

# Beam-energy dependence of the anisotropy scaling functions for identified particle species

*Roy A. Lacey  
Stony Brook University*

## Takeaway

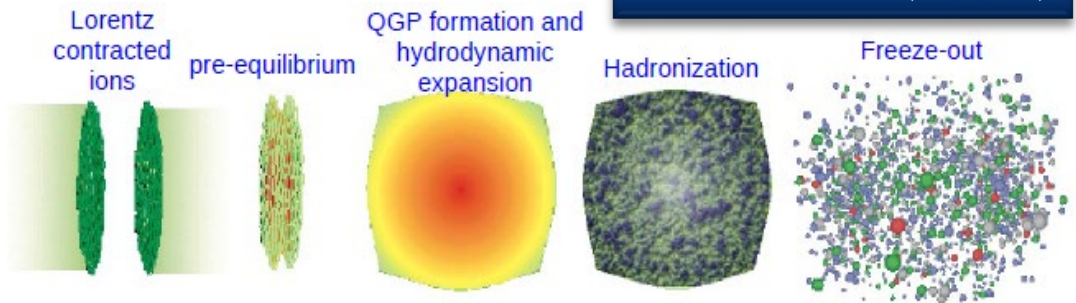
*The  $\sqrt{s_{NN}}$ -dependence of the anisotropy scaling functions for PID species can be used to:*

- I. Delineate the respective influence of expansion dynamics and viscous attenuation*
- II. Constrain  $\frac{\eta}{s}(T, \mu_B, \mu_I, \mu_S)$ ?*
  - ✓ Give insight on a possible critical point in the nuclear matter phase diagram*

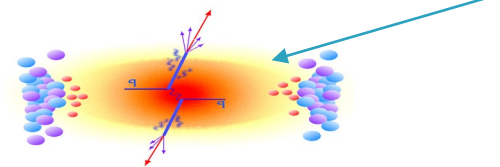


# Azimuthal Anisotropy

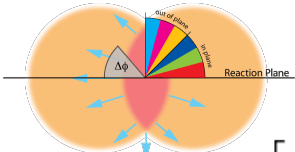
## A+A collision (S. Bass)



Drives azimuthal Anisotropy



Jet quenching – high  $p_T$



$$R_{AA}(p_T, L) \simeq \exp \left[ -\frac{2\alpha_s C_F}{\sqrt{\pi}} L \sqrt{\hat{q} \frac{\mathcal{L}}{p_T}} \right]$$

Phys. Lett. B519, 199 (2001)

$$R_{v_2}(p_T, \Delta L) = \frac{R_{AA}(90^\circ, p_T)}{R_{AA}(0^\circ, p_T)} = \frac{1 - 2v_2(p_T)}{1 + 2v_2(p_T)}$$

Specific dependencies on  $\sqrt{p_T}$ ,  $\Delta L$  and  $\hat{q}$

Reaction Plane

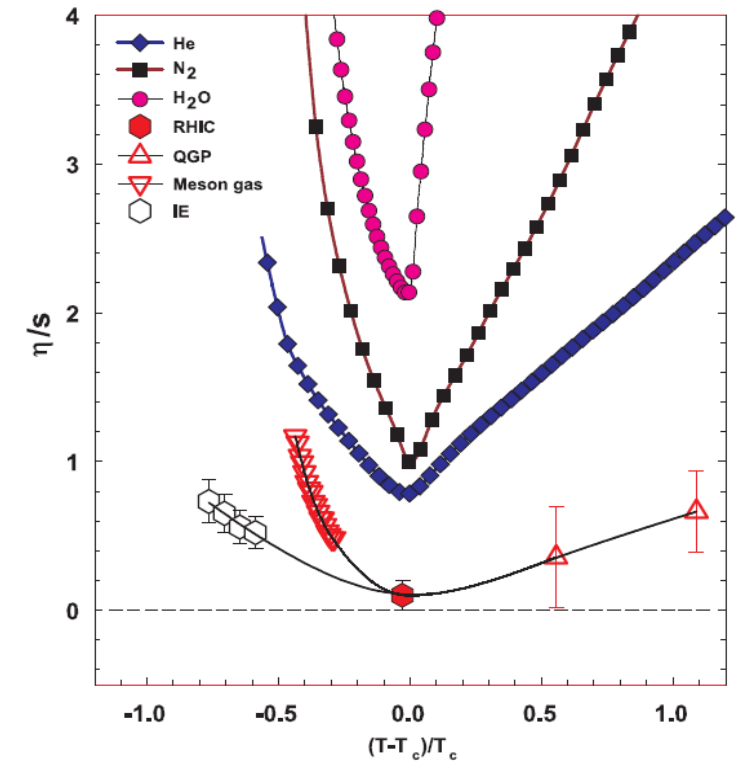
Collective flow – low  $p_T$

$$v_n = \varepsilon_n e^{-\frac{\beta}{RT} n(n+\kappa p_T^2)}, RT \equiv \mathbb{R} \propto \langle N_{\text{chg}} \rangle^{1/3}$$

Specific dependencies on  $n, \varepsilon_n, p_T, RT$  and  $\frac{\eta}{s}, \frac{\xi}{s}$

Question  $\frac{\eta}{s}(T, \mu_B, \mu_I, \mu_S)?$

Lacey et. al. Phys.Rev.Lett. 98 (2007) 092301



- Could give insights on:
  - ✓ the location of the critical point in the QCD phase diagram
  - ✓ Viscosity of particles vs. antiparticles? (influence of charged currents)

➤ Anisotropy Scaling Functions (ASF) for unidentified and identified particle species are used as constraints

## Anisotropy Scaling Functions

$$v_n(p_T, cent) = \varepsilon_n e^{-\frac{\beta}{RT} [n(n + \kappa p_T^2)]}, RT \equiv \mathbb{R} \propto \langle N_{\text{chg}} \rangle^{1/3}$$

Same harmonic with variable centrality

$$\frac{v_n(p_T, cent)}{\varepsilon_n} = \left( \frac{v'_n(p_T, cent)}{\varepsilon'_n} \right) e^{-\frac{\beta}{\mathbb{R}} [n(n + \kappa p_T^2)] \left( 1 - \frac{\mathbb{R}}{\mathbb{R}'} \right)},$$

For two harmonics at a fixed centrality

$$\frac{v_n(p_T)}{\varepsilon_n} = \left( \frac{v_m(p_T)}{\varepsilon_m} \right)^{\frac{n}{m}} e^{\frac{\beta}{\mathbb{R}}(m-n)},$$

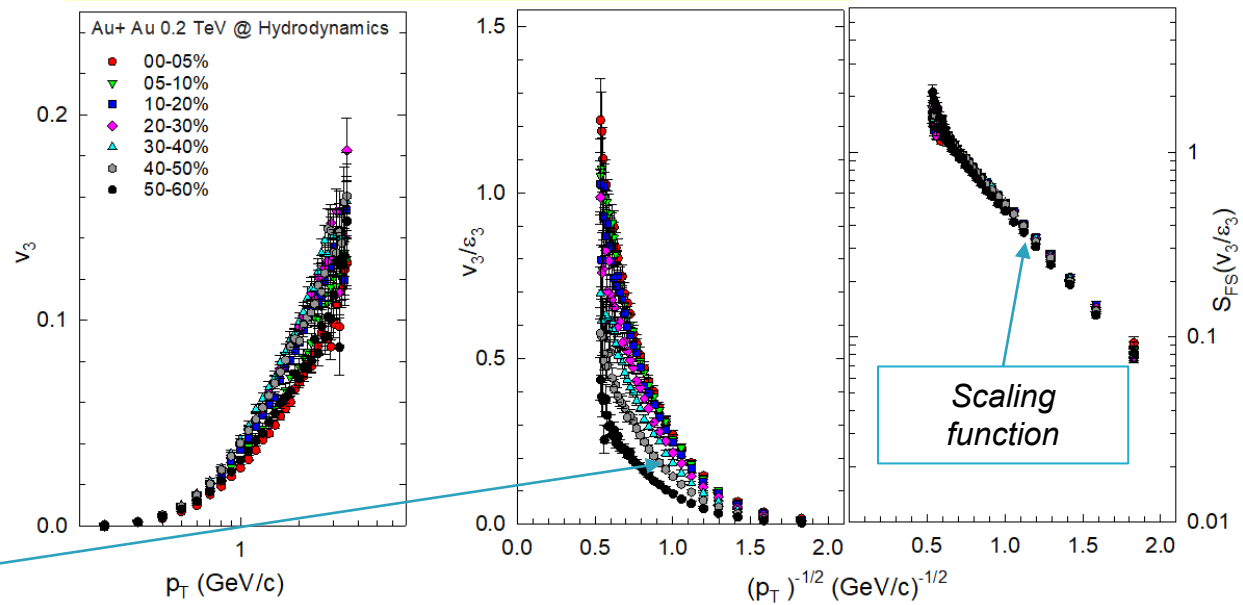
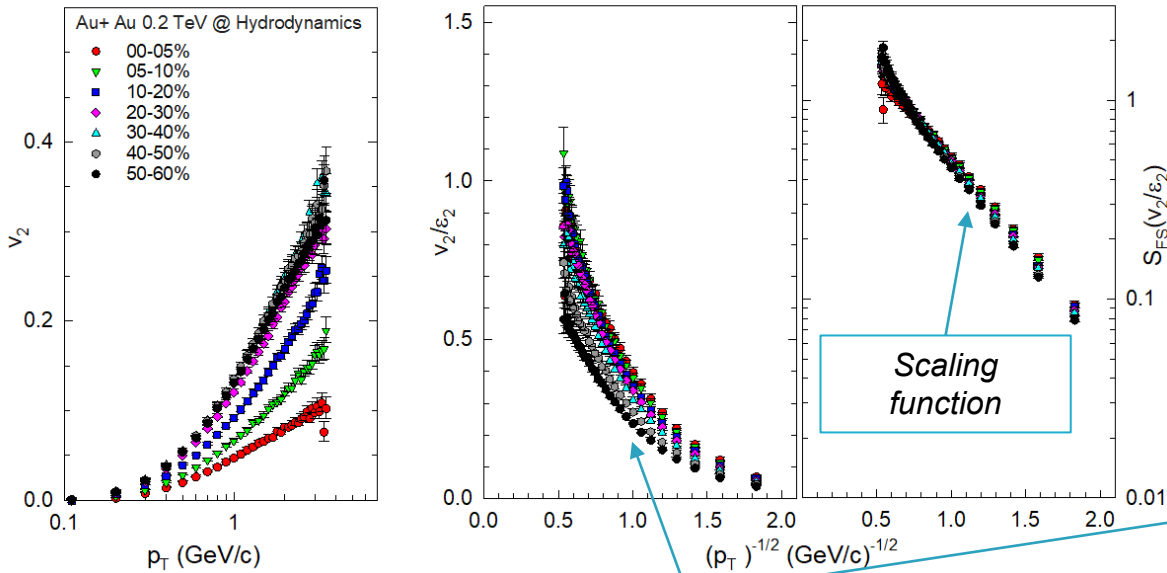
**Anisotropy Scaling Functions leverage the specific dependencies of  $\varepsilon_n$ , viscous attenuation and the expansion dynamics!**

- **Data collapse to a single curve for fully constrained scaling coefficients**
  - ✓ **Scaling coefficients give access to transport coefficients**  
 $\frac{\eta}{s}(T, \mu_B)$ , *etc.*

# Anisotropy Scaling Function – proof of principle

$$\frac{v_n(p_T, cent)}{\varepsilon_n} = \left( \frac{v'_n(p_T, cent)}{\varepsilon'_n} \right) e^{-\frac{\beta}{\mathbb{R}} \left[ n(n+\kappa p_T^2) \right] \left( 1 - \frac{\mathbb{R}}{\mathbb{R}'} \right)},$$

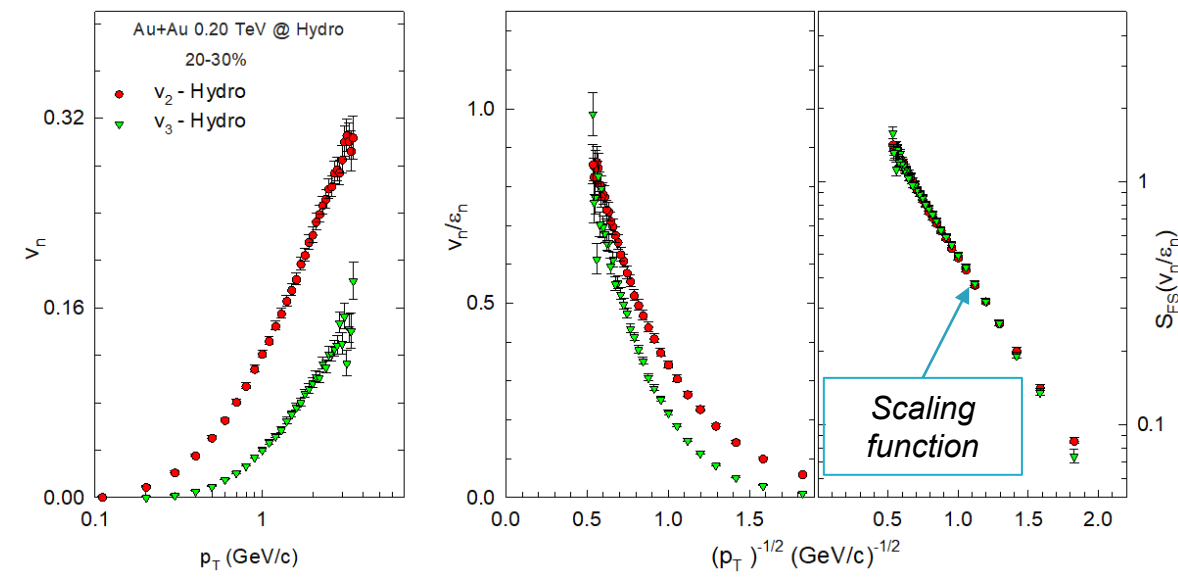
Simulated data [for charged hadrons] from Bjoern Schenke et al.



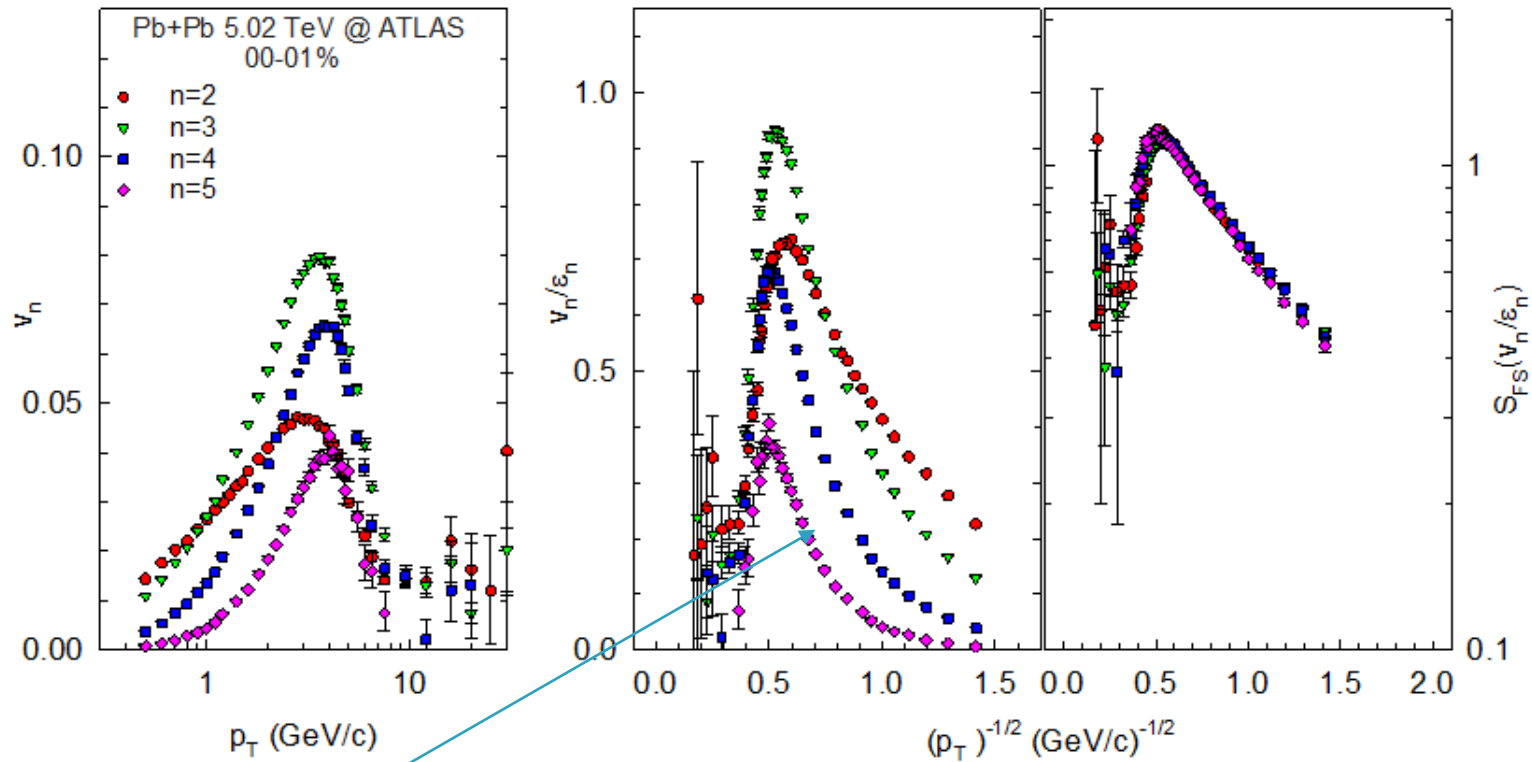
- ✓ **Eccentricity scaling (alone) is insufficient**
- Final-state interactions are crucial**
- ✓ Same  $\frac{\eta}{s}$  for  $v_2$  &  $v_3$

➤ Anisotropy data as a function of control variables collapse on to a single curve for fully constrained scaling coefficients

- ✓ Scaling coefficients are proportional to the respective transport coefficients



## $n$ & $p_T$ dependence



*$n^2$ -dependent viscous attenuation apparent*

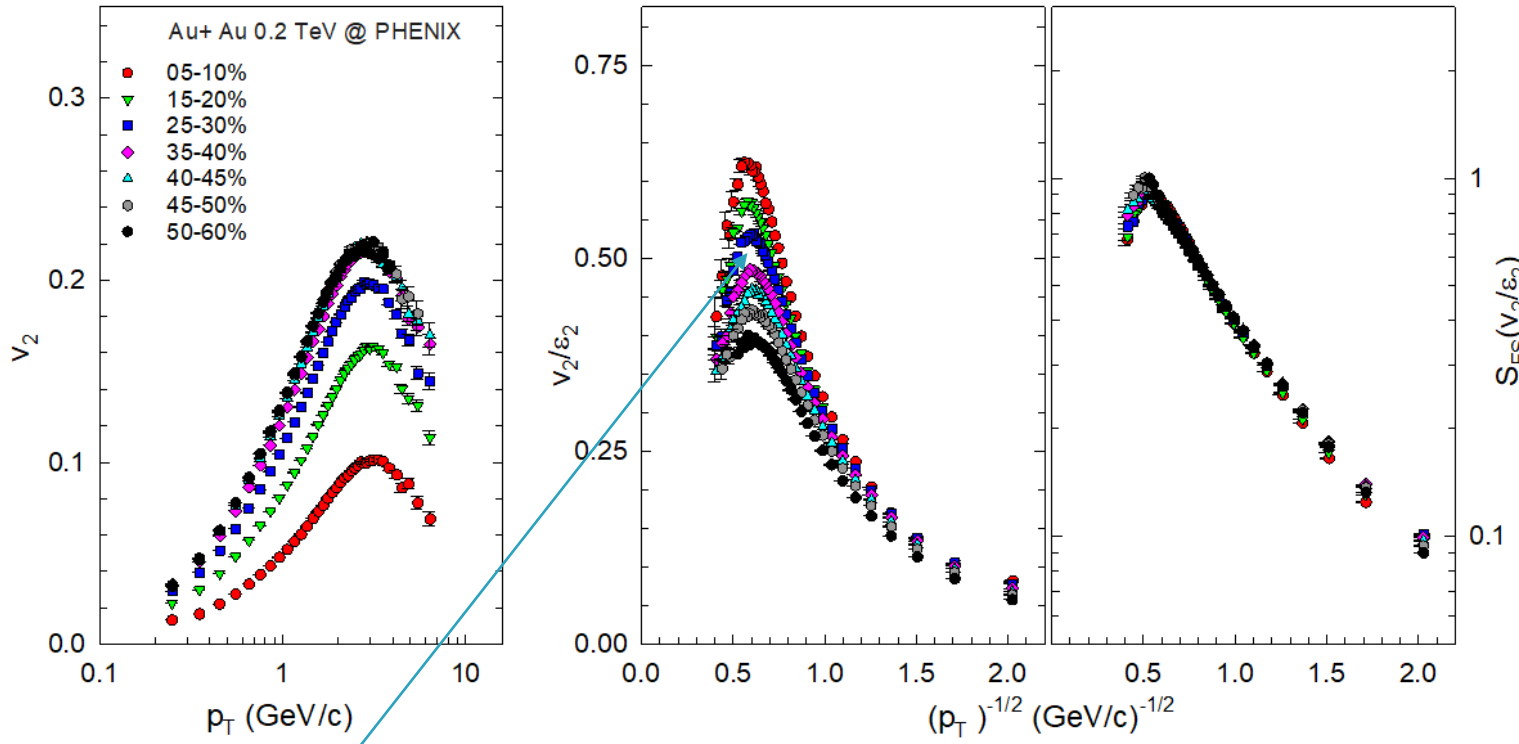
- Characteristic patterns of viscous attenuation validated
  - ✓  $n^2$  dependence of the viscous coefficient confirmed
  - ✓ Small influence from expansion dynamics
  - ✓ Scaling breaks when mode-coupled harmonics contribute
- Scaling coefficient provides constraint for  $\frac{\eta}{s}$  (T)



## Centrality & $p_T$ dependence

$$\frac{v_n(p_T)}{\varepsilon_n} = \left( \frac{v'_n(p_T)}{\varepsilon'_n} \right) e^{-\frac{\beta}{\mathbb{R}} [n(n+\kappa p_T^2)] \left( 1 - \frac{\mathbb{R}}{\mathbb{R}'} \right)},$$

$$\ln \left( \frac{v_n}{\varepsilon_n} \right) = -\frac{\beta}{\mathbb{R}} \left[ n(n + \kappa p_T^2) \right], RT \equiv \mathbb{R} \propto \langle N_{\text{ch}} \rangle^{1/3}$$

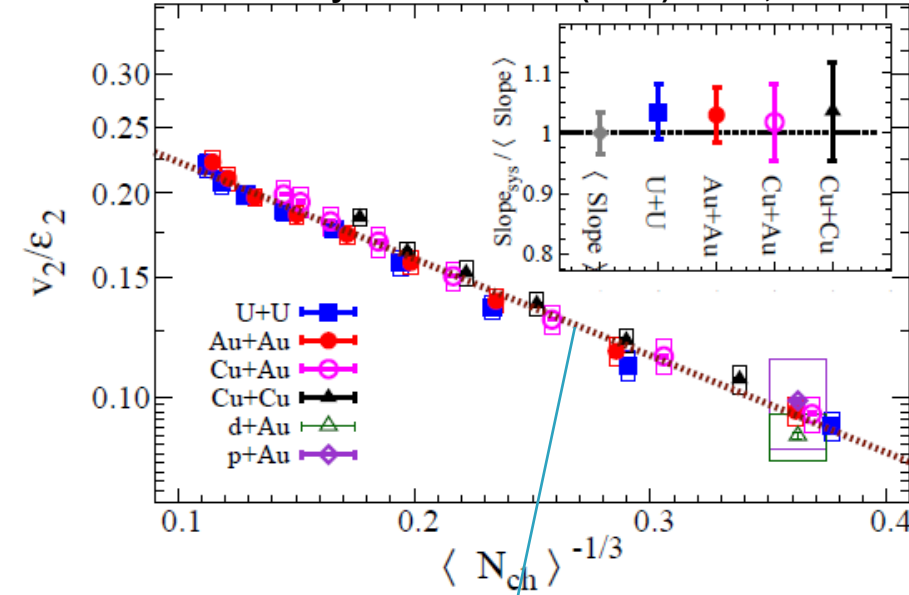


Centrality/ $p_T$ -dependent viscous attenuation

Similar patterns for other beam energies

## Collision-system & centrality dependence of $p_T$ -integrated $v_2/\varepsilon_2$

STAR - Phys.Rev.Lett. 122 (2019) no.17, 172301



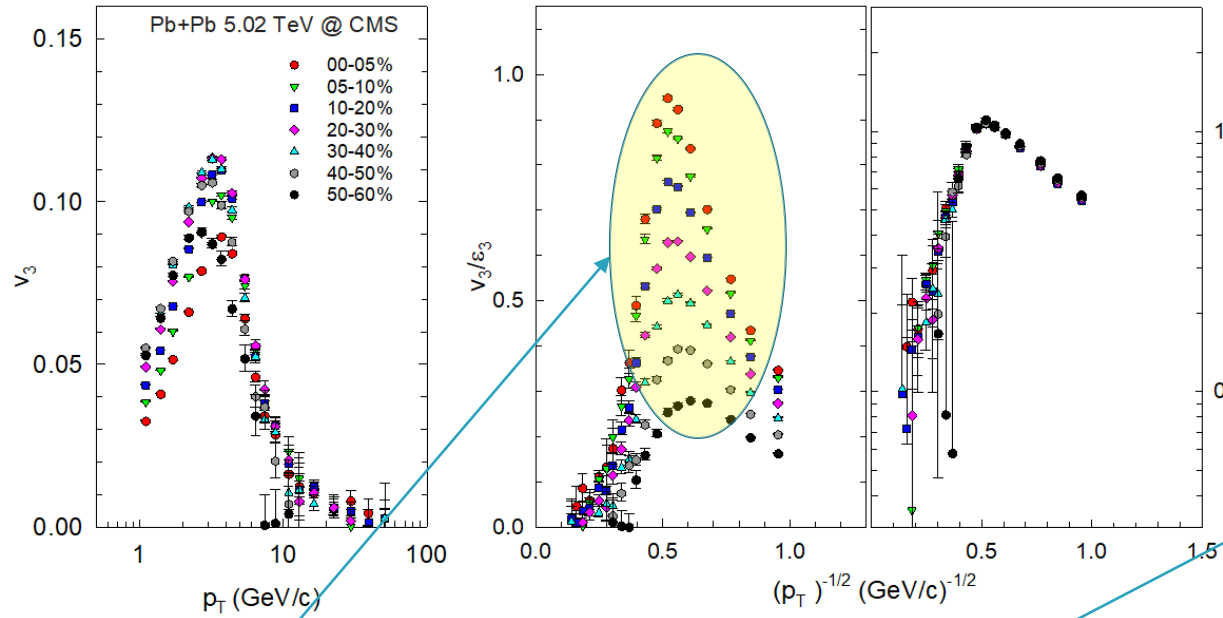
System-independent but  $RT$ -dependent viscous attenuation?

➤ Characteristic patterns of viscous attenuation validated

✓  $1/RT$  dependence of the viscous coefficient confirmed

❖ Scaling coefficient provides constraint for  $\frac{\eta}{s}$  (T)

# Anisotropy Scaling Functions

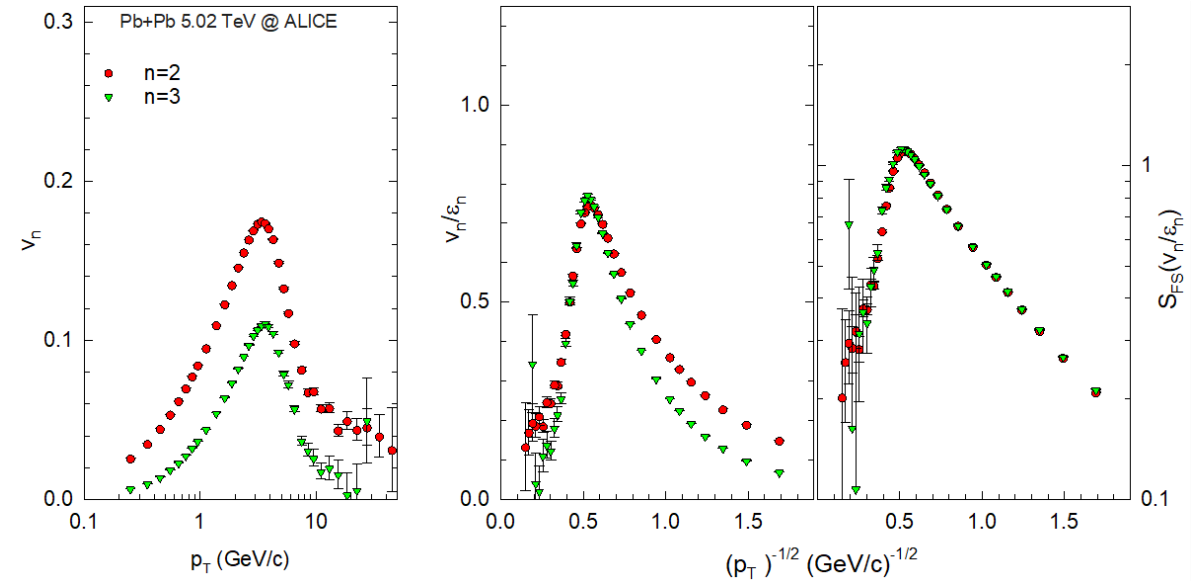
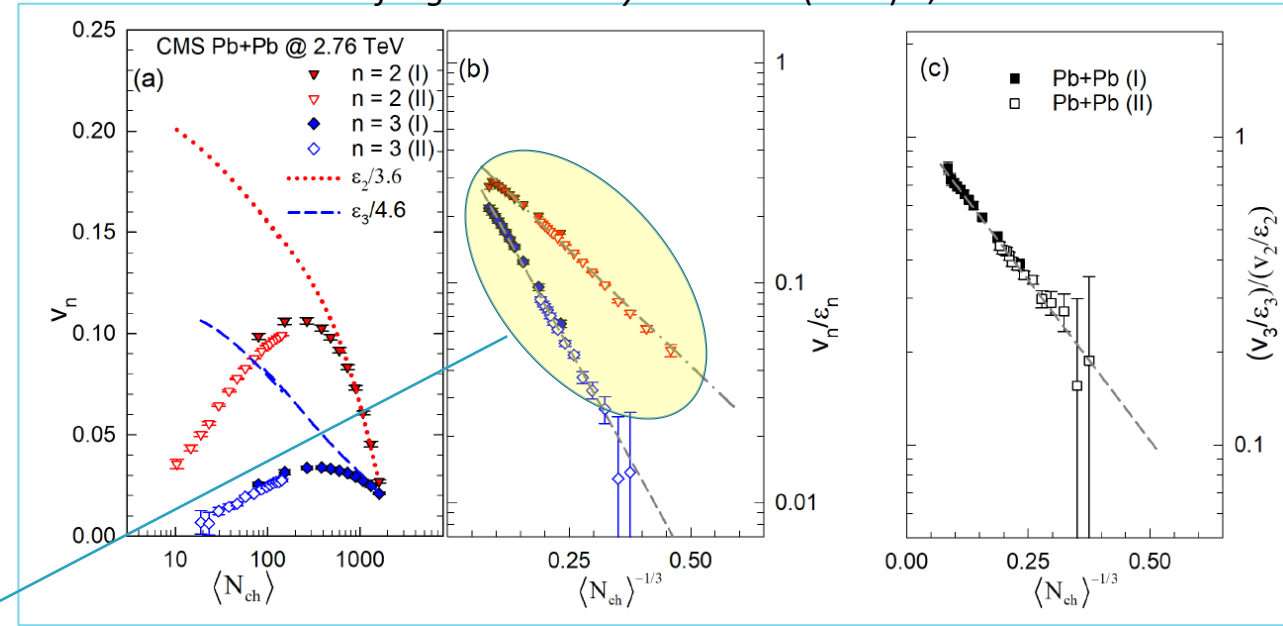


$p_T$ - &  $RT$ -dependent  
viscous attenuation?

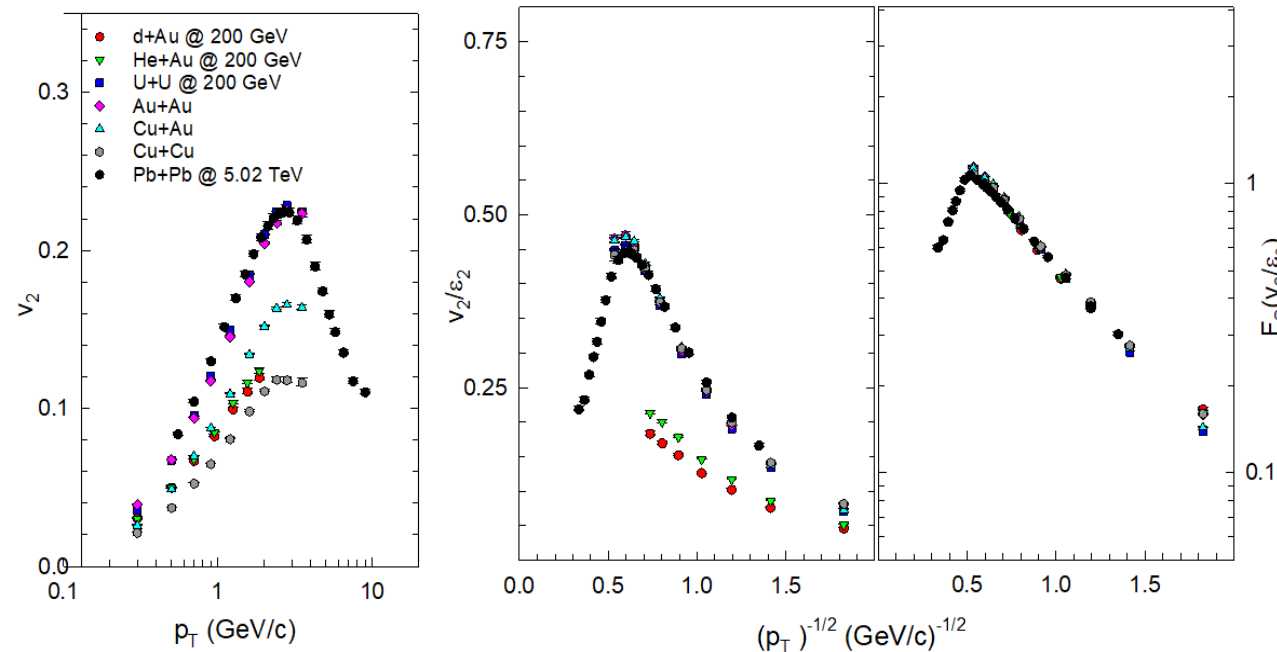
$n$ - &  $RT$ -dependent  
viscous attenuation?

➤ Characteristic patterns of viscous attenuation validated

❖ Scaling coefficient provides constraint for  $\frac{\eta}{s}$  (T)

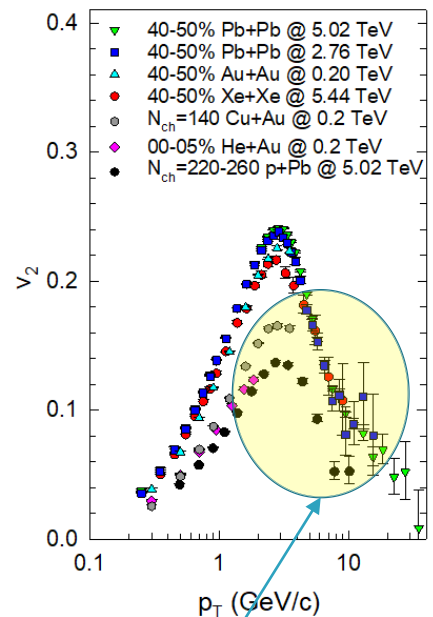


# Anisotropy Scaling Functions – Systems & Energies

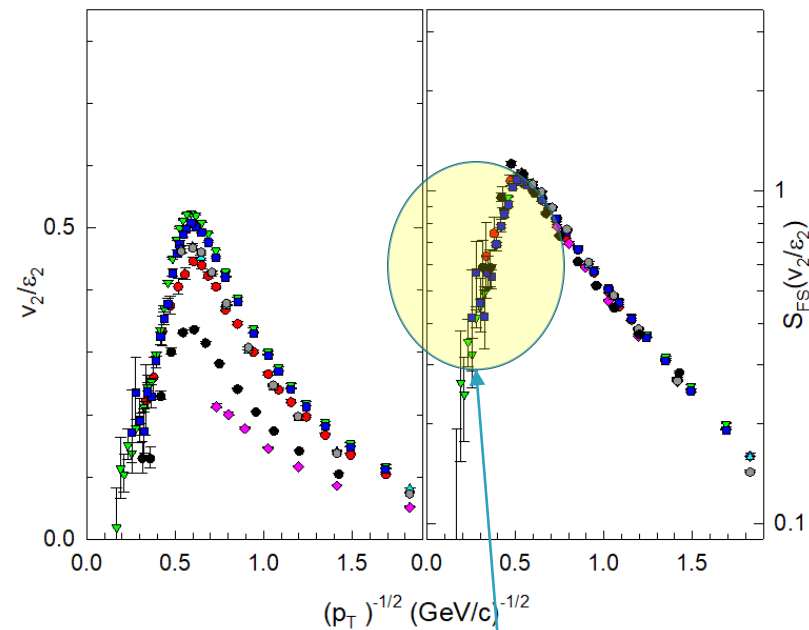


✓ Same  $\langle N_{chg} \rangle$  for U+U, Pb+Pb, Au+Au, Cu+Au and Cu+Cu

✓ Different  $\langle N_{chg} \rangle$  for d(<sup>3</sup>He)+Au



➤ High- $p_T$  p+Pb



➤ Jet quenching in p+Pb?

➤ Indications for viscous attenuation and jet quenching across systems.

✓ Signal attenuation very important for small dimensionless sizes.

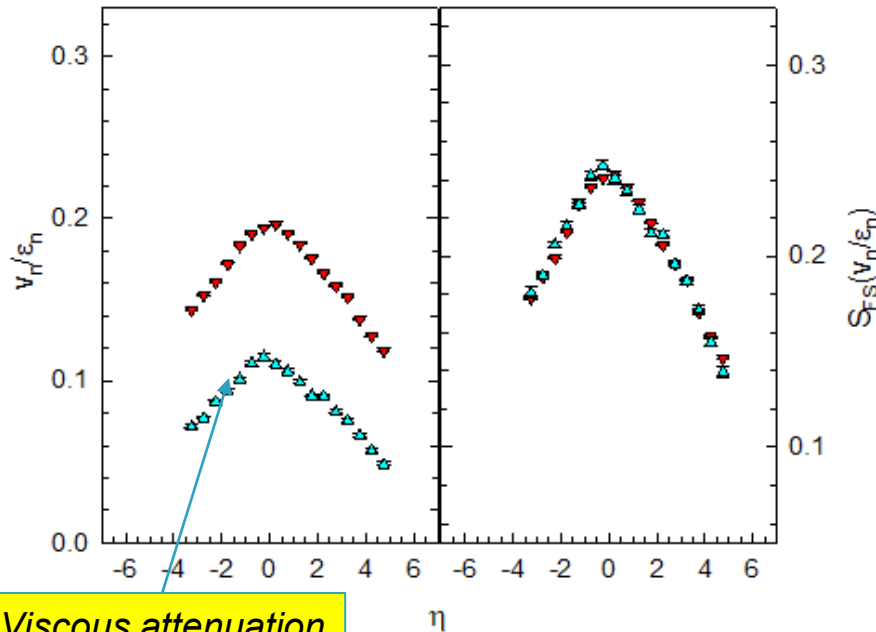
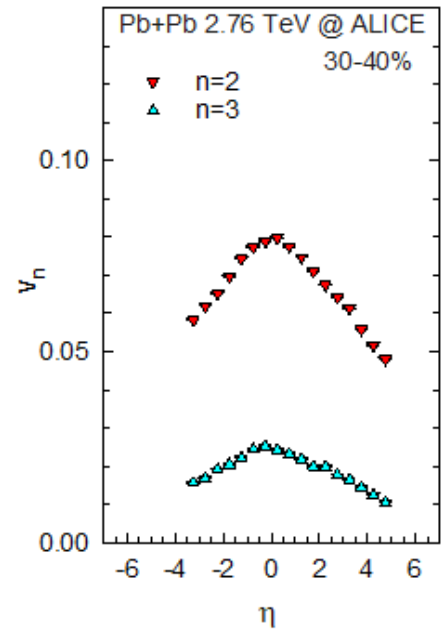
❖ *Scaling coefficients indicate;*

✓ *an increase in  $\eta/s$  from RHIC to LHC.*

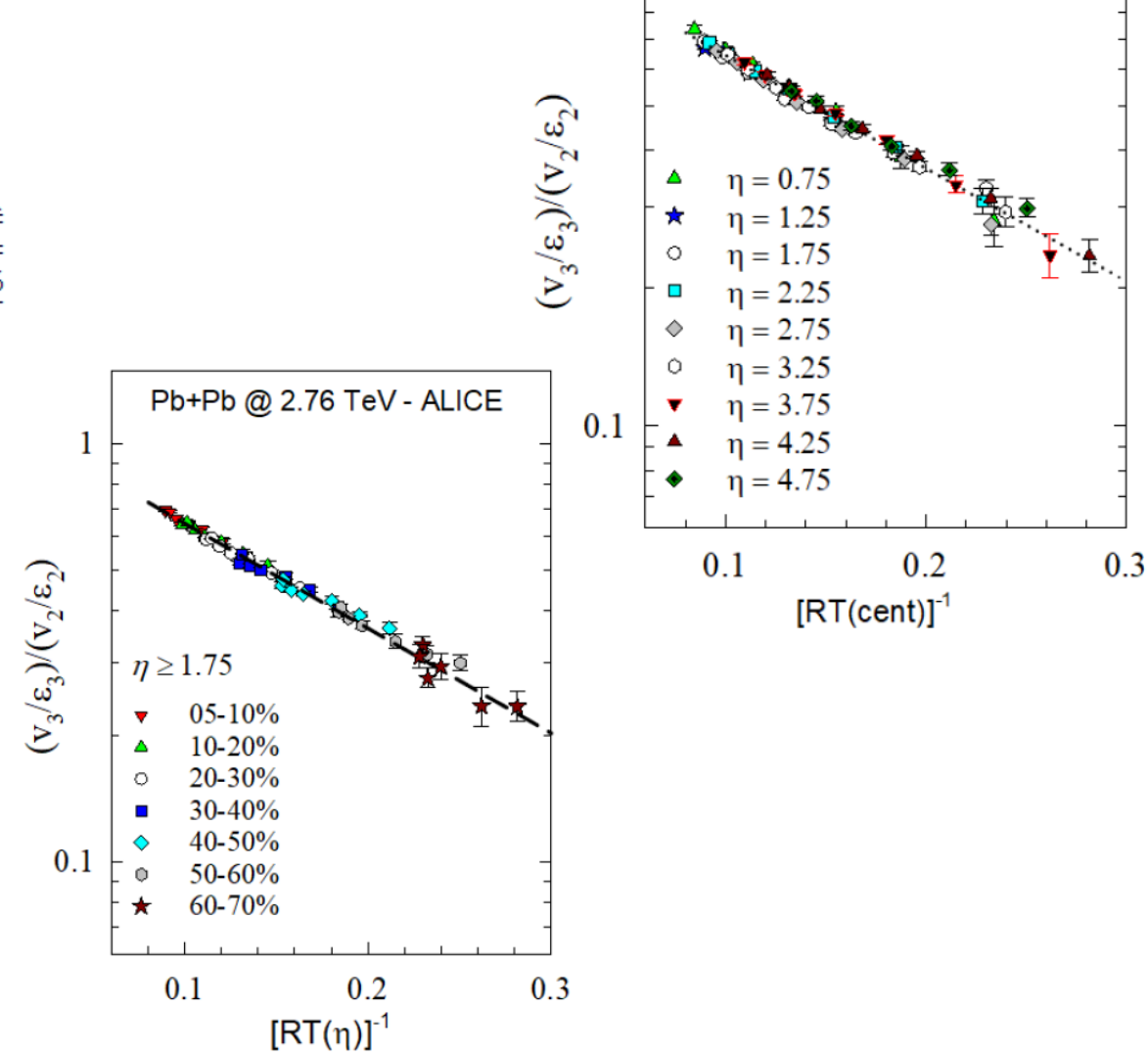
✓ *A modest increase in  $\eta/s$  from large to small systems.*



## $\eta$ - dependence



Viscous attenuation difference

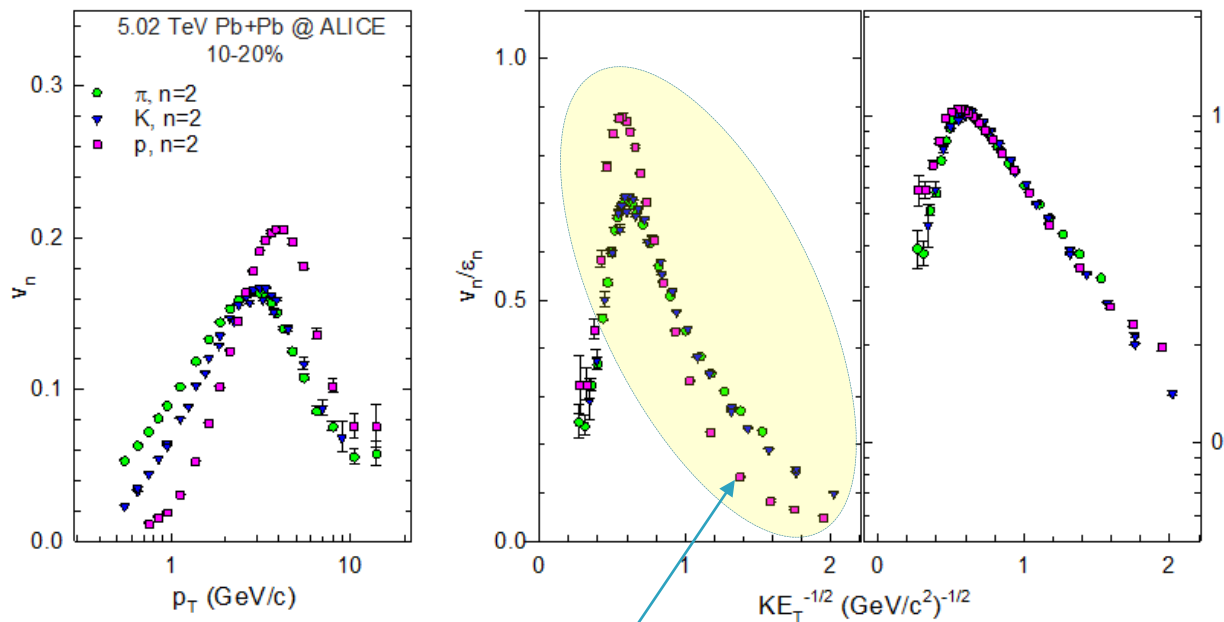


➤  $\eta$  – dependent patterns of viscous attenuation validated

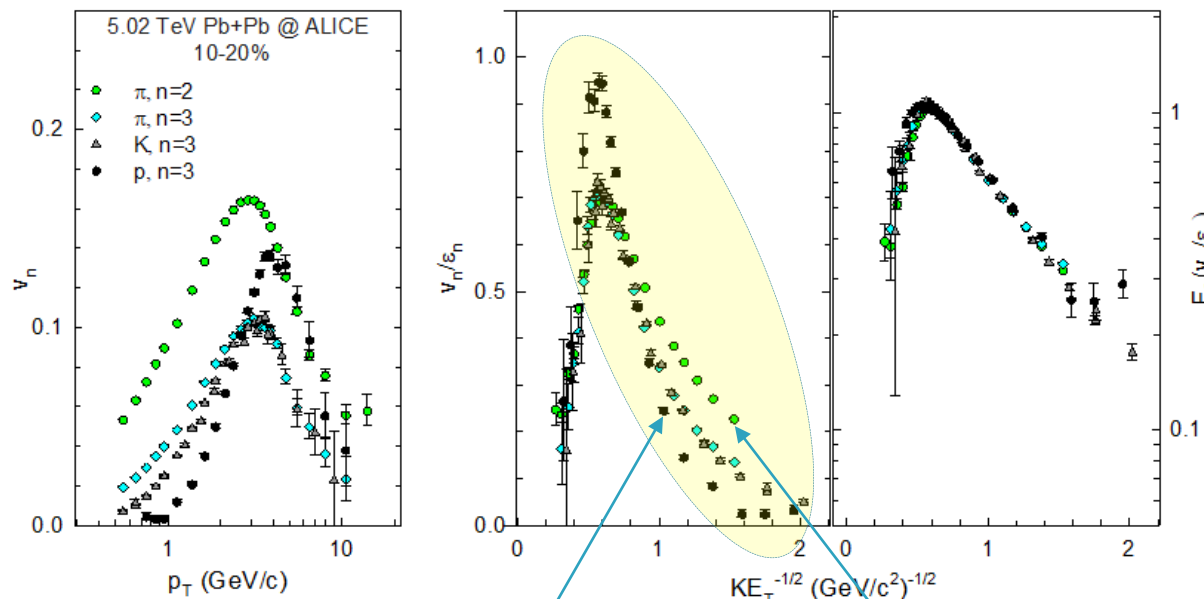
- ✓ Scaling coefficient provide a further constraint for  $\frac{\eta}{s}$  (T)
- ✓ Factorization across  $\eta$  does not hold

Viscous\_attenuation(cent,  $p_T$ ,  $\eta$ ) understood!

# Anisotropy Scaling Function for PID species



Compare  $v_2$  and  $v_3$



Effects of expansion dynamics  
 ✓ Scaling for this effect required

✓ Same  $\frac{\eta}{s}$

Start @ 5 TeV

- ✓  $\mu_{B,S,I} \sim 0$
- ✓ PID-independent control variables
- ✓ PID-dependent expansion dynamics

Effects of expansion dynamics

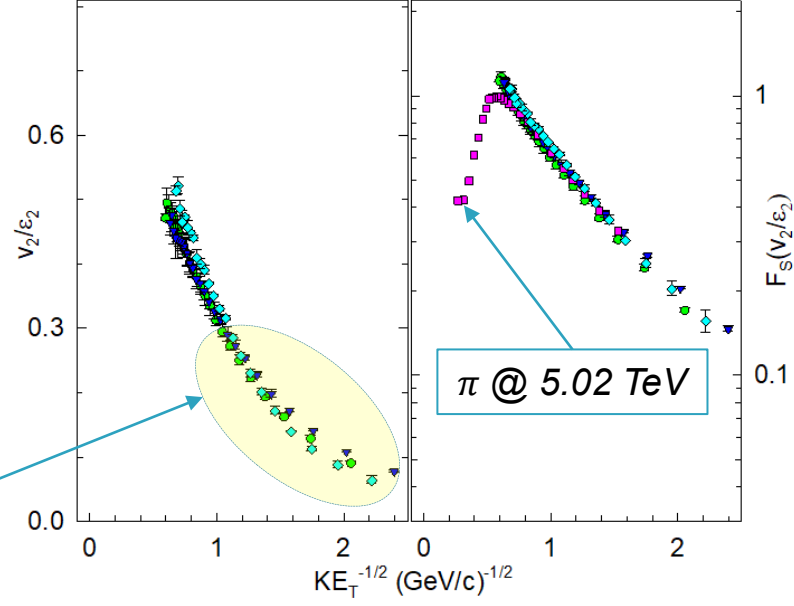
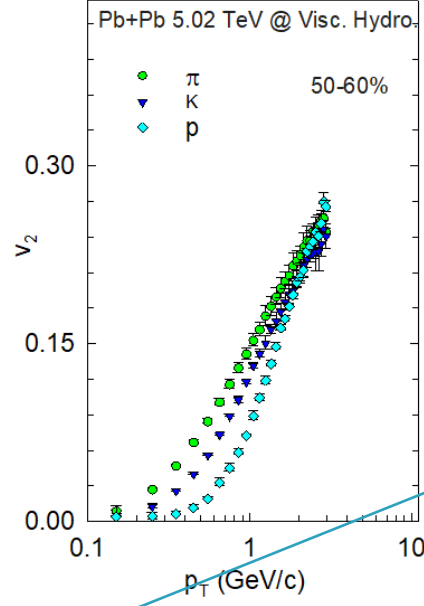
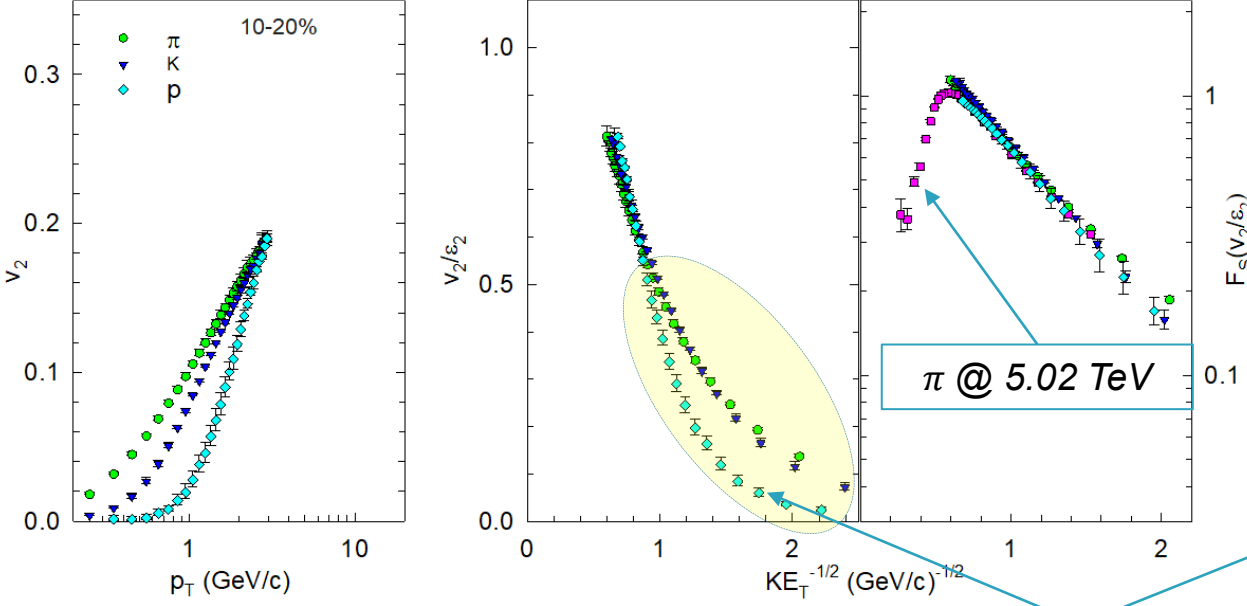
Effects of Viscous attenuation

➤ Scaling for viscous attenuation and expansion dynamics validated.

# PID Scaling Functions – Further proof of principle

Simulated data [for identified particles] from Huichao Song et al.

Pb+Pb 5.02 TeV @ Hydrodynamics

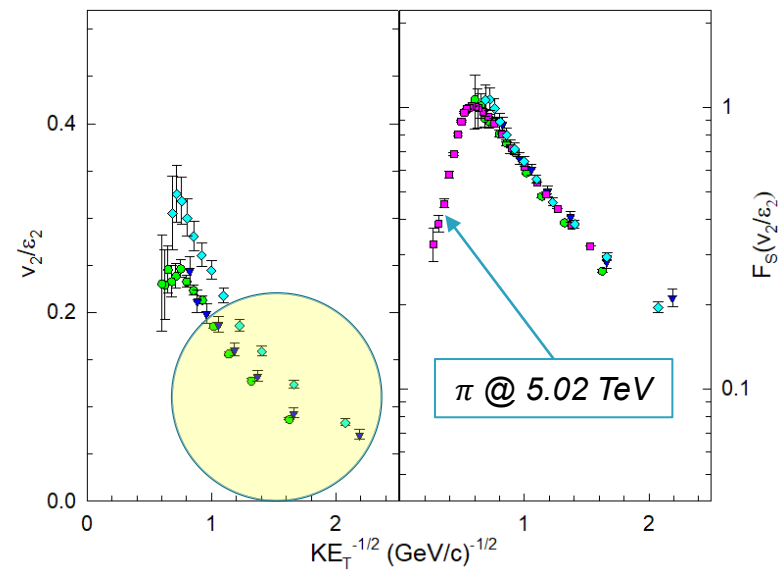
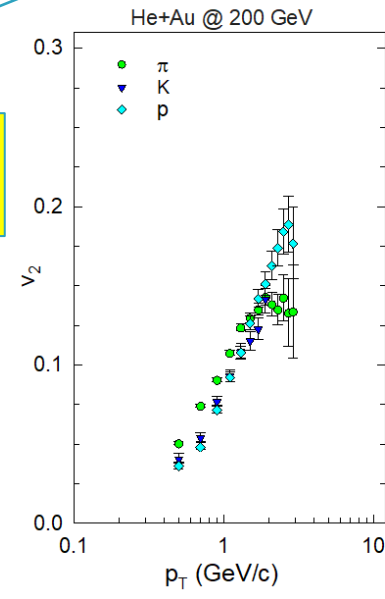


- ✓  $\mu_{B,S,I} \sim 0$
- ✓ PID-independent control variables
- ✓ PID-dependent expansion dynamics

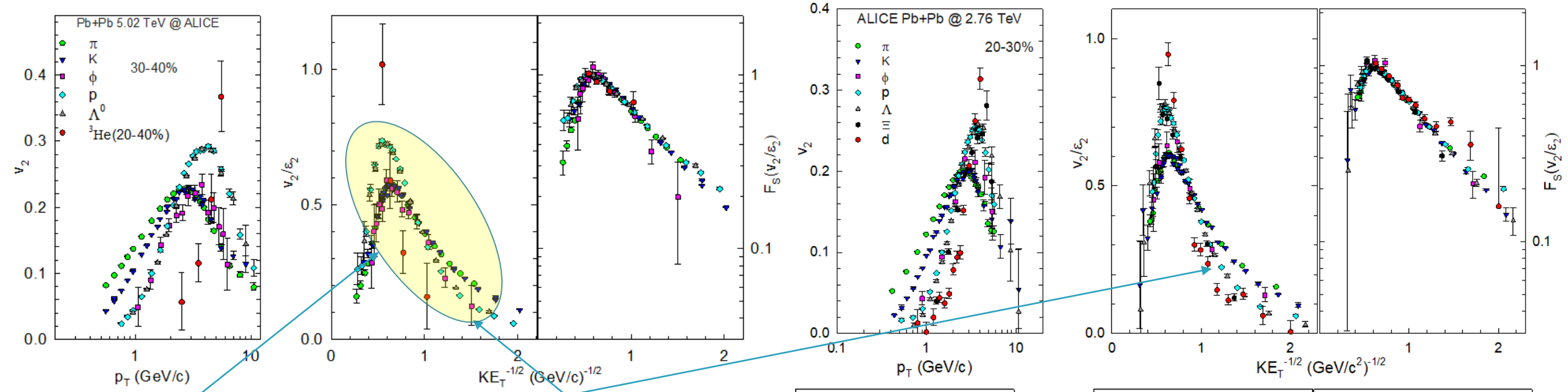
Diff. expansion dynamics  
✓ Same  $\eta/s$

➤ Scaling for viscous attenuation and different expansion dynamics validated.

✓ Expansion dynamics is centrality-, system- and beam-energy dependent



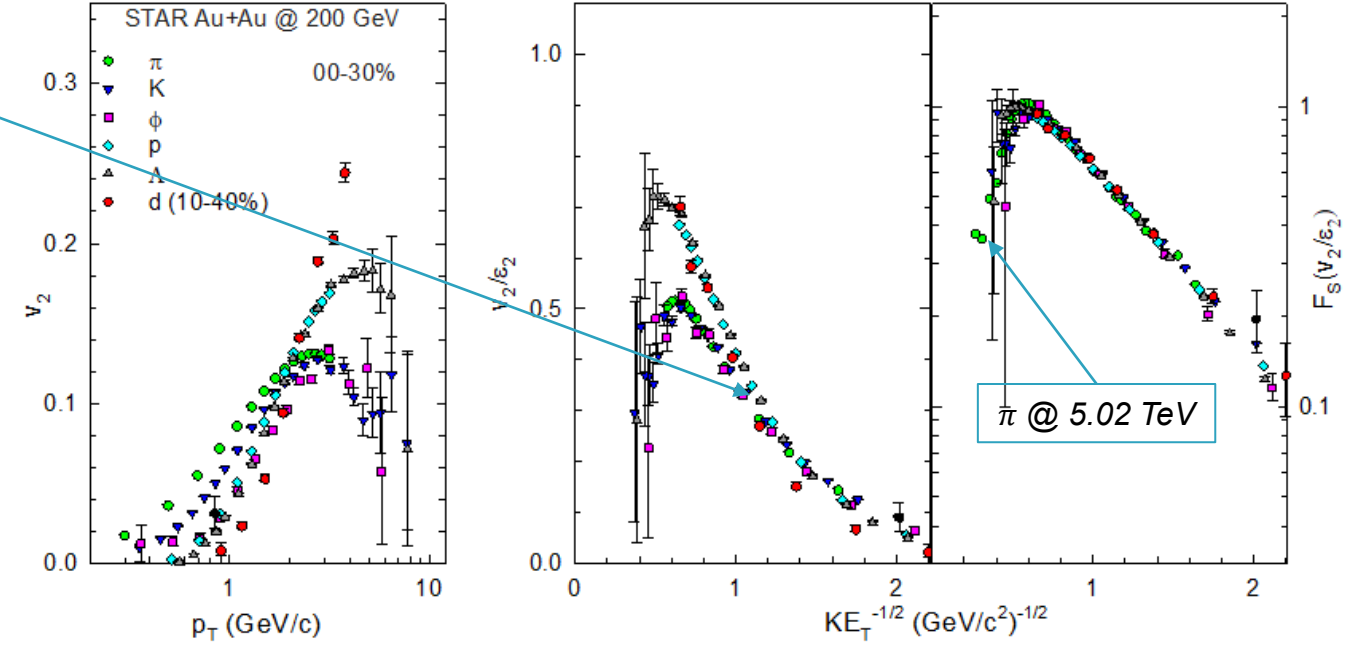
# Anisotropy Scaling Functions – species across systems & energies



Diff. expansion dynamics  
 ✓ Same  $\eta/s$  ?

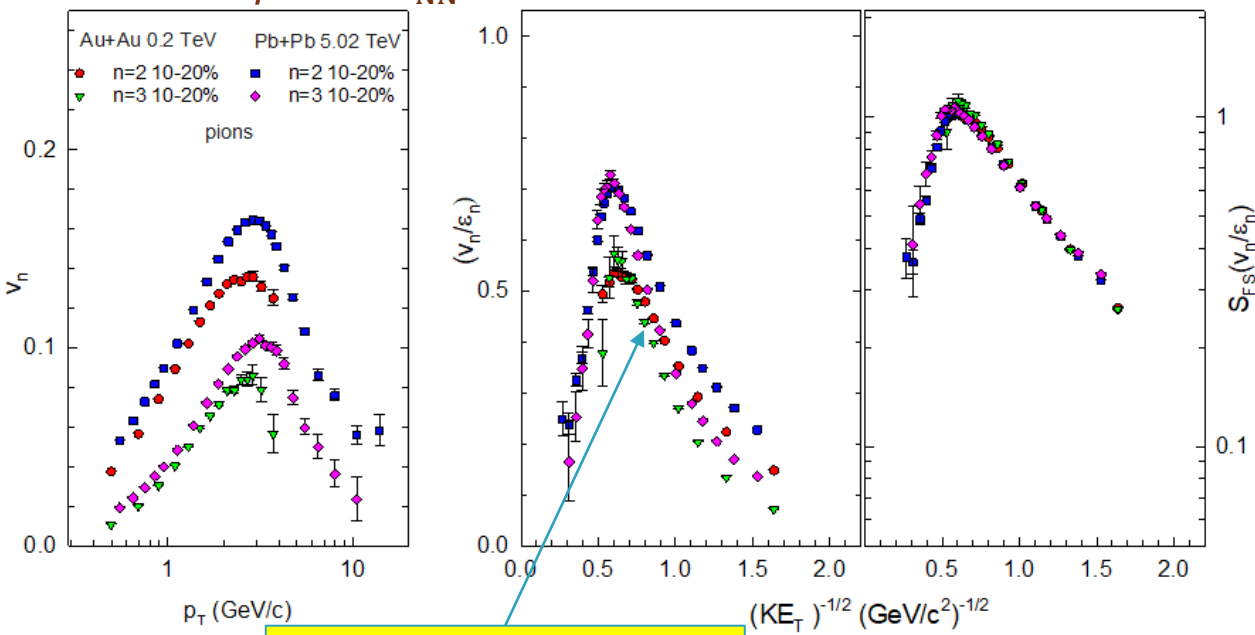
✓ 1 meson branch  
 ✓ 2 baryon branches

- **Scaling for viscous attenuation and expansion dynamics validated for all beam energies studied**
  - ✓ **Scaling observed for all particle species (multiple mechanisms superfluous)**
  - ✓ **Baryon-number-dependent expansion dynamics - not mass**



# PID Scaling Functions – Systems, species & Energies

Compare  $\sqrt{s_{NN}} = 0.2$  and 5.02 TeV



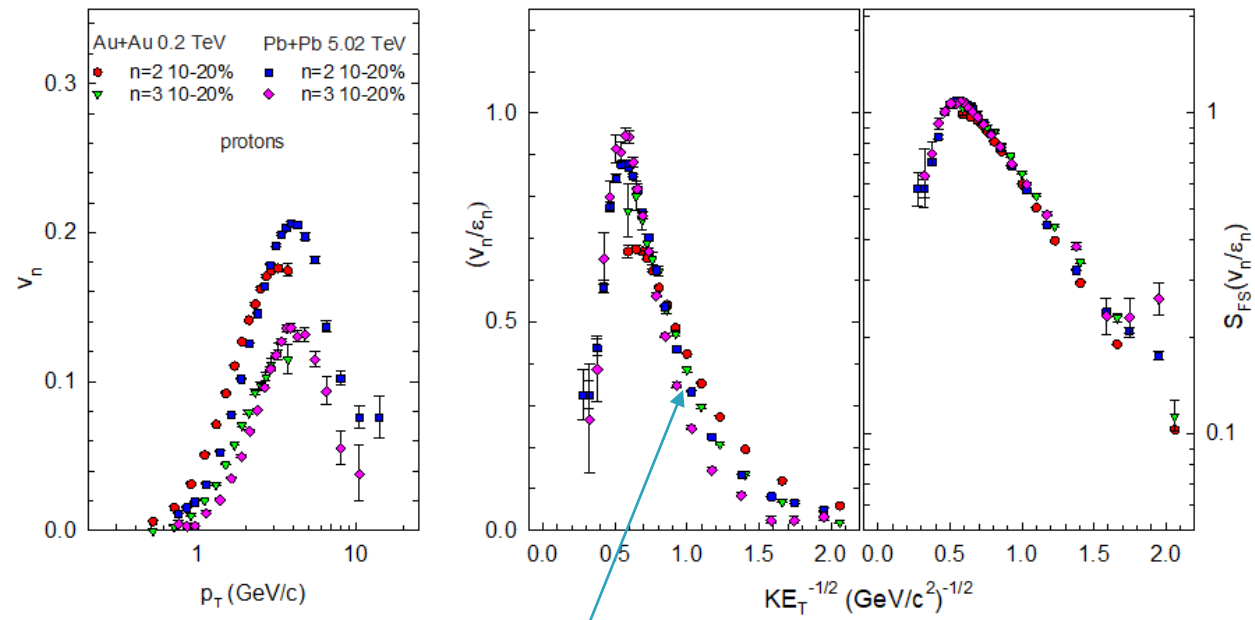
Indications for diff.  $\eta/s$

➤ Scaling for viscous attenuation and expansion dynamics validated across beam energies and systems

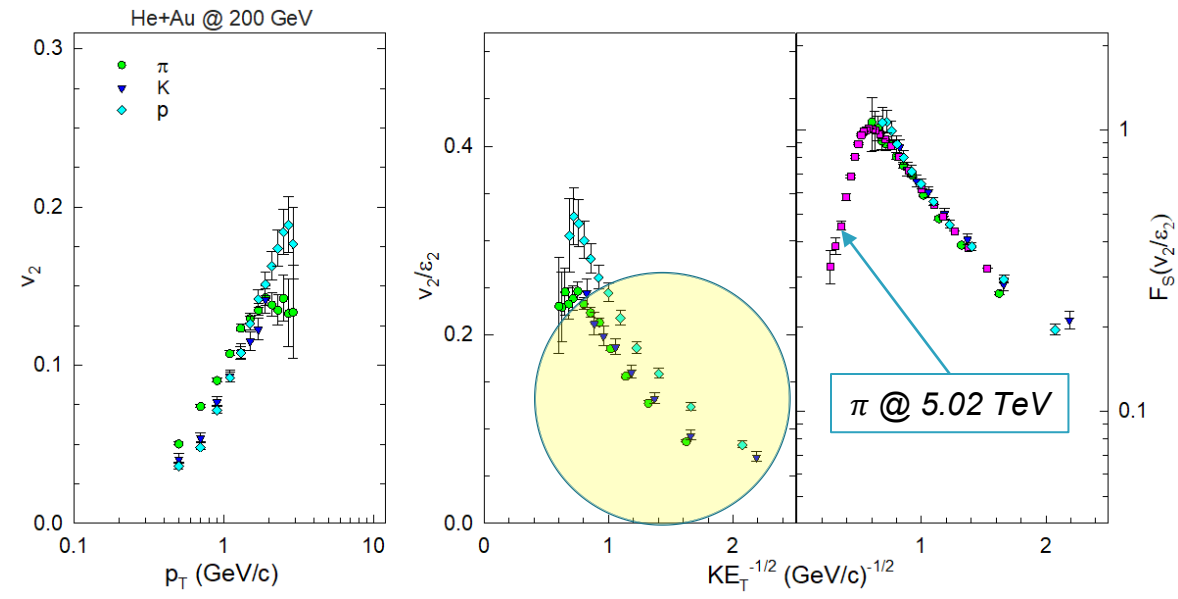
✓ Scaling observed for all particle species (multiple mechanisms superfluous)

❖ Scaling coefficients indicate;

- ✓ an increase in  $\eta/s$  from RHIC to LHC.
- ✓ A modest increase in  $\eta/s$  from large to small systems.

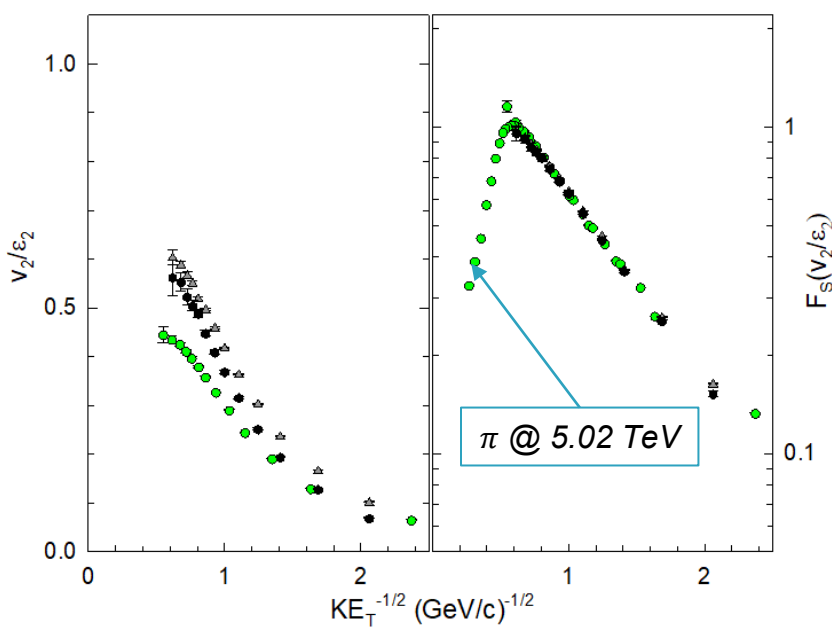
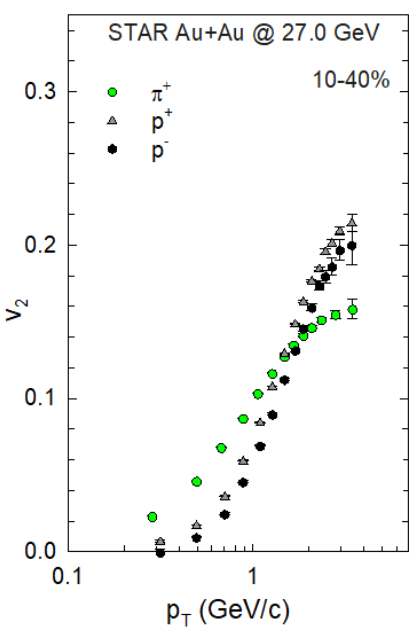
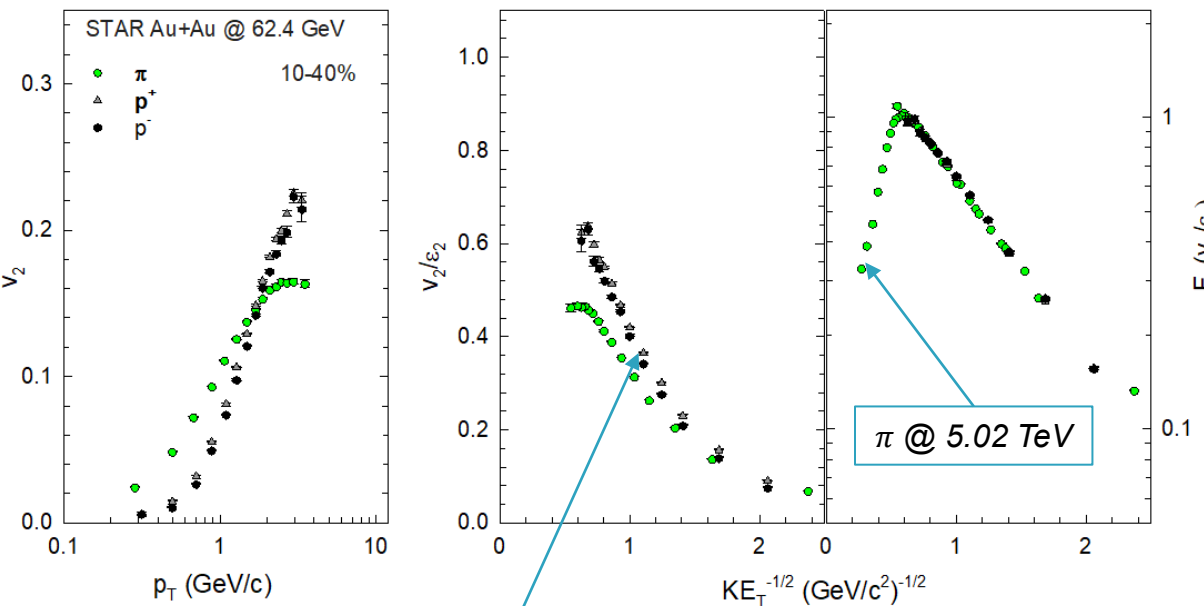


Diff.  $\eta/s$  & expansion dynamics



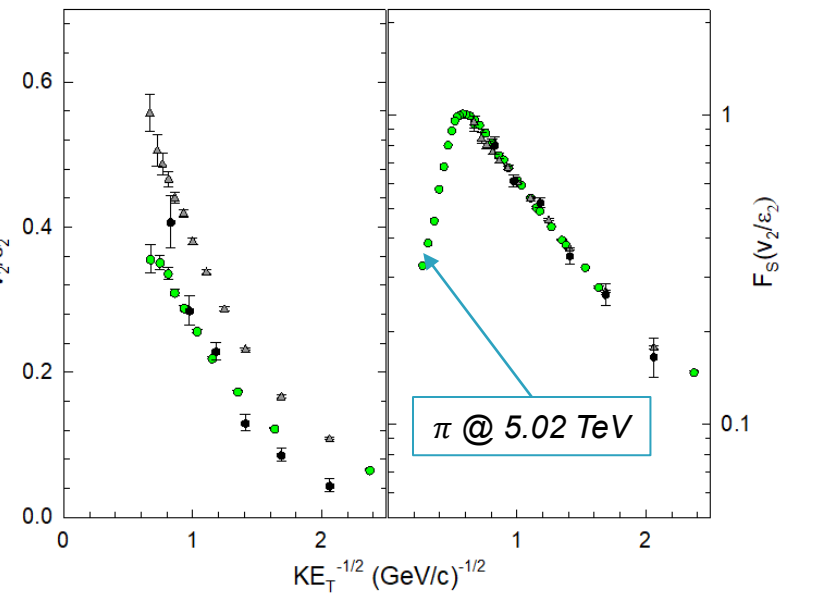
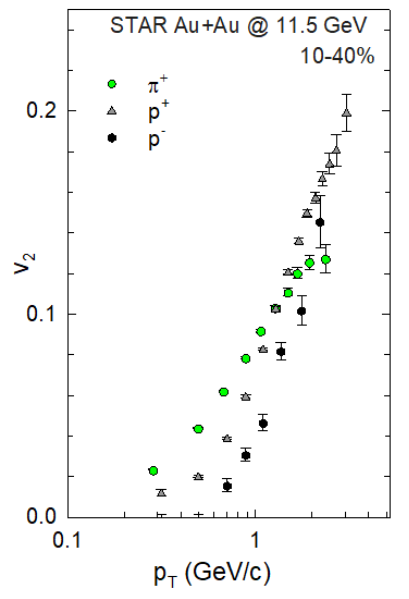


# PID Scaling Functions – particles vs. anti-particles



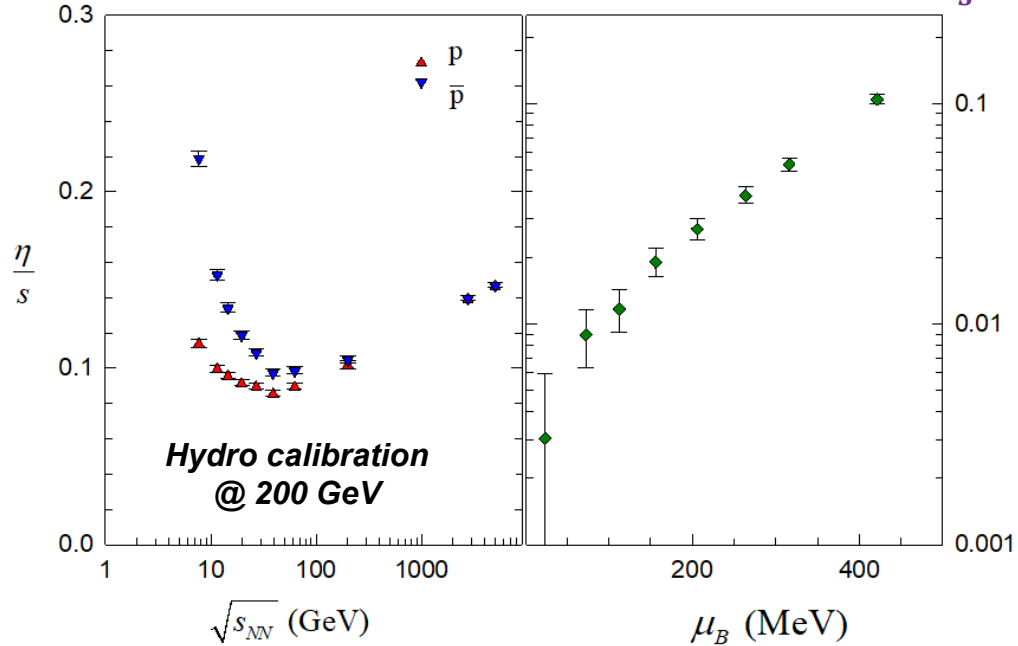
**Particle/Anti-particle difference**  
 ✓ Same expansion dynamics

- **Scaling for viscous attenuation and expansion dynamics validated for particles and anti-particles across beam energies**
- ✓ **Scaling coefficients indicate particle/anti-particle-dependent viscosity**

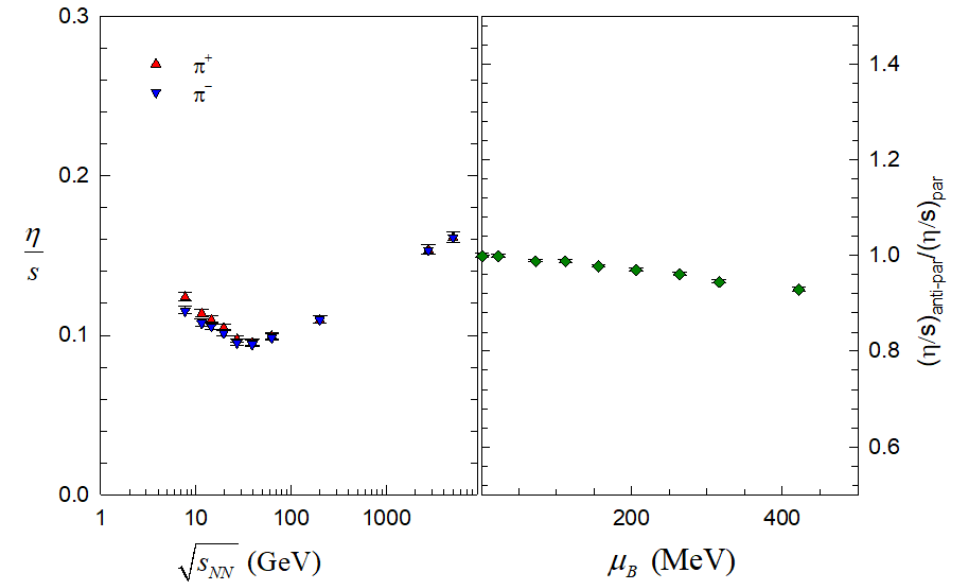
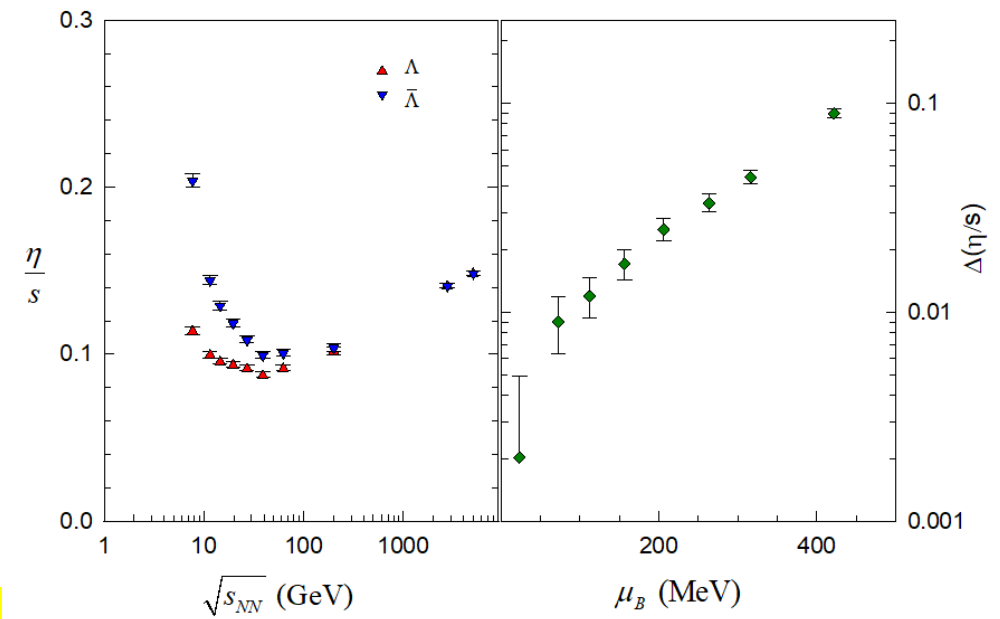


# Beam-energy-dependent specific viscosity

$\sqrt{s_{NN}}$  dependence of the extracted values of  $\frac{\eta}{s}$



$$\frac{\eta}{s}(T, \mu_B, \mu_S, \mu_I)$$



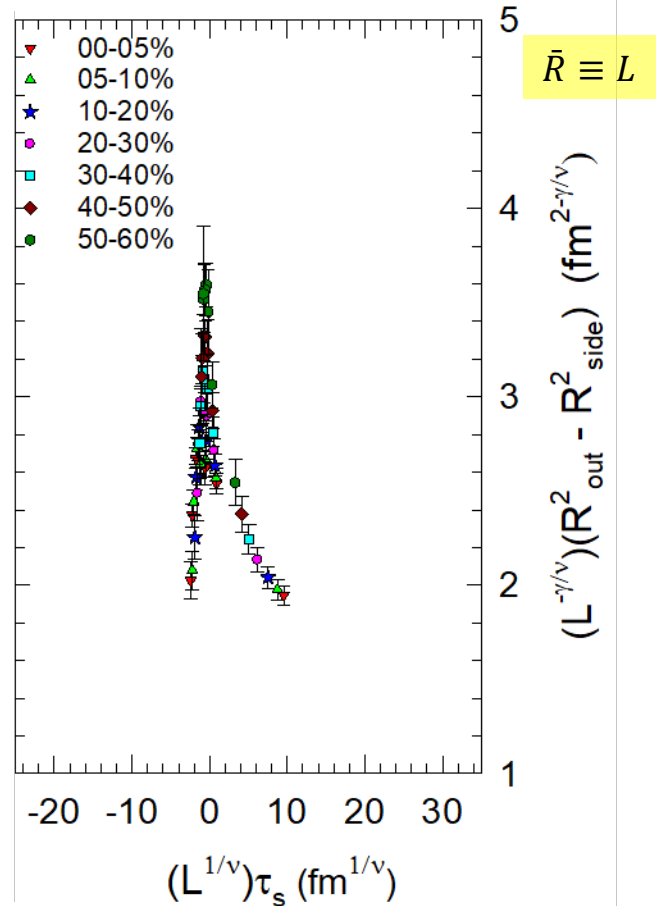
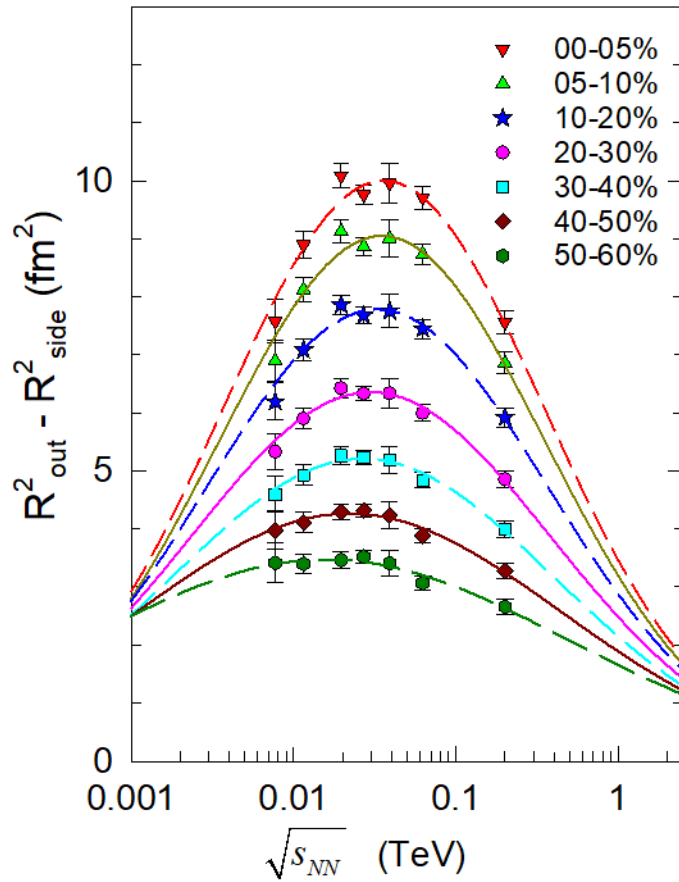
- Specific viscosity extracted across beam energies
  - ✓ Nonmonotonic patterns suggestive of critical behavior?
  - ✓  $\mu_B$  –dependent particle/anti-particle dependence

Charged currents drive particle/anti-particle viscosity difference



# Succeptibility Scaling Function

$$L^{-\gamma/\nu} \chi(s, L) = f_2^s(sL^{1/\nu})$$



$$\sqrt{s_{CEP}} = 45 \text{ GeV}$$

$$\nu \sim 0.66$$

$$\gamma \sim 1.2$$

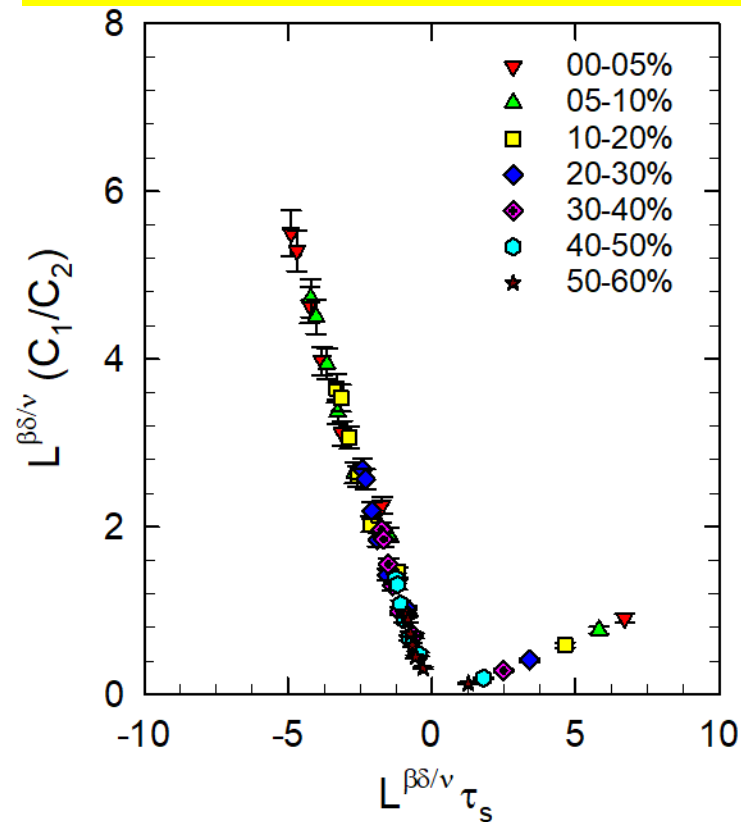
$$s = (\sqrt{s} - \sqrt{s_{CEP}}) / \sqrt{s_{CEP}}$$

**Finite-size scaling leads to data collapse for 3D Ising critical exponents.**

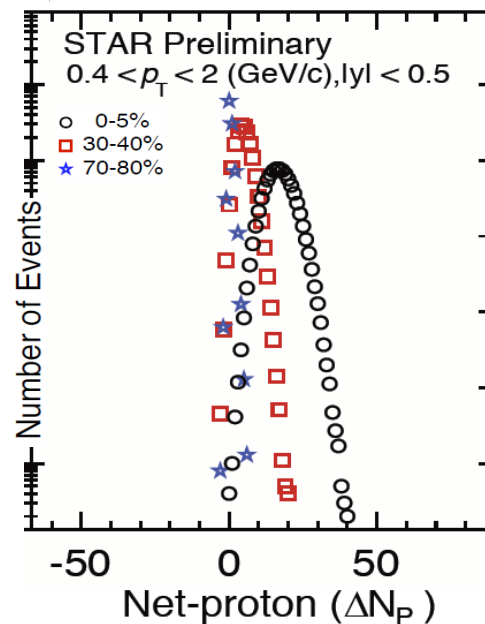
✓ **Slow quench in KZ language**

# Scaling Function for Net baryon Susceptibility ratio

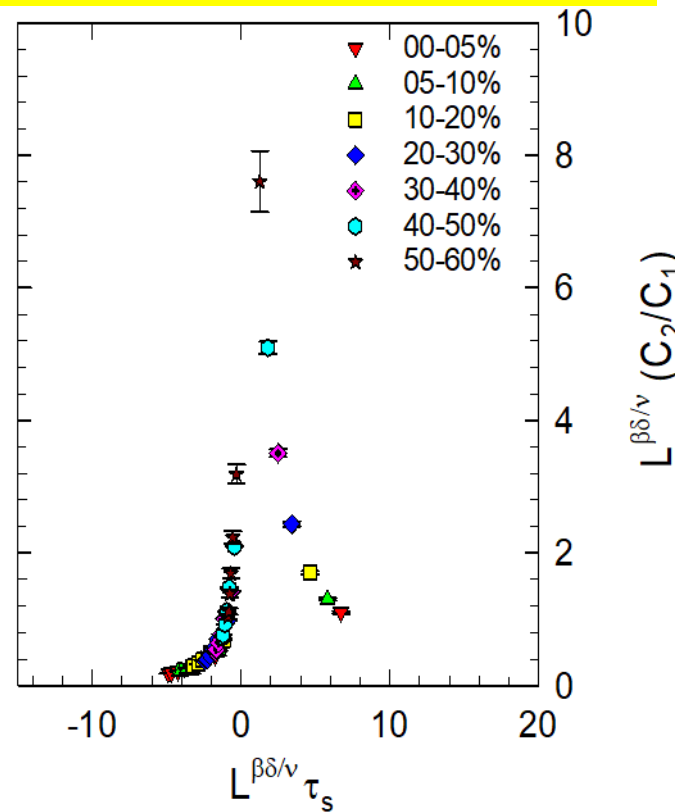
$$\chi(\mu_s, L) = L^{-\beta\delta/\nu} f_1^\mu(\mu_s L^{\beta\delta/\nu})$$



$C_n^{\Delta N_p}$  extracted  
from distributions



$$\chi(\mu_s, L) = L^{\beta\delta/\nu} f_1^\mu(\mu_s L^{\beta\delta/\nu})$$



$$\nu \sim 0.66$$

$$\beta \sim 0.33$$

$$\delta \sim 4.8$$

$$\kappa_T \propto \frac{\langle C_2^{\Delta N_p} \rangle - \langle C_2^{\Delta N_p} \rangle^2}{\langle \Delta N_p \rangle} = \frac{C_2^{\Delta N_p}}{C_1^{\Delta N_p}}$$

Finite-size scaling leads to data collapse for 3D Ising critical exponents.

✓ Slow quench in KZ language



## Summary

The  $\sqrt{s_{\text{NN}}}$ -dependence of the anisotropy scaling functions for PID species can be used to:

- Delineate the respective influence of expansion dynamics and viscous attenuation
- Constrain  $\frac{\eta}{s}(T, \mu_B, \mu_I, \mu_S)$ ?

The scaling functions extracted from the wealth of the flow data indicate:

- ❖  $\mu$  –dependent particle/anti-particle  $\frac{\eta}{s}$  dependence
- ❖ non-monotonic patterns for;
  - ✓  $\frac{\eta}{s}(T, \mu_B)$ ,
  - ✓  $\frac{\eta}{s}(T, \mu_S)$ ,
  - ✓  $\frac{\eta}{s}(T, \mu_I)$ ,

Consistent with earlier indications for the CEP

*End*