Tolerances for vibration of quadrupoles

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Vibration of quadrupoles

The vertical displacement of a beam caused by a quadrupole vibrating with an amplitude Δy_q and an angular frequency ω_q at the vertical phase advance ϕ_q from the IP:

$$\begin{aligned}\Delta y^* &= \sum_n \sqrt{\beta^* \beta_q} \exp(-nT_0/\tau_y + i\omega_q n T_0) \sin(\phi_q + n\mu_y) k_q \Delta y_q \\ &= \sum_n \sqrt{\beta^* \beta_q} \exp(-n\alpha_y + in\mu_q) \sin(\phi_q + n\mu_y) k_q \Delta y_q \,,\end{aligned} \tag{1}$$

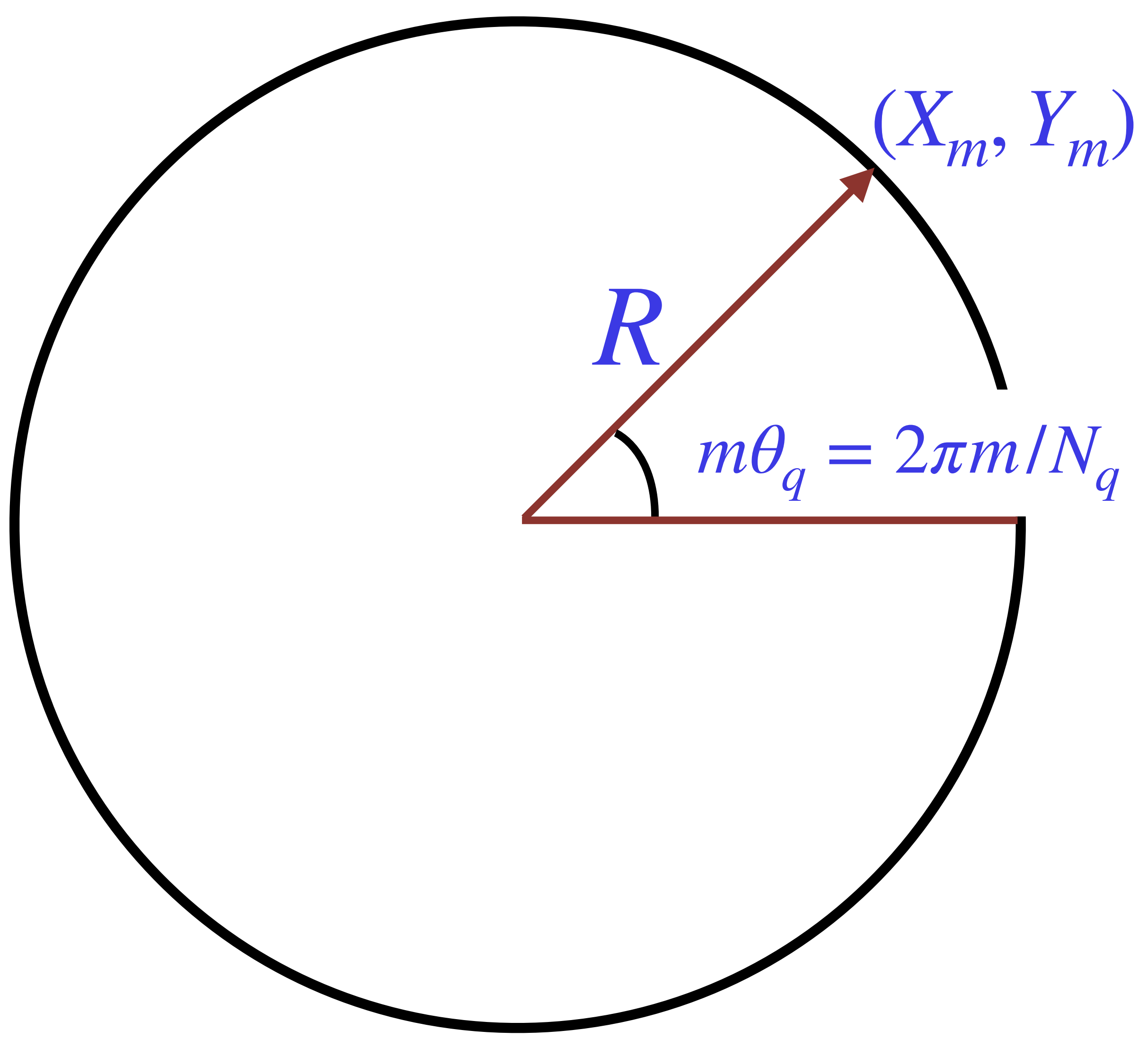
where μ_y, T_0 & τ_y, are the vertical betatron angular tune, the revolution & damping times, and $\alpha_y \equiv T_0/\tau_y$, $\mu_q \equiv \omega_q T_0$. β^*, β_q, k_q are the beta functions at the IP and the quadrupole, and the focusing strength of the quadrupole.

1.1 Vibration due to seismic motion

The vibration amplitude Δy_q can be random to each quad, or coherent due to external seismic motion. First let us evaluate the coherent part by assuming that the quads are distributed over the ring uniformly with the betatron phase $\phi_q = m\Delta\phi_q$, and also physically located over a ring of the radius R with a constant separation azimuthal angle θ_q, *i.e.*,

$$X_m + iY_m = R \exp(im\theta_q) \,, \tag{2}$$

where m runs over 1 through N_q, the number of quads per ring.



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Then if the quads follow the seismic wave in the ground, the displacement Δy_m of the m-th quadrupole is written as

$$\Delta y_m = u \exp\left(i(k_X X_m + k_Y Y_m - \omega_q t)\right) , \tag{3}$$

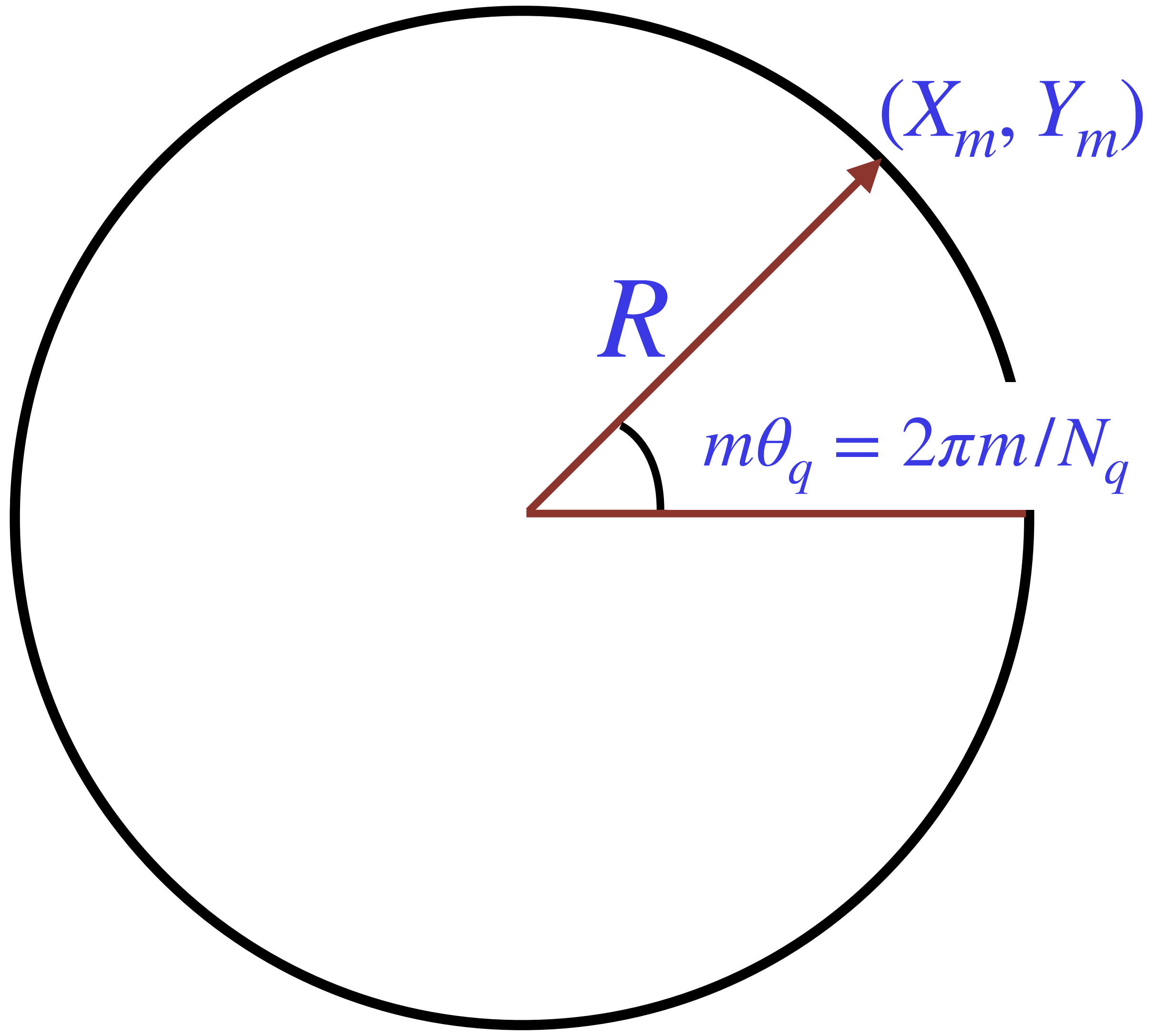
where $k_{X,Y}$ are the components of the seismic wave number vector, and u represents the amplitude. Here we just set $k_X = k$ and $k_Y = 0$ for simplicity without losing generality if the ring is nearly a circle. So we may sum up the term $\sin(\phi_q + n\mu_y)\Delta y_q$ in Eq. (1) over quads as

$$\begin{aligned} d_s &= \sum_m^{N_q} \sin(\phi_q + n\mu_y)\Delta y_m \\ &= \sum_m^{N_q} \sin(m\Delta\phi_q + n\mu_y) u \exp\left(i(kR\cos m\theta_q - \omega_q t)\right) \\ &= u \sum_{\ell=-\infty}^{\infty} \sum_m^{N_q} \sin(m\Delta\phi_q + n\mu_y) J_\ell(kR) i^\ell \exp(i\ell m\theta_q - i\omega_q t) , \end{aligned} \tag{4}$$

where we have applied $\exp(ix\cos z) = \sum_\ell i^l J_\ell(x) \exp i\ell z$. Although there may be a resonance in Eq. (4) at $\ell \sim \pm\Delta\phi_q/\theta_q$, the index ℓ becomes too large in the case of FCC-ee Z, where $\Delta\phi_q = 60\,\text{deg}$, $\theta_q = 360/1450 \sim 0.248\,\text{deg}$, and $\ell \sim 242$. As for N_q we have taken only QD's into account. Thus the coefficient J_ℓ becomes infinitesimal for such a large ℓ, so the resonant effect is negligible.

$$\exp(i(\boldsymbol{k}\cdot\boldsymbol{x} - \omega_q t))$$

$$\boldsymbol{k} = (k_X, k_Y) = (k, 0)$$



The term $\ell = 0$ in Eq. (4) gives

$$d_{s0} = uJ_0(kR)\frac{\sin(\mu_y/2)\sin(n\mu_y + (\mu_y - \Delta\phi_q)/2)}{\sin(\Delta\phi_q/2)}\,. \tag{5}$$

We know $J_0(x) \leq 1$, and the rests of the rhs of Eq. (5) are not far from 1. Then magnitude of the coherent component looks smaller than the random component:

$$|d_s| \ll \sqrt{N_q}u\,. \tag{6}$$

1.2 Resonance with the betatron frequency

$$= \sum_n \sqrt{\beta^*\beta_q}\exp(-n\alpha_y + in\mu_q)\sin(\phi_q + n\mu_y)k_q\Delta y_q\,,$$

Then the expectation value of the vibration of the beam at the IP, $\langle|\Delta y^*|^2\rangle$ is obtained by averaging Eq. ,(1) over ϕ_q as

$$\begin{aligned}\langle|\Delta y^*|^2\rangle &= \frac{1}{2\pi}\int|\Delta y^*|^2 d\phi_q \\ &= \frac{\beta^*\beta_q k_q^2\langle\Delta y_q^2\rangle}{4}\frac{\exp(\alpha)(\cosh\alpha - \cos\mu_q\cos\mu_y)}{(\cosh\alpha - \cos(\mu_q-\mu_y))(\cosh\alpha - \cos(\mu_q+\mu_y))}\,.\end{aligned} \tag{7}$$

Thus the vibration at the IP has resonances at $\mu_q = \pm\mu_y + 2m\pi$ with an integer m.

At each resonance, by assuming the spectrum of $\langle \Delta y_q^2 \rangle$ is uniform, the vibration at the IP can be evaluated as:

$$\langle |\Delta y^*|^2 \rangle = \frac{\beta^* \beta_q k_q^2}{8\alpha T_0} \sum_m S((\pm\mu_y \pm 2m\pi)/T_0) \,, \tag{8}$$

where $S(\omega)$ is the power spectrum density of $\langle \Delta y_q^2(\omega) \rangle$, and we have assumed $\cos\mu_q \cos\mu_y \sim 1/2$ and $\alpha \ll 1$.

A measurement of ground vibration tells that[1],

$$S(\omega) = \sigma\omega^{-4} \sim 10^{-15} \left(\frac{\omega}{2\pi \mathrm{Hz}}\right)^{-4} \mathrm{m}^2/\mathrm{Hz} \,, \tag{9}$$

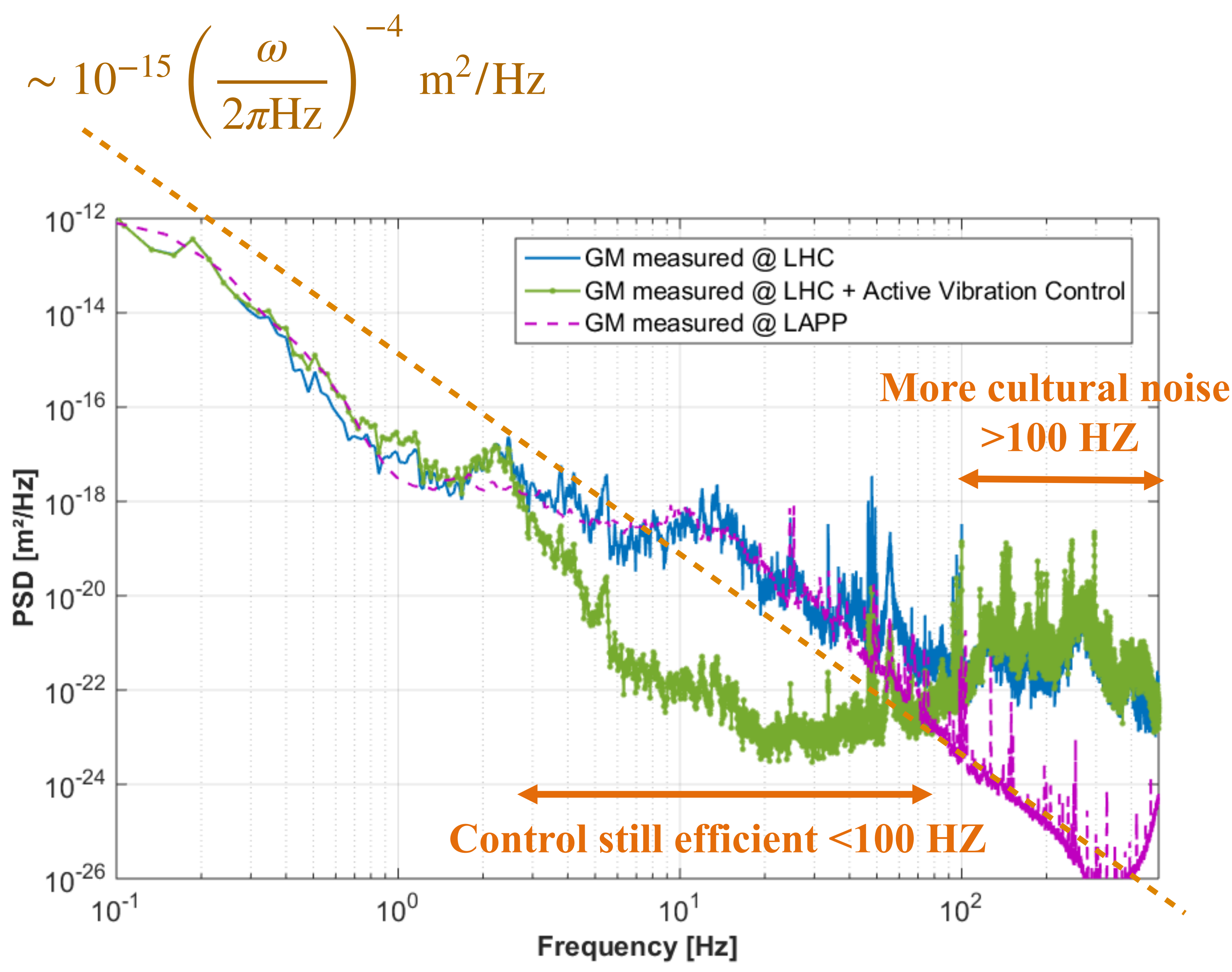
with a coefficient σ, then among the resonances only the lowest one $m \sim \mu_y/2\pi$ will matter. In the case of FCC-ee, it is at

$$\omega/2\pi = \omega_r/2\pi \sim (1.2, 1.8)\,\mathrm{kHz} \,, \tag{10}$$

corresponding to $[\mu_y/2\pi] \sim (0.4, 0.6)$, resulting in

$$S(\omega_r) \sim (4.8, 0.95) \times 10^{-28}\,\mathrm{m}^2/\mathrm{Hz} \,. \tag{11}$$

[1] https://indico.cern.ch/event/694811/contributions/2863859/attachments/1595533/2526938/2018_02_06_FCCee_MDI_workshop_Serluca.pdf



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If we plugin numbers at FCC-ee Z:

$$\begin{aligned} \beta^* &= 0.8\,\text{mm}, & \beta &= 60\,\text{m}, \\ k_q &= 0.037\,1/\text{m}, & T_0 &= 300\,\mu\text{s}, \\ \alpha &= 4\times 10^{-4}\,\text{s}, \end{aligned} \tag{12}$$

into Eq, (8) and multiply the number of quadrupoles in the arc N_q=1450, we get

$$\sqrt{\Delta y^{*2}} \sim 7.8\,\text{pm}\,, \tag{13}$$

which is far smaller than the IP vertical beam size.

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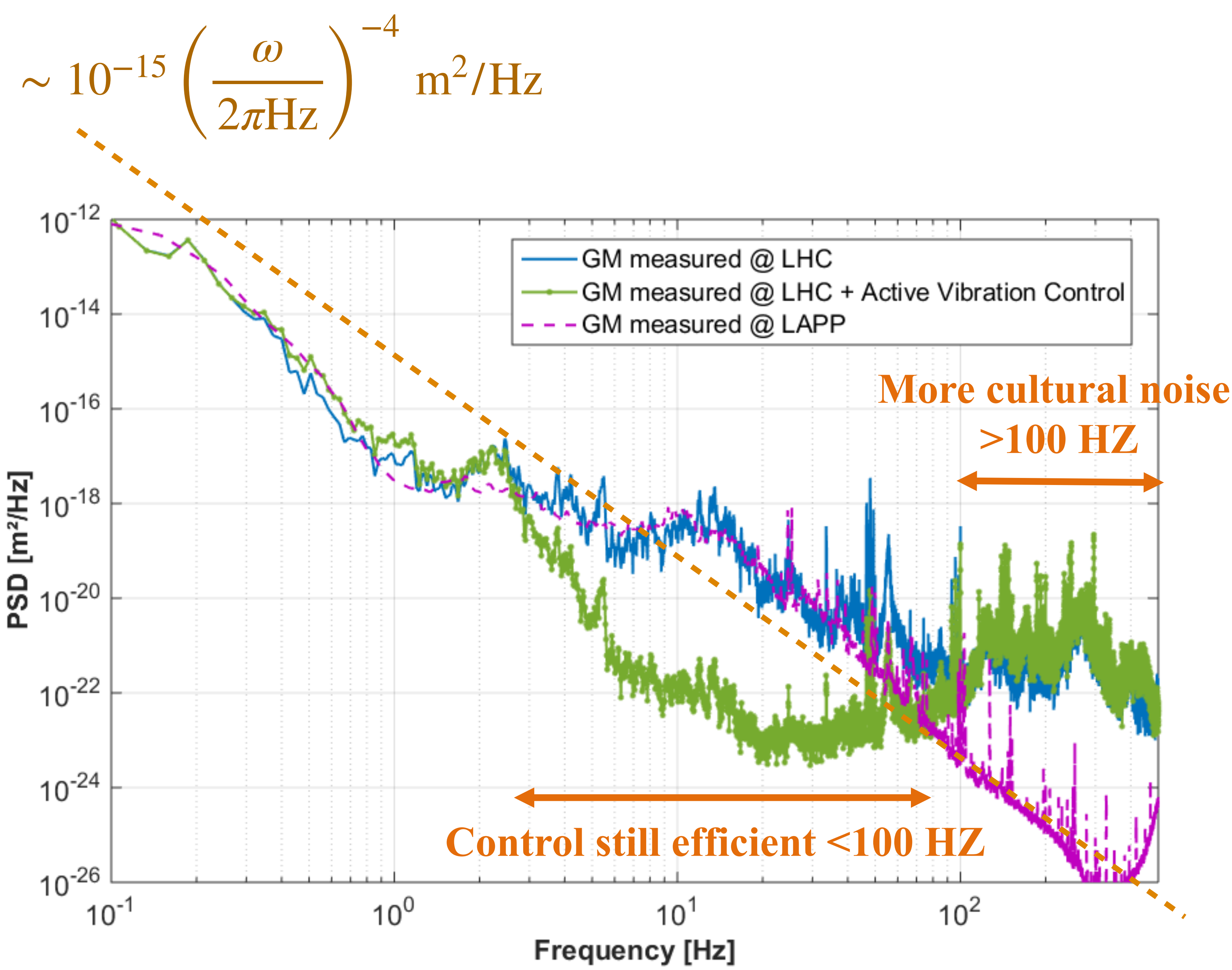

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1.3 Non-resonant vibration

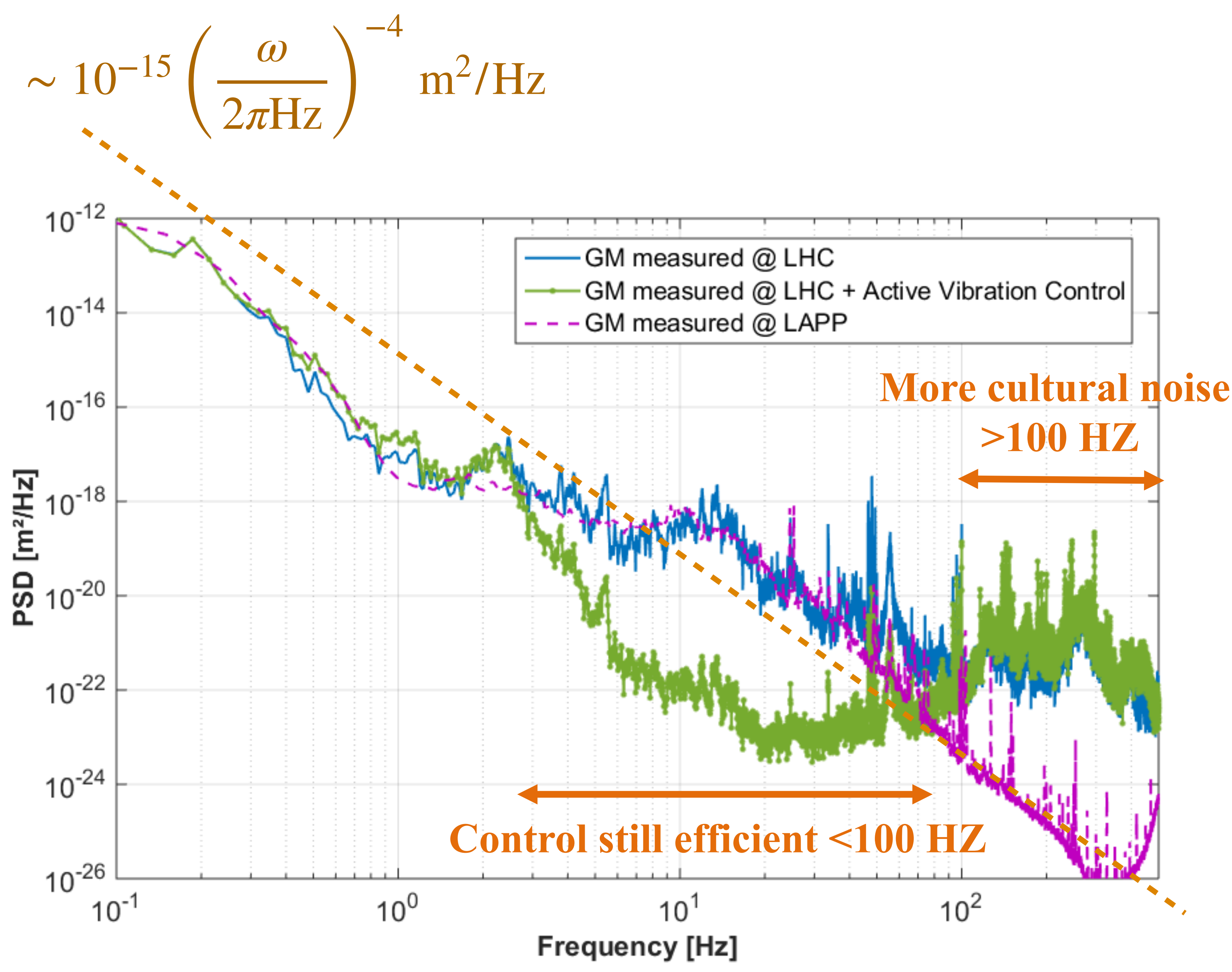
Next let us look at the off-resonant contribution of Eq. (7), If we roughly approximtate the tune-dependent term by 1, the integrated power spectrum in a range $\omega \geq \omega_c$ is given as

$$\sqrt{\Delta y^{*2}} = \frac{N_q \beta^* \beta_q k_q^2}{4} \int_{\omega_c}^{\infty} S(\omega) \frac{d\omega}{2\pi} = \frac{N_q \beta^* \beta_q k_q^2 \sigma}{24\pi\omega_c^3} . \tag{14}$$

In the case for the previous measurement, we estimate $\sigma \sim 1.6 \times 10^{-12}\,\mathrm{m}^2/\mathrm{Hz}$, then

$$\sqrt{\Delta y^{*2}} \sim 2.8\,\mathrm{nm} \tag{15}$$

for $\omega_c = 2\pi \times 1\,\mathrm{Hz}$. The assumption here is that below the critical frequency ω_c, an orbit feedback suppresses the beam oscillation perfectly.



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Summary

Tolerances for the vibration of quadrupoles are evaluated for three cases:

- A seismic wave has smaller effects than random motion of each quadrupole for an equal amplitude.
- Resonance with the betatron frequencY: weak, as the betatron frequency is in the range of kHz.
- Non-resonant, incoherent vibration of each quad produces ~3 nm vertical motion at the IP for ≧1 Hz.
  - Assuming each quad follows the ground motion measured at LHC & LAPP.
  - No amplification of the mechanical motion of the girders has been assumed.
  - Below 1 Hz, a vertical orbit feedback is required.