MSTW Heavy Flavour Results

Robert Thorne

November 29th, 2010

University College London

Associate of IPPP Durham

In collaboration with A.D. Martin, W.J. Stirling and G. Watt

Heavy Quark issues

Will discuss Charm $\sim 1.4\mathrm{GeV}$, bottom $\sim 4.75\mathrm{GeV}$ as heavy flavours.

Quick reminder.

Two distinct regimes:

Near threshold $Q^2 \sim m_H^2$ massive quarks not partons. Created in final state. Described using **Fixed Flavour Number Scheme** (FFNS).

$$F(x,Q^2) = C_k^{FF,n_f}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2)$$

Note that n_f is effective number of light quarks.

Does not sum $\alpha_S^n \ln^n Q^2/m_H^2$ terms in perturbative expansion. Usually achieved by definition of heavy flavour parton distributions and solution of evolution equations.

Additional problem FFNS known up to NLO (Laenen *et al*), but are not defined at NNLO – $\alpha_S^3 C_{2,Hg}^{FF,3}$ not fully known.

Recent progress by Blümlein *et al* for high Q^2

Variable Flavour

High scales $Q^2 \gg m_H^2$ massless partons. Behave like up, down (strange always in this regime. Sum $\ln(Q^2/m_H^2)$ terms via evolution. **Zero Mass Variable Flavour Number Scheme** (ZM-VFNS). Ignores $\mathcal{O}(m_H^2/Q^2)$ corrections.

$$F(x,Q^2) = C_j^{ZM,n_f} \otimes f_j^{n_f}(Q^2).$$

Partons in different number regions related to each other perturbatively.

$$f_j^{n_f+1}(Q^2) = A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2),$$

Perturbative matrix elements $A_{jk}(Q^2/m_H^2)$ (Buza *et al* $\mathcal{O}(\alpha_S^2)$, Blümlein *et al* $\mathcal{O}(\alpha_S^3)$) containing $\ln(Q^2/m_H^2)$ terms relate $f_i^{n_f}(Q^2)$ and $f_i^{n_f+1}(Q^2)$ $\rightarrow$ correct evolution for both.

We use a **General-Mass Variable Flavour Number Scheme** (VFNS) taking one from the two well-defined limits of $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$.

Particular definition. More on this later.

Dependence on m_c at NLO in 2008 fits.

m_c (GeV)	χ^2_{global} 2699 pts	$\chi^2_{F_2^c}$ 83 pts	$\alpha_s(M_Z^2)$
1.1	2728	263	0.1182
1.2	2625	188	0.1188
1.3	2563	134	0.1195
1.4	2543	107	0.1202
1.45	2541	100	0.1205
1.5	2545	97	0.1209
1.6	2574	104	0.1216
1.7	2627	128	0.1223

Correlation between m_c and $\alpha_S(M_Z^2)$ at best fit.

For low m_c overshoot low Q^2 medium x data badly.

Preference for $m_c = 1.45\text{GeV}$. A little low for pole mass determination.

BCDMS and NMC data prefer lower m_c, lower α_S and quicker threshold evolution respectively.

Dependence on m_c at NNLO in 2008 fits.

m_c (GeV)	χ^2_{global} 2615 pts	$\chi^2_{F_2^c}$ 83 pts	$\alpha_s(M_Z^2)$
1.1	2499	114	0.1158
1.2	2463	88	0.1162
1.26	2546	82	0.1165
1.3	2457	82	0.1166
1.4	2480	95	0.1171
1.5	2527	125	0.1175
1.6	2589	167	0.1180
1.7	2666	217	0.1184

Less correlation between m_c and $\alpha_S(M_Z^2)$.

For high m_c undershoot moderate Q^2 data badly.

Preference for low value of $m_c = 1.26\mathrm{GeV}$.

Newer data seem to prefer higher mass.

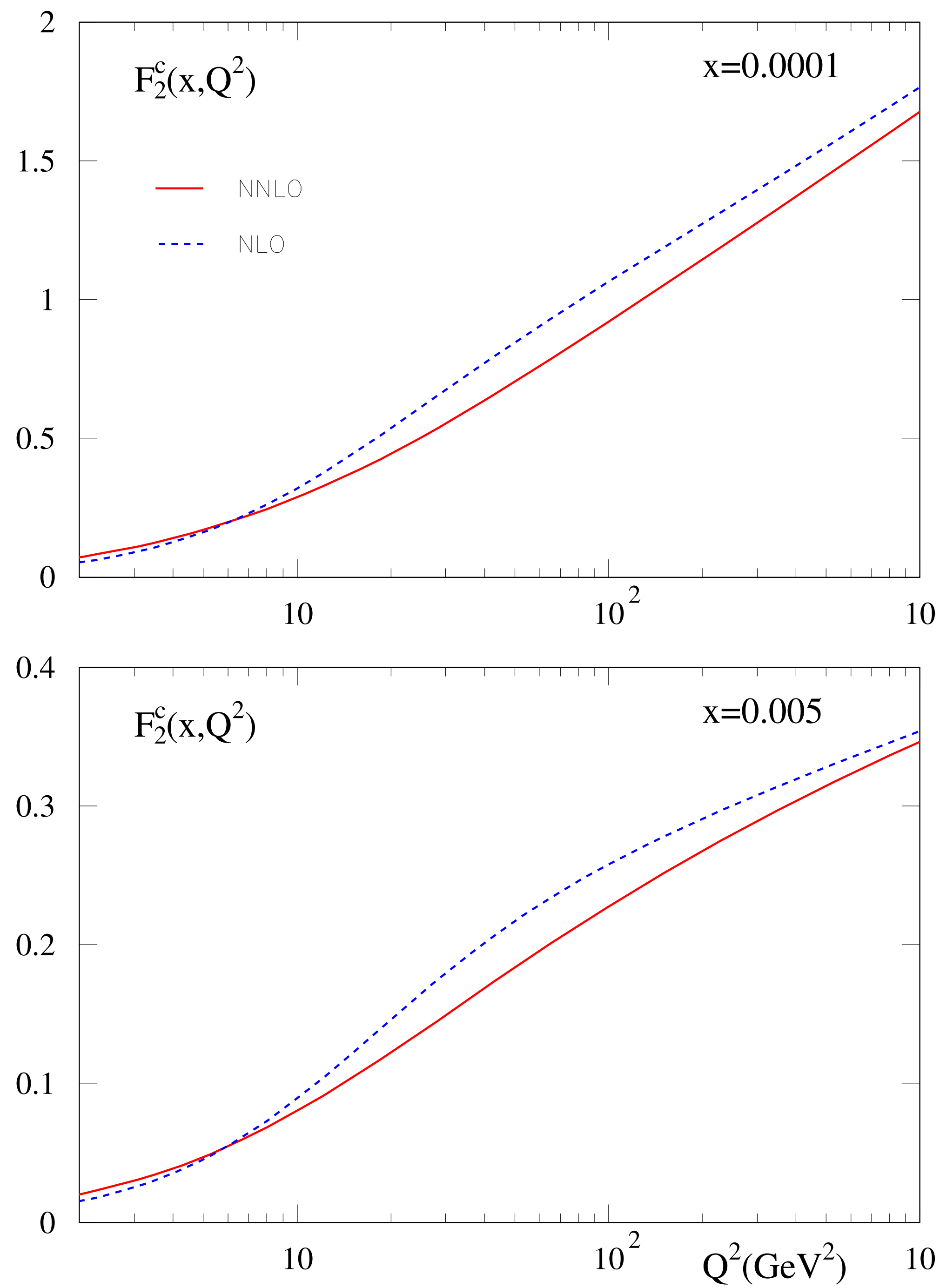
Difference partially due to low Q^2 region, though competing effects here - NNLO larger but smoother.

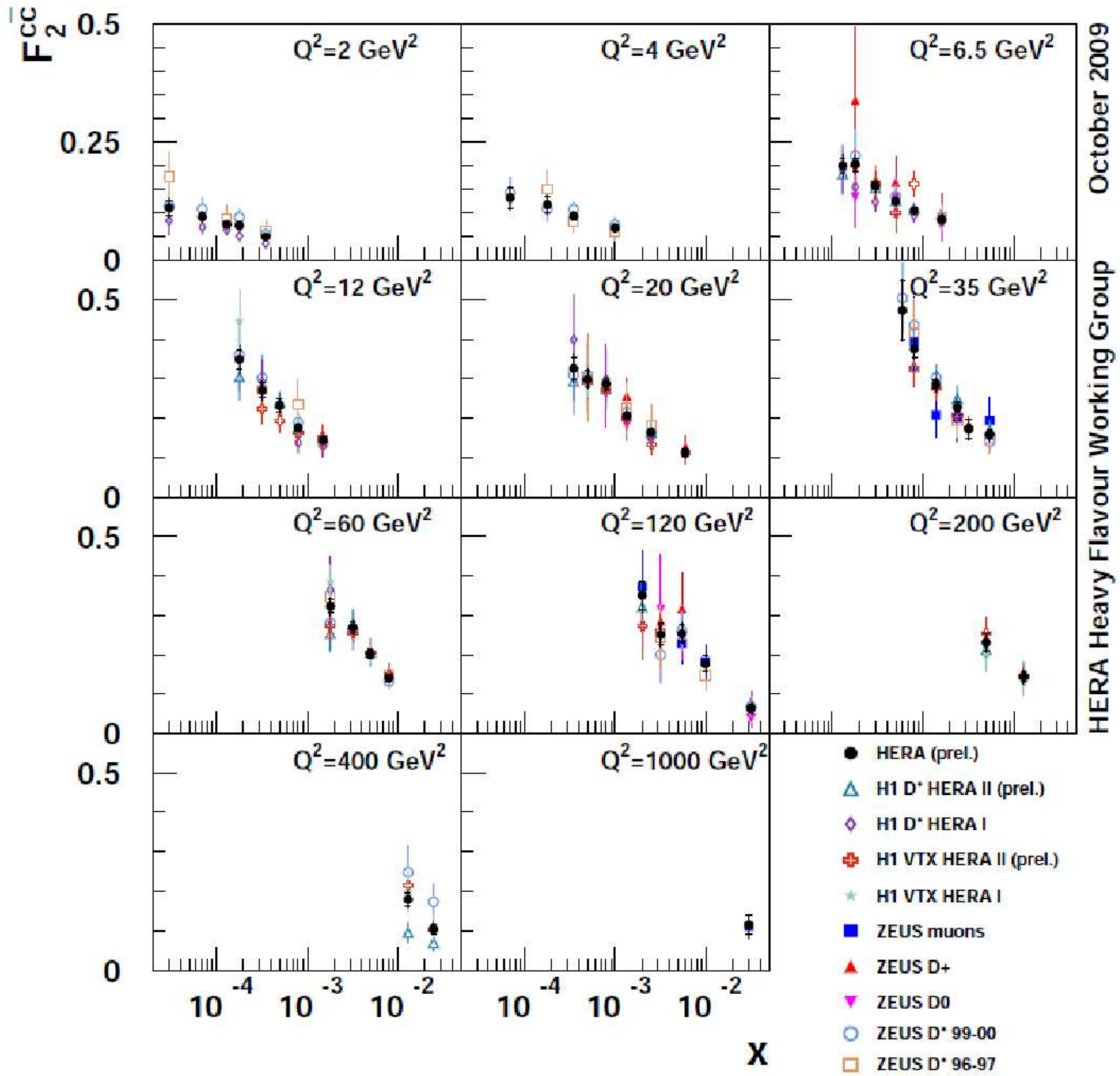
Largely due to general shape of $dF_2^c/d\ln Q^2$ at NNLO compared to NLO.

NNLO $F_2^c(x,Q^2)$ starts from higher value at low Q^2.

At high Q^2 dominated by $(c+\bar{c})(x,Q^2)$. This has started evolving from negative value at $Q^2=m_c^2$. Remains lower than at NLO for similar evolution.

General trend – $F_2^c(x,Q^2)$ flatter in Q^2 at NNLO than at NLO. Important effect on gluon distribution going from one to other.





Newer combined data seem to prefer higher mass (largely because the data in Eur.Phys.J.C38:447-459,2005 not released in structure function form).

Dependence on m_b at NLO in 2008 fits.

Vary m_b in steps of $0.25\mathrm{GeV}$.

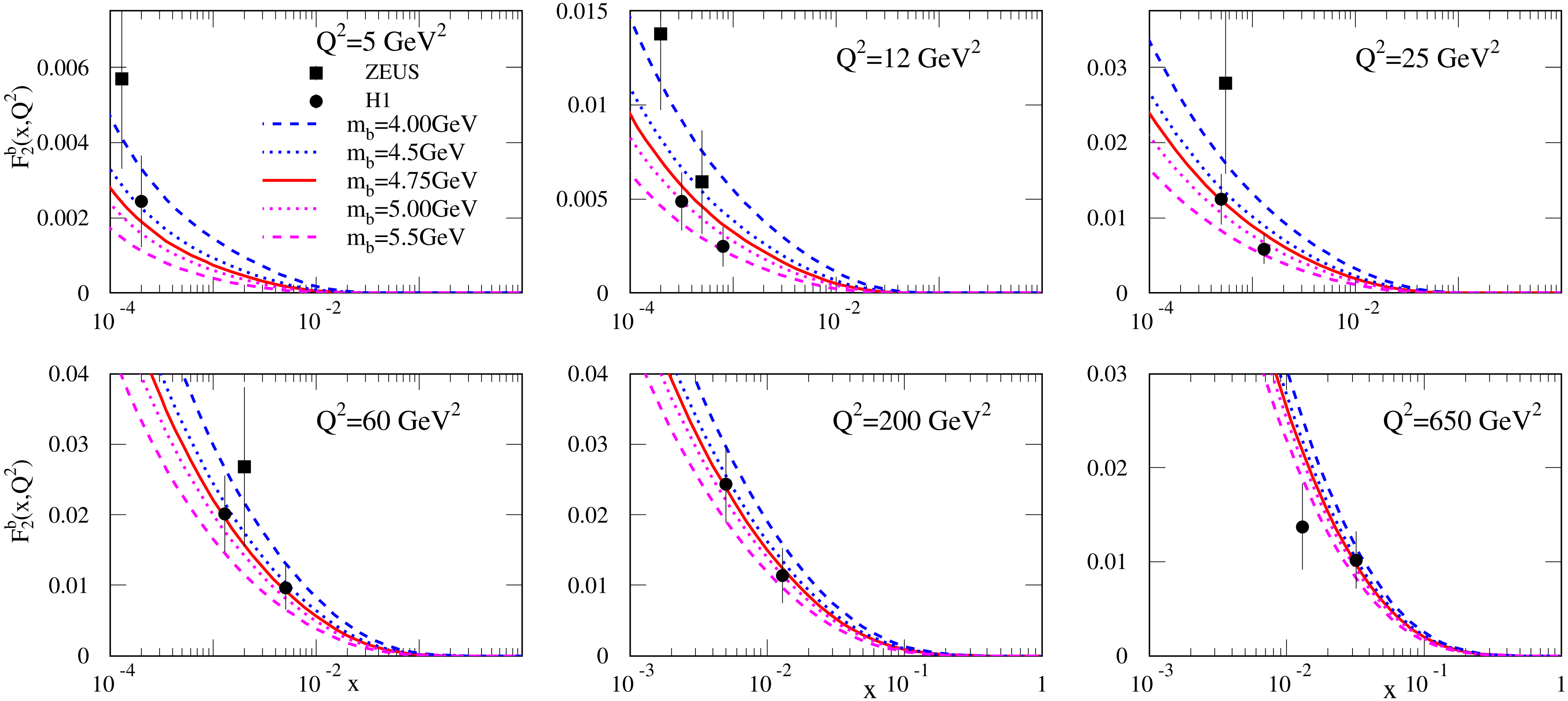
m_b (GeV)	χ^2_{global} 2699 pts	$\alpha_s(M_Z^2)$
4.00	2537	0.1201
4.25	2539	0.1202
4.50	2541	0.1202
4.75	2543	0.1202
5.00	2544	0.1201
5.25	2547	0.1201
5.50	2549	0.1200

Stays fairly flat all the way down to $m_b = 3\mathrm{GeV}$.

For lower m_b slightly better fit to HERA data, including $F_2^c(x, Q^2)$.

Similar at NNLO, but with about half the change in χ^2.

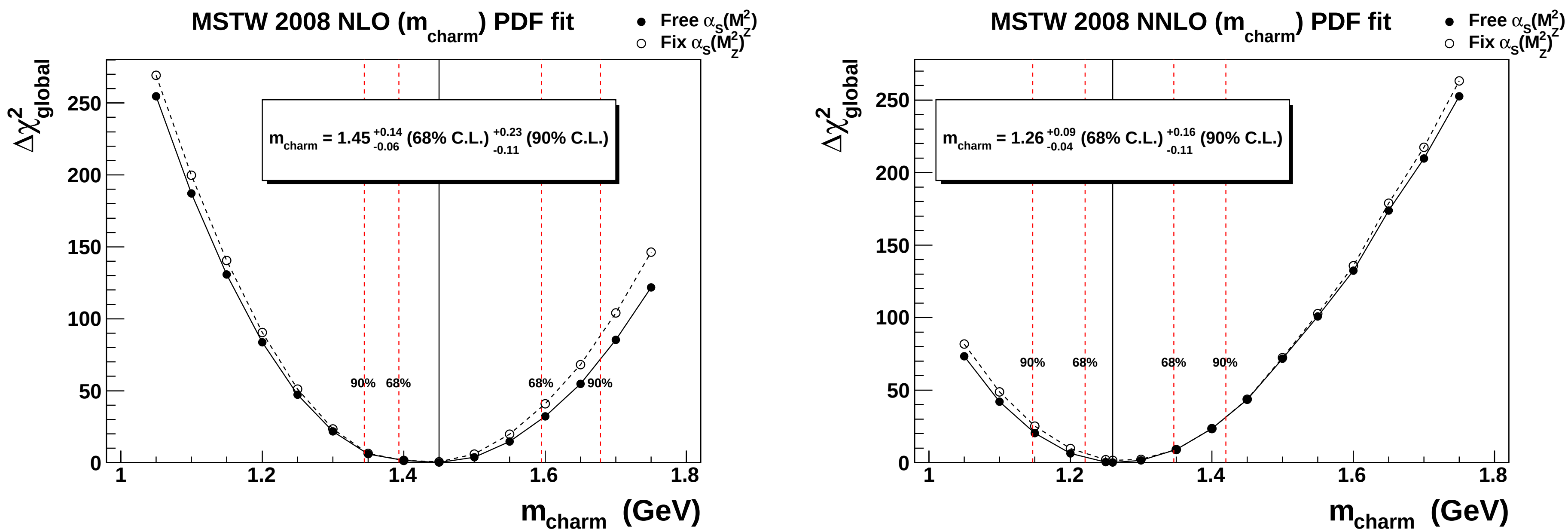
NLO comparisons to Beauty data (not in global fit) for varying m_b



Distinct preference for $m_b \approx 4.75 - 5\text{GeV}$.

Overall global fit, even including current beauty data, would prefer fairly near current default $= 4.75\text{GeV}$.

Correlation with α_S not actually very strong. Rather little tightening in χ^2 if it is kept fixed at best fit value.



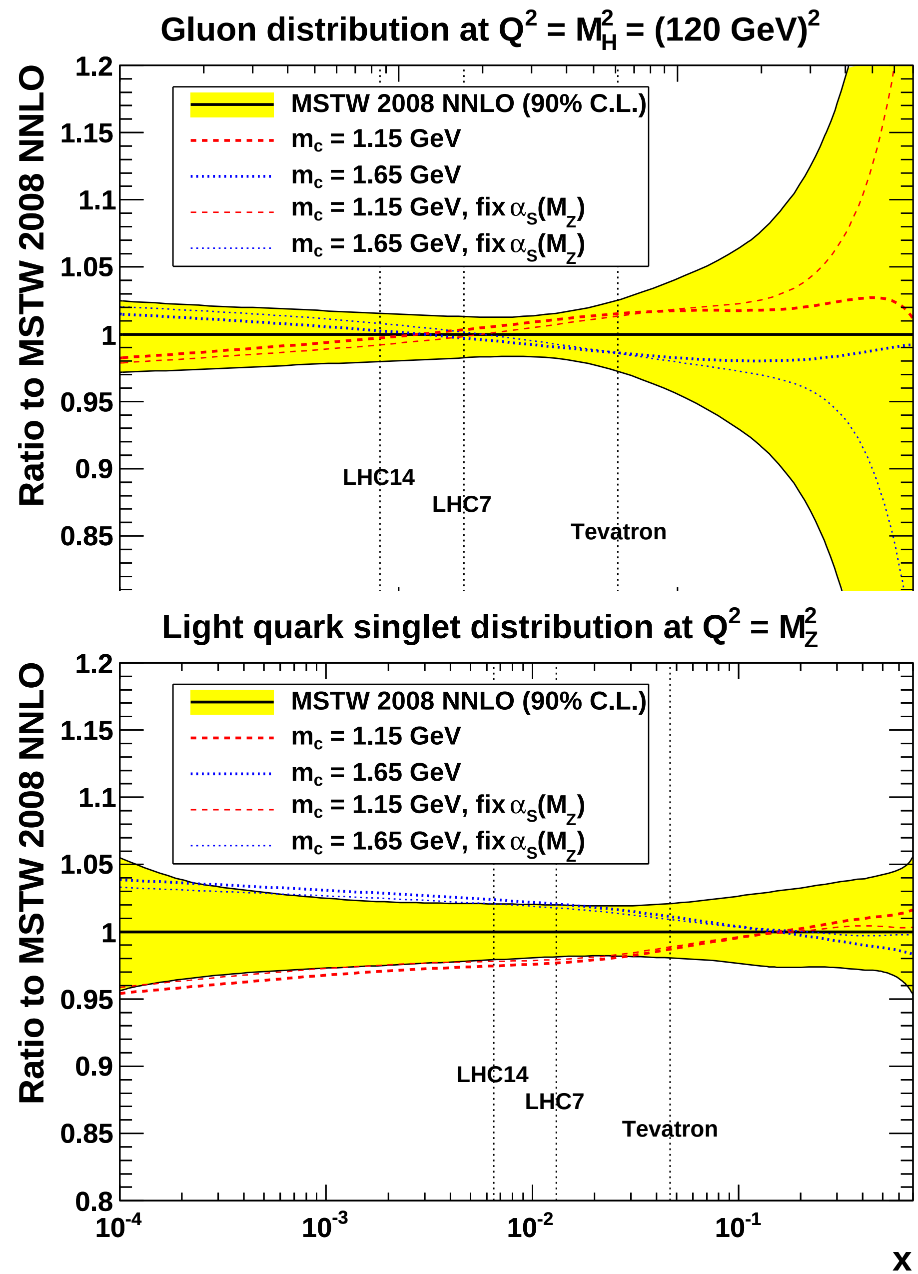
Just about consistency between NLO and NNLO values.

Can be improved slightly.

Ratio of partons when m_c is varied either with or without varying α_S

Very little change. To fairly good approx. can treat α_S and masses as uncorrelated.

Cannot determine mass uncertainty in same manner as parton parameter or even α_S because not obtaining best value from fit.



Uncertainties to to m_c and m_b

decide for moment to add uncertainties in quadrature with PDF parameter and α_S combined uncertainty.

Tevatron, $\sqrt{s} = 1.96$ TeV	$B_{\ell\nu} \cdot \sigma^W$	$B_{\ell^+\ell^-} \cdot \sigma^Z$	σ^H
Central value	2.747 nb	0.2507 nb	0.9550 pb
PDF only uncertainty	$^{+1.8\%}_{-1.5\%}$	$^{+1.9\%}_{-1.6\%}$	$^{+3.1\%}_{-3.3\%}$
PDF+α_S uncertainty	$^{+2.2\%}_{-1.7\%}$	$^{+2.2\%}_{-1.8\%}$	$^{+5.4\%}_{-4.8\%}$
PDF+α_S+$m_{c,b}$ uncertainty	$^{+2.3\%}_{-1.8\%}$	$^{+2.3\%}_{-2.0\%}$	$^{+5.6\%}_{-5.1\%}$

NNLO predictions for W, Z and Higgs ($M_H = 120$ GeV) total cross sections at the Tevatron.

Masses have very little impact at Tevatron. PDF uncertainties dominant.

LHC, $\sqrt{s} = 7$ TeV	$B_{\ell\nu} \cdot \sigma^W$	$B_{\ell^+\ell^-} \cdot \sigma^Z$	σ^H
Central value	10.47 nb	0.958 nb	15.50 pb
PDF only uncertainty	$^{+1.7\%}_{-1.6\%}$	$^{+1.7\%}_{-1.5\%}$	$^{+1.1\%}_{-1.6\%}$
PDF+α_S uncertainty	$^{+2.5\%}_{-1.9\%}$	$^{+2.5\%}_{-1.9\%}$	$^{+3.7\%}_{-2.9\%}$
PDF+α_S+$m_{c,b}$ uncertainty	$^{+2.7\%}_{-2.2\%}$	$^{+2.9\%}_{-2.4\%}$	$^{+3.7\%}_{-2.9\%}$

LHC, $\sqrt{s} = 14$ TeV	$B_{\ell\nu} \cdot \sigma^W$	$B_{\ell^+\ell^-} \cdot \sigma^Z$	σ^H
Central value	21.72 nb	2.051 nb	50.51 pb
PDF only uncertainty	$^{+1.7\%}_{-1.7\%}$	$^{+1.7\%}_{-1.6\%}$	$^{+1.0\%}_{-1.6\%}$
PDF+α_S uncertainty	$^{+2.6\%}_{-2.2\%}$	$^{+2.6\%}_{-2.1\%}$	$^{+3.6\%}_{-2.7\%}$
PDF+α_S+$m_{c,b}$ uncertainty	$^{+3.0\%}_{-2.7\%}$	$^{+3.1\%}_{-2.8\%}$	$^{+3.7\%}_{-2.8\%}$

NNLO predictions for W, Z and Higgs ($M_H = 120$ GeV) total cross sections or 7 TeV LHC and 14 TeV LHC.

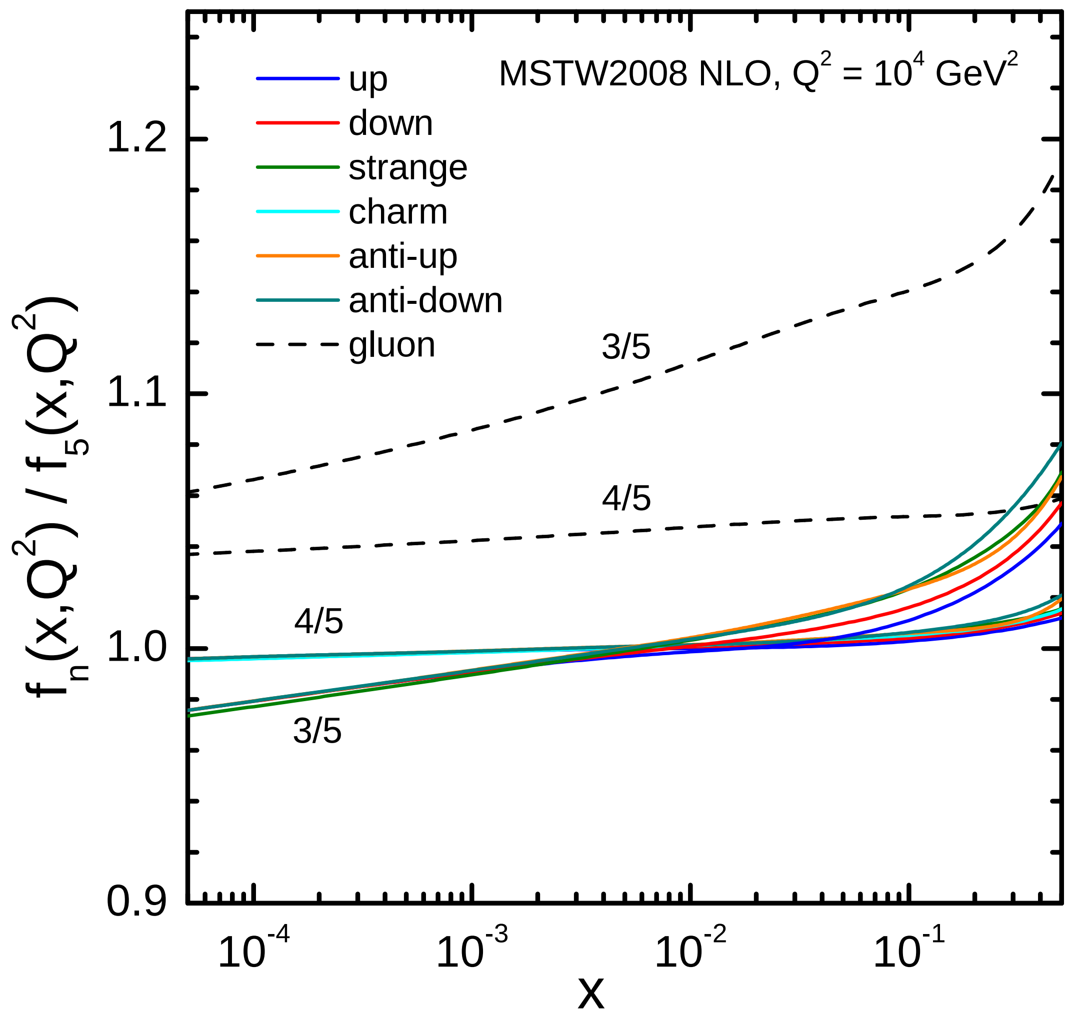
α_S uncertainties now more important, particularly for Higgs. Mass uncertainties significant, but least important of three effects, particularly for Higgs. (Not necessarily the case in supersymmetric production where b coupling enhanced.)

3 and 4 Flavour Scheme PDFs.

Generated from same input as the variable flavour number versions.

Moderate effect on quarks, due to change in coupling.

Major change in gluon due to splitting to fewer quarks. Compensated for by coupling.

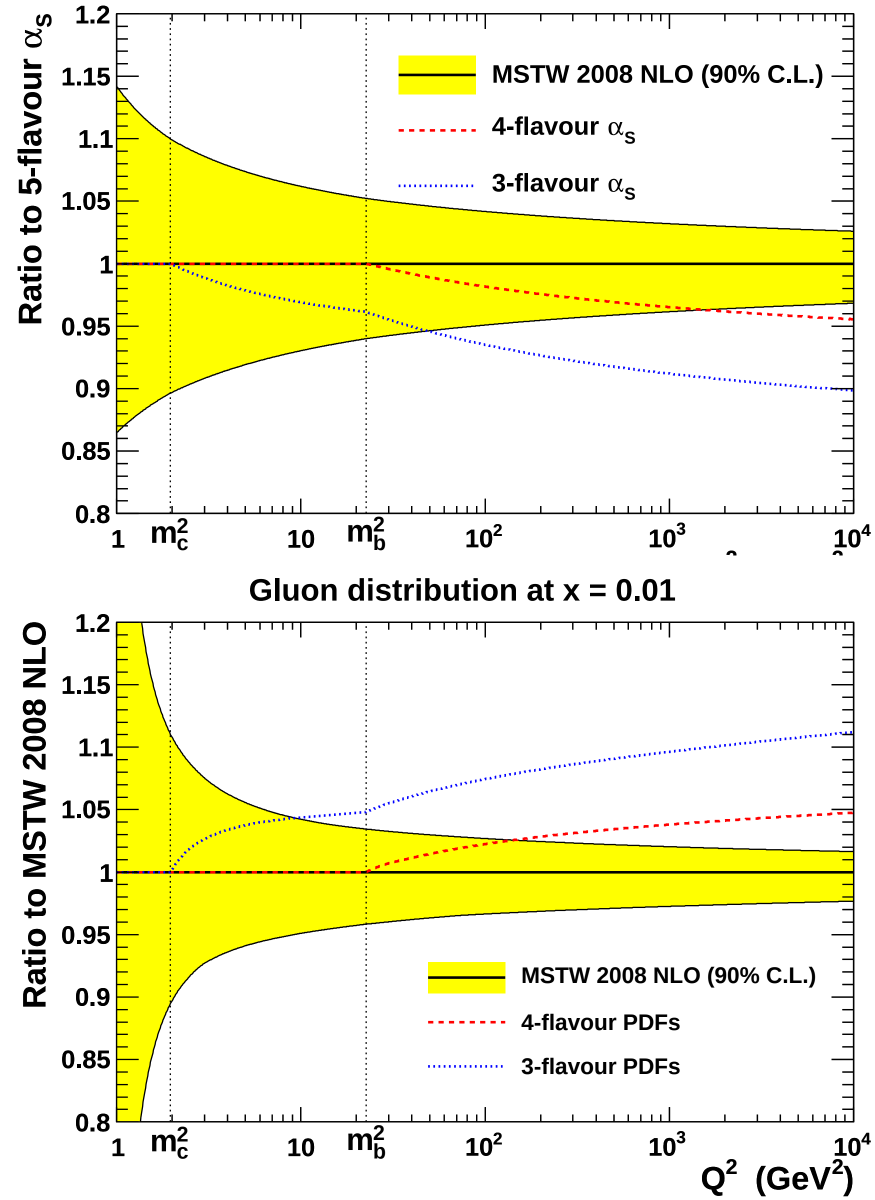


Use appropriate number of flavour in coupling for standard definition of FFNS coefficient functions (depends on renormalisation scheme one defines), i.e. same number as in PDFs at all times.

Doing this decrease in coupling compared to variable flavour, larger β-function.

To lowest order good compensation between behaviour of coupling and gluon splitting. Leads to (approx.) invariance of quantities $\propto \alpha_S g(x,Q^2)$, e.g. light flavour evolution, Higgs cross-section.

Best near $x = 0.01$.



Considerations of the GM-VFNS - Our Definition

The GM-VFNS can be defined by demanding equivalence of the n_f light flavour and n_f+1 light flavour descriptions at all orders – above transition point $n_f \to n_f+1$

$$F(x,Q^2) = C_k^{FF,n_f}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2) = C_j^{VF,n_f+1}(Q^2/m_H^2) \otimes f_j^{n_f+1}(Q^2)$$

$$\equiv \quad C_j^{VF,n_f+1}(Q^2/m_H^2) \;\otimes\; A_{jk}(Q^2/m_H^2) \;\otimes\; f_k^{n_f}(Q^2).$$

Hence, the VFNS coefficient functions satisfy

$$C_k^{FF,n_f}(Q^2/m_H^2) = C_j^{VF,n_f+1}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2),$$

which at $\mathcal{O}(\alpha_S)$ gives

$$C_{2,Hg}^{FF,n_f,(1)}(\frac{Q^2}{m_H^2}) = C_{2,HH}^{VF,n_f+1,(0)}(\frac{Q^2}{m_H^2}) \otimes P_{qg}^0 \ln(Q^2/m_H^2) + C_{2,Hg}^{VF,n_f+1,(1)}(\frac{Q^2}{m_H^2}),$$

The VFNS coefficient functions tend to the massless limits as $Q^2/m_H^2 \to \infty$.

However, $C_j^{VF}(Q^2/m_H^2)$ only uniquely defined in this limit.

Can swap $\mathcal{O}(m_H^2/Q^2)$ terms between $C_{2,HH}^{VF,0}(Q^2/m_H^2)$ and $C_{2,g}^{VF,1}(Q^2/m_H^2)$.

Also have the freedom to modify the heavy quark coefficient function, by default

$$C^{VF,0}_{2,HH}(Q^2/m_H^2, z) = \delta(z - x_{\max}).$$

Appears in convolutions for higher order subtraction terms, so do not want complicated x dependence. Simple choice.

$$C^{VF,0}_{2,HH}(Q^2/m_H^2, z) \to (1 + b(m_H^2/Q^2)^c)\delta(z - x_{\max})),$$

where again c really encompasses (m_H^2/Q^2) with logarithmic corrections.

Can also modify argument of δ-function, as in Intermediate Mass (IM) scheme of Nadolsky, Tung. Let argument of heavy quark contribution change like

$$\xi = x/x_{\max} \to x\big(1 + (x(1 + 4m_H^2/Q^2))^d 4m_H^2/Q^2\big),$$

so kinematic limit stays the same, but if $d > 0$ small x less suppressed, or if $d < 0$ (must be > -1) small x more suppressed.

In TR version of the GMVFNS have frozen term

$$\alpha_S^n(m_H^2) \sum_i C_{2,i}^{\rm FFNS}(m_H^2) \otimes f_i(m_H^2)$$

due to different order of α_S in FFNS and ZM-VFNS definition.

Depends on size of PDFs at low scales, so rather small effect at large Q^2.

However, not strictly necessary. Frozen in original TR prescription from exact condition on derivative of $d\,F_2/d, \ln Q^2$. Could have instead

$$\left(\frac{m_H^2}{Q^2}\right)^a \alpha_S^n(m_H^2) \sum_i C_{2,i}^{\rm FF}(m_H^2) \otimes f_i(m_H^2) \;\; \text{or} \;\; \left(\frac{m_H^2}{Q^2}\right)^a \alpha_S^n(Q^2) \sum_i C_{2,i}^{\rm FF}(Q^2) \otimes f_i(Q^2),$$

Any $a > 0$ provides both exactly correct asymptotic limits, though strictly should have $(m_H^2/Q^2)k\big(\ln(Q^2/m_H^2)\big)$ from factorization theorem.

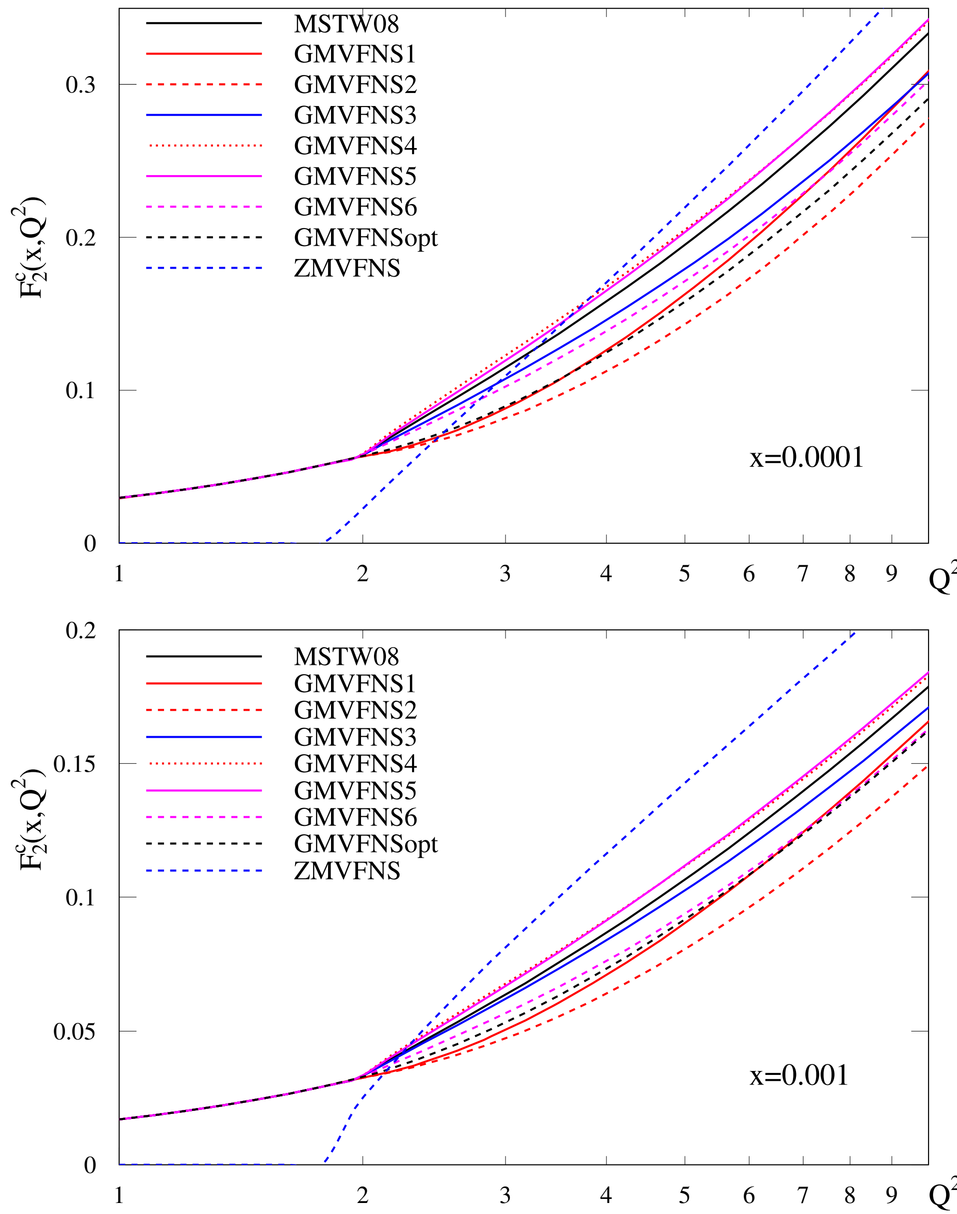
Default a, b, c, d all zero. Limit either by fit quality or *sensible* choices.

6 extreme variations tried, along with ZM-VFNS

At NLO extremes determined by same sort of deterioration in fit as required for eigenvector definitions (mainly for steepening), or *sensible* limits (more for flattening), e.g. if using $(1 - am_c^2/Q^2)$ factor, max. $a = 1$.

Variations in $F_2^c(x, Q^2)$ near the transition point at NLO due to different choices of GM-VFNS.

Optimal, $a = 1, b = -2/3, c = 1$, smooth behaviour.

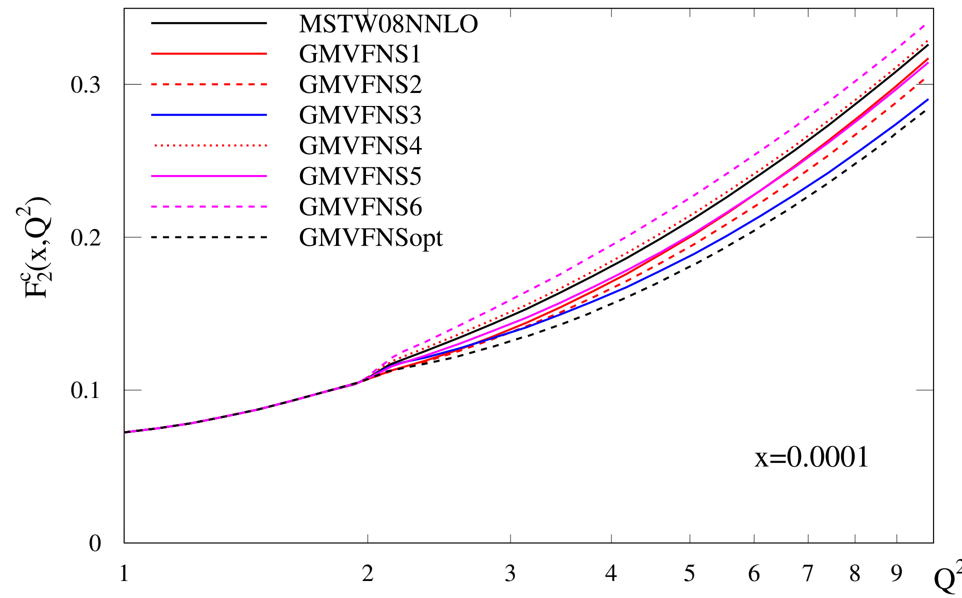


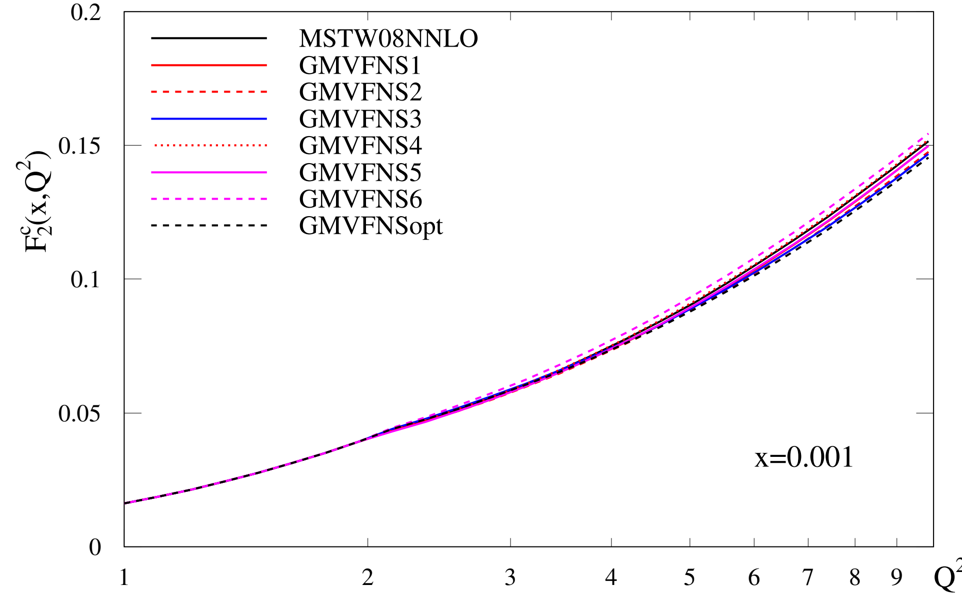
Variations in $F_2^c(x, Q^2)$ near the transition point due to different choices of GM-VFNS at NNLO.

Use limits on parameters determined at NLO. Changes in χ^2 very much smaller so not a useful method.

Very much reduced variation, almost zero variation until very small x.

Shows that NNLO evolution effects most important in this regime.





Variations in partons extracted from global fit due to different choices of GM-VFNS at NLO. Default at low end.

Initial χ^2 can change by 250.

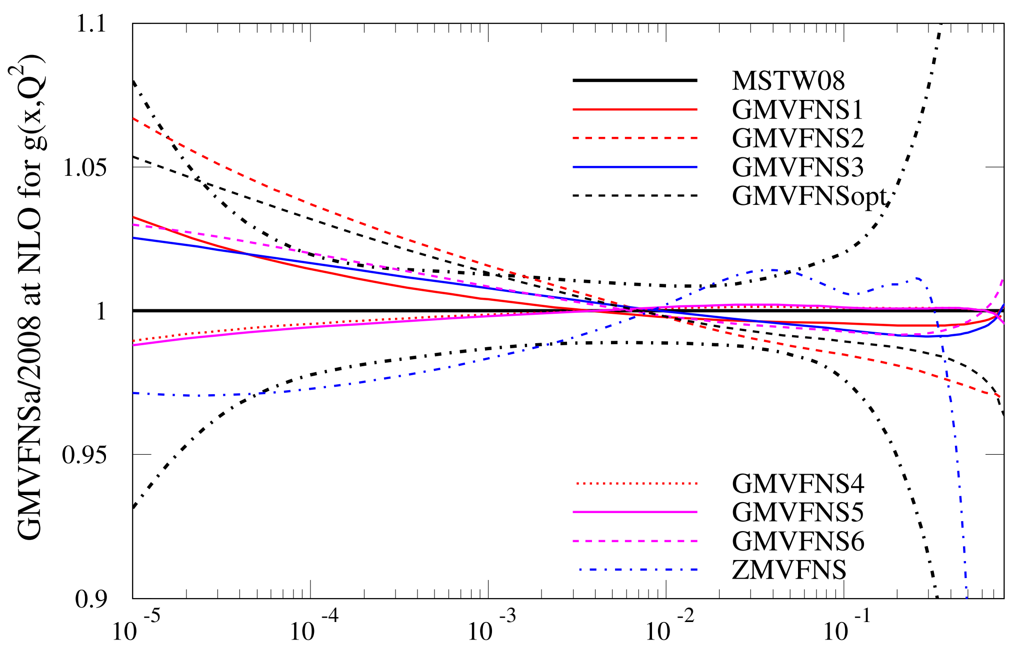
Converges to within 20 of original.

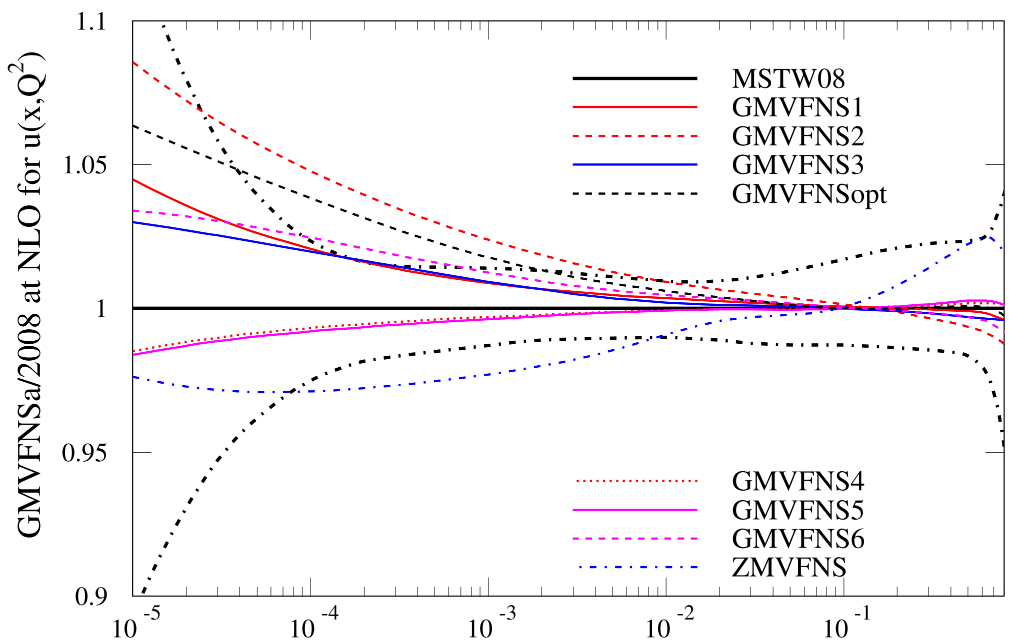
Better fit for GMVFNS1, GMVFNS3 and GMVFNS6. Best for optimal scheme.

Some changes in PDFs larger than one-sigma *uncertainty*.

If *optimal* used as centre variation a bit smaller since limit on χ^2 tighter.

Changes in $\alpha_S \sim 0.0004$, except for ZMVFNS (-0.0015).





Variations in partons extracted from global fit due to different choices of GM-VFNS at NNLO.

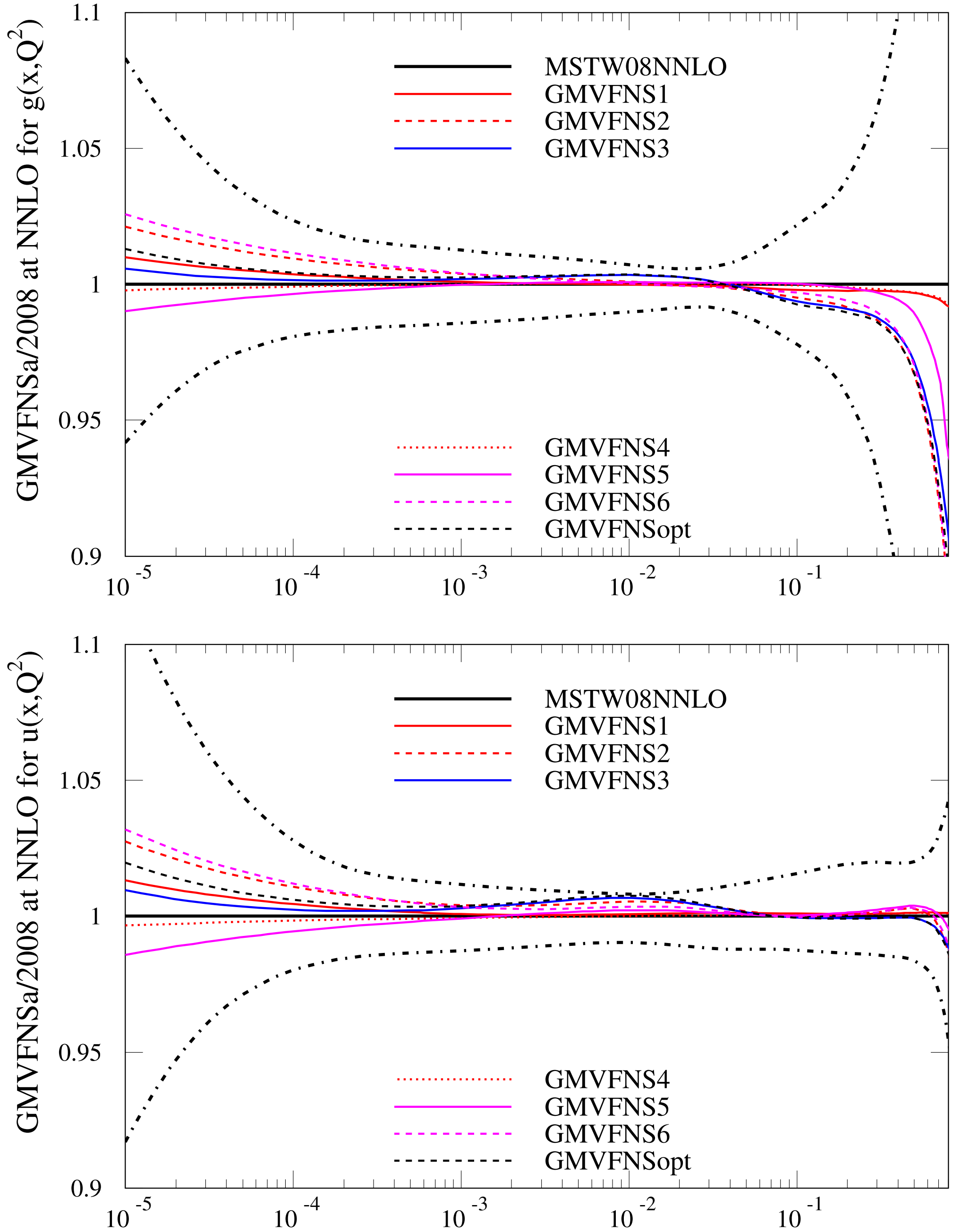
Initial changes in $\chi^2 < 20$.

Converge to about 10. None a marked improvement.

At worst changes approach $uncertainty$.

Biggest variation in high-x gluon, which has large uncertainty.

Variations in $\alpha_S(M_Z^2) \sim 0.0003$.



Cross-Section Results

At most 1.5% variation at Tevatron in σ_Z.

Up to $+3\%$ and -0.5% variation in σ_Z at the LHC. About half as much in σ_H due to higher average x sampled.

ZMVFNS clear outlier at LHC, but not the 8% from ZMVFNS to GMVFNS in CTEQ6.

At NNLO, other than from model dependence on α_S^3 FFNS term, maximum variations of order 0.5% at LHC. High-x gluon leads to 1% on σ_H at Tevatron.

Model uncertainties can be $> 1\%$ from region at very small x and low Q^2. Can perhaps input more small-x knowledge here. Effect far smaller in *optimal* scheme types.

Variation in best value of m_c with scheme.

Using optimal scheme at NLO best fit obtained for $m_c = 1.35\mathrm{GeV}$ (default $m_c = 1.45\mathrm{GeV}$). Uncertainty similar but more symmetric.

Fit ≈ 25 better than in normal fit.

Using optimal scheme at NNLO best fit obtained for $m_c = 1.23\mathrm{GeV}$ (default $m_c = 1.26\mathrm{GeV}$). Uncertainty similar but again a little more symmetric.

Fit almost identical to normal fit.

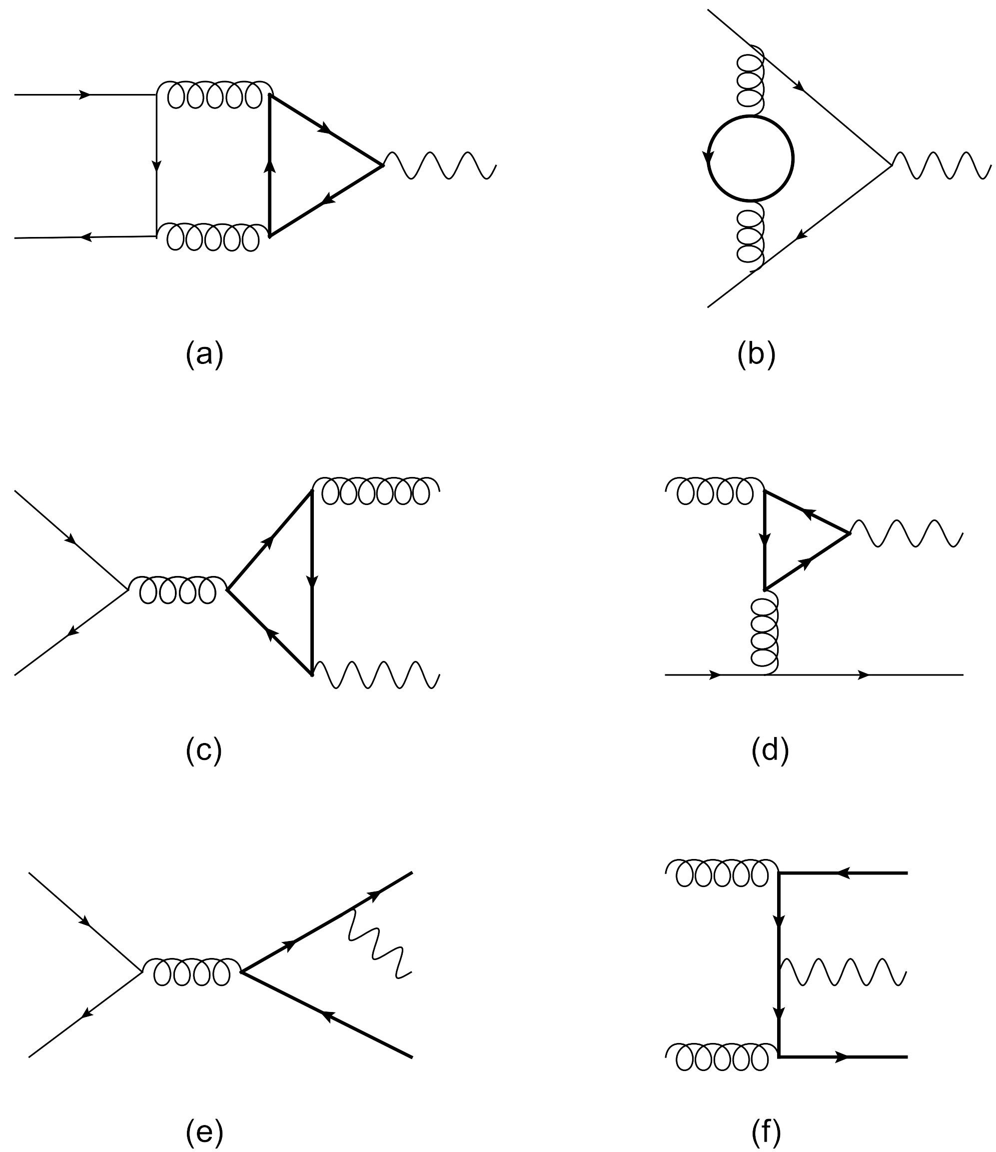
Slightly better agreement between NLO and NNLO values, but a bit lower.

Production of $Z + b\bar{b}$ in different schemes.

In 4FS diagrams including final state bottom quarks appear at $\mathcal{O}(\alpha_S^2)$.

Explicit expressions by P.J. Rijken, W.L. van Neerven.

Last two contribute to $b\bar{b}$ quarks in the final state, and are by orders of magnitude the dominant diagrams.



subprocess	Tevatron	LHC, 7 TeV	LHC, 14 TeV
$q+\bar{q}\to Z$	5.230×10^{-6}	-2.124×10^{-5}	-6.440×10^{-5}
$q+\bar{q}\to Z+g$	4.901×10^{-5}	6.185×10^{-5}	9.701×10^{-5}
$q(\bar{q})+g\to Z+q(\bar{q})$	-2.862×10^{-5}	-1.456×10^{-4}	-2.632×10^{-4}
$q+\bar{q}\to Z+b+\bar{b}$	3.754×10^{-4}	1.450×10^{-3}	3.382×10^{-3}
$g+g\to Z+b+\bar{b}$	2.090×10^{-4}	5.287×10^{-3}	1.997×10^{-2}
total	6.100×10^{-4}	6.632×10^{-3}	2.312×10^{-2}

Additional $\mathcal{O}(\alpha_S^2)$ contributions to the total Z 4FS NNLO cross section in nb (multiplied by leptonic branching ratio) at the Tevatron and LHC arising from real and virtual b-quark processes.

Clearly the $g\to b\bar{b}$ initiated process is very dominant at the LHC.

Tevatron, 1.96 TeV	$B \cdot \sigma^Z_{\rm NLO}(\rm 4FS)$	$B \cdot \sigma^Z_{\rm NLO}(\rm 5FS)$	$B \cdot \sigma^Z_{\rm NLO}(\rm 5FS, b)$
σ_0^Z	0.1989	0.1990	0.0012
σ_1^Z	0.0413	0.0436	-0.0002
total	0.2402	0.2426	0.0010

LHC, 7 TeV	$B \cdot \sigma^Z_{\rm NLO}(\rm 4FS)$	$B \cdot \sigma^Z_{\rm NLO}(\rm 5FS)$	$B \cdot \sigma^Z_{\rm NLO}(\rm 5FS, b)$
σ_0^Z	0.7846	0.8023	0.0205
σ_1^Z	0.1206	0.1285	-0.0020
total	0.9052	0.9308	0.0185

LHC, 14 TeV	$B \cdot \sigma^Z_{\rm NLO}(\rm 4FS)$	$B \cdot \sigma^Z_{\rm NLO}(\rm 5FS)$	$B \cdot \sigma^Z_{\rm NLO}(\rm 5FS, b)$
σ_0^Z	1.6922	1.7545	0.0656
σ_1^Z	0.2303	0.2465	-0.0050
total	1.9225	2.0009	0.0601

NLO predictions for the Z cross section (in nb), multiplied by leptonic branching ratio, at the Tevatron and LHC using MSTW 2008 NLO PDF, broken down into α_S^n $(n = 0, 1)$ contributions, in the 4FS and 5FS calculation. The final column gives the contribution in the 5FS from processes where the Z couples directly to b quarks.

Tevatron, 1.96 TeV	$B \cdot \sigma^Z_{\rm NNLO}(\rm 4FS)$	$B \cdot \sigma^Z_{\rm NNLO}(\rm 5FS)$	$B \cdot \sigma^Z_{\rm NNLO}(\rm 5FS, b)$
σ_0^Z	0.2013	0.2016	0.0012
σ_1^Z	0.0409	0.0431	-0.0002
σ_2^Z	0.0063	0.0060	-0.0003
total	0.2485	0.2507	**0.0008**
$\Delta_b \sigma^Z$	**0.0006**	–	
total $+ \Delta_b \sigma^Z$	0.2491	0.2507	

NNLO predictions for the total Z cross section (in nb), multiplied by leptonic branching ratio at the Tevatron using MSTW 2008 NNLO PDFs as input, broken down into the α_S^n $(n = 0, 1, 2)$ contributions. The final column gives the contribution to the 5FS cross sections from processes where the Z couples directly to b quarks. The additional $\mathcal{O}(\alpha_S^2)$ contributions to the cross section arising from real and virtual b-quark processes are added to the 4FS cross section in the last line.

Good overall agreement.

LHC, 7 TeV	$B \cdot \sigma^Z_{\rm NNLO}(\rm 4FS)$	$B \cdot \sigma^Z_{\rm NNLO}(\rm 5FS)$	$B \cdot \sigma^Z_{\rm NNLO}(\rm 5FS, b)$
σ^Z_0	0.8083	0.8266	0.0202
σ^Z_1	0.1239	0.1322	-0.0020
σ^Z_2	0.0037	-0.0002	-0.0037
total	0.9359	0.9586	**0.0145**
$\Delta_b \sigma^Z$	**0.0066**	–	
total + $\Delta_b \sigma^Z$	0.9426	0.9586	

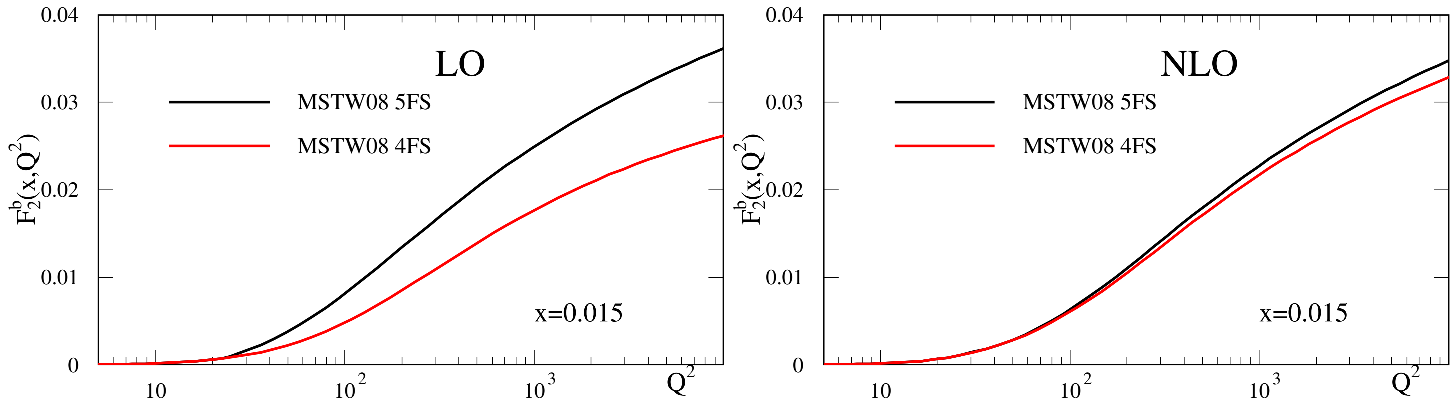
NNLO predictions for the total Z cross section (in nb), multiplied by leptonic branching ratio at the LHC (7TeV) using MSTW 2008 NNLO PDFs as input, broken down into the α_S^n ($n = 0, 1, 2$) contributions. The final column gives the contribution to the 5FS cross sections from processes where the Z couples directly to b quarks. The additional $\mathcal{O}(\alpha_S^2)$ contributions to the cross section arising from real and virtual b-quark processes are added to the 4FS cross section in the last line of each sub-table.

Pretty good agreement for light flavours, but 5FS more than twice 4FS for b contribution.

LHC, 14 TeV	$B \cdot \sigma^Z_{\rm NNLO}(\rm 4FS)$	$B \cdot \sigma^Z_{\rm NNLO}(\rm 5FS)$	$B \cdot \sigma^Z_{\rm NNLO}(\rm 5FS, b)$
σ^Z_0	1.7472	1.8110	0.0641
σ^Z_1	0.2384	0.2557	-0.0050
σ^Z_2	-0.0047	-0.0153	-0.0107
total	1.9809	2.0514	**0.0484**
$\Delta_b \sigma^Z$	**0.0231**	–	
total + $\Delta_b \sigma^Z$	2.0040	2.0514	

NNLO predictions for the total Z cross section (in nb), multiplied by leptonic branching ratio at the LHC (14TeV) using MSTW 2008 NNLO PDFs as input, broken down into the α_S^n $(n = 0, 1, 2)$ contributions. The final column gives the contribution to the 5FS cross sections from processes where the Z couples directly to b quarks. The additional $\mathcal{O}(\alpha_S^2)$ contributions to the cross section arising from real and virtual b-quark processes are added to the 4FS cross section in the last line of each sub-table.

Pretty good agreement for light flavours, but 5FS twice 4FS for b contribution.



At LO for relevant $x \sim 0.015$ the lack of resummation in 4FS leads to the structure function (driven mainly by $g \to b\bar{b}$) being suppressed to only $\sim 70\%$ of 5FS result. This should be squared in hadron-hadron process, hence factor of ~ 2.

At NLO double log corrects most of this, only $\sim 90\%$ suppression in structure functions.

However, only one of the incoming gluons has double-log correction in hadron-hadron process at NLO ($\mathcal{O}(\alpha_S^3)$). Expect correction factor of about 1.5 at NLO.

Cross Section	$m_b \neq 0$ (pb) [ratio]	$m_b = 0$ (pb) [ratio]
σ^{LO}	2.21[−]	2.37[−]
σ^{NLO} *inclusive*	3.40[1.54]	3.64[1.54]
σ^{NLO} *exclusive*	2.80[1.27]	3.01[1.27]

Indeed, much as seen in recent Febres Cordero, Reina and Wackeroth calculation.

4FS still about 70% that of 5FS calculation.

Slower convergence of 4FS calculations in hadron-hadron processes. Similar results seen for Higgs cross-sections.

Conclusions

Using our current default GM-VFNS MSTW have looked at the results of varying both the charm and bottom quark masses in the context of the MSTW2008 global fit. m_c determined with good precision, but rather lower at NNLO than NLO. Reduced difference in *optimal* scheme. We have provided 3- and 4-flavour sets for the variety of masses.

Uncertainties from mass variations significant but certainly not a dominant effect. Recommend $\Delta m_c = \pm 0.15\text{GeV}$ and $\Delta m_b = \pm 0.25\text{GeV}$ and adding uncertainties in quadrature.

Discussed variations in definition of GMVFNS. *Optimal* version the smoothest near threshold and best fit at NLO. Little variation in smoothness or fit quality at NNLO. At NNLO PDFs usually (well) within uncertainties, and cross-sections rarely change by 1%. GMVFNS variation more significant source of uncertainty at NLO. Uncertainty can be more systematically estimated in future.

Compared 4FS and 5FS contributions to Z production. Little variation between two for light quark contributions, but significant for bottom quarks. Bigger discrepancy and slower convergence than for structure functions. Understood in terms of manner of log summation.

Other constraints on Masses

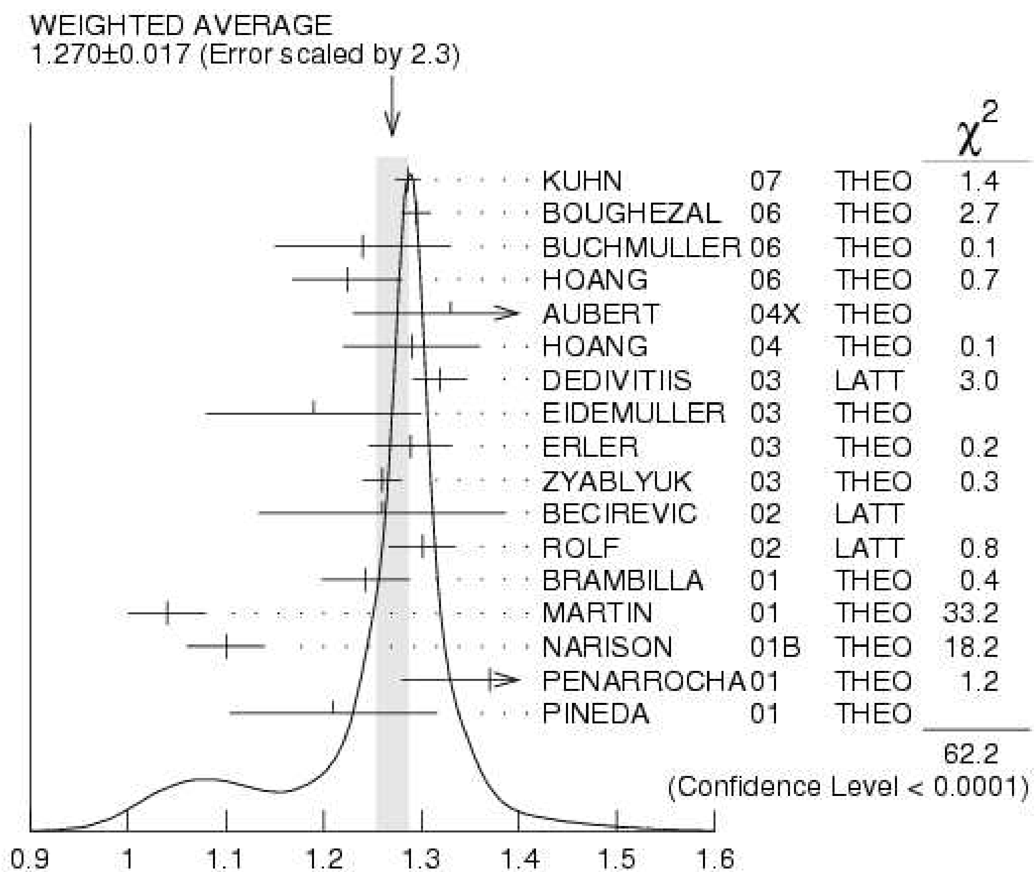
We use pole mass definition since the perturbative transition matrix elements $A_{ij}(\mu^2/m_h^2)$ (Buzu *et al*, Blümlein *et al*) which give boundary conditions for evolution and coefficient functions $C_{ij}^{FFNS}(z, m_h^2)$ (Laenen *et al*) used in definition of GM-VFNS defined in "on mass-shell"" renormalization scheme.

Could convert to other schemes, but not aware that anyone does. Would lose very convenient decoupling properties.

Is a *pseudo*-physical definition since it is not dependent on order of perturbation series or scale, but suffers from fact that there are no free quarks.

Latter point leads to significant power corrections – $\Lambda^2_{\rm QCD}/m_h^2$ and higher powers, i.e. leading twist definitions/determinations contaminated by renormalon ambiguities.

Accurate determinations of m_c and m_b nearly always given using $\overline{MS}$ definition – good apparent perturbative stability. However, even this gives individual determinations with much greater spread than quoted uncertainties, e.g. from 2008 PDG



PDG quotes $m_c(\mu = m_c) = 1.27^{+0.07}_{-0.11}$ and $m_b(\mu = m_b) = 4.20^{+0.17}_{-0.07}$.

In principle know the conversion from $\overline{MS}$ definition to pole mass to $\mathcal{O}(\alpha_S^3)$ (Chetyrkin and Steinhauser, Melnikov and van Ritbergen).

Using MSTW NNLO α_S value for bottom

$$m_b^{\rm pole} = m_b^{\overline{MS}}(\mu = m_b) * (1 + 0.095 + 0.045 + 0.035 + \cdots) = 4.9 {\rm GeV}$$

with moderate convergence of the series.

For charm the equation is

$$m_c^{\rm pole} = m_c^{\overline{MS}}(\mu = m_c) * (1 + 0.16 + 0.14 + 0.18 + \cdots)$$

So no apparent convergence at all due to larger coupling and less gluon-light-quark loop cancellation in coefficients (naively get $1.88 {\rm GeV}$).

Conversion severely renormalon contaminated. For bottom assume $\mathcal{O}(\alpha_S^3)$ is smallest term in series so is the point where the series is truncated and this term is the approx. size of power correction

$$\rightarrow m_b^{\rm pole} = 4.9{\rm GeV} \pm 0.15{\rm GeV}$$

Uncertainty similar to renormalon calculation estimate Beneke and Braun – 1994.

Not even clear where series for m_c starts to diverge (immediately?).

However, conversion for $m_b - m_c$ has cancellation of leading power correction, and $m_c - m_b = 3.4{\rm GeV}$ with very small error (Hoang and Manohar). Using this

$$m_c^{\rm pole} = 1.5{\rm GeV} \pm 0.17{\rm GeV}$$

Considering these constraints together with our fit results we suggest using

$m_c = 1.4{\rm GeV}$ with uncertainty $0.15{\rm GeV}$ for 68% C.L or $0.25{\rm GeV}$ for 90% C.L

$m_b = 4.75$ with uncertainty $0.25{\rm GeV}$ for 68% C.L or $0.5{\rm GeV}$ for 90% C.L (in latter case take round value for convenience).

Model $\mathcal{O}(\alpha_S^3)$ at low Q^2 using known leading threshold logarithms (Laenen and Moch) and leading $\ln(1/x)$ term from k_T-dependent impact factors Catani, *et al*.

Include latter in form

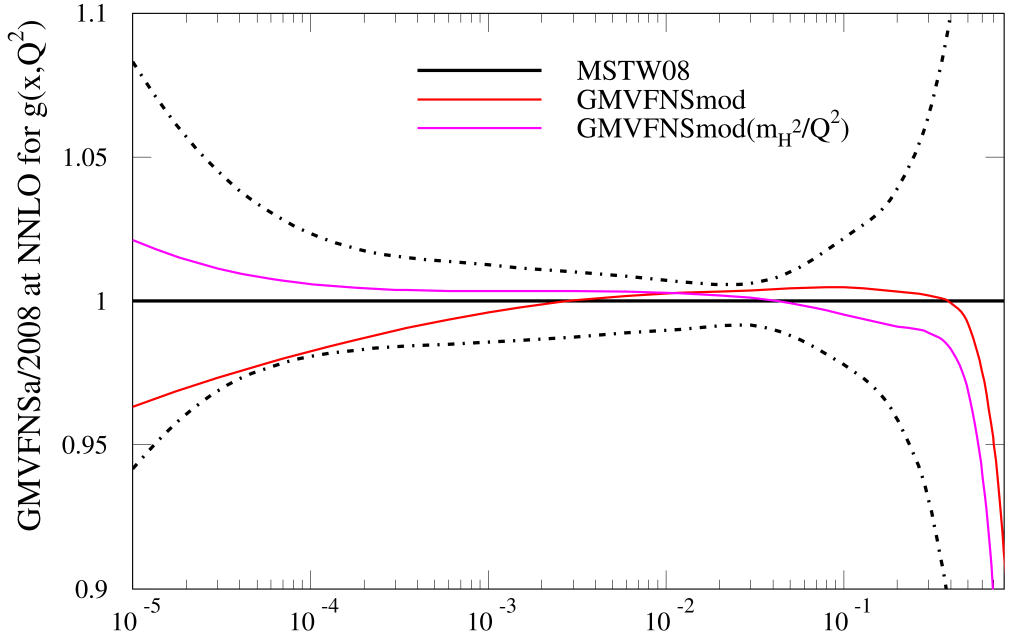
$$\propto (1 - z/x_{\max})^a(\ln(1/z) - b)/z,$$

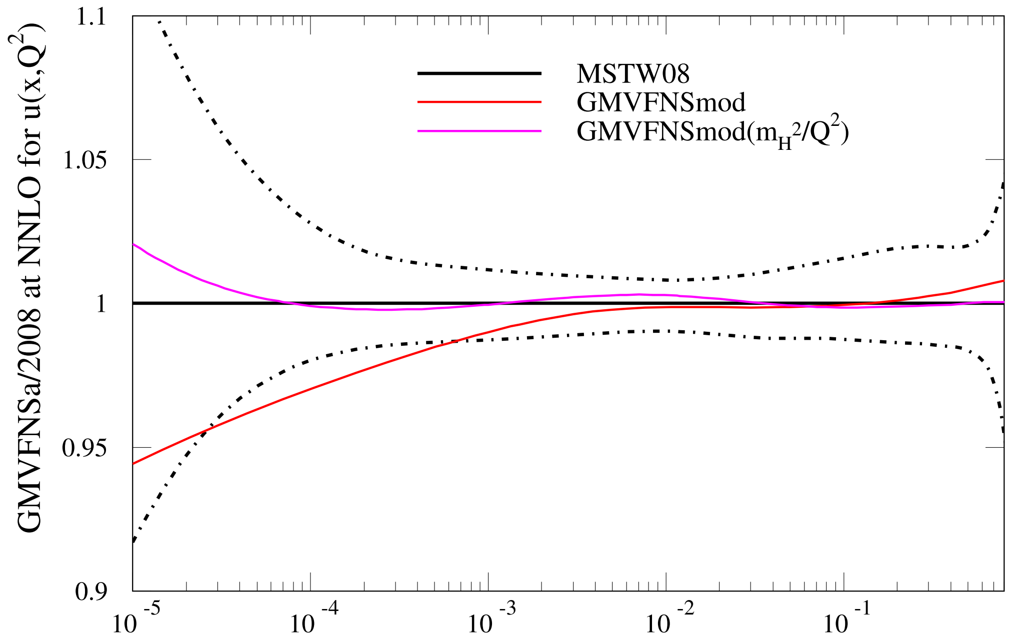
where default $a = 20, b = 4$.

Variations in a make little difference. Maximum *sensible* variation of $b = 2$ leads to effect in PDFs shown.

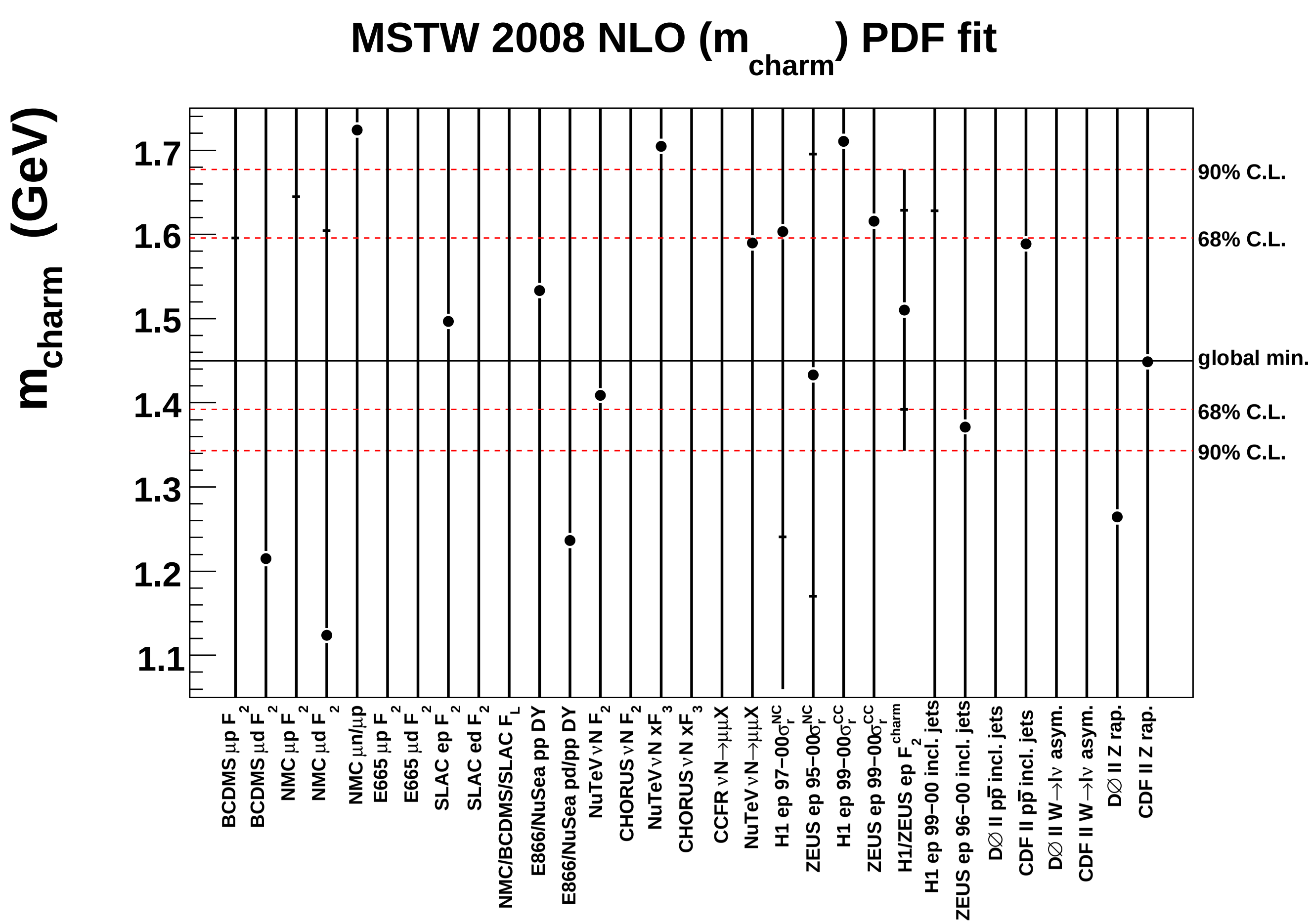
Major effect at smallest x.

Moderated significantly if $\mathcal{O}(\alpha_S^3)$ falls away rather than frozen.





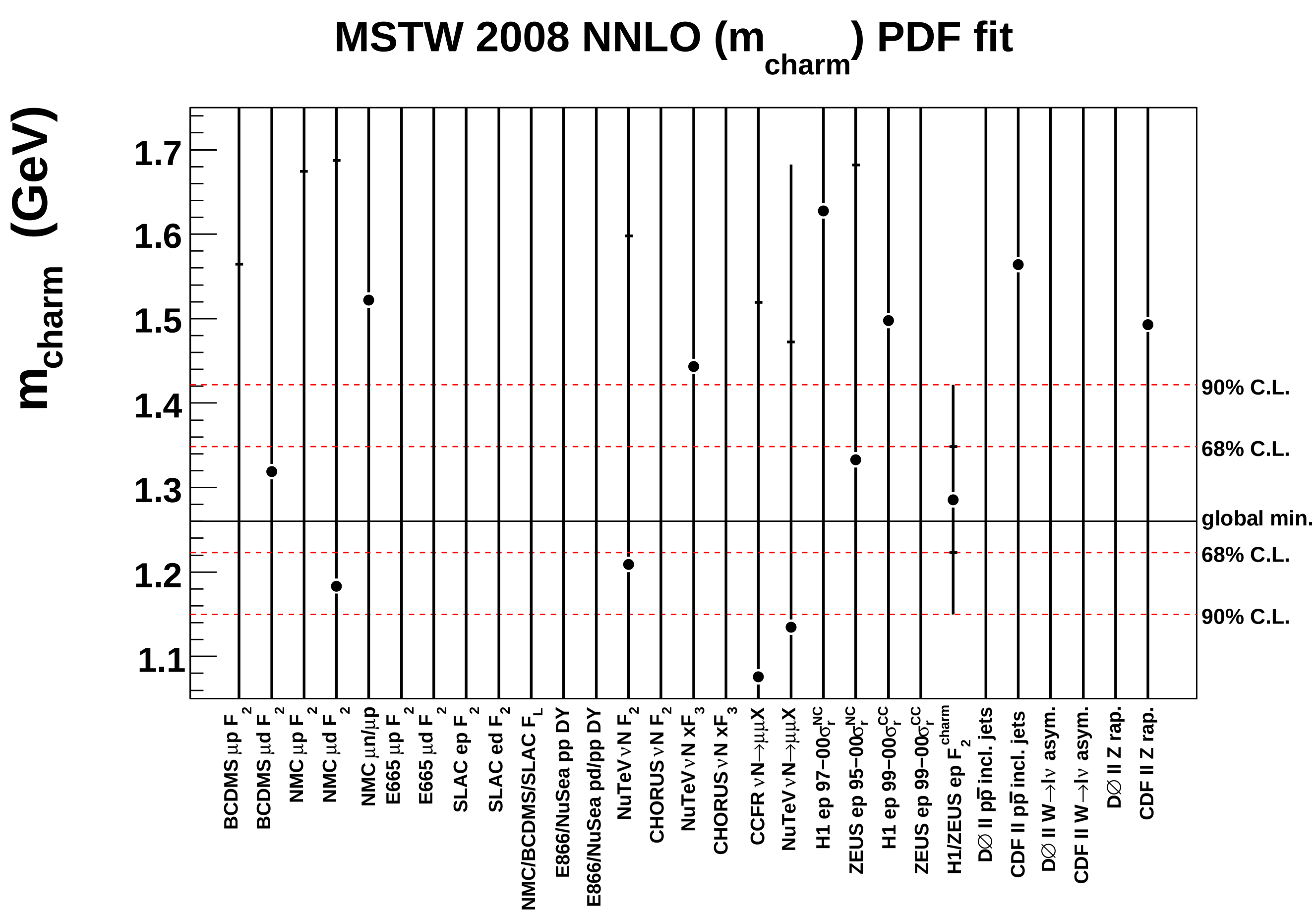
Preference of each data set.



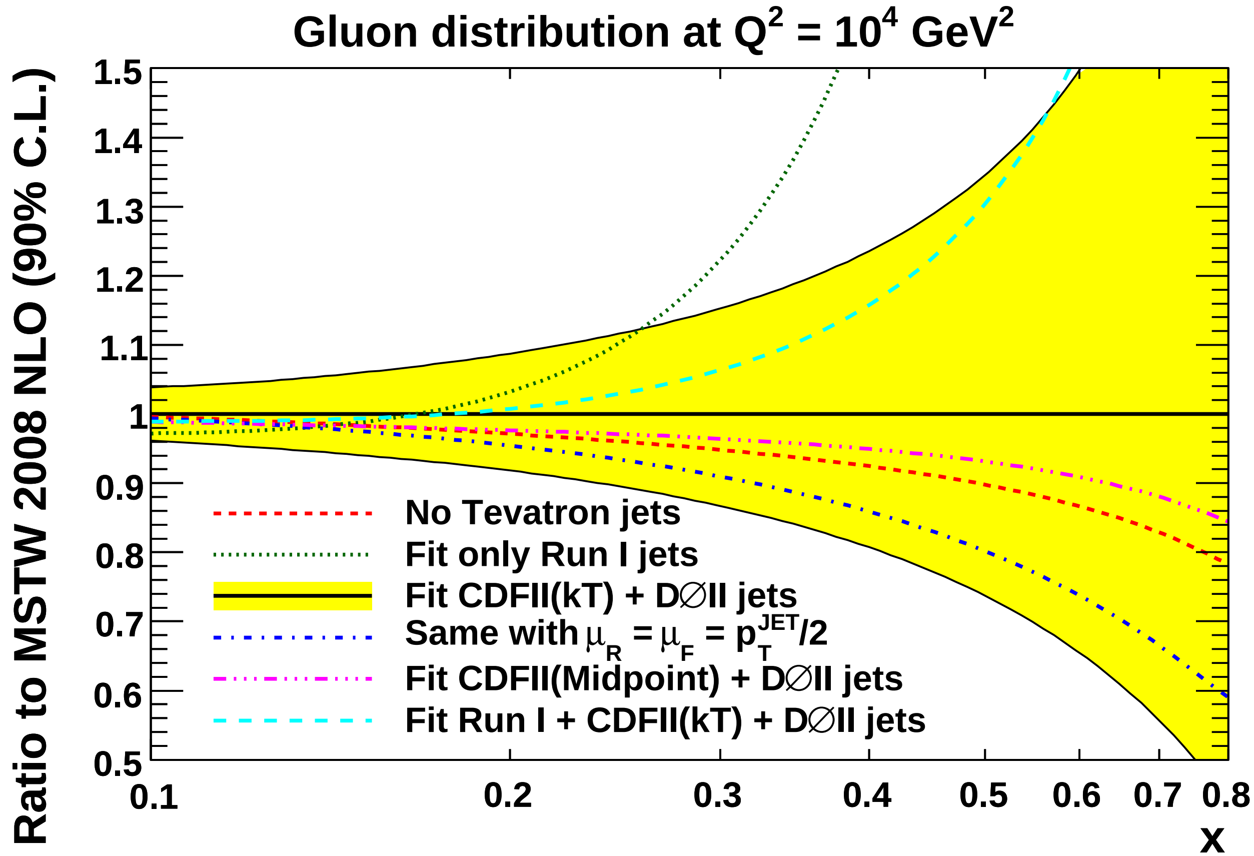
$F_2^c(x, Q^2)$ data most discriminating. In HERA inclusive data changes in m_c compensated for by change in gluon and α_S.

NMC data prefer lower m_c - quicker evolution near threshold. BCDMS mainly prefer correlated lower α_S.

Preference of each data set.



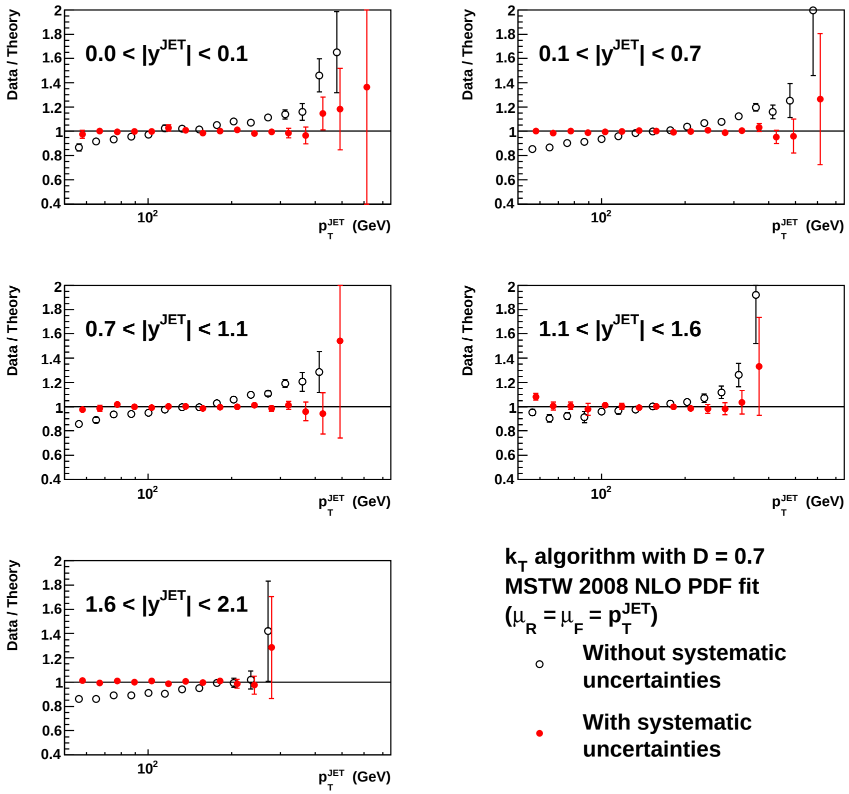
$F_2^c(x, Q^2)$ data most discriminating. In HERA inclusive data changes in m_c compensated for by change in gluon and α_S.



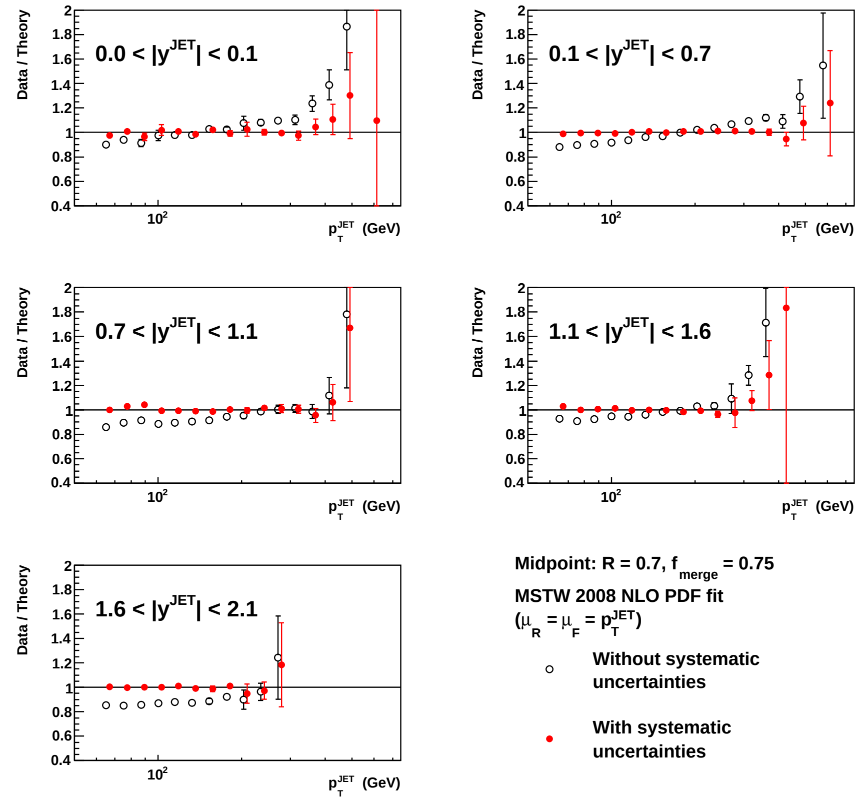
Change of scale to $p_T/2$ for Tevatron jets changes gluon above $x \sim 0.4$ by a bit more than one σ uncertainty, but fit to D0 data deteriorates by amount equivalent to this change. Change to $2p_T$ acceptable fit, slightly higher very high-x gluon. Little change below $x = 0.2$ in either case.

When changing gluon shape k_T-algorithm CDF jet data by far most constraining jet data (seen implicitly from eigenvector constraints).

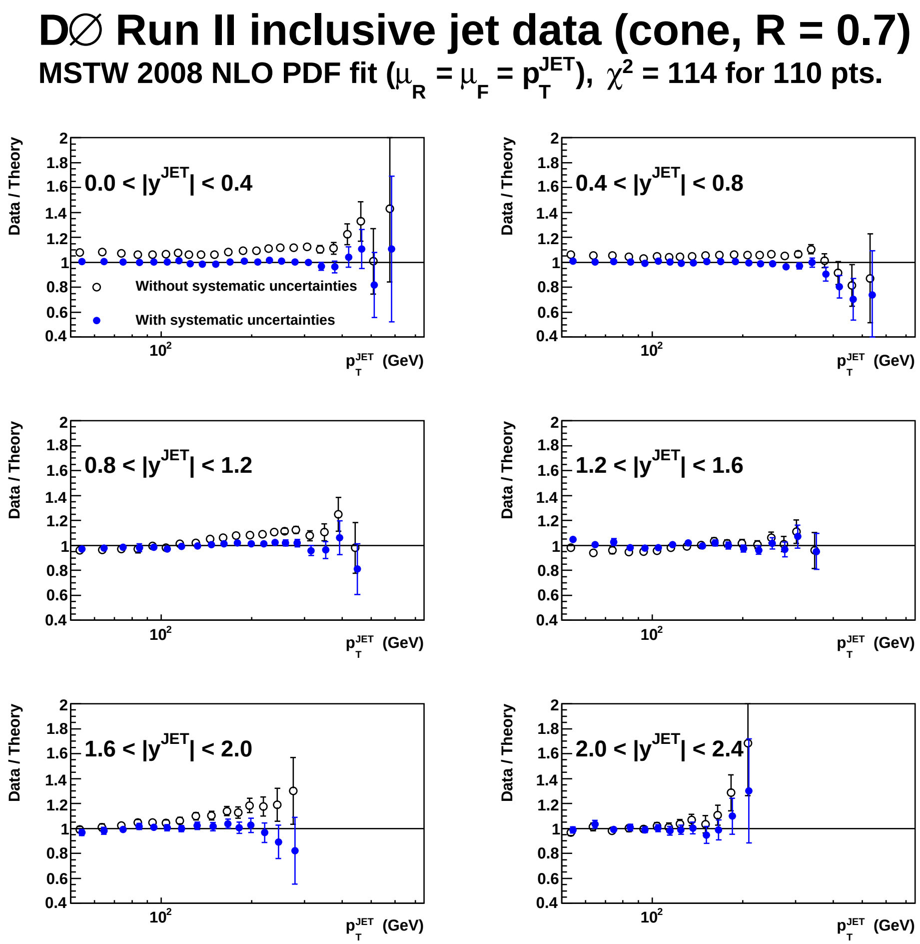
CDF Run II inclusive jet data, χ^2 = 56 for 76 pts.

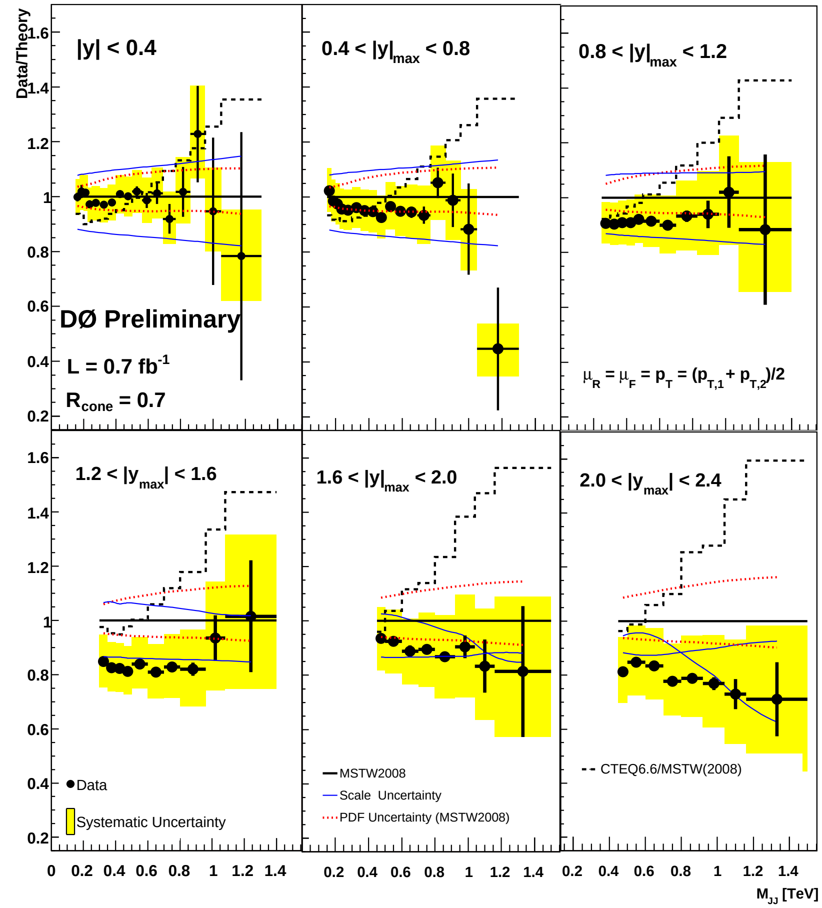


CDF Run II inclusive jet data, χ^2 = 108 for 72 pts.

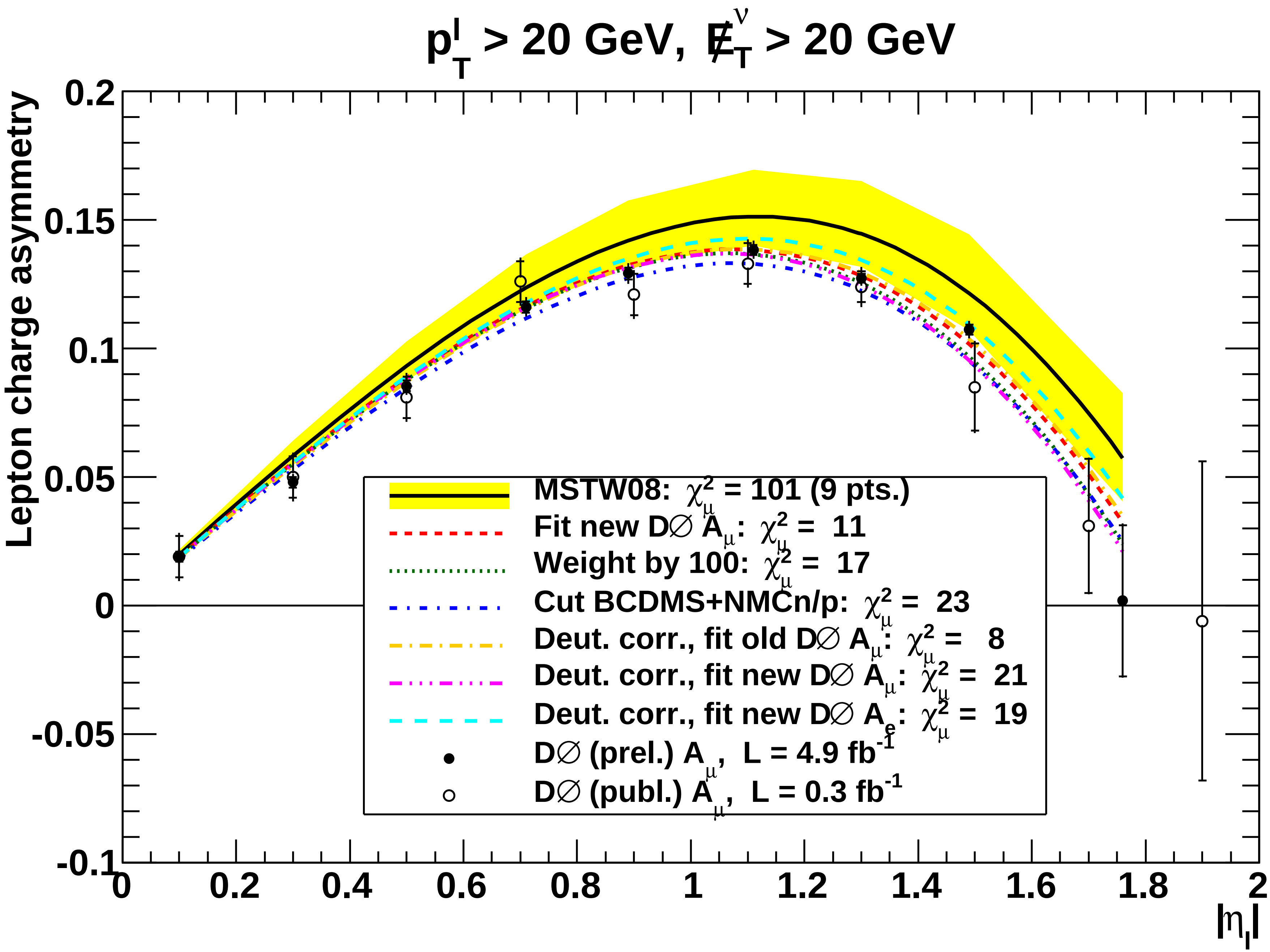


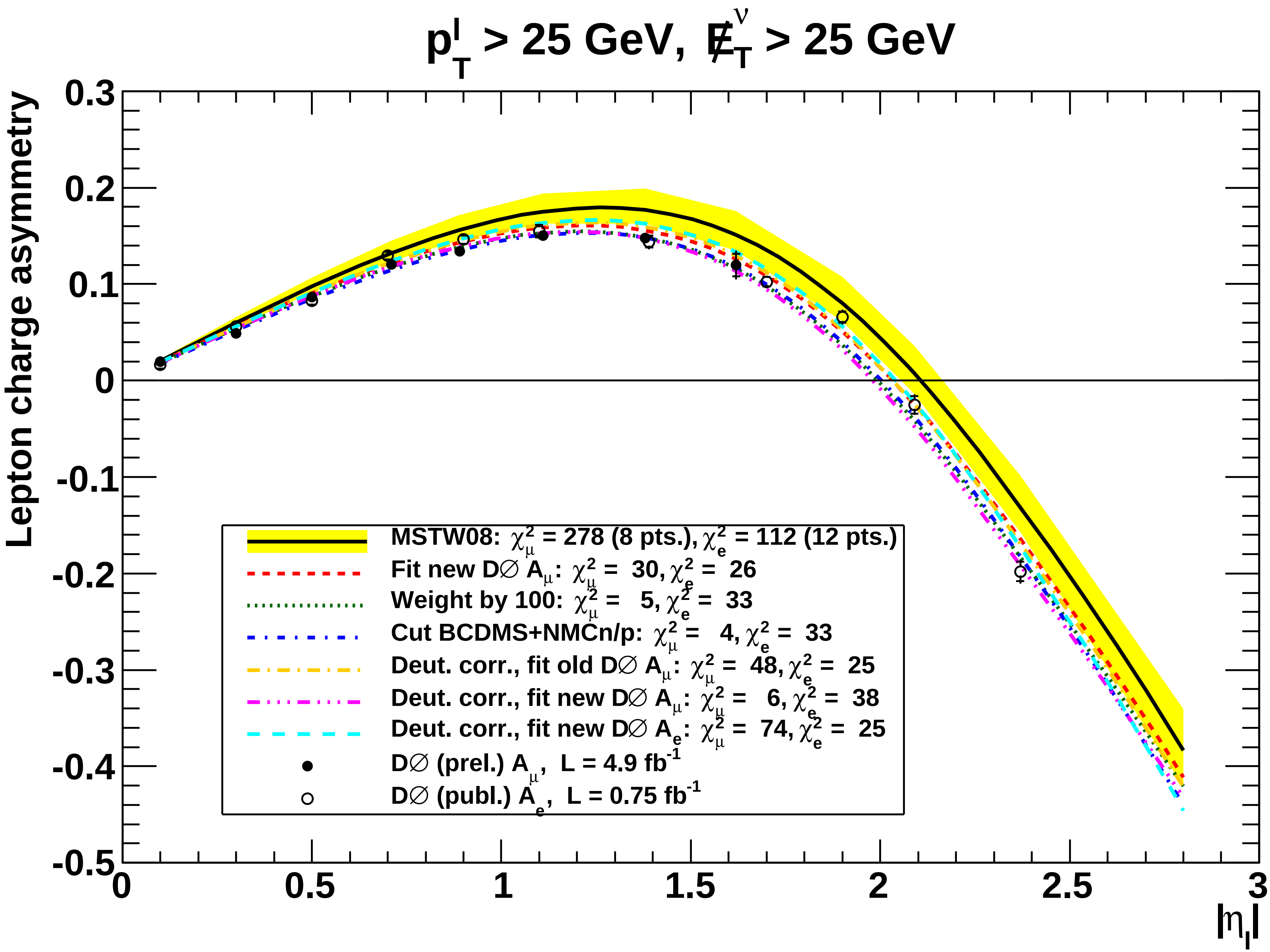
Two different analyses of CDF jet data lead to very similar data/theory.



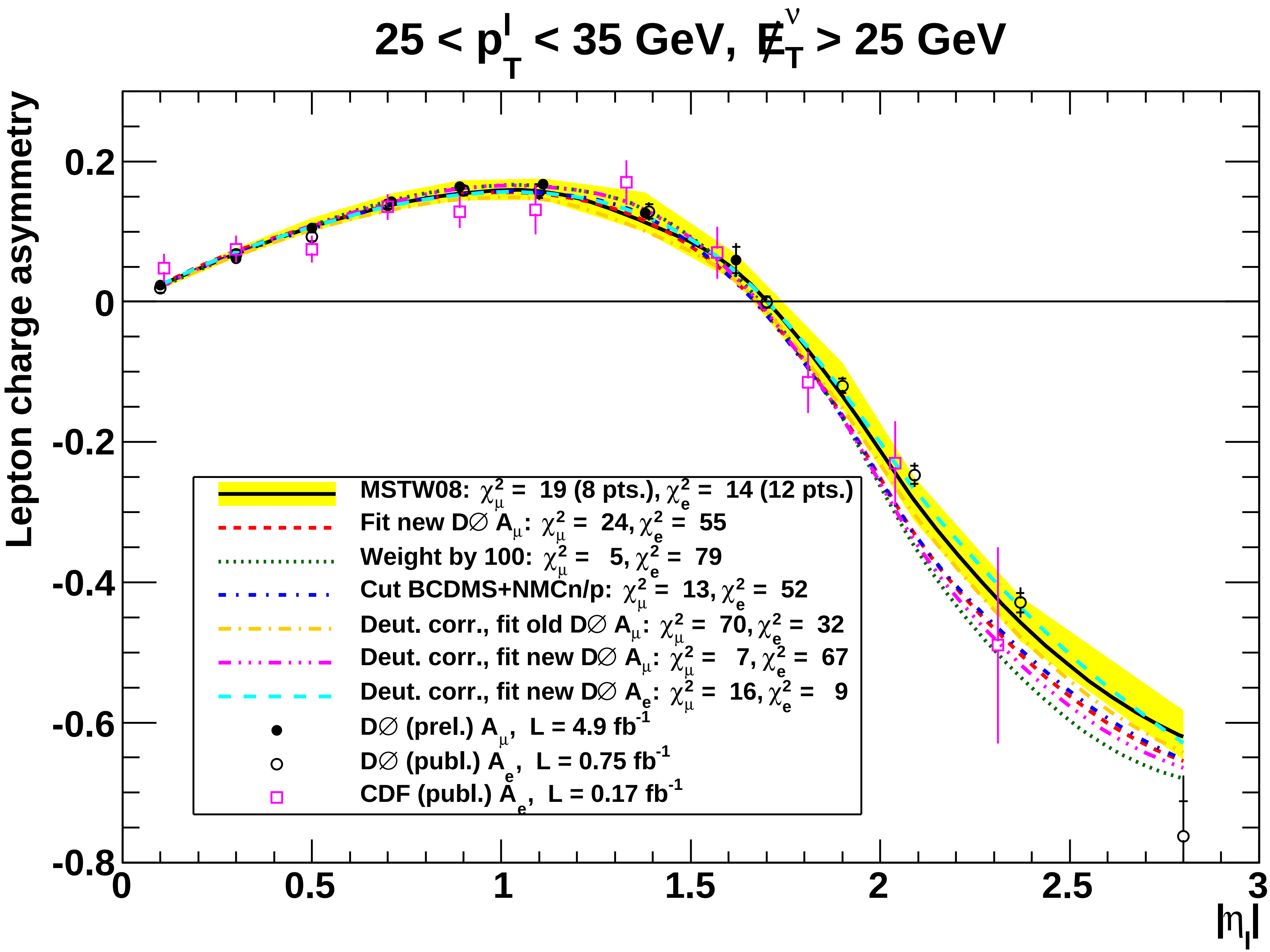


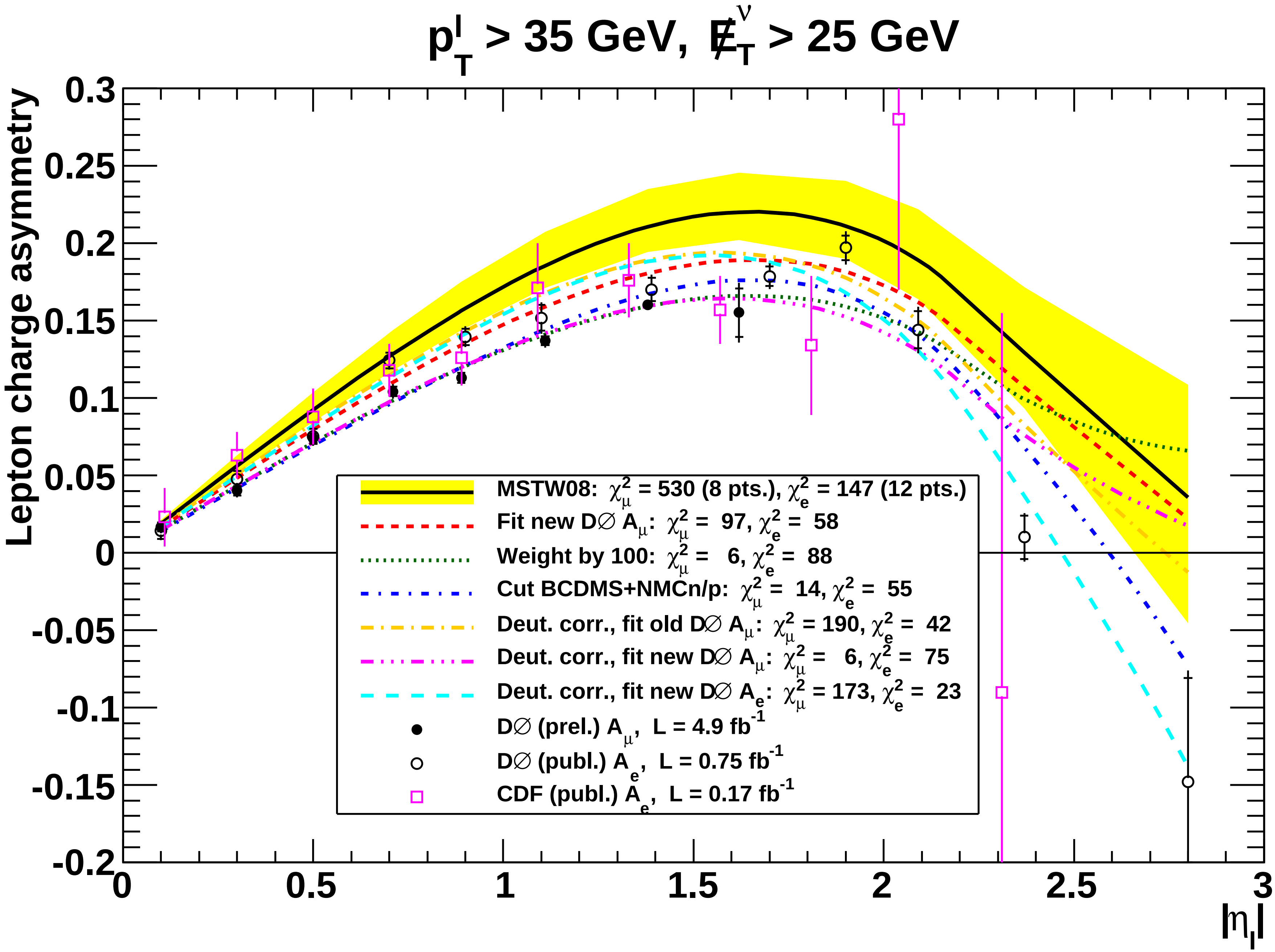
Two different analyses of D0 jet data lead to rather different data/theory. Scale variation peculiar as a function of rapidity for dijets.


$p_T^l > 20$ GeV, $\not{E}_T^\nu > 20$ GeV
Lepton charge asymmetry
$|\eta_l|$
MSTW08: χ^2_μ = 101 (9 pts.)
Fit new D∅ A_μ: χ^2_μ = 11
Weight by 100: χ^2_μ = 17
Cut BCDMS+NMCn/p: χ^2_μ = 23
Deut. corr., fit old D∅ A_μ: χ^2_μ = 8
Deut. corr., fit new D∅ A_μ: χ^2_μ = 21
Deut. corr., fit new D∅ A_e: χ^2_μ = 19
D∅ (prel.) A_μ, L = 4.9 fb^{-1}
D∅ (publ.) A_μ, L = 0.3 fb^{-1}

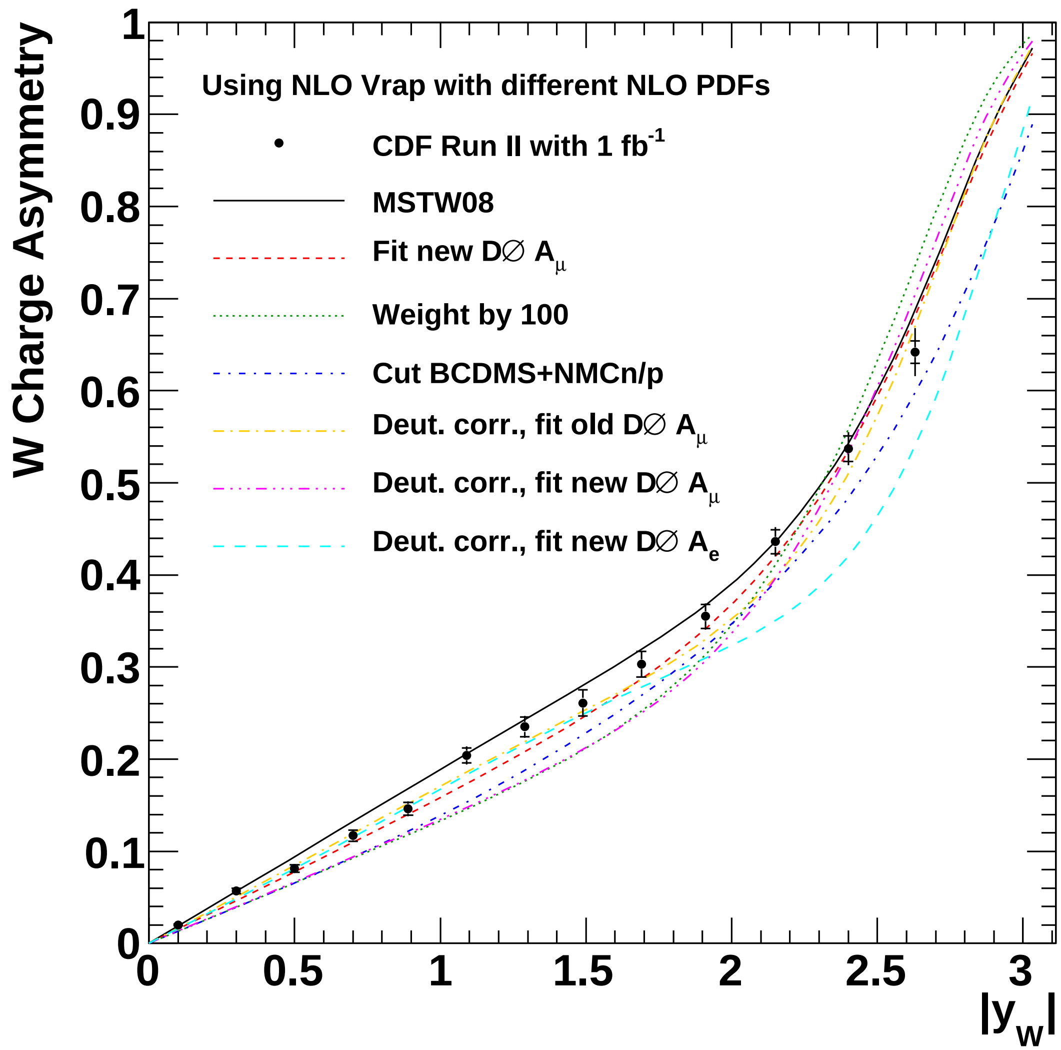


$p_T^l > 25$ GeV, $\not{E}_T^{\nu} > 25$ GeV
Lepton charge asymmetry
$|\eta_l|$
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
0.5
1
1.5
2
2.5
3
MSTW08: χ^2_μ = 278 (8 pts.), χ^2_e = 112 (12 pts.)
Fit new DØ A_μ: χ^2_μ = 30, χ^2_e = 26
Weight by 100: χ^2_μ = 5, χ^2_e = 33
Cut BCDMS+NMCn/p: χ^2_μ = 4, χ^2_e = 33
Deut. corr., fit old DØ A_μ: χ^2_μ = 48, χ^2_e = 25
Deut. corr., fit new DØ A_μ: χ^2_μ = 6, χ^2_e = 38
Deut. corr., fit new DØ A_e: χ^2_μ = 74, χ^2_e = 25
DØ (prel.) A_μ, L = 4.9 fb^{-1}
DØ (publ.) A_e, L = 0.75 fb^{-1}


25 < p_T^l < 35 GeV, $\not{E}_T^\nu$ > 25 GeV
Lepton charge asymmetry
0.2
0
-0.2
-0.4
-0.6
-0.8
0
0.5
1
1.5
2
2.5
3
$|\eta_l|$
MSTW08: χ^2_μ = 19 (8 pts.), χ^2_e = 14 (12 pts.)
Fit new DØ A_μ: χ^2_μ = 24, χ^2_e = 55
Weight by 100: χ^2_μ = 5, χ^2_e = 79
Cut BCDMS+NMCn/p: χ^2_μ = 13, χ^2_e = 52
Deut. corr., fit old DØ A_μ: χ^2_μ = 70, χ^2_e = 32
Deut. corr., fit new DØ A_μ: χ^2_μ = 7, χ^2_e = 67
Deut. corr., fit new DØ A_e: χ^2_μ = 16, χ^2_e = 9
DØ (prel.) A_μ, L = 4.9 fb^{-1}
DØ (publ.) A_e, L = 0.75 fb^{-1}
CDF (publ.) A_e, L = 0.17 fb^{-1}



p_T^l > 35 GeV, E̸_T^ν > 25 GeV
Lepton charge asymmetry
|η_l|
MSTW08: χ²_μ = 530 (8 pts.), χ²_e = 147 (12 pts.)
Fit new DØ A_μ: χ²_μ = 97, χ²_e = 58
Weight by 100: χ²_μ = 6, χ²_e = 88
Cut BCDMS+NMCn/p: χ²_μ = 14, χ²_e = 55
Deut. corr., fit old DØ A_μ: χ²_μ = 190, χ²_e = 42
Deut. corr., fit new DØ A_μ: χ²_μ = 6, χ²_e = 75
Deut. corr., fit new DØ A_e: χ²_μ = 173, χ²_e = 23
DØ (prel.) A_μ, L = 4.9 fb⁻¹
DØ (publ.) A_e, L = 0.75 fb⁻¹
CDF (publ.) A_e, L = 0.17 fb⁻¹


Using NLO Vrap with different NLO PDFs
CDF Run II with 1 fb-1
MSTW08
Fit new DØ Aμ
Weight by 100
Cut BCDMS+NMCn/p
Deut. corr., fit old DØ Aμ
Deut. corr., fit new DØ Aμ
Deut. corr., fit new DØ Ae
W Charge Asymmetry
|yW|
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.5
1.5
2
2.5
3