 2017; Sun-Ko 2017; Wei-Deng-XGH 2019; Xie-Wang-Csernai 2017; Karpenko-Becattini 2016)

(See also: Sun-Ko etal 2019; Xie-Wang-Csernai etal 2018-2021; Ivanov etal 2017-2019; Liao etal 2018-2021; Deng-XGH-Ma 2021; ……)

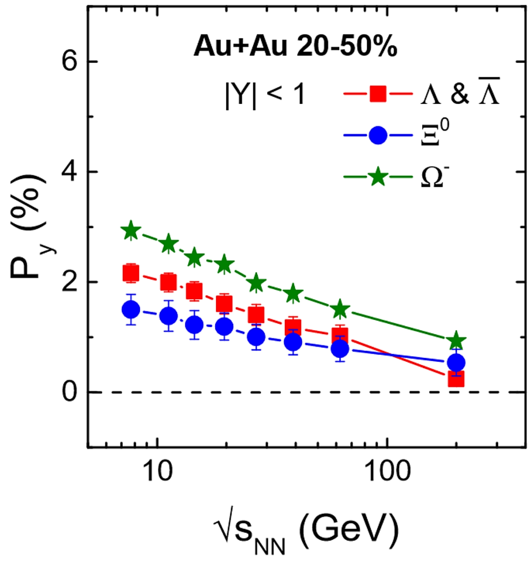
Global spin polarization: Vorticity

Ξ, Ω hyperons: Experiment = Theory

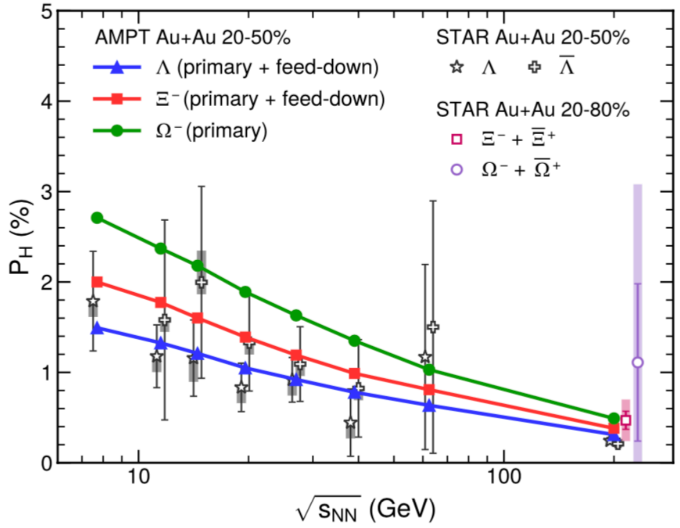
STAR, PRL126, 162301 (2021)



(Wei-Deng-XGH 2019)



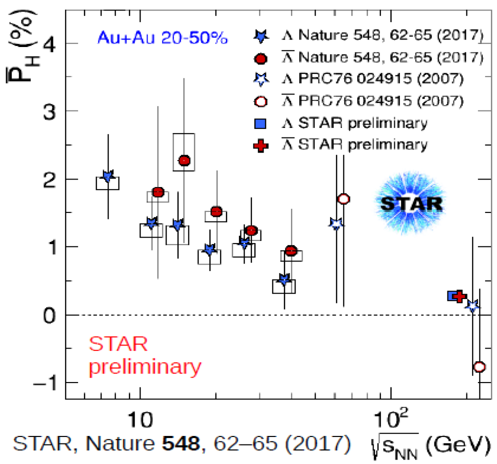
(Li-Xia-XGH-Huang 2021)



Vorticity interpretation of global spin polarization works well!

Global spin polarization: Magnetic field?

Magnetic field distinguish particles and antiparticles



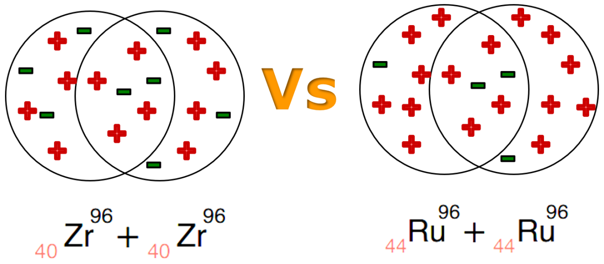
Though with big error bar, a difference between $P_y(\Lambda)$ and $P_y(\bar{\Lambda})$ is seen. Magnetic field?

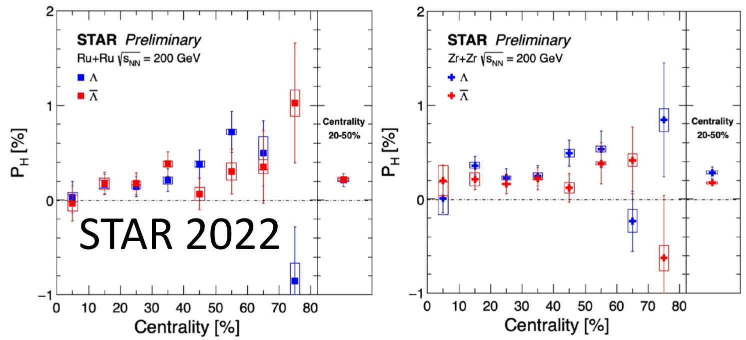
$$\langle B\rangle = \left\langle \frac{T}{2|\mu_\Lambda|}(P_{\bar{\Lambda}} - P_\Lambda)\right\rangle_{\sqrt{s}=7-200\mathrm{GeV}}$$

$$\approx (6.0 \pm 5.5) \times 10^{17}\ \mathrm{G}$$

$$\mu_\Lambda = -0.613\mu_N$$

Isobar collisions: same vorticity but 10% different B field

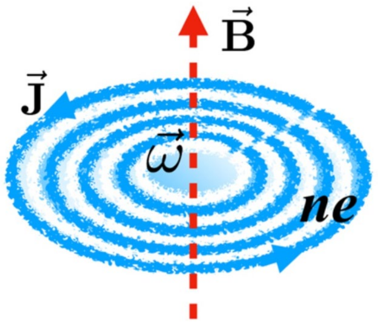




Isobar collisions: No significant difference within error bar

- Omega may be more sensitive to B: $\mu_{\Omega^-} = -2.02\mu_N$
- Rotation can induce magnetic field in a charged fluid (Barnett effect) (Guo-Liao-Wang 2019)
- Finite baryon chemical potential (Fang-Pang-Wang-Wang 2016), spatial dependent hadronization of Λ and $\bar{\Lambda}$ (Vitiuk-Bravina-Zabrodin 2019; Ayala etal 2021), mesonic potential (Csernai-Kapusta-Welle 2018),

… …



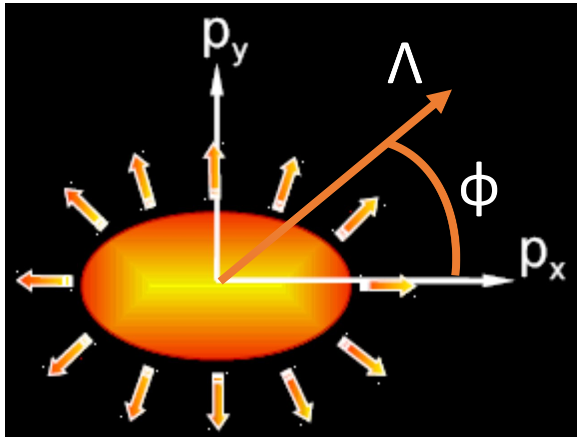
Local spin polarization

Local Λ spin polarization

The global Λ polarization reflects the total amount of angular momentum retained in the mid-rapidity region. How is it distributed in different ϕ ?

- Spin harmonic flow: $$\frac{dP_{y,z}}{d\phi} = \frac{1}{2\pi}[P_{y,z} + 2f_{2y,z}\sin(2\phi) + 2g_{2y,z}\cos(2\phi) + \cdots]$$

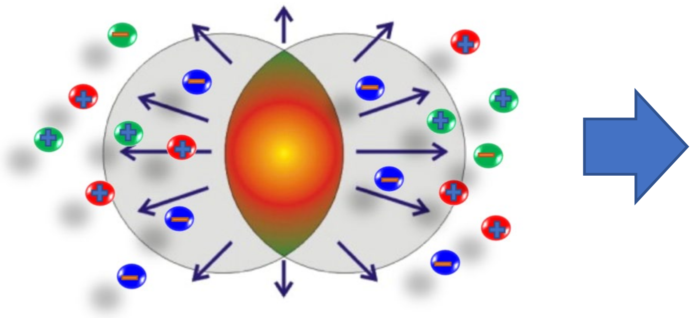
Azimuthal angle ϕ

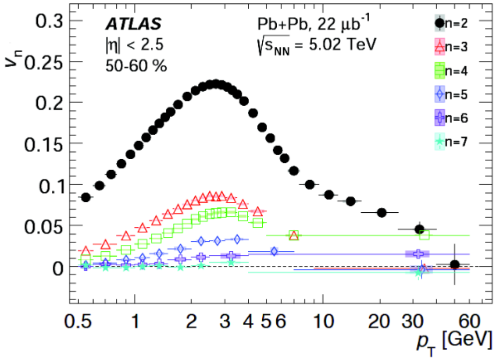


$\odot$ z direction

f_2, g_2: Spintronics analogue of elliptic flows

$$\frac{dN_{\rm ch}}{d\phi} \propto 1 + 2v_1\cos(\phi - \Psi_1) + 2v_2\cos[2(\phi - \Psi_2)] + \cdots$$





Local Λ spin polarization

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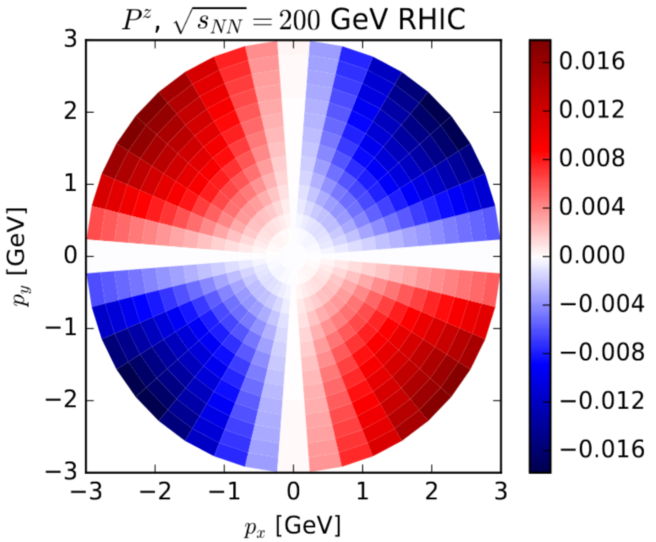
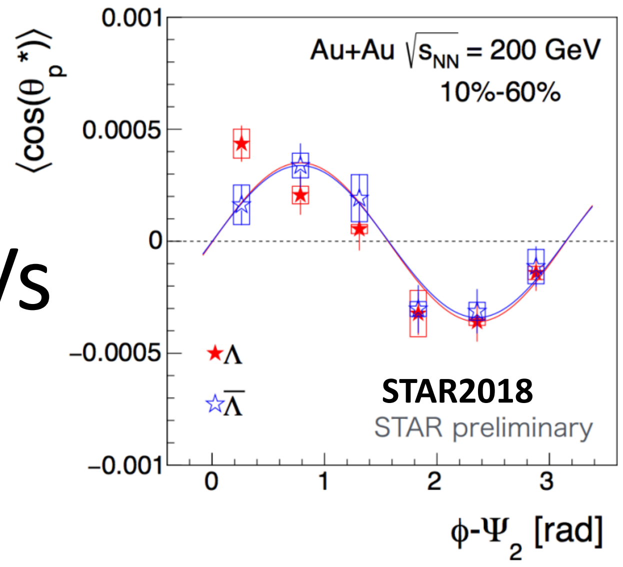
1) longitudinal polarization vs ϕ

(Becattini-Karpenko 2018)



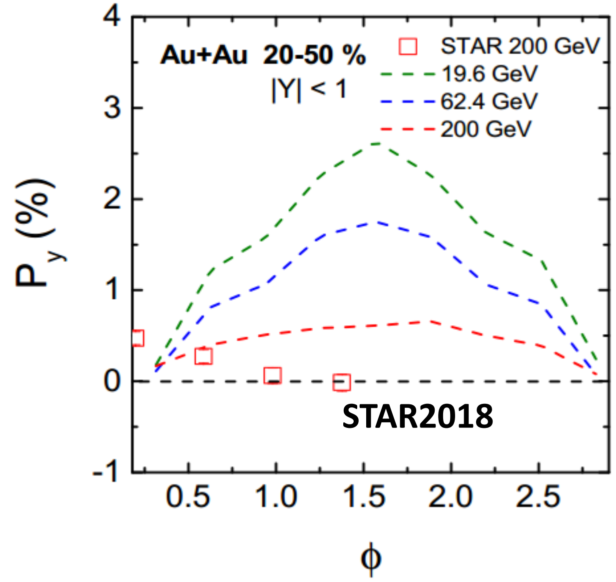
Vs



$f_{2z}^{ther} < 0$ $f_{2z}^{exp} > 0$

2) Transverse polarization vs ϕ

(Wei-Deng-XGH 2019)



$g_{2y}^{ther} < 0, g_{2y}^{exp} > 0$

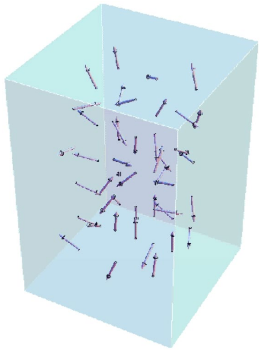
How to resolve the local spin polarization puzzles

Attack the spin sign problem from theory side:

- Understand the vorticity (☺)
- Effect of feed-down decays is not enough (☺)
- Go beyond equilibrium treatment (spin as a dynamic d.o.f)
  spin hydrodynamics
  spin kinetic theory
- Initial condition
  (Initial polarization, initial flow, … …)
- Other possibilities
  (chiral vortical effect (Liu-Sun-Ko 2019), mesonic mean-field(Csernai-Kapusta-Welle 2019), other spin chemical potential (Wu-Pang-XGH-Wang 2019, Florkowski etal 2019), contribution from shear flow (Becattini etal 2021, Fu-Liu-Pang-Song-Yin 2021, Yi-Pu-Yang 2021), contribution from gluons, … …)

Revisit spin polarization formula

- Consider a local Gibbs state for spin-1/2 fermions* (Zubarev etal 1979, Van Weert 1982, Becattini etal 2013)



Canonical stress tensor — Canonical spin tensor

$$\hat{\rho}_{\rm LG} = \frac{1}{Z_{\rm LG}} \exp\left\{ - \int_{\Xi} d\Xi_\mu(y) \left[\hat{\Theta}^{\mu\nu}(y)\beta_\nu(y) - \frac{1}{2}\hat{\Sigma}^{\mu\rho\sigma}(y)\mu_{\rho\sigma}(y) \right] \right\}$$

Thermal flow vector — Spin potential

- The corresponding Wigner function

$$W(x,p) = {\rm Tr}\left[\hat{\rho}_{\rm LG}\hat{W}(x,p)\right] = {\rm Tr}\left[\hat{\rho}_{\rm LG} \int d^4 s e^{-ip\cdot s} \hat{\bar{\psi}}\left(x+\frac{s}{2}\right) \otimes \hat{\psi}\left(x-\frac{s}{2}\right)\right]$$

- The canonical spin vector in phase space

$$S^\mu(x,p) = -\frac{1}{6}\epsilon^{\mu\nu\rho\sigma}\Sigma_{\nu\rho\sigma}(x,p) = -\frac{1}{24}\epsilon^{\mu\nu\rho\sigma}{\rm Tr}_{\rm D}\left[\{\gamma_\nu, \Sigma_{\rho\sigma}\} W(x,p)\right]$$

---

* Obtained by maximizing Von Neumann entropy under local constraints of stress and angular momentum tensors:

$s = -{\rm Tr}\left(\hat{\rho}\ln\hat{\rho}\right)$ with $n_\mu {\rm Tr}(\hat{\rho}\,\hat{\Theta}^{\mu\nu}) = n_\mu \Theta^{\mu\nu}$ and $n_\mu {\rm Tr}(\hat{\rho}\,\hat{\Sigma}^{\mu\rho\sigma}) = n_\mu \Sigma^{\mu\rho\sigma}$

Revisit spin polarization formula

- Mean spin vector for Dirac Fermion(on-shell, for particle branch) (Liu-XGH 2021; Buzzegoli 2021)

$$\bar{S}_\mu(p) = \bar{S}_{5\mu}(p) - \frac{1}{8\int d\Xi\cdot p\ \ n_F}\int d\Xi\cdot p\frac{n_F(1-n_F)}{E_p}\left\{\epsilon_{\mu\nu\alpha\beta}p^\nu\mu^{\alpha\beta} + 2\frac{\varepsilon_{\mu\nu\rho\sigma}p^\rho n^\sigma}{p\cdot n}\left[p_\lambda(\xi^{\nu\lambda}+\Delta\mu^{\nu\lambda})+\partial^\nu\alpha\right]\right\}$$

$$\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu)$$: Thermal shear tensor

$$\alpha = -\beta\mu$$: Baryon chemical potential

$$\Delta\mu_{\rho\sigma} = \mu_{\rho\sigma} - \varpi_{\rho\sigma}$$ with $$\varpi_{\rho\sigma} = \frac{1}{2}(\partial_\sigma\beta_\rho - \partial_\rho\beta_\sigma)$$ thermal vorticity tensor

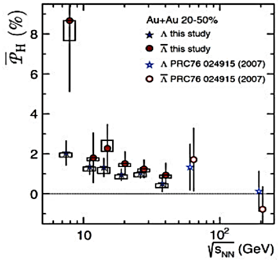
$\bar{S}_5^\mu$ is the polarization induced by finite chirality

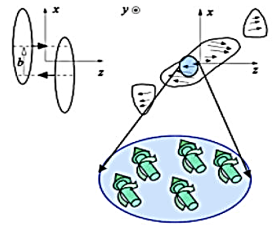
- This is a Cooper-Frye type formula for spin polarization

Recall Cooper-Frye formula for number spectrum:

$$N(p) = \int d\Xi_\mu \frac{p^\mu}{E_p} f(T(x), u^\mu(x), \mu(x))$$

$$\bar{S}^\mu(p) \quad \Leftarrow \quad T(x), u^\alpha(x), \mu(x), \mu_{\alpha\beta}(x)$$





Revisit spin polarization formula

- It is worth writing down different components in non-rel. form in phase space:

$$\boldsymbol{S} = \boldsymbol{S}_{(\mu)} + \boldsymbol{S}_{(\omega)} + \boldsymbol{S}_{(\sigma)} + \boldsymbol{S}_{(T)} + \boldsymbol{S}_{(\alpha)}$$

- Spin potential: (Buzzegoli 2021, Liu-XGH 2021)

$$S^i_{(\mu)}(x,\mathbf{p}) = \left[\frac{\mu^i}{2} - \frac{\mathbf{p}^2\mu^i - \boldsymbol{\mu}\cdot\mathbf{p}\,p^i}{2E_p^2}\right] n_F(1-n_F)$$ with $n_F = n_F(\alpha + \beta E_p)$

- Vorticity: (Becattini etal 2013, Fang etal 2016, Liu etal 2020)

$$S^i_{(\omega)}(x,\mathbf{p}) = \frac{\mathbf{p}^2\omega^i - \boldsymbol{\omega}\cdot\mathbf{p}\,p^i}{2T\,E_p^2} n_F(1-n_F)$$ with $\boldsymbol{\omega} = (\boldsymbol{\nabla}\times\boldsymbol{v})/2$

- Shear tensor: (Becattini-Buzzegoli-Palermo 2021, Liu-Yin 2021)

$$S^i_{(\sigma)}(x,\mathbf{p}) = \frac{\epsilon^{ijk}\,p^j p^l\,\sigma_{kl}}{2T\,E_p^2} n_F(1-n_F)$$ with $\sigma_{ij} = (\partial_i v_j + \partial_j v_i + 2\delta_{ij}\boldsymbol{\nabla}\cdot\boldsymbol{v}/3)/2$

- T gradient: (Becattini etal 2013, Fang etal 2016, Liu etal 2020)

$$S^i_{(T)}(x,\mathbf{p}) = -\frac{(\mathbf{p}\times\boldsymbol{\nabla}T)^i}{2T^2\,E_p} n_F(1-n_F)$$

- Chemical potential: (Fang etal 2016, Liu-XGH 2021, Yi etal 2021, Fu etal 2022)

$$S^i_{(\alpha)}(x,\mathbf{p}) = \frac{(\mathbf{p}\times\boldsymbol{\nabla}\alpha)^i}{2E_p^2} n_F(1-n_F)$$

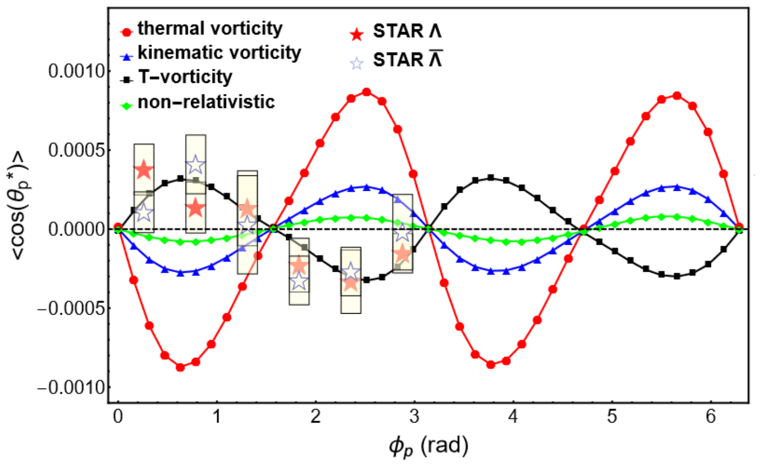
Temperature vorticity as spin chemical potential

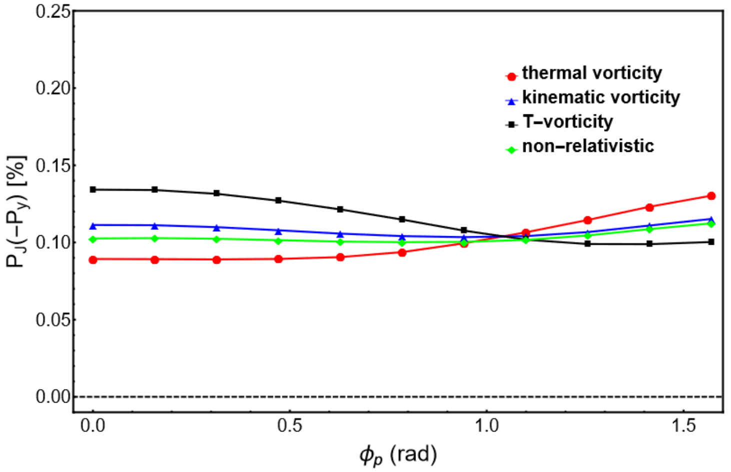
- Recall

$$S^\mu(x,p) = -\frac{1}{E_p}\left[\frac{1}{4}\epsilon^{\mu\nu\alpha\beta}p_\nu\mu_{\alpha\beta} + \frac{1}{2p\cdot n}\epsilon^{\mu\nu\alpha\beta}\left(\xi_{\nu\lambda}+\Delta\mu_{\nu\lambda}\right)n_\beta p_\alpha p^\lambda\right]n_F(1-n_F) + O(\mu^2_{\rho\sigma},\partial^2)$$

- Relax the global equilibrium condition (1)

$$\partial_\mu\beta_\nu + \partial_\nu\beta_\mu = 0 \qquad \mu_{\rho\sigma} = \frac{1}{2T^2}\left[\partial_\sigma(Tu_\rho) - \partial_\rho(Tu_\sigma)\right]$$





(Wu-Pang-XGH-Wang 2019)

Shear tensor contribution

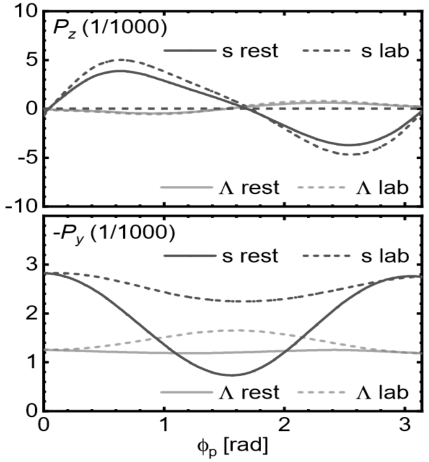
- Recall

$$S^\mu(x,p) = -\frac{1}{E_p}\left[\frac{1}{4}\epsilon^{\mu\nu\alpha\beta}p_\nu\mu_{\alpha\beta} + \frac{1}{2p\cdot n}\epsilon^{\mu\nu\alpha\beta}\left(\xi_{\nu\lambda} + \Delta\mu_{\nu\lambda}\right)n_\beta p_\alpha p^\lambda\right] n_F(1-n_F) + O(\mu_{\rho\sigma}^2, \partial^2)$$

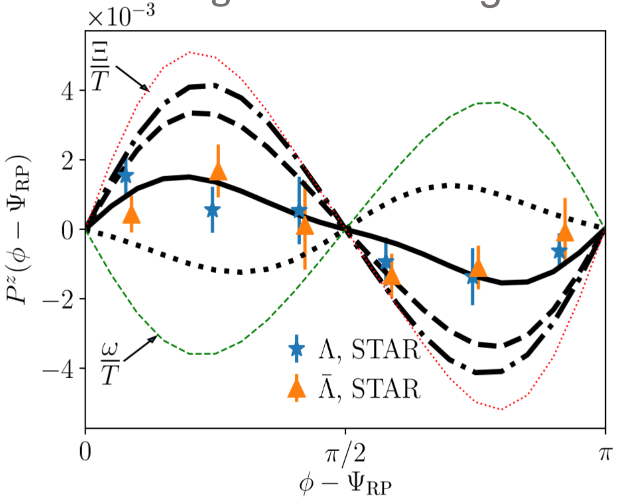
- Relax the global equilibrium condition (2) (Becattini-Buzzegoli-Palermo 2021, Liu-Yin 2021)

$$\partial_\mu\beta_\nu + \partial_\nu\beta_\mu \neq 0 \qquad \mu_{\rho\sigma} = \varpi_{\rho\sigma} = \frac{1}{2}\left(\partial_\sigma\beta_\rho - \partial_\rho\beta_\sigma\right)$$

(Fu-Liu-Song-Yin 2021)



(Becattini-Buzzegoli-Palermo-Inghirami-Karpenko 2021)

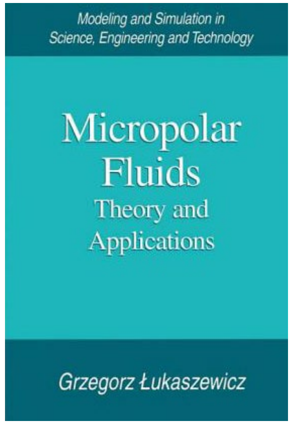


(See also Yi-Pu-Yang 2021; Florkowski-Kumar-Mazeliauskas-Ryblewski 2021; Alzhrani-Ryu-Shen 2022)

Spin hydrodynamics

- A dynamic theory for spin potential is promising: spin hydrodynamics

  - Widely used in non-relativistic spintronics, micropolar fluid, … …



- Hydrodynamics: low-energy effective theory for conserved quantities
  - Hydro modes relax at $\tau_{\mathrm{hydro}} = 1/\omega_{\mathrm{hydro}}(k) \to \infty$ when $k \to 0$
  - Hydro is constructed by gradient expansion
  - Typical hydro modes: energy density, momentum density, baryon density, …

Ideal spin hydrodynamics?

- If spin current is conserved, hydro equations would be

$$\text{Charge conservation}: \quad \partial_\mu J^\mu(x) = 0,$$
$$\text{Energy}-\text{momentum conservation}: \quad \partial_\mu \Theta^{\mu\nu}(x) = 0,$$
$$\text{Spin conservation}: \quad \partial_\mu \Sigma^{\mu\nu\rho}(x) = 0,$$

with J^μ, $\Theta^{\mu\nu}$, and $\Sigma^{\mu\nu\rho}$ expanded order by order in gradient giving constitutive relations

$$J^\mu = n u^\mu + O(\partial),$$
$$\Theta^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p\eta^{\mu\nu} + O(\partial),$$
$$\Sigma^{\mu\nu\rho} = \sigma^{\nu\rho} u^\mu + O(\partial)$$

where $O(1)$ terms usually correspond to ideal hydrodynamics (Florkowski etal 2018)

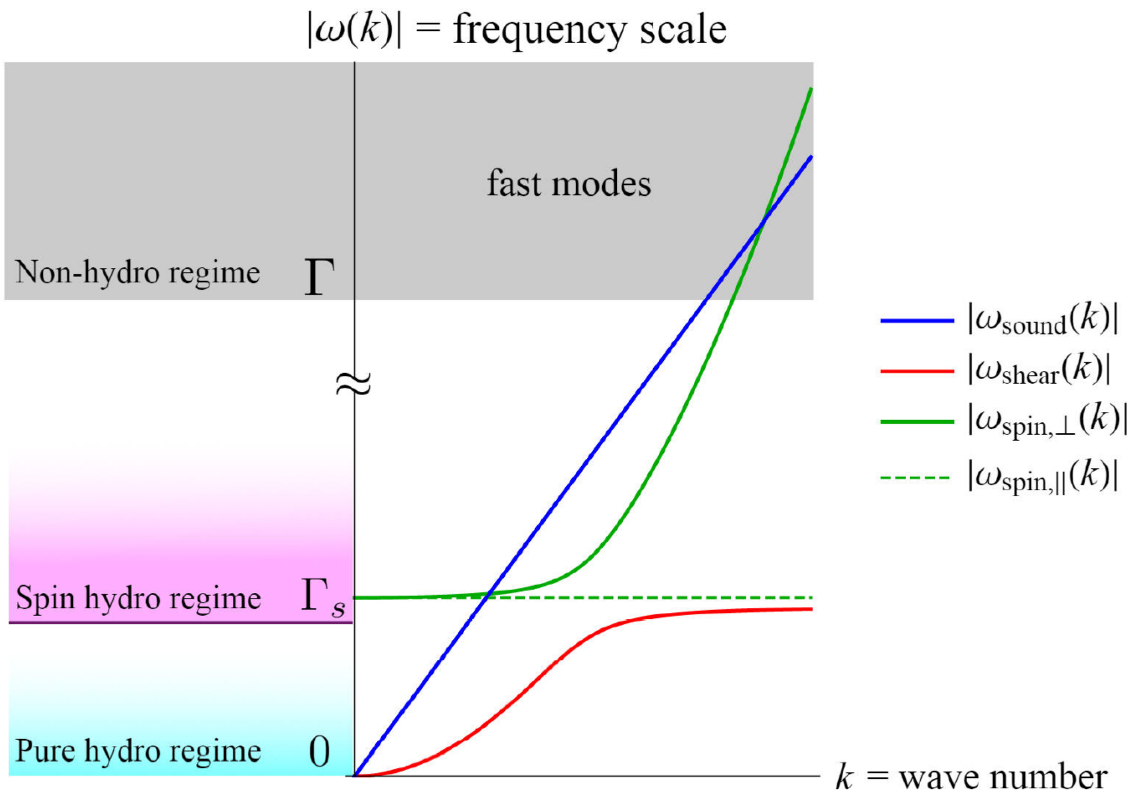
- But spin is not conserved in general (and thus not strict hydro mode)

$$\partial_\mu J^{\mu\nu\rho} = 0, \quad J^{\mu\nu\rho} = x^\nu \Theta^{\mu\rho} - x^\rho \Theta^{\mu\nu} + \Sigma^{\mu\nu\rho} \quad \Rightarrow \quad \partial_\mu \Sigma^{\mu\nu\rho}(x) = \Theta^{\rho\nu} - \Theta^{\nu\rho}$$

- The conversion between spin and orbital AM is dissipative in general

Spin hydrodynamic regime

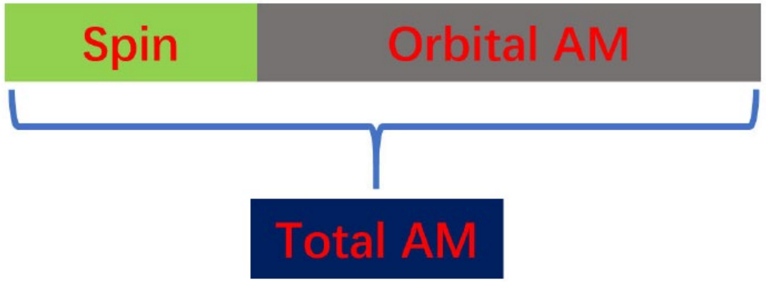
- Even though spin is not conserved, when spin relaxation rate is much smaller than other non-hydro modes, we could formulate a hydro+ for spin: Relativistic dissipative spin hydrodynamics



(Hongo-XGH-Kaminski-Stephanov-Yee 2021)

Ambiguity in definition of spin current

- The definition of spin current is ambiguous



- The pseudo-gauge transformation: preserves total conserved charges and conservation law (Becattini-Florkowski-Speranza 2018)

$$\Sigma^{\mu\nu\rho} \to \Sigma^{\mu\nu\rho} - \Phi^{\mu\nu\rho},$$
$$\Theta^{\mu\nu} \to \Theta^{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi^{\lambda\mu\nu} - \Phi^{\mu\lambda\nu} - \Phi^{\nu\lambda\mu}\right)$$

- Formulation of spin hydro depends on the pseudo-gauge choice

(Florkowski etal 2017; Montenegro etal 2017; Hattori etal 2019; Gallegos etal 2020; Bhadury etal 2020; Li-Stephanov-Yee 2020; Hu 2020-2022; Fukushima-Pu 2020; She etal 2021; Weickgenannt etal 2022; Cao-Hattori-Hongo-XGH-Taya 2022; … …)

- Fix the pseudo-gauge by coupling spin to torsion (or spin connection)

(Hongo-XGH-Kaminski-Stephanov-Yee 2021; Gallegos etal 2020; 2022)

Stress tensor and spin current

- The stress tensor and spin current

$$\Theta^{\mu}{}_{a}(x) \equiv \frac{1}{e(x)} \frac{\delta S}{\delta e_{\mu}{}^{a}(x)}\bigg|_{\omega}, \quad \Sigma^{\mu}{}_{ab}(x) \equiv -\frac{2}{e(x)} \frac{\delta S}{\delta \omega_{\mu}{}^{ab}(x)}\bigg|_{e}$$

- For QCD

$$\Theta^{\mu}{}_{a} = \frac{1}{2}\bar{q}\left(\gamma^{\mu}\overrightarrow{D}_{a} - \overleftarrow{D}_{a}\gamma^{\mu}\right)q + 2\mathrm{tr}\left(G^{\mu\rho}G_{a\rho}\right) + \mathcal{L}_{\mathrm{QCD}}e_{a}{}^{\mu},$$

$$\Sigma^{\mu}{}_{ab} = -\frac{i}{2}\bar{q}e^{\mu}{}_{c}\{\gamma^{c}, \Sigma_{ab}\}q$$

- Equations of motion (Ward-Takahashi identities for diffeomorphisim and local Lorentz invariance)($G_{\mu} = T^{\nu}{}_{\nu\mu}$)

$$(D_{\mu} - G_{\mu})\Theta^{\mu}{}_{a} = -\Theta^{\mu}{}_{b}T^{b}{}_{\mu a} + \frac{1}{2}\Sigma^{\mu}{}_{b}{}^{c}R^{b}{}_{c\mu a} + F_{a\mu}J^{\mu},$$

$$(D_{\mu} - G_{\mu})\Sigma^{\mu}{}_{ab} = -(\Theta_{ab} - \Theta_{ba})$$

Construction of spin hydrodynamics

- Step 1: Identify (quasi-)hydro modes

  - Eight (quasi-)hydro variables: $\epsilon, n, u^a, \sigma_{ab}$ (or $\sigma_a = \varepsilon^{abcd} u_b \sigma_{cd}/2$) with constraints $u^2 = -1$, $\sigma^a u_a = \sigma_{ab} u^b = 0$.
  - Local first law of thermodynamics: $s = \beta(\epsilon + P - \mu n - \mu_{ab}\sigma^{ab}/2)$ and $T ds = d\epsilon - \mu dn - \mu^{ab} d\sigma_{ab}/2$.
  - Conjugate variables: inverse temperature $\beta \equiv \frac{\partial s}{\partial \epsilon}$, chemical potentials $\mu = \frac{\partial s}{\partial n}$, $\mu^{ab} = -\frac{T}{2}\frac{\partial s}{\partial \sigma_{ab}}$.
  - Power counting scheme

$$\{\beta, n, u^a, e_\mu{}^a\} = O(\partial^0) \quad \text{and} \quad \{\mu^{ab}, \sigma_{ab}, \omega_\mu{}^{ab}\} = O(\partial)$$

- Step 2: Tensor decomposition

$$\Theta^\mu{}_a = \epsilon u^\mu u_a + p\Delta^\mu_a + u^\mu \delta q_a - \delta q^\mu u_a + \delta\Theta^\mu{}_a,$$
$$\Sigma^\mu{}_{ab} = \varepsilon^\mu{}_{abc}(\sigma^c + \delta\sigma u^c)$$

Construction of spin hydrodynamics

- Step 3: Calculate the entropy production rate

$$(\nabla_\mu - G_\mu)s^\mu = (\nabla_\mu - G_\mu)(\delta s^\mu + \beta\mu\delta J^\mu) - \delta\Theta^\mu{}_a\big|_{(s)}(D_\mu\beta^a - T^a{}_{\mu b}\beta^b)$$
$$- \delta\Theta^\mu{}_a\big|_{(a)}(D_\mu\beta^a - T^a{}_{\mu b}\beta^b - \beta\mu_\mu{}^a) - \delta J^\mu[\nabla_\mu(\beta\mu) - F_{\mu\nu}\beta^\nu] + O(\partial^3)$$

- Step 4: Second law of local thermodynamics $(\nabla_\mu - G_\mu)s^\mu \geq 0$

(Hongo-XGH-Kaminski-Stephanov-Yee 2021, 2022)

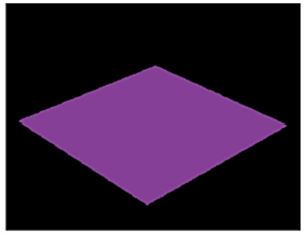
$$\delta\Theta^\mu{}_a\big|_{(s)} = -\eta^{\mu}{}_a{}^{\nu}{}_b(D_\nu u^b - T^b{}_{\nu c}u^c),$$
$$\delta\Theta^\mu{}_a\big|_{(a)} = -(\eta_s)^{\mu}{}_a{}^{\nu}{}_b(D_\nu u^b - T^b{}_{\nu c}u^c - \mu_\nu{}^b)$$
$$\eta^{\mu}{}_a{}^{\nu}{}_b = 2\eta\left(\frac{1}{2}(\Delta^{\mu\nu}\Delta_{ab} + \Delta^\mu_b\Delta^\nu_a) - \frac{1}{3}\Delta^\mu_a\Delta^\nu_b\right) + \zeta\Delta^\mu_a\Delta^\nu_b,$$
$$(\eta_s)^{\mu}{}_a{}^{\nu}{}_b = \frac{1}{2}\eta_s(\Delta^{\mu\nu}\Delta_{ab} - \Delta^\mu_b\Delta^\nu_a).$$

with $\eta \geq 0$ shear, $\zeta \geq 0$ bulk, and $\eta_s \geq 0$ rotational viscosities.

- With equation of state $p = p(\epsilon, n, \sigma_{ab})$, the equations are closed

(Quasi-)hydro modes

Perturbation about global static thermal equilibrium

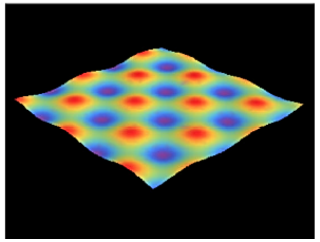


$\epsilon = \epsilon_0$

$u^\mu = (1, \mathbf{0})$

$\sigma^a = 0$

perturb $\Rightarrow$



$\epsilon = \epsilon_0 + \delta\epsilon$

$u^\mu = (1, \mathbf{0}) + \delta u^\mu$

$\sigma^a = 0 + \delta\sigma^a$

$$\begin{cases} \bullet \text{ One pair of sound modes : } \omega_{\text{sound}}(\boldsymbol{k}) = \pm c_s|\boldsymbol{k}| - \frac{i}{2}\gamma_{\parallel}\boldsymbol{k}^2 + O(\boldsymbol{k}^3), \\ \bullet \text{ One longitudinal spin mode : } \omega_{\text{spin},\parallel}(\boldsymbol{k}) = -i\Gamma_s, \\ \bullet \text{ Two shear modes : } \omega_{\text{shear}}(\boldsymbol{k}) = -i\gamma_\perp \boldsymbol{k}^2 + O(\boldsymbol{k}^4), \\ \bullet \text{ Two transverse spin modes : } \omega_{\text{spin},\perp}(\boldsymbol{k}) = -i\Gamma_s - i\gamma_s\boldsymbol{k}^2 + O(\boldsymbol{k}^4). \end{cases}$$

where we introduced a set of static/kinetic coefficients as

$$c_s^2 \equiv \frac{\partial p}{\partial \epsilon}, \quad \gamma_\parallel \equiv \frac{1}{\epsilon_0 + p_0}\left(\zeta + \frac{4}{3}\eta\right), \quad \gamma_\perp \equiv \frac{\eta}{\epsilon_0 + p_0}$$

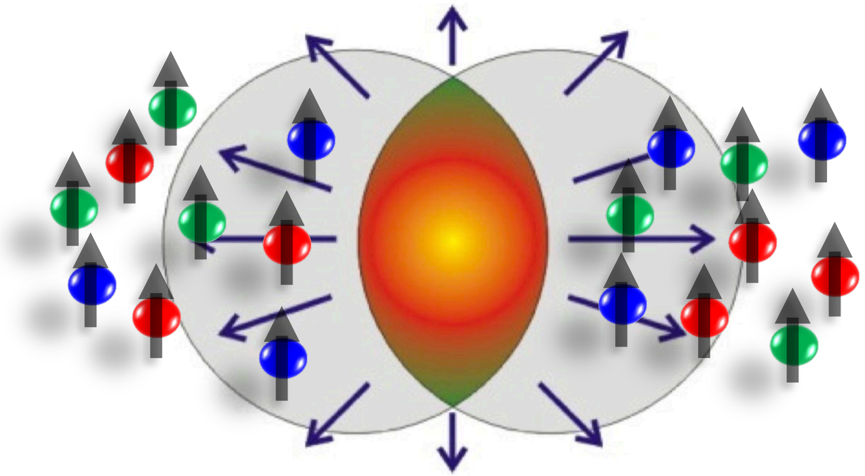
$$\chi_s \delta_{ij} \equiv \frac{\partial \sigma_i}{\partial \mu^j}, \quad \gamma_s \equiv \frac{\eta_s}{2(\epsilon_0 + p_0)}, \quad \boxed{\Gamma_s \equiv \frac{2\eta_s}{\chi_s}}$$

Spin relaxation rate

(see Hongo-XGH-Kaminski-Stephanov-Yee 2022 for pQCD computation of spin relaxation rate)

Spin hydrodynamics

- If you are interested in numerical spin hydrodynamics for Λ polarization



Spin alignment of vector mesons

Spin density matrix

- Spin state is conveniently described by spin density matrix (SDM)
- Choose a direction (e.g. y-axis in lab frame) to quantize spin.
- For spin-1/2 particles (e.g. quarks), a state with only polarization in y-direction:

$$\rho^q = \frac{1}{2}\left(1 + P_y^q \sigma_y\right) = \frac{1}{2}\begin{pmatrix} 1 + P_y^q & 0 \\ 0 & 1 - P_y^q \end{pmatrix}$$

- A spin-1 particle (e.g. ϕ meson), SDM is 3×3 matrix in basis $|1\rangle, |0\rangle, |{-1}\rangle$

$$\rho^V = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1-1} \\ \rho_{01} & \rho_{00} & \rho_{0-1} \\ \rho_{-11} & \rho_{-10} & \rho_{-1-1} \end{pmatrix}$$

- Hermiticity and unit-trace constrain that only 8 elements are independent: 3 form a vector and 5 form a rank-2 spherical tensor

Spin density matrix

- Consider recombination $q + \bar{q} \rightarrow \phi$
- ρ^V is obtained by $\rho^q \otimes \rho^{\bar{q}}$ projected onto spin-1 subspace

$$\rho^V = \begin{pmatrix} \frac{(1+P_y^q)(1+P_y^{\bar{q}})}{3+P_y^q p_y^{\bar{q}}} & 0 & 0 \\ 0 & \frac{1-P_y^q P_y^{\bar{q}}}{3+P_y^q p_y^{\bar{q}}} & 0 \\ 0 & 0 & \frac{(1-P_y^q)(1-P_y^{\bar{q}})}{3+P_y^q p_y^{\bar{q}}} \end{pmatrix}$$

- Vorticity dominance: $P_q = P_{\bar{q}}$, $\rho_{00} < \frac{1}{3}$

  Magnetic-field dominance: $P_q = -P_{\bar{q}}$, $\rho_{00} > \frac{1}{3}$
- The results of Λ polarization suggests vorticity dominance

(Liang-Wang 2004)

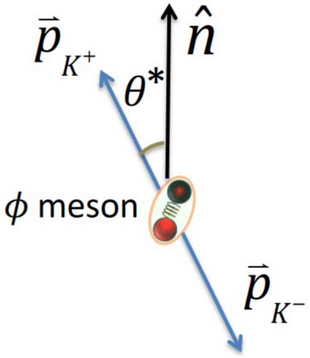
$$\rho_{00} = \frac{1 - P_y^2}{3 + P_y^2} \approx \frac{1}{3} - \frac{4}{9} P_y^2$$

- Spin alignment: $\rho_{00} - \frac{1}{3}$

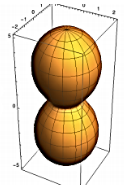
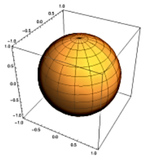
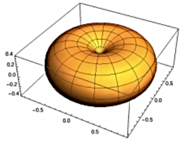
Expectation: spin alignment is a 10^{-4} level phenomenon

Spin alignment

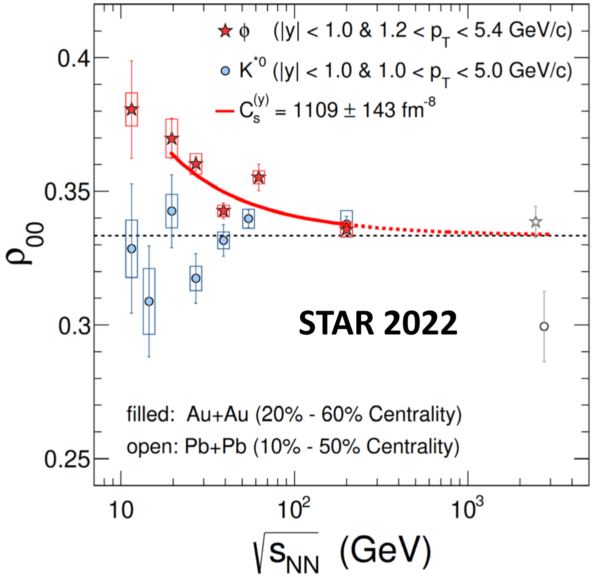
- Main decay channels, parity-even strong decay: $\phi \rightarrow KK,\ K^{*0} \rightarrow K\pi$



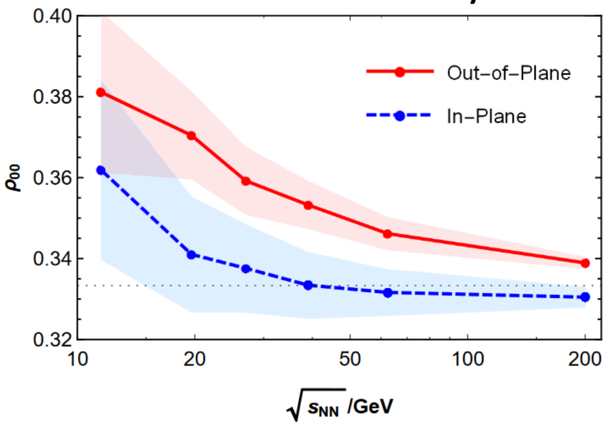
$$\frac{dN}{d(\cos\theta^*)} = N_0 \times \left[(1-\rho_{00}) + (3\rho_{00}-1)\cos^2\theta^*\right]$$

$\rho_{00}>1/3$:  $\rho_{00}=1/3$:  $\rho_{00}<1/3$: 

- Global spin alignment **Puzzle: ϕ-meson $\rho_{00} > \frac{1}{3}$ and too big!**



A recent theory



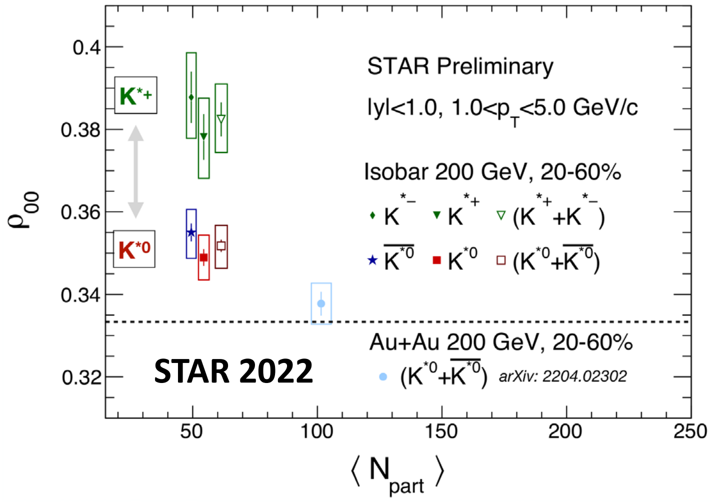
$$\rho_{00}(x,\mathbf{0}) - \frac{1}{3} \propto \left\langle (g_\phi \mathbf{B}^\phi_{x(y)})^2 \right\rangle$$

$$\left\langle (g_\phi \mathbf{E}^\phi_{x(y)})^2 \right\rangle$$

(Sheng-Oliva-Liang-Wang-Wang 2022)

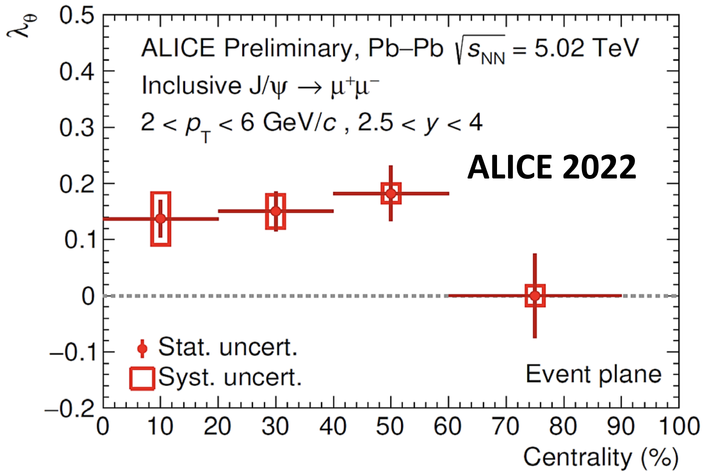
Spin alignment

- Global spin alignment



Puzzle: charge sensitive but $K^{*+}(u\bar{s})$ larger than $K^{*0}(d\bar{s})$, opposite to B-effect

- Global spin alignment of $J/\psi \to l^+ l^-$

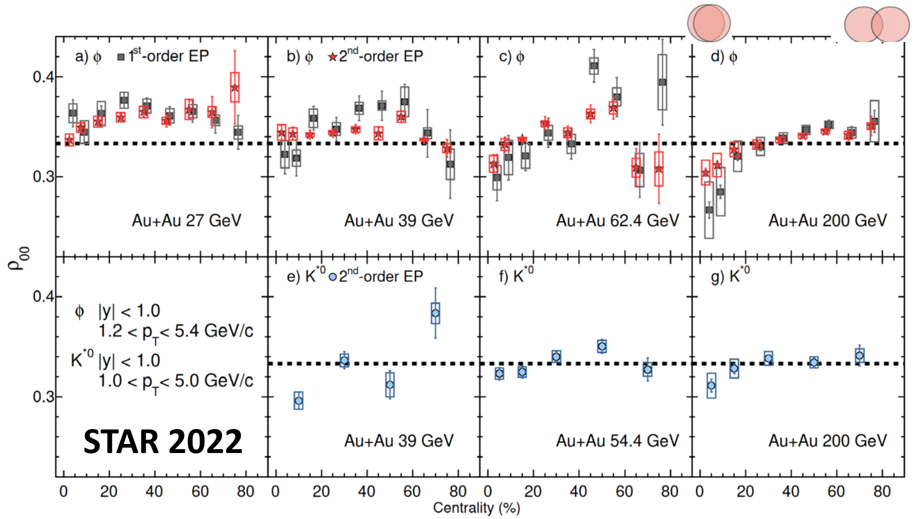


$$W(\theta) \propto \frac{1}{3+\lambda_\theta}\left(1+\lambda_\theta \cos^2\theta\right) \qquad \rho_{00} - \frac{1}{3} = \frac{2}{3}\frac{\lambda_\theta}{3+\lambda_\theta}$$

Puzzle: Λ polarization at LHC is very small but a big J/ψ spin alignment

Local spin alignment

- Centrality dependence

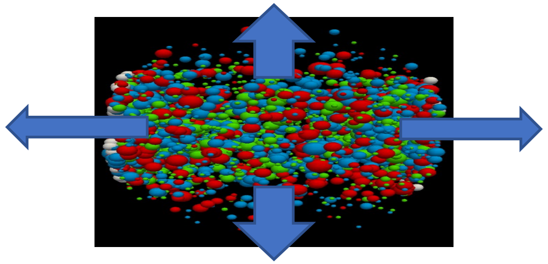


Central: $\rho_{00} < \frac{1}{3}$

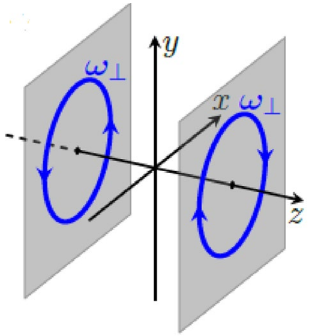
Noncentral: $\rho_{00} > \frac{1}{3}$

More significant at higher energies

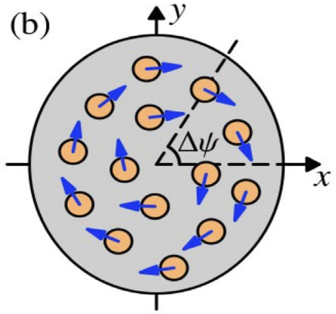
- Local spin alignment



Central collisions



$$\mathbf{P}^{q,\bar{q}} = (P_x^{q,\bar{q}}, P_y^{q,\bar{q}}, P_z^{q,\bar{q}})$$



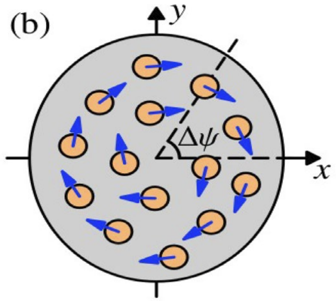
Quark spin density matrix:

$$\rho^{q,\bar{q}} = \frac{1}{2}\begin{pmatrix} 1 + P_y^{q,\bar{q}} & P_z^{q,\bar{q}} - iP_x^{q,\bar{q}} \\ P_z^{q,\bar{q}} + iP_x^{q,\bar{q}} & 1 - P_y^{q,\bar{q}} \end{pmatrix}$$

More significant at higher energies

Local spin alignment

- Vector meson spin density matrix element



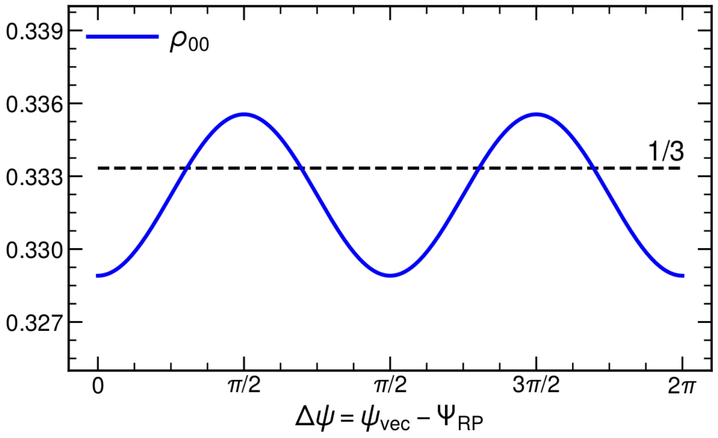
$$P_x^{q,\bar{q}}(\Delta\psi) = F_\perp \sin(\Delta\psi)$$
$$P_y^{q,\bar{q}}(\Delta\psi) = -F_\perp \cos(\Delta\psi)$$
$$P_z^{q,\bar{q}}(\Delta\psi) = 0$$

$$\Rightarrow \quad \rho_{00} = \frac{1 - P_y^q P_y^{\bar{q}} + P_x^q P_x^{\bar{q}} + P_z^q P_z^{\bar{q}}}{3 + \mathbf{P}^q \cdot \mathbf{P}^{\bar{q}}} \approx \frac{1}{3} - \frac{F_\perp^2}{9} - \frac{F_\perp^2}{3}\cos(2\Delta\psi)$$

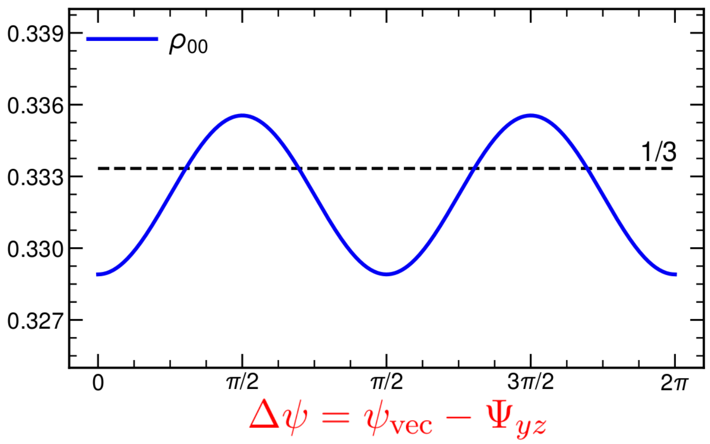
(Xia-Li-XGH-Huang 2020)

- More experimental verification of this scenario is needed

1) Measure azimuthal angle dependence



2) Measure ρ_{00} w.r.t other plane, e.g., yz plane



Local spin alignment unchanged, but global one may change significantly

Summary

Global angular momentum

Inhomogeneous expansion

⋮

→ Vortical strongly interacting matter →

Hyperon spin polarization

Vector meson spin alignment

… …

↓

Global hyperon polarization ☺

Local hyperon polarization ☹

Vector meson spin alignment ☹

Thank you

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