Complementary Two-Particle Correlation Observables for Relativistic Nuclear Collisions

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Motivation

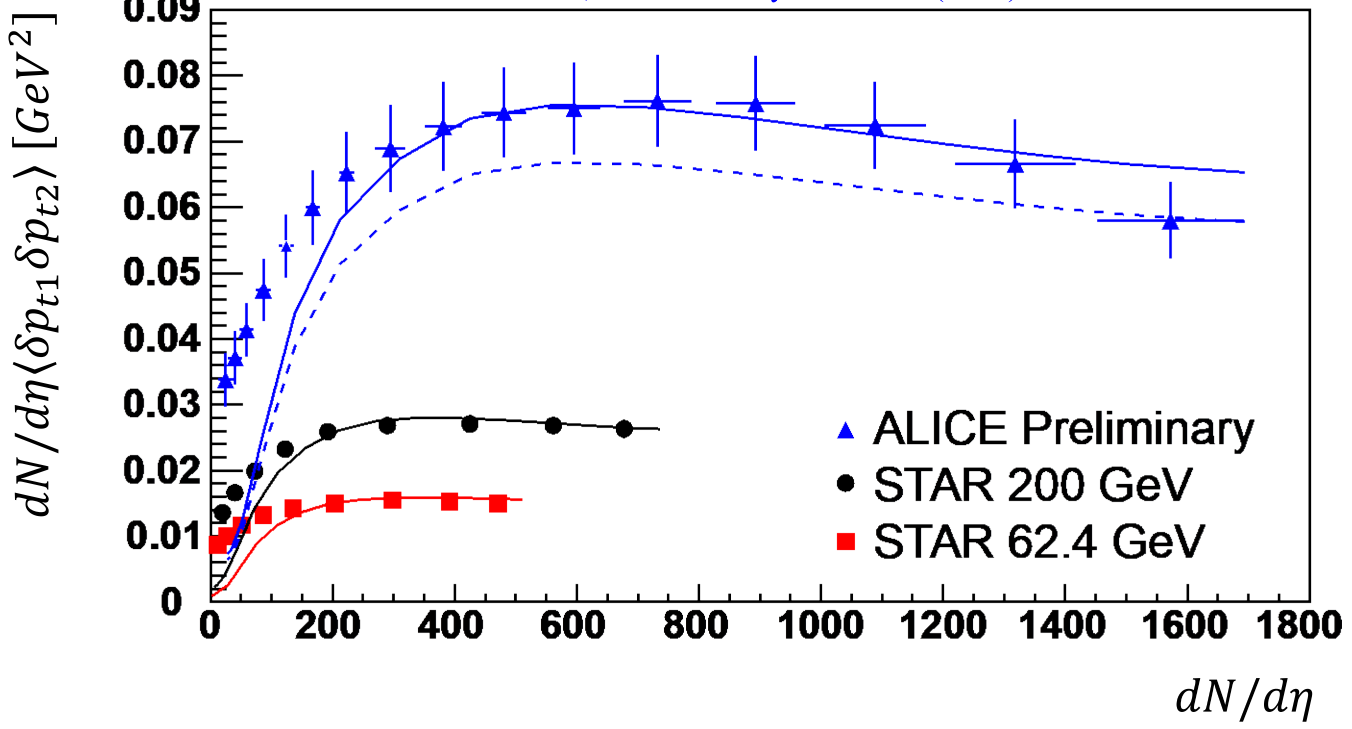
What are the observable consequences of incomplete thermalization of the medium created in nuclear collisions?

Correlations of transverse momentum fluctuations

$$\langle \delta p_{T1} \delta p_{T2} \rangle = \frac{\langle \sum_i \sum_{i \neq j} \delta p_{Ti} \delta p_{Tj} \rangle}{\langle N(N-1) \rangle}$$

$$\delta p_{Ti} = p_{Ti} - \langle p_T \rangle$$

Gavin, Moschelli Phys.Rev. C85 (2012) 014905



STAR, Phys.Rev.C 72 (2005) 044902
ALICE preliminary since published in Eur. Phys. J. C 74 (2014) 3077

Boltzmann-Langevin Equation

Boltzmann Eq.
Relaxation Time Approx.

$$\left(\frac{\partial}{\partial t} + \mathbf{v_p} \cdot \nabla\right) f(\mathbf{p}, \mathbf{x}, t) = -\nu[\, f(\mathbf{p}, \mathbf{x}, t) - f^{eq}(\mathbf{p}, \mathbf{x}, t)\,]$$

Relaxation time ν^{-1}
Drift velocity $\mathbf{v_p} = \mathbf{p}/E$
Temperature T
Velocity $\boldsymbol{u}$
Chemical Potential μ

Local equilibrium distribution

$$f^{eq} = \exp\{-\gamma(E - \mathbf{p} \cdot \boldsymbol{u} - \mu)/T\}$$

Conservation laws require we choose T, $\boldsymbol{u}$, μ so that f^{eq} gives the same energy, momentum, and particle density as f. Use eigenfunctions φ_i with zero eigenvalue corresponding to the conserved quantities to define a projection operator.

$$P = \sum_{i=1}^{5} c(\mathbf{x}, t)\varphi_i$$

$$Pf = f^{eq}$$

Linearized Boltzmann-Langevin Equation:

$$f(t + \Delta t) - f(t) = -\nu(1 - P)f(t)\Delta t + \Delta W(t)$$

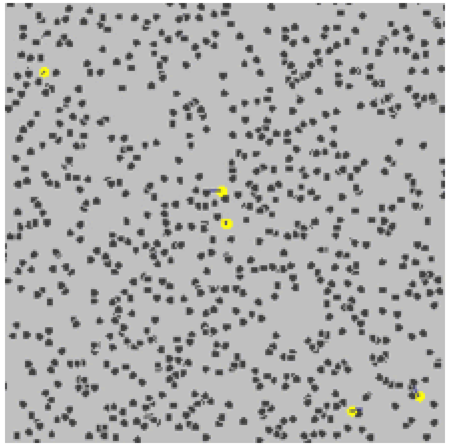
Stochastic term $\Delta W(t)$ represents a random change to f at each time step.

Gavin, Moschelli, Zin Phys. Rev. C95 (2017) 6, 064901

Brownian Motion

$$\mathrm{v}(t+\Delta t)-\mathrm{v}(t)=-\gamma \mathrm{v}(t)\Delta t+\Delta W(t)$$

change in velocity — friction — collisions

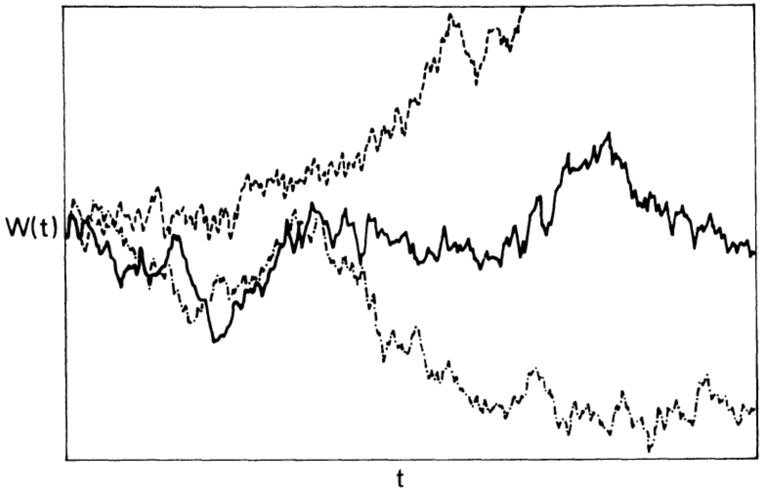


Average of noise $\langle \Delta W \rangle = 0$

Variance of noise $\langle \Delta W^2 \rangle = \Gamma \Delta t$

"strength" of the noise

Using the fluctuation dissipation theorem one can find $\Gamma = 2\gamma/m$



Gardiner, Handbook of Stochastic Methods 3ed

Single Particle Equation

Averaging over noise $$f(t+\Delta t) - f(t) = -\nu(1-P)f(t)\Delta t + \Delta W(t)$$

Gives $$\frac{d}{dt}\langle f\rangle = -\nu(1-P)\langle f\rangle$$

Solve using the method of characteristics to find

$$f(\mathbf{p},\mathbf{x},t) = f_0(\mathbf{p},\mathbf{x}-\mathbf{v_p}t)S(t,t_0) + f^{eq}(\mathbf{p},\mathbf{x}-\mathbf{v_p}t)(1-S(t,t_0))$$

initial conditions — equilibrium — survival probability

S is the probability particles escape the collision volume without suffering any collisions

$$S(t,t_0) = e^{-\int_{t_0}^{t}\nu(t')dt'}$$

As thermalization proceeds $S \to 0$.

Two-Particle Equation

Two-particle correlations

$$G_{12} = \langle f_1 f_2 \rangle - \langle f_1 \rangle \langle f_2 \rangle - \langle f_1 \rangle \delta(1-2)$$

$$\delta(1-2) = \delta(\mathbf{x}_1 - \mathbf{x}_2)\delta(\mathbf{p}_1 - \mathbf{p}_2)$$

Using the Itô product rule:

$$\Delta\langle f_1 f_2 \rangle = \underbrace{\langle f_2 \Delta f_1 \rangle + \langle f_1 \Delta f_2 \rangle}_{-\nu(2-P_1-P_2)\langle f_1 f_2 \rangle} + \underbrace{\langle \Delta f_1 \Delta f_2 \rangle}_{\Gamma \Delta t}$$

$$\Gamma = \nu P_1 P_2 \big(\langle f_1 \rangle - f_1^{eq}\big)\delta(1-2)$$

Using the Itô product rule and the Boltzmann-Langevin Eq.

$$\left(\frac{d}{dt} + \nu(2 - P_1 - P_2)\right) G_{12} = \nu P_1 P_2 \big(\langle f_1 \rangle - f_1^{eq}\big)\delta(1-2)$$

Using the method of characteristics again

$$G_{12} = G_{12}^{eq} + A_{12} S + B_{12} S^2$$

The initial phase space distribution determines the coefficients A_{12} and B_{12} as functions of the momenta and initial positions

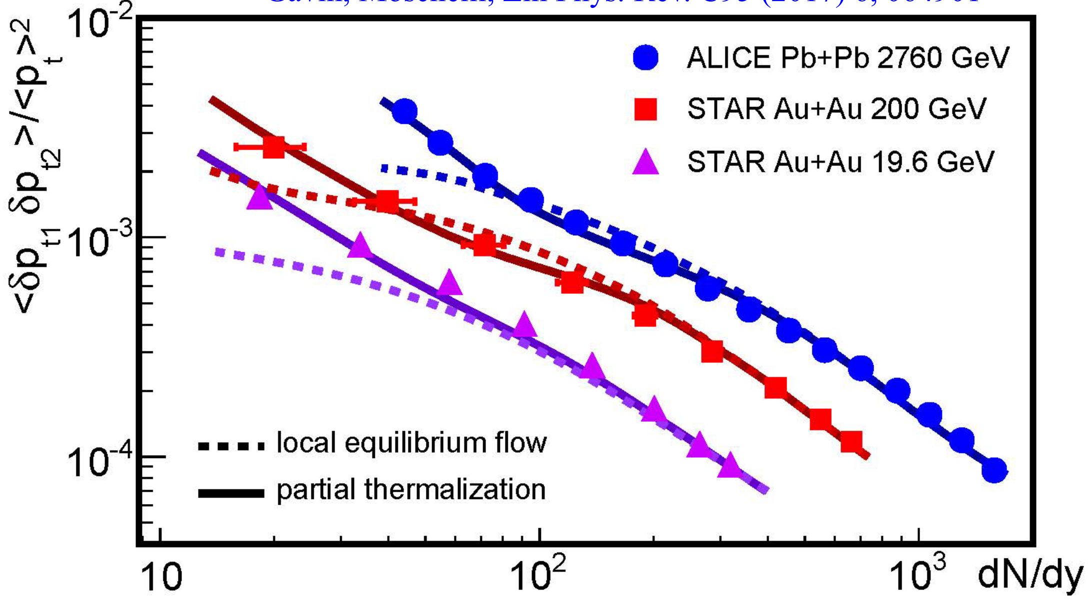
Observing Thermalization

$$\langle \delta p_{T1} \delta p_{T2} \rangle = \frac{\langle \sum_i \sum_{i \neq j} \delta p_{Ti} \delta p_{Tj} \rangle}{\langle N(N-1) \rangle}$$

$$\langle \delta p_{T1} \delta p_{T2} \rangle = \langle \delta p_{T1} \delta p_{T2} \rangle_0 S^2 + \langle \delta p_{T1} \delta p_{T2} \rangle_{eq} (1 - S^2)$$

S = survival probability
$S = 1 \rightarrow$ no interaction
$S = 0 \rightarrow$ local equilibrium

Gavin, Moschelli, Zin Phys. Rev. C95 (2017) 6, 064901



Look for updates coming soon!
Zoulfekar PhD thesis.

ALICE: Eur. Phys. J C74, 3077 (2014)
STAR: Phys. Rev. C72, 044902 (2005)
STAR: Phys. Rev. C99, 044918 (2019)

Use Multiple Observables to Constrain Theory

Complementary Observables

$$(1+\mathcal{R})\langle\delta p_{T1}\delta p_{T2}\rangle - \mathcal{C} + 2\langle p_T\rangle\mathcal{D} + \langle p_T\rangle^2\mathcal{R} = 0$$

Cody, Gavin, Koch, Kocherovsky, Mazloum, Moschelli arXiv: 2110.04884

$\mathcal{R}$	Multiplicity Fluctuations
$\langle\delta p_{T1}\delta p_{T2}\rangle$	Correlations of Transverse Momentum Fluctuations
$\mathcal{C}$	Transverse Momentum Correlations
$\mathcal{D}$	Multiplicity-Momentum Correlations **(New)**

Why are these complementary? All derived from the same distribution of two-particle correlations

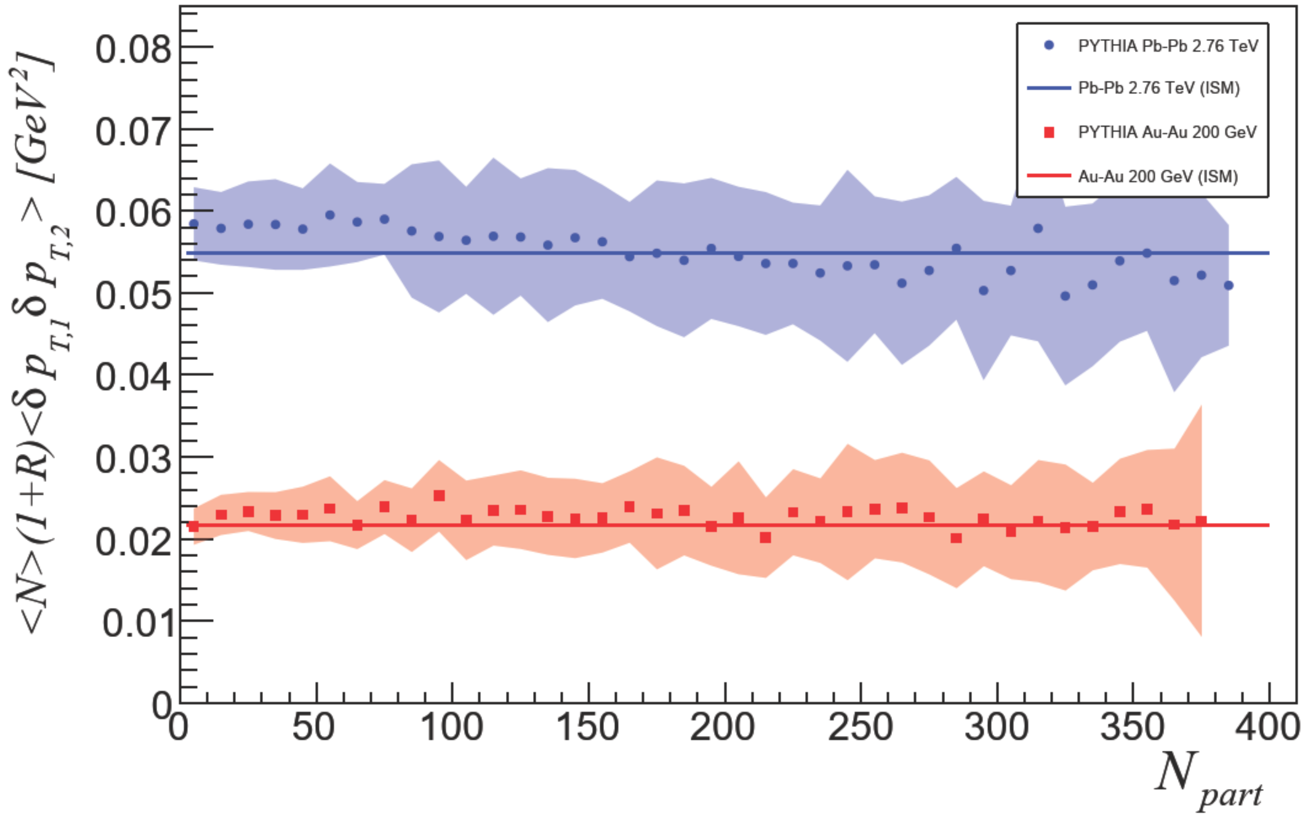
Correlations of Transverse Momentum Fluctuations

$$\langle \delta p_{T1} \delta p_{T2} \rangle = \frac{\langle \sum_i \sum_{i \neq j} \delta p_{Ti} \delta p_{Tj} \rangle}{\langle N(N-1) \rangle}$$

$$\langle \delta p_{T1} \delta p_{T2} \rangle = \langle \delta p_{T1} \delta p_{T2} \rangle_0 S^2 + \langle \delta p_{T1} \delta p_{T2} \rangle_{eq} (1 - S^2)$$

- $\delta p_{Ti} = p_{Ti} - \langle p_T \rangle$
- Used to search for QCD critical point
- Sensitive to temperature fluctuations
- Removes multiplicity fluctuations

$0.15 < p_T < 2\ GeV$
$|\eta| < 1$



Lines – independent source model
Points - PYTHIA/Angantyr

PYTHIA Comput.Phys.Commun. 191 (2015) 159-177, arXiv:1410.3012
Angantyr 10 (2018) 134, arXiv:1806.10820JHEP

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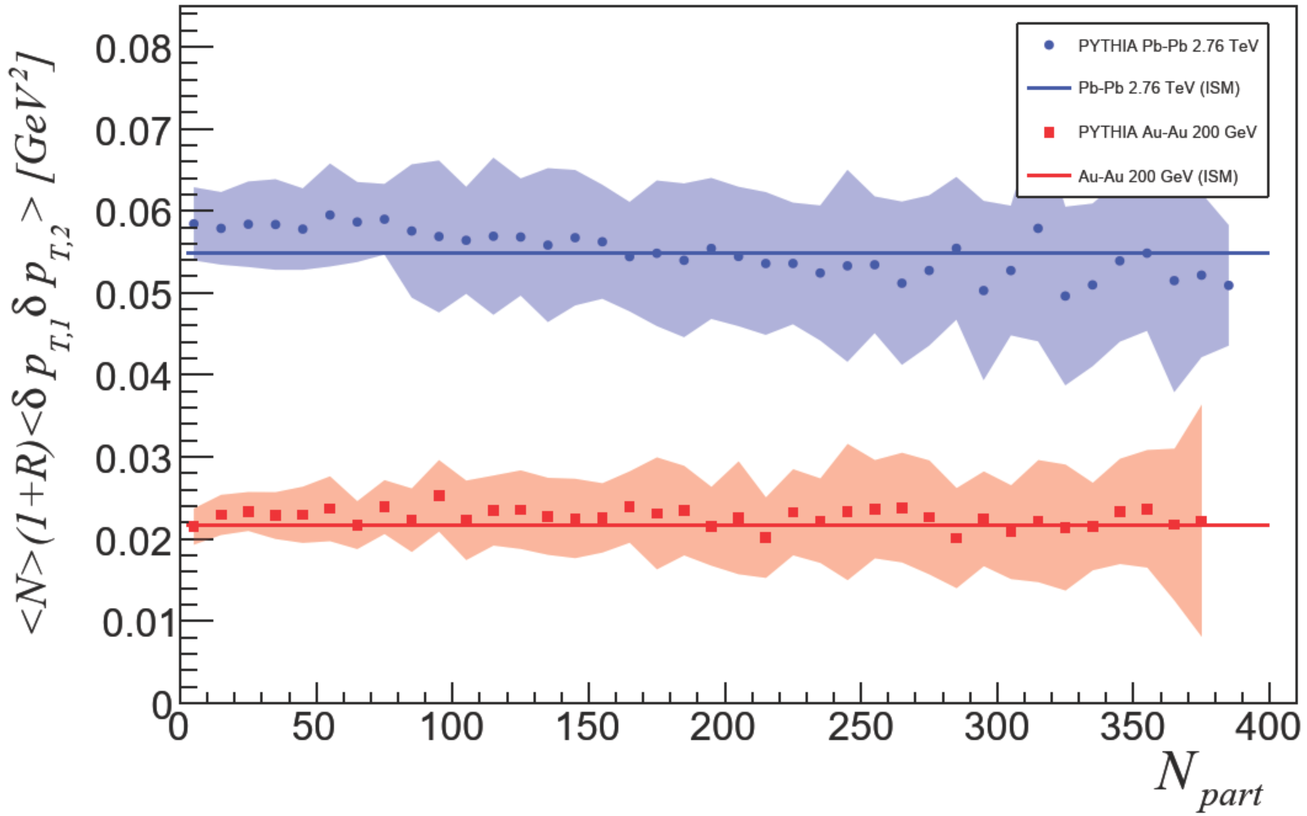
Independent Source Model

$$\langle \delta p_{t1} \delta p_{t2} \rangle = \frac{2 \langle \delta p_{t1} \delta p_{t2} \rangle_{pp}}{\langle N_{part} \rangle} \frac{(1 + \mathcal{R}_{pp})}{(1 + \mathcal{R})}$$

$0.15 < p_T < 2\ GeV$
$|\eta| < 1$



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$0.15 < p_T < 2\ GeV$
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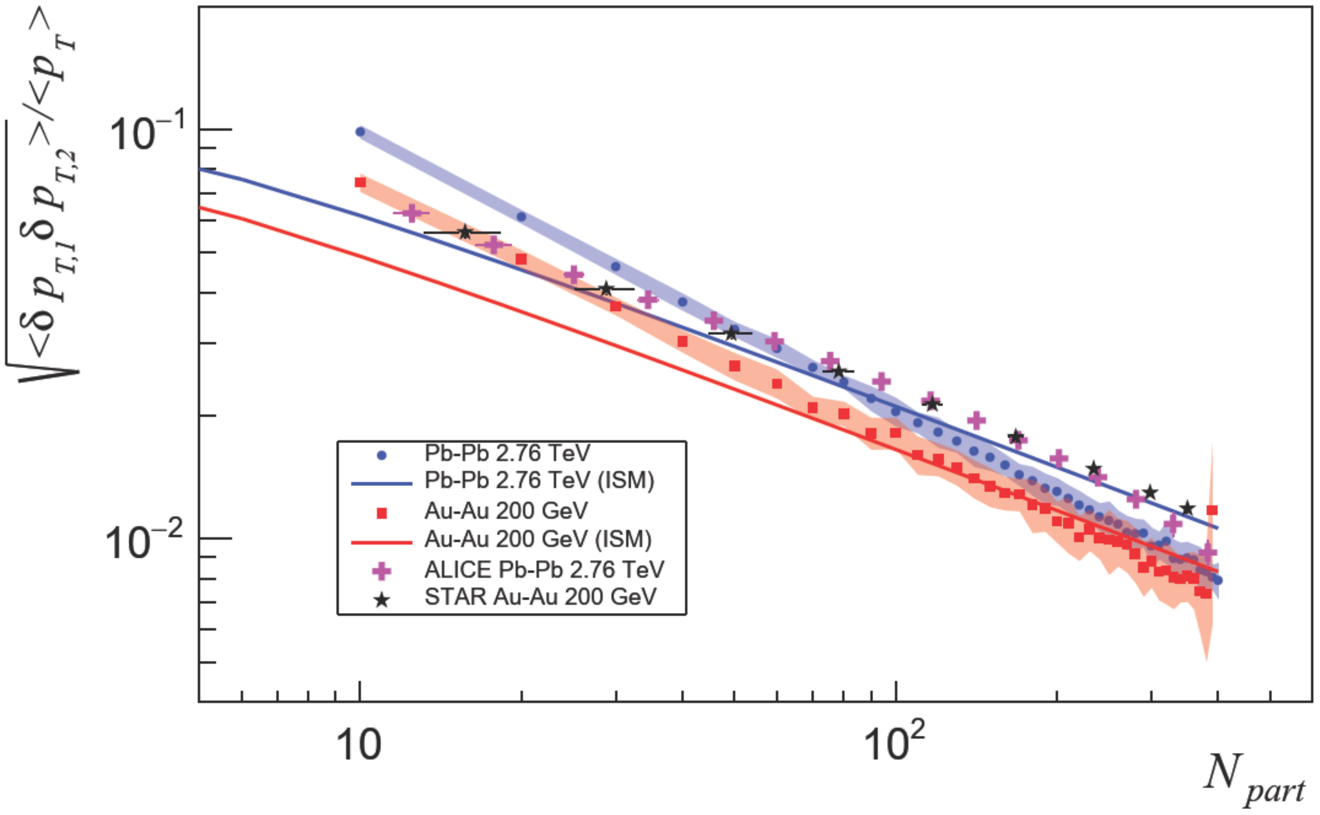


ALICE Eur.Phys.J.C 74 (2014) 10, 3077
STAR Phys.Rev.C 99 (2019) 4, 044918

Multiplicity – Momentum Correlations

$$\mathcal{D} = \frac{\langle \sum_i \sum_{i \neq j} \delta p_{Ti} \rangle}{\langle N \rangle^2}$$

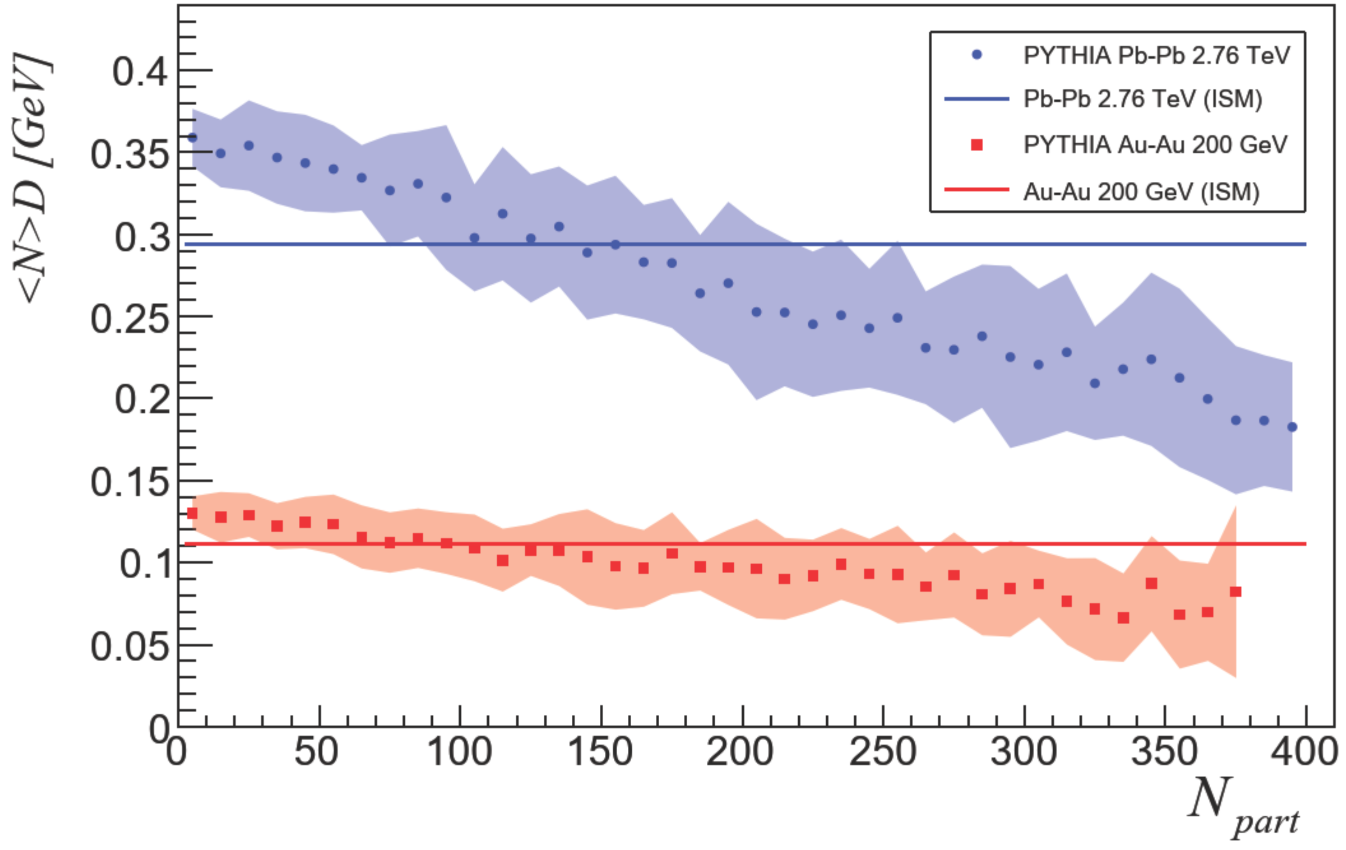
$$\mathcal{D} = \frac{Cov(P_T, N) - \langle p_T \rangle Var(N)}{\langle N \rangle^2}$$

$$\mathcal{D} = \mathcal{D}_0 S + \mathcal{D}_{eq}(1 - S)$$

- Removes multiplicity fluctuations
- $\mathcal{D} = 0$ in thermal equilibrium (Grand Canonical Ensemble)
- PYTHIA: positive, nonzero $\mathcal{D}$
- Consistent with increase in $\langle p_T \rangle$ with multiplicity

$0.15 < p_T < 2\ GeV$

$|\eta| < 1$



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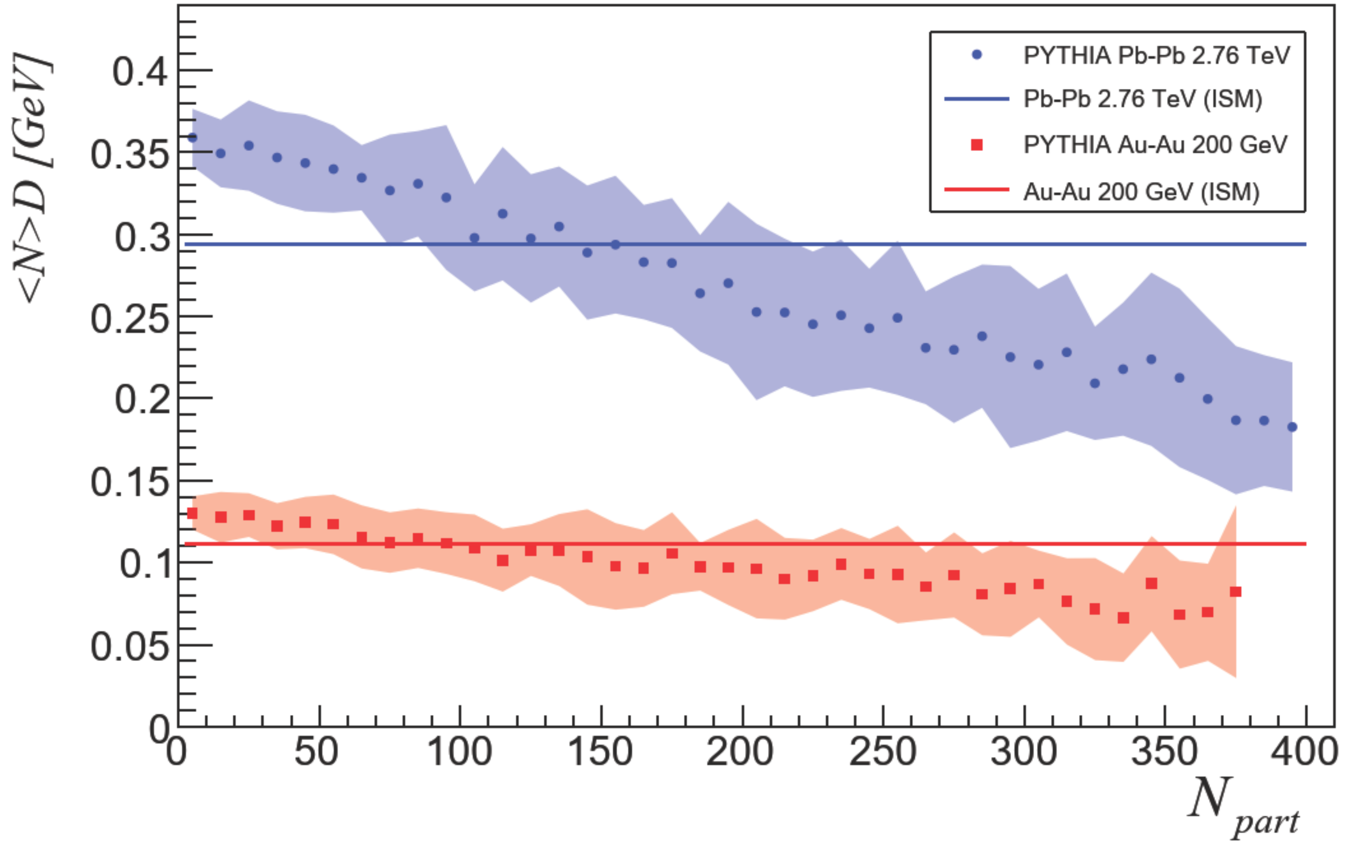
$$\mathcal{D} = \mathcal{D}_0 S + \mathcal{D}_{eq}(1 - S)$$

Independent Source Model

$$\mathcal{D} = \frac{2\mathcal{D}_{pp}}{\langle N_{part} \rangle}$$

$0.15 < p_T < 2\ GeV$
$|\eta| < 1$



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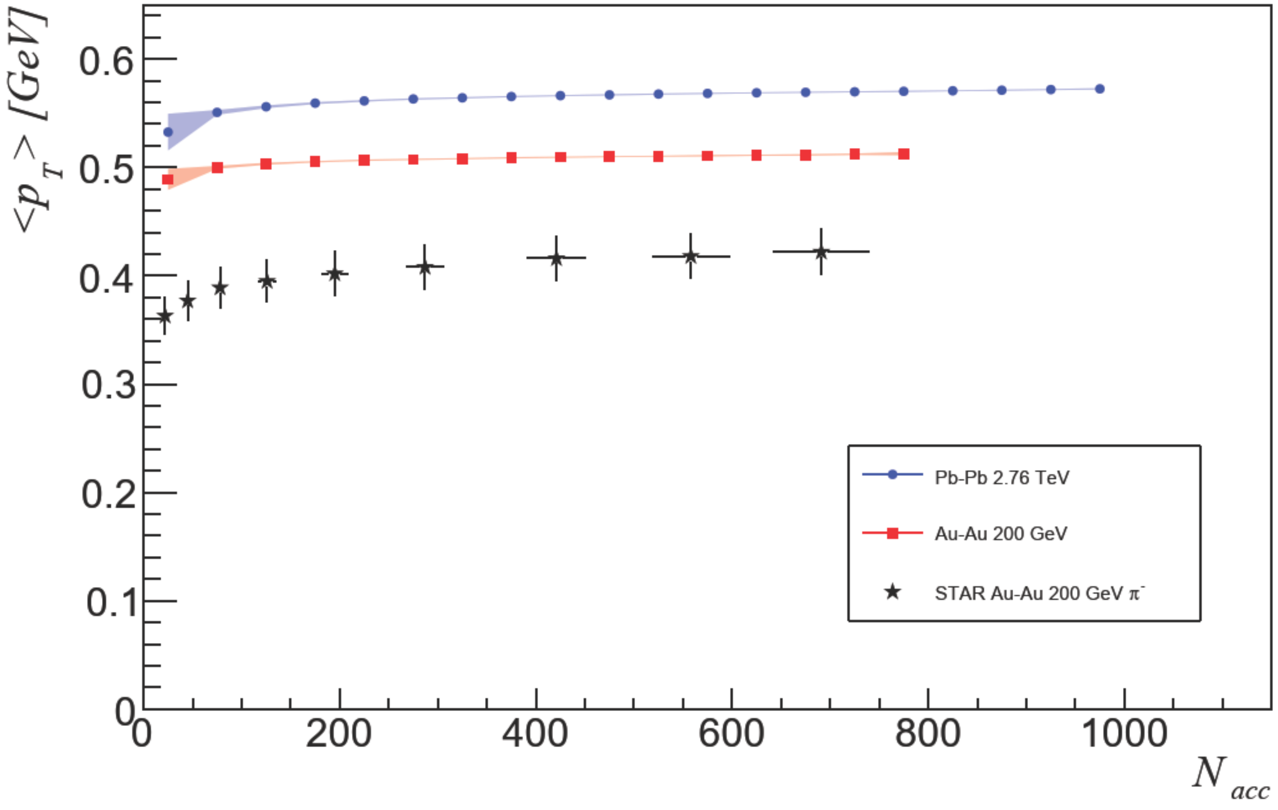
$$\mathcal{D} = \frac{Cov(P_T, N) - \langle p_T \rangle Var(N)}{\langle N \rangle^2}$$

$$\mathcal{D} = \mathcal{D}_0 S + \mathcal{D}_{eq}(1 - S)$$

Independent Source Model

$$\mathcal{D} = \frac{2\mathcal{D}_{pp}}{\langle N_{part} \rangle}$$

$0.15 < p_T < 2\ GeV$
$|\eta| < 0.5$



STAR Phys.Rev.C 79 (2009) 034909

Multiplicity Fluctuations

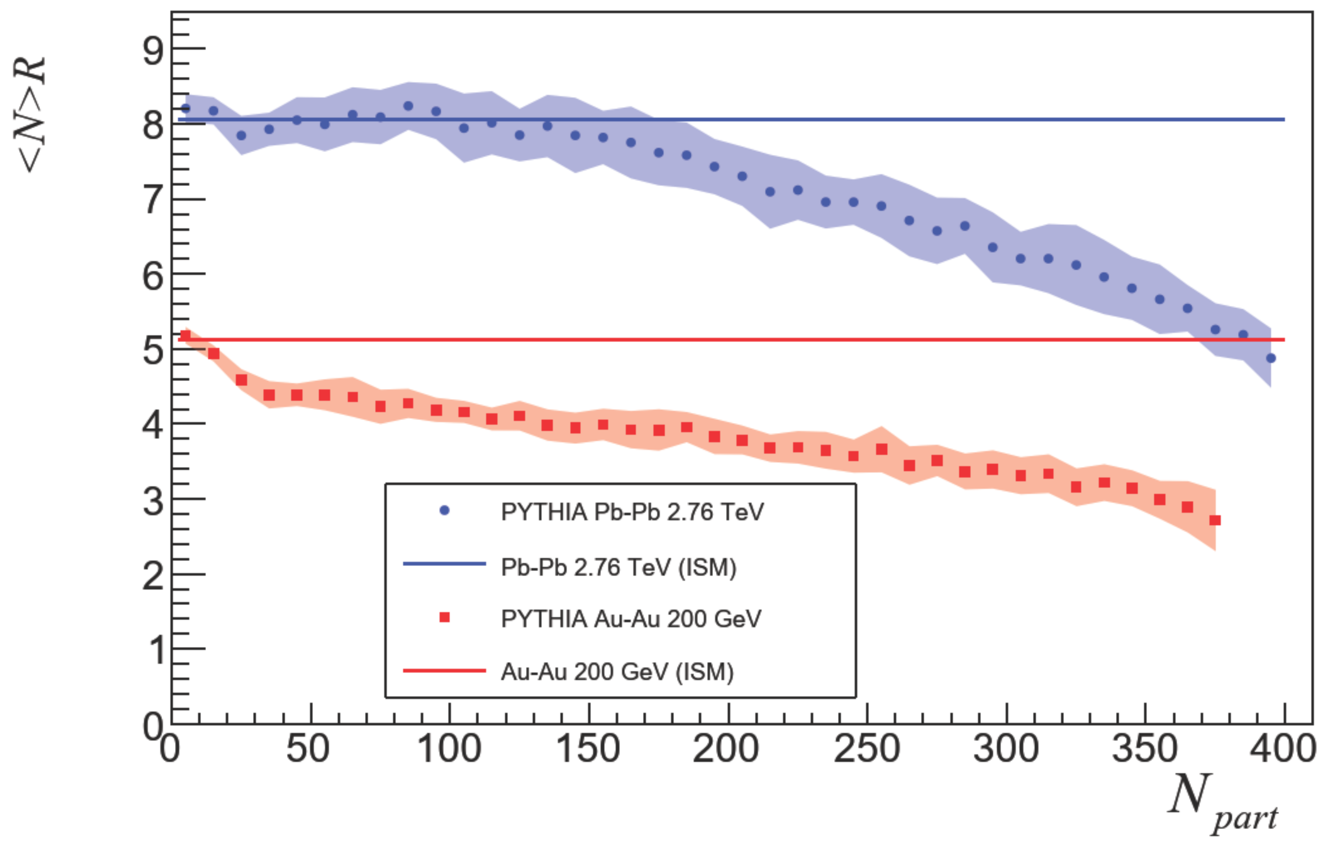
$$\mathcal{R} = \frac{\langle N(N-1)\rangle - \langle N\rangle^2}{\langle N\rangle^2} = \frac{Var(N) - \langle N\rangle}{\langle N\rangle^2}$$

No dependence on Survival probability S

- $\mathcal{R}$ measures correlations, sets an overall scale
- $\mathcal{R} = 0$ for independent particle production
- $\mathcal{R} \propto 1/\langle N\rangle$ for non-Poissonian distributions
- Indicates "volume fluctuations"

Zhou, Jia, Phys.Rev.C 98 (2018) 4, 044903 arXiv:1803.01812

$0.15 < p_T < 2\ GeV$
$|\eta| < 1$



Multiplicity Fluctuations

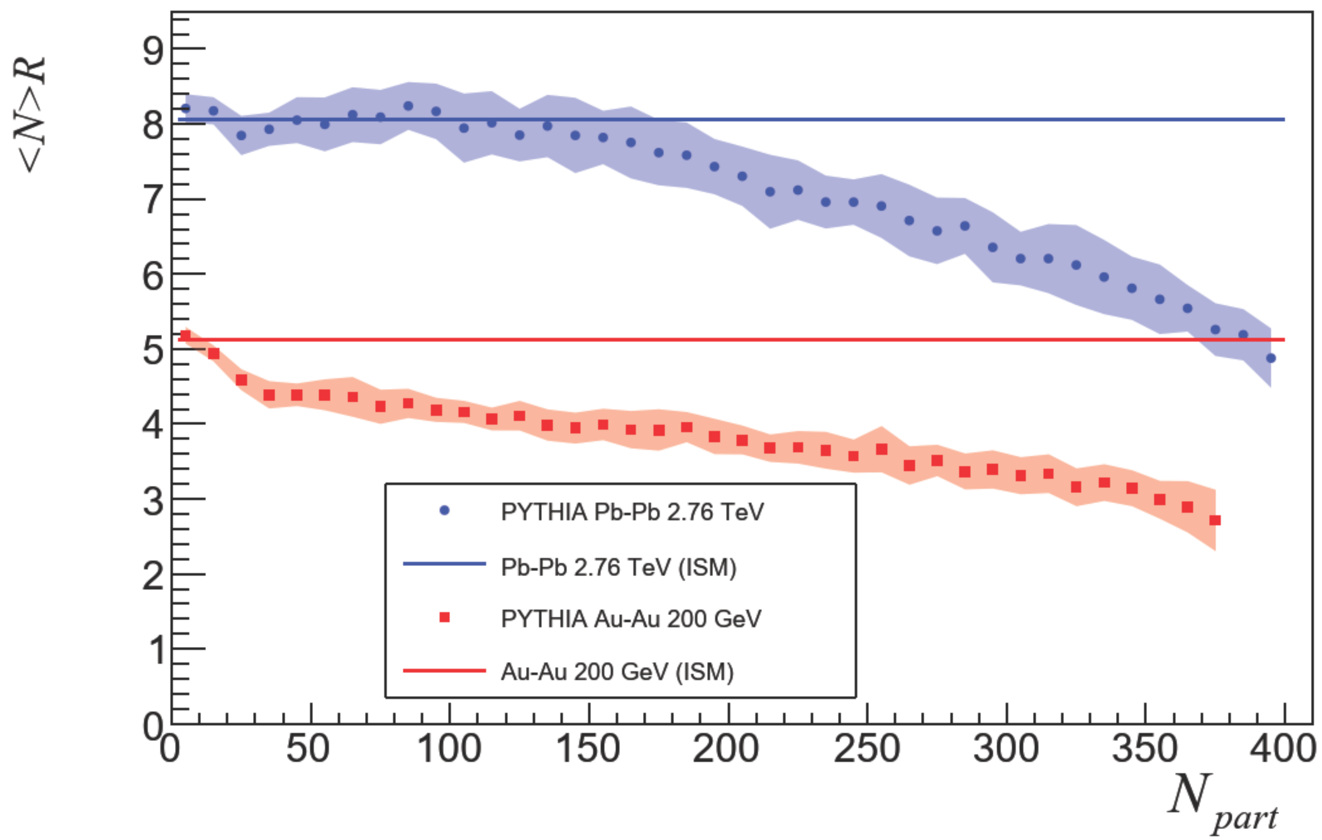
$$\mathcal{R} = \frac{\langle N(N-1)\rangle - \langle N\rangle^2}{\langle N\rangle^2} = \frac{Var(N) - \langle N\rangle}{\langle N\rangle^2}$$

No dependence on Survival probability S

Independent Source Model

$$\mathcal{R} = \frac{2\mathcal{R}_{pp}}{\langle N_{part}\rangle} + \frac{\langle N_{part}^2\rangle - \langle N_{part}\rangle^2}{\langle N_{part}\rangle^2}$$

$0.15 < p_T < 2\ GeV$
$|\eta| < 1$



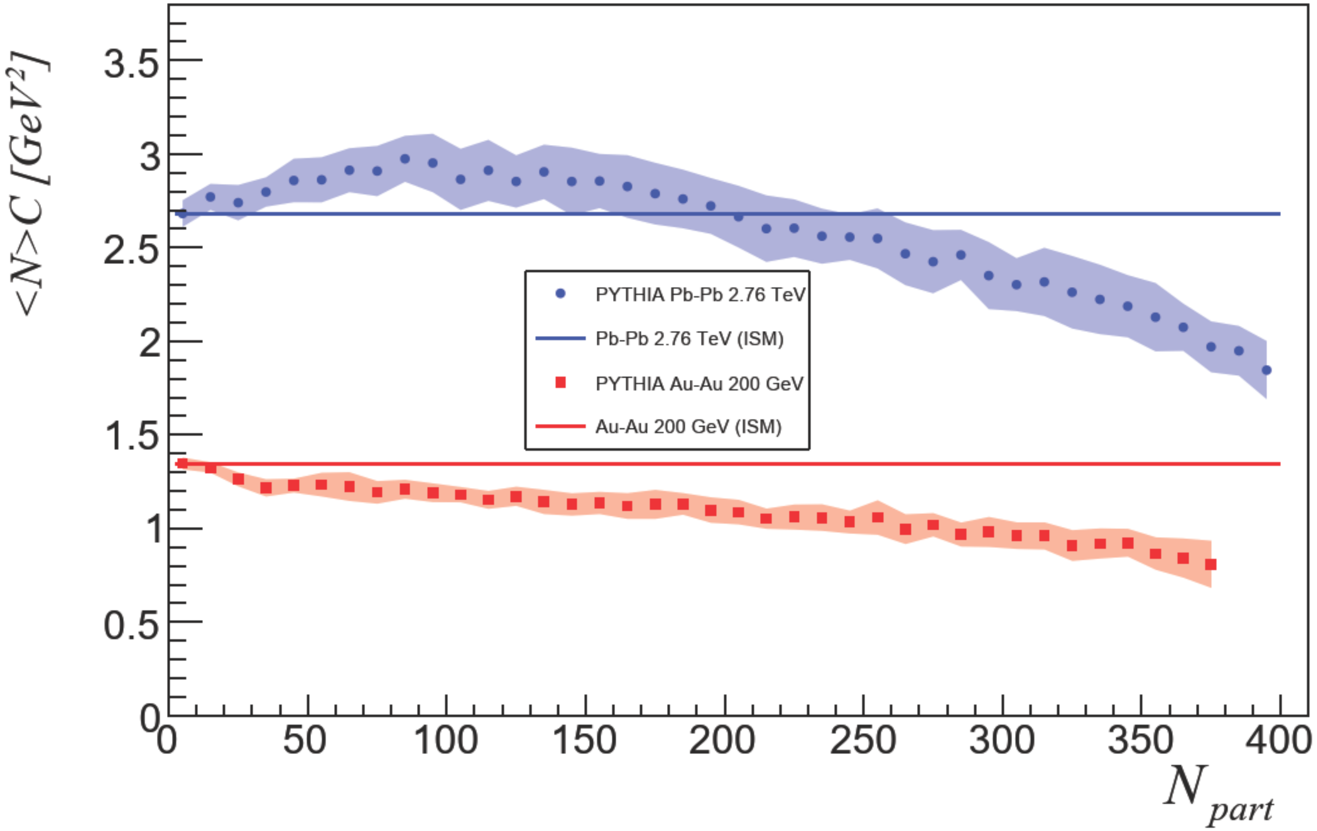
Transverse Momentum Correlations

$$\mathcal{C} = \frac{\left\langle \sum_i \sum_{i \neq j} p_{Ti} p_{Tj} \right\rangle - \langle P_T \rangle^2}{\langle N \rangle^2}$$

$$\mathcal{C} = \mathcal{C}_0 S^2 + \mathcal{C}_{eq}(1 - S^2) + 2\langle p_T \rangle (\mathcal{D}_0 - \mathcal{D}_{eq}) S (1 - S)$$

- $\langle P_T \rangle = \langle \sum_i p_{Ti} \rangle$
- $\mathcal{C}$ is the momentum weighted version of $\mathcal{R}$
- sensitive to both number density fluctuations and transverse momentum fluctuations
- Used to estimate shear viscosity and shear relaxation time

$0.15 < p_T < 2\ GeV$
$|\eta| < 1$



Phys Rev C94, 024921 (2016) arXiv:1606.02692
Nucl Phys A982, 311 (2019) arXiv:1807.06532

Transverse Momentum Correlations

$$\mathcal{C} = \frac{\langle \sum_i \sum_{i\neq j} p_{Ti} p_{Tj} \rangle - \langle P_T \rangle^2}{\langle N \rangle^2}$$

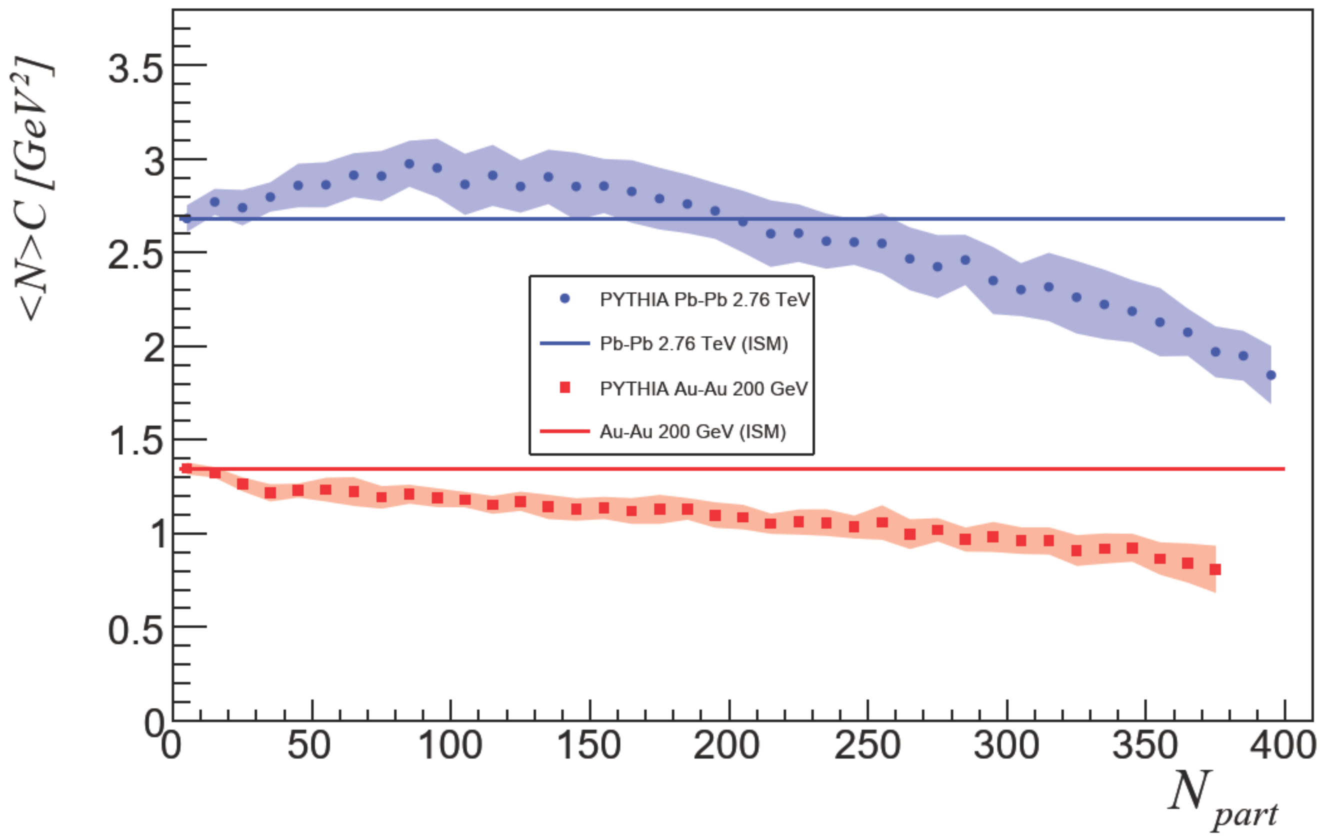
$$\mathcal{C} = \mathcal{C}_0 S^2 + \mathcal{C}_{eq}(1 - S^2) + 2\langle p_T \rangle (\mathcal{D}_0 - \mathcal{D}_{eq}) S (1 - S)$$

Independent Source Model

$$\mathcal{C} = \frac{2\mathcal{C}_{pp}}{\langle \mathrm{N}_{part} \rangle} + \left(\frac{\langle N_{part}^2 \rangle - \langle N_{part} \rangle^2}{\langle N_{part} \rangle^2} \right) \langle p_t \rangle^2$$

$0.15 < p_T < 2\ GeV$

$|\eta| < 1$



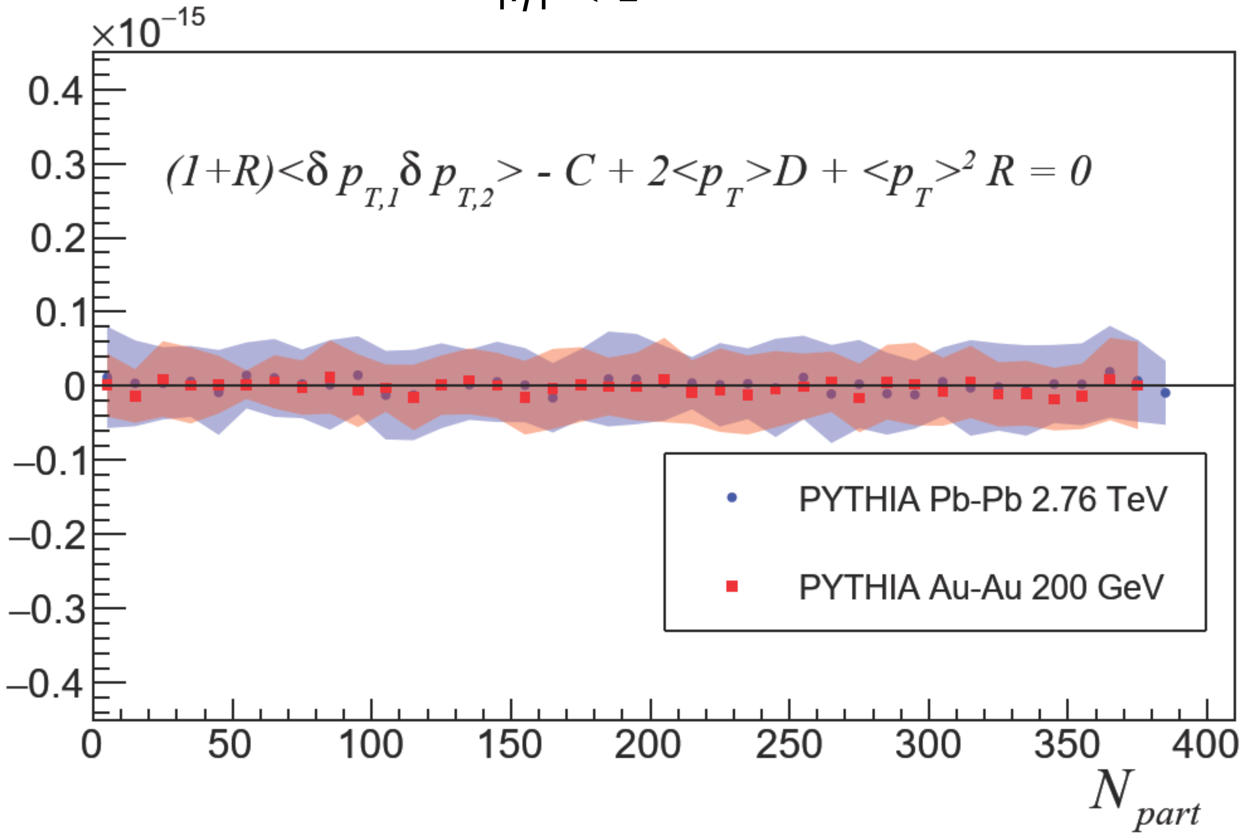
Complementary Correlations

$$(1+\mathcal{R})\langle \delta p_{T1}\delta p_{T2}\rangle - \mathcal{C} + 2\langle p_T\rangle\mathcal{D} + \langle p_T\rangle^2\mathcal{R} = 0$$

- Validates consistent calculation of observables using PYTHIA
- Theories or models that explain one observable should be able to explain all.
- Can interpret one observable in terms of the physics contributions of the others
- Example:

$$\langle \delta p_{T1}\delta p_{t2}\rangle = \frac{\mathcal{C}-2\langle p_T\rangle\mathcal{D}-\langle p_T\rangle^2\mathcal{R}}{(1+\mathcal{R})}$$

$0.15 < p_T < 2\ GeV$
$|\eta| < 1$



PYTHIA Comput.Phys.Commun. 191 (2015) 159-177, arXiv:1410.3012
Angantyr 10 (2018) 134, arXiv:1806.10820JHEP

Summary

New observable, multiplicity-momentum correlations, $\mathcal{D}$

- $\mathcal{D} = 0$ expected in equilibrium
- Positive in PYTHIA

Complementary observables $(1 + \mathcal{R})\langle \delta p_{T1} \delta p_{T2} \rangle - \mathcal{C} + 2\langle p_T \rangle \mathcal{D} + \langle p_T \rangle^2 \mathcal{R} = 0$

- $\mathcal{R}$ Multiplicity Fluctuations
- $\mathcal{C}$ Transverse Momentum Correlations
- $\langle \delta p_{T1} \delta p_{T2} \rangle$ Correlations of Transverse Momentum Fluctuations
- $\mathcal{D}$ Multiplicity-Momentum Correlations
- All derived from the same parent correlation function

- Use for validation of measurement or calculation of observables
- Challenge theories and models to address all observables simultaneously
- Interpret one observable in terms of physics contributions of the others

Boltzman-Langevin evolution of correlations is sensitive to incomplete thermalization

- $\mathcal{R}$ has no dependence on survival probability S
- $\mathcal{D}$ depends on S
- $\langle \delta p_{T1} \delta p_{T2} \rangle$ depends on S^2
- $\mathcal{C}$ depends on S and S^2
- Simultaneous comparison to multiple observables with different powers of S constrains extraction of S

Independent Source Model (pp source)

$$\mathcal{R} = \frac{2\mathcal{R}_{pp}}{\langle N_{part}\rangle} + \frac{\langle N_{part}^2\rangle - \langle N_{part}\rangle^2}{\langle N_{part}\rangle^2}$$

$$\mathcal{D} = \frac{2\mathcal{D}_{pp}}{\langle N_{part}\rangle}$$

$$\mathcal{C} = \frac{2\mathcal{C}_{pp}}{\langle \mathrm{N}_{part}\rangle} + \left(\frac{\langle N_{part}^2\rangle - \langle N_{part}\rangle^2}{\langle N_{part}\rangle^2}\right)\langle p_t\rangle^2$$

$$\langle \delta p_{t1}\delta p_{t2}\rangle = \frac{2\langle \delta p_{t1}\delta p_{t2}\rangle_{pp}}{\langle N_{part}\rangle}\frac{(1+\mathcal{R}_{pp})}{(1+\mathcal{R})}$$

Correlations and Fluctuations

Momentum Density of Particles $\rho_1(\mathbf{p}_1)$

Pair Momentum Density of Particles $\rho_2(\mathbf{p}_1, \mathbf{p}_2) = \rho_1(\mathbf{p}_1)\rho_1(\mathbf{p}_2) + r(\mathbf{p}_1, \mathbf{p}_2)$

Correlated Pair Distribution $r(\mathbf{p}_1, \mathbf{p}_2) = \rho_2(\mathbf{p}_1, \mathbf{p}_2) - \rho_1(\mathbf{p}_1)\rho_1(\mathbf{p}_2)$

In the case of no correlations $\rho_2(\mathbf{p}_1, \mathbf{p}_2) = \rho_1(\mathbf{p}_1)\rho_1(\mathbf{p}_2)$

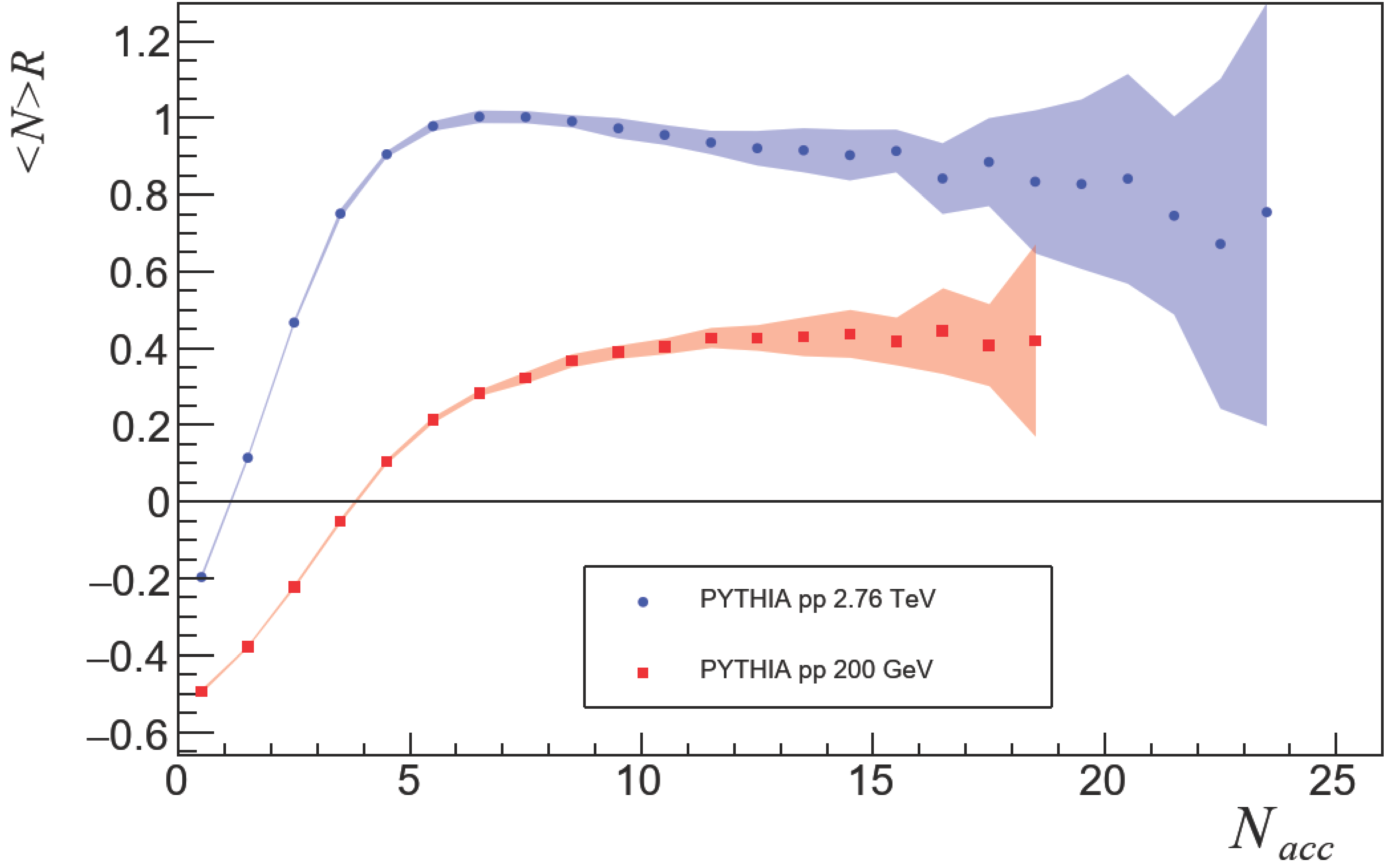
$$r(\mathbf{p}_1, \mathbf{p}_2) = 0$$

$$\mathcal{R} = \frac{1}{\langle N \rangle^2} \int \int r(\mathbf{p}_1, \mathbf{p}_2) d^3\mathbf{p}_1 \mathrm{d}^3\mathbf{p}_2$$

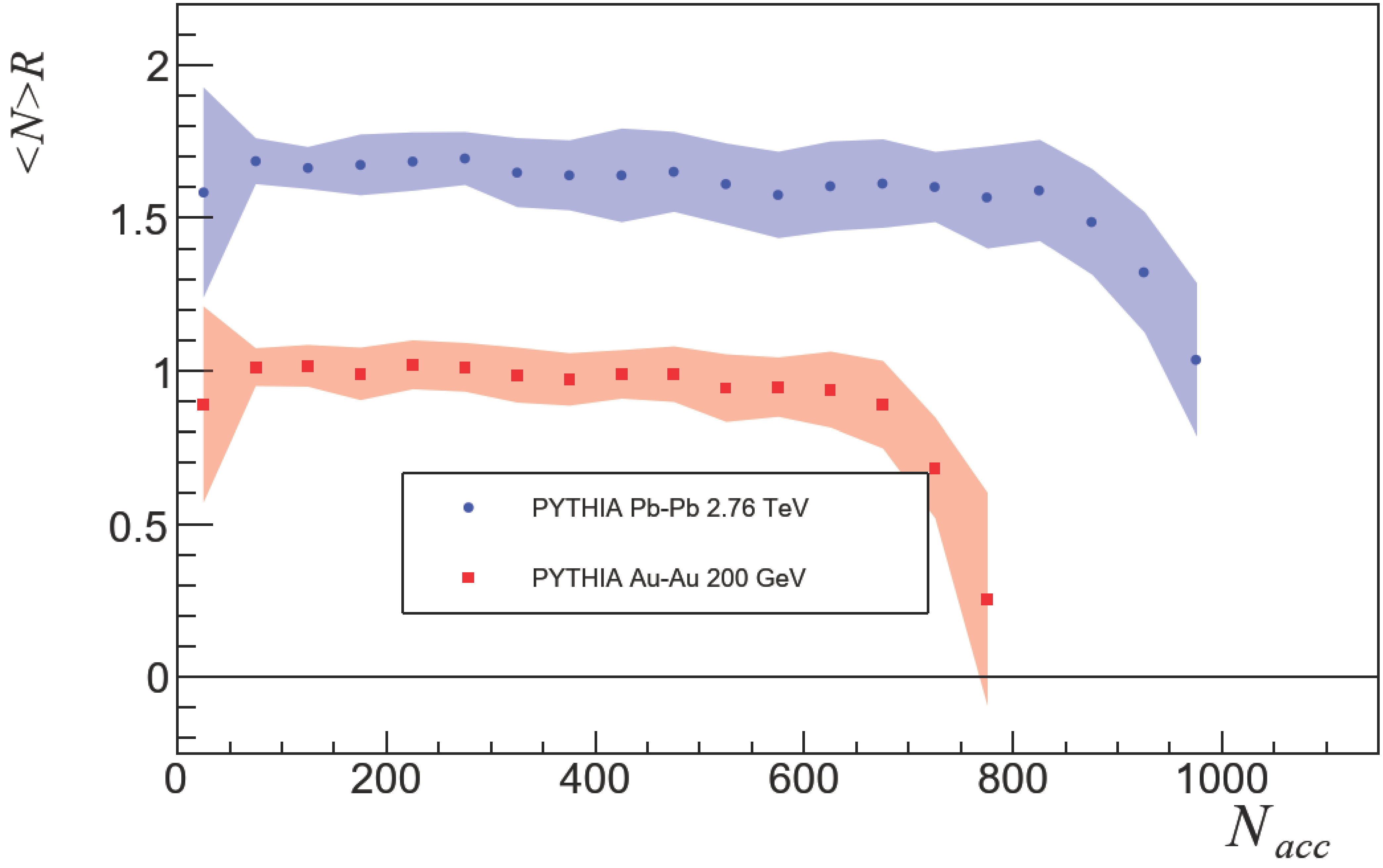
$$\mathcal{C} = \frac{1}{\langle N \rangle^2} \int \int r(\mathbf{p}_1, \mathbf{p}_2) p_{T1} p_{T2} d^3\mathbf{p}_1 \mathrm{d}^3\mathbf{p}_2$$

$$\langle \delta p_{T1} \delta p_{T2} \rangle = \frac{1}{\langle N(N-1) \rangle} \int \int r(\mathbf{p}_1, \mathbf{p}_2)\ \delta p_{T1} \delta p_{T2} d^3\mathbf{p}_1 \mathrm{d}^3\mathbf{p}_2$$

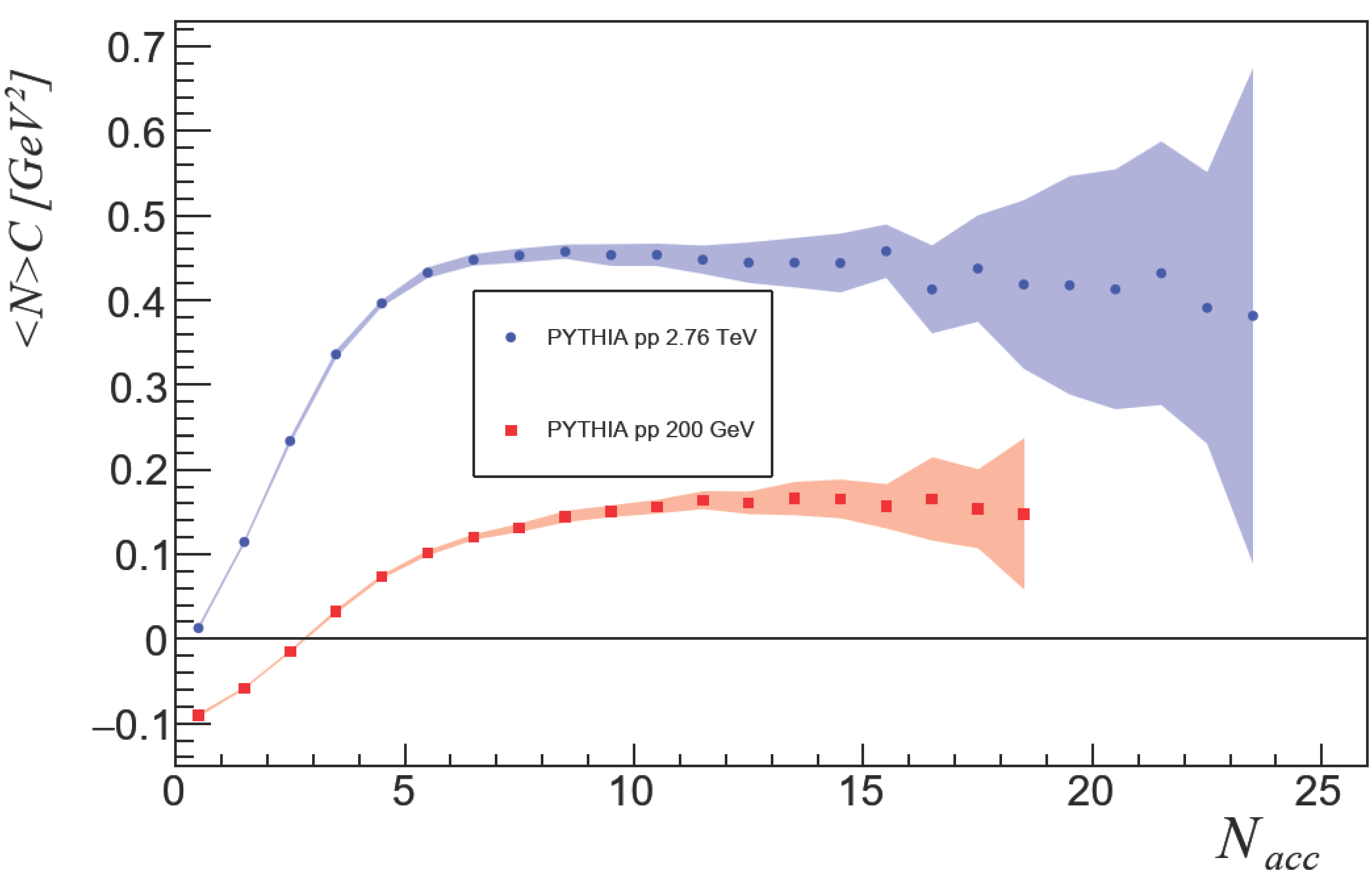
$$\mathcal{D} = \frac{1}{\langle N \rangle^2} \int \int r(\mathbf{p}_1, \mathbf{p}_2) \delta p_{T1} d^3\mathbf{p}_1 \mathrm{d}^3\mathbf{p}_2$$



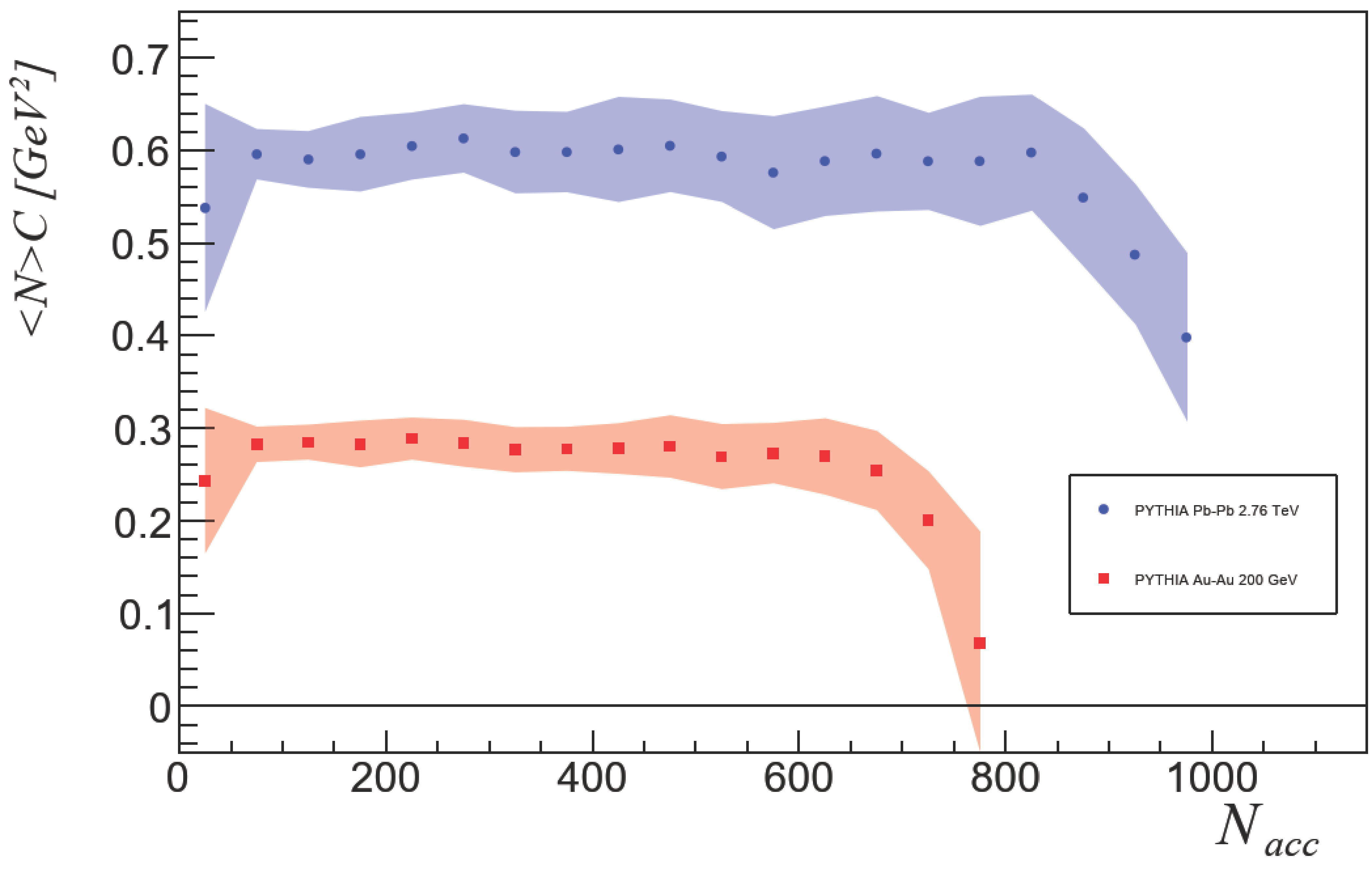
$<N>R$
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
20
25
N_{acc}
PYTHIA pp 2.76 TeV
PYTHIA pp 200 GeV



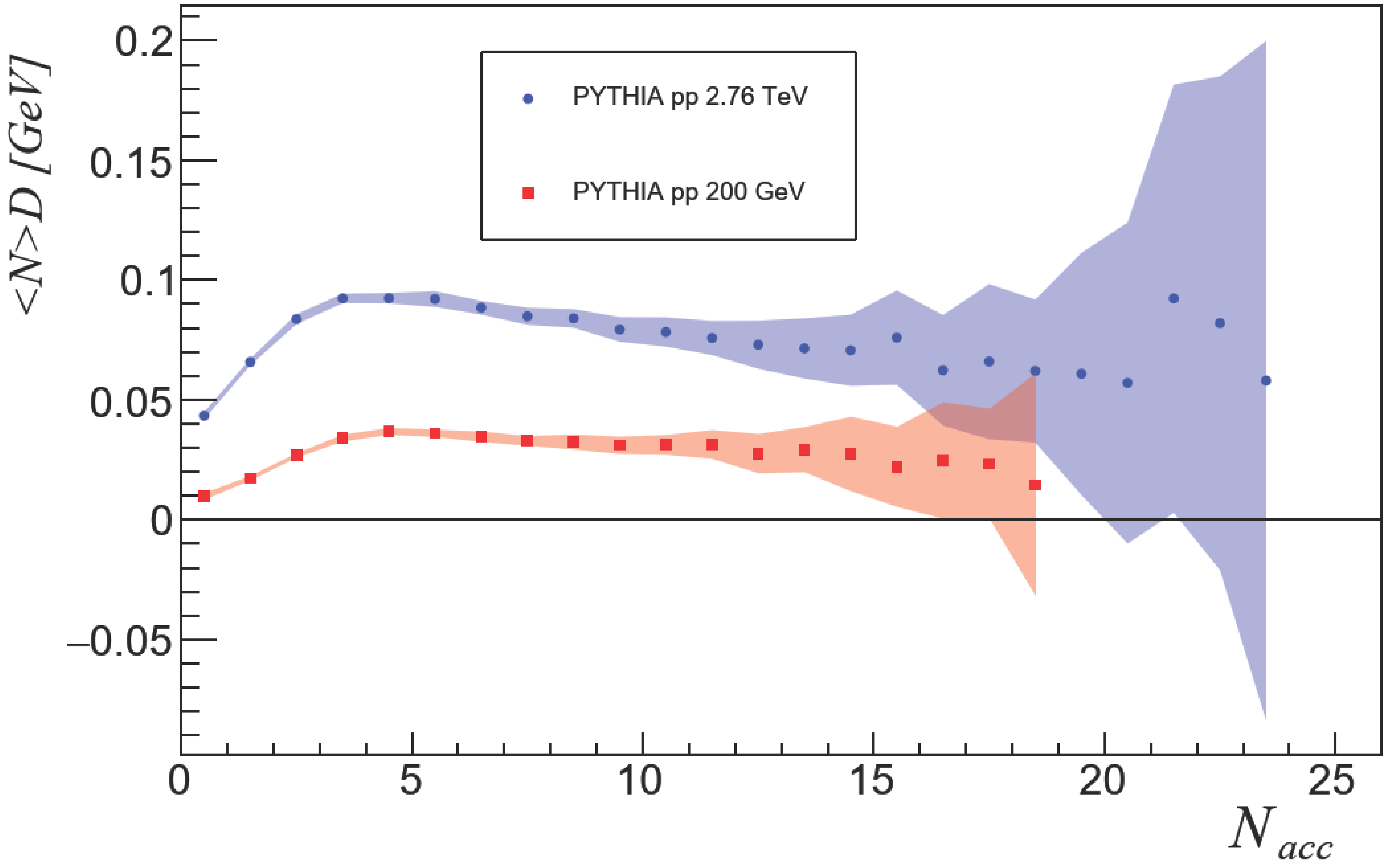
<N>R
2
1.5
1
0.5
0
0
200
400
600
800
1000
N_acc
PYTHIA Pb-Pb 2.76 TeV
PYTHIA Au-Au 200 GeV



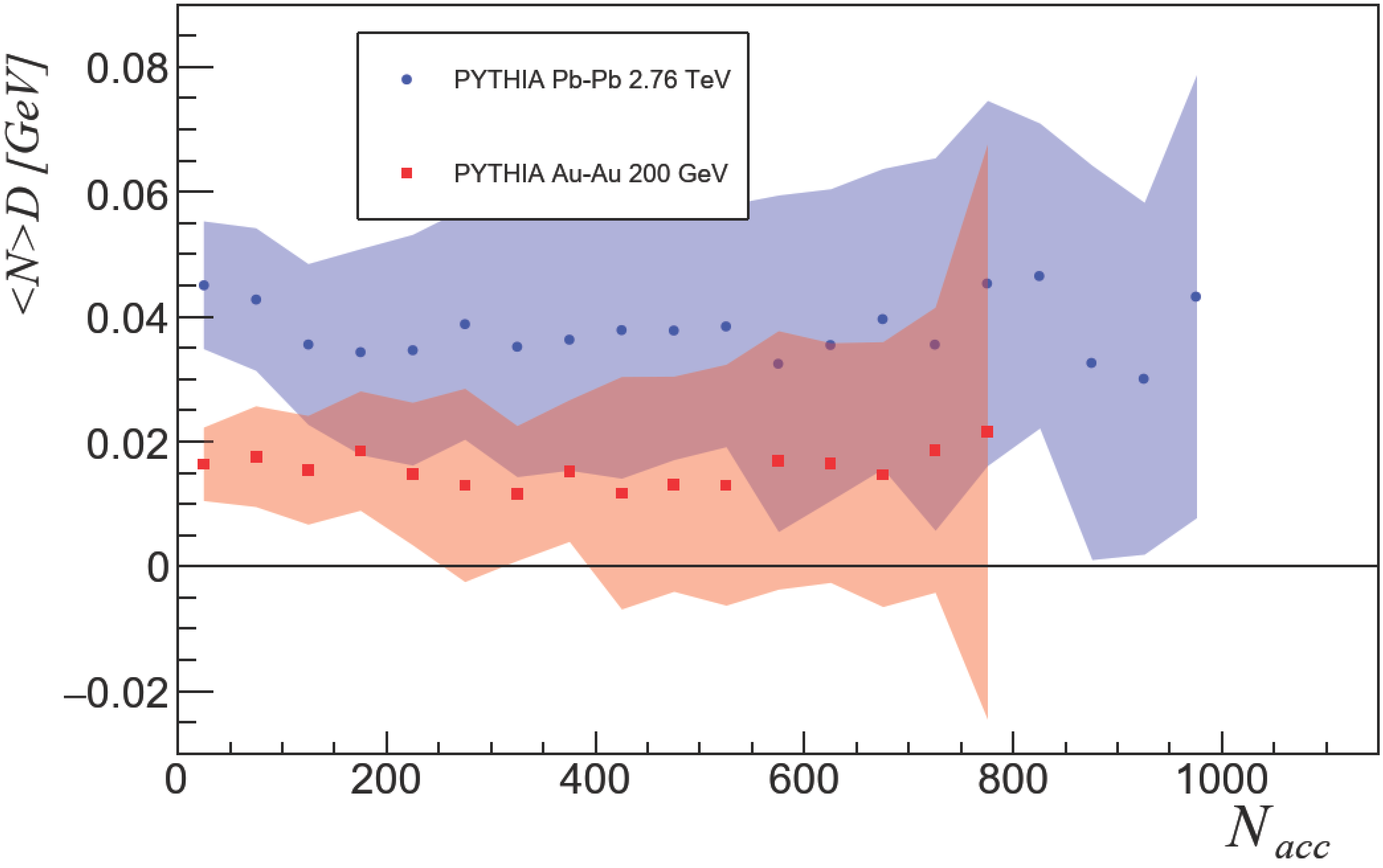
PYTHIA pp 2.76 TeV
PYTHIA pp 200 GeV
<N>C [GeV²]
N_acc



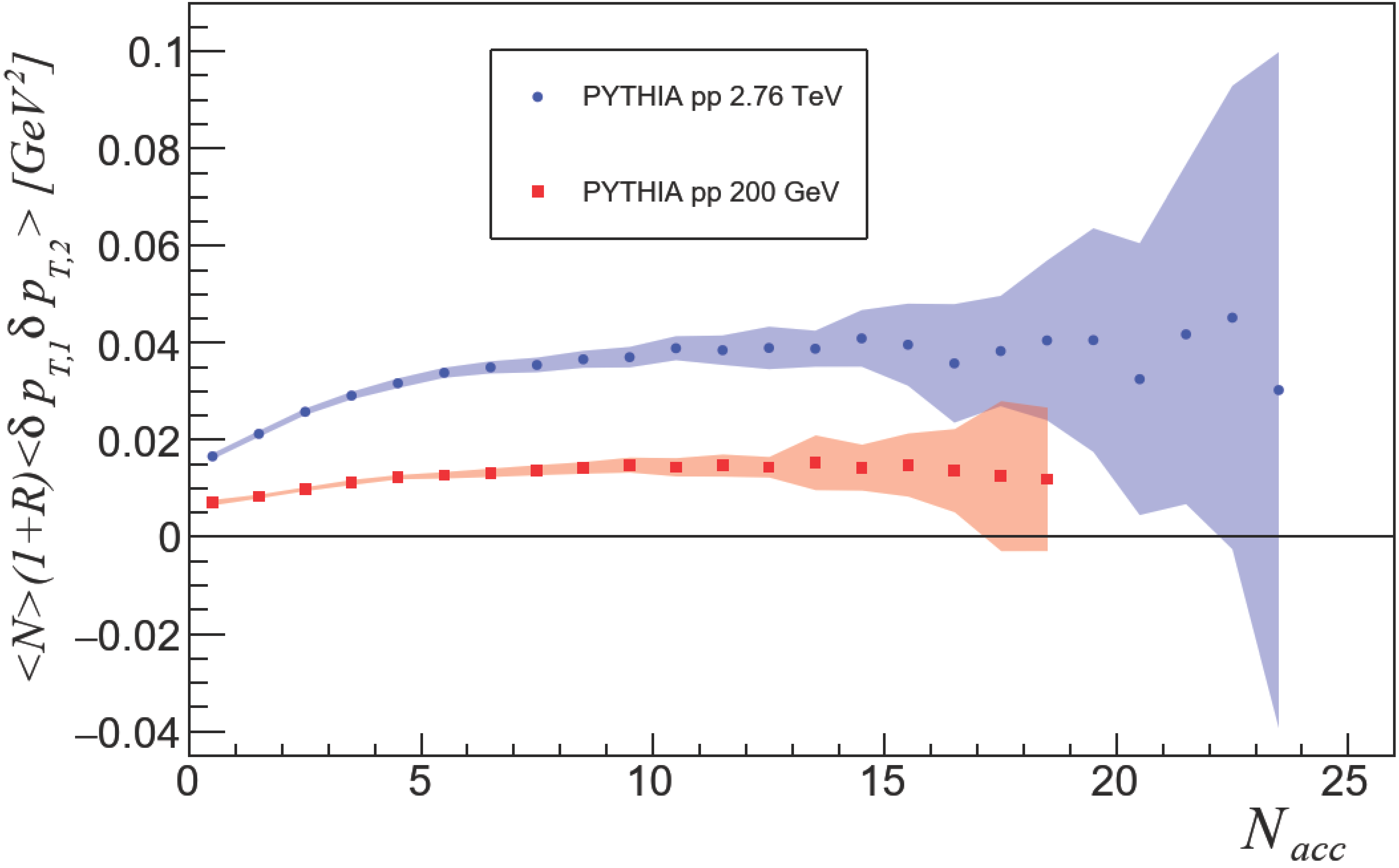
<N>C [GeV^2]
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
200
400
600
800
1000
N_acc
PYTHIA Pb-Pb 2.76 TeV
PYTHIA Au-Au 200 GeV

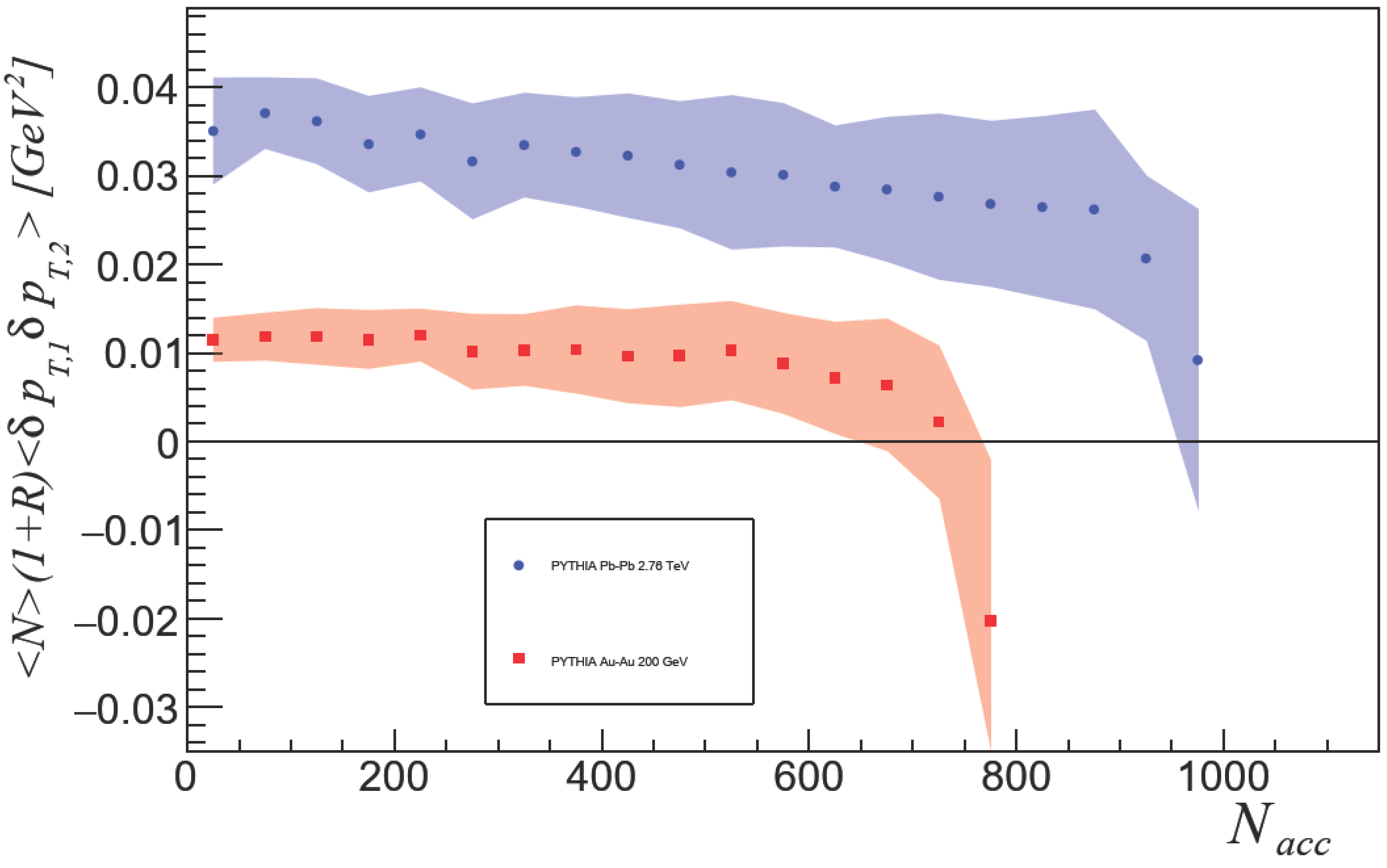


PYTHIA pp 2.76 TeV
PYTHIA pp 200 GeV
<N>D [GeV]
0.2
0.15
0.1
0.05
0
−0.05
0
5
10
15
20
25
N_{acc}

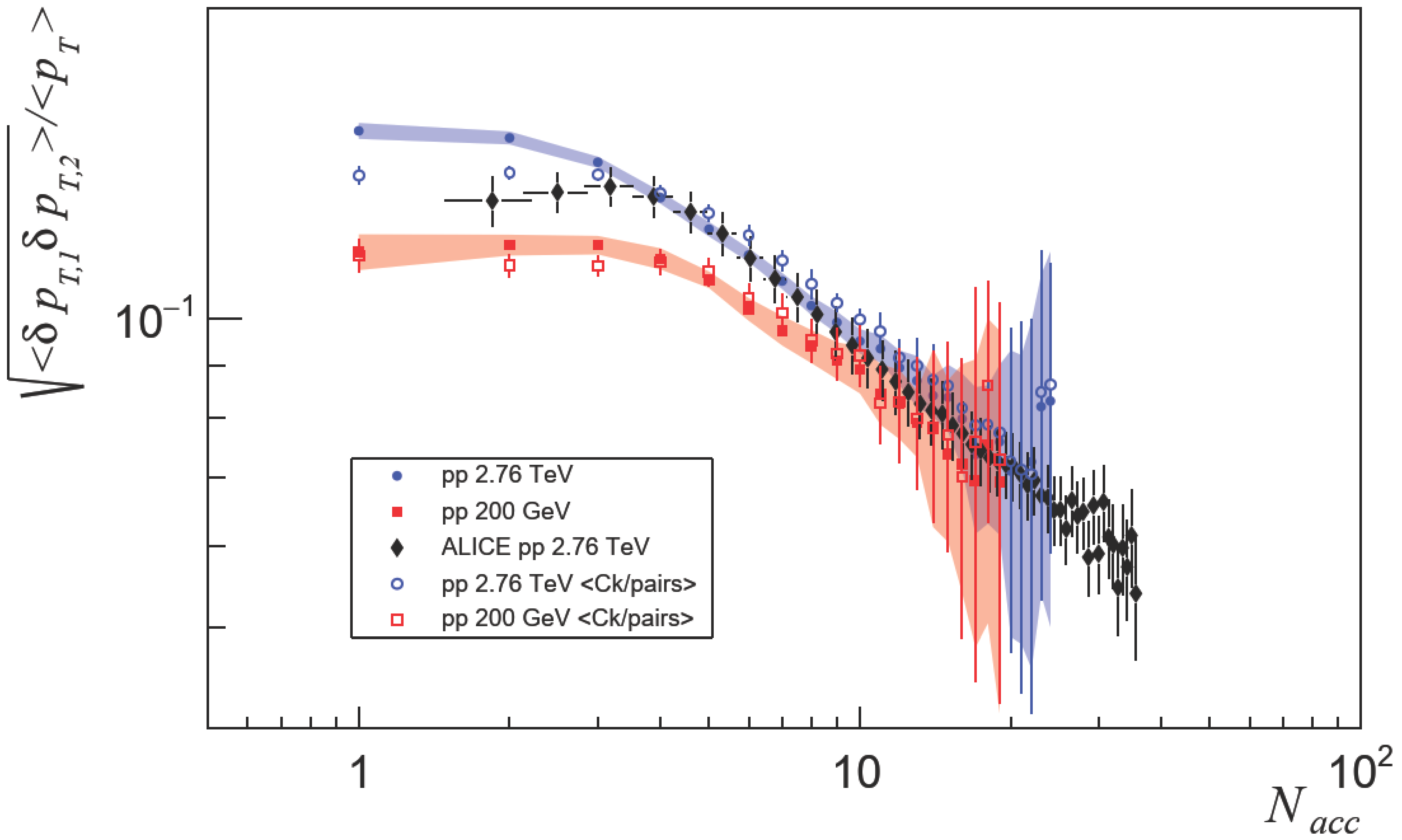


PYTHIA Pb-Pb 2.76 TeV
PYTHIA Au-Au 200 GeV
<N>D [GeV]
0.08
0.06
0.04
0.02
0
–0.02
0
200
400
600
800
1000
N acc

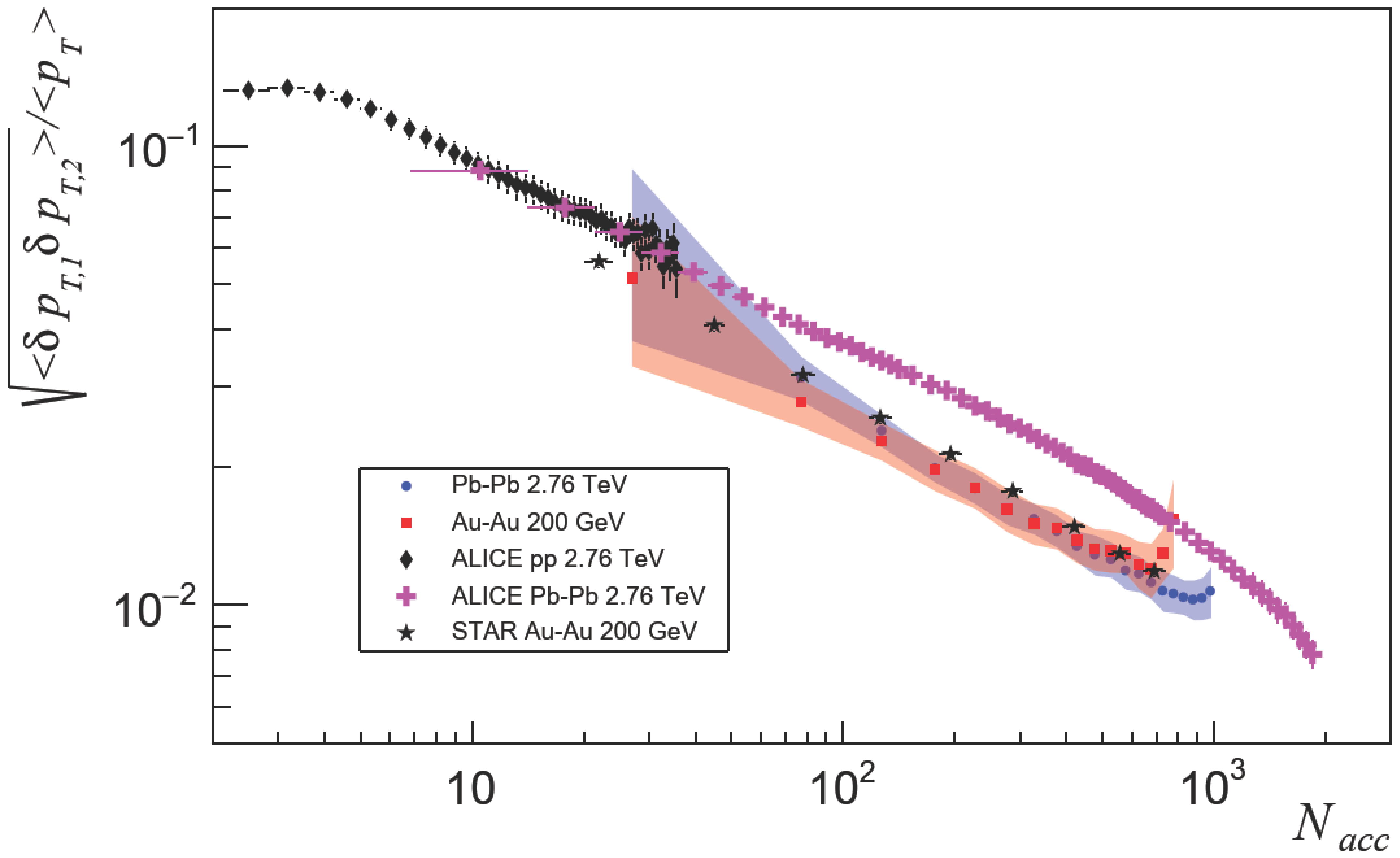

PYTHIA pp 2.76 TeV
PYTHIA pp 200 GeV
<N>(1+R)<δ p_T,1 δ p_T,2> [GeV²]
N_acc
0.1
0.08
0.06
0.04
0.02
0
−0.02
−0.04
0
5
10
15
20
25



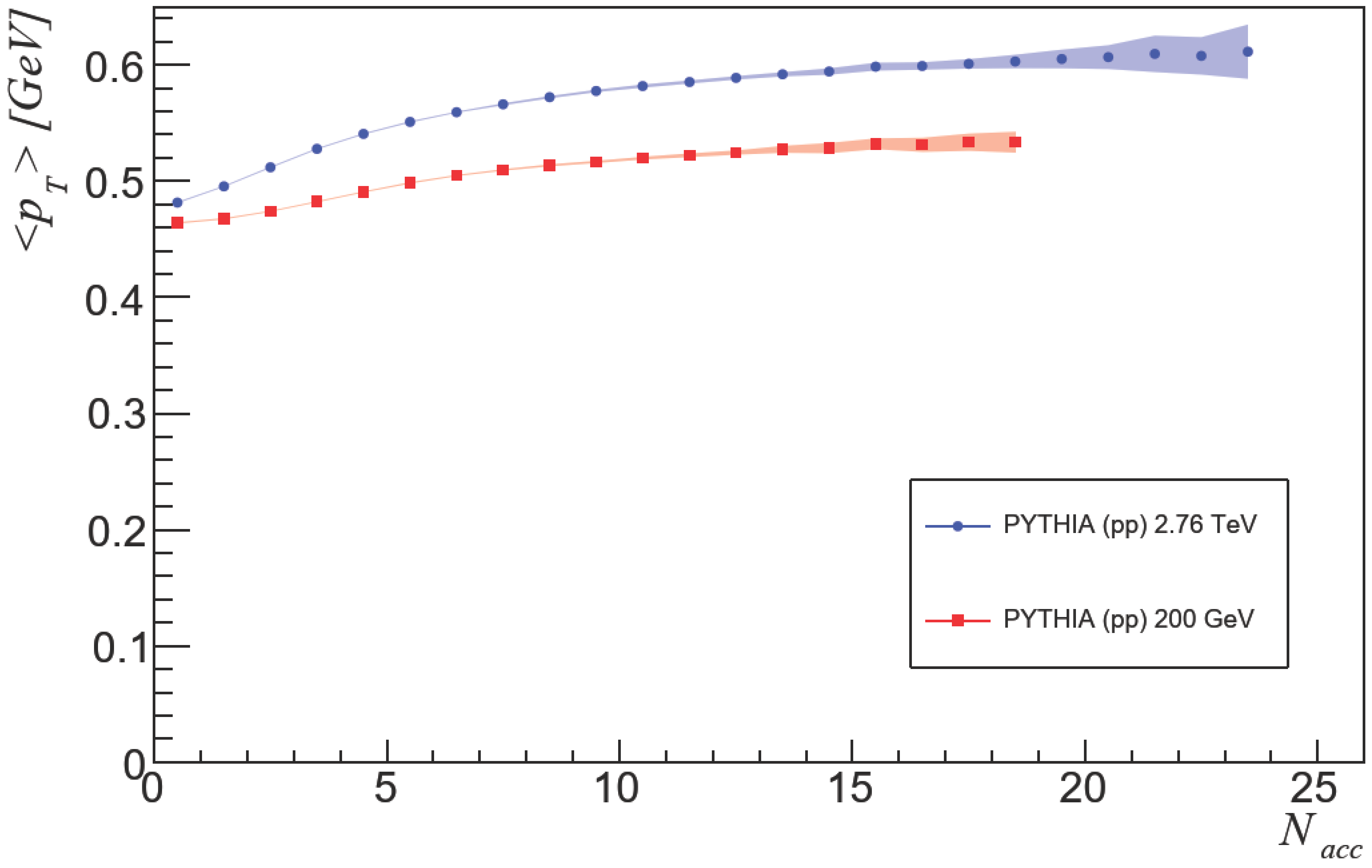
$<N>(1+R)<\delta p_{T,1}\delta p_{T,2}>\ [GeV^2]$
0.04
0.03
0.02
0.01
0
−0.01
−0.02
−0.03
0
200
400
600
800
1000
N_{acc}
PYTHIA Pb-Pb 2.76 TeV
PYTHIA Au-Au 200 GeV



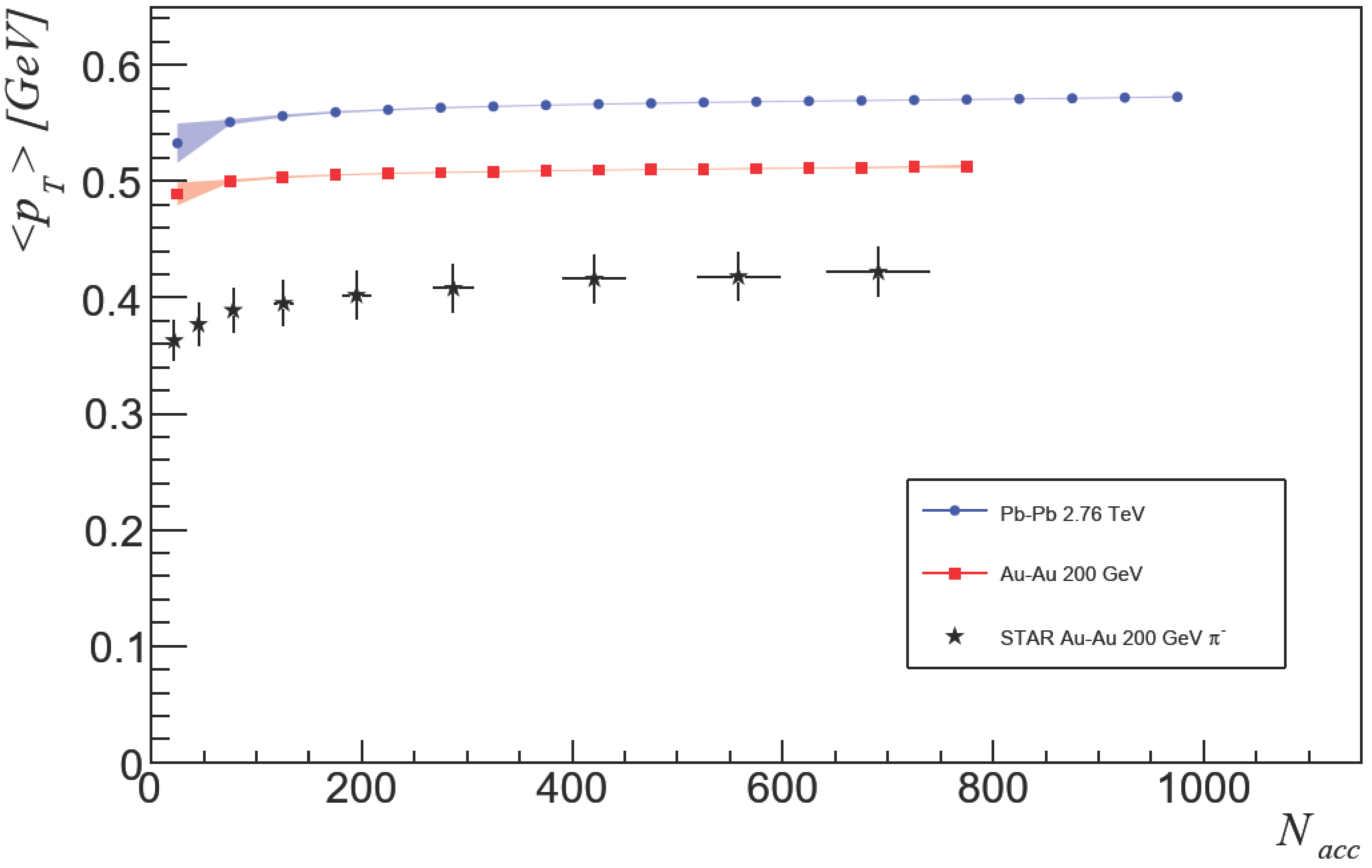
$\sqrt{\langle \delta p_{T,1} \delta p_{T,2} \rangle} / \langle p_T \rangle$
10^{-1}
1
10
10^2
N_{acc}
pp 2.76 TeV
pp 200 GeV
ALICE pp 2.76 TeV
pp 2.76 TeV <Ck/pairs>
pp 200 GeV <Ck/pairs>



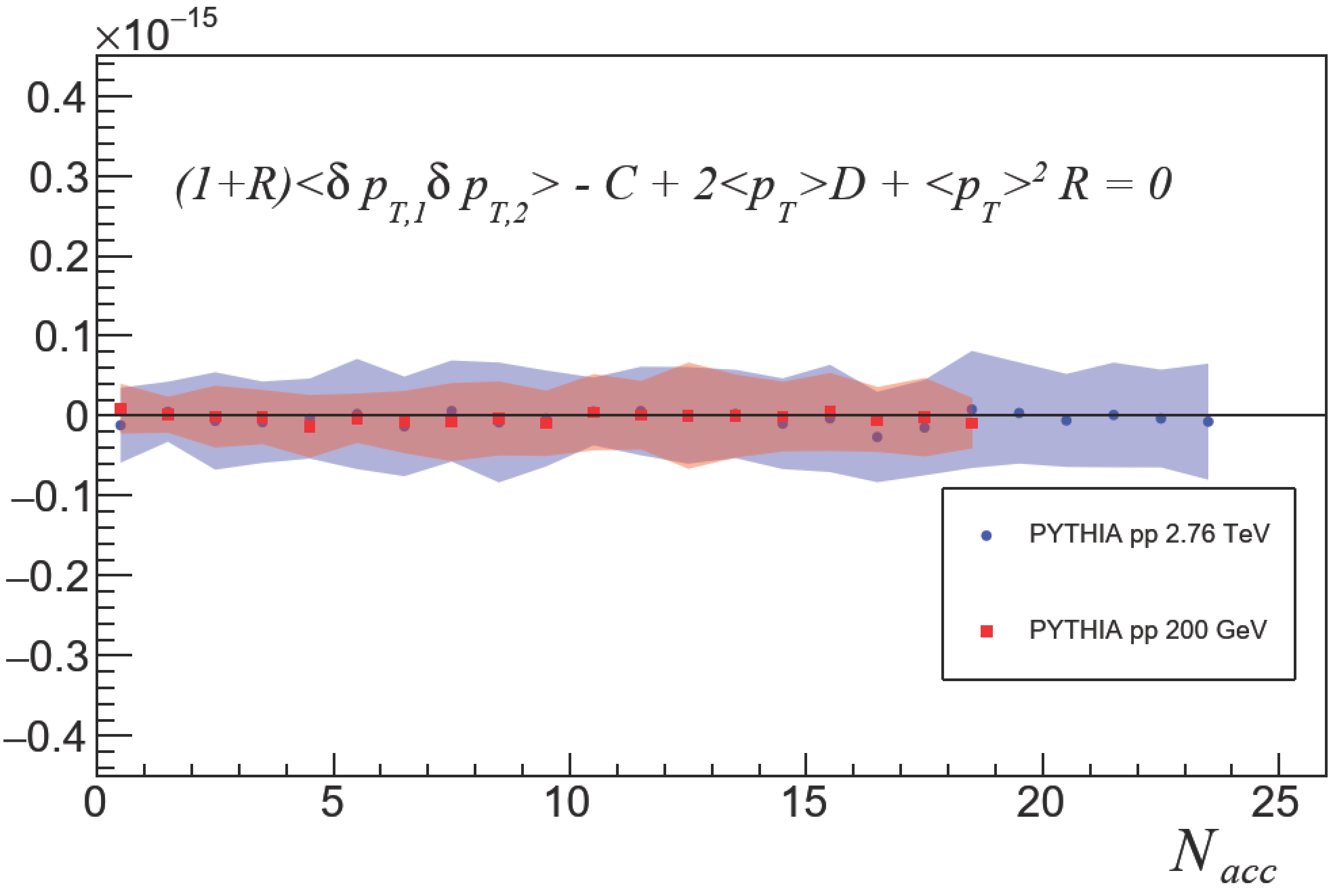
$\sqrt{\langle\delta p_{T,1}\delta p_{T,2}\rangle}/\langle p_T\rangle$
10^{-1}
10^{-2}
10
10^2
10^3
N_{acc}
Pb-Pb 2.76 TeV
Au-Au 200 GeV
ALICE pp 2.76 TeV
ALICE Pb-Pb 2.76 TeV
STAR Au-Au 200 GeV



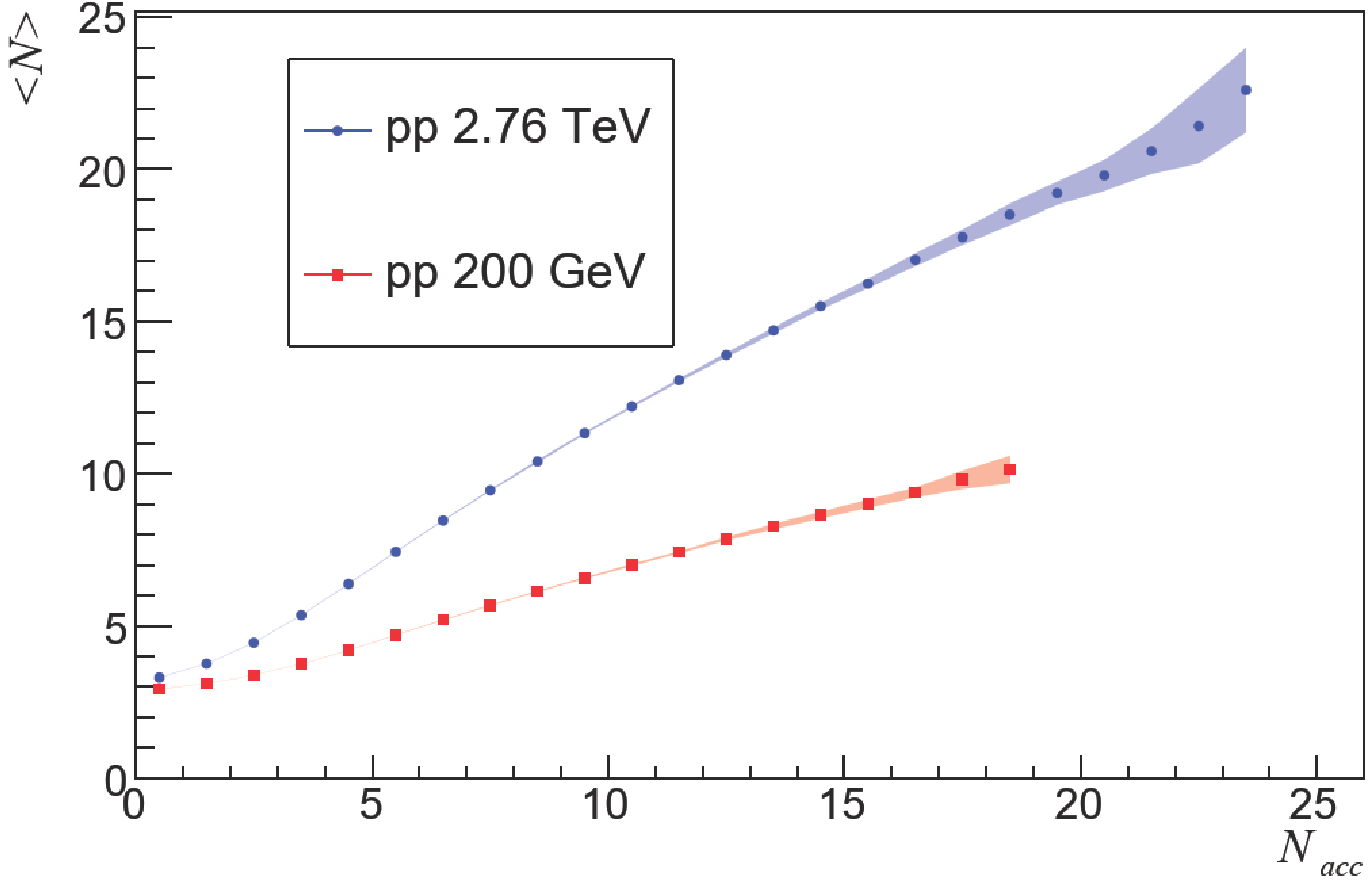
$<p_T>$ [GeV]
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
N_{acc}
PYTHIA (pp) 2.76 TeV
PYTHIA (pp) 200 GeV



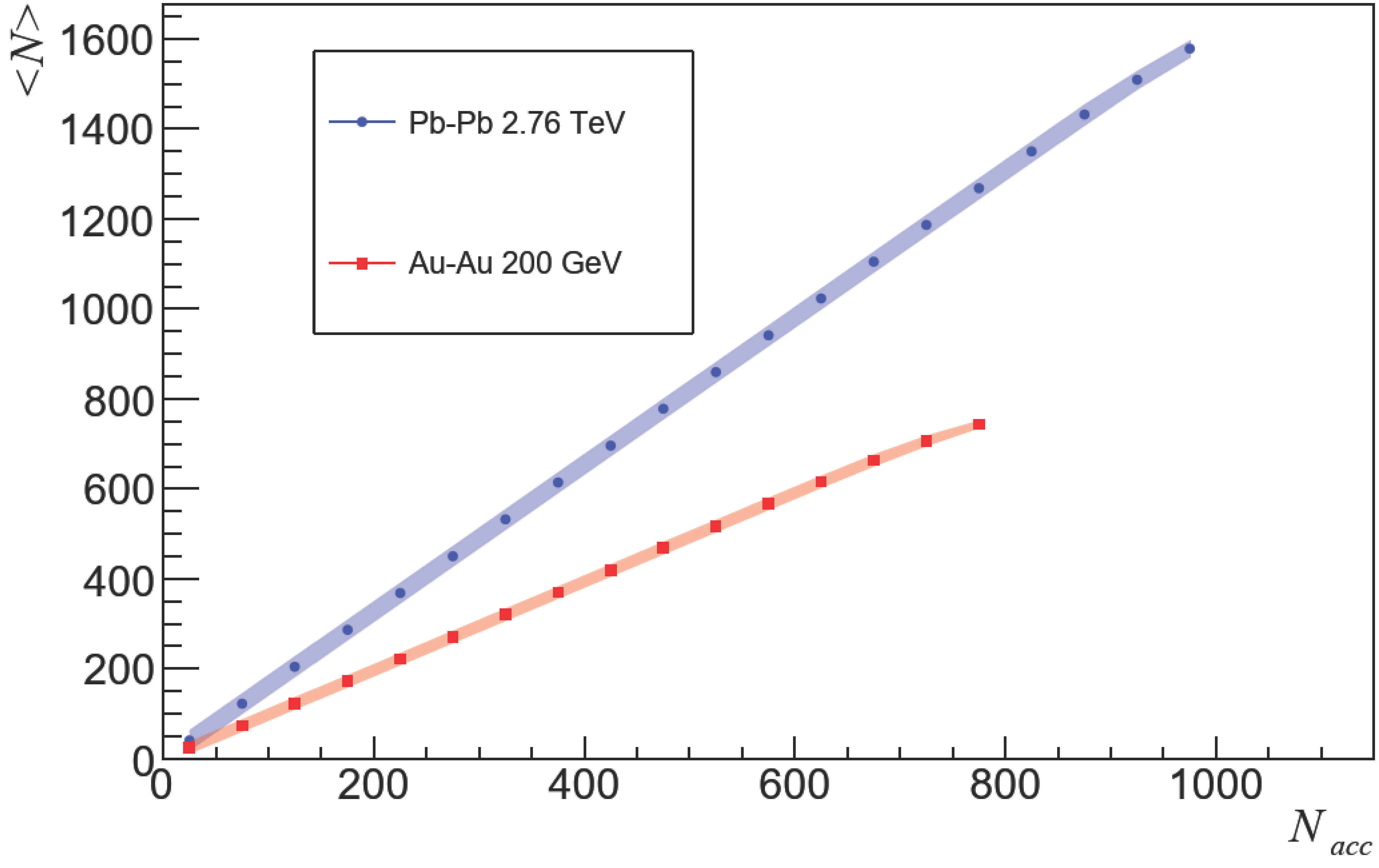
<p_T> [GeV]
0.6
0.5
0.4
0.3
0.2
0.1
0
0
200
400
600
800
1000
N acc
Pb-Pb 2.76 TeV
Au-Au 200 GeV
STAR Au-Au 200 GeV π-



×10^−15
(1+R)<δ p_T,1 δ p_T,2> - C + 2<p_T>D + <p_T>^2 R = 0
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
0
5
10
15
20
25
N_acc
PYTHIA pp 2.76 TeV
PYTHIA pp 200 GeV



<N>
25
20
15
10
5
0
0
5
10
15
20
25
N acc
pp 2.76 TeV
pp 200 GeV



<N>
1600
1400
1200
1000
800
600
400
200
0
0
200
400
600
800
1000
N acc
Pb-Pb 2.76 TeV
Au-Au 200 GeV