

Complementary Two-Particle Correlation Observables for Relativistic Nuclear Collisions

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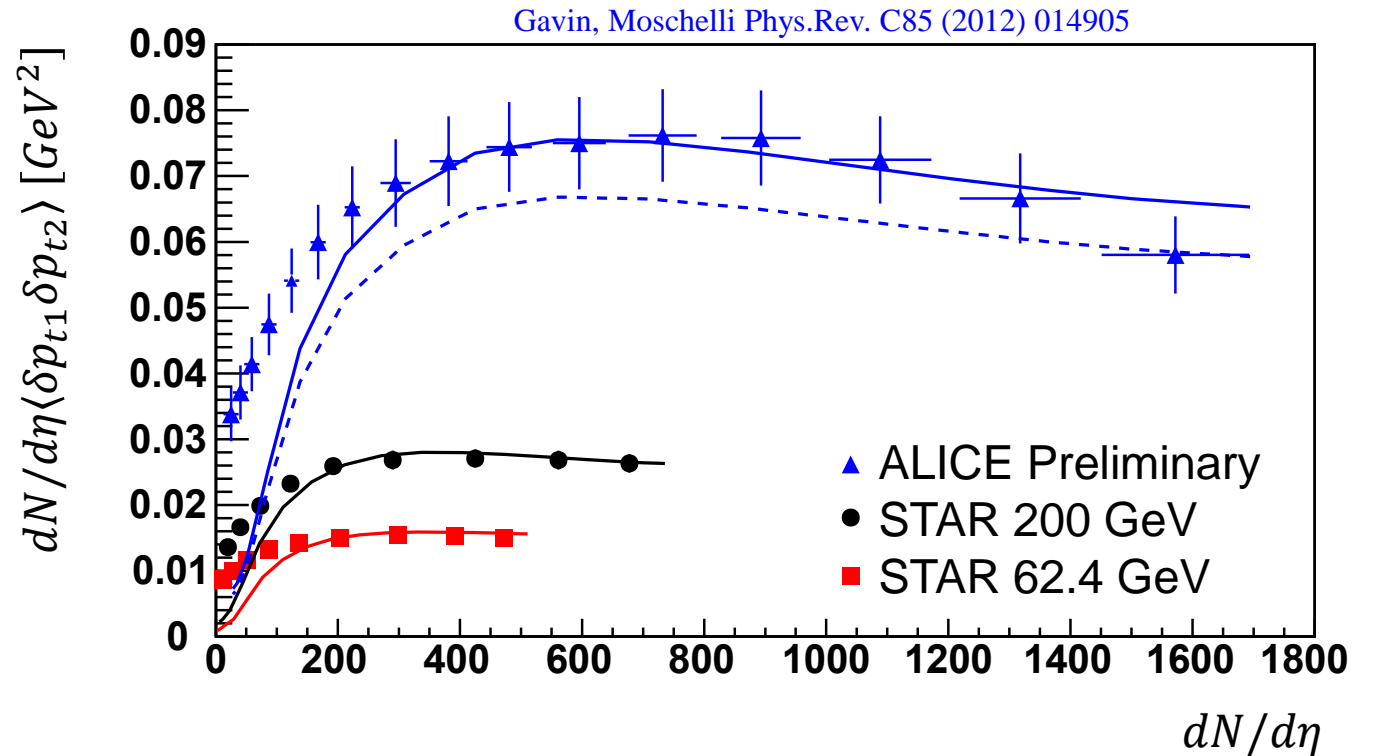
Motivation

What are the observable consequences of incomplete thermalization of the medium created in nuclear collisions?

Correlations of transverse momentum fluctuations

$$\langle \delta p_{T1} \delta p_{T2} \rangle = \frac{\langle \sum_i \sum_{i \neq j} \delta p_{Ti} \delta p_{Tj} \rangle}{\langle N(N-1) \rangle}$$

$$\delta p_{Ti} = p_{Ti} - \langle p_T \rangle$$



STAR, Phys.Rev.C 72 (2005) 044902

ALICE preliminary since published in Eur. Phys. J. C 74 (2014) 3077

Boltzmann-Langevin Equation

Boltzmann Eq.

Relaxation Time Approx.

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_p \cdot \nabla \right) f(\mathbf{p}, \mathbf{x}, t) = -\nu [f(\mathbf{p}, \mathbf{x}, t) - f^{eq}(\mathbf{p}, \mathbf{x}, t)]$$

Relaxation time ν^{-1}
Drift velocity $\mathbf{v}_p = \mathbf{p}/E$
Temperature T
Velocity \mathbf{u}
Chemical Potential μ

Local equilibrium
distribution

$$f^{eq} = \exp\{-\gamma(E - \mathbf{p} \cdot \mathbf{u} - \mu)/T\}$$

Conservation laws require we choose T , \mathbf{u} , μ so that f^{eq} gives the same energy, momentum, and particle density as f . Use eigenfunctions φ_i with zero eigenvalue corresponding to the conserved quantities to define a projection operator.

$$P = \sum_{i=1}^5 c(\mathbf{x}, t) \varphi_i$$

$$Pf = f^{eq}$$

Linearized Boltzmann-Langevin Equation:

$$f(t + \Delta t) - f(t) = -\nu(1 - P)f(t)\Delta t + \Delta W(t)$$

Stochastic term $\Delta W(t)$ represents a random change to f at each time step.

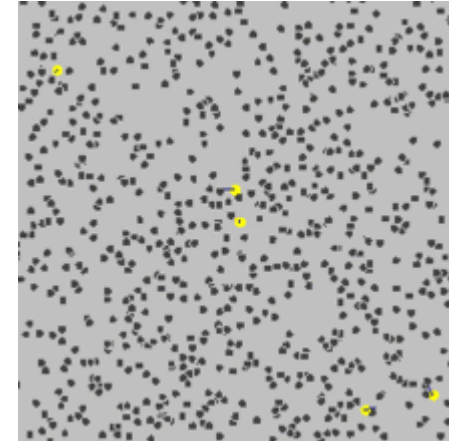
Brownian Motion

$$v(t + \Delta t) - v(t) = -\gamma v(t)\Delta t + \Delta W(t)$$

change in velocity

friction

collisions

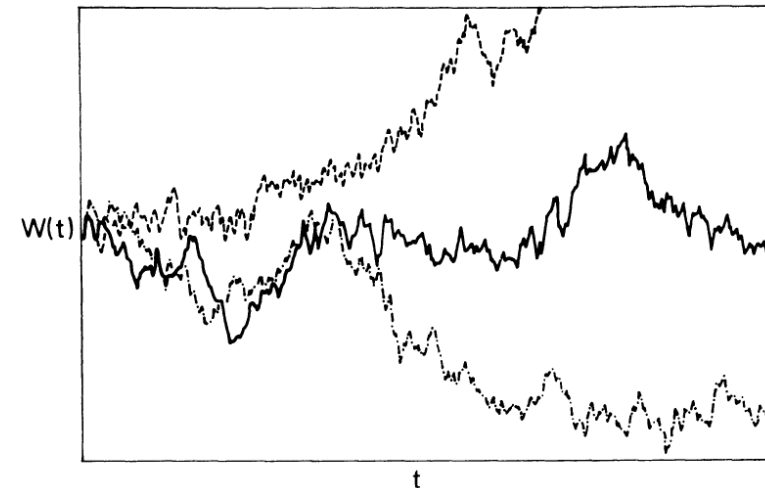


Average of noise $\langle \Delta W \rangle = 0$

Variance of noise $\langle \Delta W^2 \rangle = \Gamma \Delta t$

“strength” of the noise

Using the fluctuation dissipation theorem one can find $\Gamma = 2\gamma/m$



Single Particle Equation

Averaging over noise

$$f(t + \Delta t) - f(t) = -\nu(1 - P)f(t)\Delta t + \Delta W(t)$$

Gives

$$\frac{d}{dt}\langle f \rangle = -\nu(1 - P)\langle f \rangle$$

Solve using the method of characteristics to find

$$f(\mathbf{p}, \mathbf{x}, t) = f_0(\mathbf{p}, \mathbf{x} - \mathbf{v}_p t) S(t, t_0) + f^{eq}(\mathbf{p}, \mathbf{x} - \mathbf{v}_p t) (1 - S(t, t_0))$$

initial conditions

equilibrium

survival probability

S is the probability particles escape the collision volume without suffering any collisions

$$S(t, t_0) = e^{-\int_{t_0}^t \nu(t') dt'}$$

As thermalization proceeds $S \rightarrow 0$.

Two-Particle Equation

Two-particle correlations

$$G_{12} = \langle f_1 f_2 \rangle - \langle f_1 \rangle \langle f_2 \rangle - \langle f_1 \rangle \delta(1 - 2)$$

$$\delta(1 - 2) = \delta(\mathbf{x}_1 - \mathbf{x}_2) \delta(\mathbf{p}_1 - \mathbf{p}_2)$$

Using the Itô product rule:

$$\Delta \langle f_1 f_2 \rangle = \underbrace{\langle f_2 \Delta f_1 \rangle + \langle f_1 \Delta f_2 \rangle}_{-v(2 - P_1 - P_2) \langle f_1 f_2 \rangle} + \underbrace{\langle \Delta f_1 \Delta f_2 \rangle}_{\Gamma \Delta t}$$

$$\Gamma = v P_1 P_2 (\langle f_1 \rangle - f_1^{eq}) \delta(1 - 2)$$

Using the Itô product rule and the Boltzmann-Langevin Eq.

$$\left(\frac{d}{dt} + v(2 - P_1 - P_2) \right) G_{12} = v P_1 P_2 (\langle f_1 \rangle - f_1^{eq}) \delta(1 - 2)$$

Using the method of characteristics again

$$G_{12} = G_{12}^{eq} + A_{12} S + B_{12} S^2$$

The initial phase space distribution determines the coefficients A_{12} and B_{12} as functions of the momenta and initial positions

Observing Thermalization

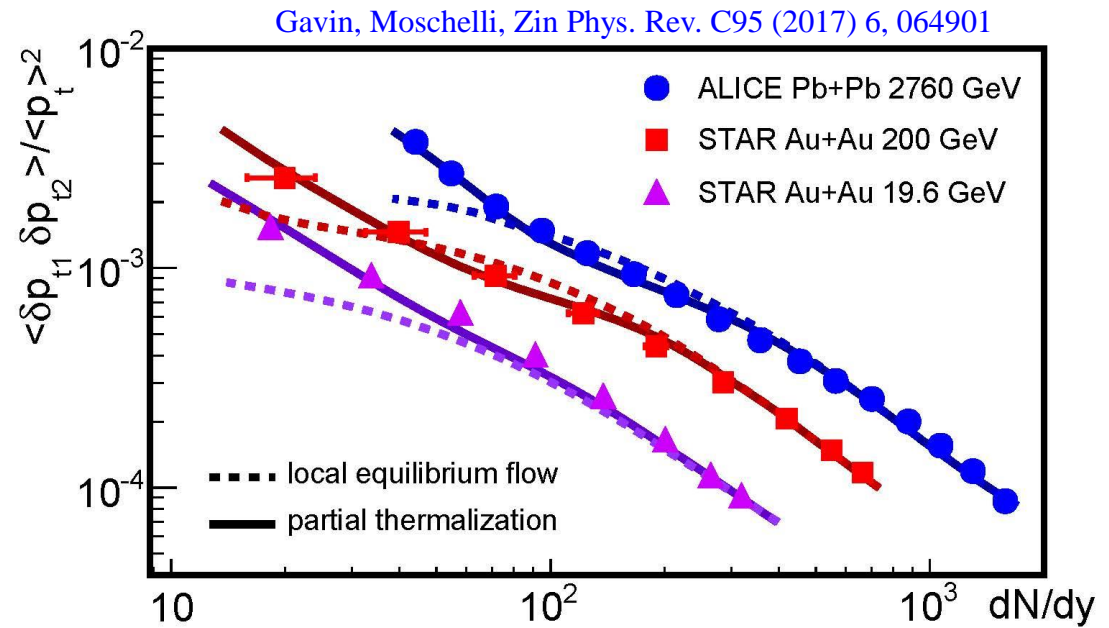
$$\langle \delta p_{T1} \delta p_{T2} \rangle = \frac{\langle \sum_i \sum_{i \neq j} \delta p_{Ti} \delta p_{Tj} \rangle}{\langle N(N-1) \rangle}$$

$$\langle \delta p_{T1} \delta p_{T2} \rangle = \langle \delta p_{T1} \delta p_{T2} \rangle_0 S^2 + \langle \delta p_{T1} \delta p_{T2} \rangle_{eq} (1 - S^2)$$

S = survival probability

$S = 1 \rightarrow$ no interaction

$S = 0 \rightarrow$ local equilibrium



Look for updates coming soon!

Zoulfekar PhD thesis.

ALICE: Eur. Phys. J C74, 3077 (2014)

STAR: Phys. Rev. C72, 044902 (2005)

STAR: Phys. Rev. C99, 044918 (2019)

Use Multiple Observables to Constrain Theory

Complementary Observables

$$(1 + \mathcal{R})\langle\delta p_{T1}\delta p_{T2}\rangle - \mathcal{C} + 2\langle p_T\rangle\mathcal{D} + \langle p_T\rangle^2\mathcal{R} = 0$$

Cody, Gavin,
Koch, Kocherovsky,
Mazloun, Moschelli
arXiv: 2110.04884

\mathcal{R}	Multiplicity Fluctuations
$\langle\delta p_{T1}\delta p_{T2}\rangle$	Correlations of Transverse Momentum Fluctuations
\mathcal{C}	Transverse Momentum Correlations
\mathcal{D}	Multiplicity-Momentum Correlations (New)

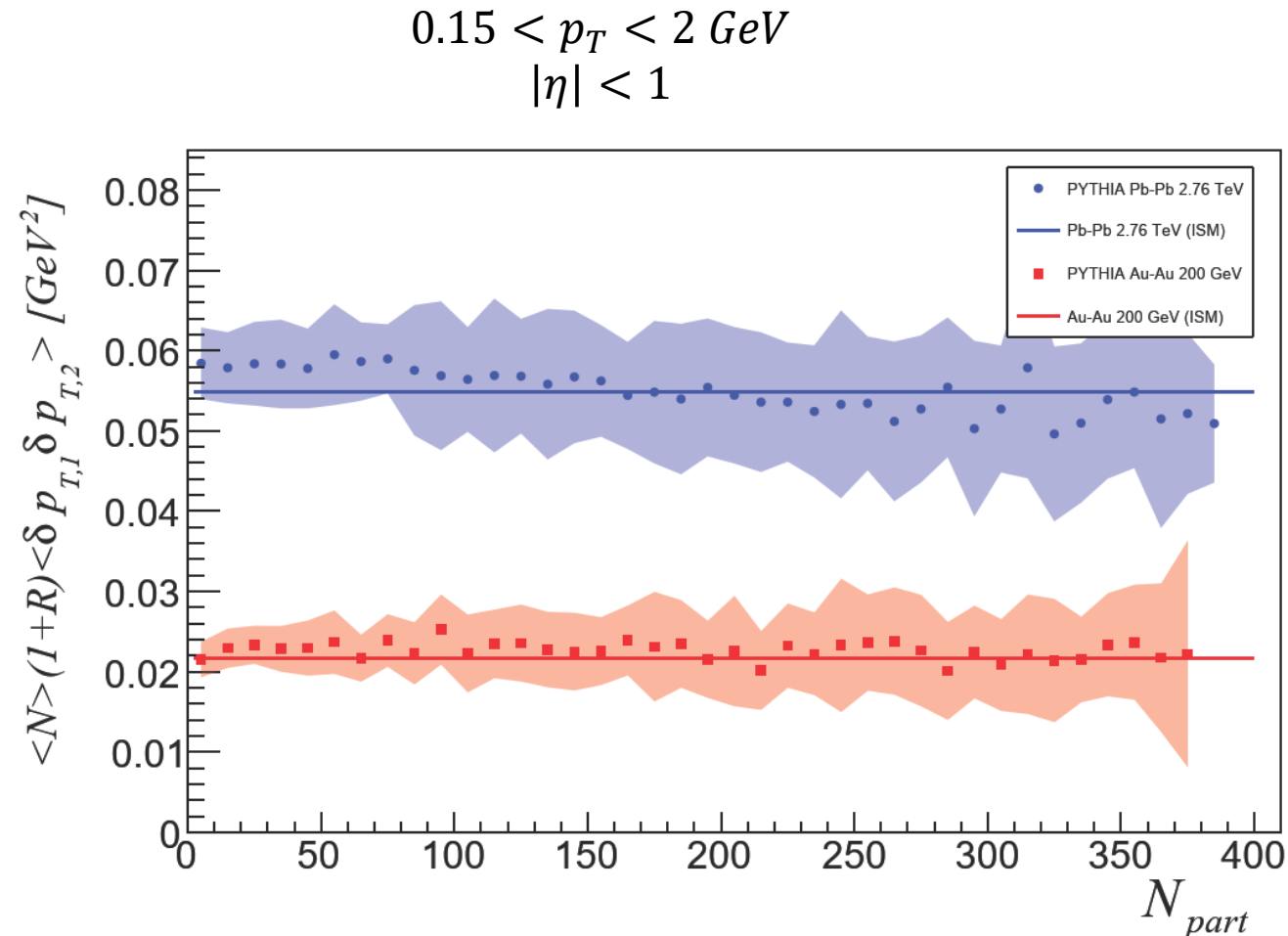
Why are these complementary? All derived from the same distribution of two-particle correlations

Correlations of Transverse Momentum Fluctuations

$$\langle \delta p_{T1} \delta p_{T2} \rangle = \frac{\langle \sum_i \sum_{i \neq j} \delta p_{Ti} \delta p_{Tj} \rangle}{\langle N(N-1) \rangle}$$

$$\langle \delta p_{T1} \delta p_{T2} \rangle = \langle \delta p_{T1} \delta p_{T2} \rangle_0 S^2 + \langle \delta p_{T1} \delta p_{T2} \rangle_{eq} (1 - S^2)$$

- $\delta p_{Ti} = p_{Ti} - \langle p_T \rangle$
- Used to search for QCD critical point
- Sensitive to temperature fluctuations
- Removes multiplicity fluctuations



Lines – independent source model

Points - PYTHIA/Angantyr

PYTHIA [Comput.Phys.Commun. 191 \(2015\) 159-177, arXiv:1410.3012](#)

Angantyr [10 \(2018\) 134, arXiv:1806.10820JHEP](#)

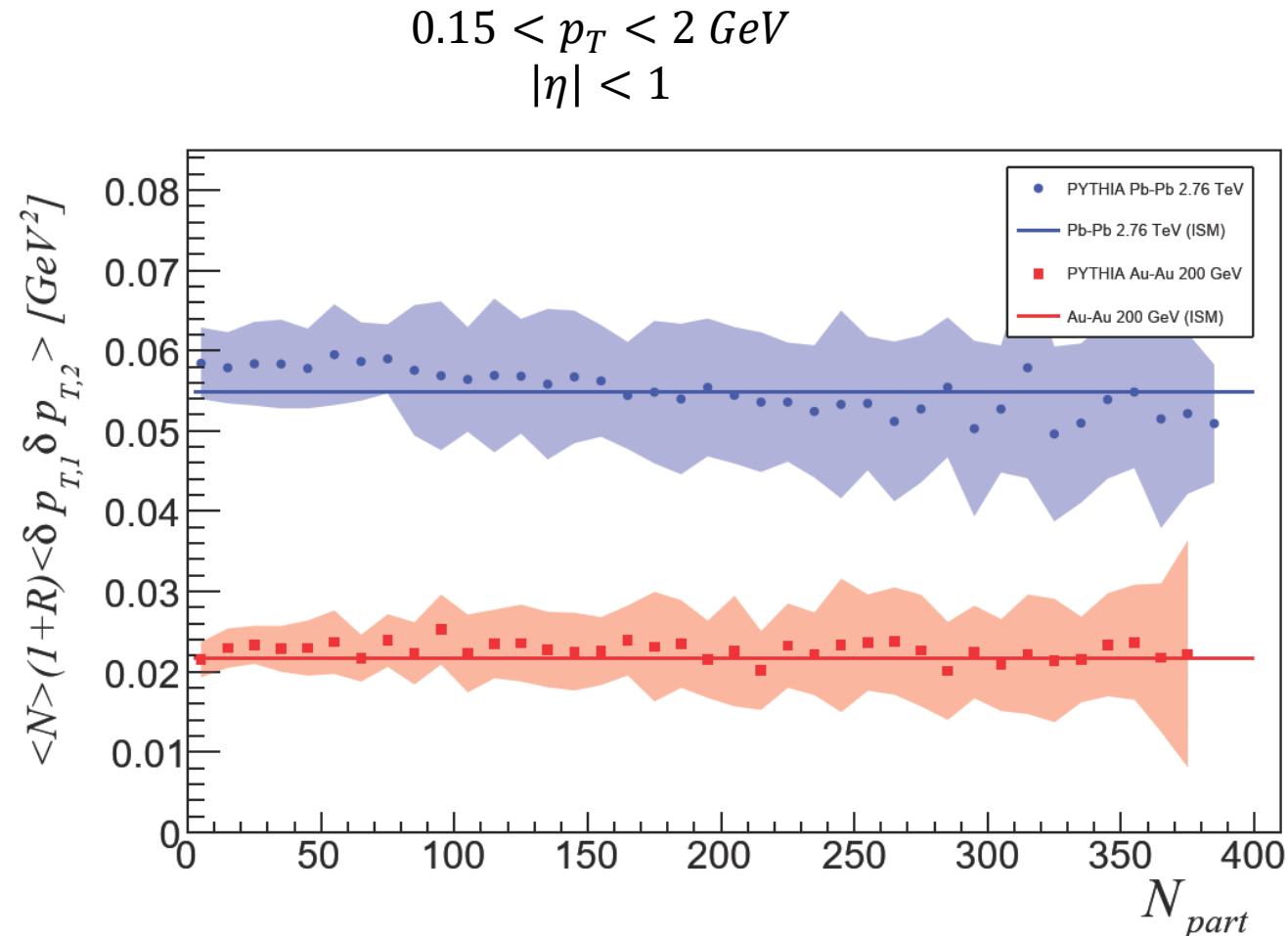
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Independent Source Model

$$\langle \delta p_{t1} \delta p_{t2} \rangle = \frac{2 \langle \delta p_{t1} \delta p_{t2} \rangle_{pp} (1 + \mathcal{R}_{pp})}{\langle N_{part} \rangle (1 + \mathcal{R})}$$



Lines – independent source model

Points - PYTHIA/Angantyr

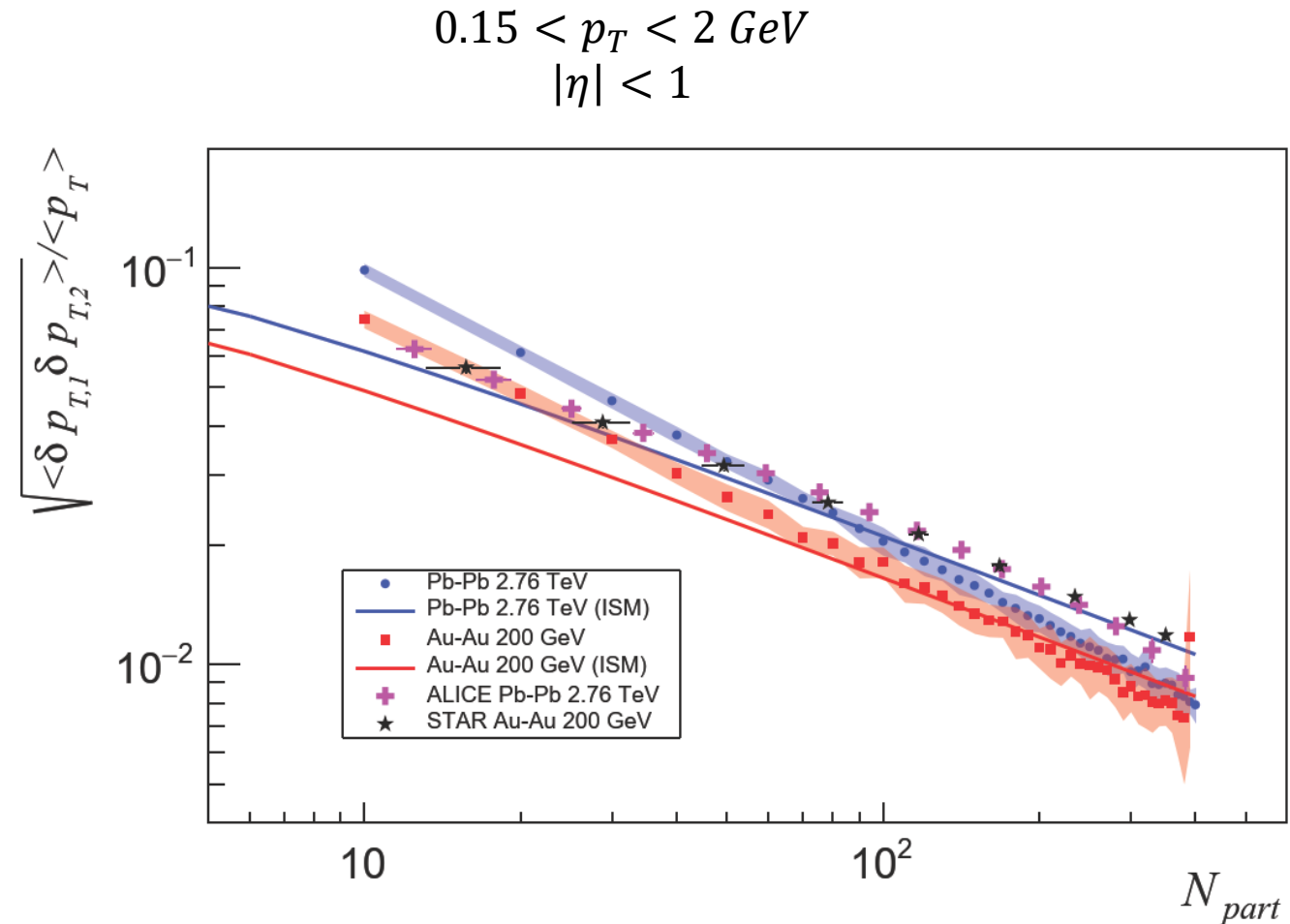
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Independent Source Model

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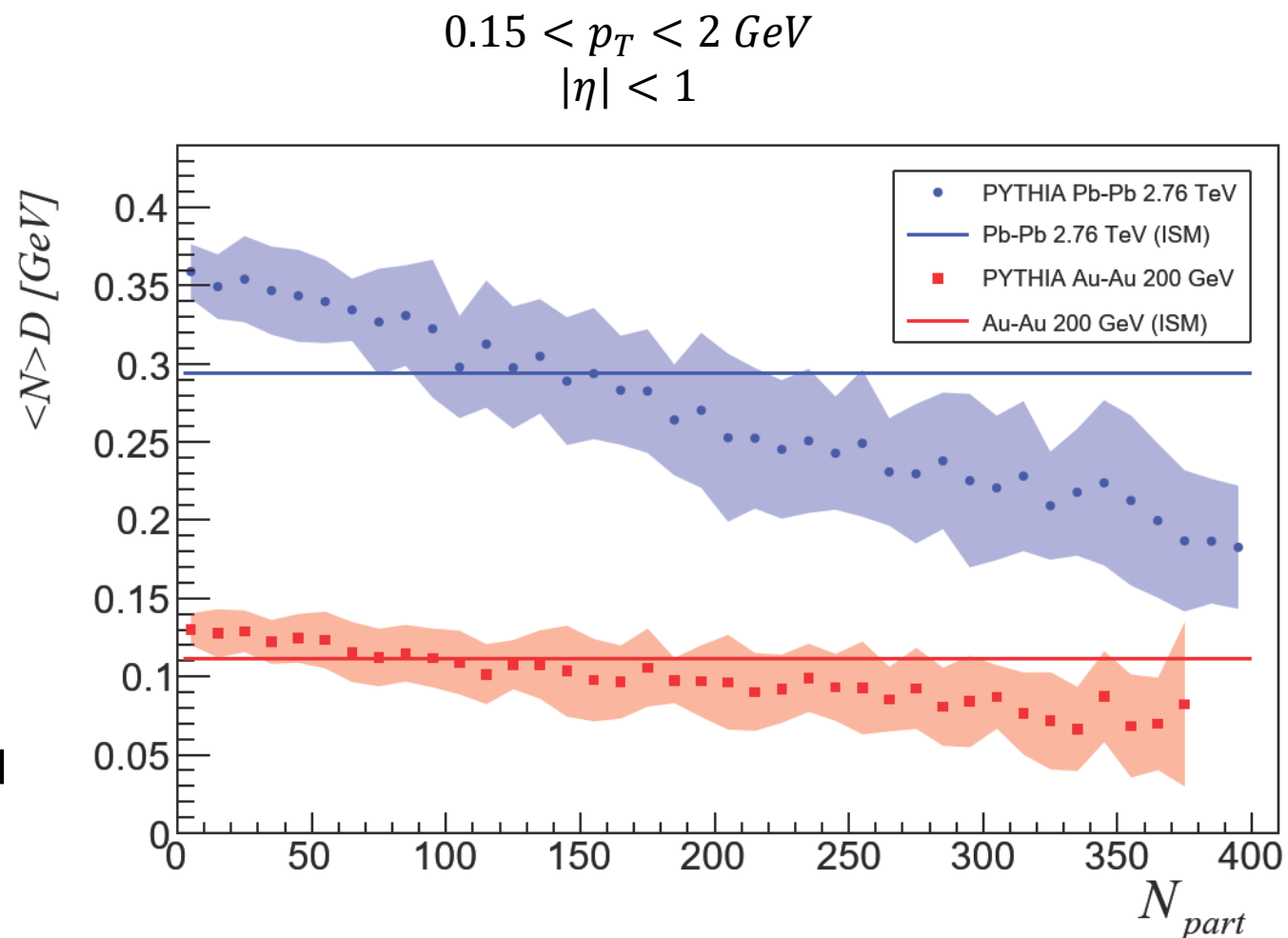
Multiplicity – Momentum Correlations

$$\mathcal{D} = \frac{\langle \sum_i \sum_{i \neq j} \delta p_{Ti} \delta p_{Tj} \rangle}{\langle N \rangle^2}$$

$$\mathcal{D} = \frac{\text{Cov}(P_T, N) - \langle p_T \rangle \text{Var}(N)}{\langle N \rangle^2}$$

$$\mathcal{D} = \mathcal{D}_0 S + \mathcal{D}_{eq} (1 - S)$$

- Removes multiplicity fluctuations
- $\mathcal{D} = 0$ in thermal equilibrium (Grand Canonical Ensemble)
- PYTHIA: positive, nonzero \mathcal{D}
- Consistent with increase in $\langle p_T \rangle$ with multiplicity



Multiplicity – Momentum Correlations

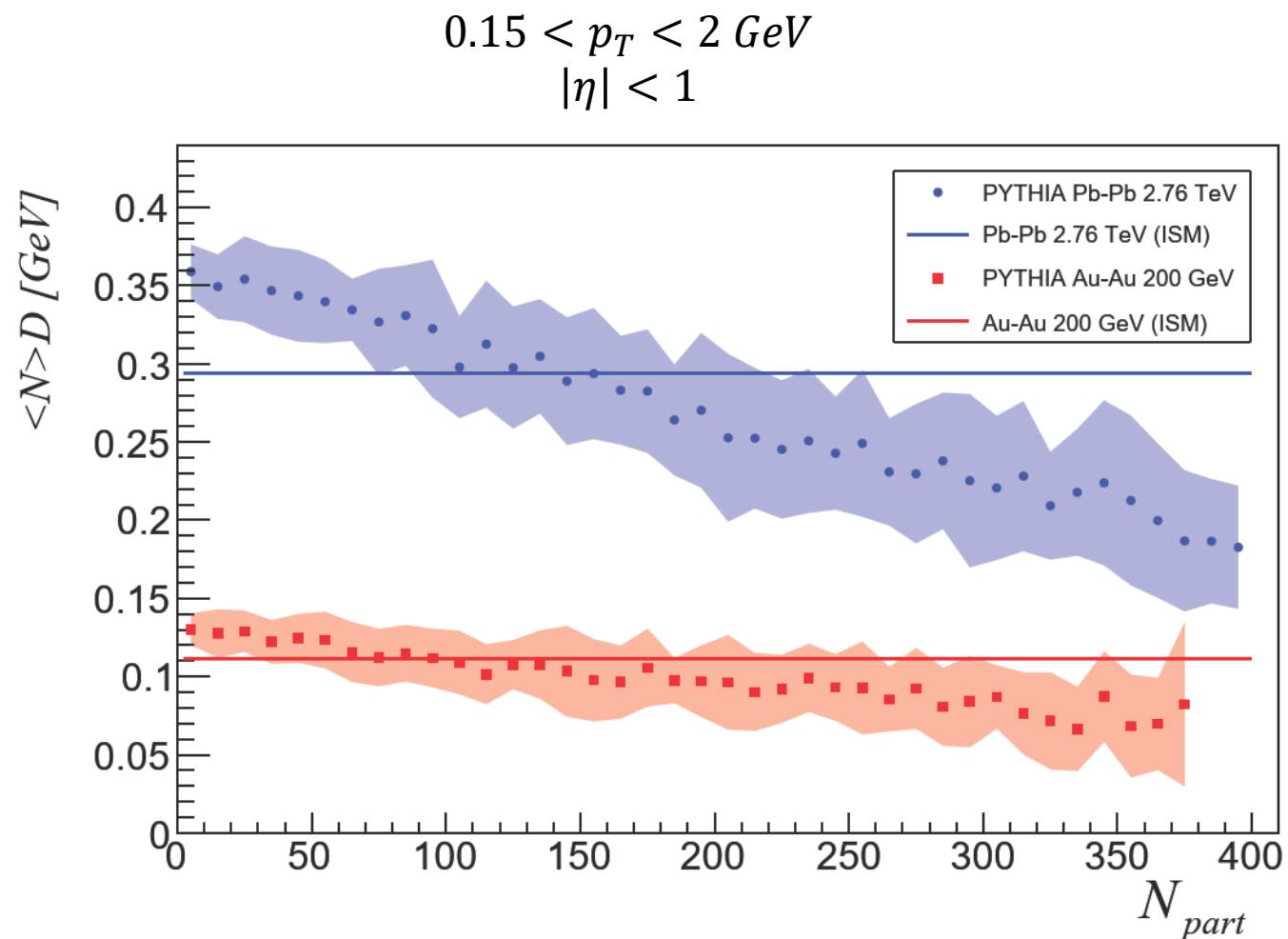
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$$\mathcal{D} = \mathcal{D}_0 S + \mathcal{D}_{eq} (1 - S)$$

Independent Source Model

$$\mathcal{D} = \frac{2\mathcal{D}_{pp}}{\langle N_{part} \rangle}$$



Multiplicity – Momentum Correlations

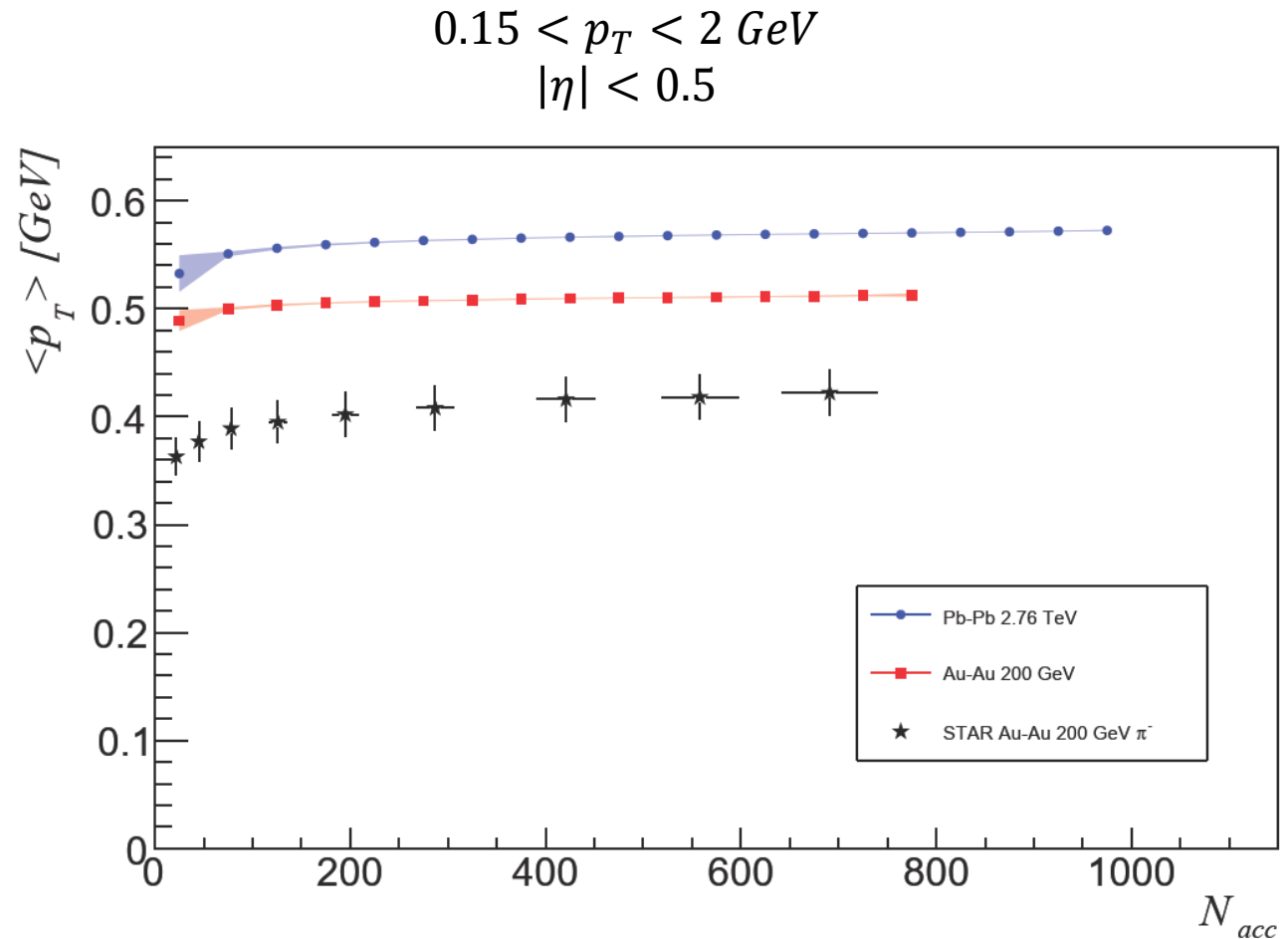
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Independent Source Model

$$\mathcal{D} = \frac{2\mathcal{D}_{pp}}{\langle N_{part} \rangle}$$



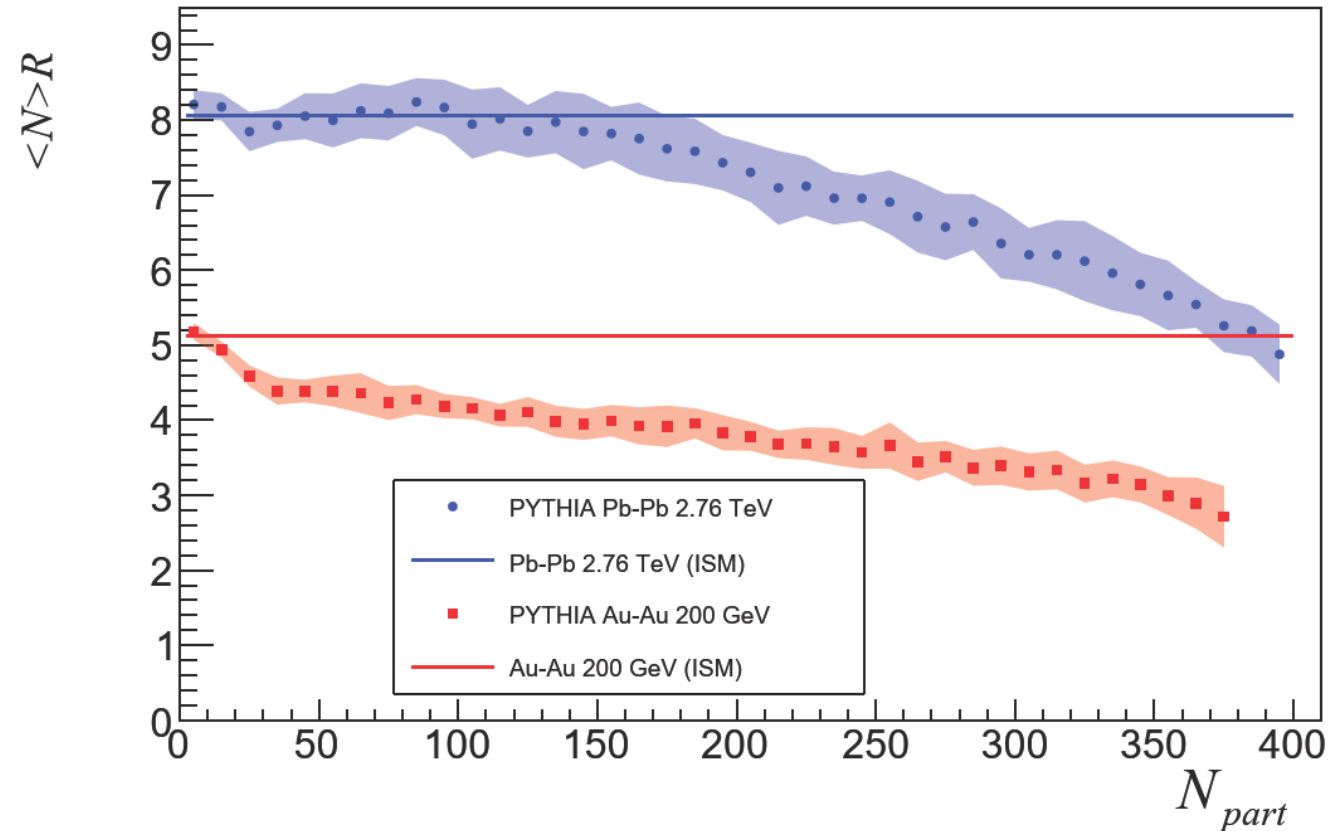
Multiplicity Fluctuations

$$\mathcal{R} = \frac{\langle N(N-1) \rangle - \langle N \rangle^2}{\langle N \rangle^2} = \frac{\text{Var}(N) - \langle N \rangle}{\langle N \rangle^2}$$

No dependence on Survival probability S

- \mathcal{R} measures correlations, sets an overall scale
- $\mathcal{R} = 0$ for independent particle production
- $\mathcal{R} \propto 1/\langle N \rangle$ for non-Poissonian distributions
- Indicates “volume fluctuations”

$$0.15 < p_T < 2 \text{ GeV} \\ |\eta| < 1$$



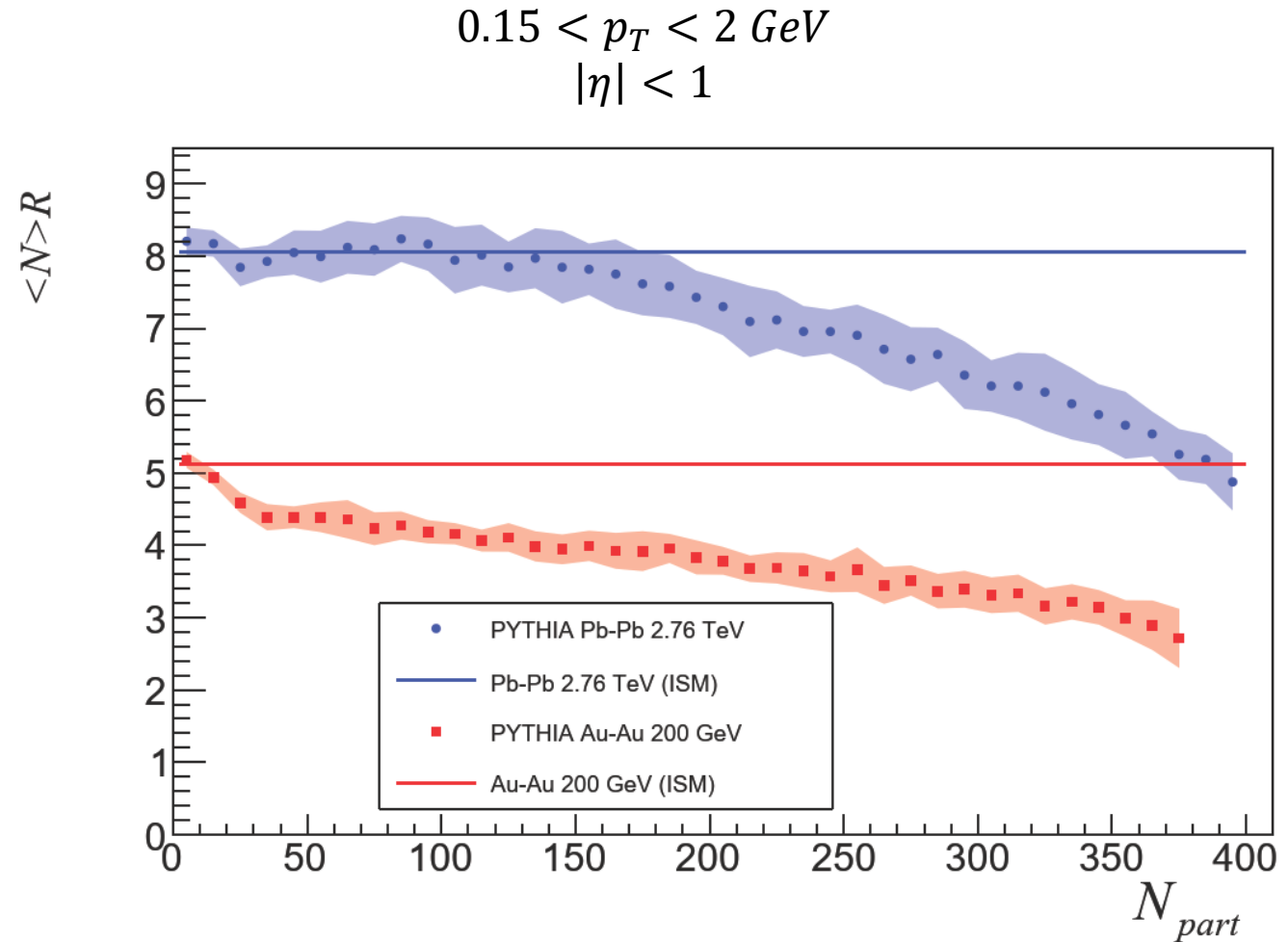
Multiplicity Fluctuations

$$\mathcal{R} = \frac{\langle N(N-1) \rangle - \langle N \rangle^2}{\langle N \rangle^2} = \frac{\text{Var}(N) - \langle N \rangle}{\langle N \rangle^2}$$

No dependence on Survival probability S

Independent Source Model

$$\mathcal{R} = \frac{2\mathcal{R}_{pp}}{\langle N_{part} \rangle} + \frac{\langle N_{part}^2 \rangle - \langle N_{part} \rangle^2}{\langle N_{part} \rangle^2}$$



Transverse Momentum Correlations

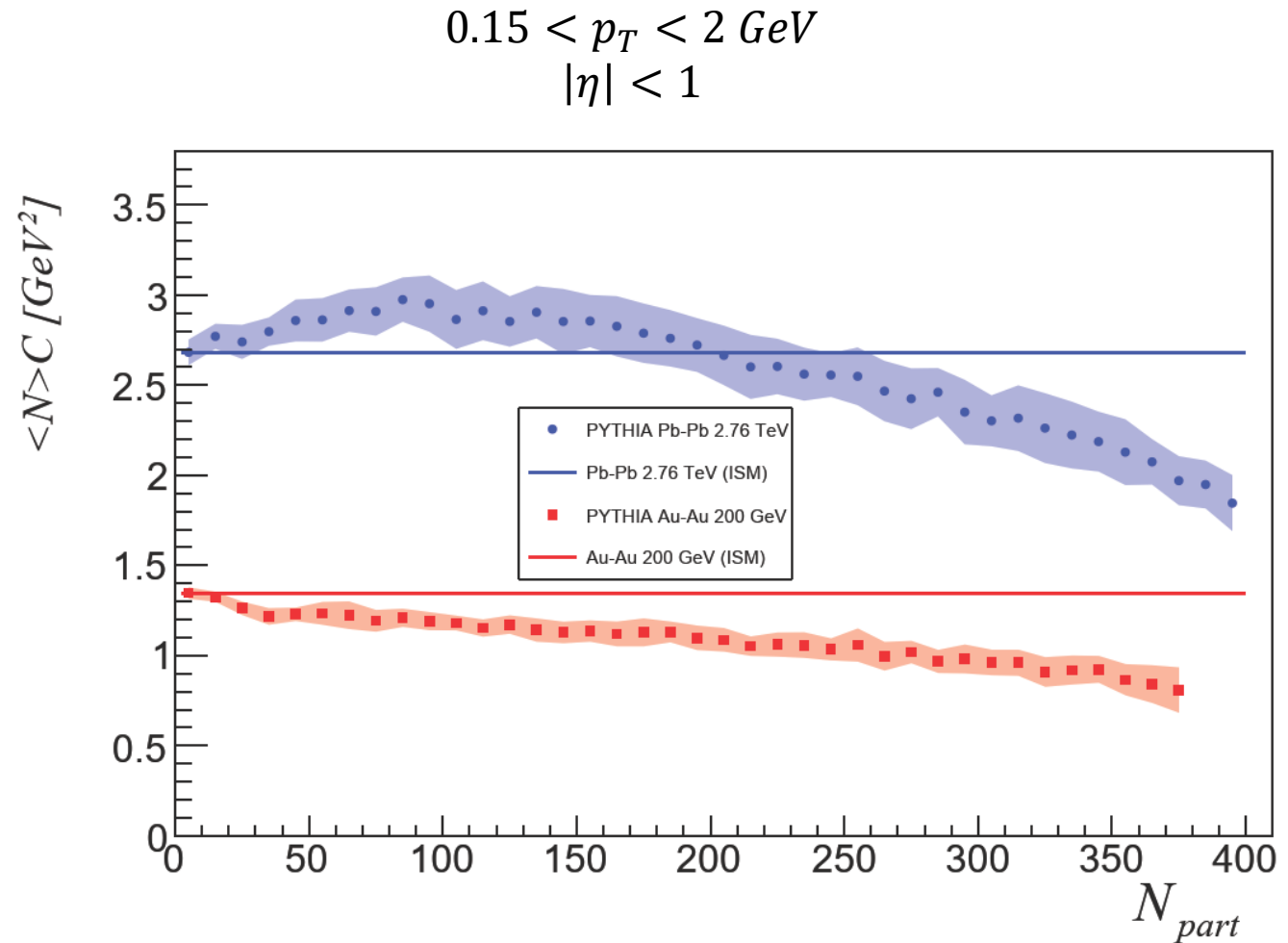
$$c = \frac{\langle \sum_i \sum_{i \neq j} p_{Ti} p_{Tj} \rangle - \langle P_T \rangle^2}{\langle N \rangle^2}$$

$$c = c_0 S^2 + c_{eq} (1 - S^2) + 2 \langle p_T \rangle (\mathcal{D}_0 - \mathcal{D}_{eq}) S (1 - S)$$

- $\langle P_T \rangle = \langle \sum_i p_{Ti} \rangle$
- c is the momentum weighted version of \mathcal{R}
- sensitive to both number density fluctuations and transverse momentum fluctuations
- Used to estimate shear viscosity and shear relaxation time

[Phys Rev C94, 024921 \(2016\) arXiv:1606.02692](#)

[Nucl Phys A982, 311 \(2019\) arXiv:1807.06532](#)



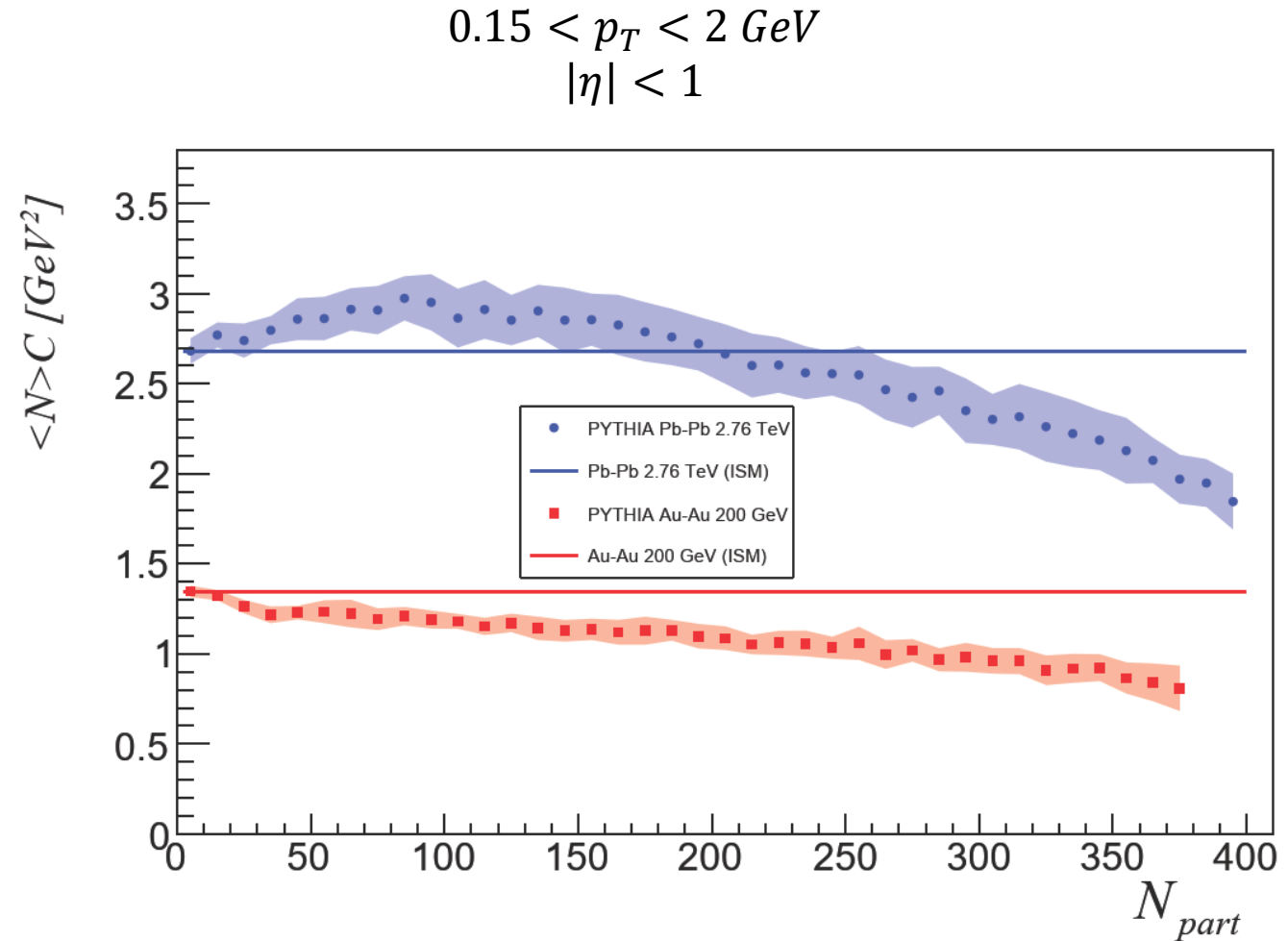
Transverse Momentum Correlations

$$c = \frac{\langle \sum_i \sum_{i \neq j} p_{Ti} p_{Tj} \rangle - \langle P_T \rangle^2}{\langle N \rangle^2}$$

$$c = c_0 S^2 + c_{eq} (1 - S^2) + 2 \langle p_T \rangle (\mathcal{D}_0 - \mathcal{D}_{eq}) S (1 - S)$$

Independent Source Model

$$c = \frac{2c_{pp}}{\langle N_{part} \rangle} + \left(\frac{\langle N_{part}^2 \rangle - \langle N_{part} \rangle^2}{\langle N_{part} \rangle^2} \right) \langle p_t \rangle^2$$

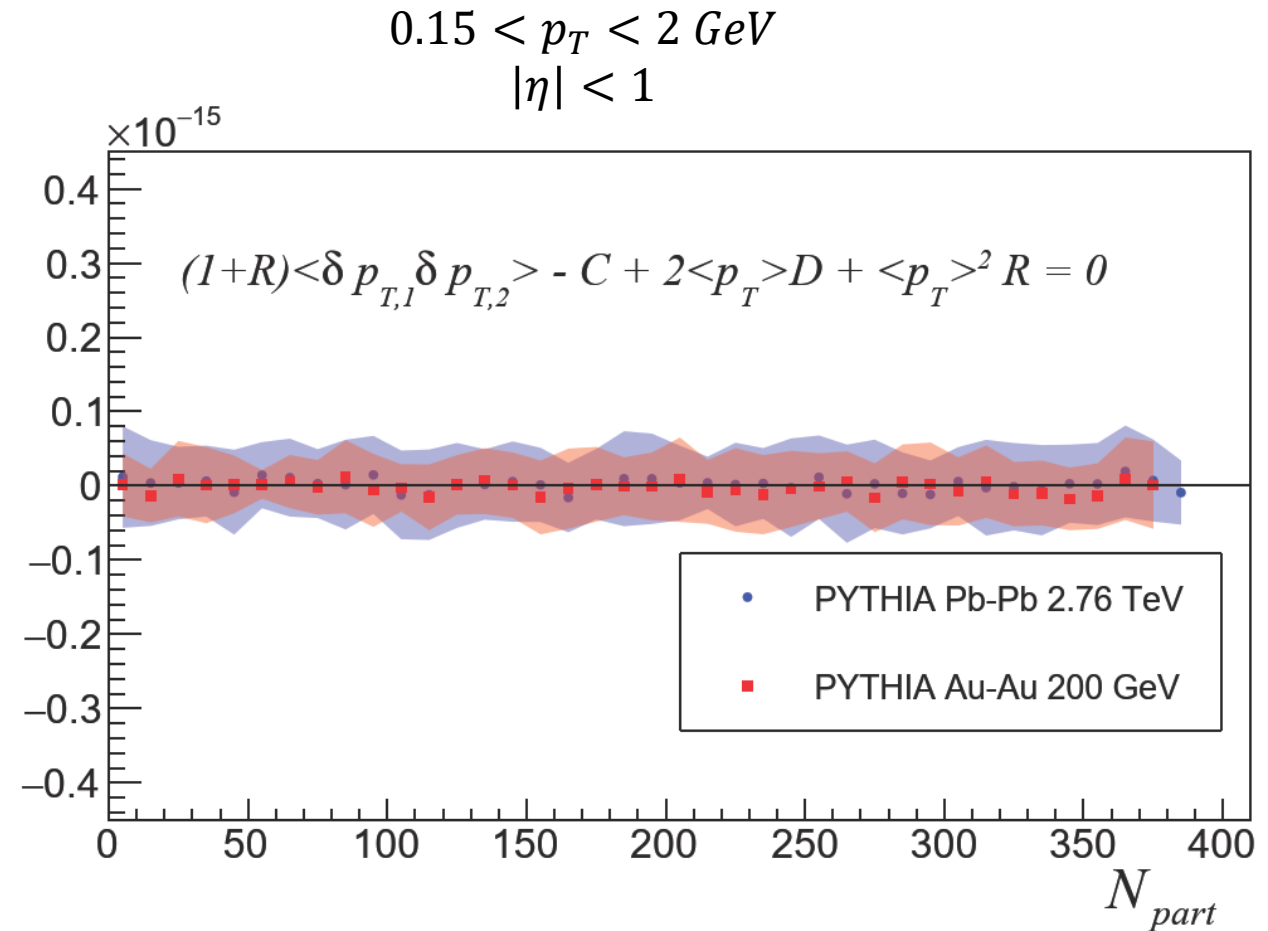


Complementary Correlations

$$(1 + \mathcal{R})\langle\delta p_{T1}\delta p_{T2}\rangle - \mathcal{C} + 2\langle p_T\rangle\mathcal{D} + \langle p_T\rangle^2\mathcal{R} = 0$$

- Validates consistent calculation of observables using PYTHIA
- Theories or models that explain one observable should be able to explain all.
- Can interpret one observable in terms of the physics contributions of the others
- Example:

$$\langle\delta p_{T1}\delta p_{t2}\rangle = \frac{\mathcal{C} - 2\langle p_T\rangle\mathcal{D} - \langle p_T\rangle^2\mathcal{R}}{(1 + \mathcal{R})}$$



Summary

New observable, multiplicity-momentum correlations, \mathcal{D}

- $\mathcal{D} = 0$ expected in equilibrium
- Positive in PYTHIA

Complementary observables $(1 + \mathcal{R})\langle\delta p_{T1}\delta p_{T2}\rangle - \mathcal{C} + 2\langle p_T\rangle\mathcal{D} + \langle p_T\rangle^2\mathcal{R} = 0$

- \mathcal{R} Multiplicity Fluctuations
- \mathcal{C} Transverse Momentum Correlations
- $\langle\delta p_{T1}\delta p_{T2}\rangle$ Correlations of Transverse Momentum Fluctuations
- \mathcal{D} Multiplicity-Momentum Correlations
- All derived from the same parent correlation function

- Use for validation of measurement or calculation of observables
- Challenge theories and models to address all observables simultaneously
- Interpret one observable in terms of physics contributions of the others

Boltzman-Langevin evolution of correlations is sensitive to incomplete thermalization

- \mathcal{R} has no dependence on survival probability S
- \mathcal{D} depends on S
- $\langle\delta p_{T1}\delta p_{T2}\rangle$ depends on S^2
- \mathcal{C} depends on S and S^2
- Simultaneous comparison to multiple observables with different powers of S constrains extraction of S

Independent Source Model (pp source)

$$\mathcal{R} = \frac{2\mathcal{R}_{pp}}{\langle N_{part} \rangle} + \frac{\langle N_{part}^2 \rangle - \langle N_{part} \rangle^2}{\langle N_{part} \rangle^2}$$

$$\mathcal{D} = \frac{2\mathcal{D}_{pp}}{\langle N_{part} \rangle}$$

$$c = \frac{2c_{pp}}{\langle N_{part} \rangle} + \left(\frac{\langle N_{part}^2 \rangle - \langle N_{part} \rangle^2}{\langle N_{part} \rangle^2} \right) \langle p_t \rangle^2$$

$$\langle \delta p_{t1} \delta p_{t2} \rangle = \frac{2\langle \delta p_{t1} \delta p_{t2} \rangle_{pp} (1 + \mathcal{R}_{pp})}{\langle N_{part} \rangle (1 + \mathcal{R})}$$

Correlations and Fluctuations

Momentum Density of Particles $\rho_1(\mathbf{p}_1)$

Pair Momentum Density of Particles $\rho_2(\mathbf{p}_1, \mathbf{p}_2) = \rho_1(\mathbf{p}_1)\rho_1(\mathbf{p}_2) + r(\mathbf{p}_1, \mathbf{p}_2)$

Correlated Pair Distribution $r(\mathbf{p}_1, \mathbf{p}_2) = \rho_2(\mathbf{p}_1, \mathbf{p}_2) - \rho_1(\mathbf{p}_1)\rho_1(\mathbf{p}_2)$

In the case of no correlations $\rho_2(\mathbf{p}_1, \mathbf{p}_2) = \rho_1(\mathbf{p}_1)\rho_1(\mathbf{p}_2)$

$$r(\mathbf{p}_1, \mathbf{p}_2) = 0$$

$$\mathcal{R} = \frac{1}{\langle N \rangle^2} \int \int r(\mathbf{p}_1, \mathbf{p}_2) d^3 \mathbf{p}_1 d^3 \mathbf{p}_2$$

$$\mathcal{C} = \frac{1}{\langle N \rangle^2} \int \int r(\mathbf{p}_1, \mathbf{p}_2) p_{T1} p_{T2} d^3 \mathbf{p}_1 d^3 \mathbf{p}_2$$

$$\langle \delta p_{T1} \delta p_{T2} \rangle = \frac{1}{\langle N(N-1) \rangle} \int \int r(\mathbf{p}_1, \mathbf{p}_2) \delta p_{T1} \delta p_{T2} d^3 \mathbf{p}_1 d^3 \mathbf{p}_2$$

$$\mathcal{D} = \frac{1}{\langle N \rangle^2} \int \int r(\mathbf{p}_1, \mathbf{p}_2) \delta p_{T1} d^3 \mathbf{p}_1 d^3 \mathbf{p}_2$$

