



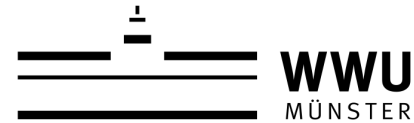
SQM2022

The 20th International Conference on Strangeness in Quark Matter
13-17 June 2022 Busan, Republic of Korea

Lecture 3

Particle production and the statistical hadronization model

A. Andronic - University of Münster



Outline

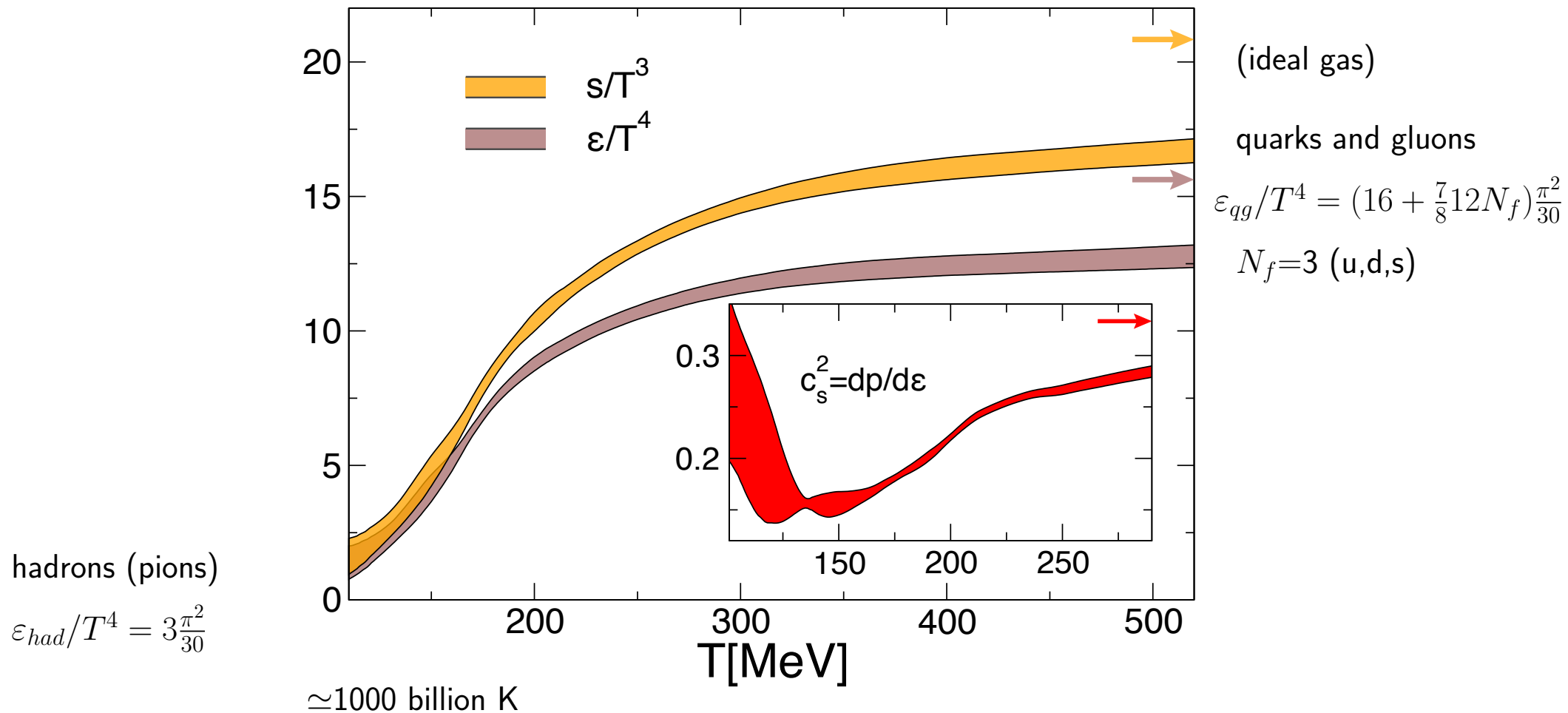
- The context
- Measurement of hadron yields (central collisions)
- The statistical (thermal) model and the thermal fits
- Thermal fits and the QCD phase diagram
- "Small systems" (pp, p-Pb collisions)
- The heavy quarks (=charm)

Further (compact) reading: A.Andronic et al., [Nature 561 \(2018\) 321](#) (and ref.therein)

Lattice QCD predicts a phase transition ($\mu_B=0$)

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Borsanyi et al., [PLB 370 \(2014\) 99](#) , A. Bazavov et al., [PLB 795 \(2019\) 15](#)

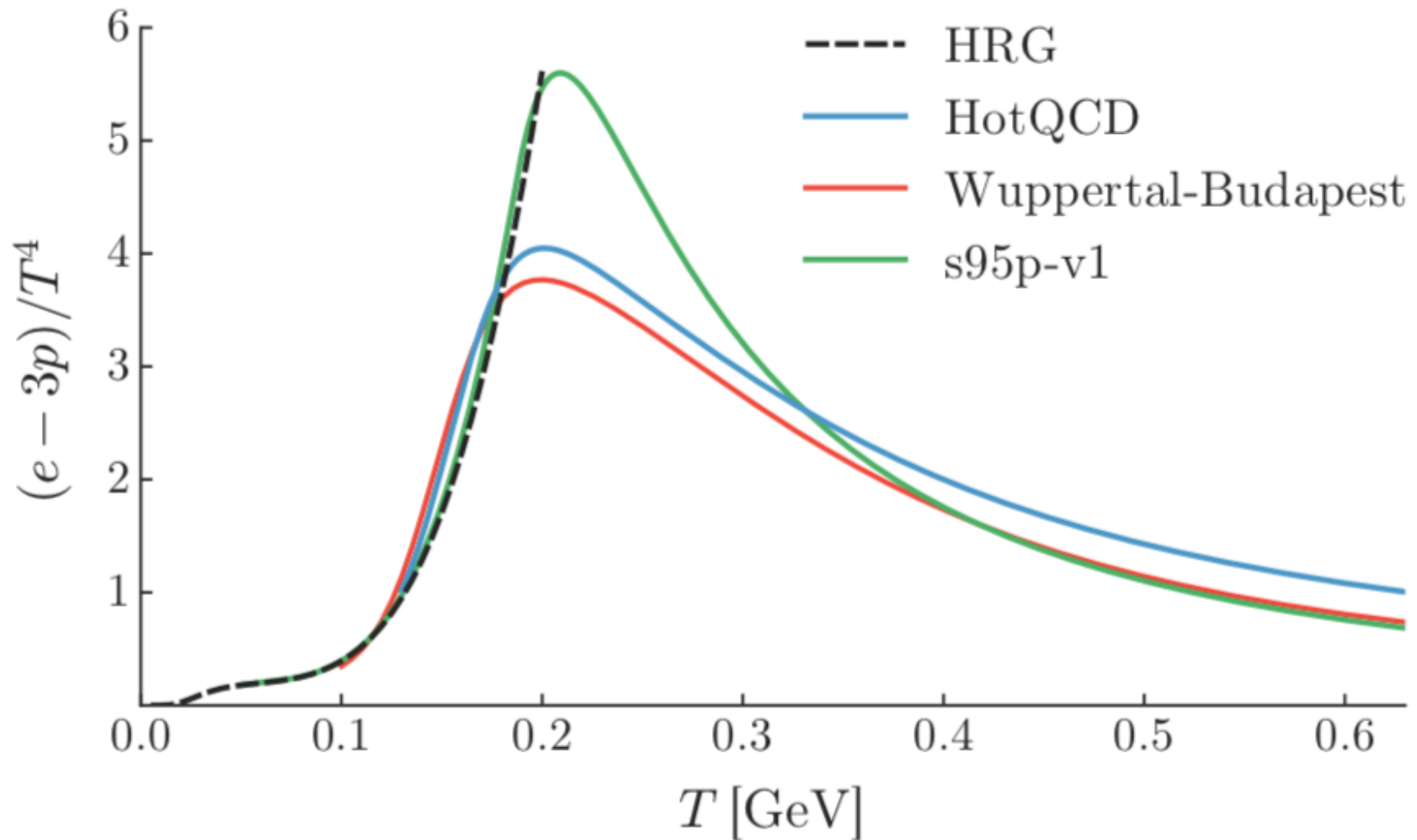
$T_c = 156.5 \pm 1.5$ MeV, $\varepsilon_c \simeq 0.4$ GeV/fm³, or $2.5\varepsilon_{nuclear}$ *crossover*

ideal-gas limit not reached at very large temperatures (GeV range explored)

Lattice QCD: EoS

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Moreland, Soltz, [PRC 93 \(2016\) 044913](#)

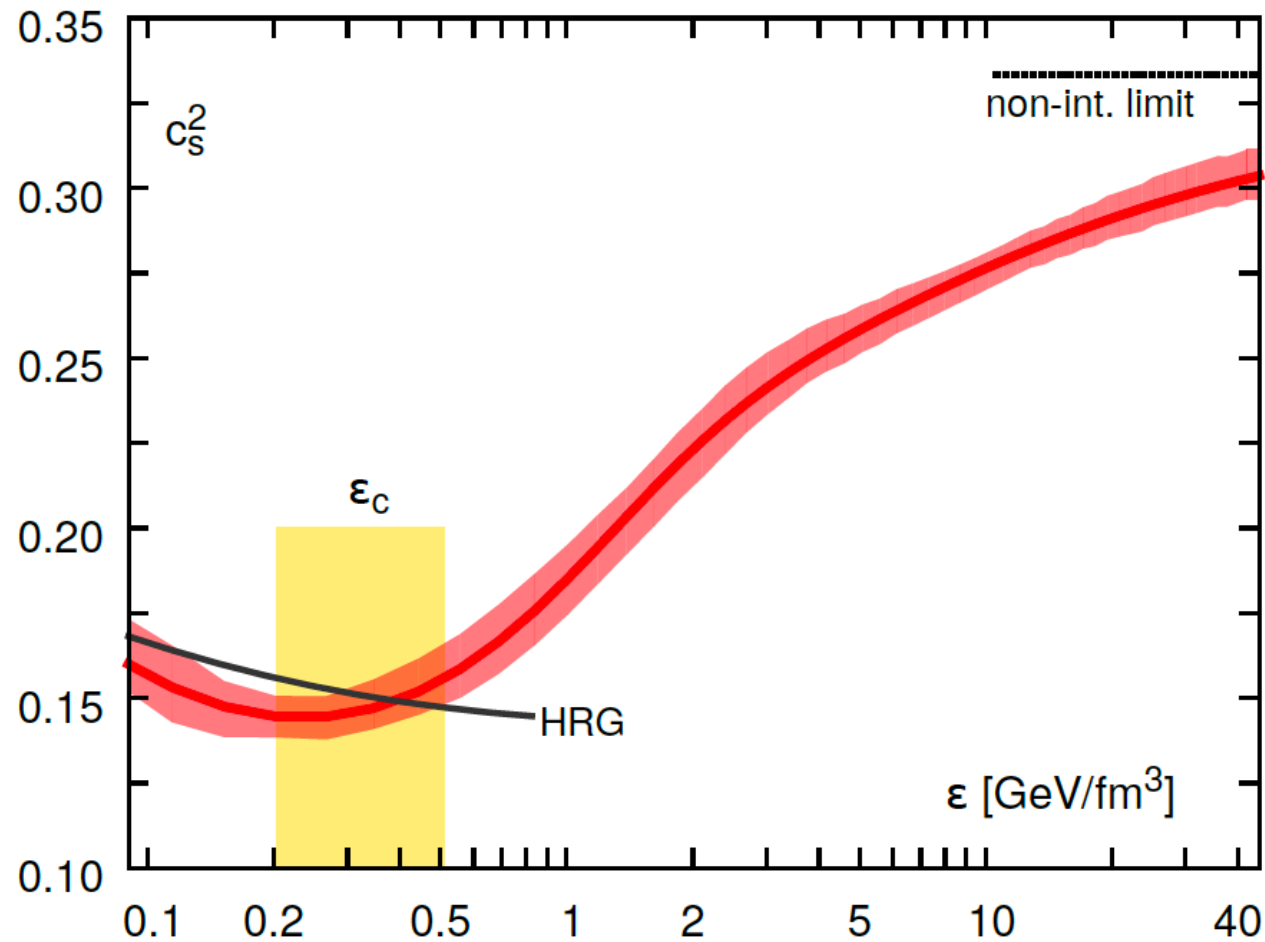
parametrization *s95p-v1* (used for some time) needs to be abandoned Lattice QCD quite precise

$(\varepsilon - 3P)/T^4$ the trace anomaly ...or *interaction measure*

Lattice QCD: EoS

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Ding, Karsch, Mukherjee, [arXiv:1504.05274](https://arxiv.org/abs/1504.05274)

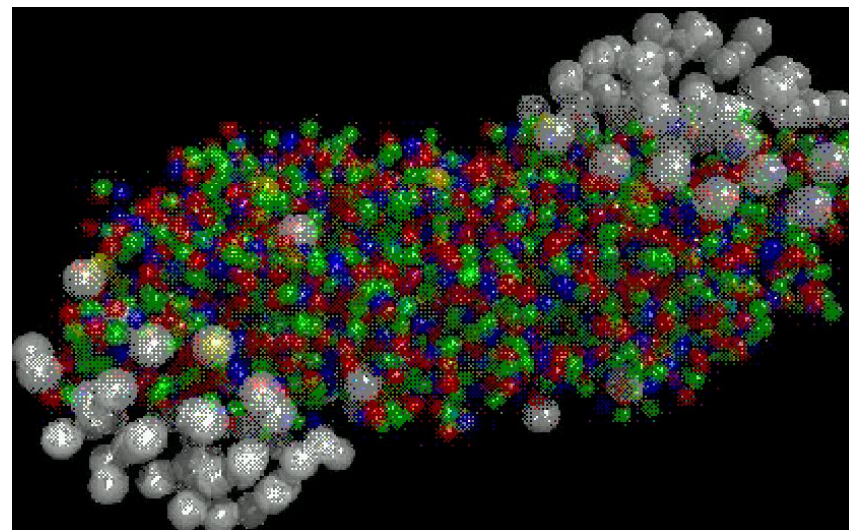
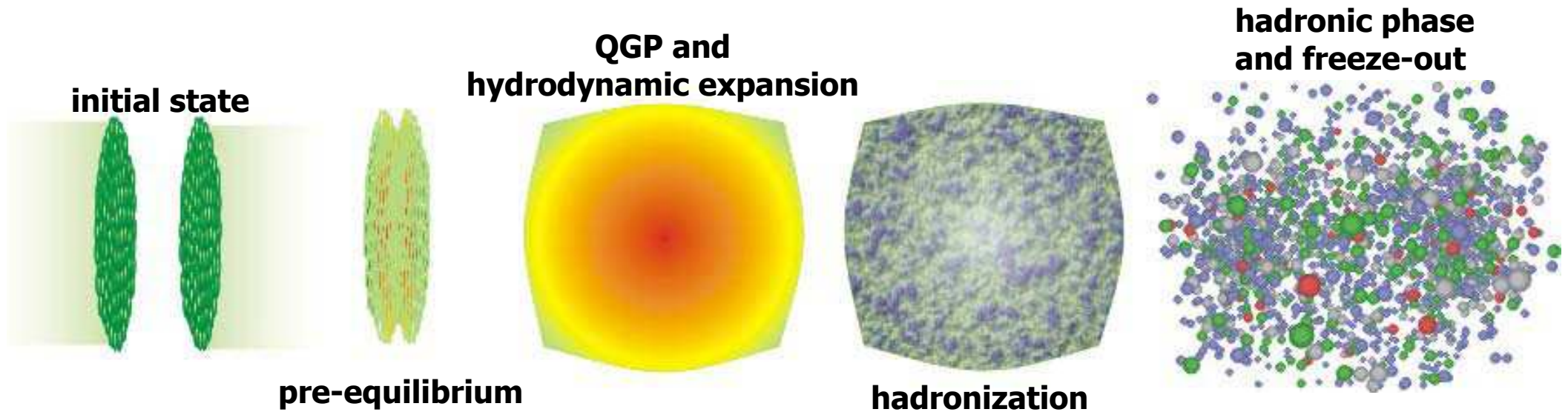
$$c_s^2 = \frac{dP}{d\epsilon} = \frac{dP/dT}{d\epsilon/dT} = \frac{s}{C_V}, \quad C_V \text{ specific heat}$$

HRG = Hadron Resonance Gas (this lecture)

From theory to experiment: QGP in the lab

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SMASH animation

Stages of a high-energy nucleus-nucleus collision

1. initial collisions ($t \leq t_{coll} = 2R/\gamma_{cm}c$)
2. thermalization: equilibrium is established ($t_{eq} \lesssim 1 \text{ fm}/c$)
3. expansion and cooling ($t_{QGP} < 10\text{-}15 \text{ fm}/c$)
4. hadronization
5. chemical freeze-out: inelastic collisions cease (close to the phase boundary?); yields (and distribution over species) are frozen
6. kinetic freeze-out: elastic collisions cease; spectra and correlations are frozen ($t_{had} = 3\text{-}5 \text{ fm}/c$)

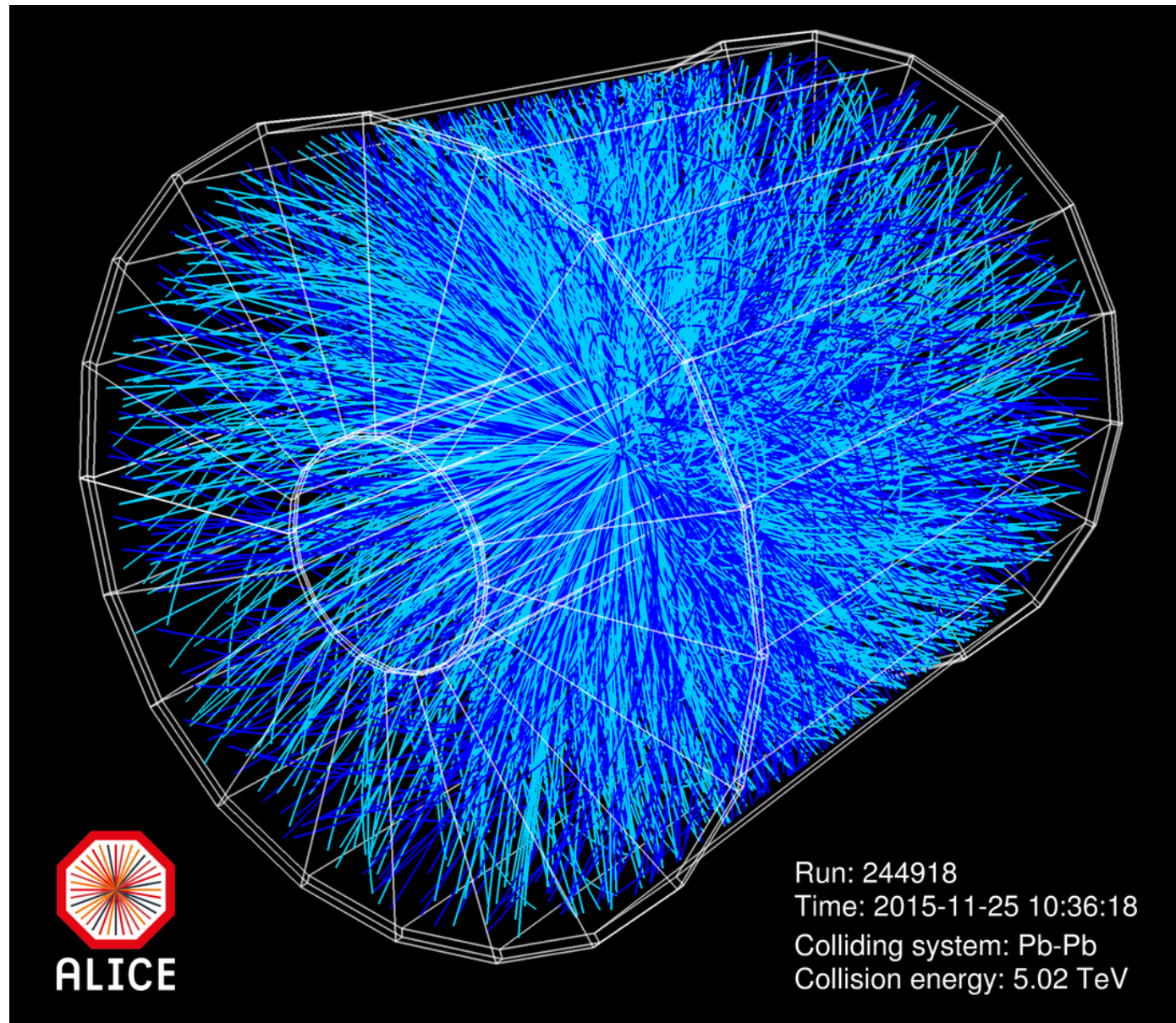
we measure 5. and 6. and want to determine properties at stages 2,3,4
...only possible via models (“run the movie backwards”)

We now try to characterize 5. via an analysis of hadron abundances ...
are they in equilibrium? connection to hadronization?

Nucleus-nucleus collisions at the LHC

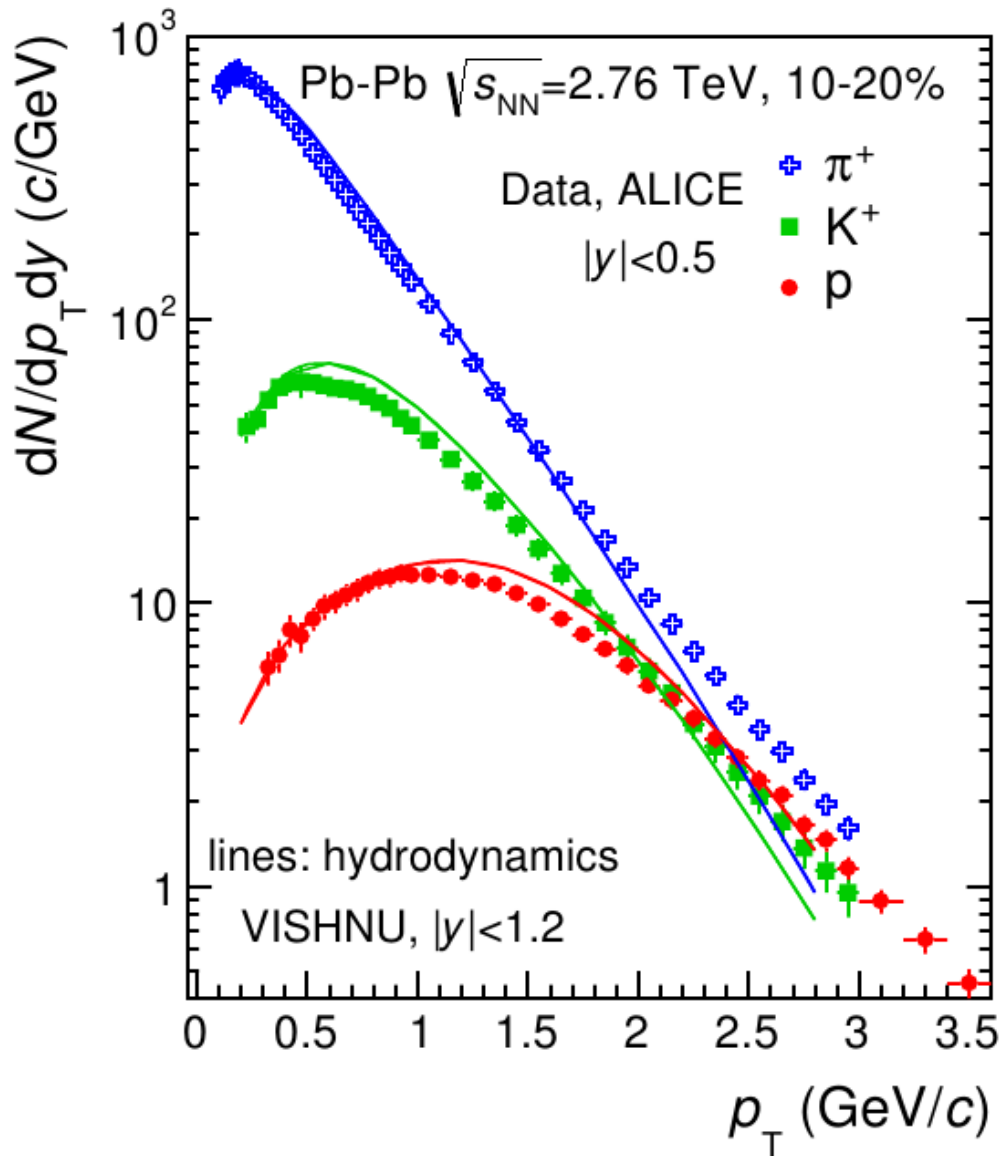
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a picture of a central collision (about 3200 primary tracks in $|\eta| < 0.9$); “Camera”: Time Projection Chamber, 5 m length, 5 m diam.; 500 mil. pixels; we take a few 100 pictures per second (and are preparing to take 50000)

Spectra



measuring spectra we observe strong collective flow ($\beta \simeq 0.6$), leading to marked mass dependence:

$$p = m\beta\gamma$$

NB: the decays (feed-down) complication

hydrodynamic models reproduce the data

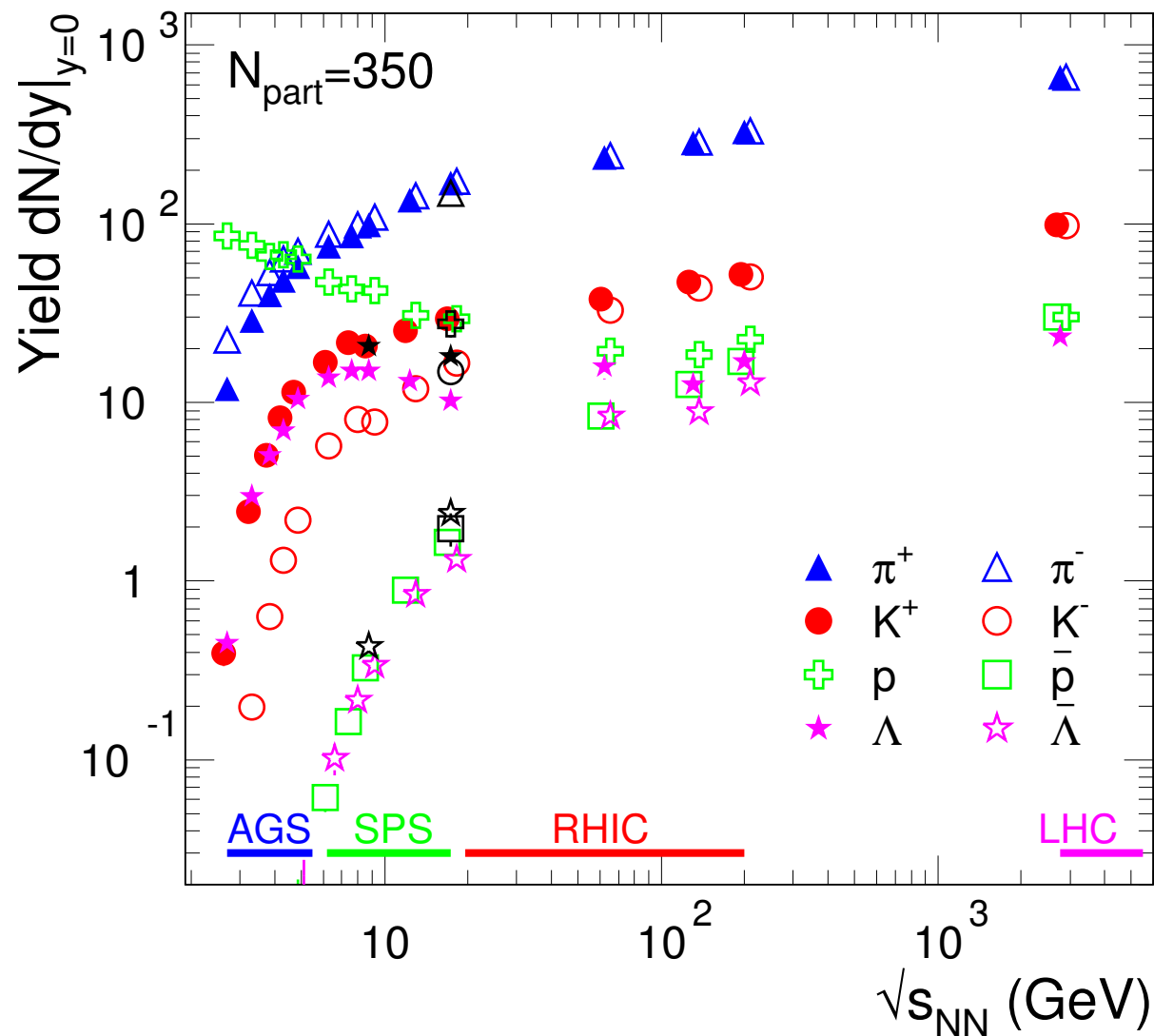
$T_0 \simeq 500$ MeV, expansion ($\Delta t \sim 10$ fm/c)

from now on we will integrate on p_T

Hadron yields at midrapidity (central collisions)

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- lots of particles, mostly newly created ($m = E/c^2$)

- a great variety of species:

π^\pm ($u\bar{d}$, $d\bar{u}$), $m=140$ MeV

K^\pm ($u\bar{s}$, $\bar{u}s$), $m=494$ MeV

p (uud), $m=938$ MeV

Λ (uds), $m=1116$ MeV

also: $\Xi(dss)$, $\Omega(sss)$...

- mass hierarchy in production (u, d quarks: remnants from the incoming nuclei)

A.Andronic, [arXiv:1407.5003](https://arxiv.org/abs/1407.5003)

...natural to think of the thermal (statistical) model ($e^{-m/T}$)

The thermal model

also known as: statistical / hadron resonance gas / statistical hadronization model (HRG / SHM)

...is in a way the simplest model

(“All models are wrong, but some are useful”, George Box)

The analysis of hadron yields within the thermal model provides a “snapshot” of a nucleus-nucleus collision at chemical freeze-out (the earliest in the collision timeline we can look with hadronic observables)

Test hypothesis of hadron abundancies in equilibrium

...but the devil is in the details ...one needs:

- a complete hadron spectrum (all species of hadrons, see [Particle Data Book](#))
- canonical approach at low energies (and smaller systems)
- to understand the data well (control fractions from weak decays)

Reminder about ensembles

microcanonical: describes an isolated system (E, V, T)

canonical and *grand canonical*: suppose a heat reservoir (temperature T) the system (our collection of hadrons) exchange energy with that

canonical: no particles are exchanged (N, V, T)

grand canonical: the system exchanges particles $(\langle N \rangle, V, T)$
chemical potentials, or fugacities, are introduced to ensure conservation, on average, of particle numbers

here use *grand canonical*

(supplemented when needed with correction for canonical)

The statistical (thermal) model

grand canonical partition function for specie (hadron) i :

$$\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

$g_i = (2J_i + 1)$ spin degeneracy factor; T temperature;

$E_i = \sqrt{p^2 + m_i^2}$ total energy; (+) for fermions (-) for bosons

$\mu_i = \mu_B B_i + \mu_{I_3} I_{3i} + \mu_S S_i + \mu_C C_i$ chemical potentials

μ ensure conservation (on average) of quantum numbers, fixed by “initial conditions”

i) isospin: $V_{cons} \sum_i n_i I_{3i} = I_3^{tot}$, with $V_{cons} = N_B^{tot} / \sum_i n_i B_i$

I_3^{tot} , N_B^{tot} isospin and baryon number of the system (=0 at high energies)

ii) strangeness: $\sum_i n_i S_i = 0$

iii) charm: $\sum_i n_i C_i = 0$.

Brief summary of cases

General ($\epsilon = +1$ for bosons, $\epsilon = -1$ for fermions; particle index i omitted):

$$N = -T \frac{\partial \ln Z}{\partial \mu} = \frac{gV}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E - \mu)/T] \pm 1}$$

$$N = \frac{g}{2\pi^2} TV m^2 \epsilon \sum_{k=1}^{\infty} \frac{\epsilon^k}{k} e^{\frac{\mu}{T} k} K_2 \left(\frac{m}{T} k \right)$$

Classical statistics:

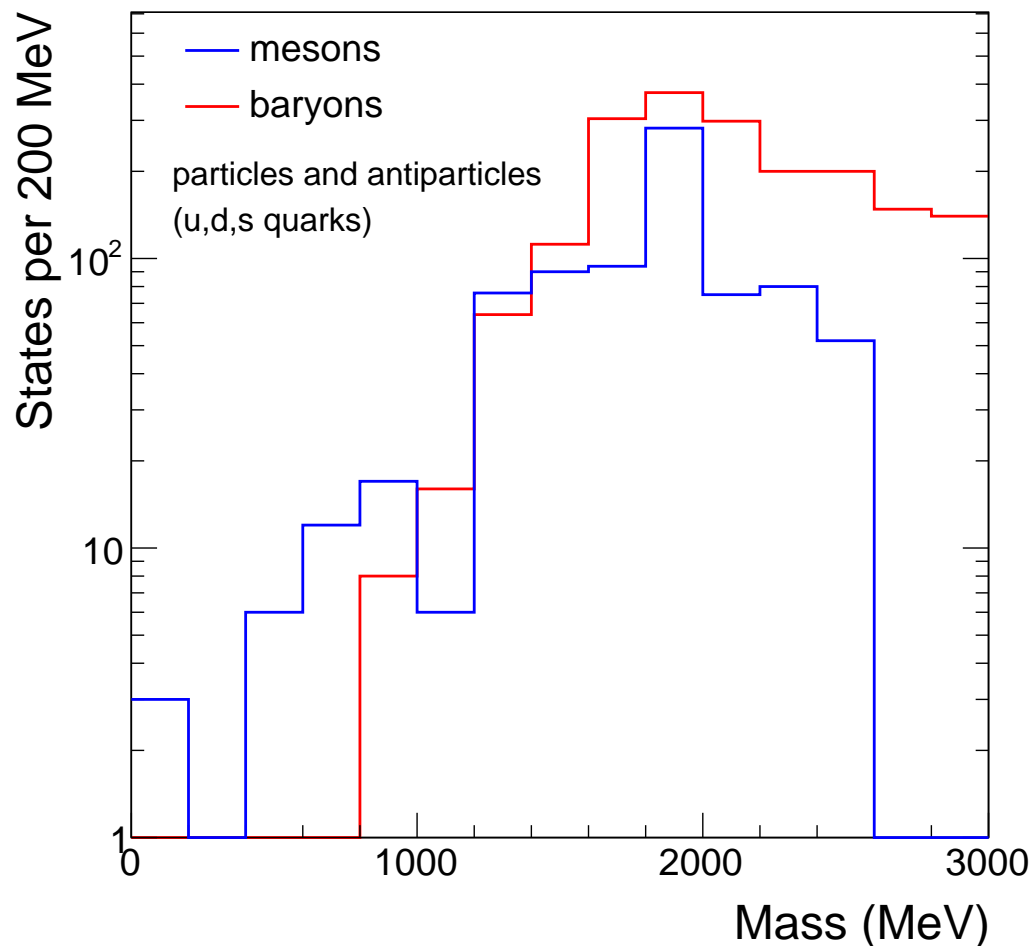
$$N = \frac{g}{2\pi^2} TV m^2 e^{\frac{\mu}{T}} K_2 \left(\frac{m}{T} \right) = gV e^{\frac{\mu}{T}} \left(\frac{mT}{2\pi} \right)^{3/2} e^{-\frac{m}{T}} \left(1 + \frac{15T}{8m} + O(T^2/m^2) \right)$$

$$K_2(x) = \sqrt{\frac{\pi}{2x}} e^{-x} \left(1 + \frac{15}{8x} + O(1/x^2) \right)$$

Model input: hadron spectrum

...embodies low-energy QCD ...*vacuum masses*

well-known for $m < 2$ GeV; many confirmed states above 2 GeV, still incomplete



for high m , BR not well known, but can be reasonably guessed

T found to be robust in fits with spectrum truncated above 1.8 GeV

$\sigma [f_0(500)]$ meson proposed recently to be discarded (reduction of π densities by 3-4%)

Giacosa, Begun, Broniowski, [arXiv:1603.07687](https://arxiv.org/abs/1603.07687)

$$\rho(m) = c \cdot m^{-a} \exp(m/T_H)$$

$$T_H \simeq 180 \text{ MeV (max } T \text{ for hadrons)}$$

$(2J + 1)$ counted in

Hadron mass spectrum and Hagedorn's bootstrap model

exponential form of mass spectrum (for large m): $\rho(m) = c \cdot m^{-a} \exp(m/T_H)$
in the statistical bootstrap model: Hagedorn, [Lect.Notes Phys 221 \(1985\) 53](#))

“fireballs (hadrons) consist of fireballs, which consist of fireballs, which...”

consider Boltzmann statistics, non-relativistic approximation

$$\ln Z_i(T, V) = \frac{V}{(2\pi)^3} \int e^{-\sqrt{p^2+m^2}/T} d^3p \simeq V \left(\frac{T}{2\pi}\right)^{2/3} m_i^{2/3} e^{-m_i/T}$$

$$\ln Z = \sum_i \ln Z_i = V \left(\frac{T}{2\pi}\right)^{2/3} \sum_i m_i^{2/3} e^{-m_i/T} \text{ (sum over all hadrons)}$$

$$\ln Z = V \left(\frac{T}{2\pi}\right)^{2/3} \int_0^\infty \rho(m) m^{3/2} e^{-m/T} dm$$

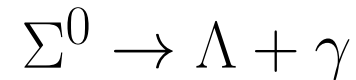
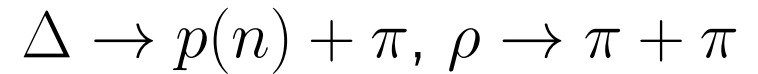
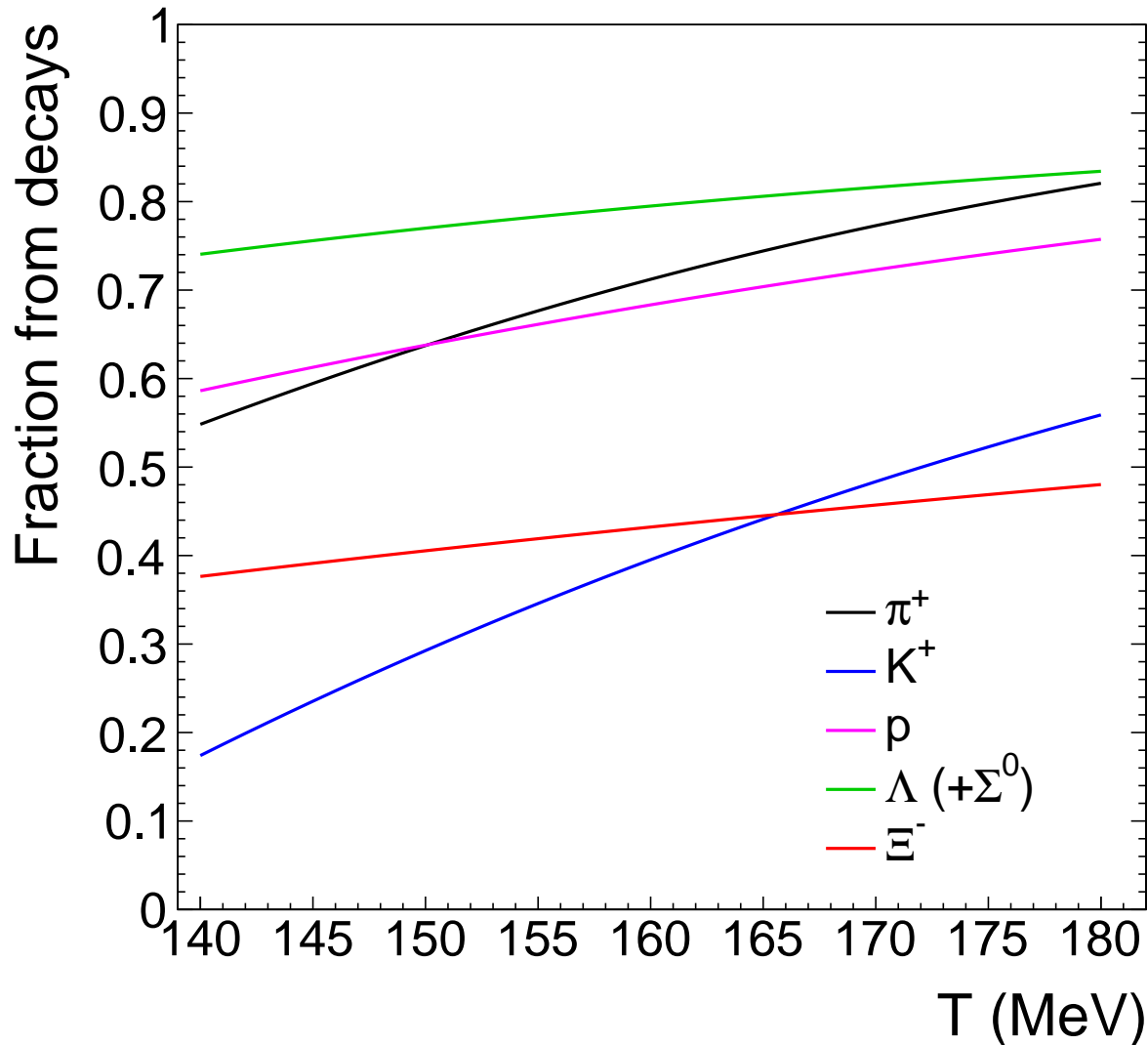
$$\varepsilon(T_H) = \frac{T^2}{V} \frac{\partial \ln Z}{\partial T}(T_H) \sim \frac{3T_H}{2} (m^{5/2-a})|_0^\infty + (m^{7/2-a})|_0^\infty + \text{finite}$$

diverge at T_H for $a < 7/2$

T_H maximal temperature of hadronic systems; beyond: QGP (Cabibbo, Parisi)

Decays

(almost all) hadrons are subject to strong and electromagnetic decays



weak decays can be treated as well ...to account for the exact experimental situation

contribution of resonances is significant (and particle-dependent)

(plot for $\mu_B=0$)

Considering widths of resonances

$$n_i = N_i/V = -\frac{T}{V} \frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1}$$

$$n_i = \frac{g_i}{2\pi^2} \frac{1}{N_{BW}} \int_{M_{thr}}^\infty dm \int_0^\infty \frac{\Gamma_i^2}{(m - m_i)^2 + \Gamma_i^2/4} \cdot \frac{p^2 dp}{\exp[(E_i^m - \mu_i)/T] \pm 1}$$

M_{thr} threshold mass for the decay channel.

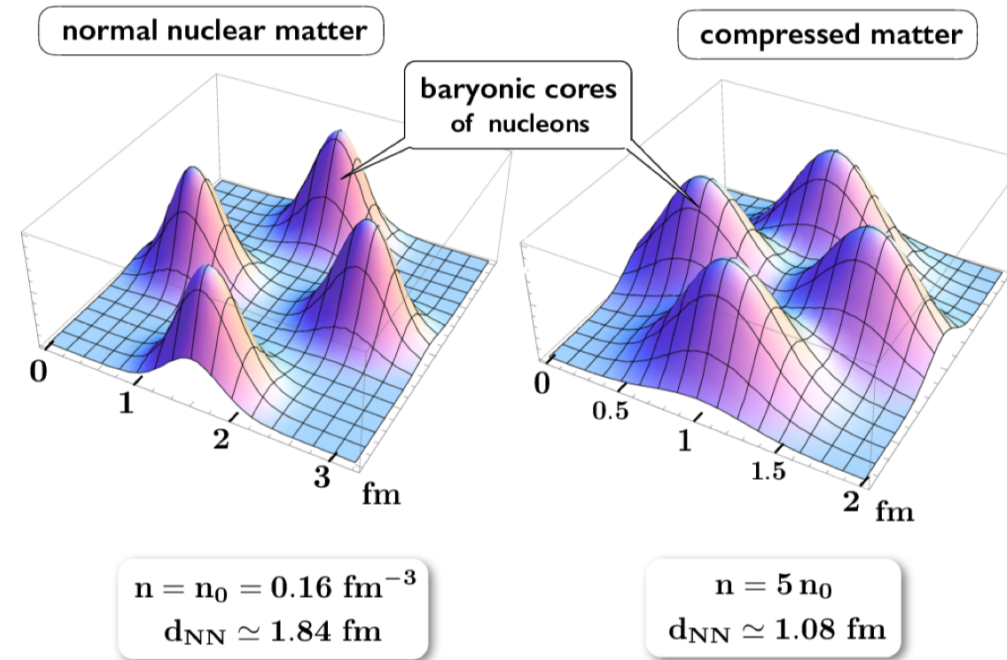
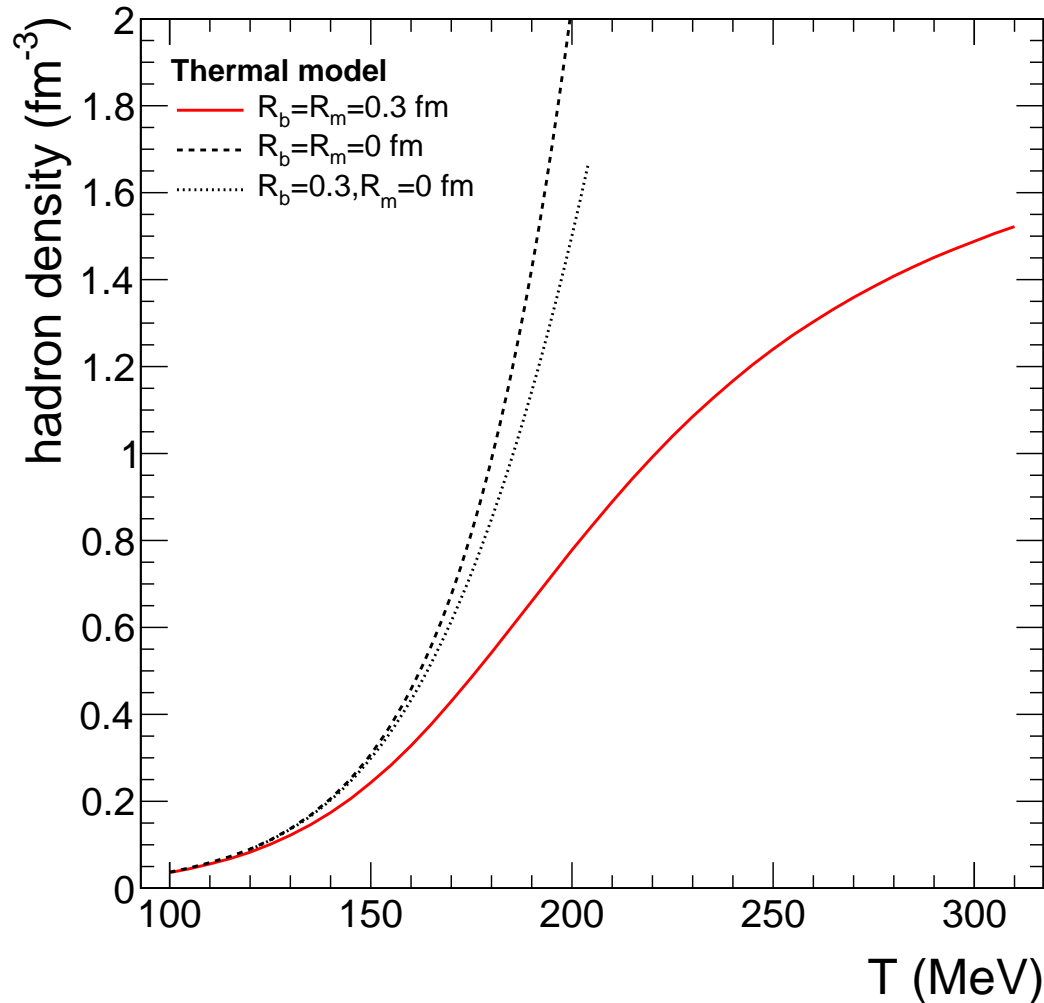
Example: for $\Delta^{++} \rightarrow p + \pi^+$, $M_{thr}=1.068$ GeV ($m_{\Delta^{++}}=1.232$ GeV)

Important mainly at “low” temperatures ($T \lesssim 150$ MeV)

Hadron densities

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Weise, [arXiv:1811.09682](https://arxiv.org/abs/1811.09682)

(baryons: gaussians, $r=0.5 \text{ fm}$)

"hadron gas": a dense system (also nuclear matter is rather a liquid than a gas)

(the usual case is $R_{baryon} = R_{meson} = 0.3 \text{ fm}$...hard-sphere repulsion)

Air at NTP: intermolecule distance $\simeq 50 \times$ molecule size

Canonical correction (“canonical suppression”)

needed whenever the abundance of hadrons with a given quantum number is very small ...so that one needs to enforce exact quantum-number conservation in AA collisions; *Example: strangeness at low energies*

$$n_{i,S}^C = n_{i,S}^{GC} \cdot \frac{I_s(N_S)}{I_0(N_S)}$$

$$N_S = V_c \cdot \sum S \cdot n_{i,S},$$

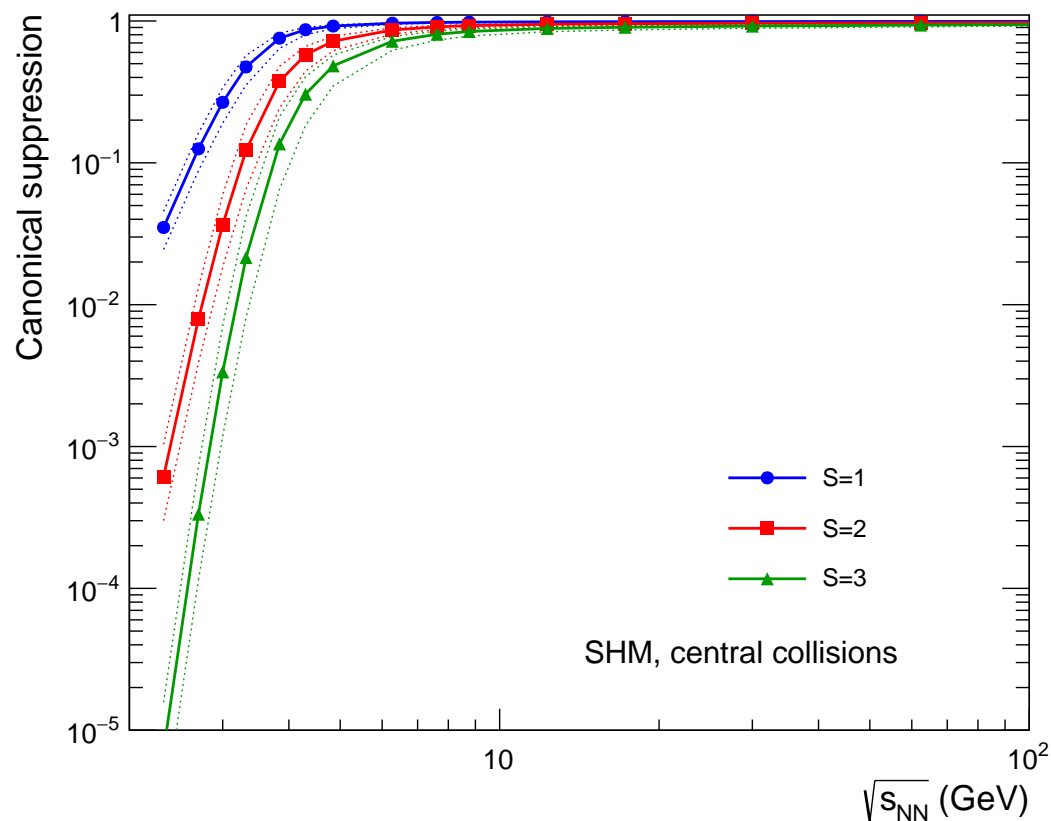
total amount of strangeness-carrying hadrons (part.+antipart.)

$$n_{K,\Lambda}^C = n_{K,\Lambda}^{GC} \cdot \frac{I_1(N_S)}{I_0(N_S)},$$

$$n_{\phi}^C = n_{\phi}^{GC}$$

...negligible for $\sqrt{s_{NN}} \gtrsim 8$ GeV

relevant for small volumes (peripheral AA, pp, p-Pb collisions), $V_c \sim dN_{ch}/dy$



Strangeness suppression factor, γ_s

...a non-thermal fit parameter, to check possible non-thermal production of strangeness

for a hadron carrying “absolute” strangeness $s = |S - \bar{S}|$: $n_i \rightarrow n_i \gamma_s^s$

Examples: $K^\pm (u\bar{s}, \bar{u}s)$: $n_K \gamma_s$, $\Lambda (uds)$: $n_\Lambda \gamma_s$,

$\Xi(dss)$: $n_\Xi \gamma_s^2$, $\Omega(sss)$: $n_\Omega \gamma_s^3$, $\phi(s\bar{s})$: $n_\phi \gamma_s^2$

in principle, usage of γ_s is to be avoided if one tests the basic thermal model

even as some models employ it ($\Rightarrow \gamma_s = 0.6 - 0.8$), all agree that it is not needed at RHIC, LHC energies (for central collisions)

here (central AA collisions) we fix $\gamma_s=1$

Thermal fits of hadron abundances

$$n_i = N_i/V = -\frac{T}{V} \frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1}$$

Latest PDG hadron mass spectrum ...quasi-complete up to $m=2$ GeV;
our code: 555 species (including fragments, charm and bottom hadrons)

for resonances, the width is considered in calculations

canonical treatment whenever needed (small abundances)

Minimize: $\chi^2 = \sum_i \frac{(N_i^{exp} - N_i^{therm})^2}{\sigma_i^2}$

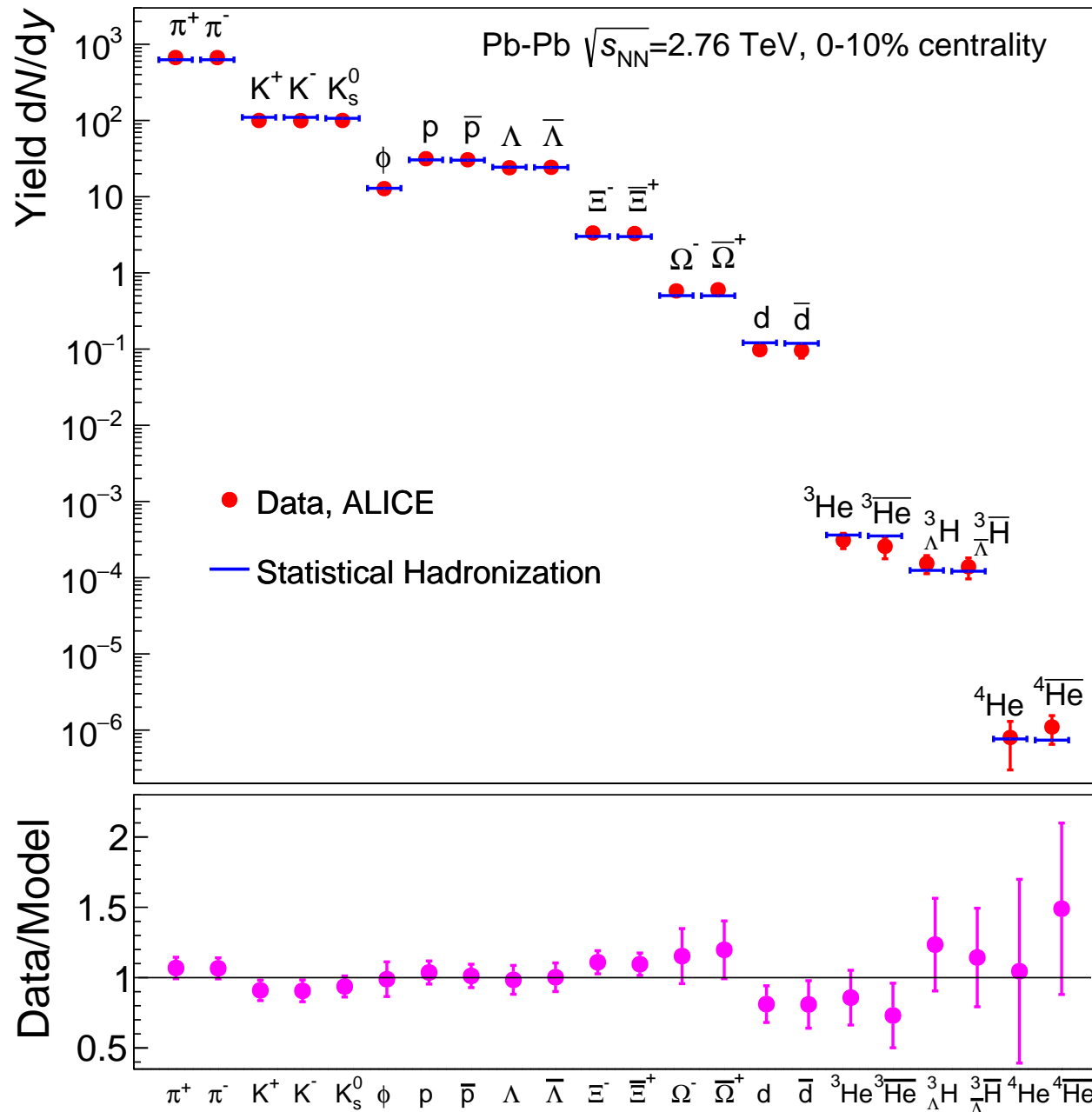
N_i hadron yield, σ_i experimental uncertainty (stat.+syst.)

$\Rightarrow (T, \mu_B, V)$...tests chemical freeze-out (chemical equilibrium)

Thermal fit – LHC, Pb–Pb, 0-10%

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matter and antimatter produced
in equal amounts

$$T_{CF} = 156.6 \pm 1.7 \text{ MeV}$$

$$\mu_B = 0.7 \pm 3.8 \text{ MeV}$$

$$V_{\Delta y=1} = 4175 \pm 380 \text{ fm}^3$$

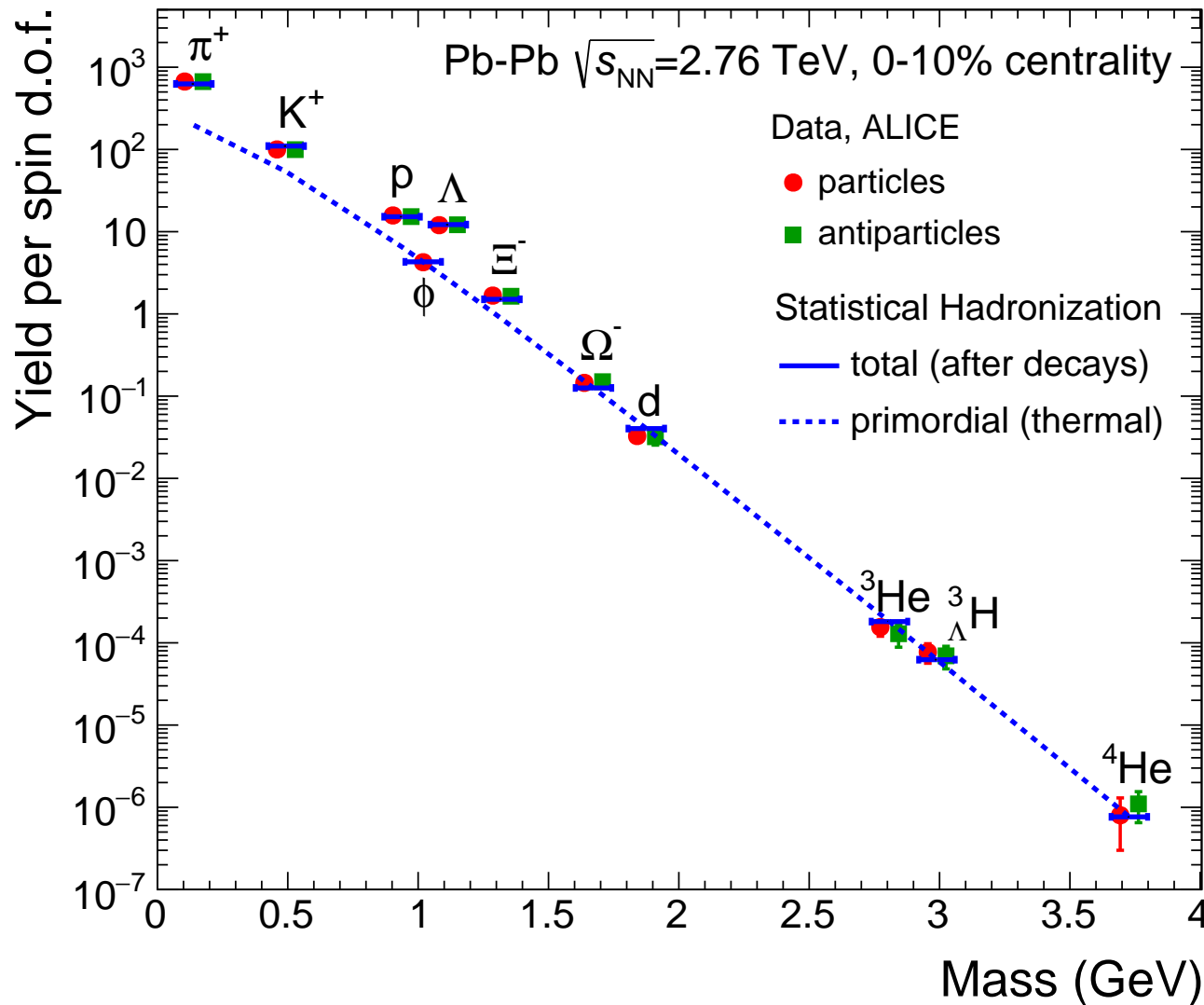
$$\chi^2/N_{df} = 16.7/19$$

remarkably, loosely-bound
objects (d , ${}^3_{\Lambda}H$) are also well
described

Model uncertainties 1. Hadron spectrum

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contribution of resonances
is significant
(and particle-dependent)

Fit of ϕ , Ω , d , ${}^3\text{He}$, ${}^3\text{H}$, ${}^4\text{He}$:

$$T_{CF} = 156 \pm 2.5 \text{ MeV}$$

$$(\chi^2/N_{df} = 7.4/8)$$

Fit of nuclei (d , ${}^3\text{He}$, ${}^4\text{He}$):

$$T_{CF} = 159 \pm 5 \text{ MeV}$$

3-4 MeV upper bound of systematic uncertainty due to hadron spectrum

Model uncertainties 2. Interactions

hadron eigenvolumes ...to mimick interactions (beyond low-density, Dashen-Ma)

$R_{meson} = 0.3, R_{baryon} = 0.3$ fm is generally considered to be a reasonable case
point-like hadrons lead to same T , but volume larger by 20-25%

an extreme case, $R_{meson} = 0, R_{baryon} = 0.3$ fm leads to
 $T = 161.0 \pm 2.0$ MeV, $\mu_B = 0$ fixed, $V = 3470 \pm 280$ fm³

NB: in this case, the result is rather sensitive on the set of hadrons in the fit
for instance, using hadrons up to Ω , cannot constrain T (unphysically large)

Vovchenko, Stöcker (et al.), [JPG 44 \(2017\) 055103](#), [arXiv:1606.06350](#)

...and anything else can be imagined, see (R dependent on mass & strangeness)
Alba, Vovchenko, Gorenstein, Stöcker, [NPA 974 \(2018\) 22](#), etc.

T -dependent Breit-Wigner resonance widths:

Vovchenko, Gorenstein, Stöcker, [PRC 98 \(2018\) 034906](#)

Interactions, rightly done

...for now only at the LHC

non-strange baryon sector treated in S-matrix formalism
(πN scattering phase shifts, *including non-resonant contributions*)

[PLB 792 \(2019\) 304](#)

solved the so-called "proton puzzle" (too many protons in the statistical model)
for $T=156$ MeV, proton yield decreased by 17% compared to point-like

recently tackled: strange baryon sector, Cleymans et al., [PRC 103 \(2021\) 014904](#)

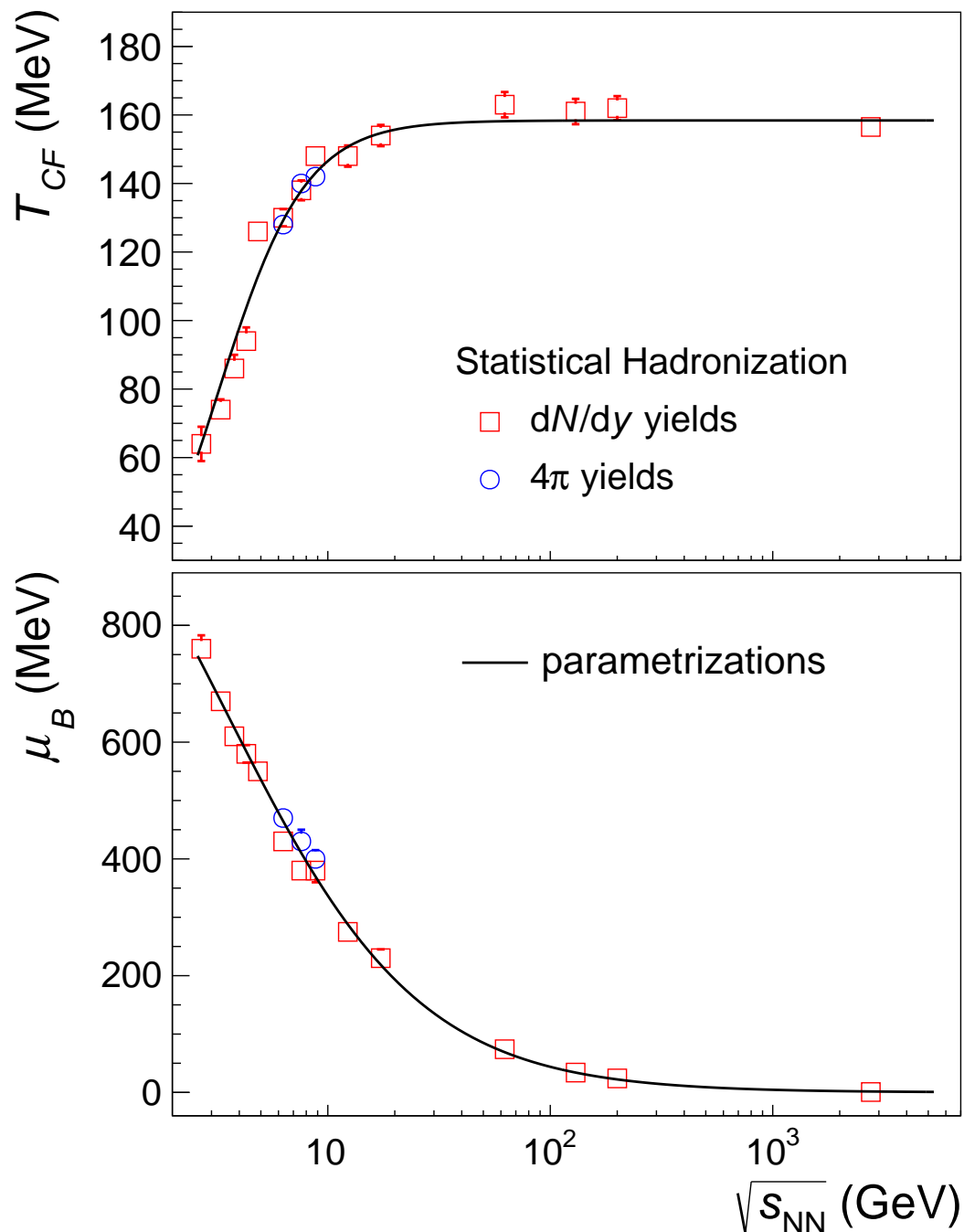
NB: presence of resonances implies interaction

(this is why moderate $R = 0.3$ fm is a reasonable choice)

Energy dependence of T , μ_B (central collisions)

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thermal fits exhibit a limiting temperature:

$$T_{lim} = 158.4 \pm 1.4 \text{ MeV}$$

$$T_{CF} = T_{lim} \frac{1}{1 + \exp(2.60 - \ln(\sqrt{s_{NN}}(\text{GeV}))/0.45)}$$

$$\mu_B[\text{MeV}] = \frac{1307.5}{1 + 0.288\sqrt{s_{NN}}(\text{GeV})}$$

NPA 772 (2006) 167, PLB 673 (2009) 142

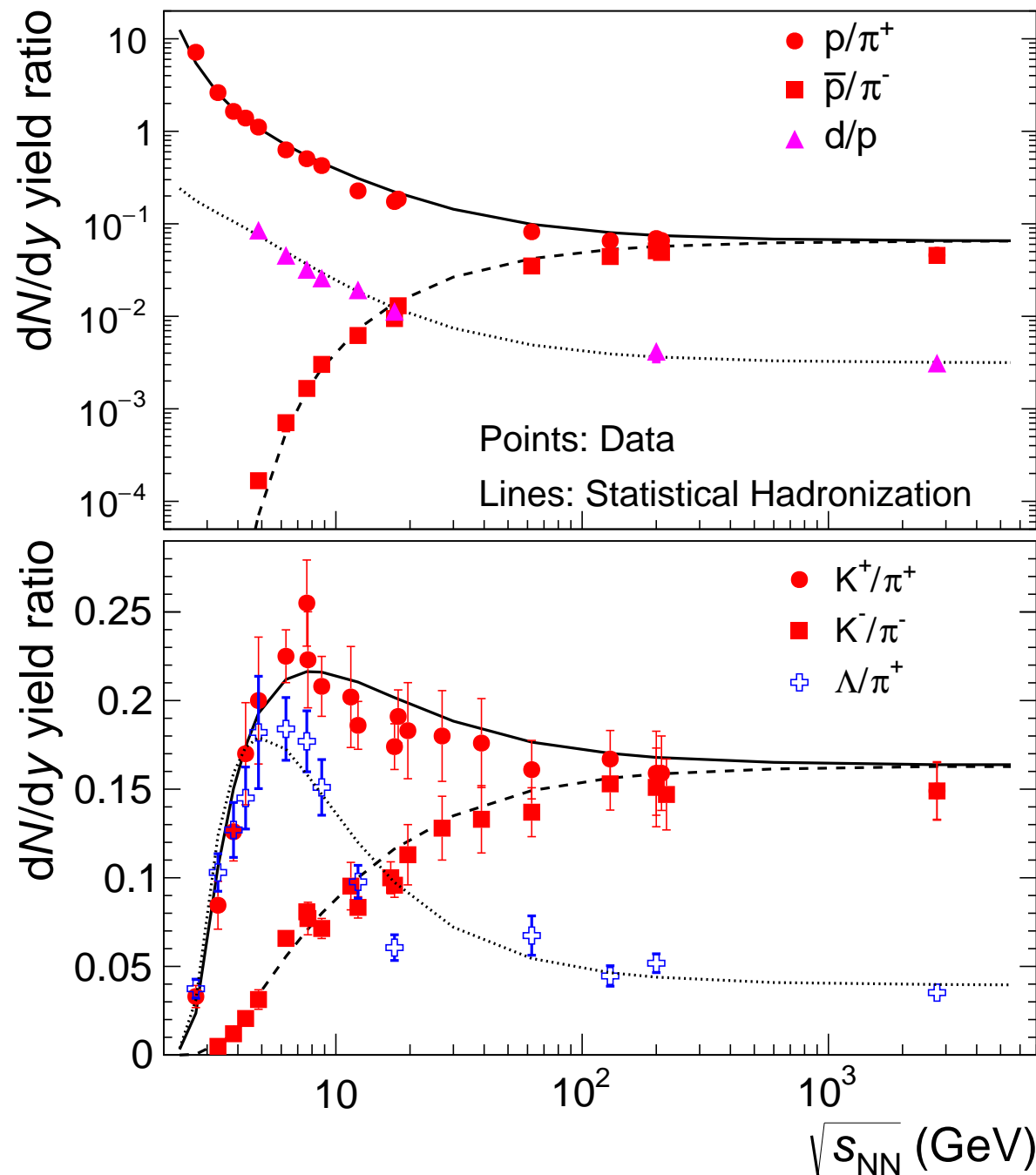
μ_B is a measure of the net-baryon density, or matter-antimatter asymmetry

determined by the "stopping" of the colliding nuclei

The grand (albeit partial) view

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Data:

AGS: E895, E864, E866, E917, E877

SPS: NA49, NA44

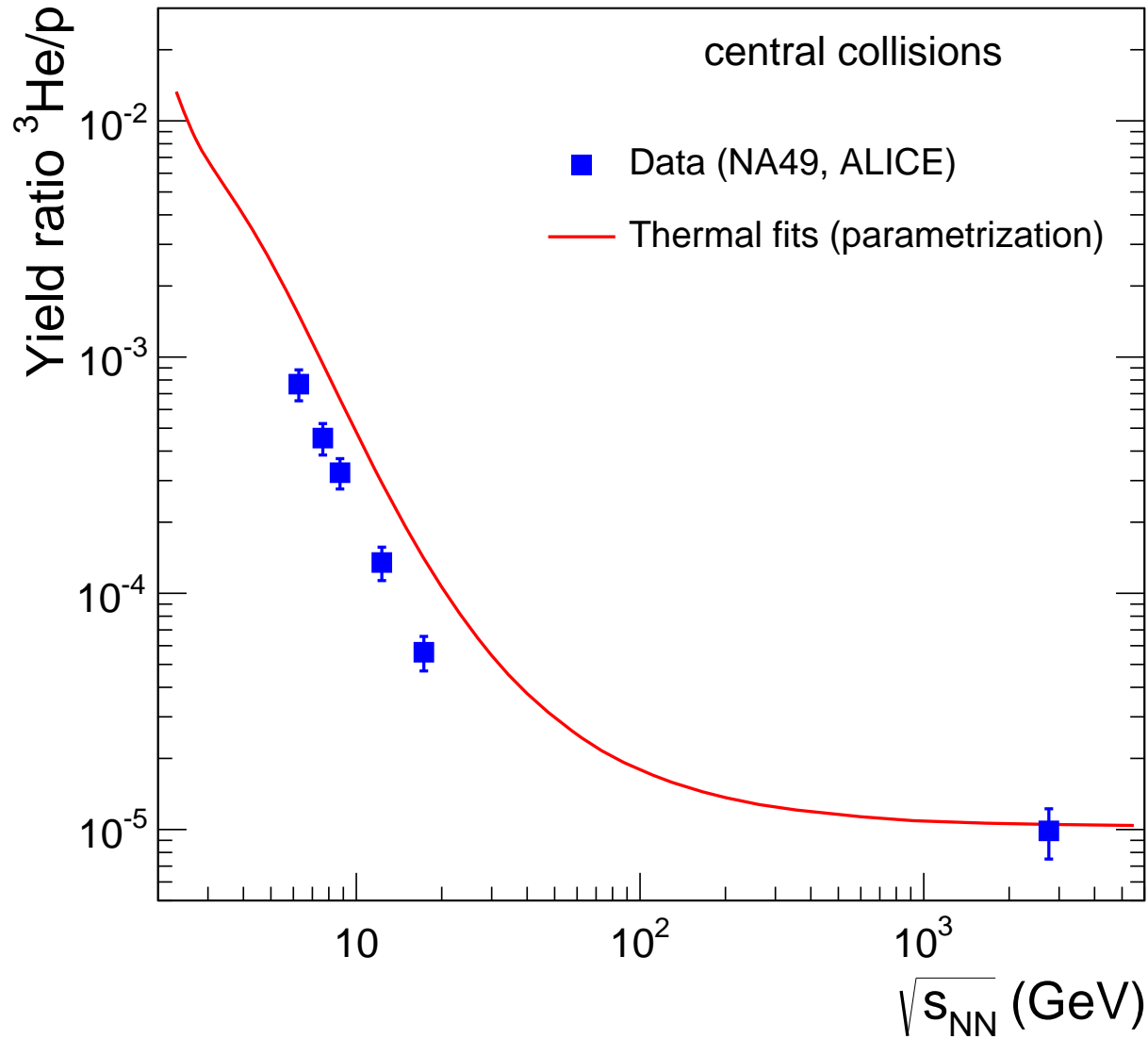
RHIC: STAR, BRAHMS

LHC: ALICE

NB: no contribution from weak decays

“structures” described by SHM
(determined by strangeness conservation)

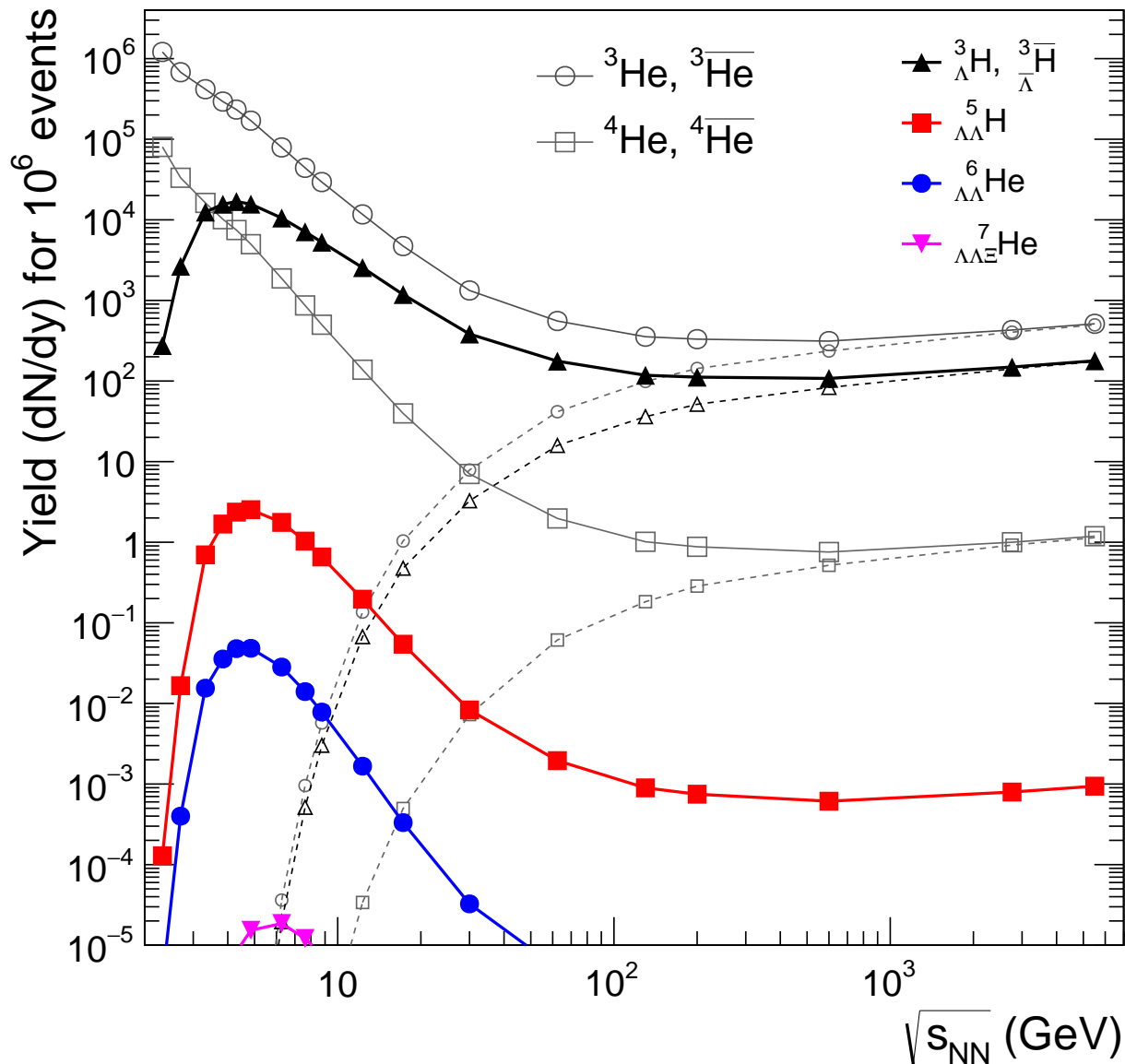
Something that doesn't work so well ?



(prelim.) STAR BES data confirm this discrepancy

Complex objects

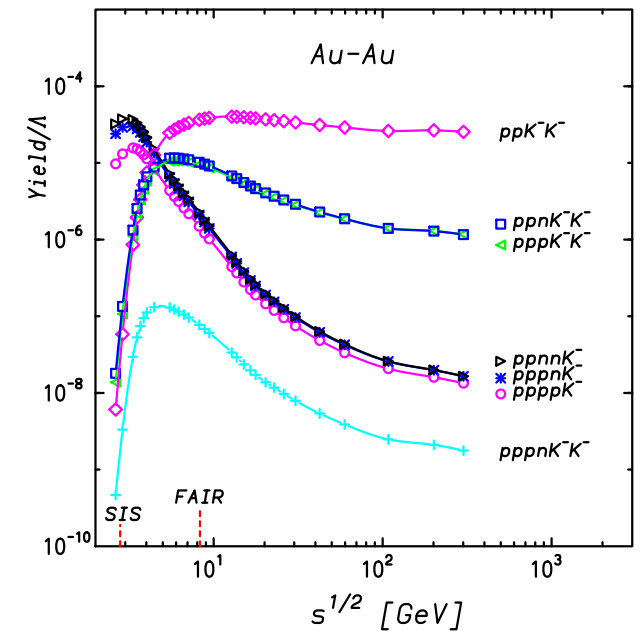
...are copiously produced at RHIC FT (and FAIR) energies



...some to be discovered

maybe also nucleon- K^- clusters?

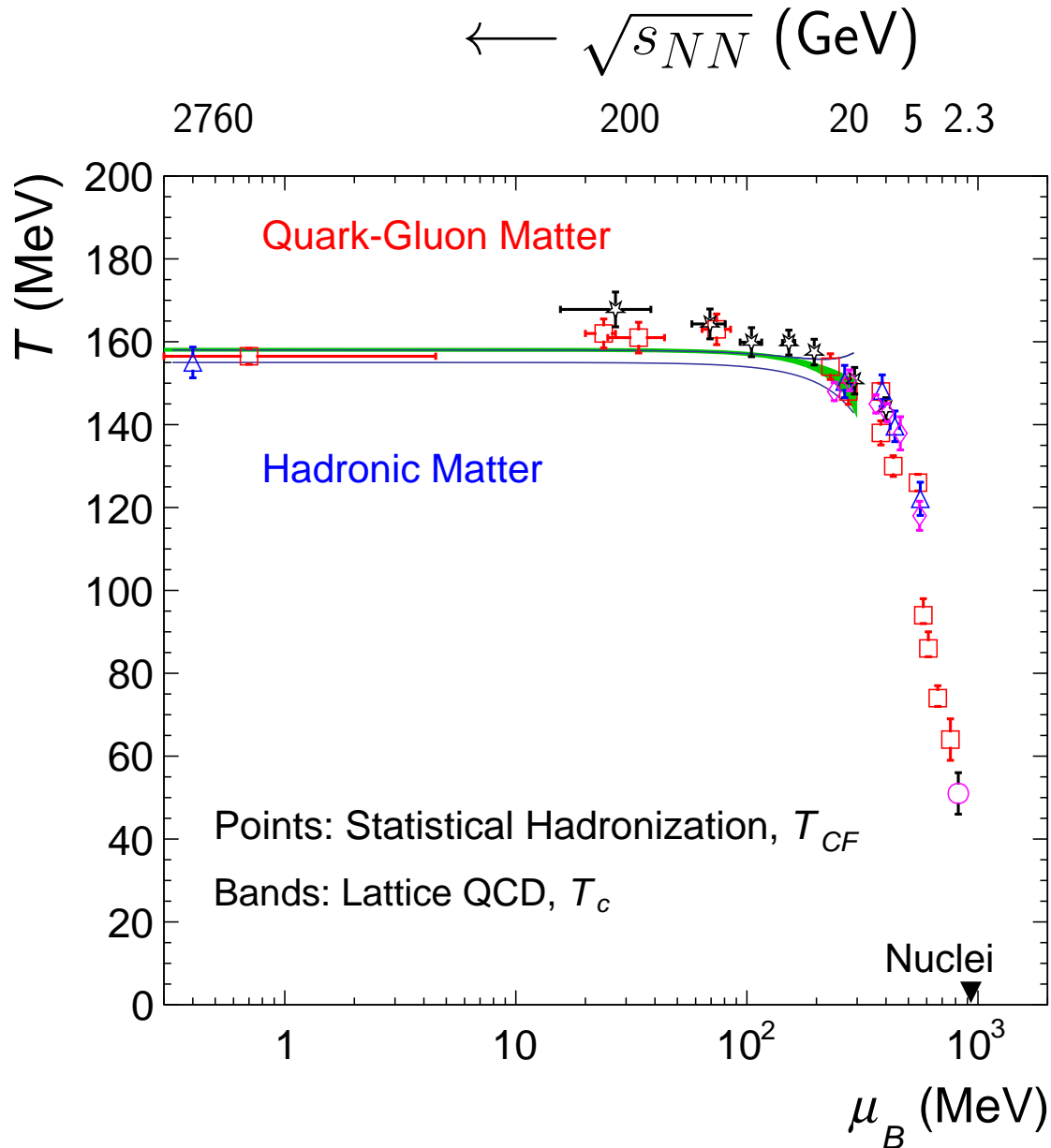
AA, PBM, K.Redlich, NPA 765 (2006) 211



The phase diagram of QCD

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at LHC, remarkable “coincidence” with Lattice QCD results

at LHC ($\mu_B \simeq 0$): purely-produced (anti)matter ($m = E/c^2$), as in the Early Universe

$\mu_B > 0$: more matter, from “remnants” of the colliding nuclei

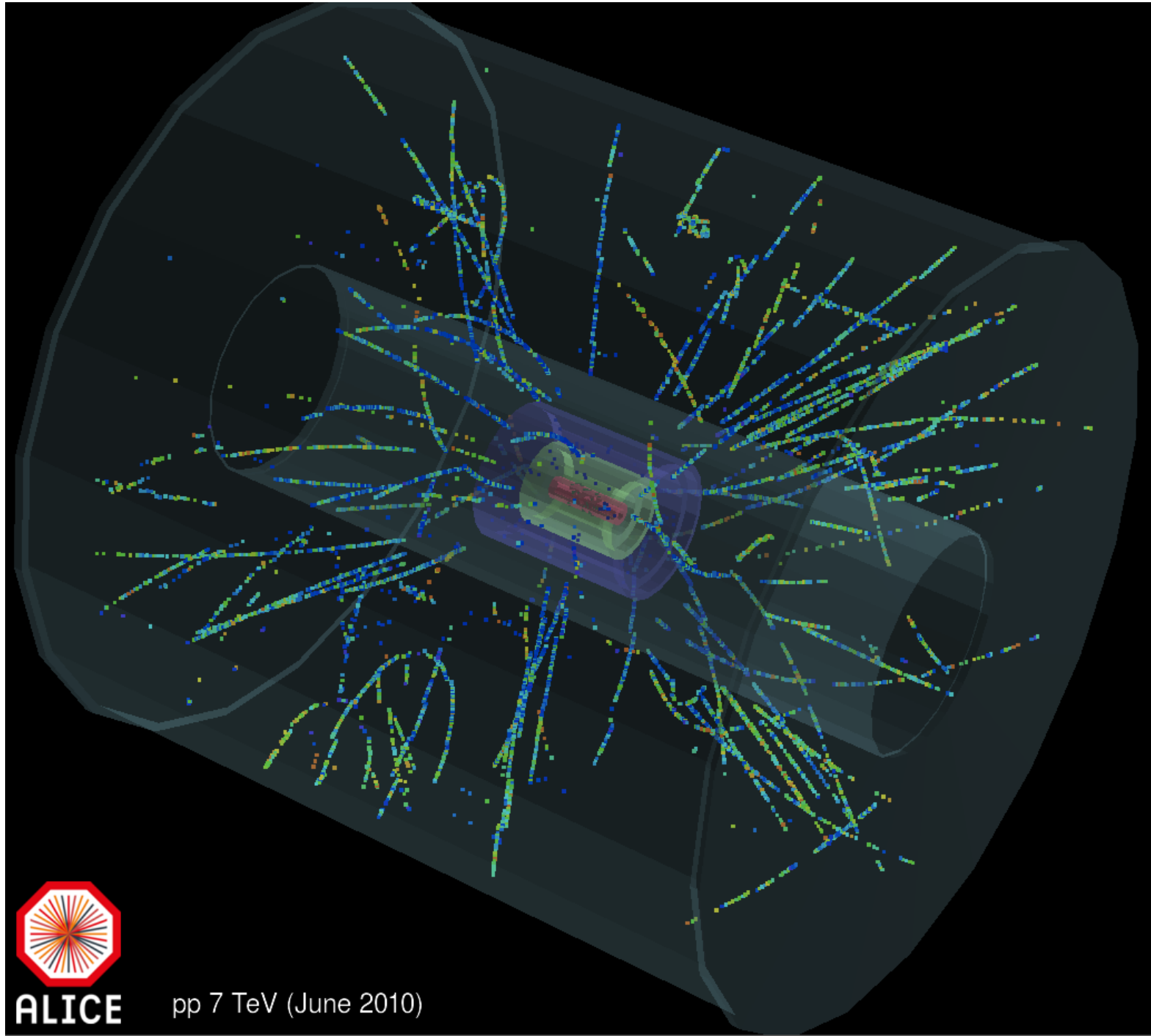
$\mu_B \gtrsim 400$ MeV: *the critical point awaiting discovery (at FAIR?)*

μ_B is a measure of the net-baryon density, or matter-antimatter asymmetry

The proton at low resolution ...in collisions at the LHC

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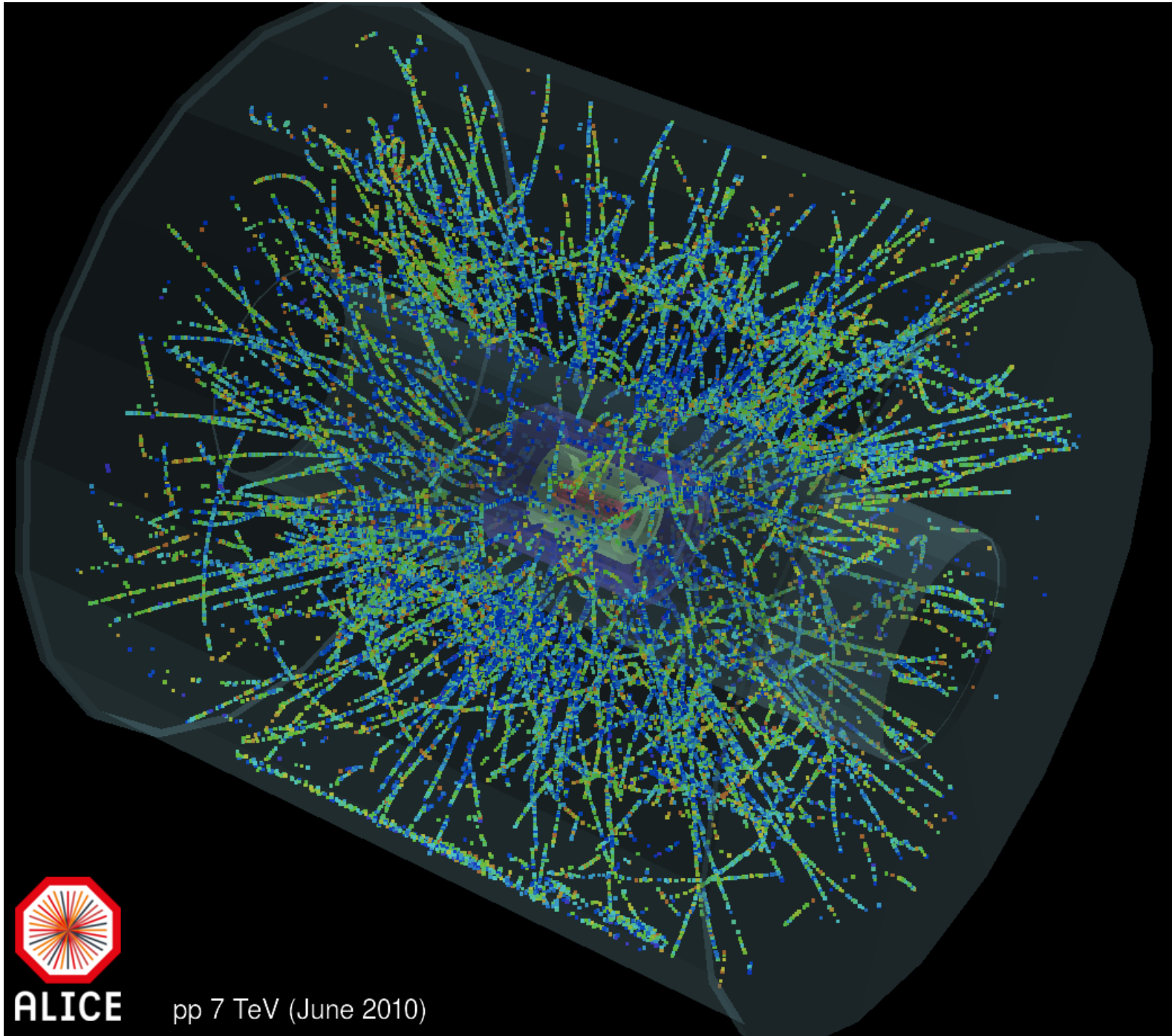
ALICE

pp 7 TeV (June 2010)

The proton at high resolution ...in collisions at the LHC

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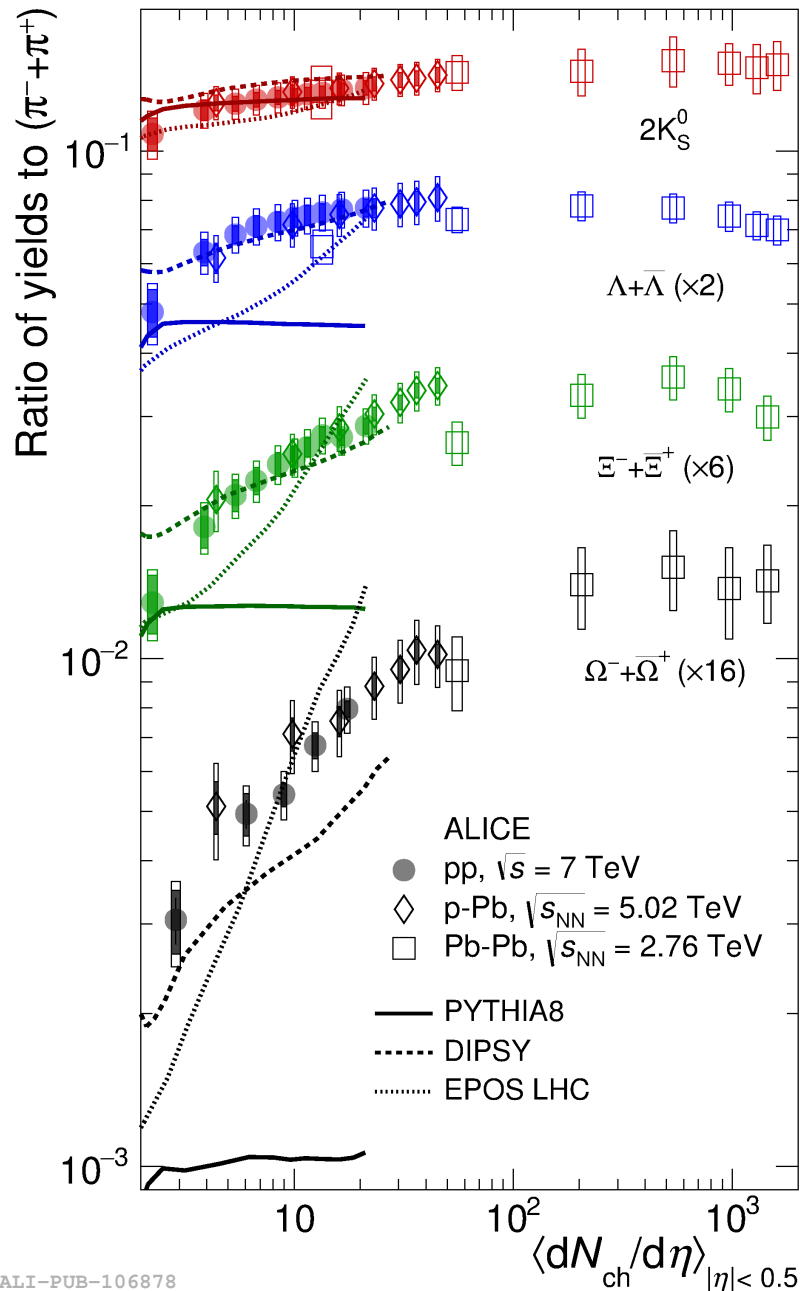
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Strangeness production - from small to large systems

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ALICE, [Nature Physics 13 \(2017\) 535](#)

(big geometric) fireball in Pb–Pb reached with violent pp and p–Pb collisions

canonical to grand-canonical strangeness production regime

Vislavicius, Kalweit, [arXiv:1610.03001](#)

is the same mechanism at work in small systems (at large multiplicities)?

string hadronization models do not describe data well

...new ideas are being put forward

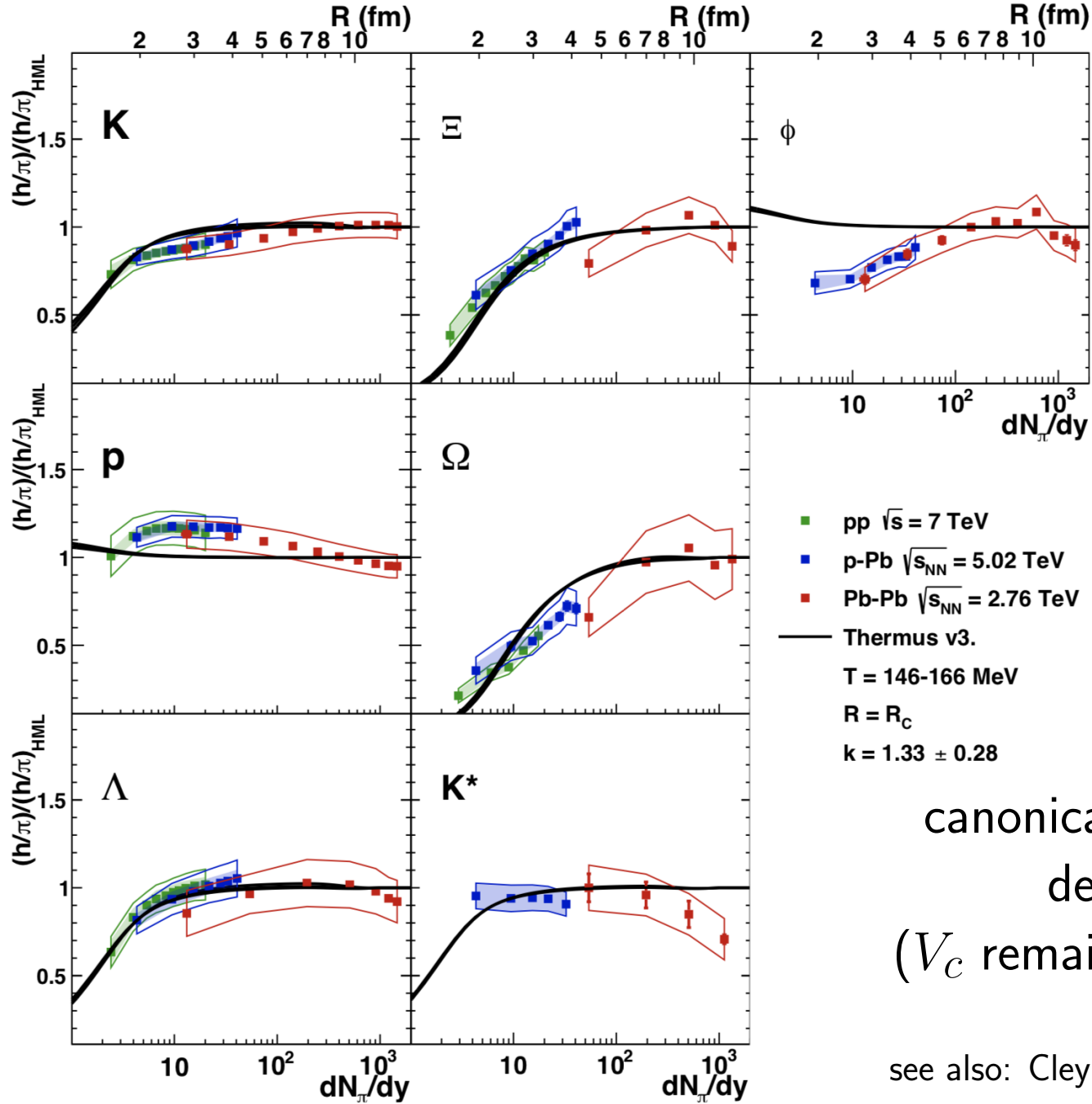
C.Bierlich et al, [arXiv:2205.11170](#)

(rope hadronization; no QGP)

Particle production - from small to large systems

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Vislavicius, Kalweit,
[arXiv:1610.03001](https://arxiv.org/abs/1610.03001)

ratios to high multiplicity
 limit (HML)

canonical statistical hadronization
 describes data well
 (V_C remains to be better constrained)

see also: Cleymans et al., [PRC 103 \(2021\) 014904](https://arxiv.org/abs/2008.08801)

Quark interlude

Up to now we only considered hadrons built with *up*, *down*, *strange* quarks ...these are light, masses from a few MeV (*u*, *d*) to ~ 90 MeV (*s*)

What about heavier ones?

...for instance *charm*, which weights about 1.2 GeV

Produced in pairs ($c\bar{c}$) in initial hard collisions, $t \sim 1/(2m_c) \leq 0.1$ fm/ c

Preserve their identity throughout the evolution of the fireball

...ideal messengers of the early stage (Lecture 1, Y.J. Lee)

- To what extent do charm (and beauty) quarks thermalize in QGP?
- Is charm-hadron formation at the QCD phase boundary a possibility?

SHM for charm (SHMc)

pQCD production, "throw in": $N_{c\bar{c}} = 13.8 \rightarrow g_c = 31.5; N_{J/\psi} \sim g_c^2$

LHC, central collisions, $\Delta y = 1$

assume:

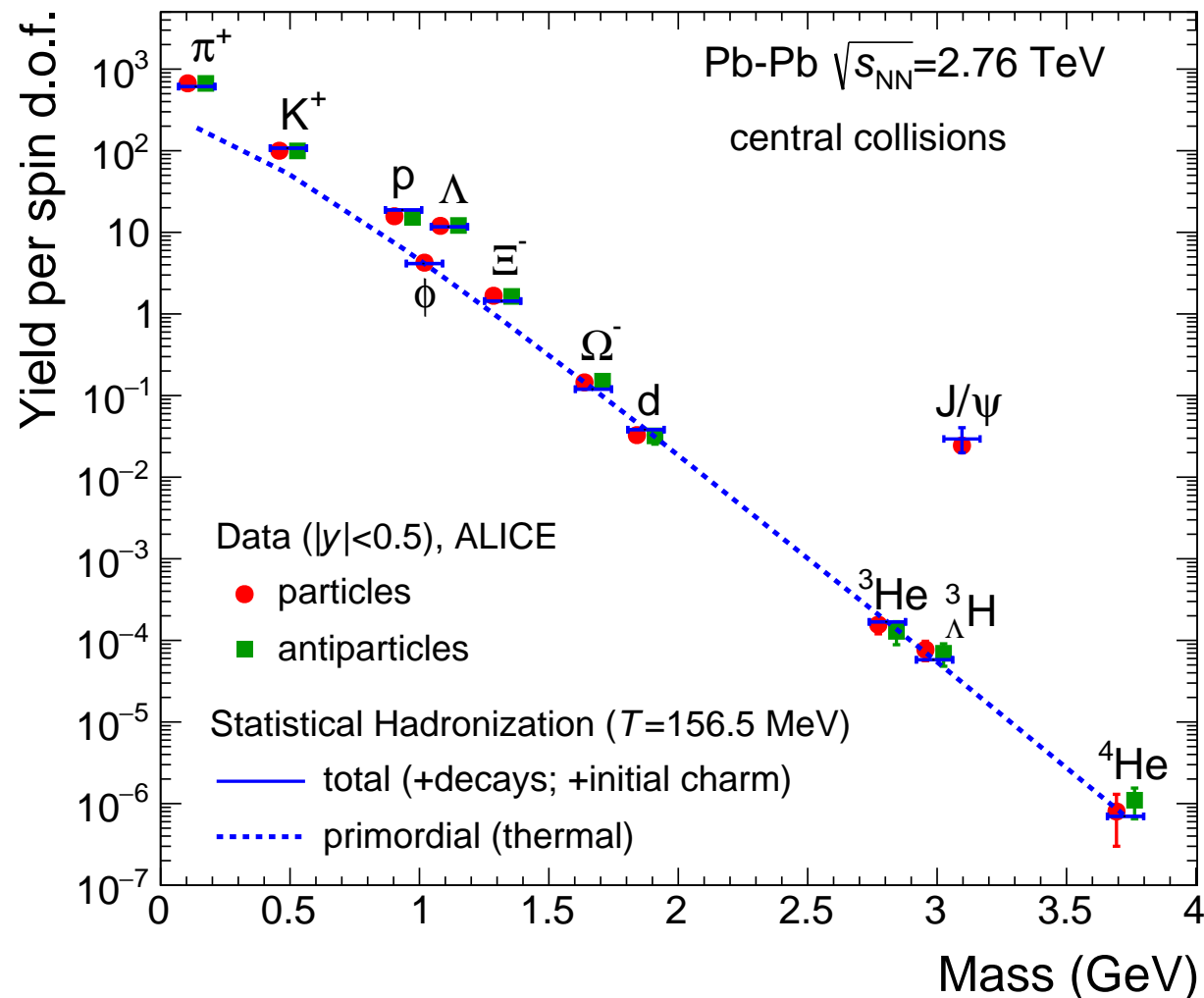
- full thermalization of c, \bar{c} ("mobility" in $V \simeq 4000 \text{ fm}^3$)

- full color screening (Matsui-Satz)

Braun-Munzinger, Stachel, [PLB 490 \(2000\) 196](#)

Model predicts all charm chemistry ($\psi(2S), X(3872)$)

π, K^\pm, K^0 from charm included in the thermal fit
(0.7%, 2.9%, 3.1% for $T=156.5 \text{ MeV}$)

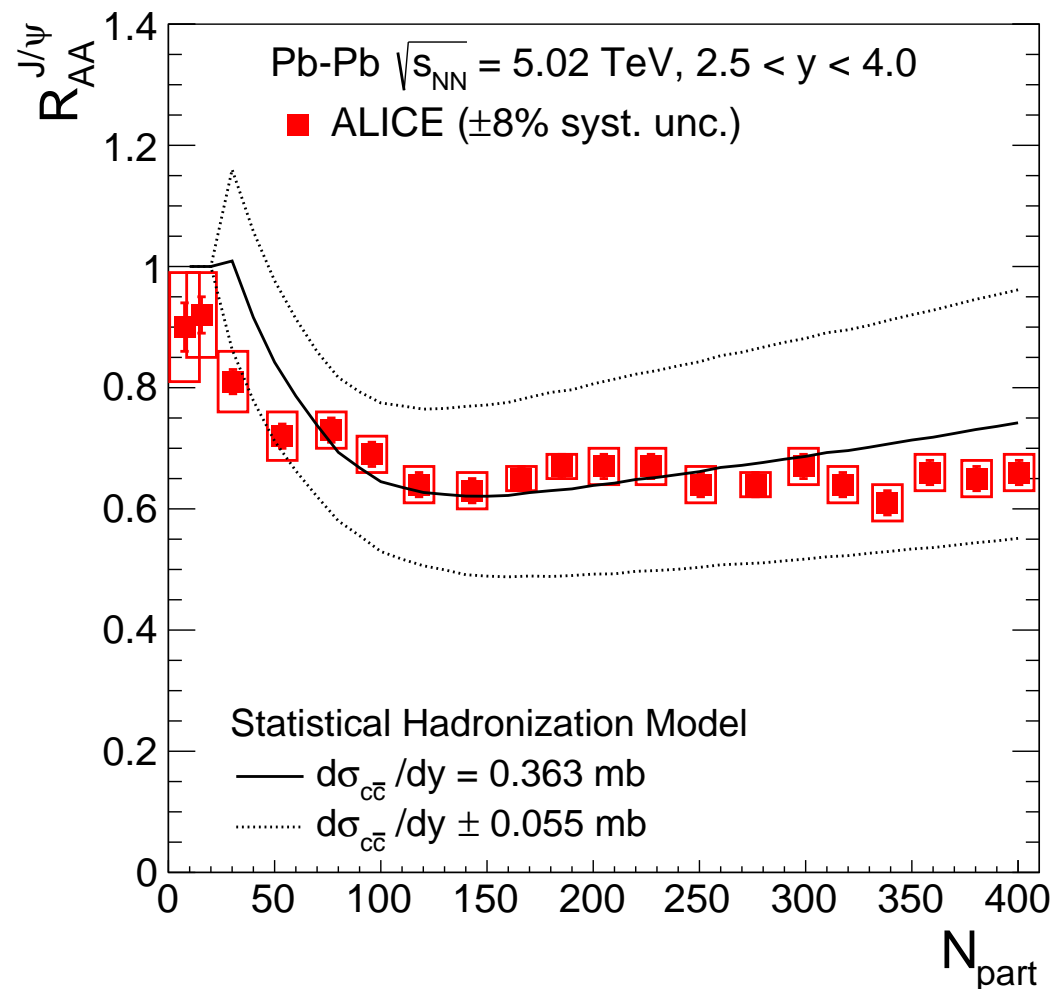
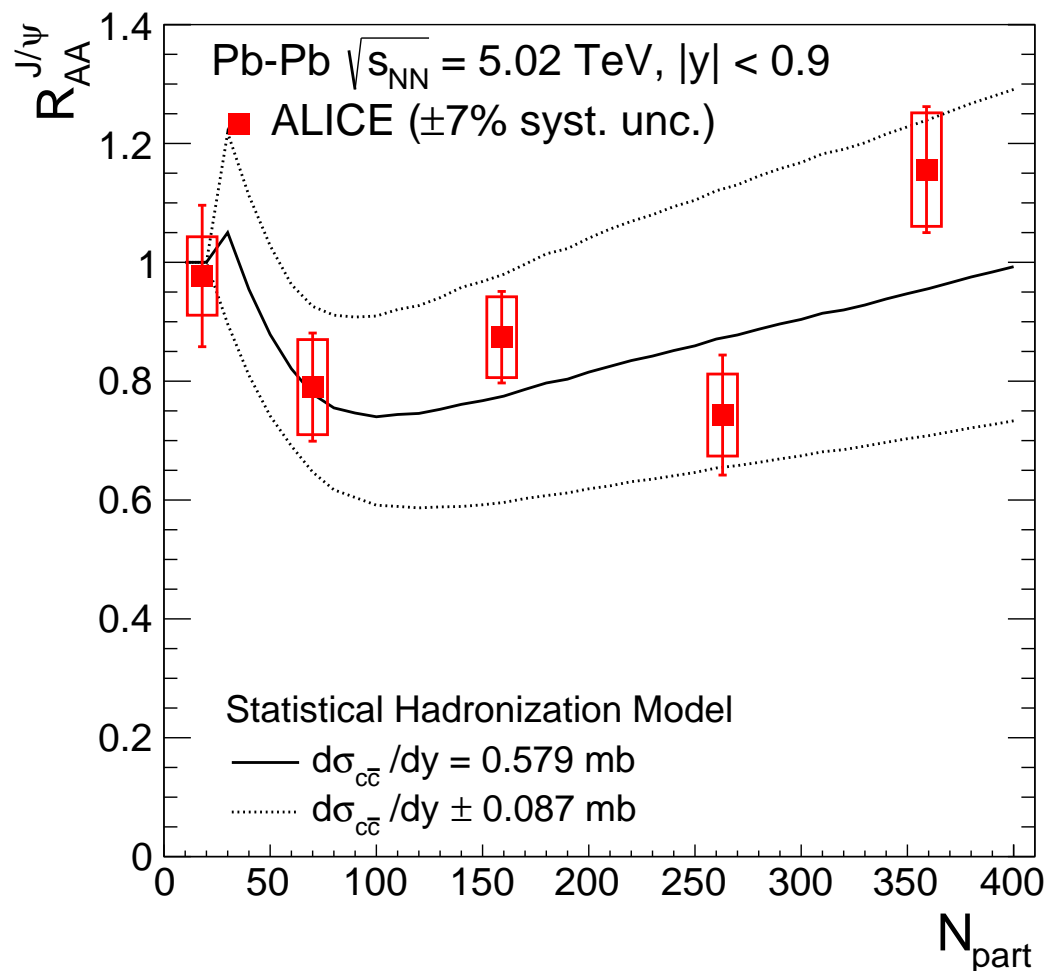


The Statistical Hadronization Model for charm (SHMc)

A. Andronic

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full thermalization of c quarks in QGP, hadronization at chemical freeze-out

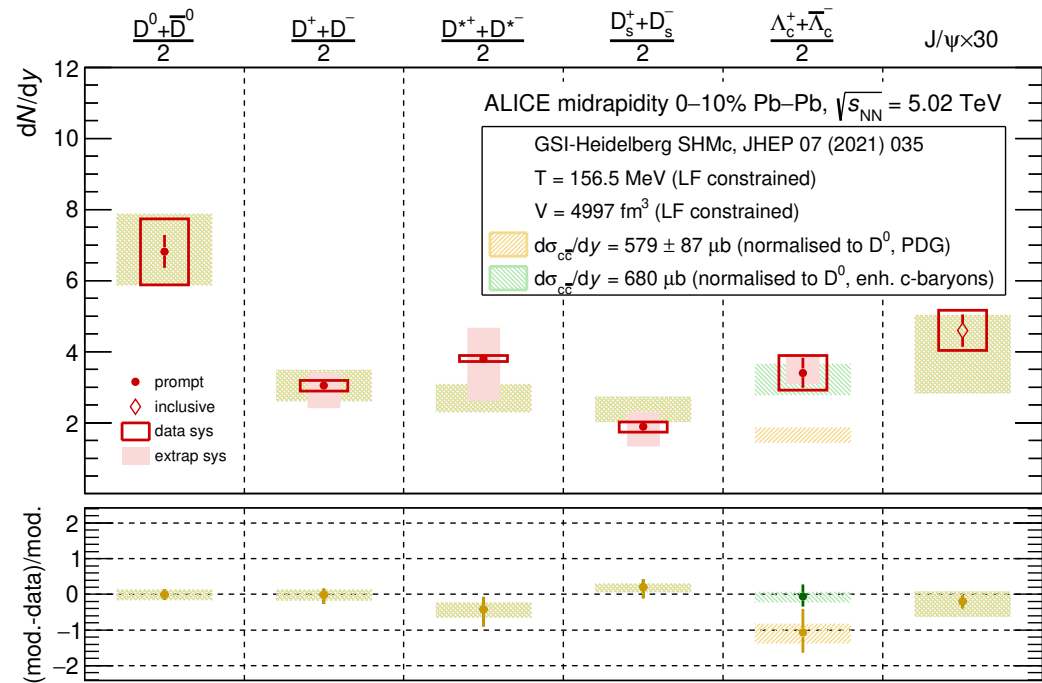
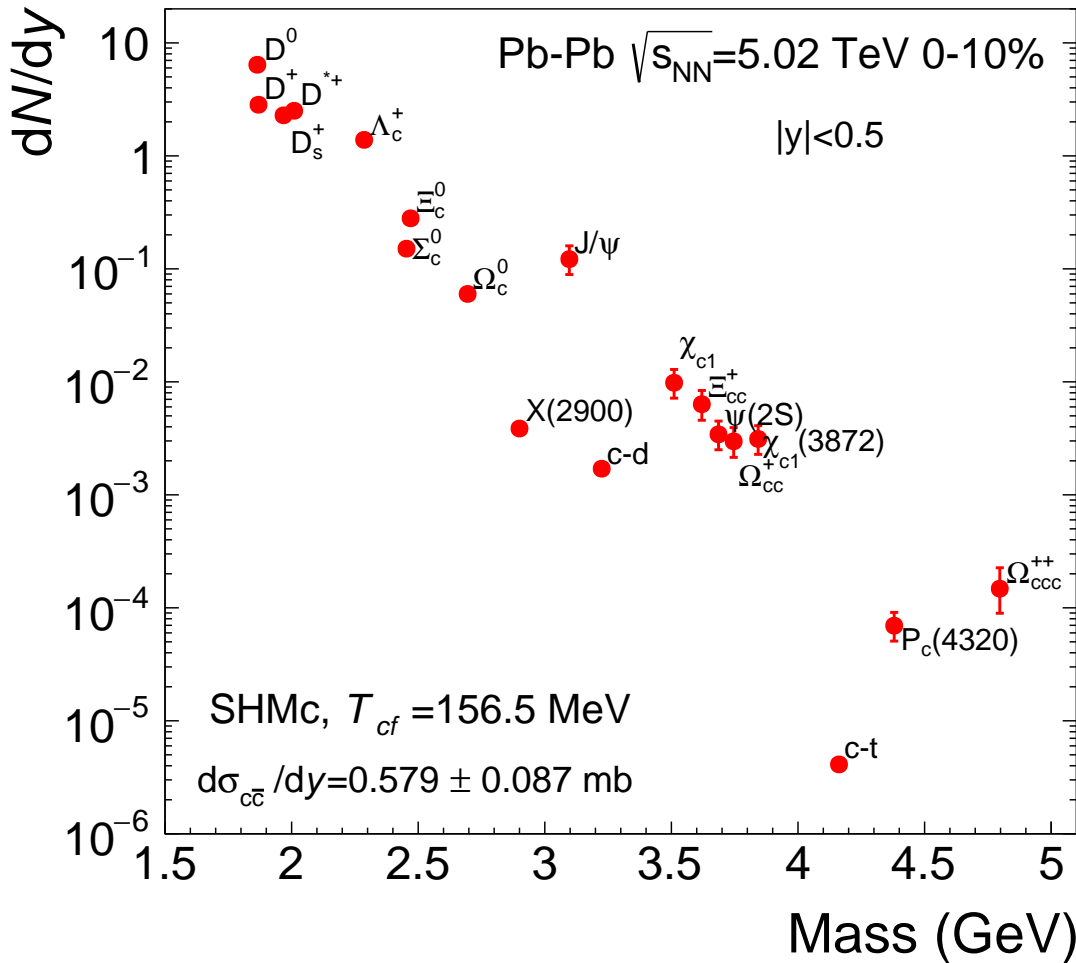


$d\sigma_{c\bar{c}}/dy$ via normalization to D^0 in Pb-Pb 0-10%, ALICE, [arXiv:2110.09420](https://arxiv.org/abs/2110.09420)

$dN/dy = 6.82 \pm 1.03$ (and assuming hadronization fractions in data as in SHMc)

SHMc: the charm zoo

The power of the model: predicting the full suite of charmed hadrons



QM'22 talk, L. Vermunt (ALICE)

AA et al, [JHEP 07 \(2021\) 035](#) (Updated slightly)

$$\frac{dN_{c\bar{c}}}{dy} = 13.8$$

- abundance of hadrons with light quarks consistent with chemical equilibration
- there is a variety of approaches ... *a personal bias: the “minimal model”*
a minimal set of parameters, means a well-constrained model
- the thermal model provides a simple way to access the QCD phase boundary
...at high energies (at low energies canonical suppression needs more care)
...but is it more than a 1st order description (of loosely-bound objects)?
...and what fundamental point does it make about hadronization?
(as a dynamical process, understanding still missing)
- more insights from higher moments (B , S conservation)
... HRG: baseline against which critical fluctuations may be discovered
- Even the much heavier charm quarks appear to thermalize in bulk in QGP
...leading to charm-hadron production (largely) at the QGP phase boundary

Supplementary material

A. Andronic

Matching HRG and Lattice QCD

A. Andronic

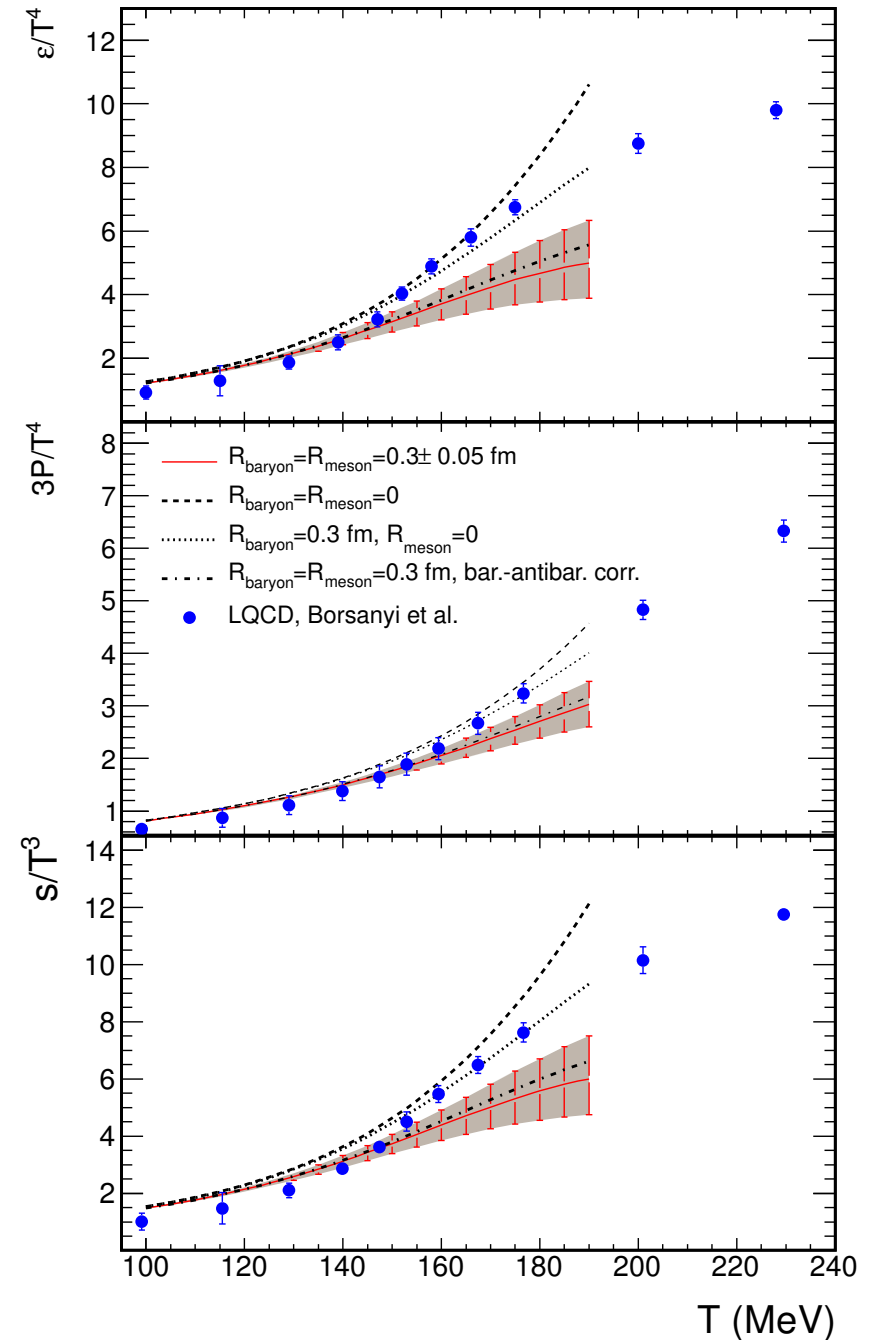
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a subject of quite some debate

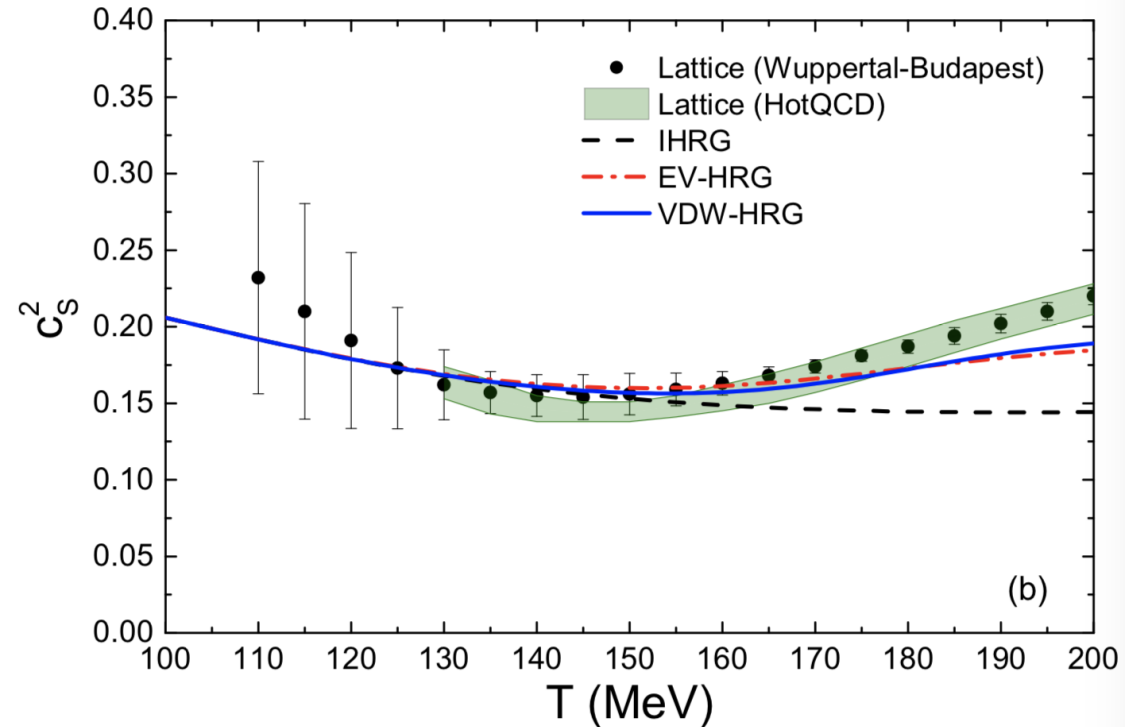
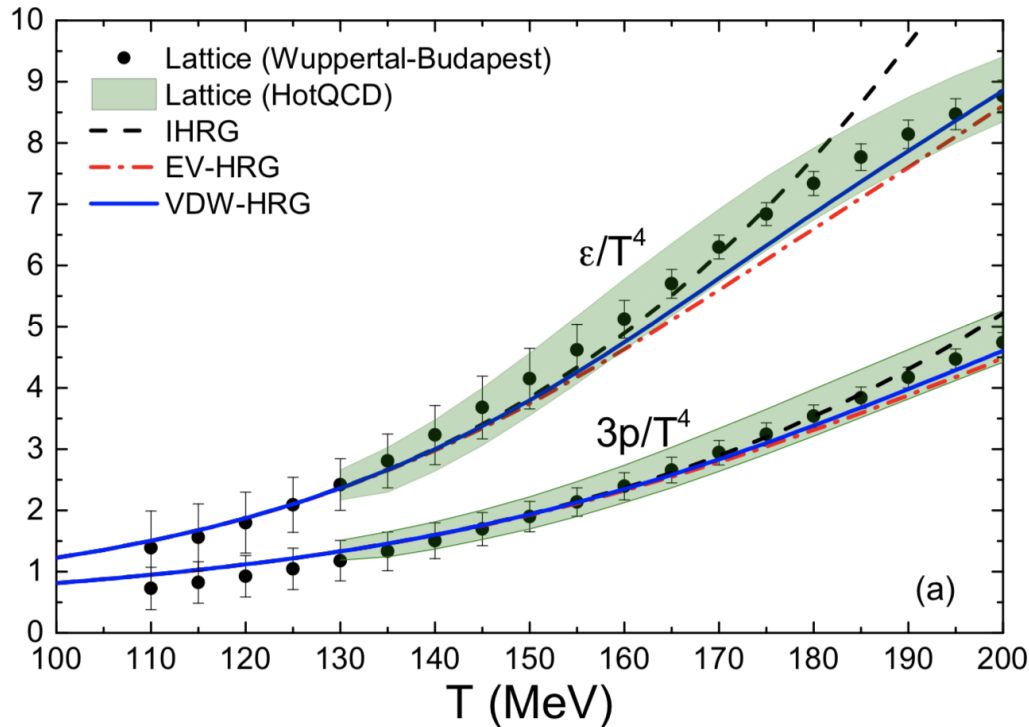
hadrons are not necessarily point-particles

an “excluded-volume” (hard sphere repulsion) is often employed

AA et al., [PLB 718 \(2012\) 80](#)



Matching HRG and LQCD - 2



Vovchenko, Gorenstein, Stöcker, [PRL 118 \(2017\) 182301](#)

$$c_s^2 = dP/d\varepsilon$$

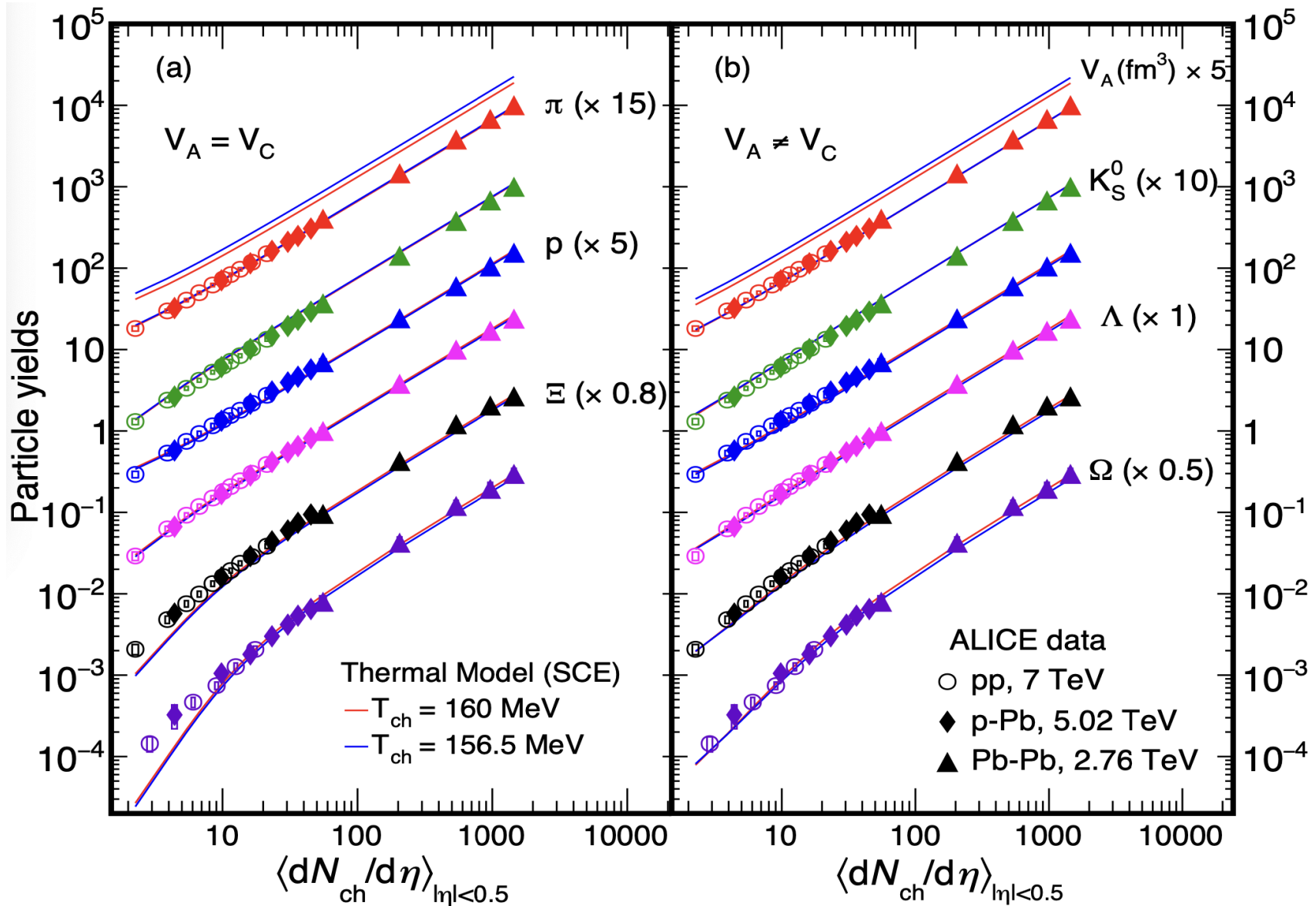
IHRG = ideal hadron resonance gas (model); EV-HRG = excluded-volume HRG

VDW = van der Waals, $a=329 \text{ MeVfm}^3$, $b=3.42 \text{ fm}^3$

critical point $T_c \simeq 19.7 \text{ MeV}$, $\mu_c \simeq 908 \text{ MeV}$ ($n_c \simeq 0.45n_0$)

describes LQCD χ_2 (susceptibilities) of conserved charges

SHM in pp, p-Pb, Pb-Pb collisions



SHM for charm: method and inputs

- Thermal model calculation (grand canonical) T, μ_B : $\rightarrow n_X^{th}$
- $N_{c\bar{c}}^{dir} = \frac{1}{2}g_c V (\sum_i n_{D_i}^{th} + n_{\Lambda_i}^{th}) + g_c^2 V (\sum_i n_{\psi_i}^{th} + n_{\chi_i}^{th})$
- $N_{c\bar{c}} \ll 1 \rightarrow$ Canonical (J.Cleymans, K.Redlich, E.Suhonen, Z. Phys. C51 (1991) 137):

$$N_{c\bar{c}}^{dir} = \frac{1}{2}g_c N_{oc}^{th} \frac{I_1(g_c N_{oc}^{th})}{I_0(g_c N_{oc}^{th})} + g_c^2 N_{c\bar{c}}^{th} \rightarrow g_c \text{ (charm fugacity)}$$

$$\text{Outcome: } N_D = g_c V n_D^{th} I_1/I_0 \quad N_{J/\psi} = g_c^2 V n_{J/\psi}^{th}$$

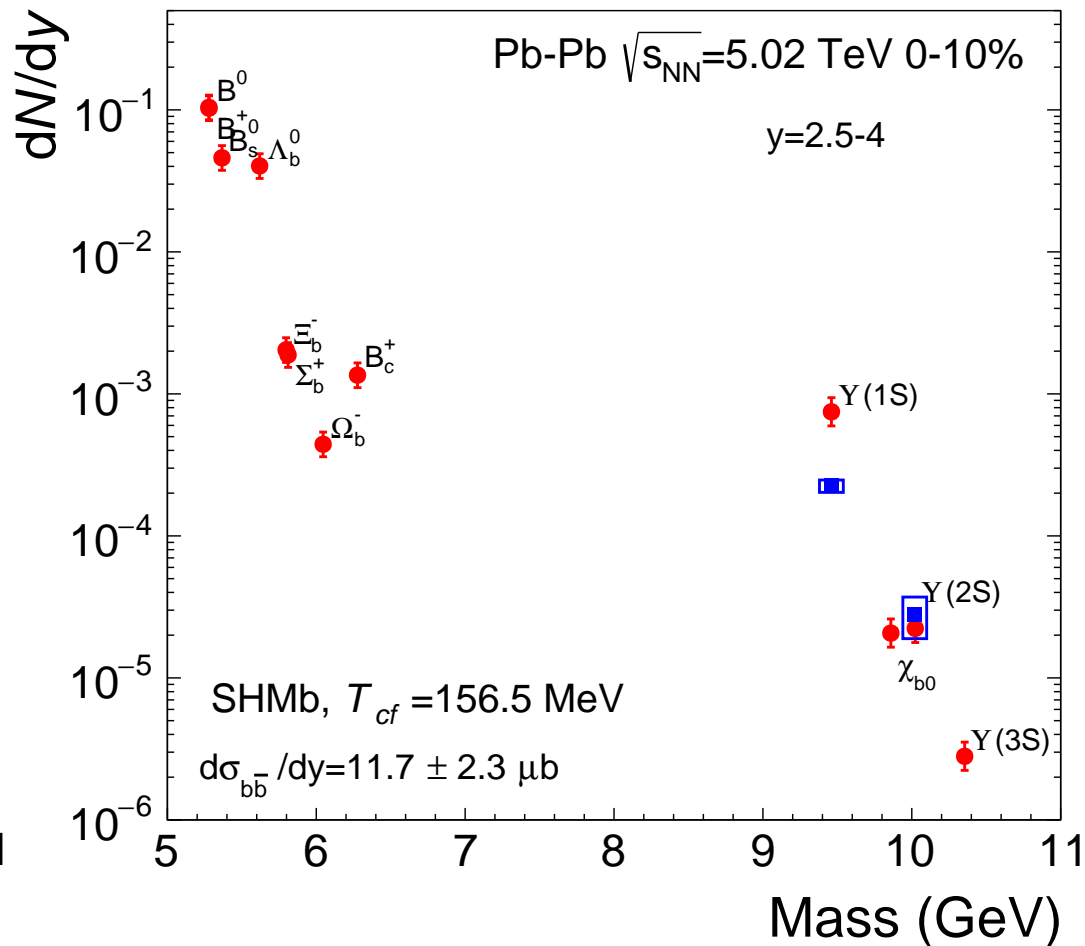
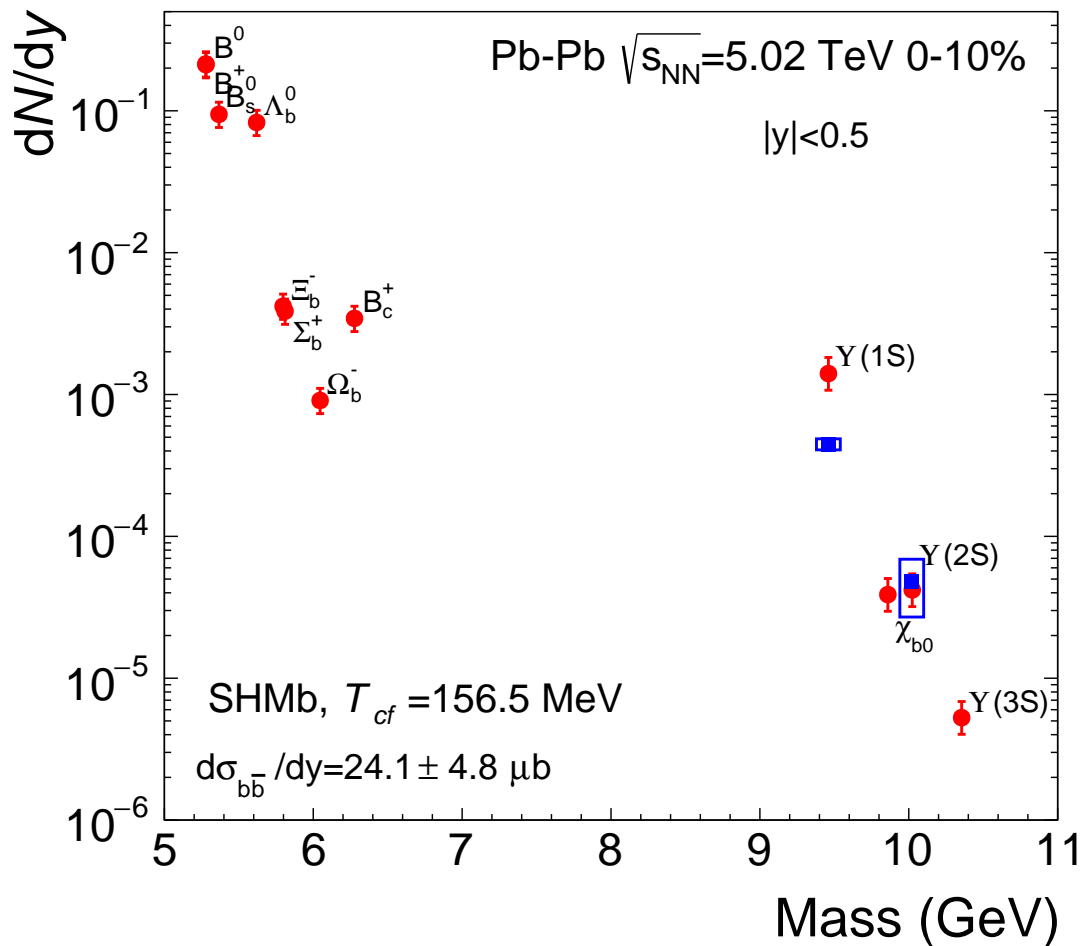
$$\text{Inputs: } T, \mu_B, \quad V_{\Delta y=1} (= (dN_{ch}^{exp}/dy)/n_{ch}^{th}), \quad N_{c\bar{c}}^{dir} \text{ (exp. or pQCD)}$$

Assumed minimal volume for QGP: $V_{QGP}^{min} = 200 \text{ fm}^3$

SHMb: the beauty zoo (0-10%)

A. Andronic

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$$g_b = 1.05 \cdot 10^9$$

$$B_c : 3.44 \cdot 10^{-3}$$

$$g_b = 0.86 \cdot 10^9$$

$$B_c : 1.36 \cdot 10^{-3}$$

Blue: Υ data (CMS, ALICE): calc. based on R_{AA} and pp (would be nice to include in publications dN/dy)