Running masses for heavy quarks in DIS

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Plan

- Talk based on results on . . .
  - . . . improved theory predictions for heavy-quark structure functions
    N.A. Lo Presti, H. Kawamura, S. M. and A. Vogt `arXiv:1008.0951`
  - . . . data analysis for heavy-quark production in DIS
    S. Alekhin and S. M. `arXiv:0811.1412`
  - . . . precise parton distribution functions from global fits
    S. Alekhin, J. Blümlein, S. Klein and S. M. `arXiv:0908.2766`
    S. Alekhin, J. Blümlein and S. M. `arXiv:1007.3657`
  - . . . the running charm-quark mass
    S. Alekhin and S. M. `arXiv:1011.5790`

Heavy-quark masses

QCD Lagrangian

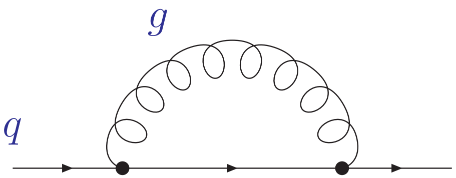
$$\mathcal{L} \quad = \quad -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{\text{flavors}} \bar{q} \, (\mathrm{i} \not{D} - m_q) \, q$$

- Covariant derivative $D_\mu = \partial_\mu + \mathrm{i} g_s A_\mu$
- Formal parameters of the theory (no observables)
  - strong coupling $\alpha_s = g_s^2/(4\pi)$
  - quark masses m_q
- Quantum corrections (loop integrals) require UV renormalization; (scheme dependence):
  - $\alpha_s \rightarrow$ asymptotic freedom, running coupling ($\overline{MS}$ scheme)
  - $m_q \rightarrow$ pole mass or running mass ($\overline{MS}$ scheme)

Pole mass

- Based on (unphysical) concept of top-quark being a free parton

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{p^2=m_q^2}$$



  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$

Running quark masses

- $\overline{MS}$ mass definition $m(\mu_R)$ realizes running mass (scale dependence)
  - renormalization group equation (mass anomalous dimension γ)

$$\left(\mu_R^2 \frac{\partial}{\partial \mu_R^2} + \beta(\alpha_s)\frac{\partial}{\partial \alpha_s}\right) m(\mu_R) \; = \; \gamma(\alpha_s) m(\mu_R)$$

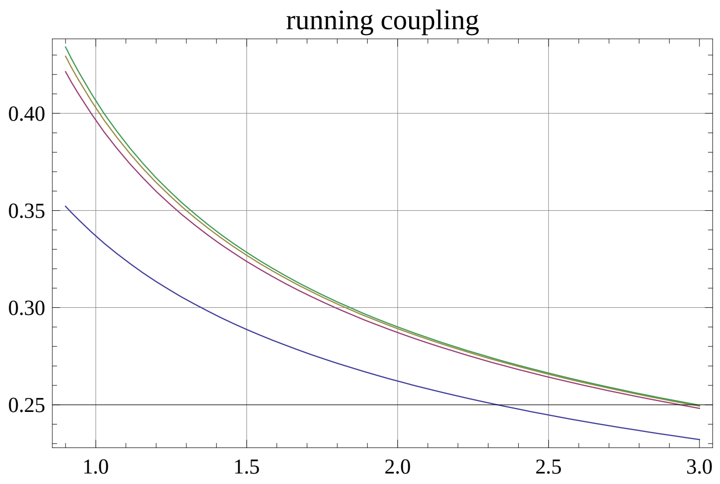
  - short distance mass probes at scale of hard scattering
    $m_{\rm pole} = m_{\rm short\ distance} + \delta m$
  - conversion between pole mass and $\overline{MS}$ mass definition in perturbation theory: $m = m(\mu_R)\left(1 + a_s(\mu_R) d^{(1)} + a_s(\mu_R)^2 d^{(2)}\right)$

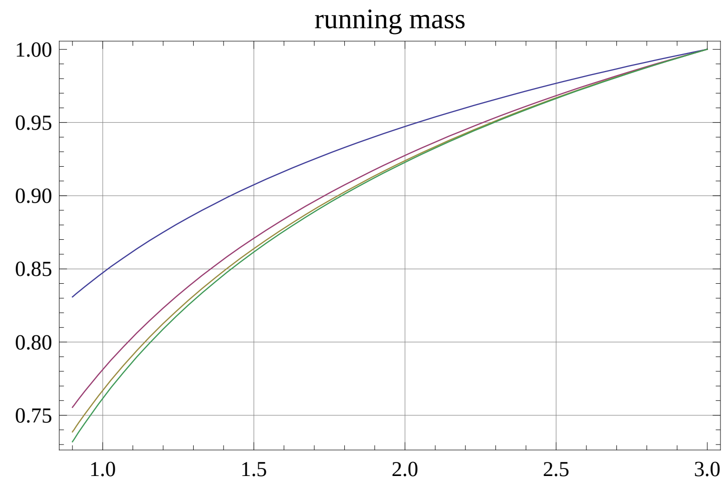
Scale dependence

- Renormalization group equation for scale dependence
  - strong coupling α_s and mass m

$$\mu^2 \frac{d}{d\mu^2} \alpha_s(\mu) \;=\; \beta(\alpha_s) \qquad\qquad \mu^2 \frac{d}{d\mu^2} m(\mu) \;=\; \gamma(\alpha_s) m(\mu)$$

- Perturbative expansion known to four loops
  - β-function van Ritbergen, Vermaseren, Larin '97 and mass anomalous dimension γ Chetyrkin '97; Larin, van Ritbergen, Vermaseren '97
  - very good convergence of perturbative series even at low scales
- Plot at low scales $\mu = 1.0 \dots 3.0$ GeV
  α_s (left) and mass ratio $m(3\text{GeV})/m(\mu)$ (right)





- Use of charm-quark mass $m_c(m_c)$ is well justified

Quark masses in PDF fits

- Choice of value for heavy-quark masses part of uncertainty
- PDF fits assume pole mass scheme for heavy-quarks
  - numerical values systematically lower than those from PDG (2-loop conversion to pole mass)

	m_c [GeV]	m_b [GeV]
ABKM	$1.5^{+0.25}_{-0.25}$	$4.5^{+0.5}_{-0.5}$
MSTW	1.4	4.75
CTEQ6.6	1.3	4.5
GJR	1.3	4.2
PDG	$1.66^{+0.09}_{-0.15}$	$4.79^{+0.19}_{-0.08}$

PDG

- PDG quotes running masses:
  charm: $m_c(m_c) = 1.27^{+0.07}_{-0.11}$ GeV, bottom: $m_b(m_b) = 4.20^{+0.17}_{-0.07}$ GeV

Illustration for top-quark mass

ILC

- Pole mass measurements are strongly order-dependent
  - e.g. threshold scan of cross section in e^+e^- collision
    Beneke, Signer, Smirnov ‘99;
    Hoang, Teubner ‘99;
    Melnikov, Yelkhovsky ‘98;
    Penin, Pivovarov ‘99;
    Yakovlev ‘99
  - LO (dotted), NLO (dashed), NNLO (solid)

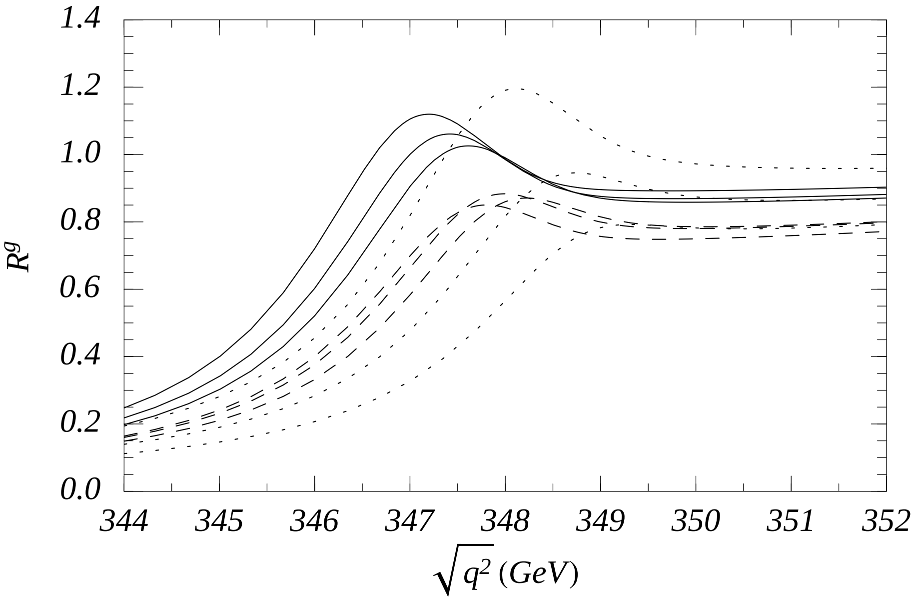
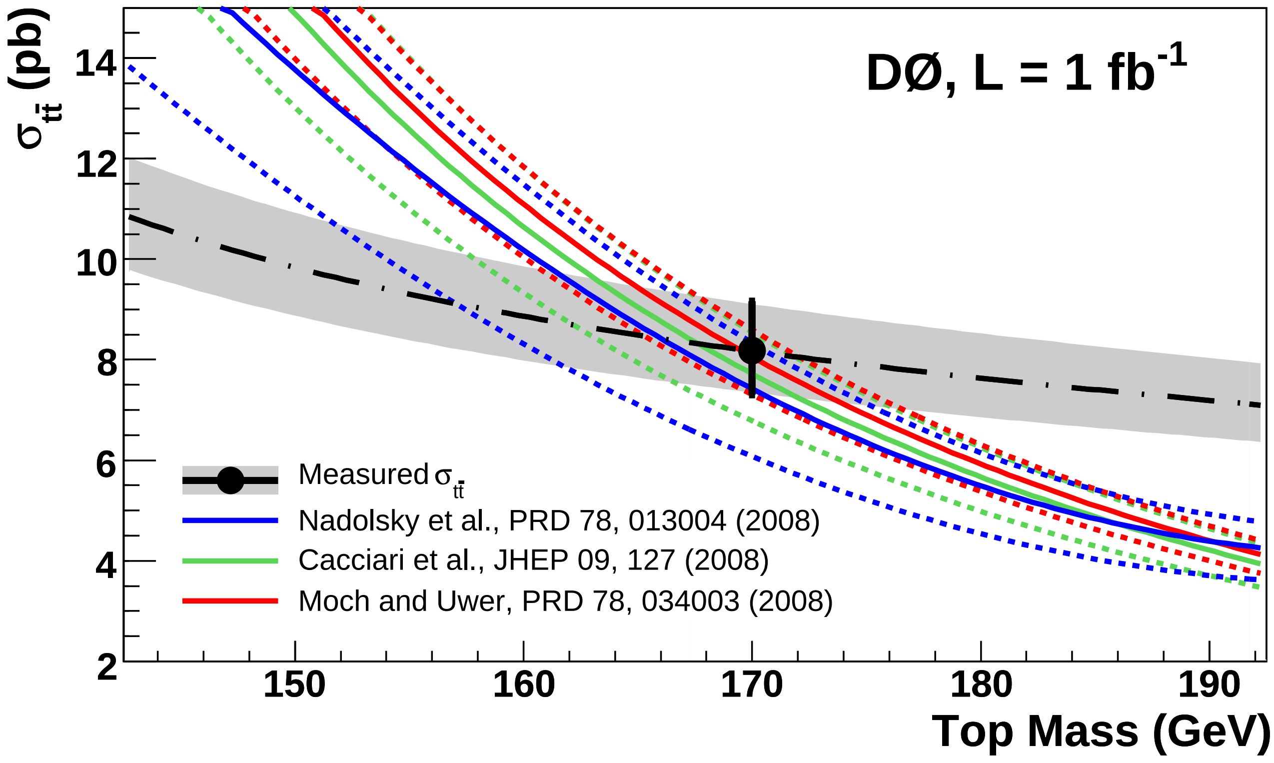


Illustration for top-quark mass

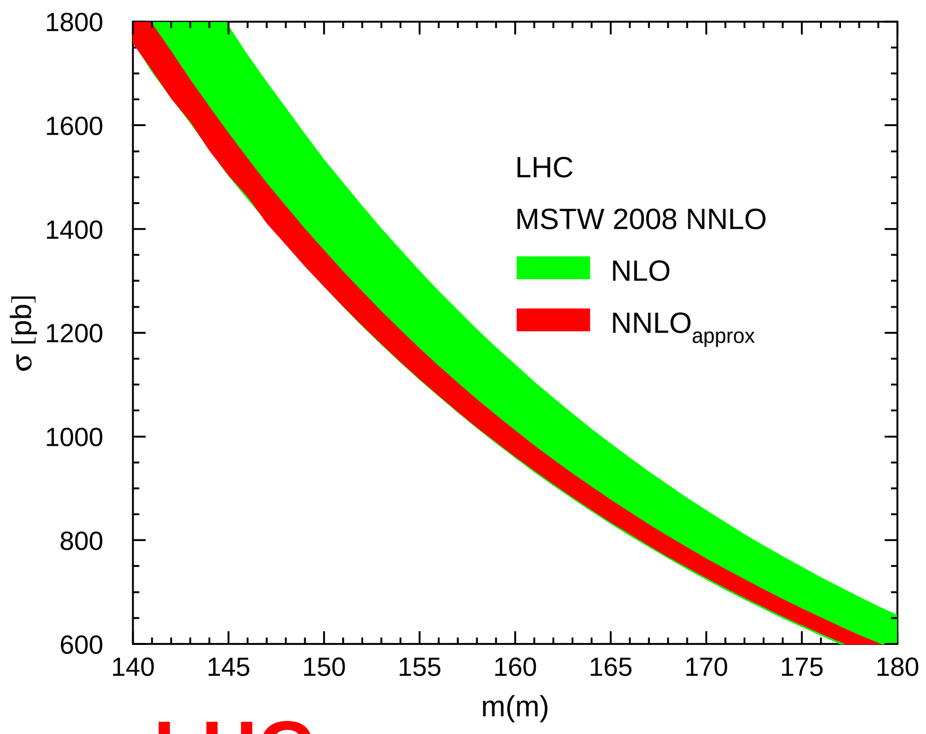
Tevatron

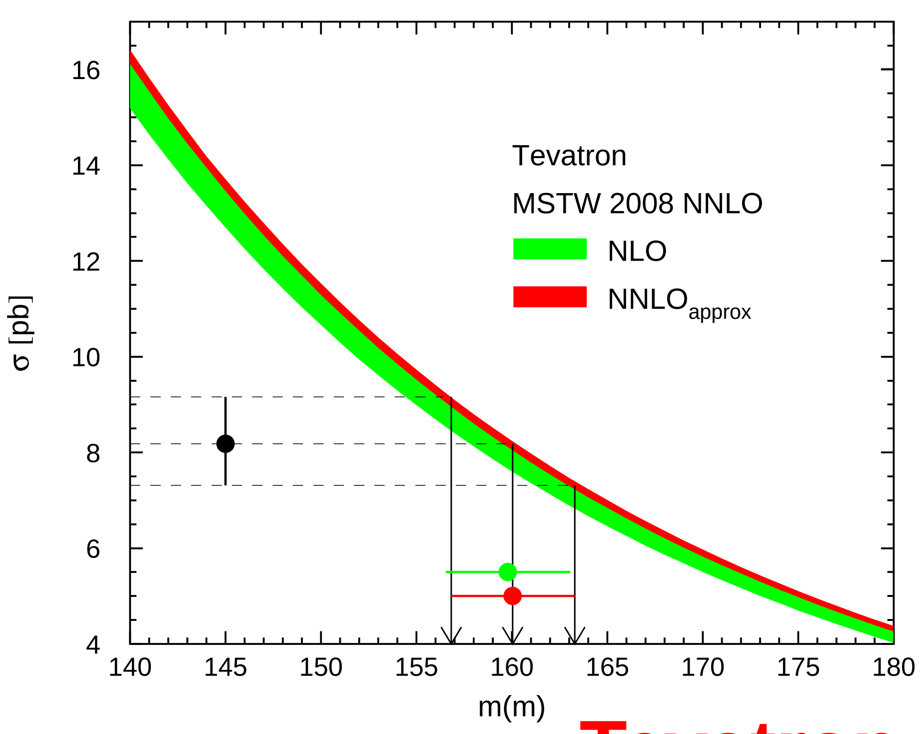
- Total cross section and different channels of Tevatron analyses (theory uncertainty band from scale variation)
- Determination of m_t from total cross section (slope $d\sigma/dm_t$)
  - e.g. DZero '09: NLO $m_t = 165.5^{+6.1}_{-5.9}$; NNLO $m_t = 169.1^{+5.9}_{-5.2}$; ...



Top quark's $\overline{MS}$ mass dependence

- Total top-quark cross section as function of $\overline{m}$
  Langenfeld, S.M., Uwer '09
  - theoretical uncertainity (band) due to variation of $\mu_R \in [\overline{m}/2, 2\overline{m}]$ for fixed set $\mu_F \in \overline{m}/2, \overline{m}, 2\overline{m}$





LHC **Tevatron**

Heavy-quark production in DIS

NLO QCD corrections

- NLO for charm structure function
  - neutral current Laenen, Riemersma, Smith, van Neerven '93
  - charged current Gottschalk '81; Glück, Kretzer, Reya '96

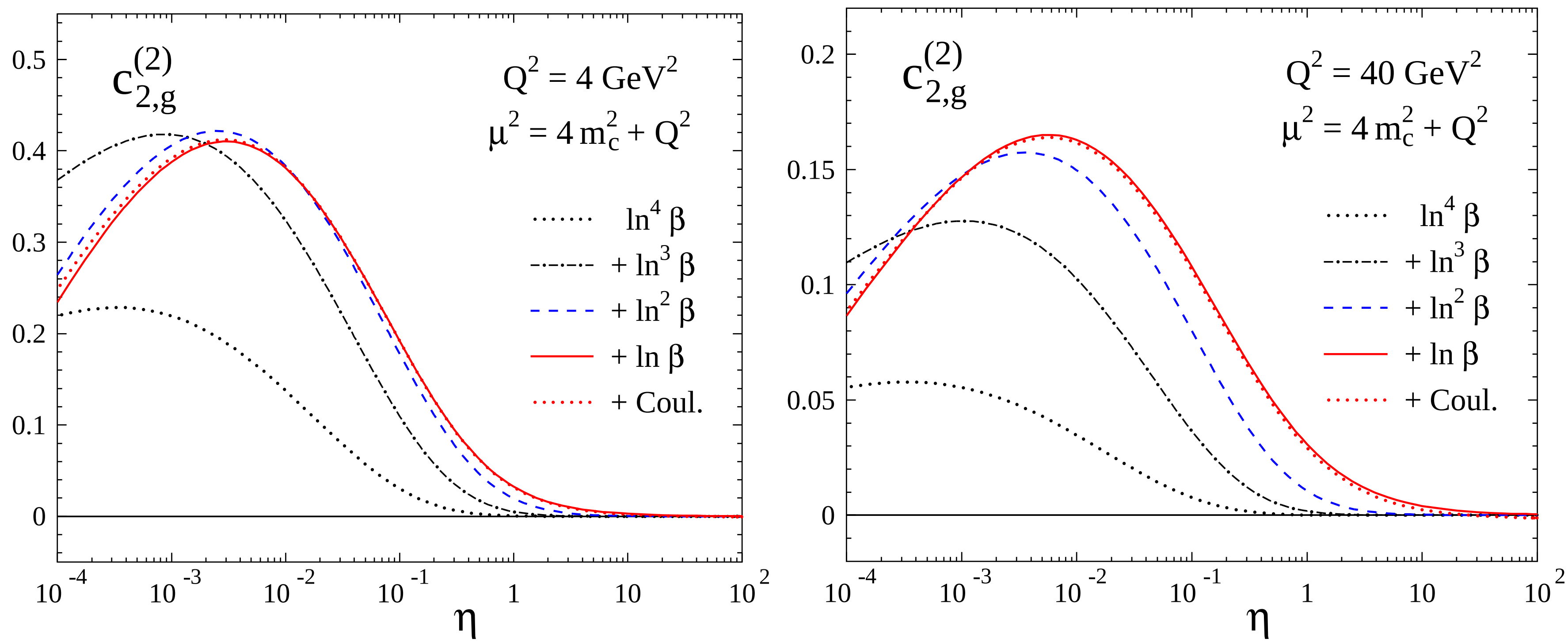
Threshold improvements beyond NLO

- Parton cross section close to threshold $s \simeq 4m^2$
  - Sudakov logarithms $\ln(\beta)$ with velocity of heavy quark
    $\beta = \sqrt{1 - 4m^2/s}$ at $n^{\rm th}$-order: $\alpha_s^n \ln^{2n}(\beta) \longleftrightarrow \alpha_s^n \ln^{2n}(N)$
- Resummation in Mellin space (renormalization group equation) predicts fixed orders in perturbation theory
  - approximate expressions to NNLO
    Laenen, S.M. '98; Alekhin, S.M. '08; Lo Presti, Kawamura, S.M., Vogt '10

Asymptotics beyond NLO

- NNLO corrections at large $Q^2 \gg m^2$ Bierenbaum, Blümlein, Klein '09

Approximate coefficient functions at NNLO



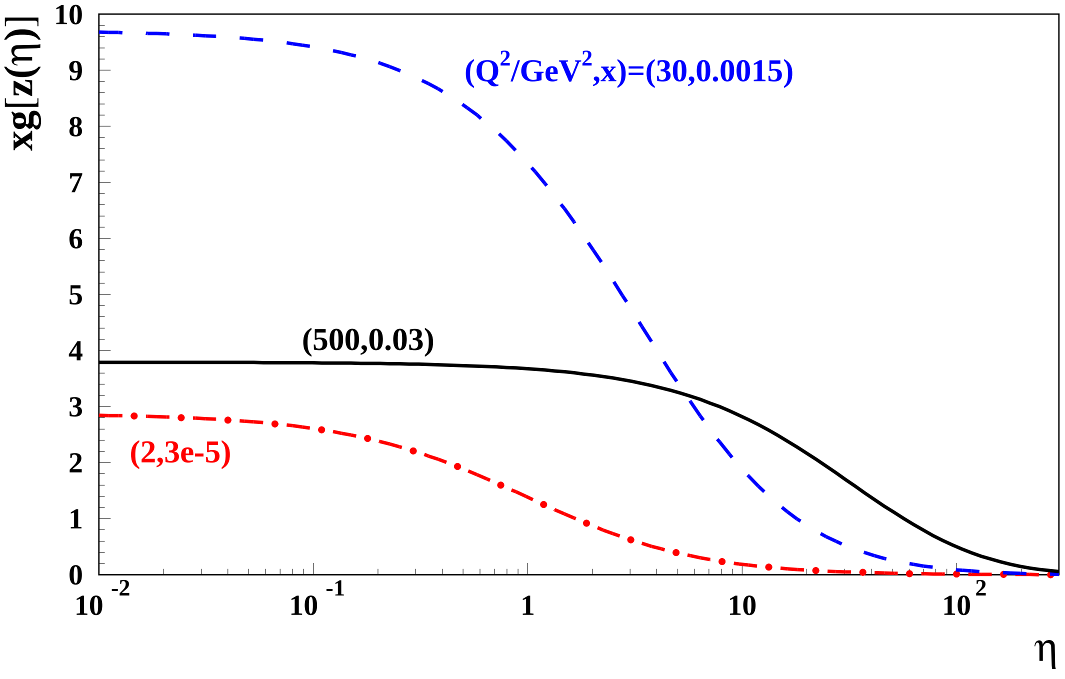
- Distance from threshold $\eta = s/(4m^2) - 1$
  - Sudakov logarithms dominant for $\eta \ll 1$ now known to NNLL
    Lo Presti, Kawamura, S.M., Vogt ‘10
- Combine Sudakov logarithms with exact scale dependence at two loops $\longrightarrow$ NNLO$_{\rm approx}$

Convolution with gluon PDF

- Recall QCD factorization

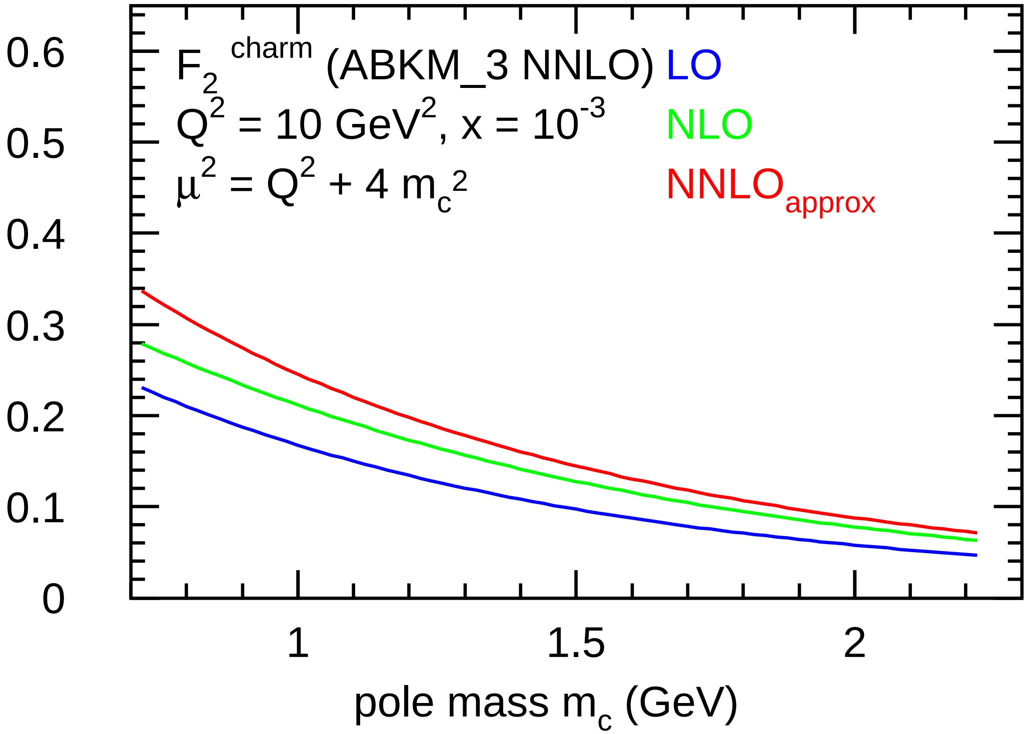
$$x^{-1} F_2^c(x, Q^2, m^2) = \frac{\alpha_s\, e_q^2}{\pi^2} \sum_{i=q,\bar{q},g} \int_0^{\eta_{max}} d\eta\; f_i\left(z(\eta), \mu^2\right)\; c_{i,k}\left(\eta, \xi, \mu^2\right)$$

  - gluon PDF gives large weight to parton dynamics near threshold for $Q^2 \lesssim 10 - 30$ GeV^2



Running quark masses in DIS

- Charm structure function Alekhin, S.M. '10
  - improved convergence
  - scale dependence reduced



- pole mass scheme for comparison

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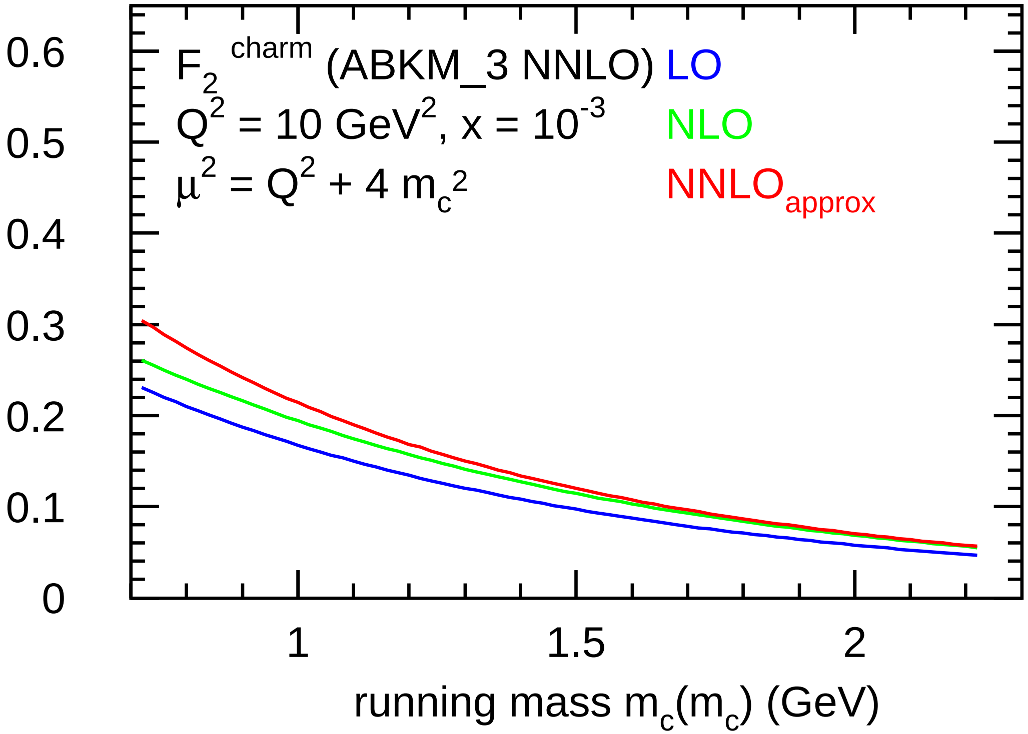

- running mass
- direct determination of $m_c(m_c)$

NLO

1.26 ± 0.09 (exp) $\pm$ 0.11 (th) GeV

NNLO$_{\text{approx}}$

1.01 ± 0.09 (exp) $\pm$ 0.03 (th) GeV

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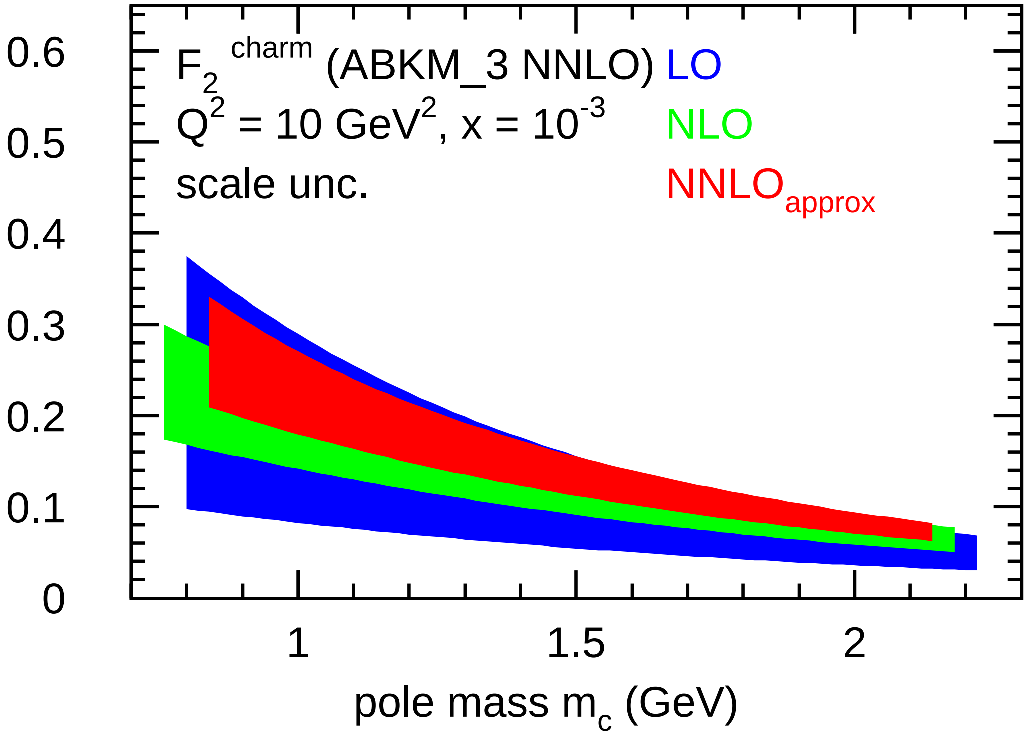

- pole mass scheme for comparison
- scale variation $0.5 \leq \kappa \leq 2$ around $\mu_R, \mu_F = \kappa\sqrt{Q^2 + 4m_c^2}$

Running quark masses in DIS

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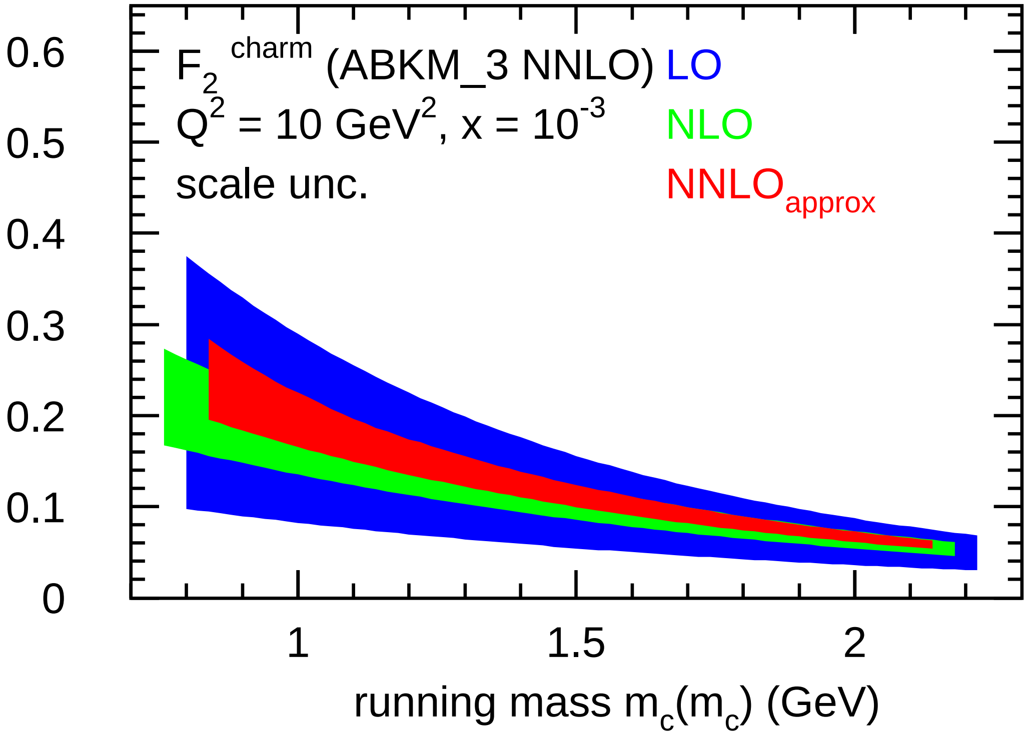
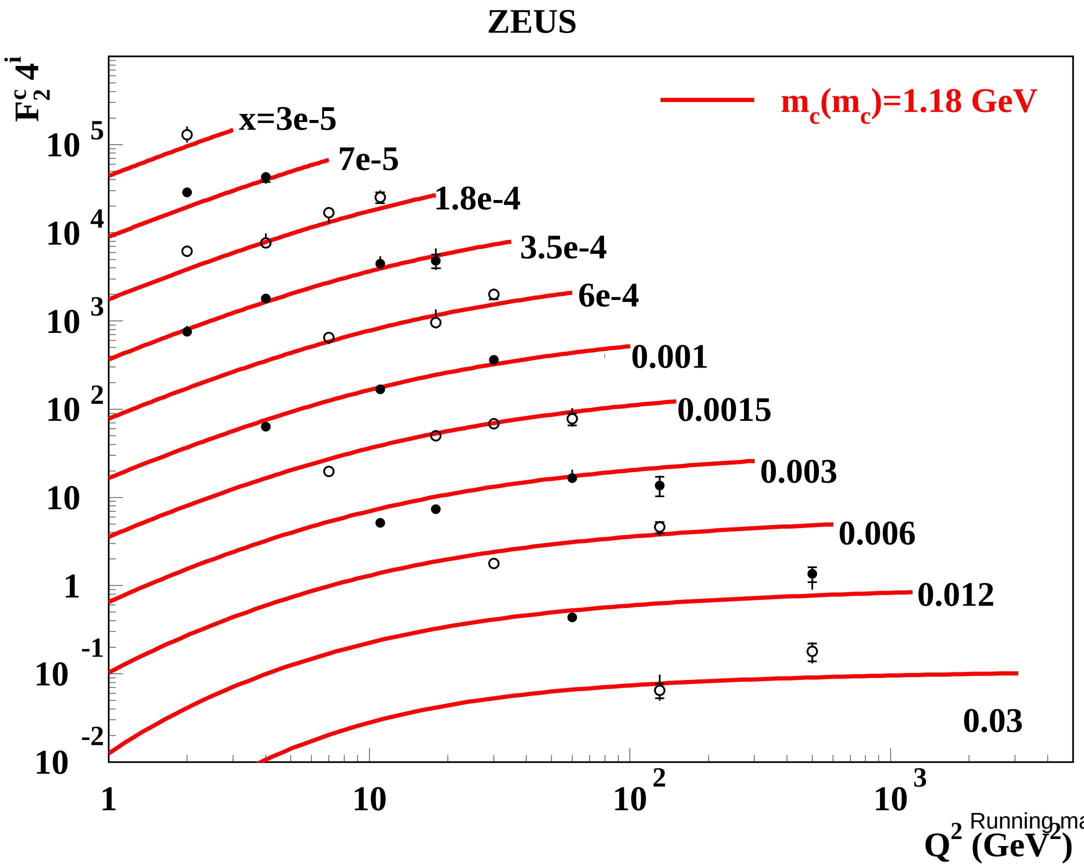


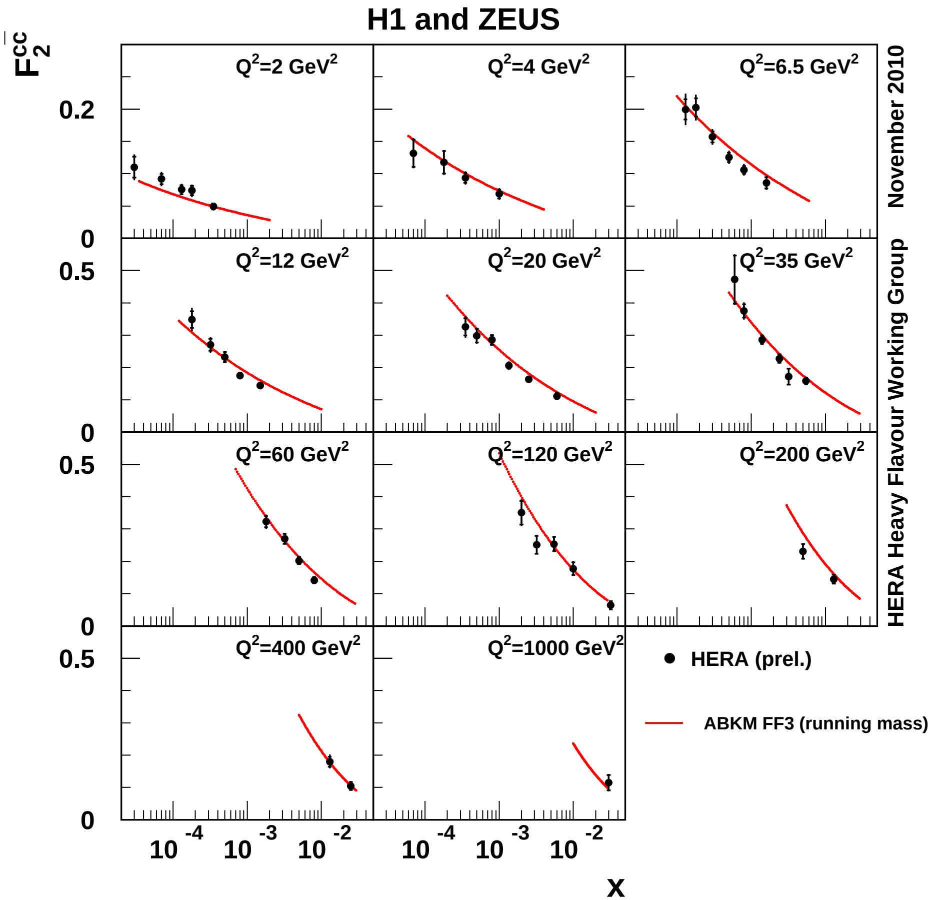
- running mass
- scale variation $0.5 \leq \kappa \leq 2$ around $\mu_R, \mu_F = \kappa\sqrt{Q^2 + 4m_c^2}$

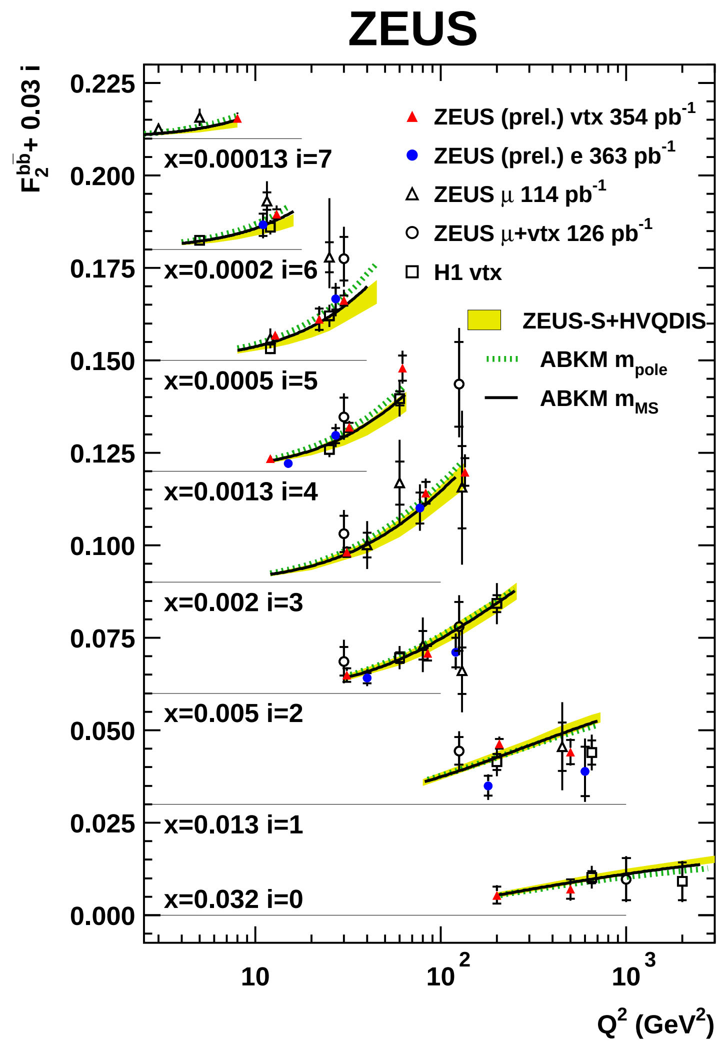
Global fit

- NNLO fit of PDFs (variant of ABKM fit Alekhin, Blümlein, Klein, Moch '09)
  - add PDG value for $m_c(m_c)$ as additional constraint
  - fit gives $m_c(m_c) = 1.18 \pm 0.06$ (exp) $\pm$ 0.03 (th) GeV
- Comparison with ZEUS data shows good agreement (theory somewhat low Q^2-region at small values of x)



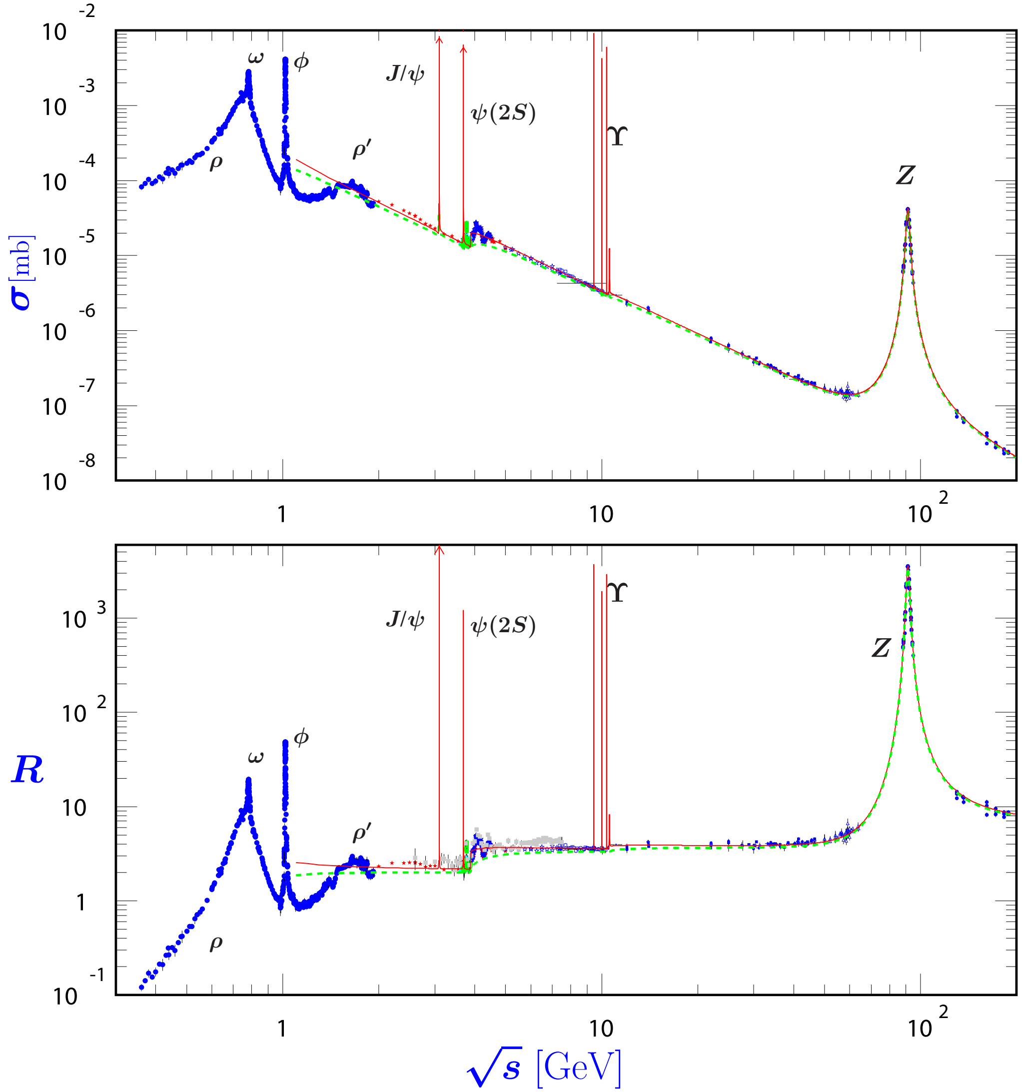
Phenomenology



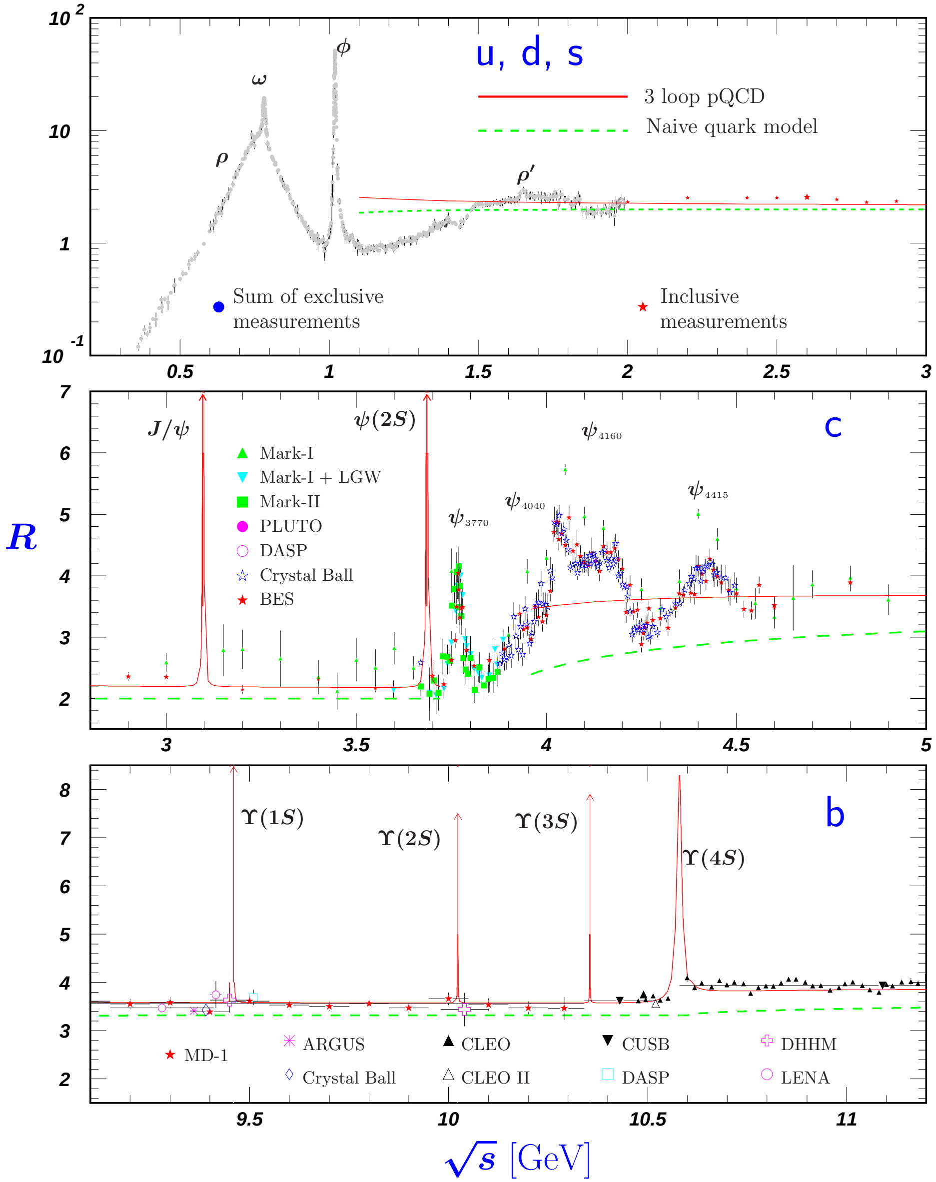


- Comparision to new data for F_2^{charm} and F_2^{bottom} from HERA

R-ratio in e^+e^--annihilation



R-ratio in e^+e^--annihilation (zoom)



- Relativistic sum rules

$$M_n = \int \frac{ds}{s^{n+1}} R(s)$$

- Most advance theory:

$$M_n^{\text{theory}} = \frac{12\pi^2}{n!} \left(\frac{d}{q^2}\right)^n \Pi(q^2)\Big|_{q^2=0}$$

  - $\Pi(q^2)$ known at $\mathcal{O}(\alpha_s^3)$
  - explicit mass dependence for heavy quarks, e.g. charm

$$\Pi_c(q^2) = e_c^2 \frac{3}{16\pi^2} \sum_{n\geq 0} C_n \left(\frac{q^2}{4m_c^2}\right)^n$$

  - precision determinations
    Chetyrkin, Kühn, Meier, Meierhofer, Marquard, Steinhauser '10
    m_c(3 GeV) = 986 MeV $\pm$ 13 MeV
    $m_c(m_c)$ = 1279 MeV $\pm$ 13 MeV
    also: Bodenstein, Bordes, Dominguez, Penarrocha, Schilcher '10

Summary

Heavy-quark DIS

- Charm structure function F_2^c
  - progress on soft gluon approximation at NNLO QCD

Heavy quark masses

- Running mass definition for heavy quark $m(\mu_R)$
- First determination of charm-quark mass from DIS data with best estimate: $m_c(m_c)$ = 1.01 $\pm$ 0.09 (exp) $\pm$ 0.03 (th) GeV

Phenomenology

- Good agreement with HERA data for $F_2^{\rm charm}$ and $F_2^{\rm bottom}$