

Photon-hadron/jet production and angular correlations  
in  
high energy proton-nucleus collisions

(from small to large  $x$ : toward a unified formalism)

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*based on:*

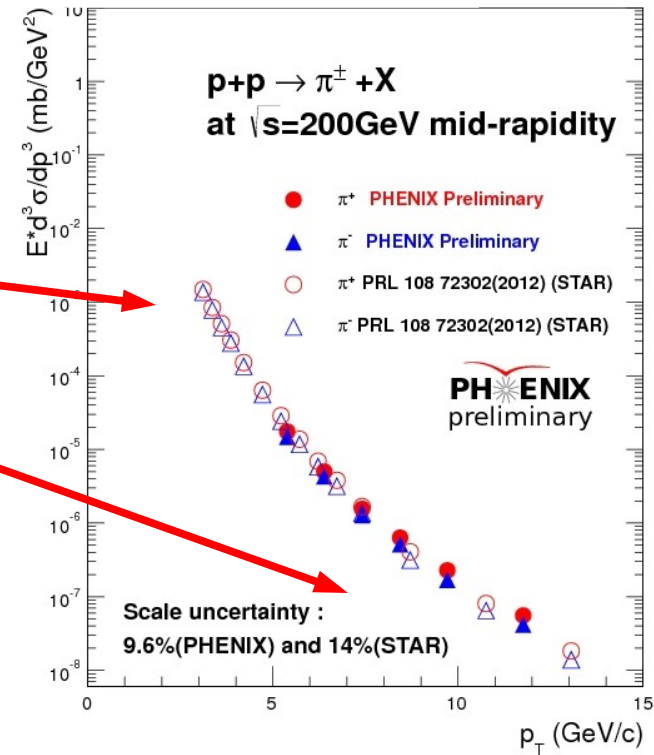
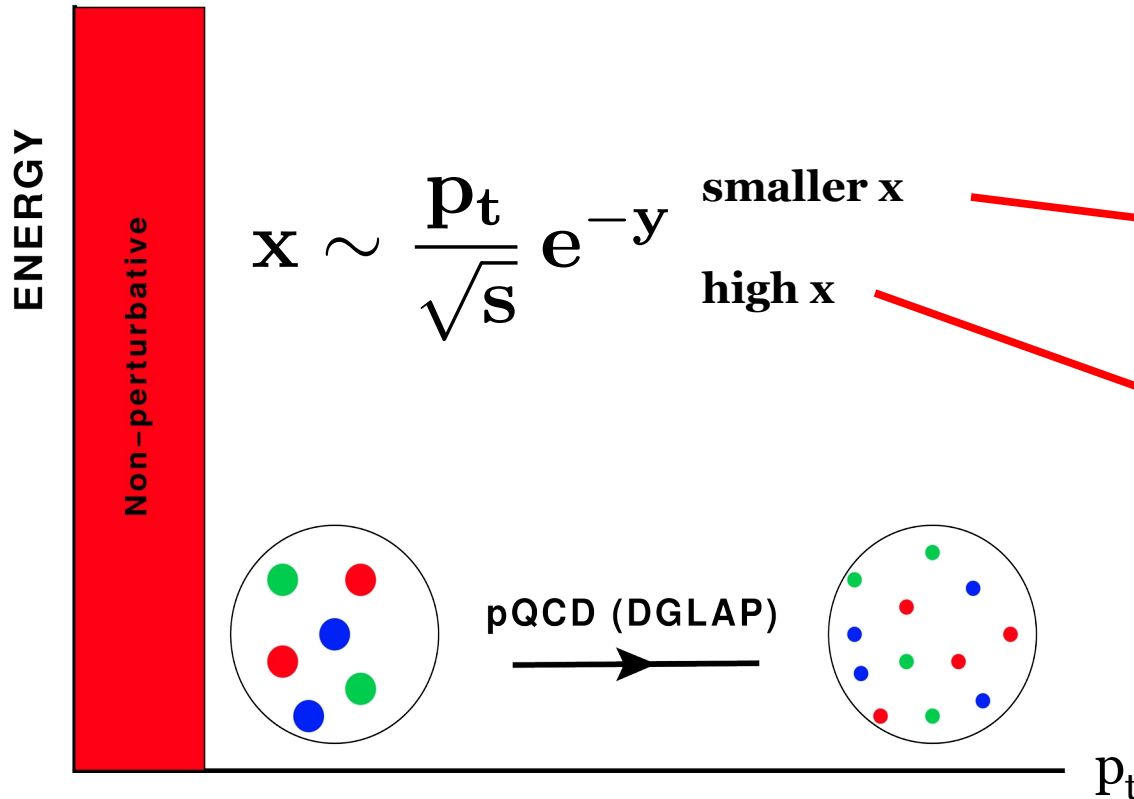
*PRD102 (2020) 1, 014008*

*PRD99 (2019) 1, 014043*

*PRD96 (2017) 7, 074020 and work in progress*

# pQCD: the standard paradigm

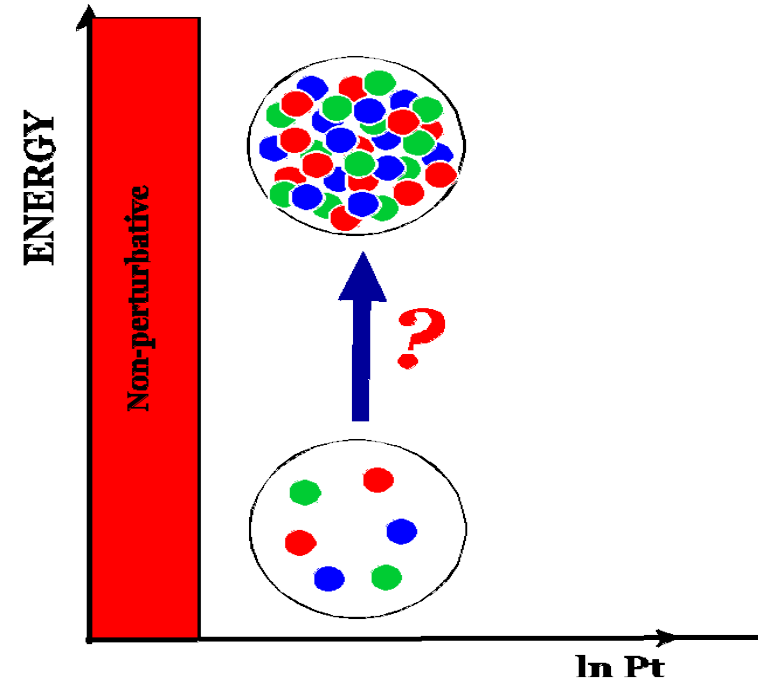
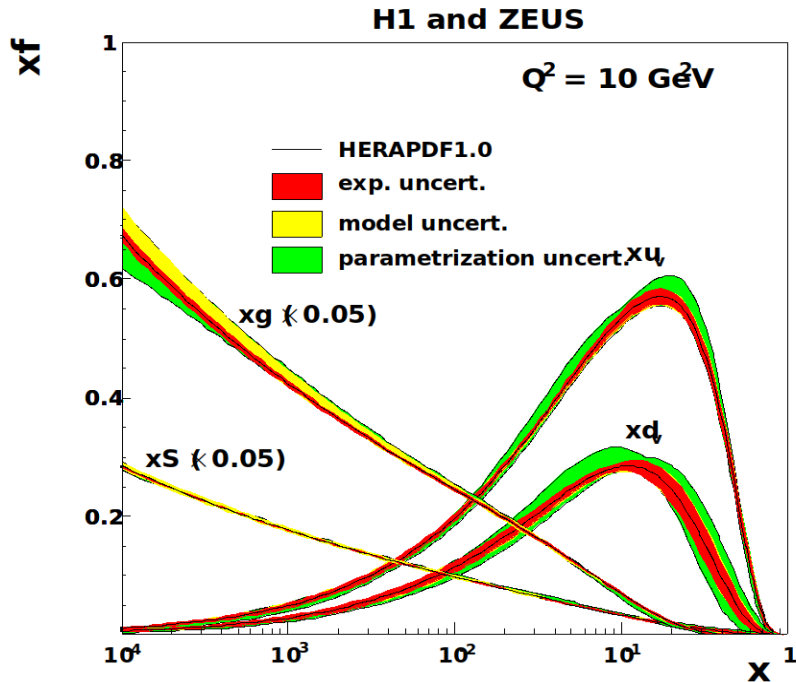
$$E \frac{d\sigma}{d^3p} \sim f_1(x, p_t^2) \otimes f_2(x, p_t^2) \otimes \frac{d\sigma}{dt} \otimes D(z, p_t^2) + \dots$$



bulk of QCD phenomena happens at low  $p_t$  (small  $x$ )



# dynamics of *universal gluonic matter*: *gluon saturation*



How does this happen ?

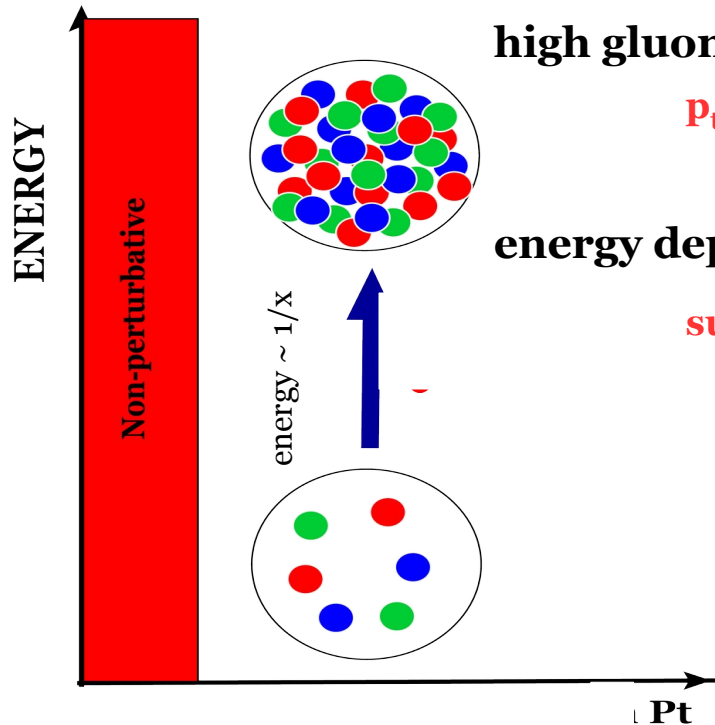
How do correlation functions evolve ?

Is there a universal fixed point for the evolution ?

Are there scaling laws ?

$$P_{gg} \sim P_{gq} \sim \frac{1}{x}$$

# QCD at high energy/small x: gluon saturation



high gluon density: Eikonal multiple scattering

$p_t$  broadening (generic to multiple scattering)

energy dependence: x-evolution via JIMWLK/BK

suppression of spectra/away side peaks

$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

$$Q_s^2(x = 3 \times 10^{-4}) \sim 1 \text{ GeV}^2$$

for a proton target (quarks)

a framework for multi-particle production in QCD at small x/low  $p_t$

*Shadowing/Nuclear modification factor*

*Azimuthal angular correlations (photon-hadron,...)*

*Long range rapidity correlations (ridge,...)*

*Initial conditions for hydro*

*Thermalization (?)*

$$x \leq 0.01$$

$$\alpha_s \ln(x_v/x) \sim 1$$

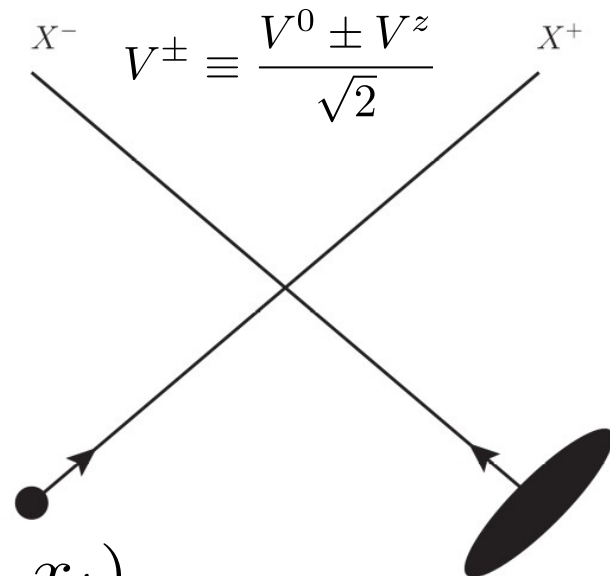
# Eikonal approximation (*dilute-dense scattering*)

$$J_a^\mu \simeq \delta^{\mu-} \rho_a$$

$$D_\mu J^\mu = D_- J^- = 0$$

$$\partial_- J^- = 0 \quad (\text{in } A^+ = 0 \text{ gauge})$$

does not depend on  $x^-$



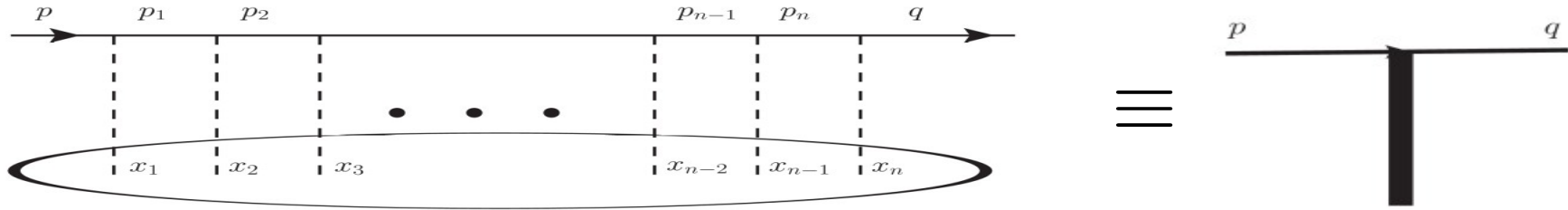
EOM:solution  $A_a^-(x^+, x_t) \equiv n^- S_a(x^+, x_t)$

$$n^\mu = (n^+ = 0, n^- = 1, n_t = 0)$$

recall (eikonal approx):  $\bar{u}(q) \gamma^\mu u(p) \rightarrow \bar{u}(p) \gamma^\mu u(p) \sim p^\mu$   
 $\bar{u}(q) \not{A} u(p) \rightarrow p \cdot A \sim p^+ A^-$

scattering of a quark from background color field  $A_a^-(x^+, x_t)$

# CGC: eikonal approximation (tree level)



$$i\mathcal{M}(p, q) = 2\pi\delta(p^+ - q^+) \bar{u}(q) \not{n} \int d^2x_t e^{-i(q_t - p_t) \cdot x_t} [V(x_t) - 1] u(p)$$

with  $V(x_t) \equiv \hat{P} \exp \left\{ ig \int_{-\infty}^{+\infty} dx^+ S_a^-(x^+, x_t) t_a \right\}$

scattering from small x gluons of the target can cause only a small angle deflection

DIS, proton-nucleus collisions: dipoles

$$\langle \text{Tr} V(x_\perp) V^\dagger(y_\perp) \rangle$$

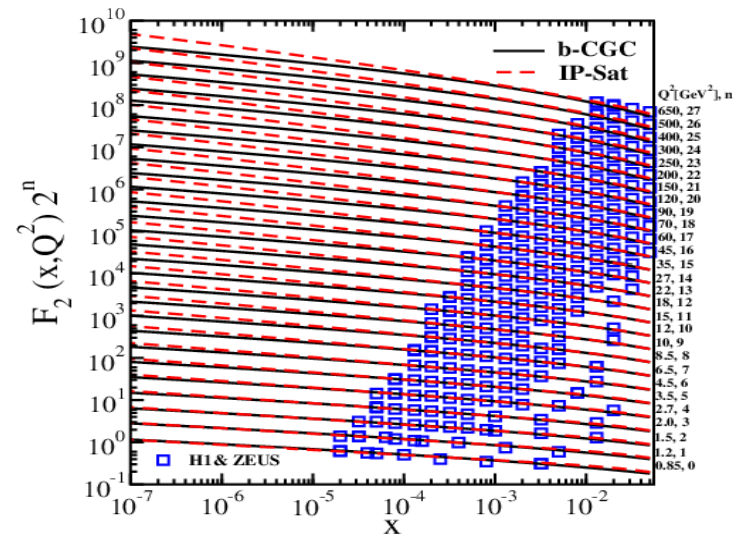
Pre-RHIC: all we know about saturation came from HERA structure function fits

HERA: large kinematic window for x  
structure functions:  $x = x_{bj}$

Still debating whether saturation is seen at HERA!

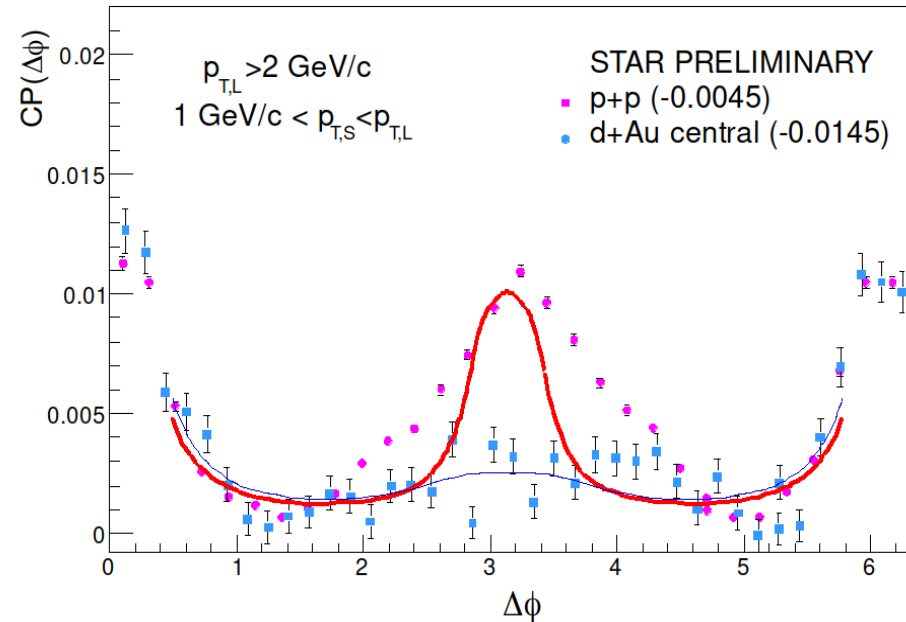
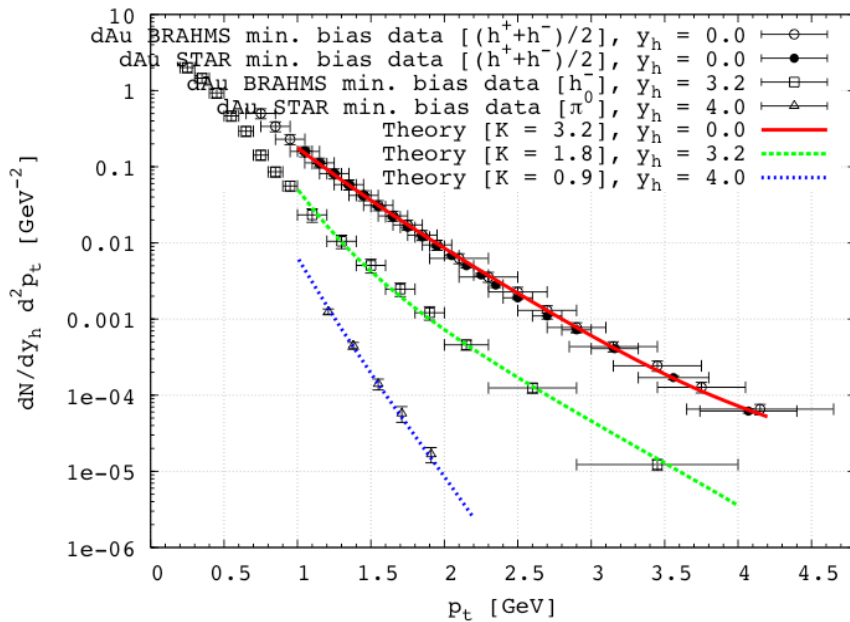
Less-inclusive observables are more discriminatory

$$x \neq x_{bj}$$



# CGC at RHIC

Single and double inclusive hadron production in dA collisions



DHJ, NPA770 (2006) 57

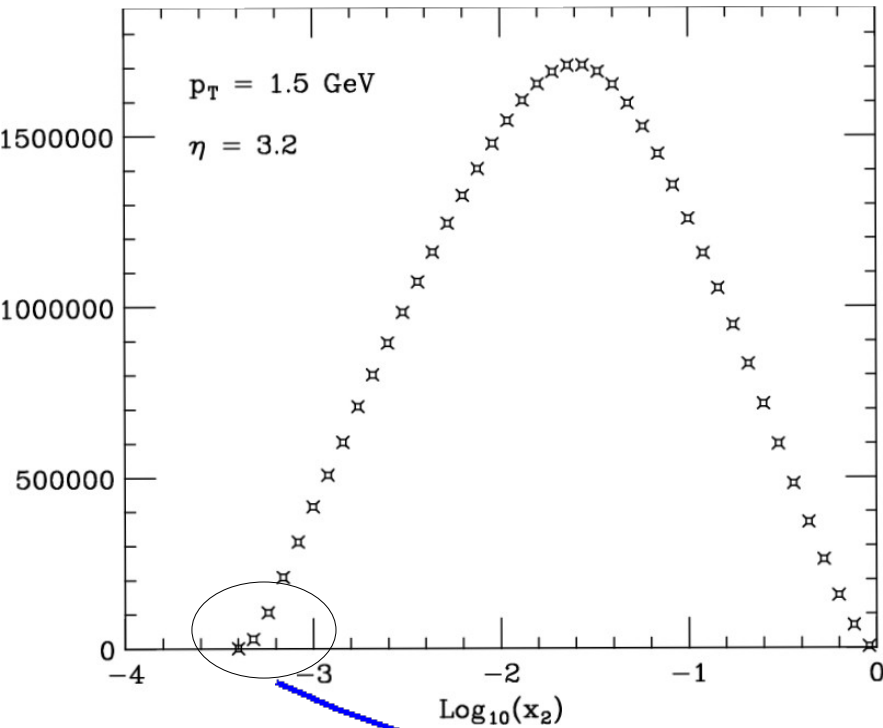
Albacete+Marquet  
PRL105 (2010) 162301

**convolution over x: is x small enough?**

# Single inclusive pion production in pp at RHIC

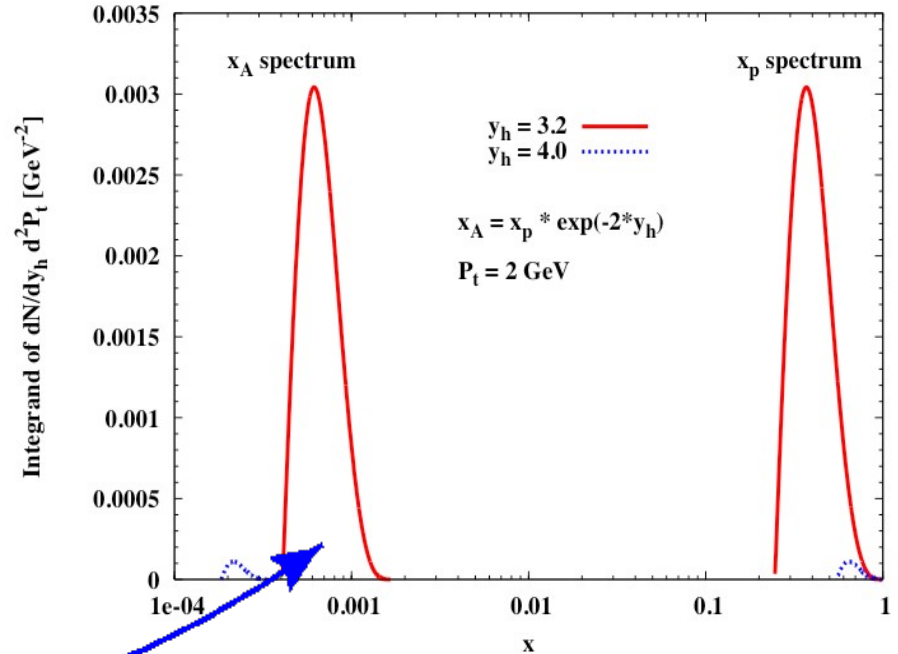
collinear factorization

GSV, PLB603 (2004) 173-183



CGC

DHJ, NPA765 (2006) 57-70



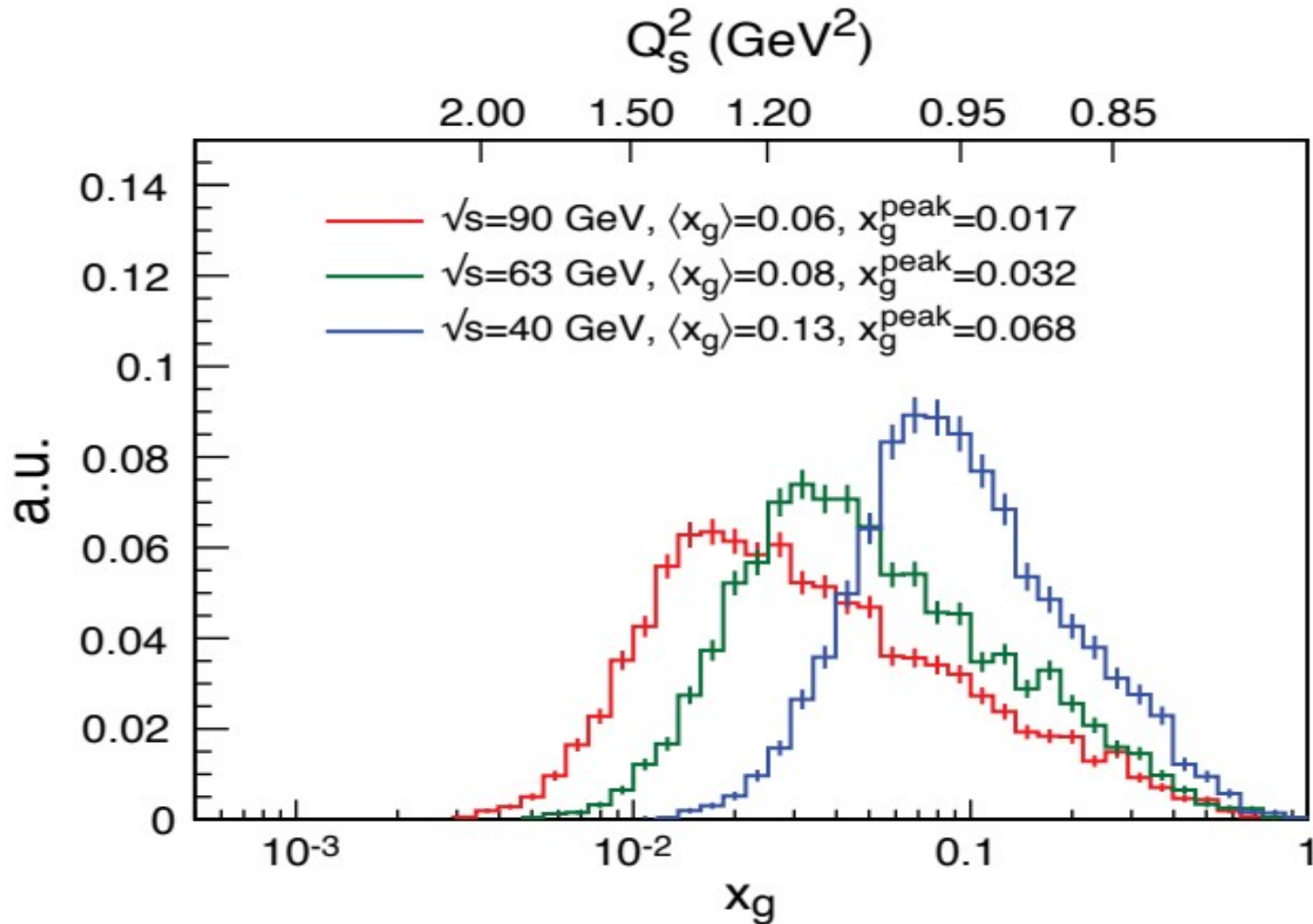
$$\int_{x_{\min}}^1 dx x G(x, Q^2) \dots \longrightarrow x_{\min} G(x_{\min}, Q^2) \dots$$

which kinematics are we in?

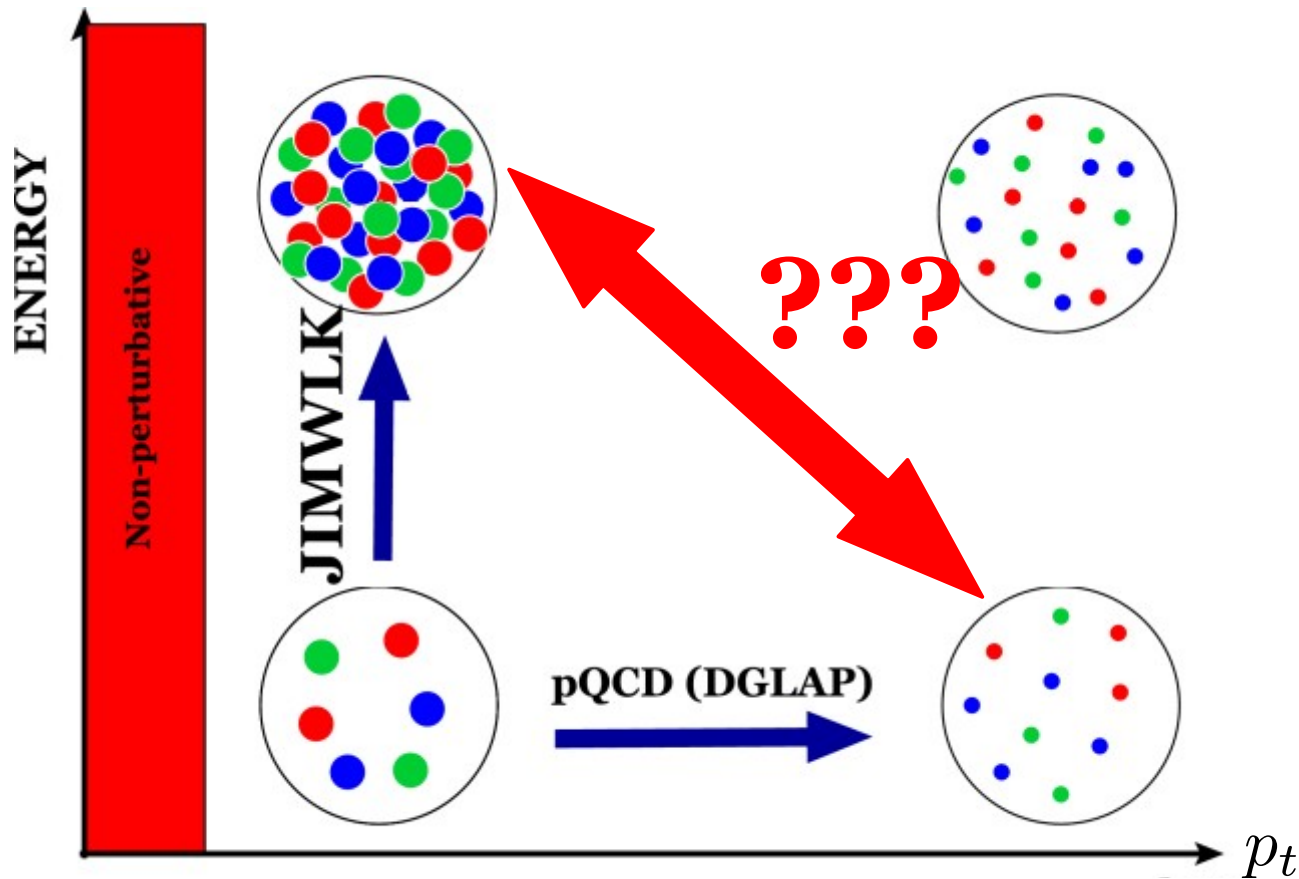


# EIC

Kinematics of double inclusive hadron production



# QCD kinematic phase space



## unifying saturation with high $p_t$ (large $x$ ) physics?

kinematics of saturation: where is saturation applicable?

structure functions at all  $Q^2$

high  $p_t$  and forward-backward correlations,

spin physics, *early time e-loss in heavy ion collisions*, .....

## Beyond eikonal approximation: longitudinal momentum exchange

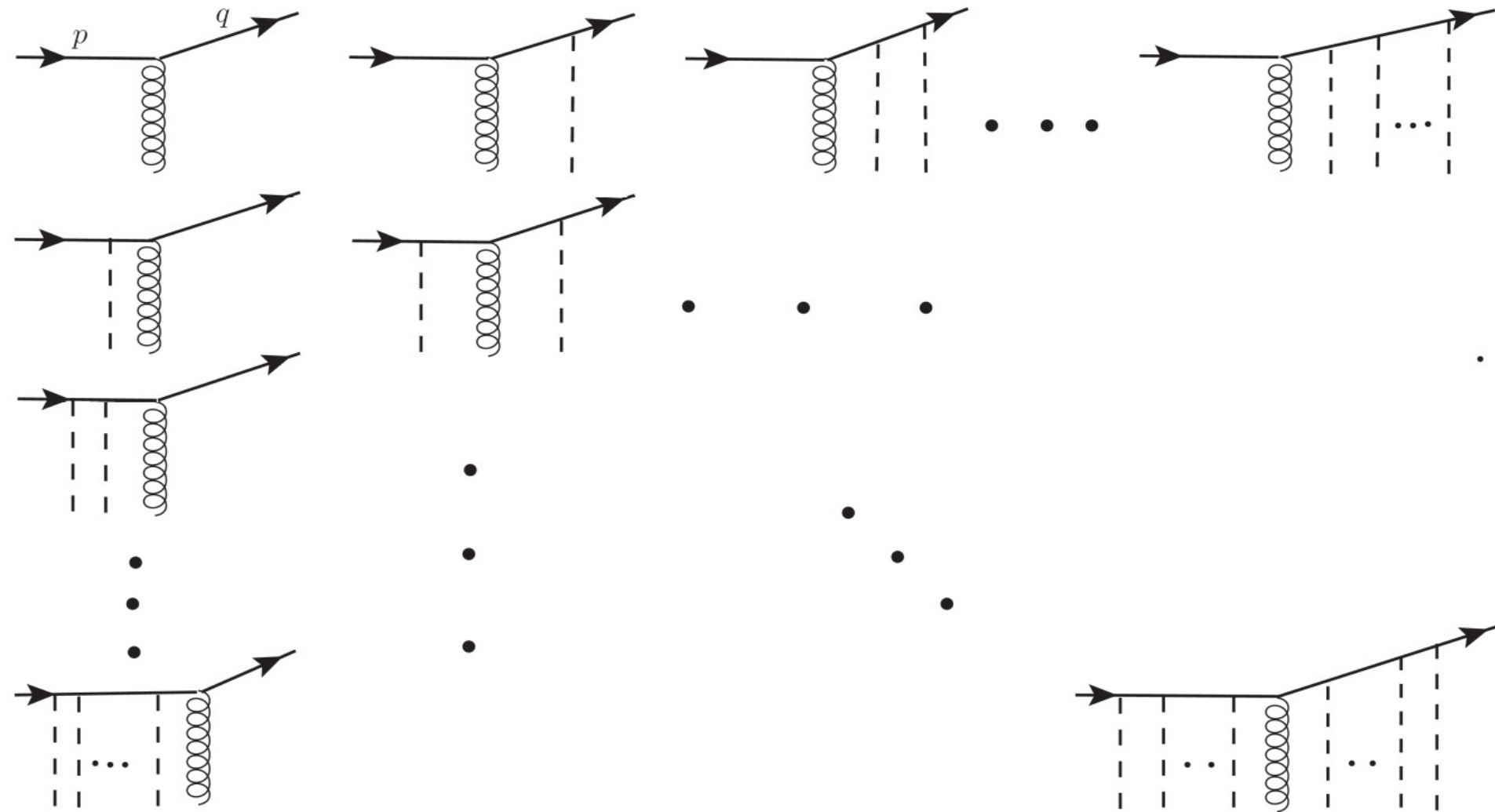
$$\mathcal{A}^\mu = \mathbf{S}^\mu + \mathbf{A}^\mu$$

single scattering from  
large  $x$  gluons of target

$$\mathbf{A}^\mu = (\mathcal{A}^\mu - \mathbf{S}^\mu)$$

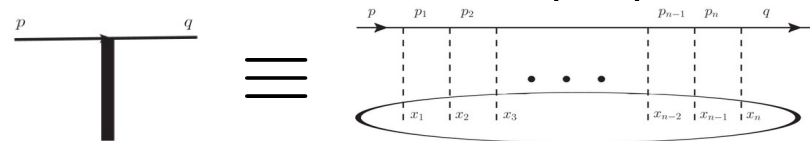
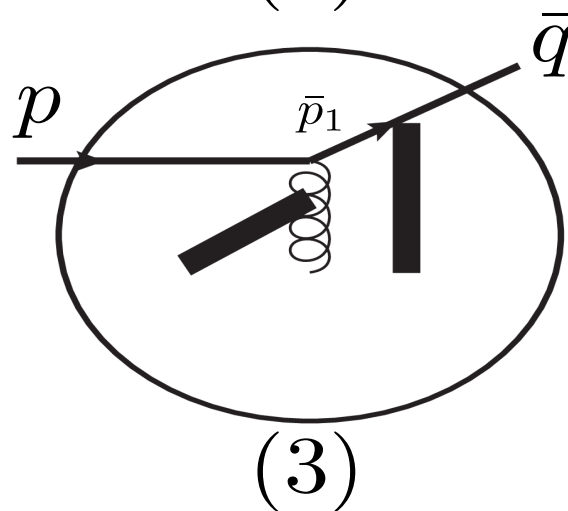
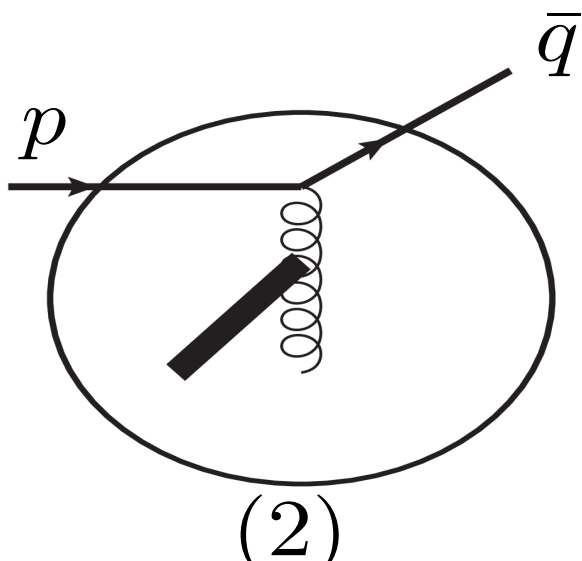
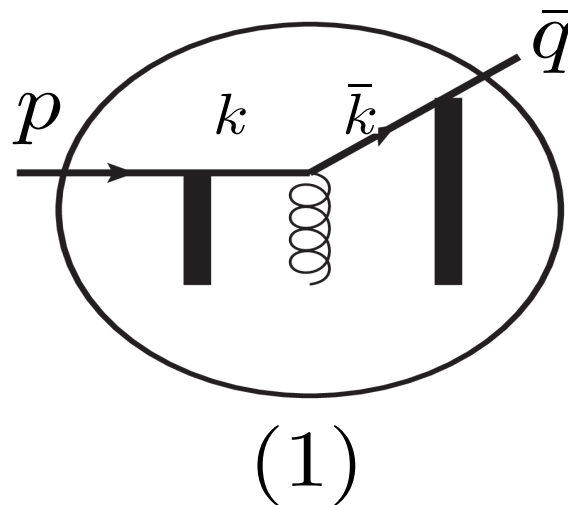
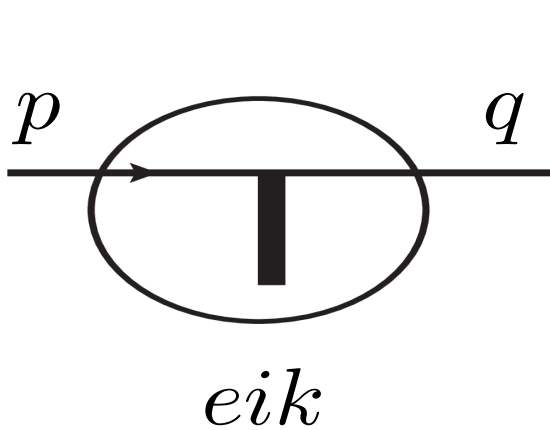
multiple scatterings from  
small  $x$  gluons of target (CGC)

$$\mathbf{S}^\mu$$



# Quark scattering: beyond small x approximation

large  $x$  partons of target can cause a large-angle deflection of the projectile



soft (eikonal) limit:  $i\mathcal{M} \longrightarrow i\mathcal{M}_{eik}$

use spinor helicity formalism: helicity amplitudes

Including large  $x$  partons of the target leads to:

longitudinal double spin asymmetries ( $A_{LL}$ )

baryon transport (beam rapidity loss), .....

**one-loop corrections: factorized cross section at all  $x$  ( $p_t$ )**

gluon radiation

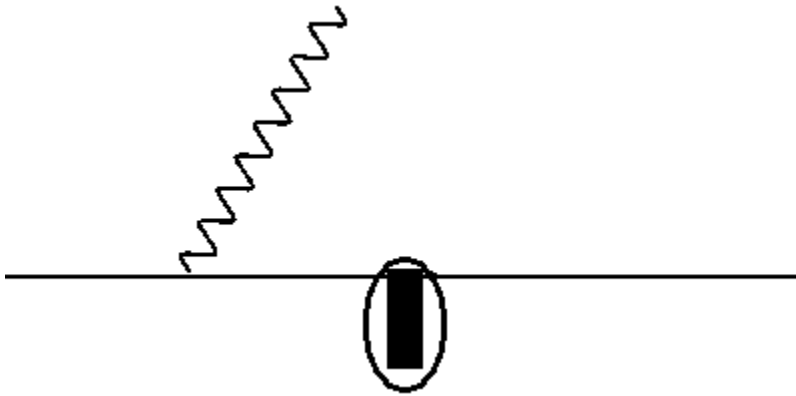
**related problem: photon production**

photon-hadron correlations:

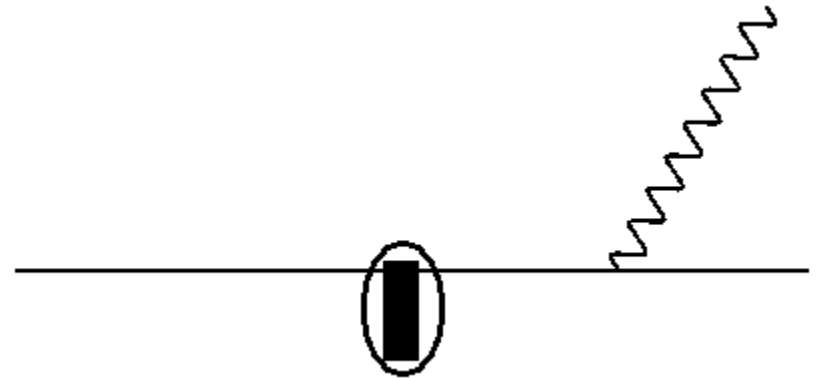
azimuthal angular correlations from low to high  $p_t$

forward-backward rapidity correlations

# photon radiation: **small x (eikonal approximation)**



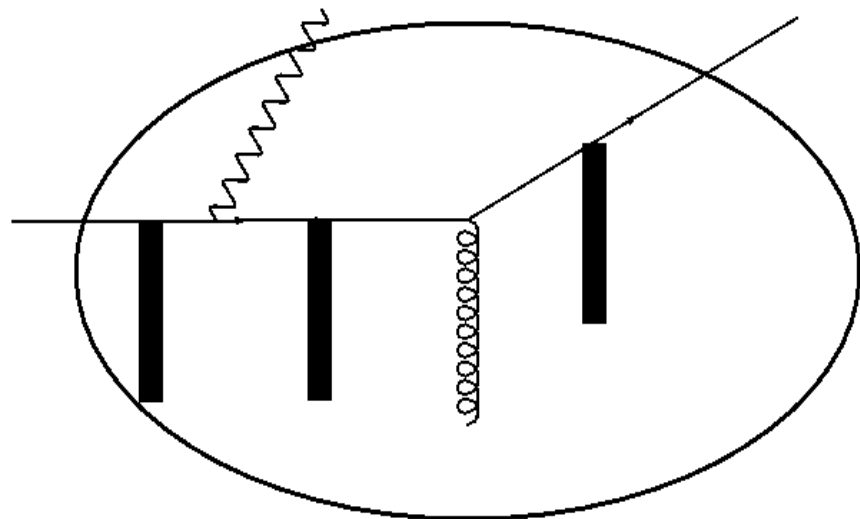
before quark scatters on the target



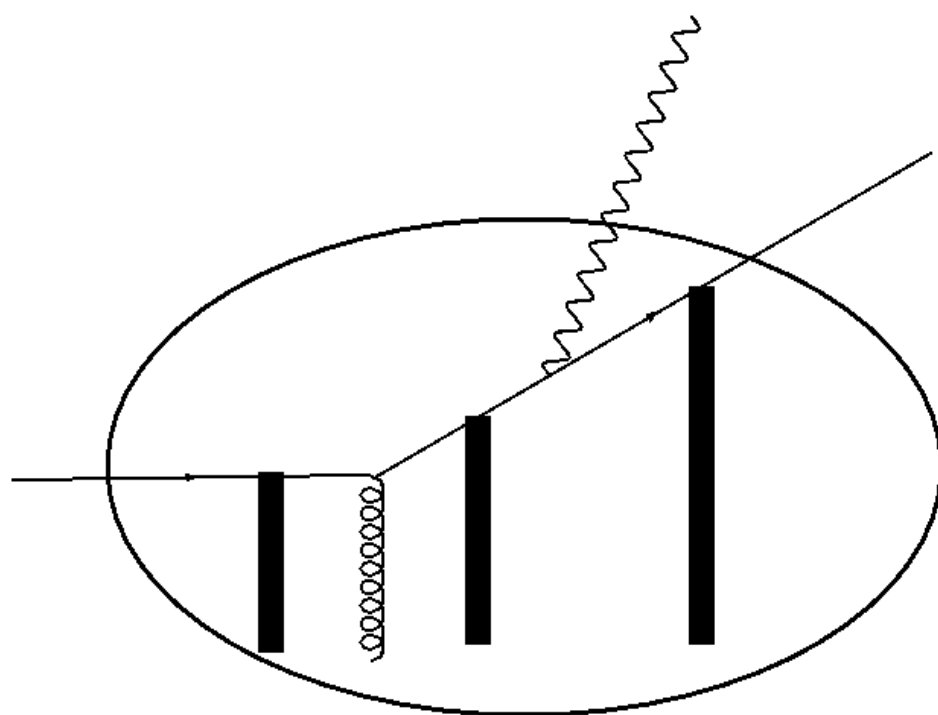
after quark scatters on the target

No radiation inside the target

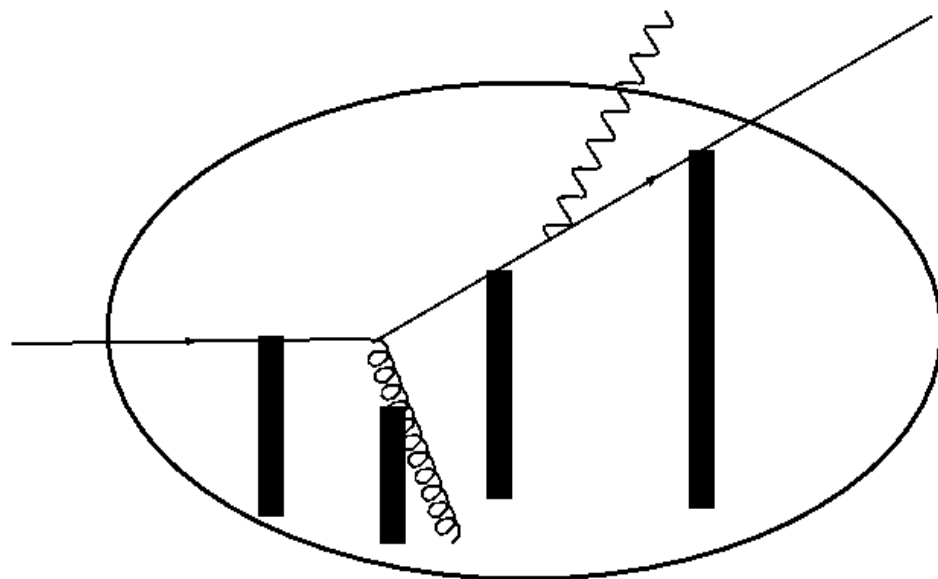
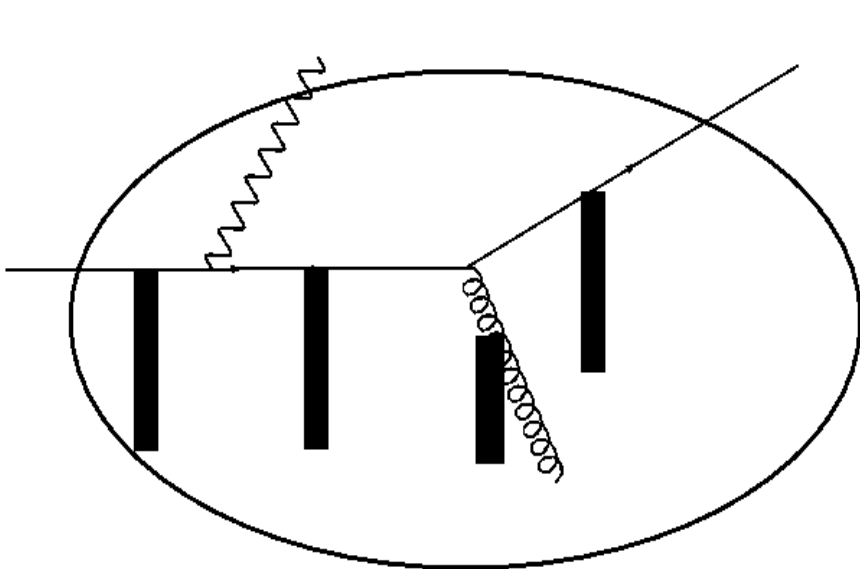
# photon radiation: **all x**



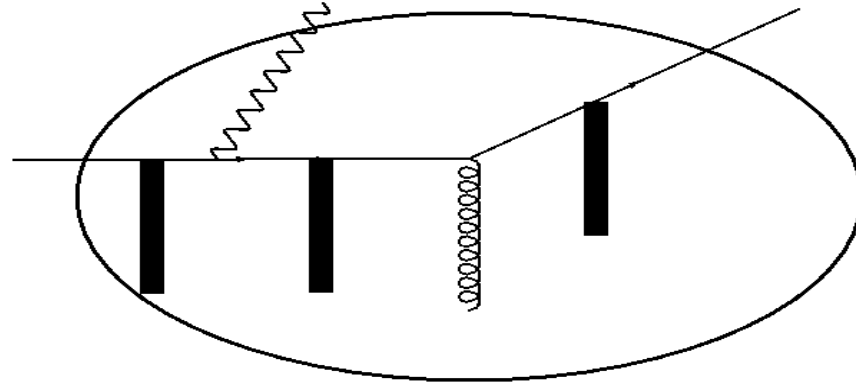
before hard scattering



after hard scattering



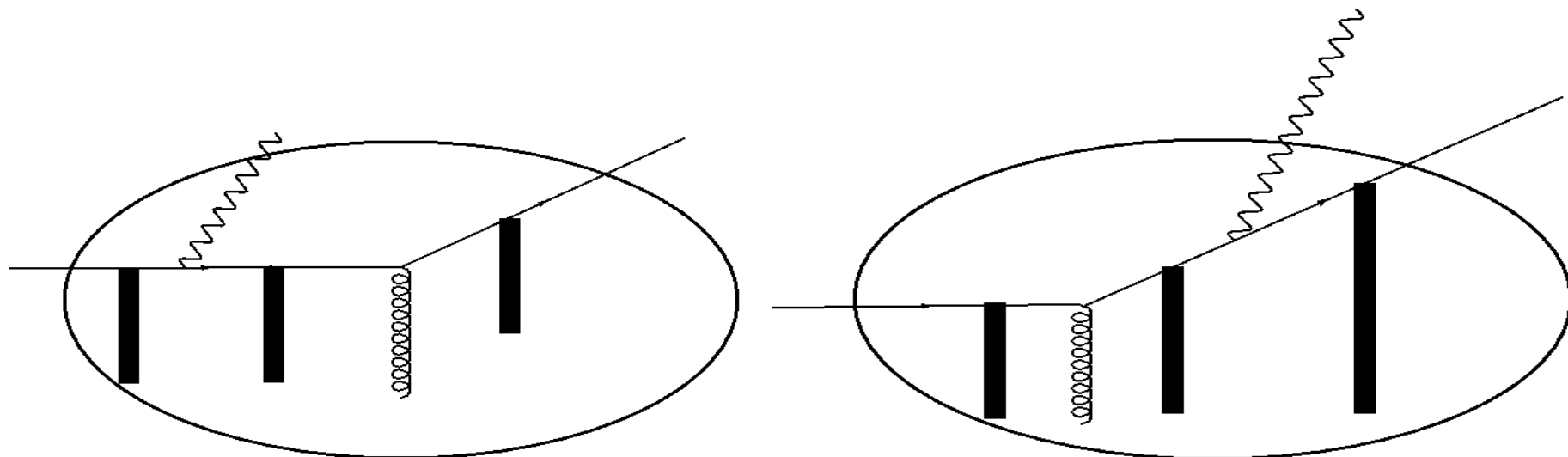
# photon radiation: all $x$



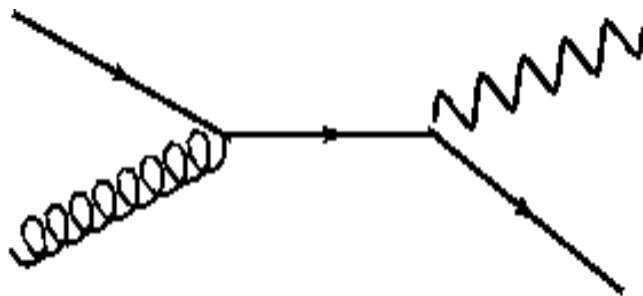
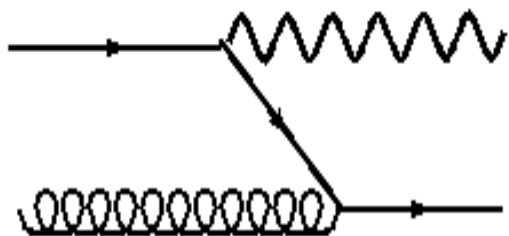
$$\begin{aligned}
 i\mathcal{M}_1(p, q, l) = & \\
 eg \int \frac{d^2 k_{2t}}{(2\pi)^2} \frac{d^2 k_{3t}}{(2\pi)^2} \frac{d^2 \bar{k}_{1t}}{(2\pi)^2} \int d^4 x d^2 y_{1t} d^2 y_{2t} d^2 \bar{y}_{1t} dz^+ \theta(x^+ - z^+) e^{i(l^+ + \bar{q}^+ - p^+)x^-} \\
 & e^{-i(\bar{q}_t - \bar{k}_{1t}) \cdot \bar{y}_{1t}} e^{-i(\bar{k}_{1t} - k_{3t}) \cdot x_t} e^{-i(k_{3t} - k_{2t}) \cdot y_{2t}} e^{-i(l_t + k_{2t} - p_t) \cdot y_{1t}} \bar{u}(\bar{q}) \bar{V}(\bar{y}_{1t}; x^+, \infty) \frac{\not{n} \bar{k}_1}{2\bar{n} \cdot \bar{q}} \\
 A(x) & \left[ \frac{\not{k}_3}{2n \cdot (p - l)} V(y_{2t}; z^+, x^+) \frac{\not{n} \not{k}_2}{2n \cdot (p - l)} + i \frac{\delta(x^+ - z^+)}{2n \cdot (p - l)} \not{n} \right] \\
 \not{\epsilon}(l) & \frac{\not{k}_1}{2n \cdot p} V(y_{1t}; -\infty, z^+) \not{n} u(p)
 \end{aligned}$$



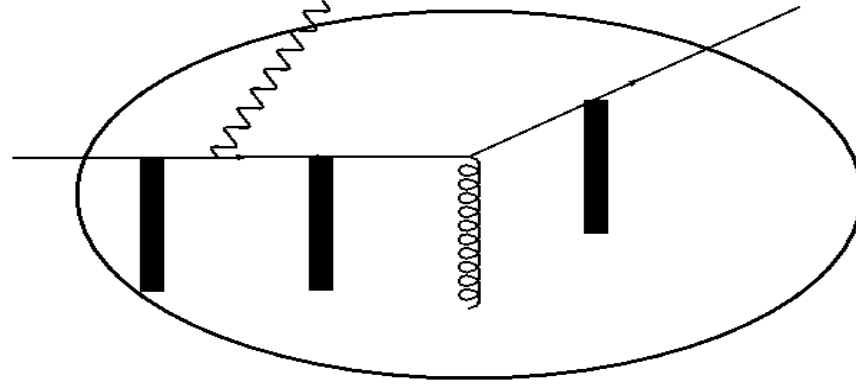
# pQCD limit (single gluon exchange):



$$V = U = 1$$



# photon radiation: helicity amplitudes



$$\mathcal{N}_{1-1} = \bar{u}(\bar{q}) \frac{\not{\epsilon} \not{k}_1}{2\bar{n} \cdot \bar{q}} \mathcal{A}(x) \frac{\not{k}_3 \not{\epsilon} \not{k}_2 \not{\epsilon}(l) \not{k}_1 \not{\epsilon}}{2n \cdot p 2n \cdot (p-l) 2n \cdot (p-l)} u(p)$$

$$\mathcal{N}_{1-2} = \bar{u}(\bar{q}) \frac{\not{\epsilon} \not{k}_1}{2\bar{n} \cdot \bar{q}} \mathcal{A}(x) \frac{\not{\epsilon} \not{\epsilon}(l) \not{k}_1 \not{\epsilon}}{2n \cdot p 2n \cdot (p-l)} u(p)$$

$$\mathcal{N}_{1-1}^{++} = (\mathcal{N}_{1-1}^{--})^* = -\sqrt{\frac{n \cdot p}{n \cdot (p-l)}} \frac{[n \cdot l k_{2\perp} \cdot \epsilon_{\perp}^* - n \cdot (p-l) l_{\perp} \cdot \epsilon_{\perp}^*]}{n \cdot l n \cdot (p-l)} \langle \bar{k}_1^+ | \mathcal{A}(x) | k_3^+ \rangle$$

$$\mathcal{N}_{1-2}^{++} = (\mathcal{N}_{1-2}^{--})^* = -\sqrt{\frac{n \cdot p}{n \cdot (p-l)}} \langle \bar{k}_1^+ | \mathcal{A}(x) | n^+ \rangle$$

$$\mathcal{N}_{1-1}^{+-} = (\mathcal{N}_{1-1}^{-+})^* = -\sqrt{\frac{n \cdot p}{n \cdot (p-l)}} \frac{[n \cdot p l_{\perp} \cdot \epsilon_{\perp} - n \cdot l k_{1\perp} \cdot \epsilon_{\perp}]}{n \cdot p n \cdot l} \langle \bar{k}_1^+ | \mathcal{A}(x) | k_3^+ \rangle$$

$$\mathcal{N}_{1-2}^{+-} = \mathcal{N}_{1-2}^{-+} = 0$$

# So far

**Classical CGC is generalized by including longitudinal momentum exchange (large angle scattering) with the target**  
*beam rapidity loss*

**Helicity amplitudes for quark and photon production are evaluated**  
*spin asymmetries*

**Relevant operators are identified**  
*products of Wilson lines and large  $x$  gluon field*  
*computing expectation values?*

**Toward a factorized cross section at all  $x$  ( $p_t$ )**  
*gluon radiation*

**Need to classify/regulate the divergences**

**Combining with small  $x$**   
*sharp boundary ( $x = 0.01$ )?*  
*matching field strengths?*

# ***SUMMARY***

***CGC is a systematic approach to high energy collisions***

***strong hints from RHIC, LHC,...***

***toward precision: NLO, sub-eikonal corrections, ...***

***CGC breaks down at large  $x$  (high  $p_t$ )***

***a significant part of EIC/RHIC/LHC phase space is at large  $x$***

***transition from large  $x$  physics to CGC (kinematics?)***

***Toward inclusion of large  $x$  physics:***

***spin asymmetries***

***beam rapidity loss***

***particle production in both small and large  $p_t$  kinematics***

***two-particle correlations: from forward-forward to forward-backward***

***one-loop correction: both collinear and CGC factorization limits***

***need to clarify/understand: gauge invariance, initial conditions, .....***