Lattice-QCD-based equations of state at finite temperature and density

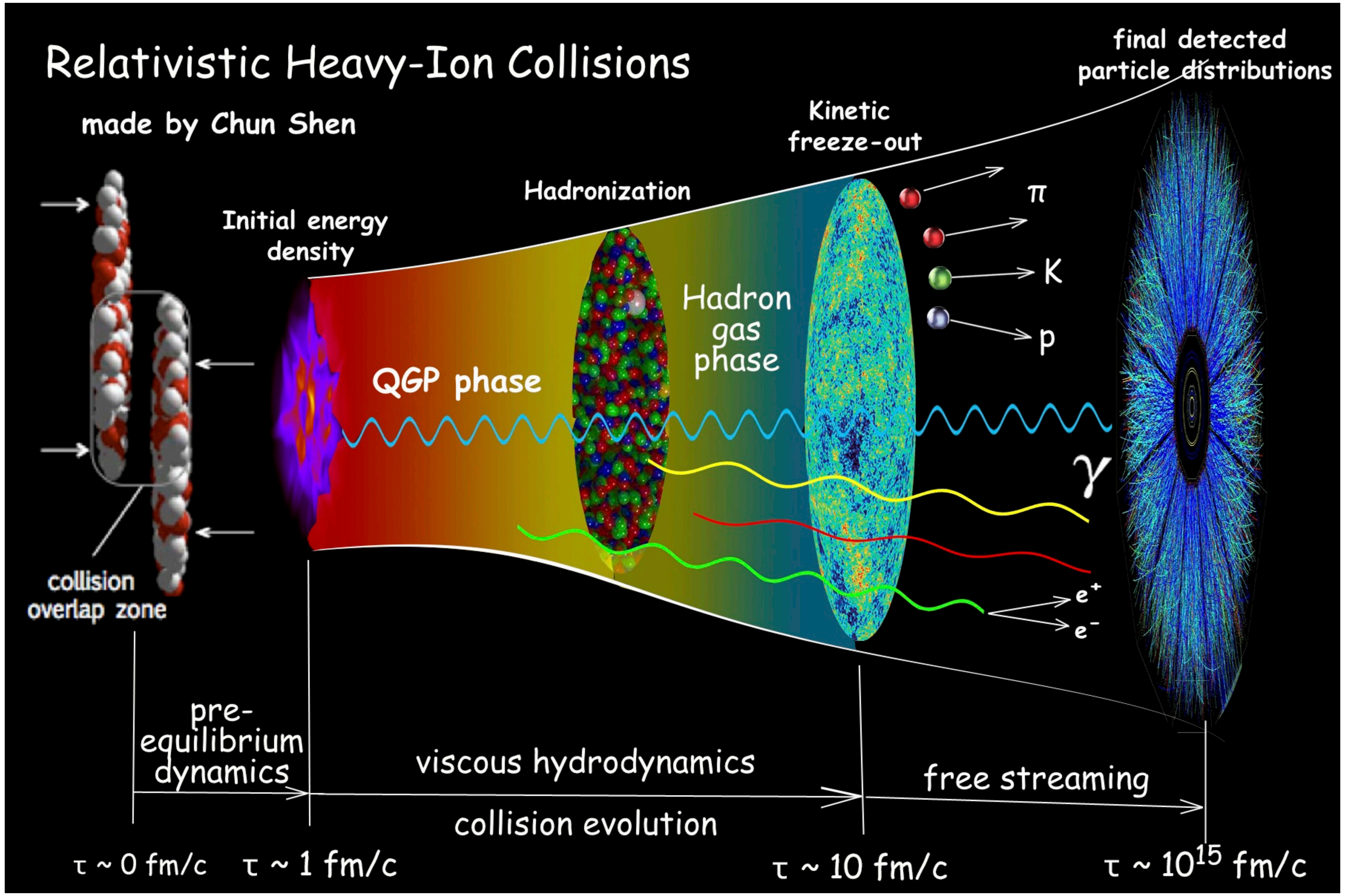
Jamie M. Karthein

In collaboration with:
Debora Mroczek, Angel Nava, Jaki Noronha-Hostler, Paolo Parotto, Damien Price, Claudia Ratti

BEST
COLLABORATION

HIC Phenomenology

- Modeling should mimic experimental conditions in all stages in order to provide robust comparisons and estimates



Lattice QCD Predictions

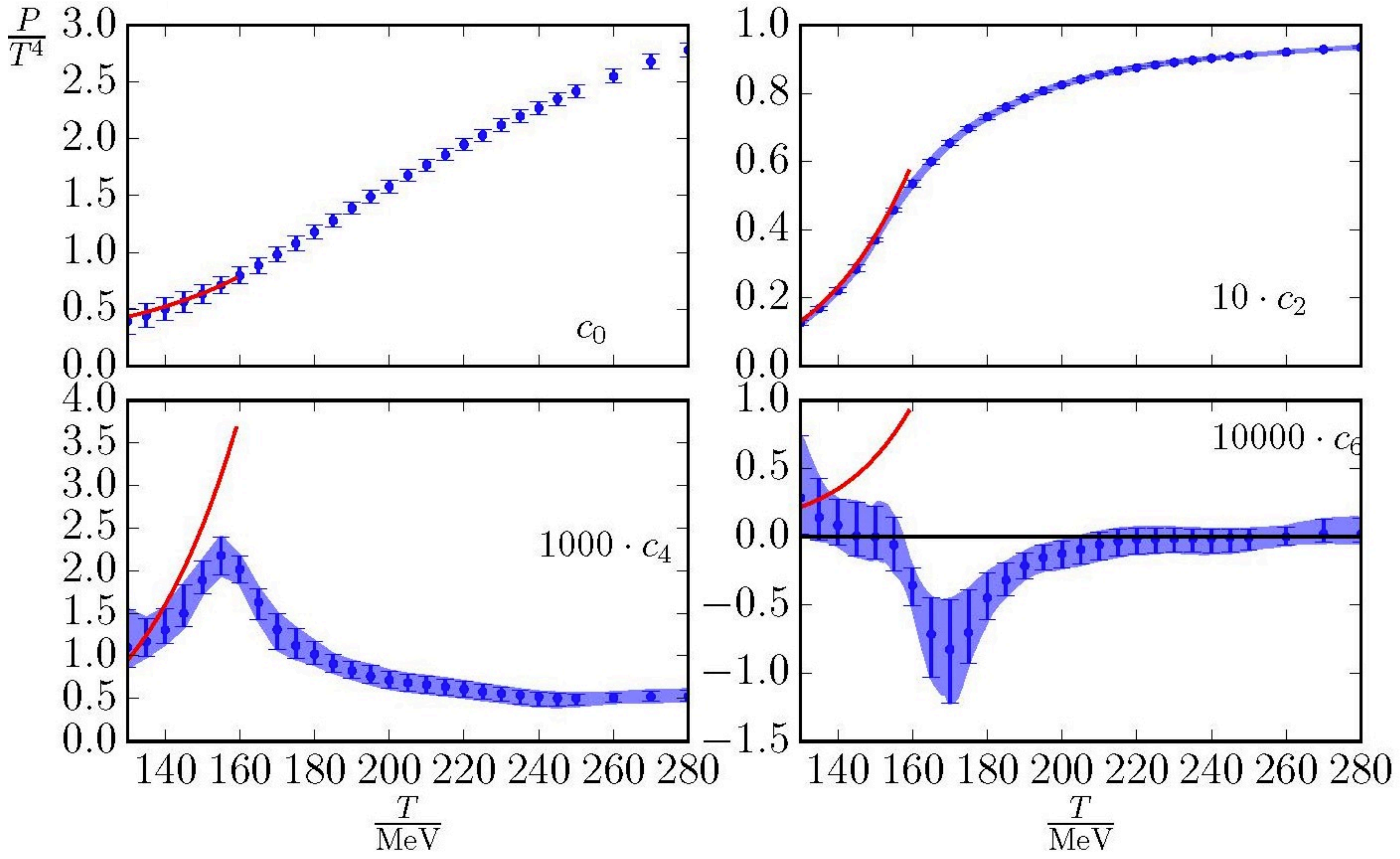
- The equation of state (EoS) for QCD has been calculated on the lattice under strangeness neutrality and fixed ratio of baryon number to electric charge, matching the heavy-ion situation

$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \frac{\mathrm{d}^{2n}(p/T^4)}{d(\frac{\mu_B}{T})^{2n}}\bigg|_{\mu_B=0} \left(\frac{\mu_B}{T}\right)^{2n}$$

$$= \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}$$

$$\langle n_s \rangle = 0$$

$$\langle n_Q \rangle = 0.4 \langle n_B \rangle$$

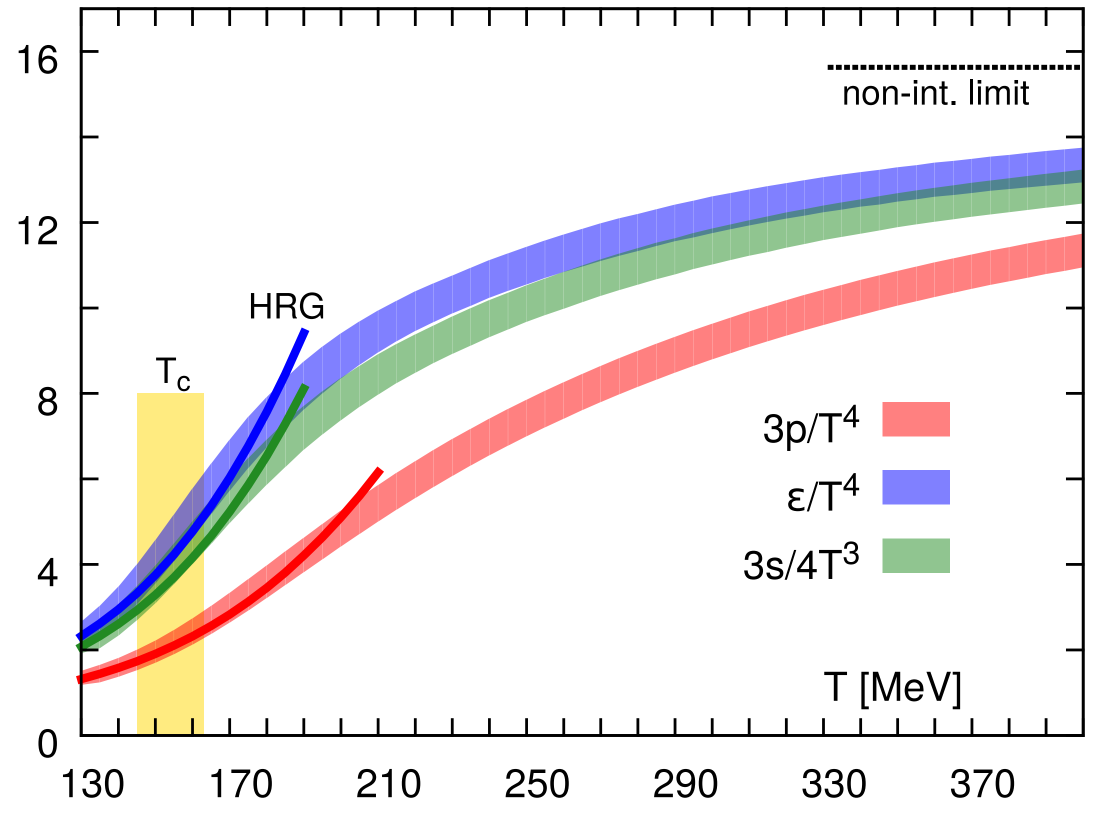


For $\mu_S = \mu_Q = 0$ see: S. Borsanyi et al, JHEP (2018)

WB: J. Guenther et al, NPA (2017)
HotQCD: A. Bazavov et al, PRD (2017)

Hadron Resonance Gas Model

- In the low-temperature regime, the system is well-described by a gas of hadrons:
  - Treat as non-interacting system of resonant states
  - Grand Canonical
  - Match experimental cuts by transforming to p_T and y



$$\frac{P}{T^4} = \frac{1}{VT^3} \sum_i \ln Z_i(T, V, \vec{\mu})$$

where: $$\ln Z_i^{M/B} = \mp \frac{V d_i}{(2\pi)^3} \int d^3p \ln\left(1 \mp \exp\left[-\left(\epsilon_i - \mu_a X_a^i\right)/T\right]\right)$$

energy $\epsilon_i = \sqrt{p^2 + m_i^2}$

conserved charges $\vec{X}_i = (B_i, S_i, Q_i)$

degeneracy d_i, mass m_i, volume V

Particle density: $\frac{n_i}{T^3} = \frac{1}{T^3}\left(\frac{\partial p}{\partial \mu_i}\right)\Big|_{T,\mu_j}$

Entropy density: $\frac{s}{T^3} = \frac{1}{T^3}\frac{\partial p}{\partial T}\Big|_{\mu_i}$

Energy density: $\frac{\epsilon}{T^4} = \frac{s}{T^3} - \frac{p}{T^4} + \sum_i \frac{\mu_i}{T}\frac{n_i}{T^3}$

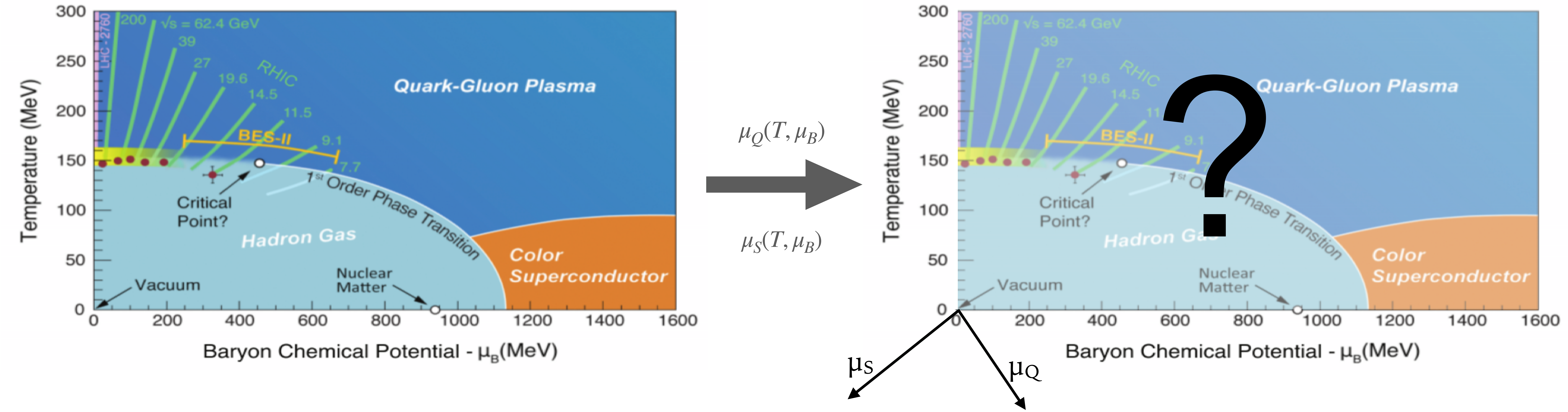
Trace anomaly: $\frac{I}{T^4} = \frac{1}{T^4}(\epsilon - 3P)$

A. Bazavov et al, PRD (2014)

Four-dimensional QCD Phase Diagram

- The strongly interacting matter present in heavy-ion collisions carries a multitude of conserved quantum numbers: baryon number, strangeness and electric charge
    - This effects thermodynamics since each charge has an associated chemical potential

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V, T, \mu_B, \mu_S, \mu_Q)$$



I. Four-dimensional BQS equation of state

Equation of State with Three Conserved Charges

- During HICs the system is not only confined to the T-μ_B plane: determine the equations that depend on μ_B, μ_Q, μ_S

$$\frac{P(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^j \left(\frac{\mu_Q}{T}\right)^k \left(\frac{\mu_S}{T}\right)^i$$

where: $$\chi_{ijk}^{BQS}(T) = \left.\frac{\partial^{i+j+k}(p/T^4)}{\partial(\frac{\mu_B}{T})^i \partial(\frac{\mu_Q}{T})^j \partial(\frac{\mu_S}{T})^k}\right|_{\mu_B,\mu_Q,\mu_S=0}$$

Lattice results only between T ~ 135 - 220 MeV for all 22 coefficients
- Utilize HRG for low T
- Impose Stefan-Boltzmann limit at high T

See also: A. Monnai et al, PRC (2019)

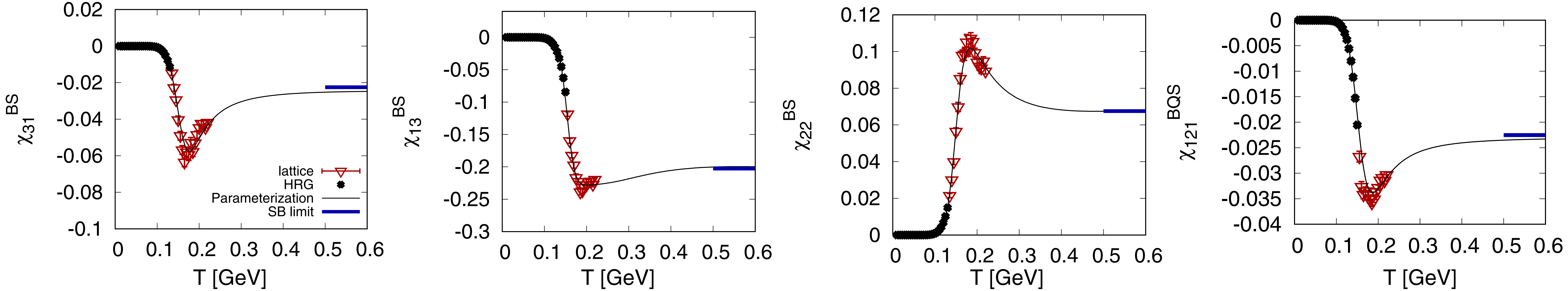
J. Noronha-Hostler, JS et al, PRC (2019)

Parametrized Taylor Coefficients

- Fit all 22 coefficients over a broad range of temperatures

$$\chi_{ijk}^{BQS}(T) = \frac{a_0^i + a_1^i/t + a_2^i/t^2 + a_3^i/t^3 + a_4^i/t^4 + a_5^i/t^5 + a_6^i/t^6 + a_7^i/t^7}{b_0^i + b_1^i/t + b_2^i/t^2 + b_3^i/t^3 + b_4^i/t^4 + b_5^i/t^5 + b_6^i/t^6 + b_7^i/t^7} + c_0$$

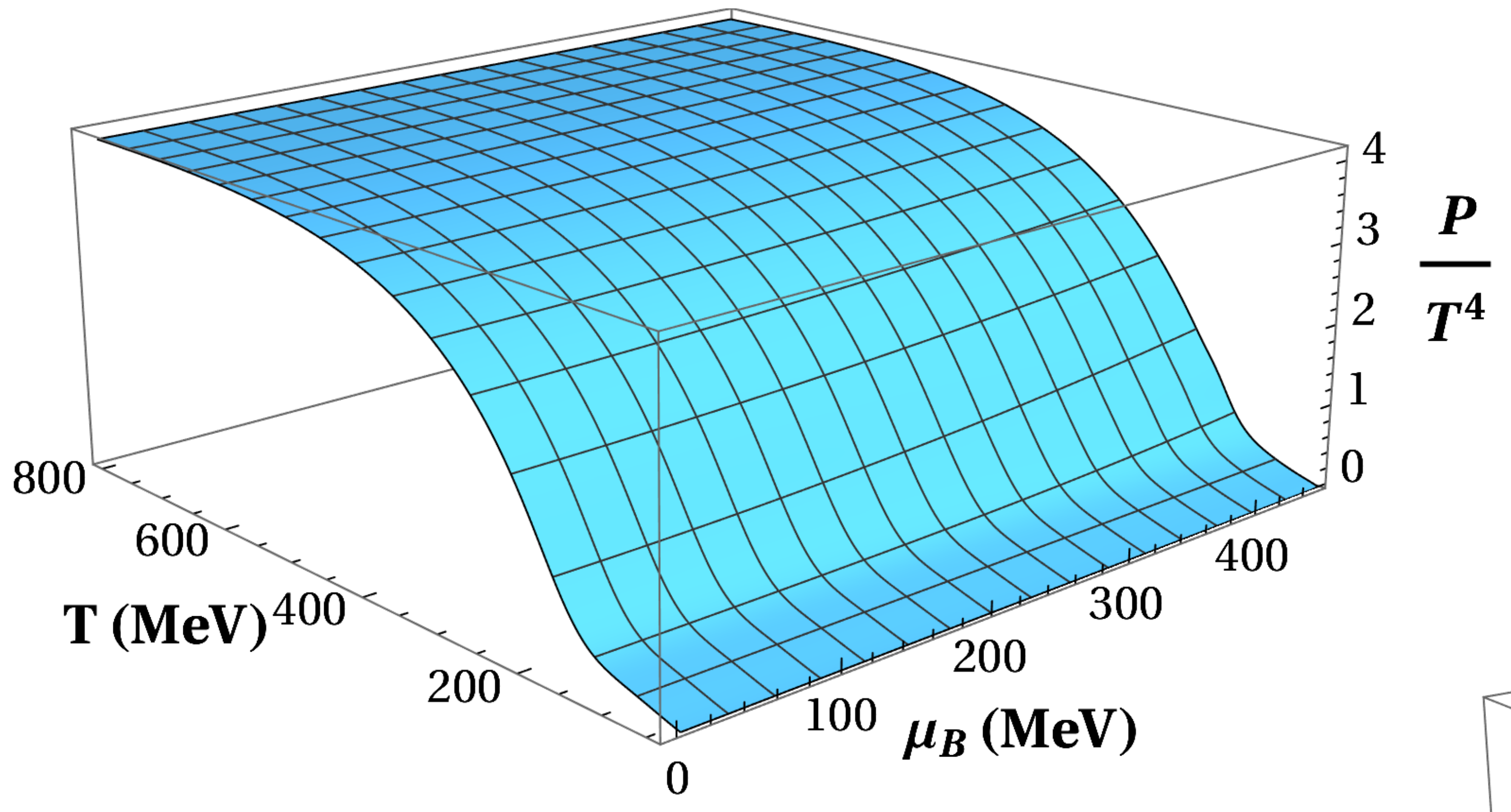
$$\chi_2^B(T) = e^{-h_1/t' - h_2/t'^2} \cdot f_3 \cdot (1 + \tanh(f_4 t' + f_5))$$

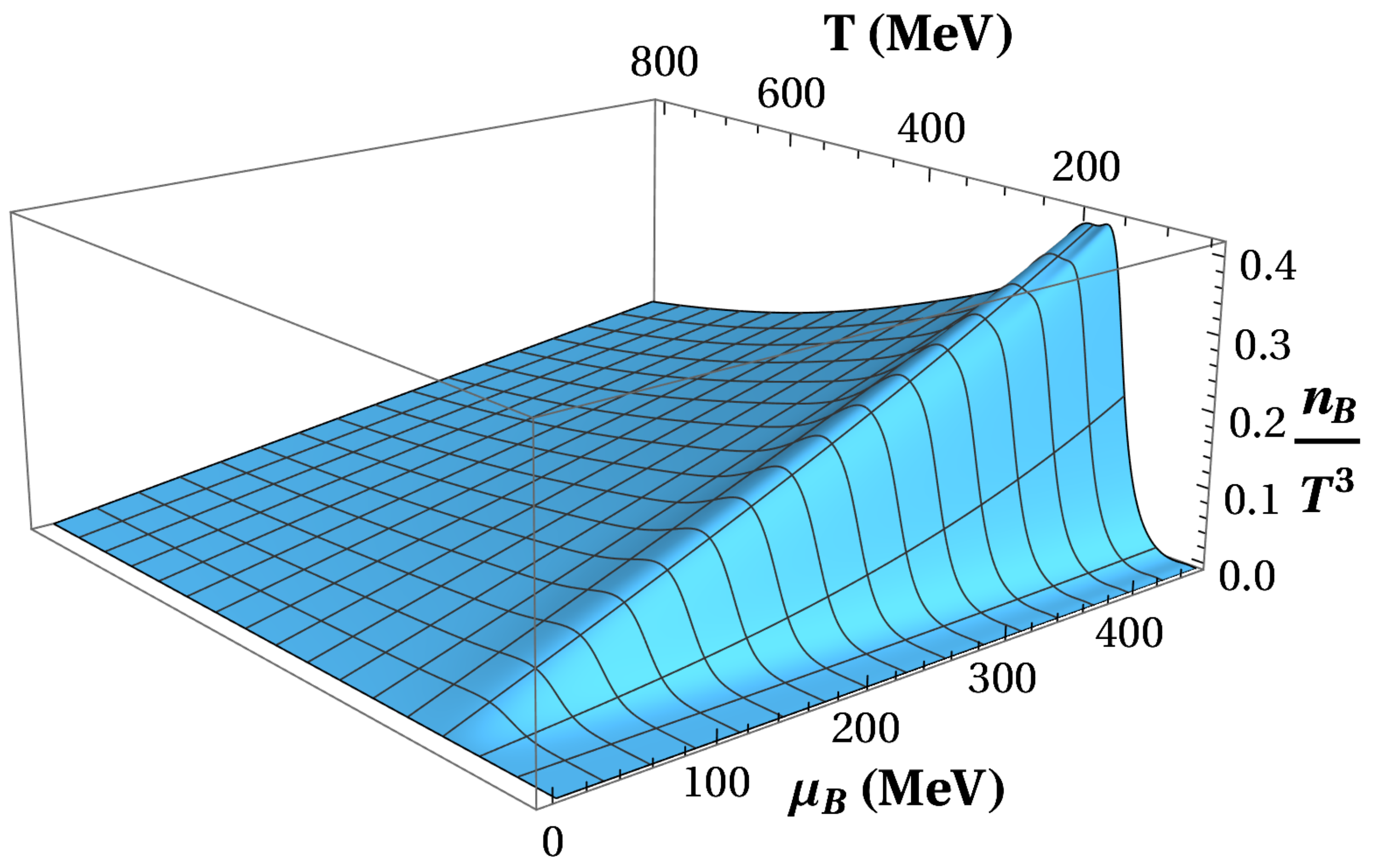


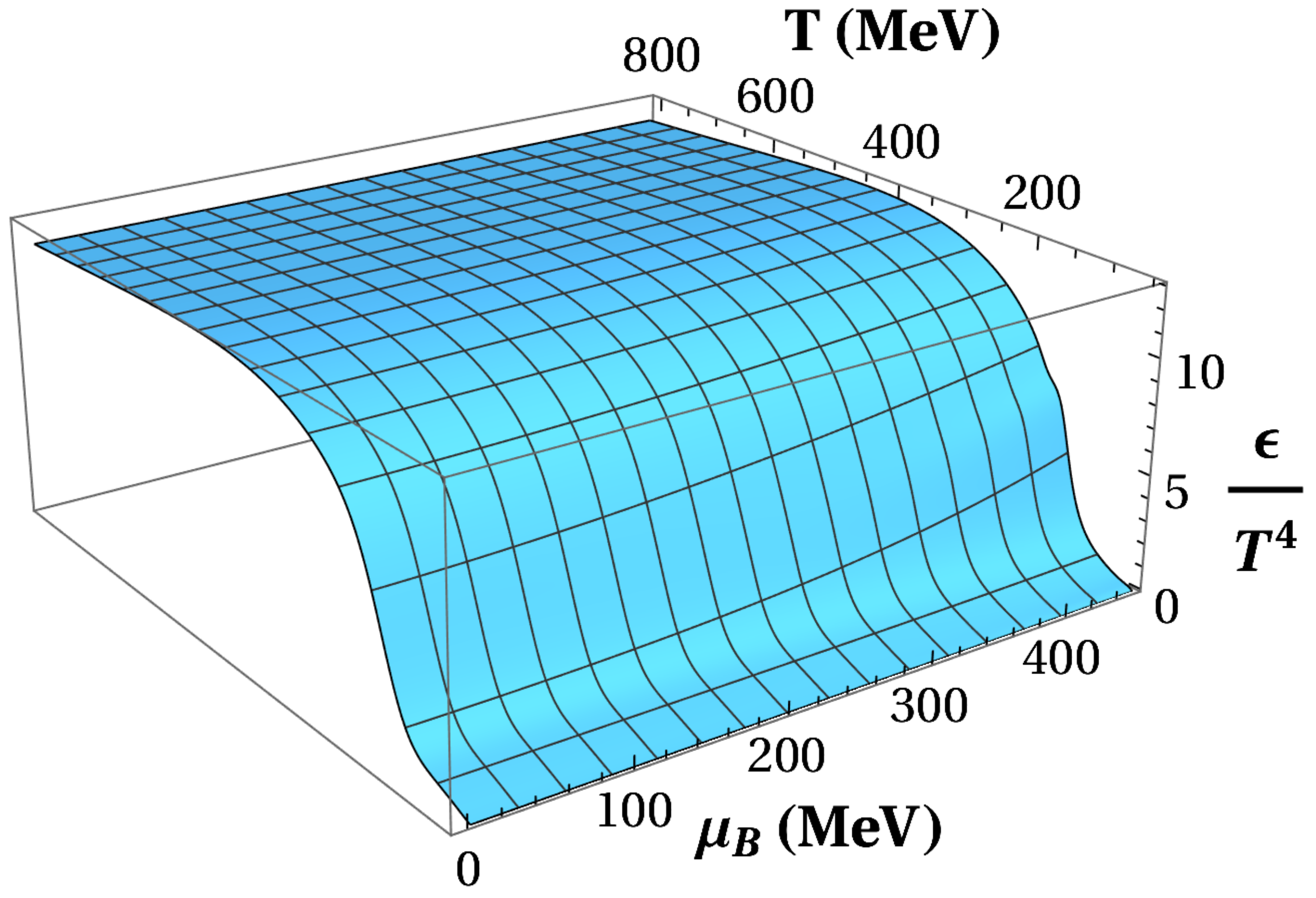
Reconstructed Taylor EoS

- Reconstruct the QCD equation of state from all diagonal and off-diagonal susceptibilities up to $\mathcal{O}(\mu_B^4)$

$$\frac{P(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!}\chi_{ijk}^{BQS}(T)\left(\frac{\mu_B}{T}\right)^j\left(\frac{\mu_Q}{T}\right)^k\left(\frac{\mu_S}{T}\right)^i$$





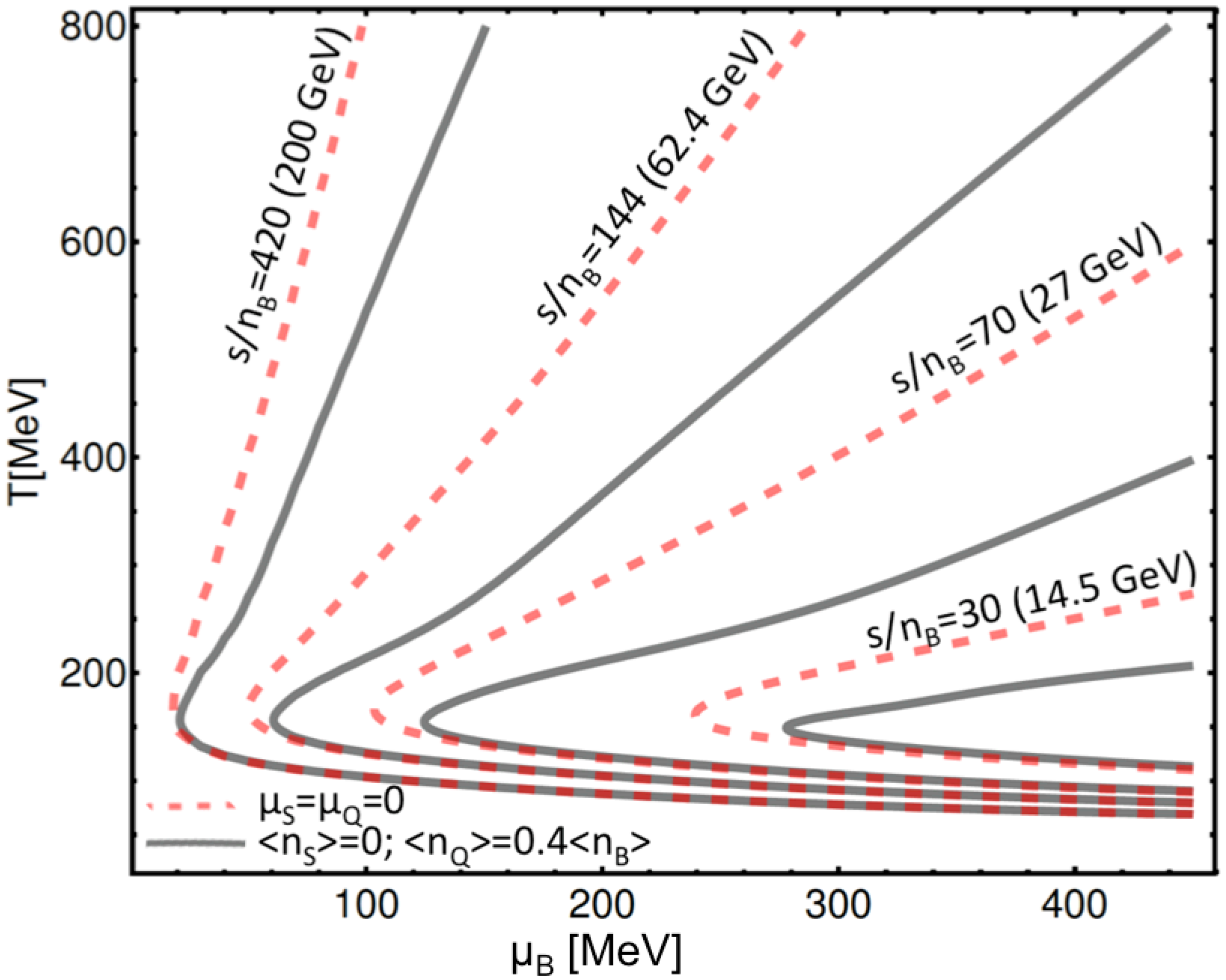


See also: A. Monnai et al, PRC (2019)

J. Noronha-Hostler, JS et al, PRC (2019)

Isentropic Trajectories

- Different paths through the phase diagram taken based on conserved charge conditions: isentropic trajectories stress importance of BQS modeling for heavy-ion phenomenology



II. Strangeness-neutral equation of state with a critical point

Equation of State with Criticality

- Update to the original EoS that first matched the Taylor expansion coefficients from Lattice QCD and implemented critical features based on universality arguments

PHYSICAL REVIEW C **101**, 034901 (2020)

QCD equation of state matched to lattice data and exhibiting a critical point singularity

Paolo Parotto,[1,2,*] Marcus Bluhm,[3,4] Debora Mroczek,[1] Marlene Nahrgang,[4] J. Noronha-Hostler,[5] Krishna Rajagopal,[6] Claudia Ratti,[1] Thomas Schäfer,[7] and Mikhail Stephanov[8]
[1]*Department of Physics, University of Houston, Houston, Texas 77204, USA*
[2]*Department of Physics, University of Wuppertal, Wuppertal D-42219, Germany*
[3]*Institute of Theoretical Physics, University of Wroclaw, 50204 Wroclaw, Poland*
[4]*SUBATECH UMR 6457 (IMT Atlantique, Université de Nantes, IN2P3/CNRS), 4 rue Alfred Kastler, 44307 Nantes, France*
[5]*Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*
[6]*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[7]*Department of Physics, North Carolina State University, Raleigh, North Carolina 27695, USA*
[8]*Physics Department, University of Illinois at Chicago, Chicago, Illinois 60607, USA*

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We construct a family of equations of state for QCD in the temperature range 30 MeV $\leqslant T \leqslant$ 800 MeV and in the chemical potential range $0 \leqslant \mu_B \leqslant$ 450 MeV. These equations of state match available lattice QCD results up to $O(\mu_B^4)$ and in each of them we place a critical point in the three-dimensional (3D) Ising model universality class. The position of this critical point can be chosen in the range of chemical potentials covered by the second Beam Energy Scan at the Relativistic Heavy Ion Collider. We discuss possible choices for the free parameters, which arise from mapping the Ising model onto QCD. Our results for the pressure, entropy density, baryon density, energy density, and speed of sound can be used as inputs in the hydrodynamical simulations of the fireball created in heavy ion collisions. We also show our result for the second cumulant of the baryon number in thermal equilibrium, displaying its divergence at the critical point. In the future, comparisons between RHIC data and the output of the hydrodynamic simulations, including calculations of fluctuation observables, built upon the model equations of state that we have constructed may be used to locate the critical point in the QCD phase diagram, if there is one to be found.

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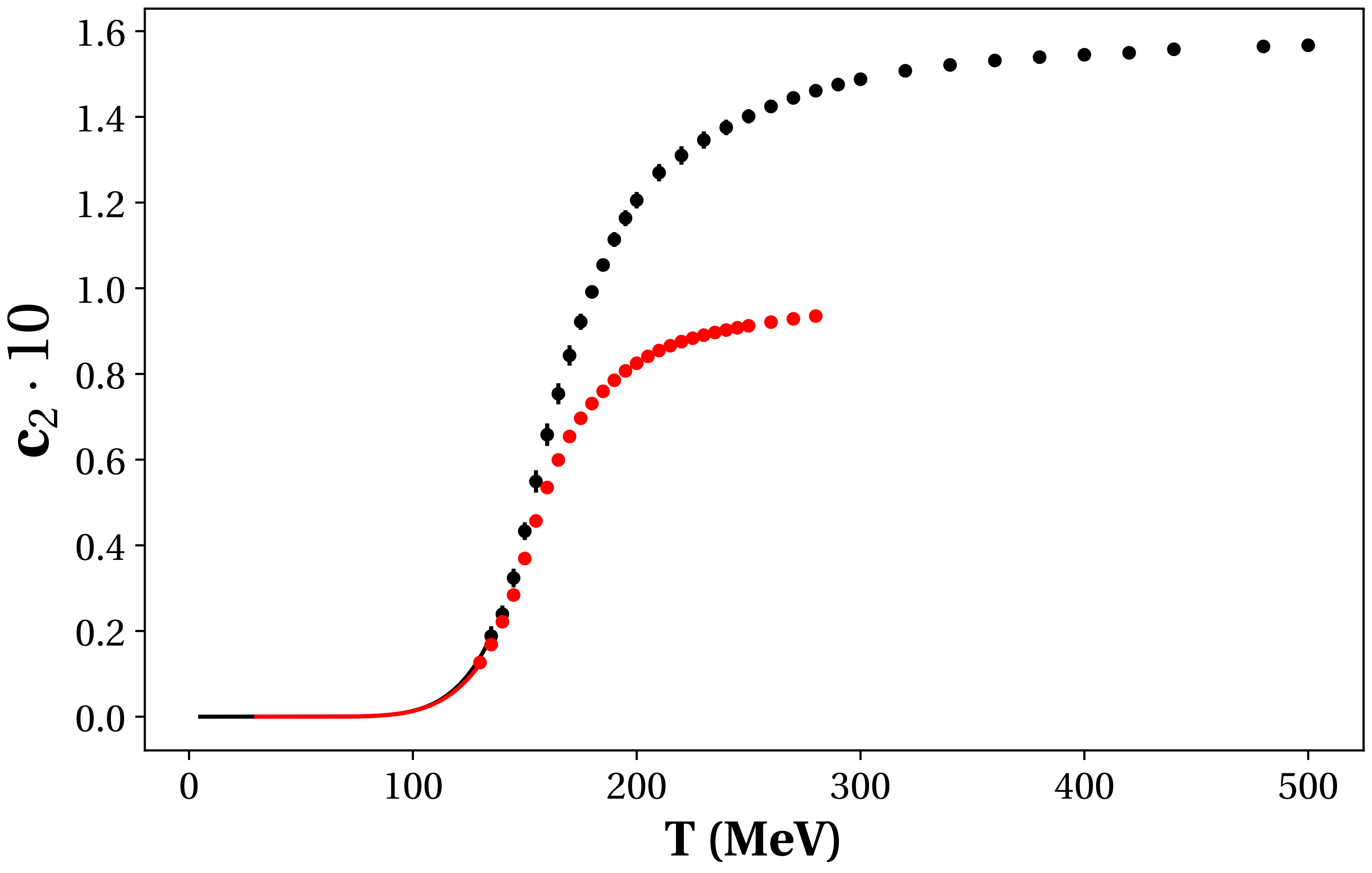
BEST COLLABORATION

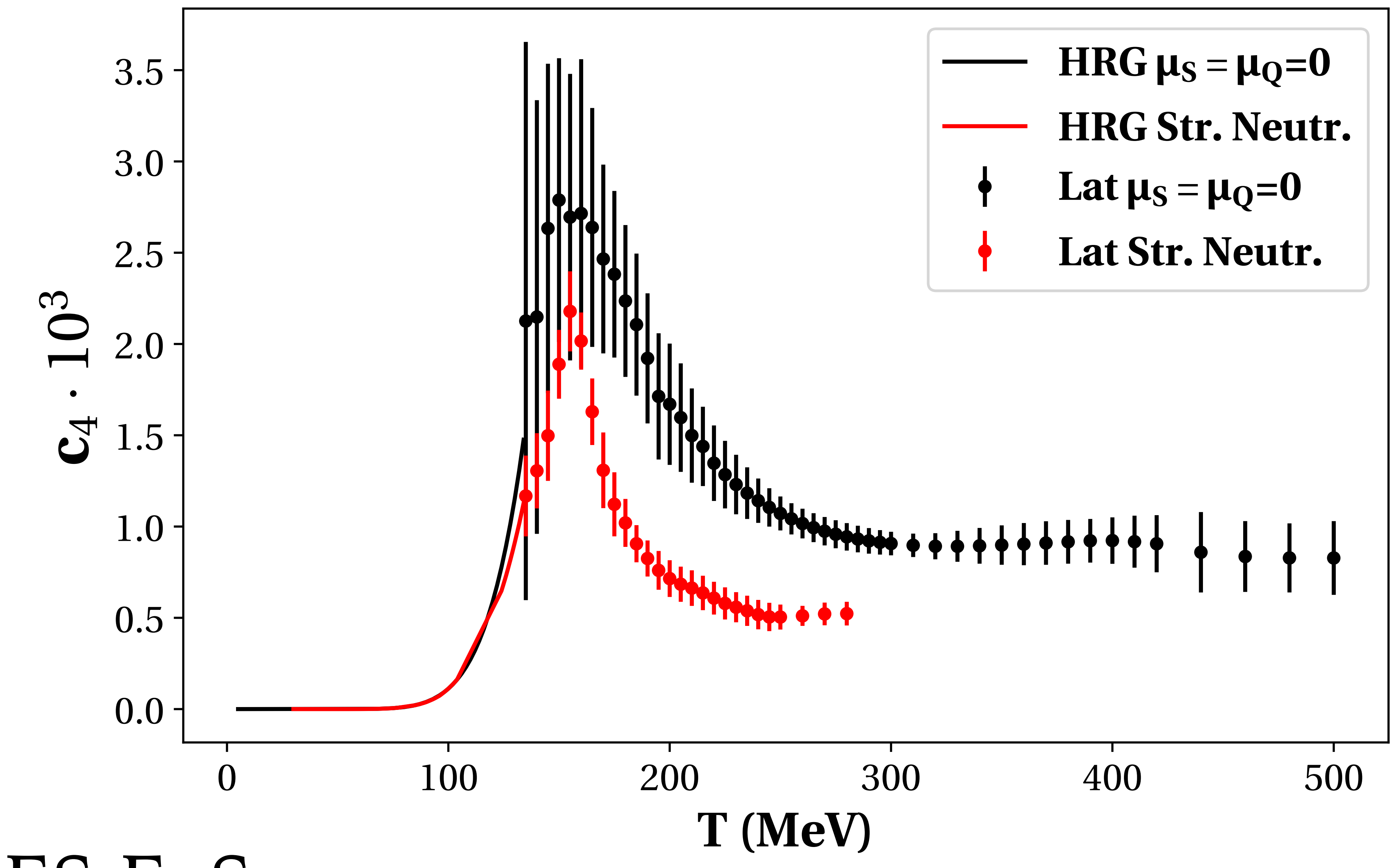
Code can be downloaded at:
https://bitbucket.org/bestcollaboration/eos_with_critical_point/src/master/

Taylor Coefficients from LQCD

- Lattice results for Taylor expansion of pressure around $\mu_B = 0$ up to $\mathcal{O}(\mu_B^4)$ are the backbone of the procedure for creating this equation of state

$$\frac{P(T,\mu_B)}{T^4} = \sum_n c_{2n}(T)\left(\frac{\mu_B}{T}\right)^{2n}$$





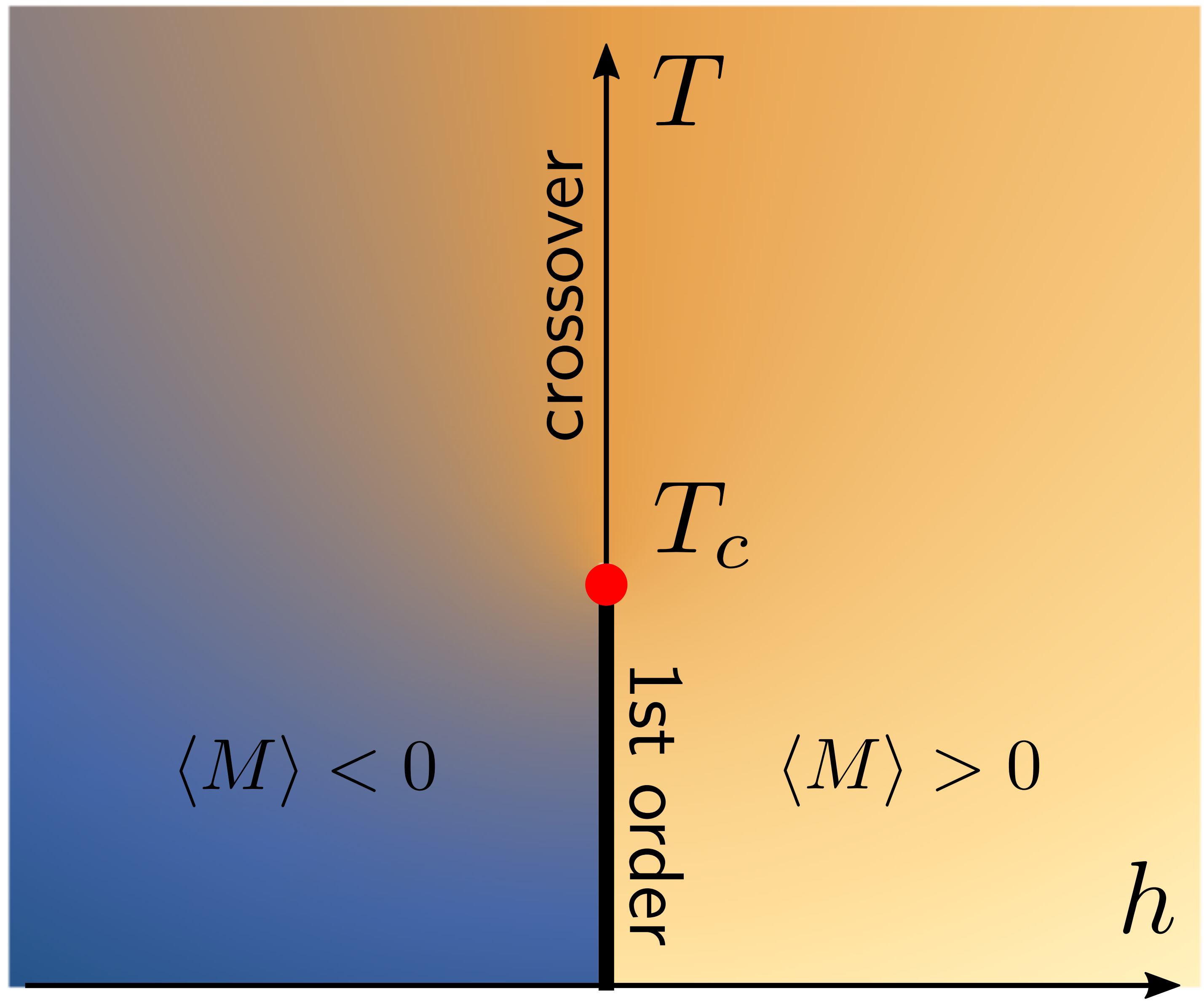
- original BES-EoS
- strangeness-neutral version

J. Guenther et al, NPA (2017)
R. Bellweid et al, PRD (2015)
See also: A. Bazavov et al, PRD (2017)

Universal Scaling EoS

- Criticality is implemented by mapping the critical point from the 3D Ising model onto the QCD phase diagram

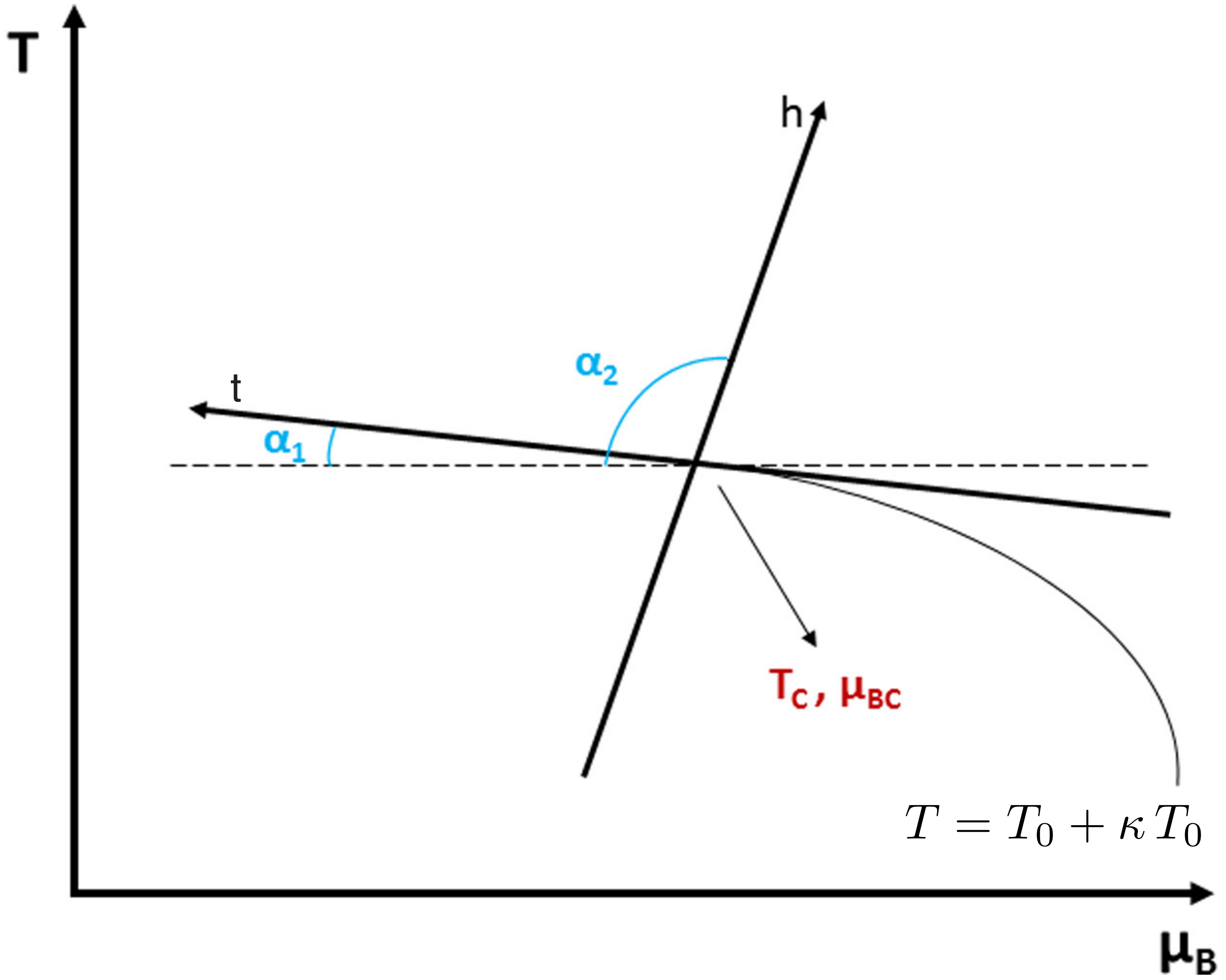
- Relevant and analogous quantities for Ising-QCD map:
  - Magnetic field, h ⟷ Baryon chemical potential, μ
  - Magnetization, M ⟷ Baryon density, n_B
  - Reduced temperature: $t = \dfrac{T - T_C}{T_C}$
  - Gibbs' free energy/thermodynamic potential = −Pressure



K. Rajagopal and F. Wilczek, Nucl. Phys. B (1993)
P. Parotto et al, PRC (2020)
A. Bzdak et al, Phys. Rep. (2020)
C. Nonaka, M. Asakawa, PRC (2005)

Mapping the 3D Ising Model onto QCD

- Phase transition along Ising temperature axis fixed onto QCD phase diagram along transition line from LQCD



$$T = T_0 + \kappa T_0 \left(\frac{\mu_B}{T_0}\right)^2 + \mathcal{O}(\mu_B^4)$$

$$\frac{T - T_C}{T_C} = w\ (t\ \rho\ sin\alpha_1 + h\ sin\alpha_2)$$

$$\frac{\mu_B - \mu_{B,C}}{T_C} = -\ w\ (t\ \rho\ cos\alpha_1 + h\ cos\alpha_2)$$

P. Parotto et al, PRC (2020)

3D Ising Model Parametrization

- Universal scaling behavior encoded in parameters (R, θ):
  - Magnetic field: $h = h_0 R^{\beta\delta} H(\theta)$
  - Reduced temperature: $t = R(1 - \theta^2)$
  - Magnetization: $M = M_0 R^{\beta} \theta$
  - Gibbs' free energy: $G = h_0 M_0 R^{2-\alpha}[g(\theta) - \theta H(\theta)]$

  where $\alpha = 0.11$, $\beta = 0.326$, $\delta = 4.8$ are 3D Ising critical exponents, H(θ) is a polynomial in odd powers of θ, and g(θ) is a polynomial in (1-θ^2).

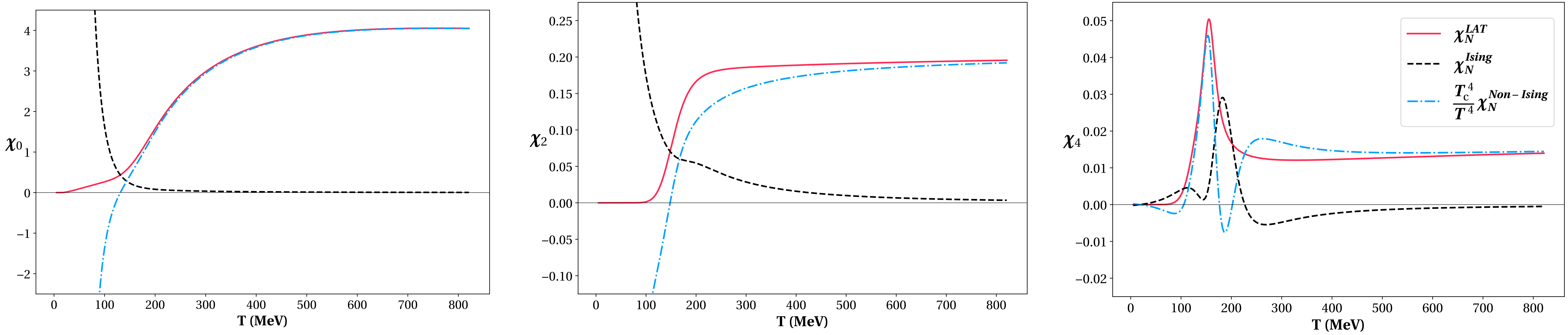
- Generally, free energy includes singular and non-singular contributions:
  - $P(T, \mu_B) = -G[R, \theta] + P_{bkg}(T, \mu_B)$

P. Parotto et al, PRC (2020)
A. Bzdak et al, Phys. Rep. (2020)
C. Nonaka, M. Asakawa, PRC (2005)
J. Zinn-Justin Quantum Field theory and Critical Phenomena

Singular and Non-singular Contributions

- Our construction requires that the total free energy (pressure) is the one from the lattice, so order-by-order we have:

$$\chi_N^{Lat}(T) = \chi_N^{Ising}(T) + \chi_N^{Non-Ising}(T)$$

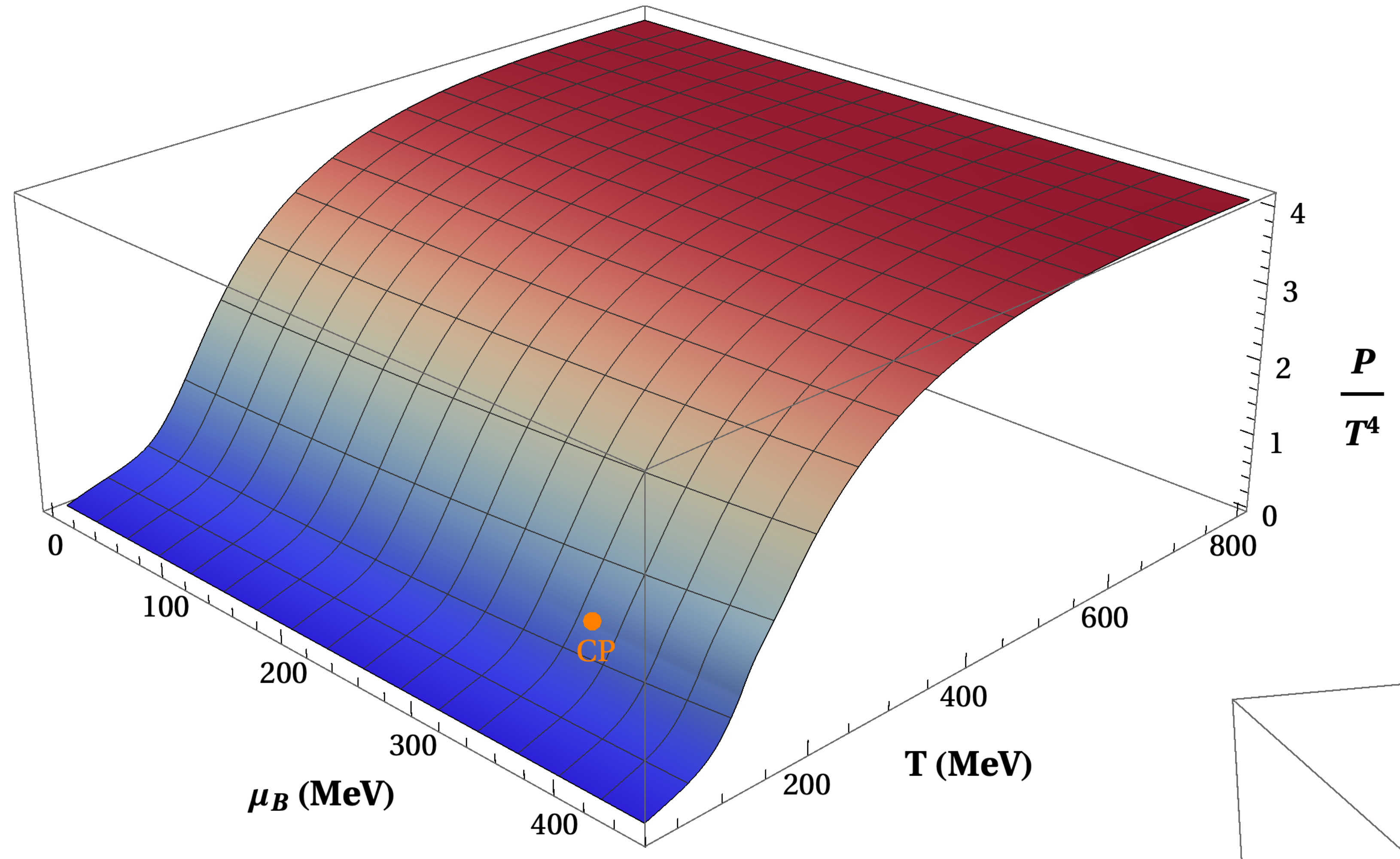


$$P(T,\mu_B) = T^4 \sum_{n=0}^{2} c_{2n}^{\text{non-Ising}}(T)\left(\frac{\mu_B}{T}\right)^{2n} + T_C^4\, P_{\text{symm}}^{\text{Ising}}(T,\mu_B)$$

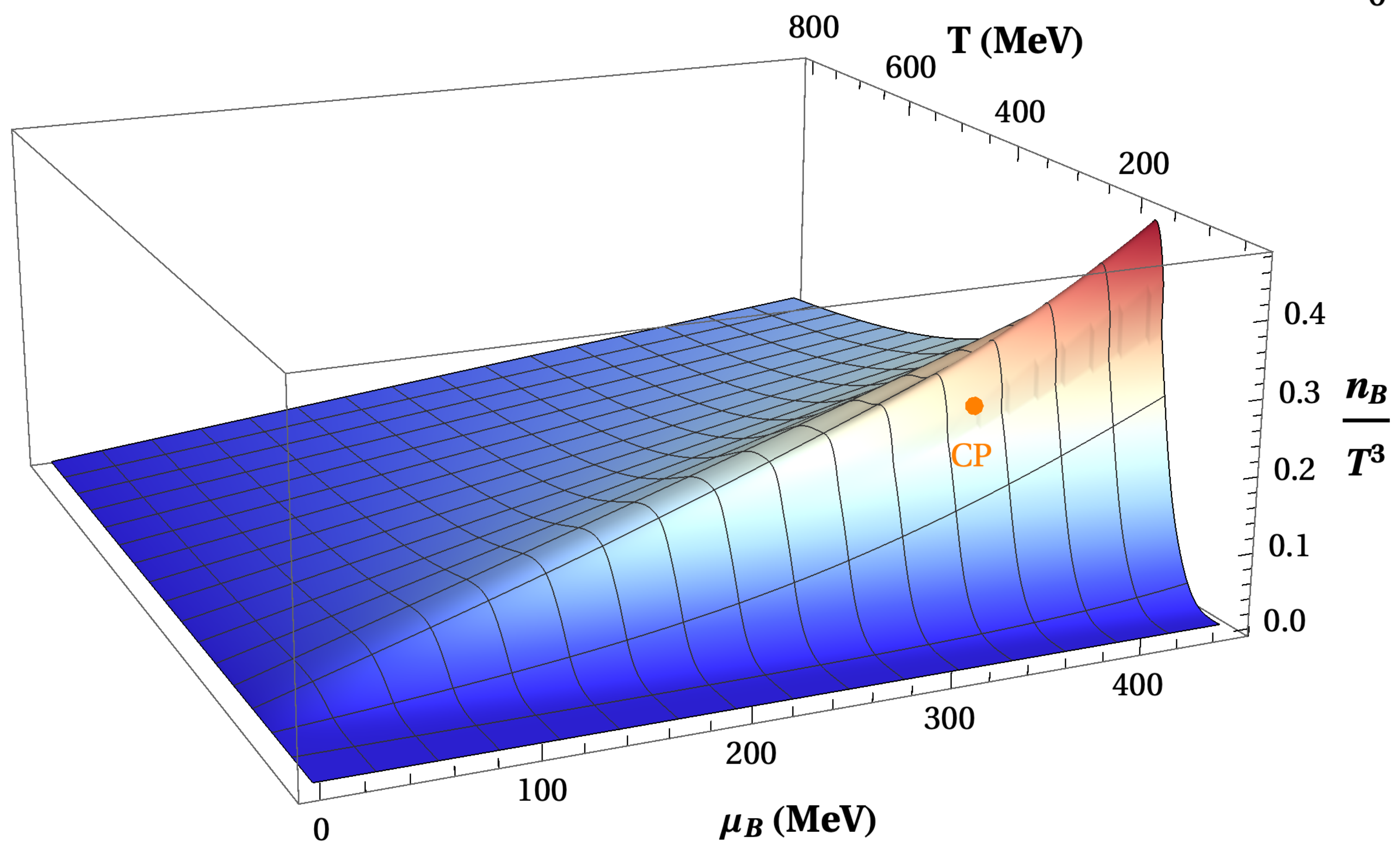
JMK et al, EPJ Plus (2021)

EoS Thermodynamic Outputs

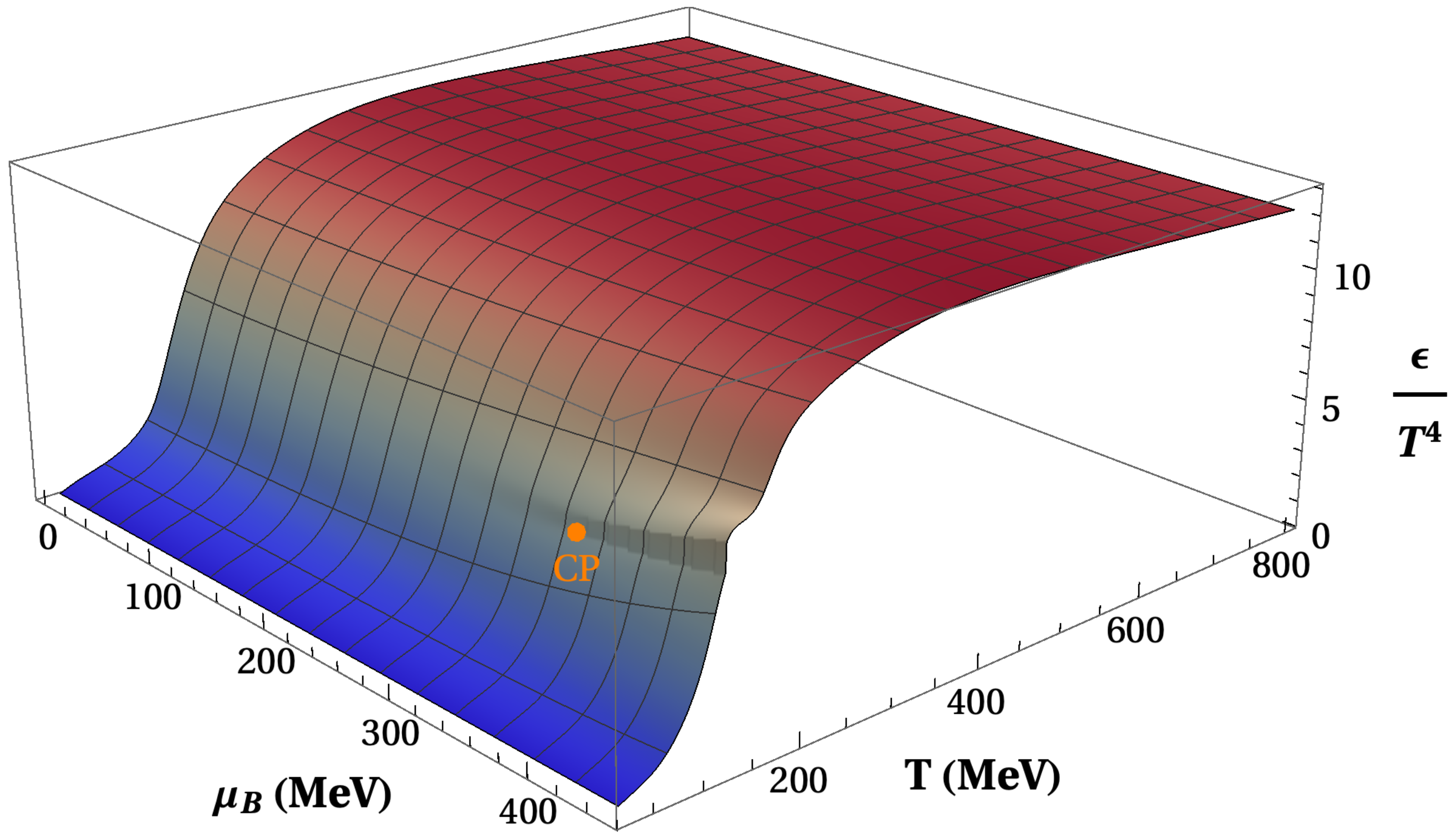
- Pressure and its derivatives show effects of critical region on these quantities: stronger effects with increasing derivatives



Pressure



Baryon density



Energy density

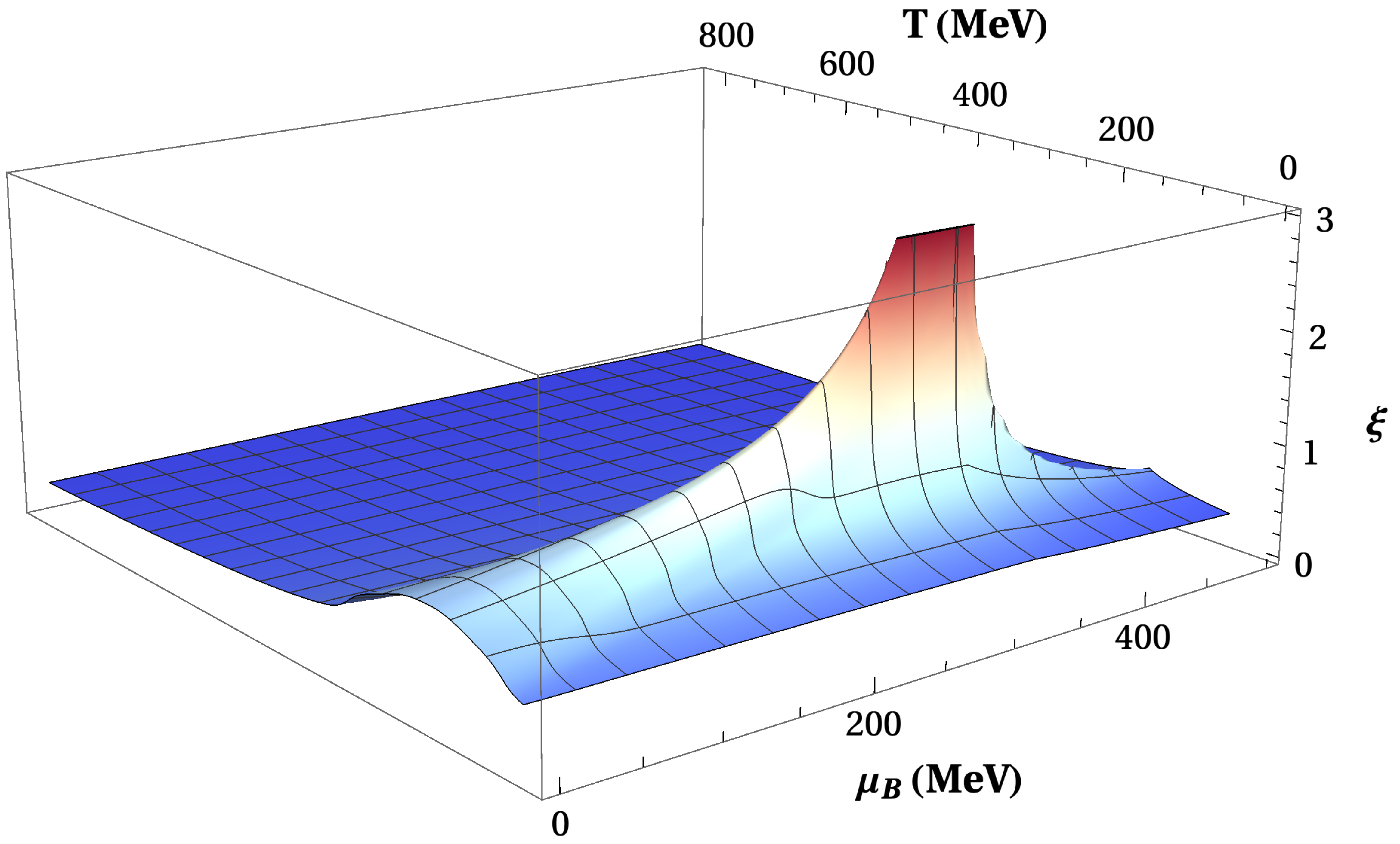
JMK et al, EPJ Plus (2021)

Correlation Length

➤ Additionally, calculate the correlation length in the 3D Ising model:

$$\xi^2(t, M) = f^2 |M|^{-2\nu/\beta} g(x)$$

where f =1fm, ν = 0.63 is the correlation length critical exponent, g(x) is the scaling function and the scaling parameter is $x = \frac{|t|}{|M|^{1/\beta}}$

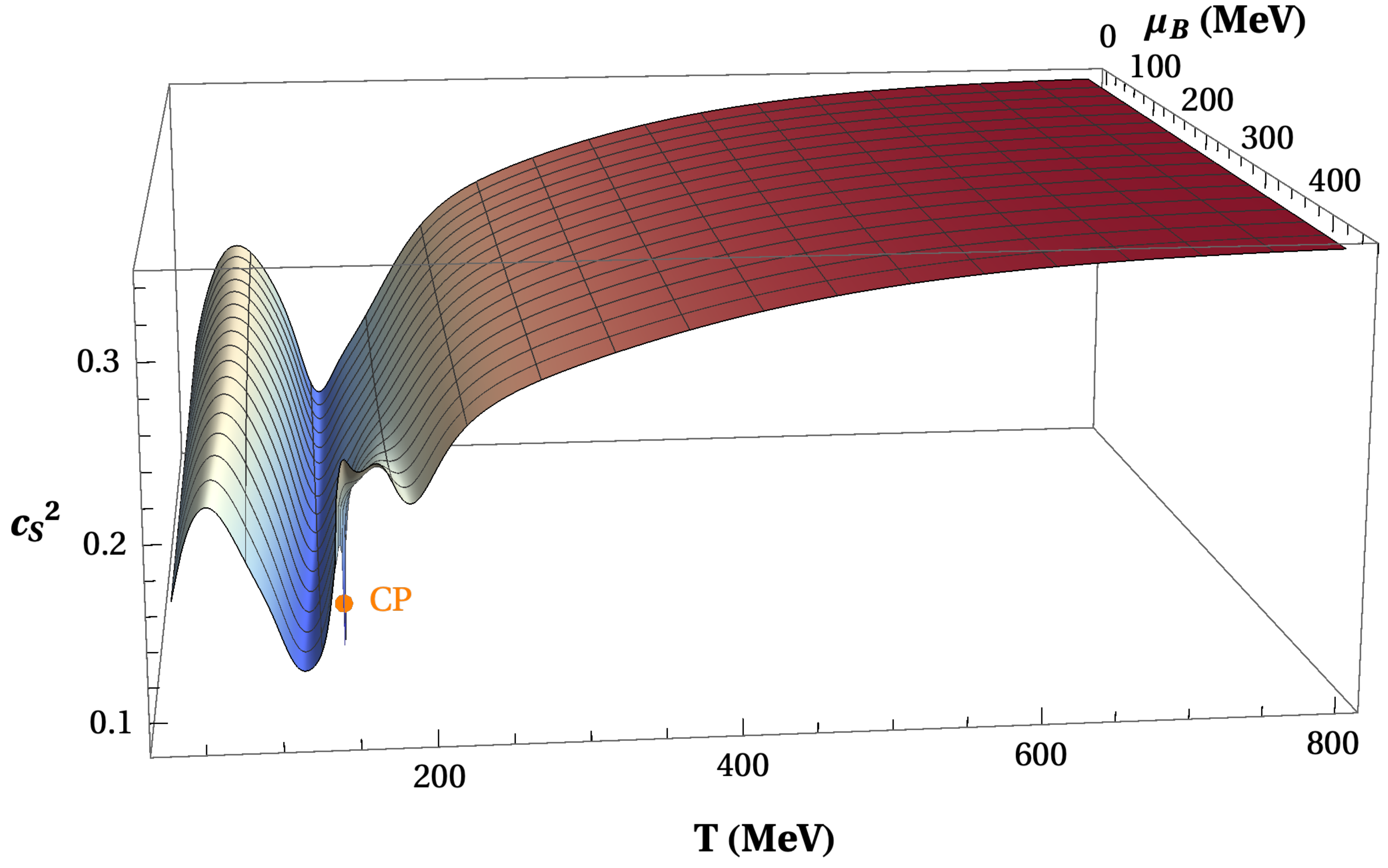


Correlation length

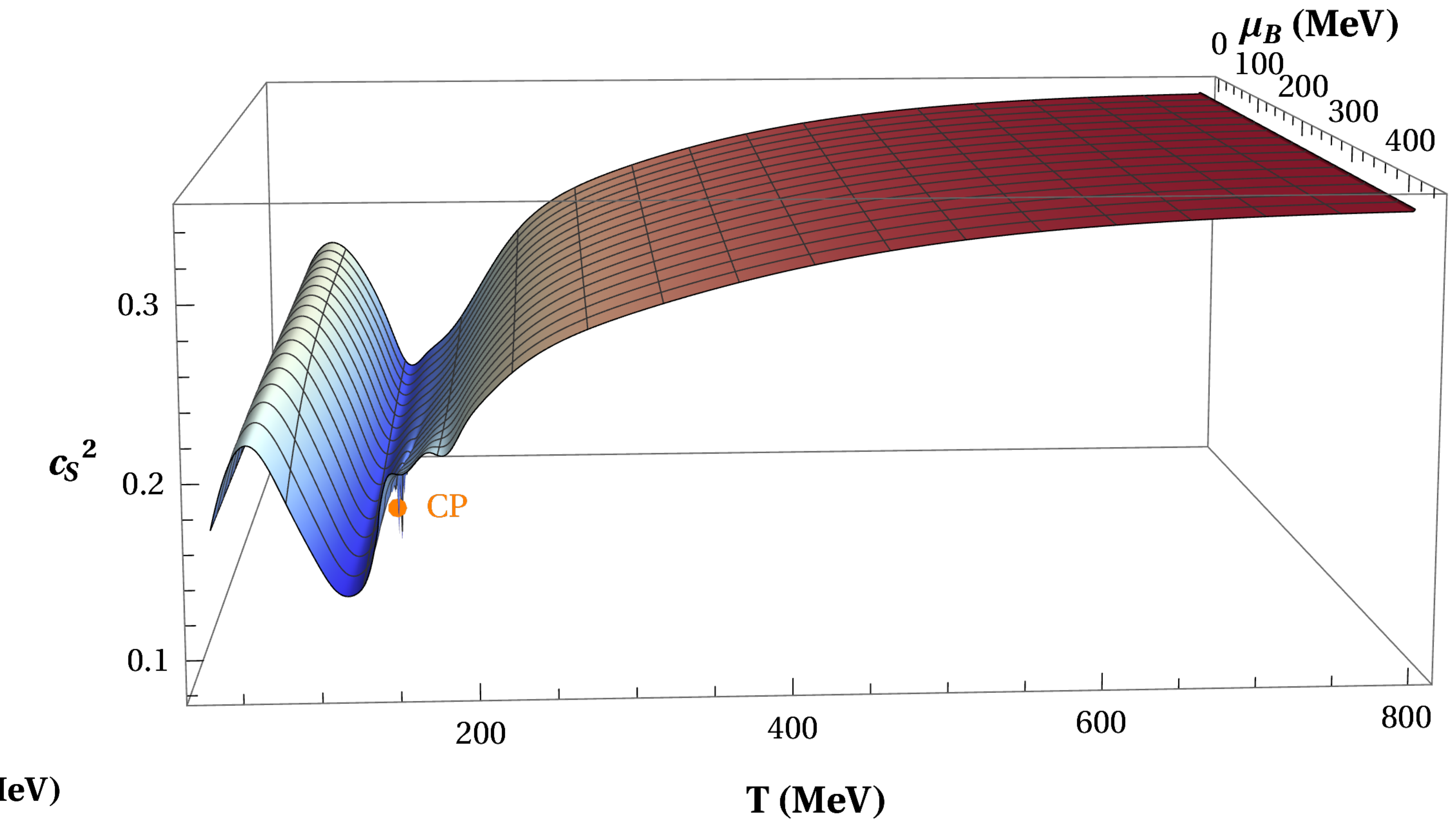
B. Berdnikov and K. Rajagopal, PRD (2000)
C. Nonaka and M. Asakawa, PRC (2005)
JMK et al, EPJ Plus (2021)

Size of Critical Region - Speed of Sound

➤ By changing the parameters of the mapping we can control the critical contribution to the overall thermodynamics

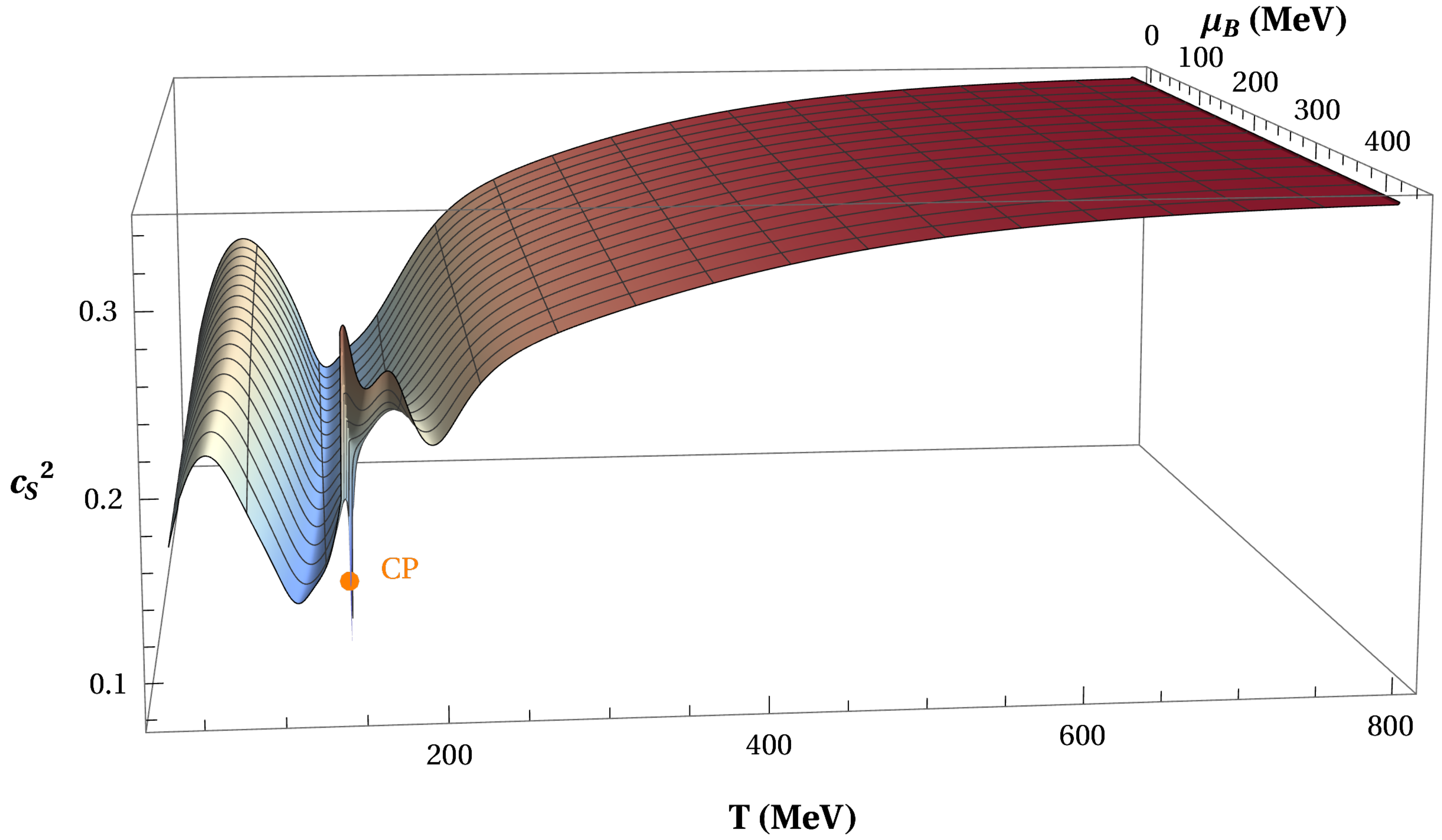


$\omega = 2$
(same as published version)



$\omega = 4$

$\omega = 0.75$

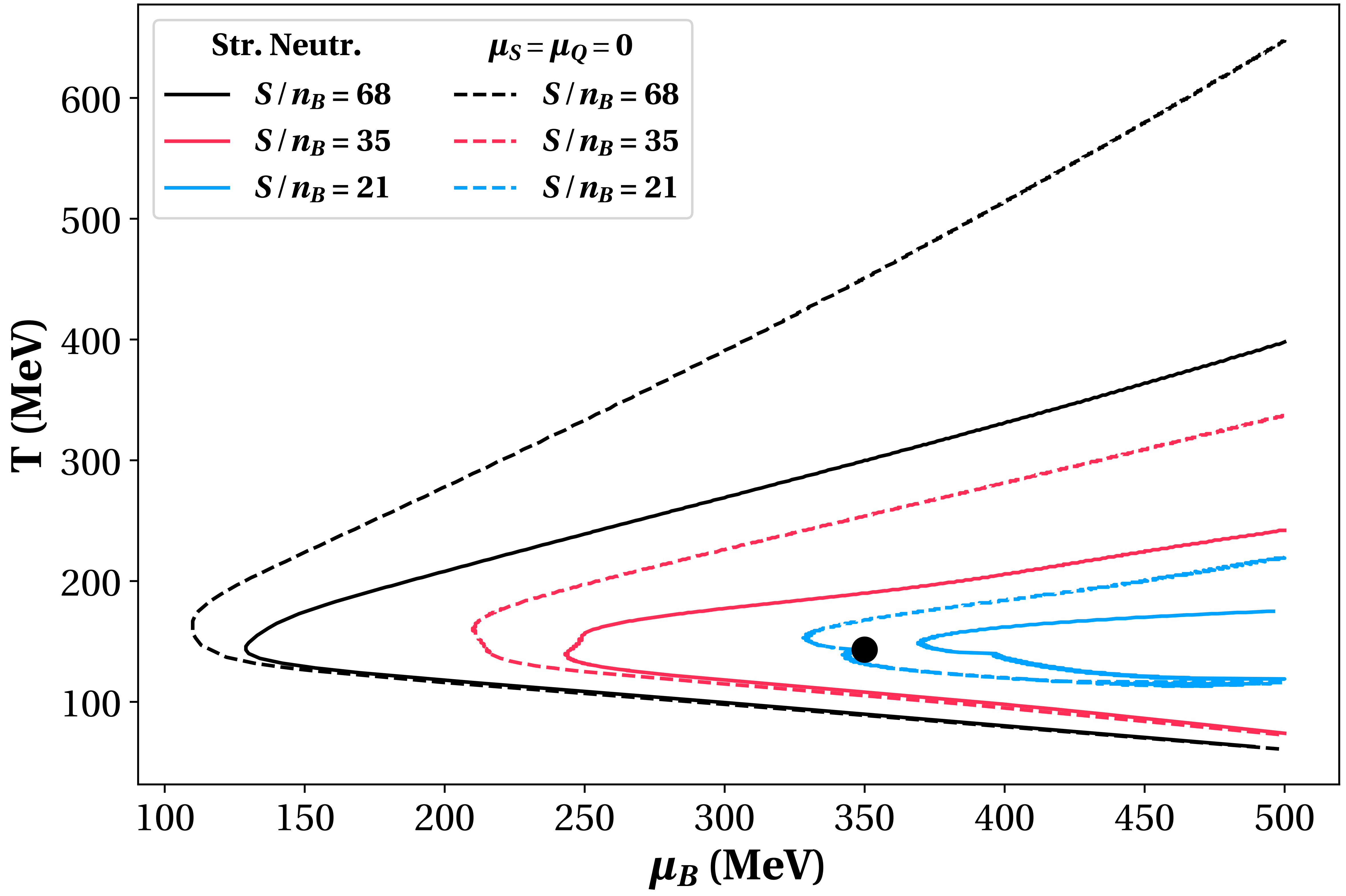


See also:
M. Pradeep and M. Stephanov, PRD (2019)
P. Parotto et al, PRC (2020)
D. Mroczek et al, PRC (2021)
Wei-jie Fu et al, PRD (2021)

JMK et al, EPJ Plus (2021)

Isentropic Trajectories

- Isentropes show the path of the HIC system through the phase diagram in the absence of dissipation
  - Different path when conserved charge conditions applied



JMK et al, EPJ Plus (2021)

Conclusions

- Realistic modeling of strongly-interacting matter for heavy-ion-collision systems should involve constraints on the conserved charges.
- We updated the BES-EoS to include strangeness neutrality conditions, which performs in a range of temperature and baryonic chemical potential relevant for BES-II.
- We see the expected critical features in the EoS and note a shift in the isentropic trajectories between the new and original versions.
- A calculation of the correlation length in the 3D Ising model has been performed.

Back-up Slides

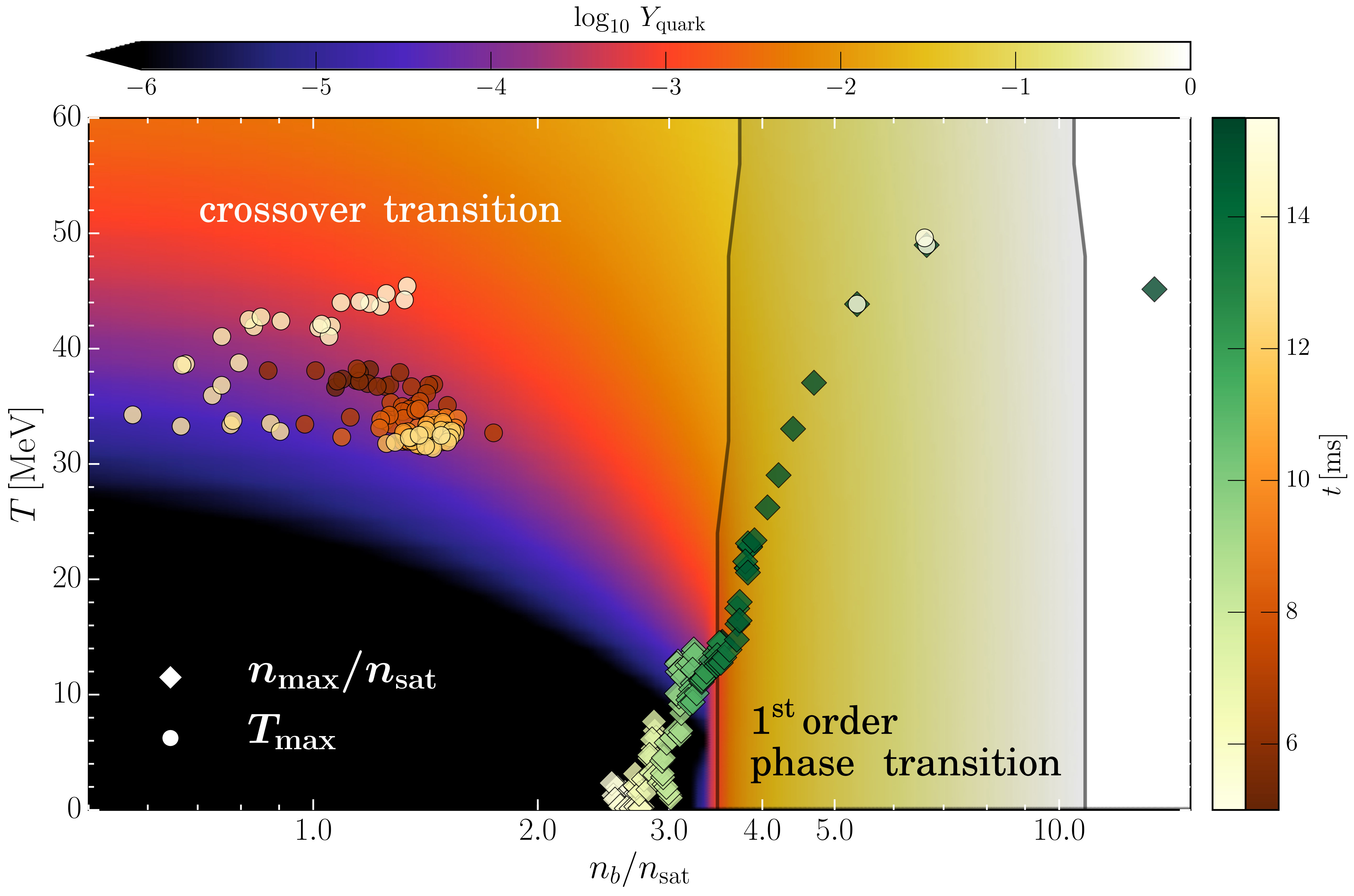
Strangeness neutrality on the lattice

$$\tilde{\chi}_n^{B,2} = (\chi_{n+2,00}^{BQS} + s_1^2\chi_{n02}^{BQS} + q_1^2\chi_{n20}^{BQS} + 2s_1\chi_{n+1,01}^{BQS} + 2q_1\chi_{n+1,10}^{BQS} + 2q_1 s_1\chi_{n11}^{BQS})/2$$

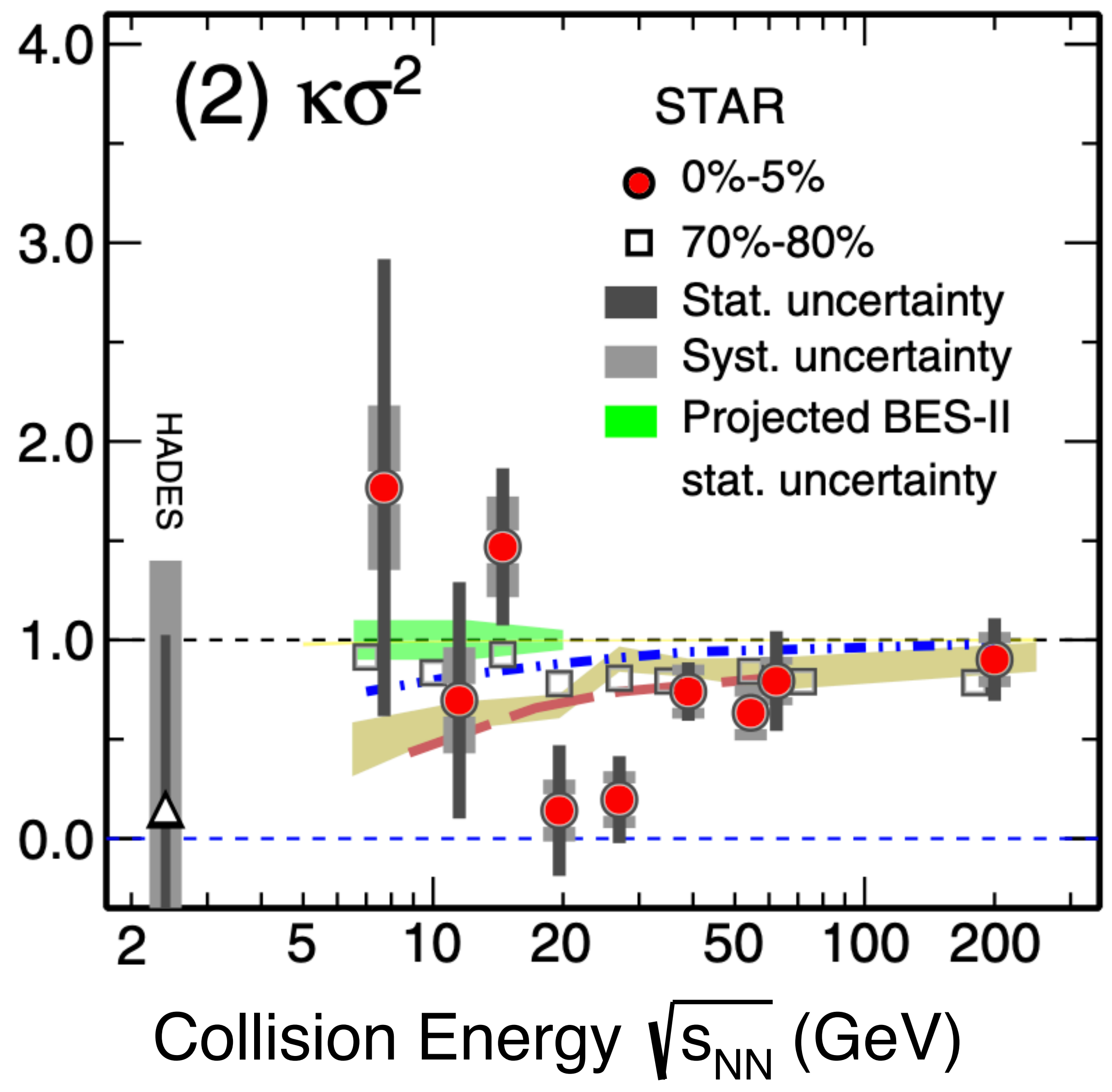
$$\begin{aligned}\tilde{\chi}_n^{B,4} = (&24s_1s_3\chi_{n02}^{BQS} + s_1^4\chi_{n04}^{BQS} + 24q_3s_1\chi_{n11}^{BQS} \\ &+ 24q_1s_3\chi_{n11}^{BQS} + 4q_1s_1^3\chi_{n13}^{BQS} + 24q_1q_3\chi_{n20}^{BQS} \\ &+ 6q_1^2s_1^2\chi_{n22}^{BQS} + 4q_1^3s_1\chi_{n31}^{BQS} + q_1^4\chi_{n40}^{BQS} \\ &+ 24s_3\chi_{n+1,01}^{BQS} + 4s_1^3\chi_{n+1,03}^{BQS} + 24q_3\chi_{n+1,10}^{BQS} \\ &+ 12q_1s_1^2\chi_{n+1,12}^{BQS} + 12q_1^2s_1\chi_{n+1,21}^{BQS} + 4q_1^3\chi_{n+1,30}^{BQS} \\ &+ 6s_1^2\chi_{n+2,02}^{BQS} + 12q_1s_1\chi_{n+2,11}^{BQS} + 6q_1^2\chi_{n+2,20}^{BQS} \\ &+ 4s_1\chi_{n+3,01}^{BQS} + 4q_1\chi_{n+3,10}^{BQS} + \chi_{n+4,00}^{BQS})/24\end{aligned}$$

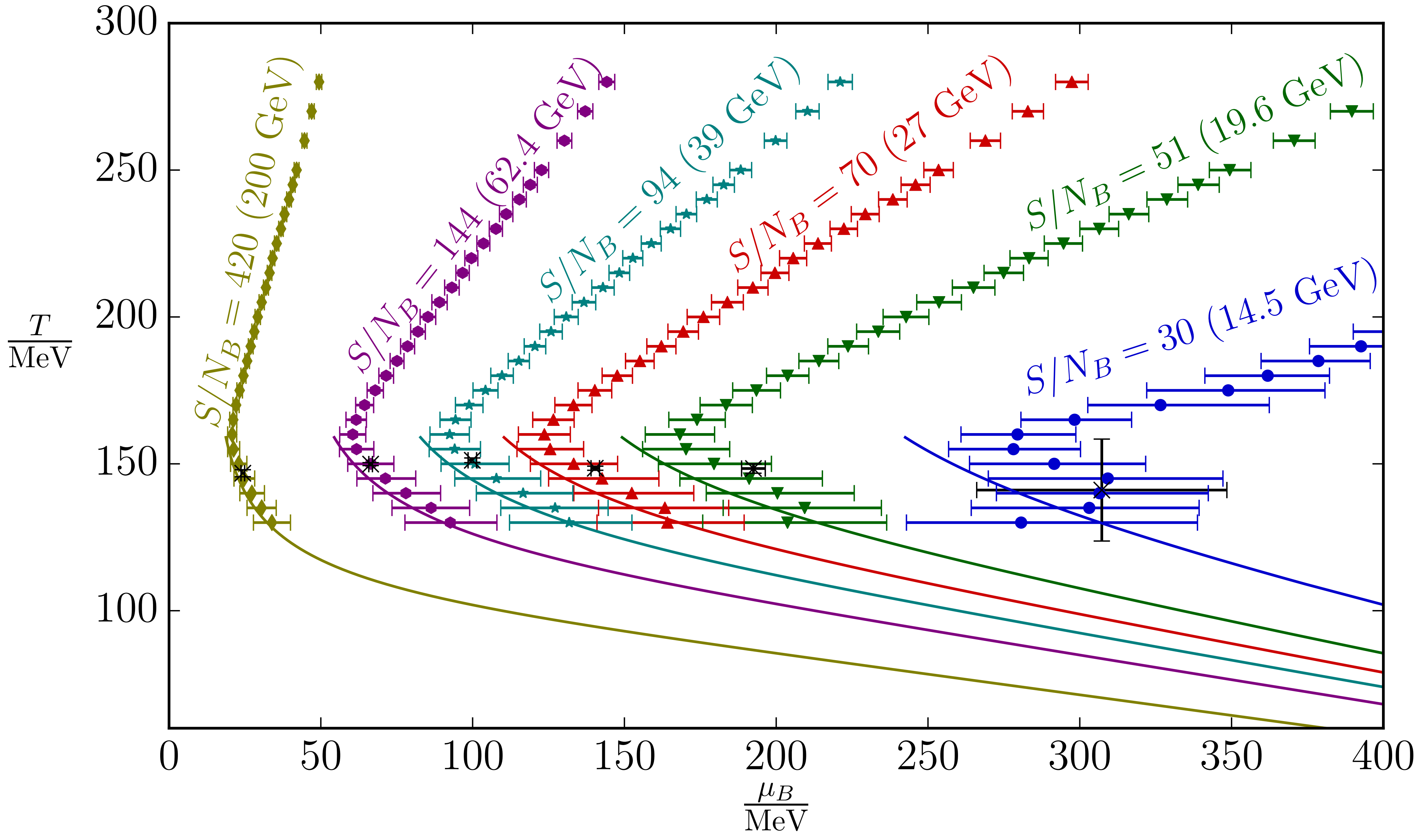
A. Bazavov et al PRD (2020)

Phase diagram in terms of number density



Relevant regions for potential criticality

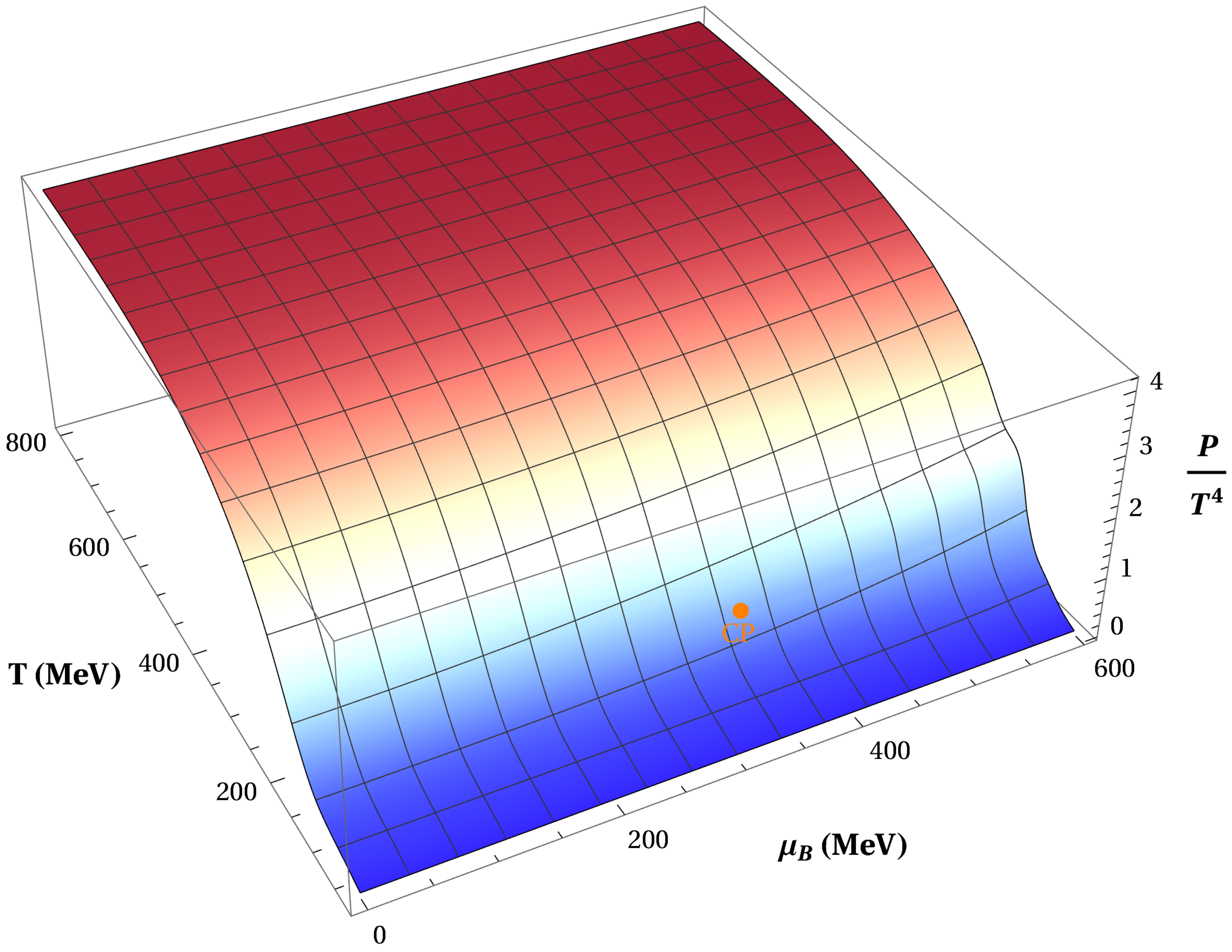




STAR collaboration PRL (2021)
R. Bellwied et al, PLB (2015)

Extension to higher μ_B and speed of sound calculation

- Computational grid of the program: $30 \leq T \leq 821$, $0 \leq \mu_B \leq 600$



$$c_s^2 = \frac{n_B^2 \partial_T^2 P - 2 S n_B \partial_T \partial_{\mu_B} P + S^2 \partial_{\mu_B}^2 P}{(\epsilon + P)\left[\partial_T^2 P \partial_{\mu_B}^2 P - (\partial_T \partial_{\mu_B} P)^2\right]}$$

Correlation length in Ising model variables