# *Finite volume and magnetic effects on pion correlation function in HICs*

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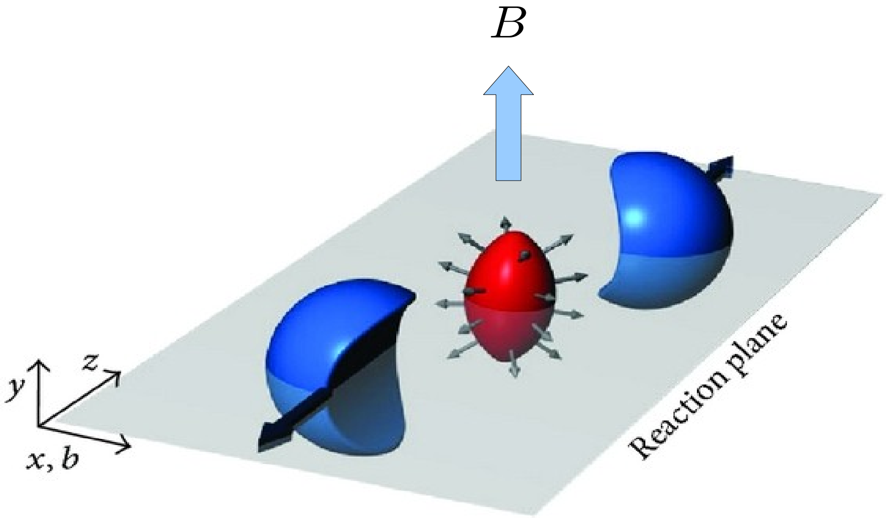
WWND, March 2022

A. Ayala, S. Bernal-Langarica, C.V, Phys. Rev. D105, 056001 (2022)

Suported by FONDECYT 1190192

## Motivatrion and goal

- Magnetic field effects on two-pion correlation
- Finite volume pion condensation effects in two-pion correlation
- Anisotropy effects (cylinder)




$$\mathrm{C}_2(P, q; T, eB, N, V)$$

## Outline

- Non-equilibrium chemical potential
- Finite size pion condensation effects
- Pion wave function in homogeneous magnetic field
- 2-pion correlator with fixed pion multiplicity
- Results
- Colnclusion and outlook

## Non-equilibrium statistics with pion condensation and finite volume effects

- Out of equilibrium effective chemical potential

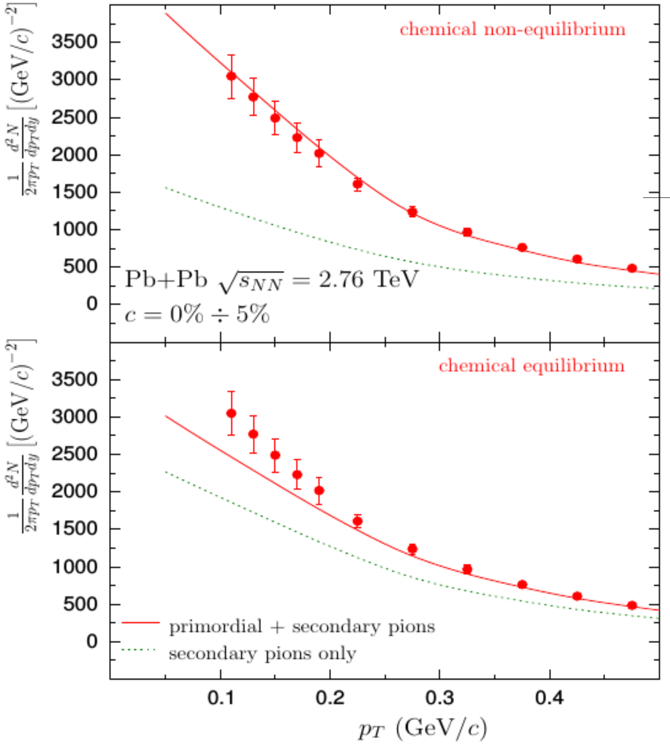
  → Approximately contant number of pions

- Finite volume increases pion ground state contribution (pion condensation)

Non-equilibrium statistical hadronization model → non-eq. chemical potential

Letessier,Rafelski,PRC59(1999)947

$$\mu_\pi = \mu_q + \mu_{\bar{q}}, \quad \mu_p = 3\mu_q, \quad \ldots$$




transverse momentum spectra of pions in the low-pT region

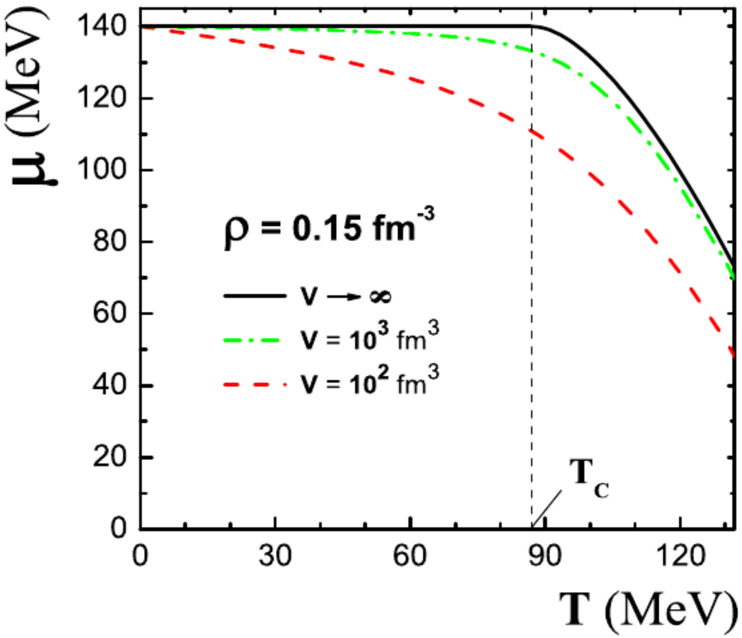
Begun,Florkowski,Rybczynski,
PRC90(2014)014906

$\mu_\pi \lesssim m_{\pi^0}$ at freezout temperatures

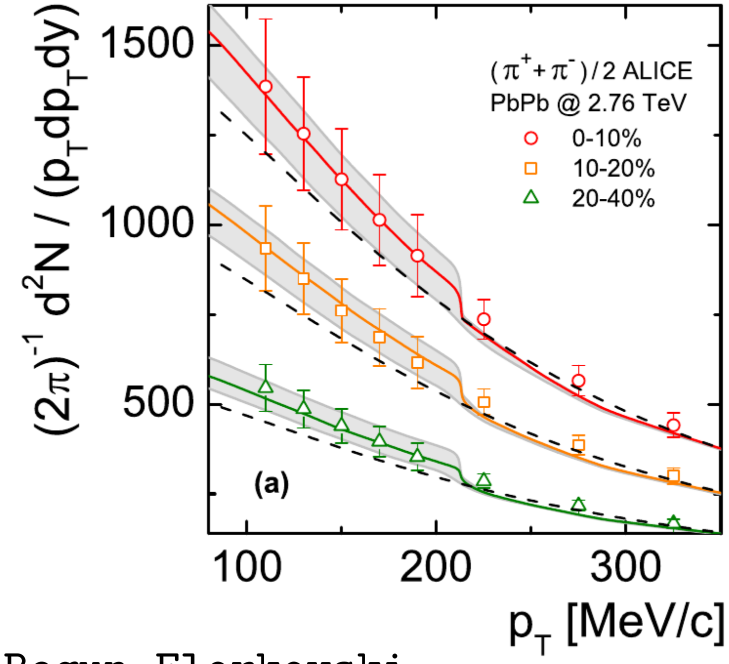
→ near condensation critical chemical potential

BEC in a box

$$\rho = \frac{N}{V} = \underbrace{\frac{1}{V}\frac{g}{e^{(E_0-\mu)/T}-1}}_{\text{condensed}} + \underbrace{\frac{1}{V}\sum_{\boldsymbol{n}\neq 0}\frac{g}{e^{(E_{\boldsymbol{n}}-\mu)/T}-1}}_{\text{normal}}$$

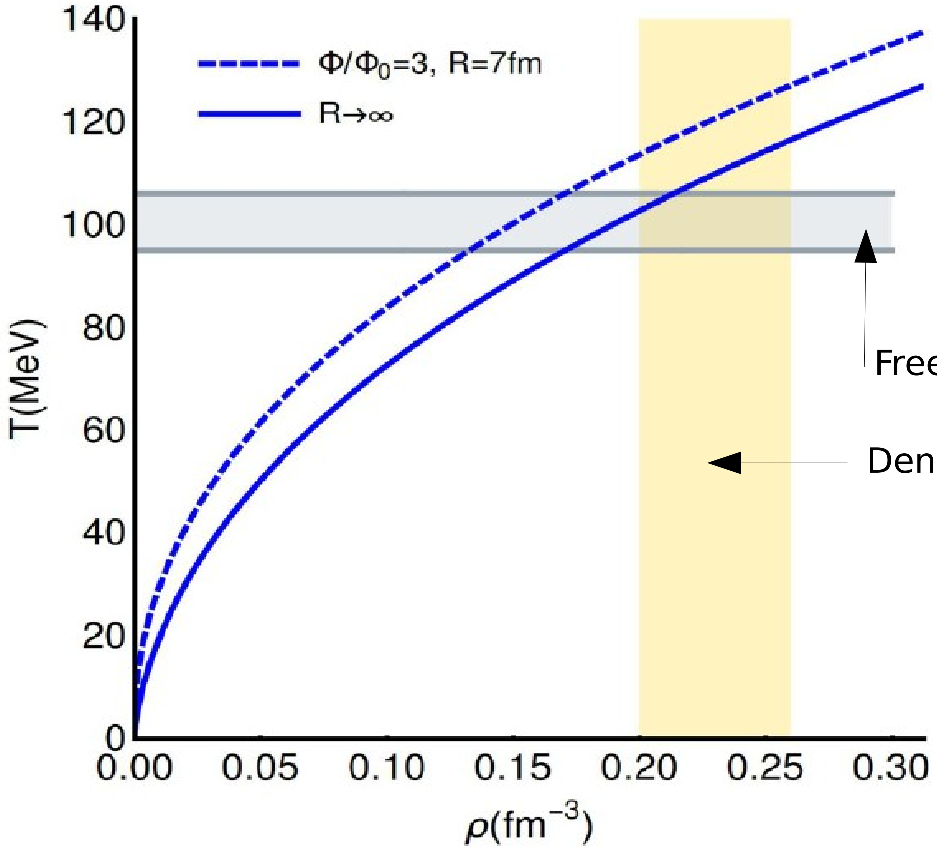



Begun, Gorenstein
PRC77(2008)064903




Begun,Florkowski,
PRC91(2015)054909

# Magnetic field effects



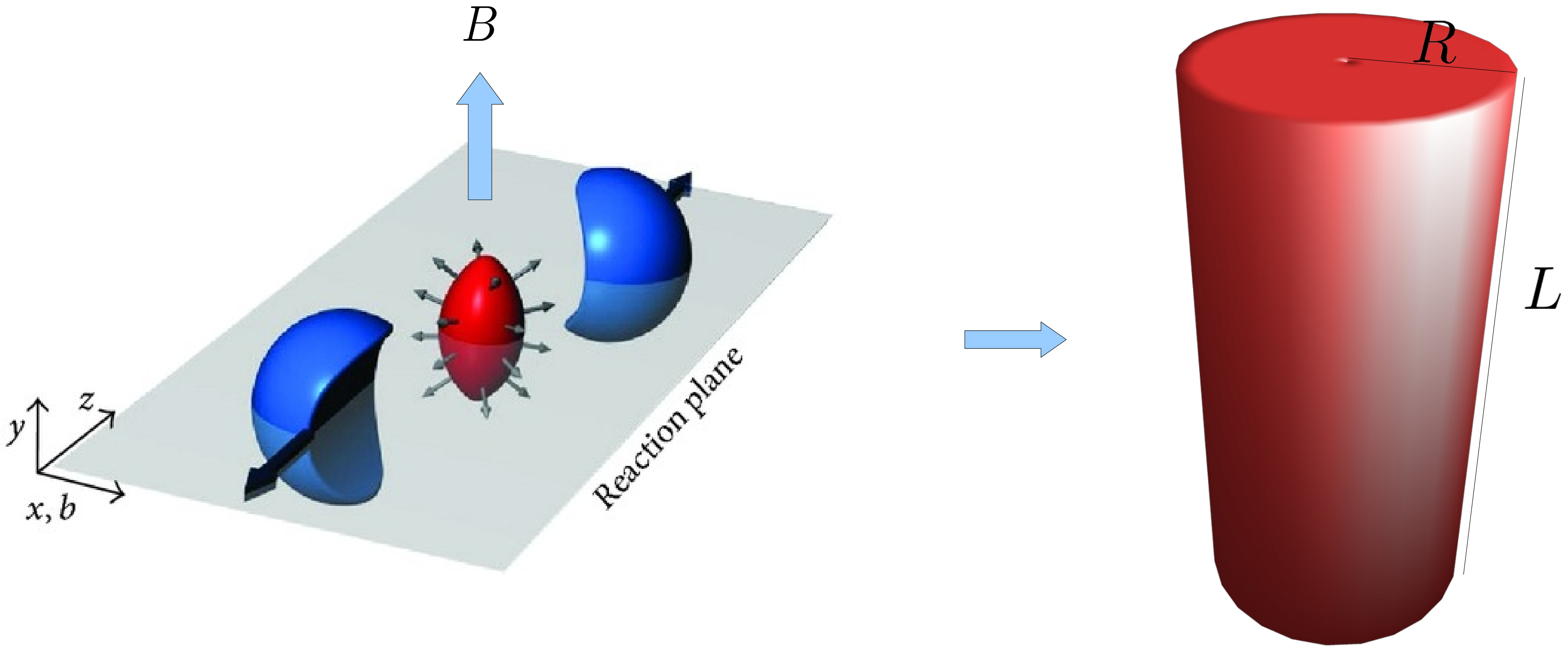

$eB \approx 0.12\, m_\pi^2$

Freezeout temperatures

Densities in semicentral collision

Ayala,Mercado,Villavicencio
PRC95(2017)014904

# Two-pion correlation function in a finite volume and within a magnetic field



# wave function quantization under cylindrical hard wall boundaries

$$\left\{-\left(i\frac{\partial}{\partial t}\right)^2+\left[-i\boldsymbol{\nabla}+q\boldsymbol{A}\right]^2+m^2\right\}\psi(\vec{r},t)=0,$$

$$\begin{aligned}\psi(r=R,t) &= 0,\\ \psi(z=\pm\tfrac{L}{2},t) &= 0.\end{aligned}$$

$$\psi_{nlj}(r,\theta,z,t) = \tilde{A}_{nlj}\,e^{-iE_{nlj}t}e^{-il\theta}\cos\left(k_j\,z\right)e^{-\frac{qBr^2}{4}}r^l\,{}_1F_1\left[-a_{nl},\,l+1;\,\frac{qBr^2}{2}\right]$$

normalization constants

$$\int d^3r\,\psi^*_{nlj}(\vec{r},t)\frac{\overleftrightarrow{\partial}}{\partial t}\psi_{nlj}(\vec{r},t)=1$$

energy eigenvalues

quantized momentum in *z*-direction

$$k_j\equiv\frac{(2j+1)\pi}{L}$$

confluent hypergeometric function

wave function quantization under cylindrical hard wall boundaries

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$${}_1F_1\left[-a_{nl},l+1;\frac{qBR^2}{2}\right]=0$$

boundary condition

$$a_{nl}=\frac{E_{nlj}^2-m^2-k_j^2}{2qB}-\frac{2l+1}{2}$$

$$\Rightarrow$$

$$E_{nlj}^2=k_j^2+m^2+qB(2l+1+2a_{nl}),$$

$$C_2(\vec{p}_1,\vec{p}_2) = \frac{P_2(\vec{p}_1,\vec{p}_2)}{P_1(\vec{p}_1)P_1(\vec{p}_2)}$$

2-pion correlation function

$$P_1(\vec{p}) \equiv \frac{d^3N}{d^3p} = \frac{1}{(2\pi)^3}\sum_\lambda 2E_\lambda N_\lambda \psi^*_\lambda(\vec{p})\psi_\lambda(\vec{p}).$$

$$\lambda = n, l, j$$

$$P_2(\vec{p}_1,\vec{p}_2) \equiv \frac{d^6N}{d^3p_1d^3p_2}$$

$$= P_1(\vec{p}_1)P_1(\vec{p}_2) + \left|\frac{1}{(2\pi)^3}2E_0N_0\psi^*_0(\vec{p}_1,t)\psi_0(\vec{p}_2,t)\right|^2 + \left|\frac{1}{(2\pi)^3}\sum_{\lambda\neq 0}2E_\lambda N_\lambda\psi^*_\lambda(\vec{p}_1,t)\psi_\lambda(\vec{p}_2,t)\right|^2$$

$$N_\lambda = \frac{1}{\exp(E_\lambda - \mu)/T - 1},$$

statistical weight

$$N = \sum_\lambda \frac{1}{\exp(E_\lambda - \mu)/T - 1}$$

chemical potential is obtainted by fixing *N*

# Results

set of external parameters $T,\ N,\ qB, L,\ R,\ \ P,\ P_z,\ q,\ q_z,\ \theta_1,\ \theta_2$

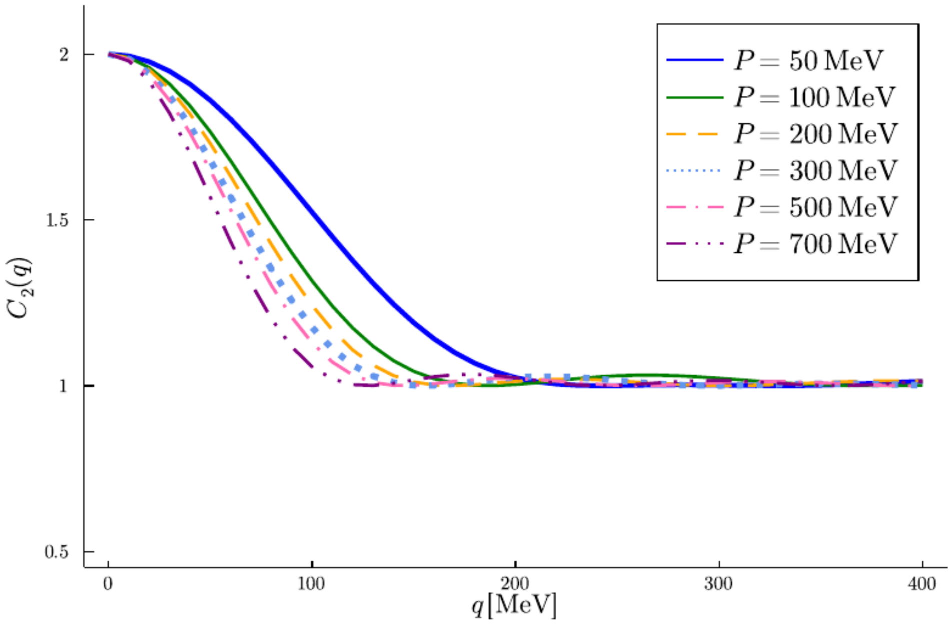
$P = \frac{1}{2}(p_{r1} + p_{r2})$ $\qquad q = |p_{r1} - p_{r2}|$

$P_z = \frac{1}{2}(p_{z1} + p_{z2})$ $\qquad q_z = |p_{z1} - p_{z2}|$

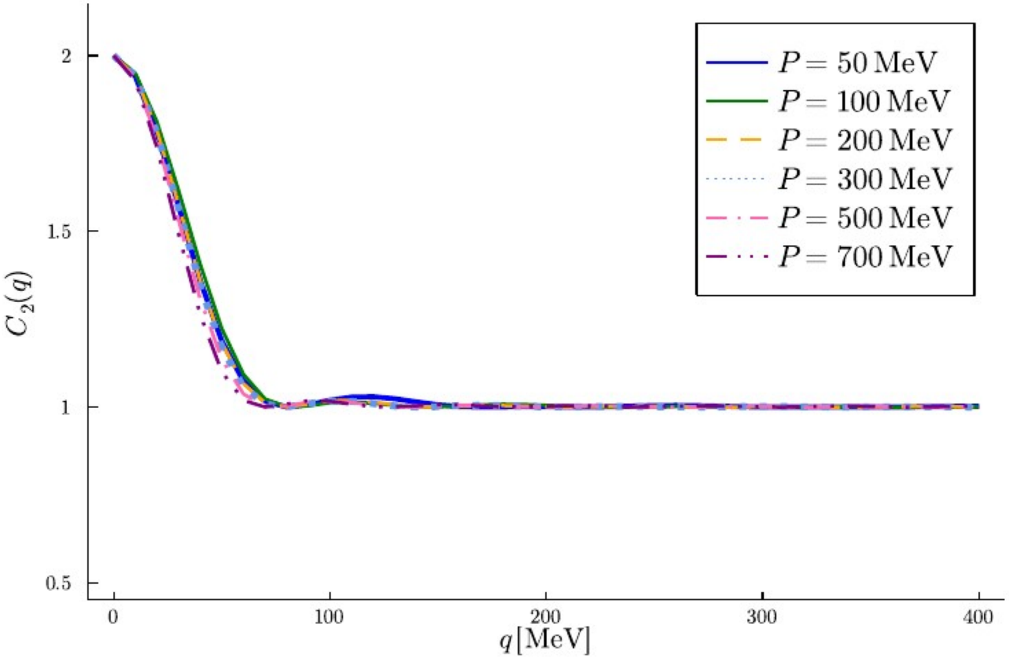
We consider $P_z \approx 0,\ \ \mathrm{q}_z \approx 0,\ \ \theta_1 \approx \theta_2$

# Results

$$p_{z1} = p_{z2} = 0, \quad \theta_2 = \theta_1, \quad T = 100\ \text{MeV}, \quad \mu = 0 \quad \text{(dilute gas)}$$




$qB = 0, \quad R = 5\ \text{fm},\ L = 10\ \text{fm}$




$qB = 0, \quad R = 10\ \text{fm},\ L = 5\ \text{fm}$

# Results

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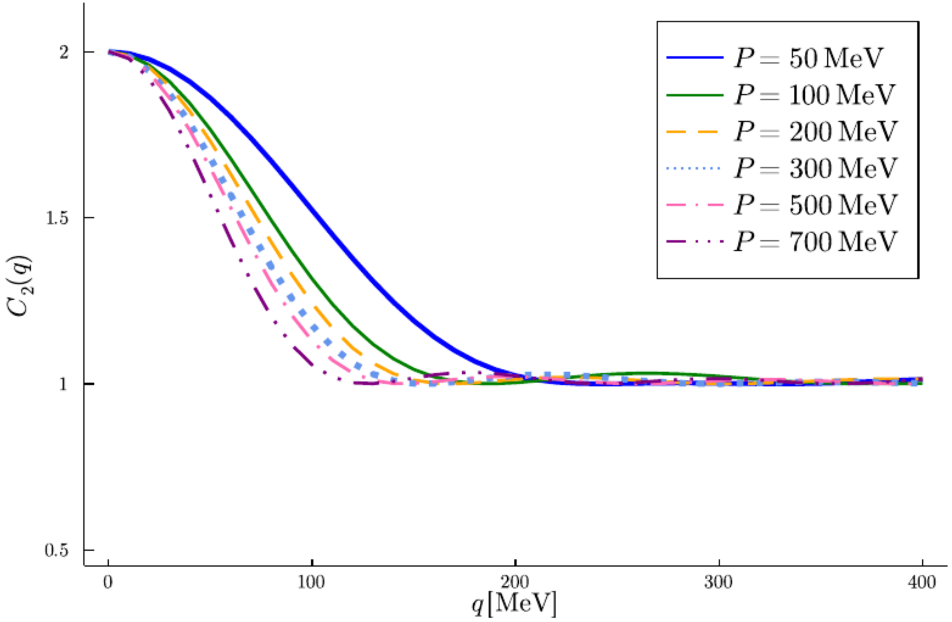
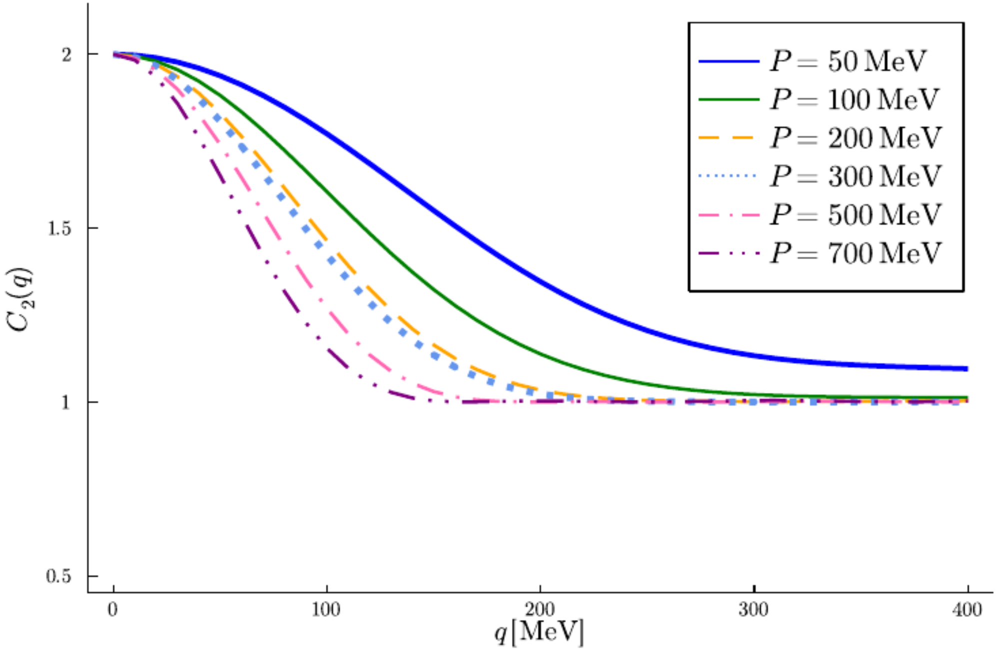



$qB = 0, \quad R = 5$ fm, $L = 10$ fm

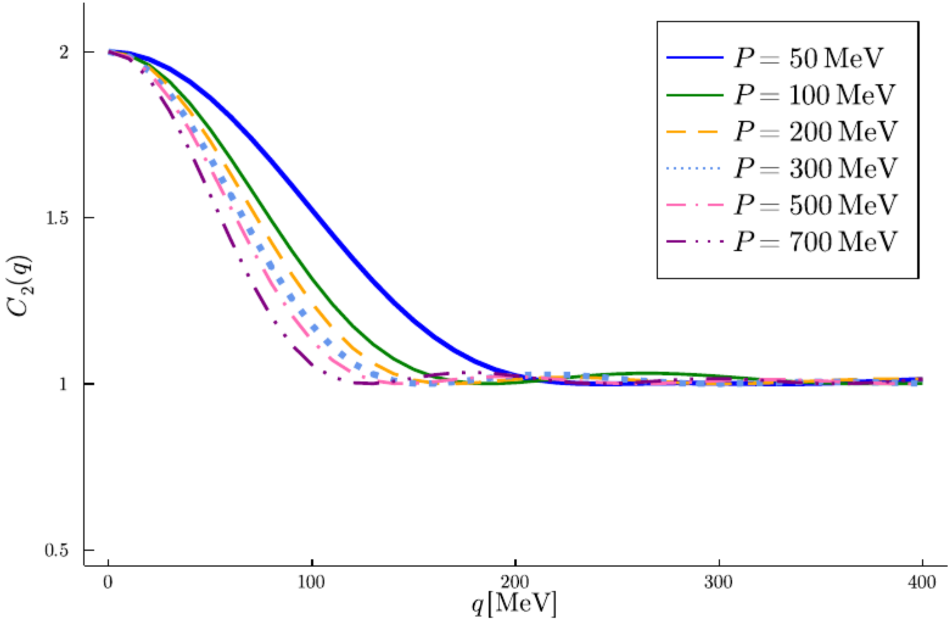



$qB = 2m_\pi^2, \quad R = 5$ fm, $L = 10$ fm

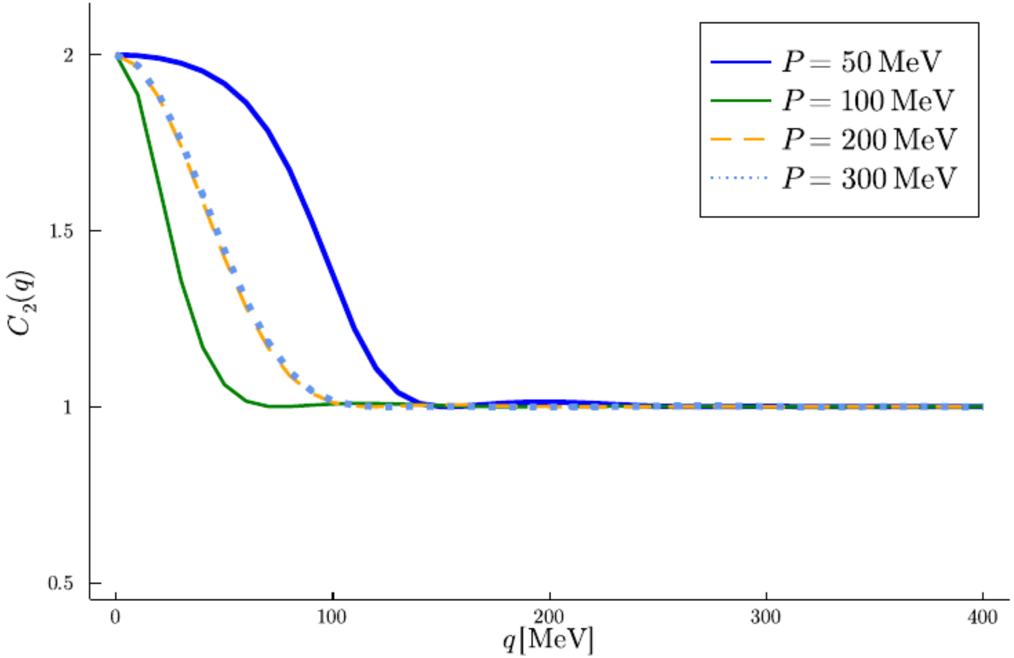
# Results

$$p_{z1} = p_{z2} = 0, \quad \theta_2 = \theta_1, \quad T = 100\text{ MeV}, \quad \mu = 0 \quad \text{(dilute gas)}$$

non monotonic function of *P*



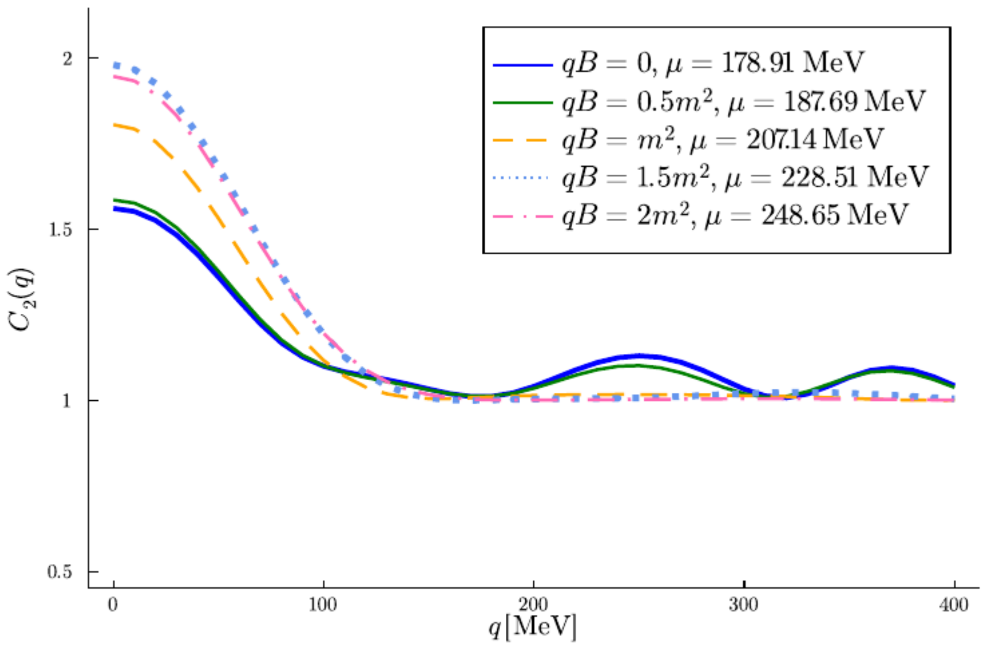

$qB = 0, \quad R = 5\text{ fm}, L = 10\text{ fm}$



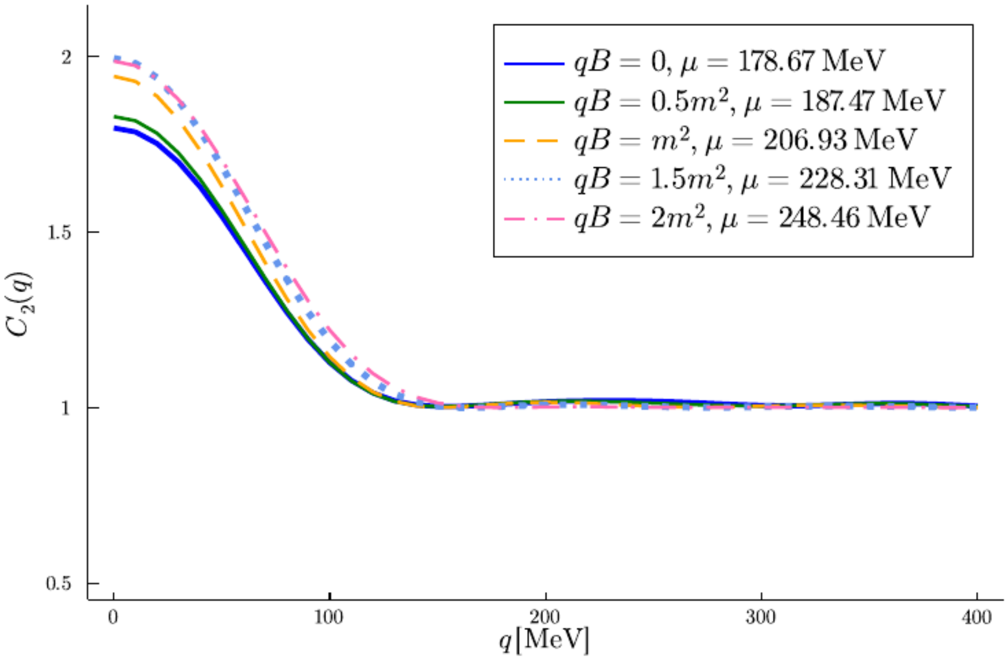

$qB = 2m_\pi^2, \quad R = 10\text{ fm}, L = 5\text{ fm}$

# Results

$$p_{z1} = p_{z2} = 0, \quad \theta_2 = \theta_1, \quad R = 5 \text{ fm}, \ L = 10 \text{ fm}, \quad N = 320 \quad \text{(fixed pion multiplicity)}$$



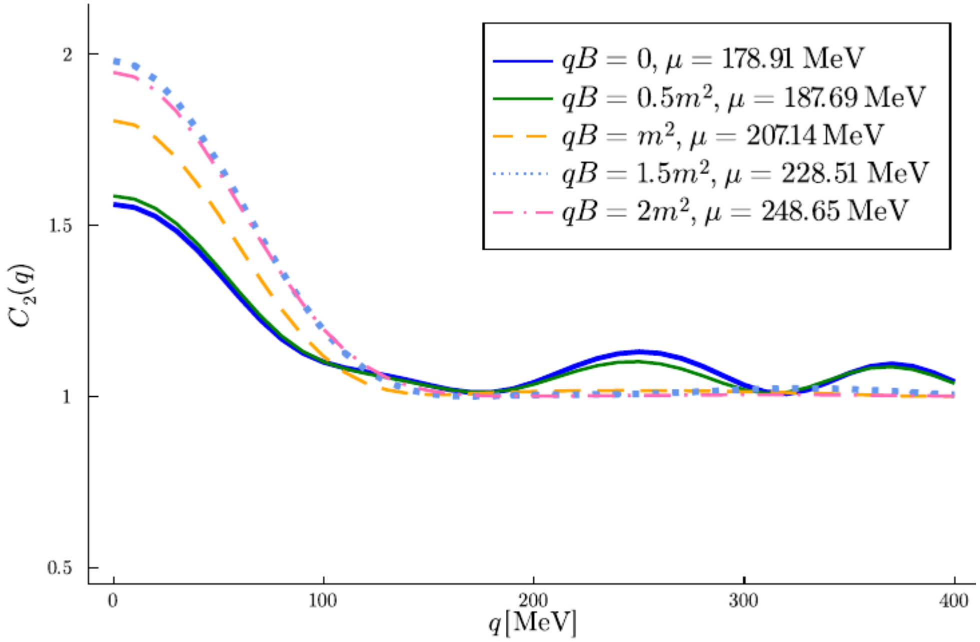

$T = 100$ MeV, $\quad P = 500$MeV



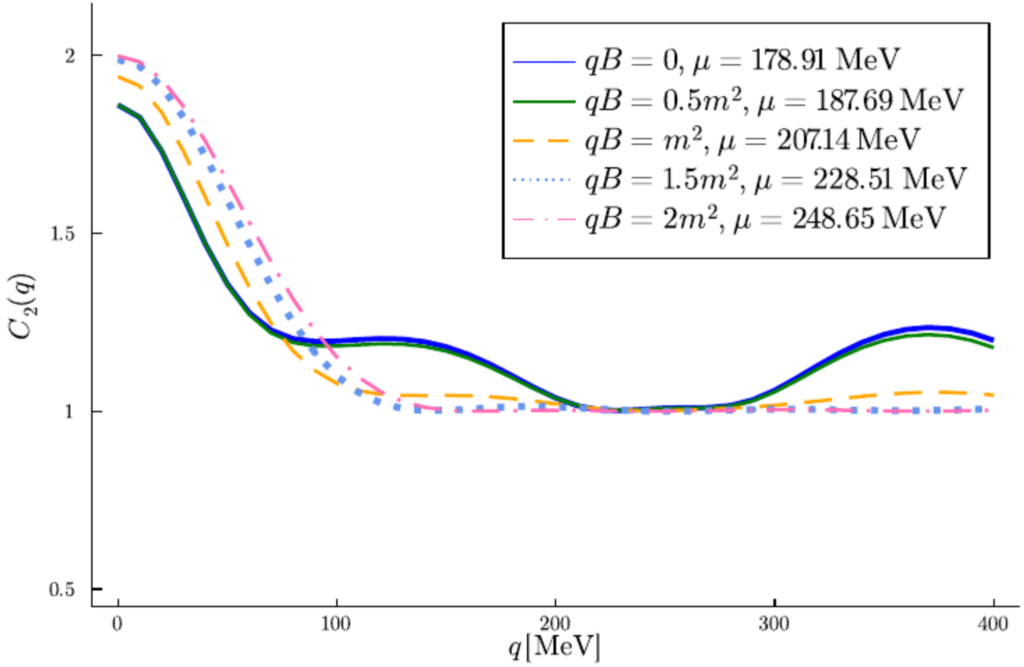

$T = 150$ MeV, $\quad P = 500$MeV

# Results

$$p_{z1} = p_{z2} = 0, \quad \theta_2 = \theta_1, \quad R = 5 \text{ fm}, \ L = 10 \text{ fm}, \quad N = 320 \quad \text{(fixed pion multiplicity)}$$



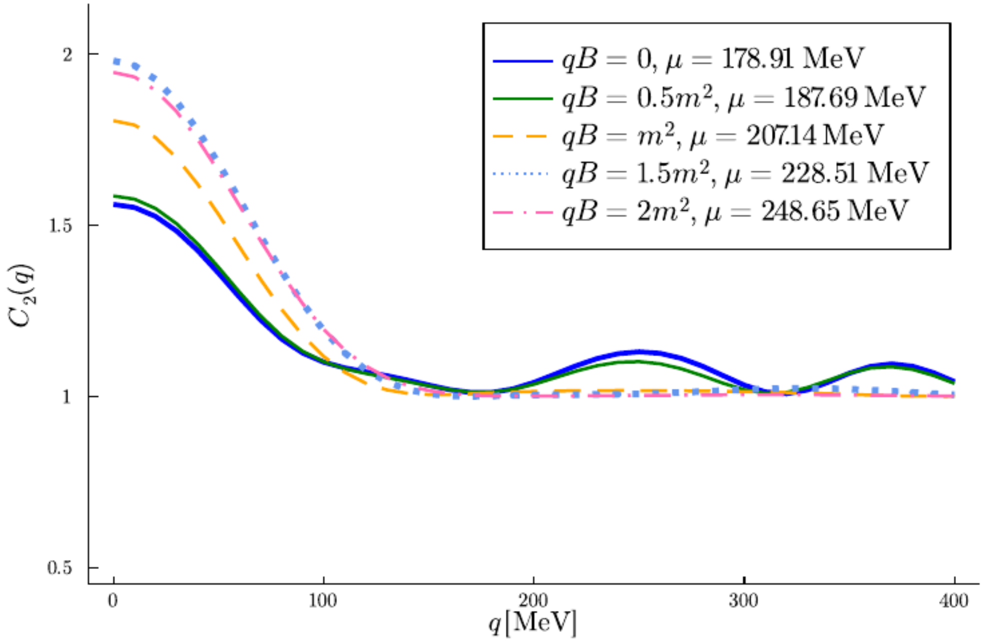

$T = 100$ MeV, $P = 500$MeV



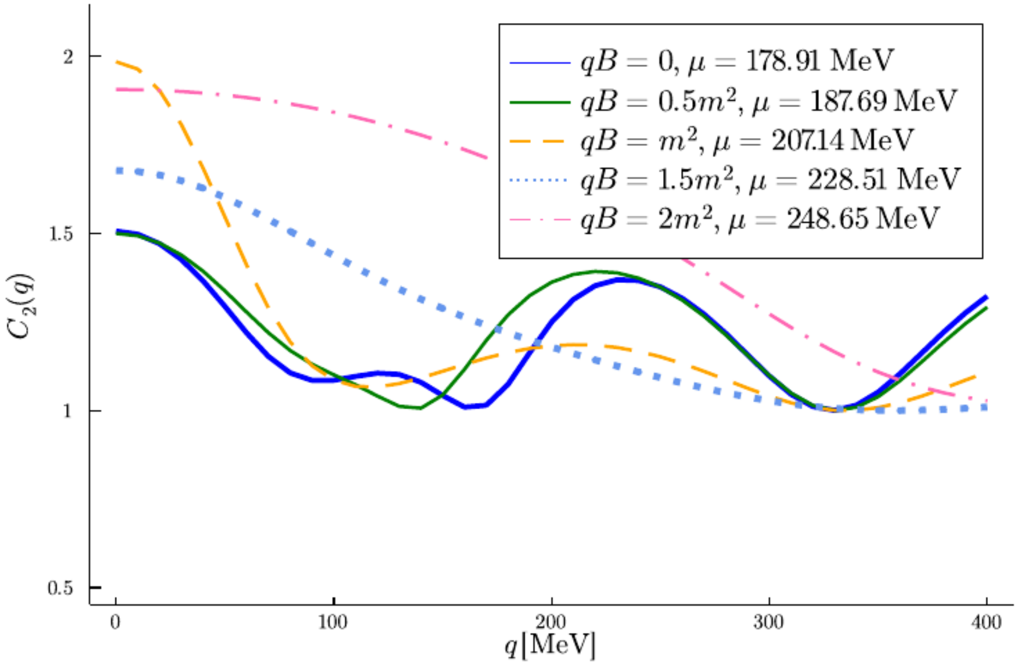

$T = 100$ MeV, $P = 700$MeV

# Results

$$p_{z1} = p_{z2} = 0, \quad \theta_2 = \theta_1, \quad R = 5 \text{ fm}, \ L = 10 \text{ fm}, \quad N = 320 \quad \text{(fixed pion multiplicity)}$$




$T = 100$ MeV, $\quad P = 500$MeV

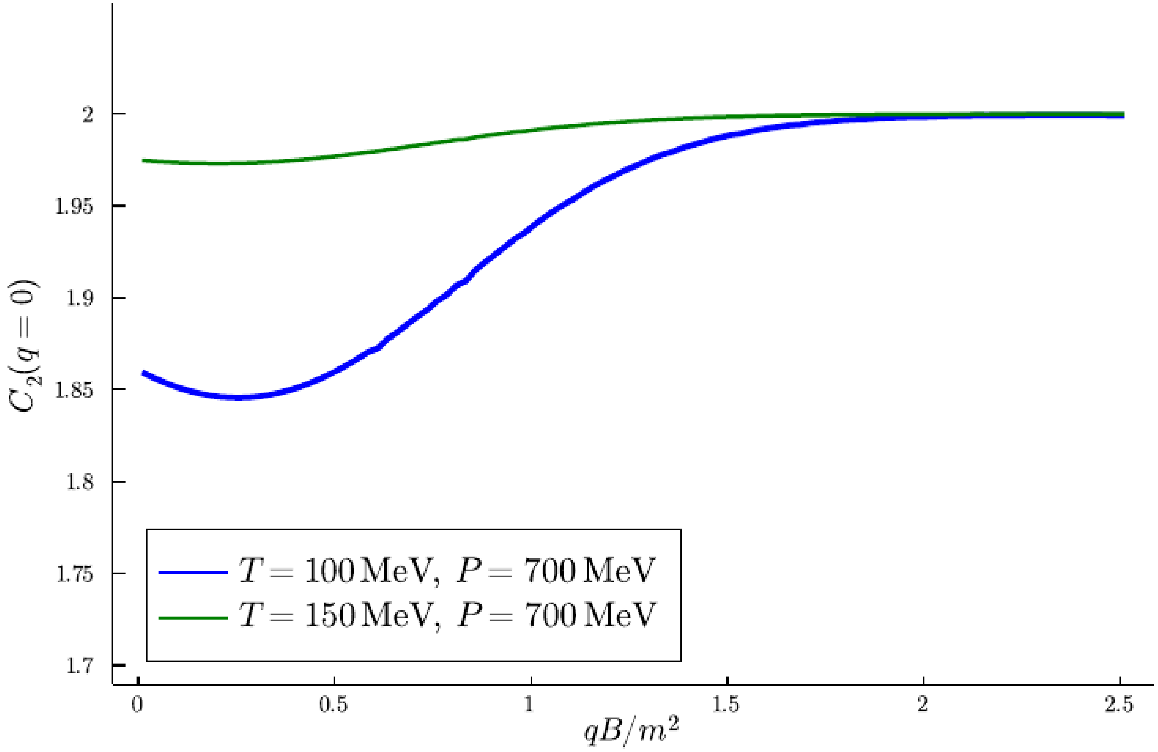


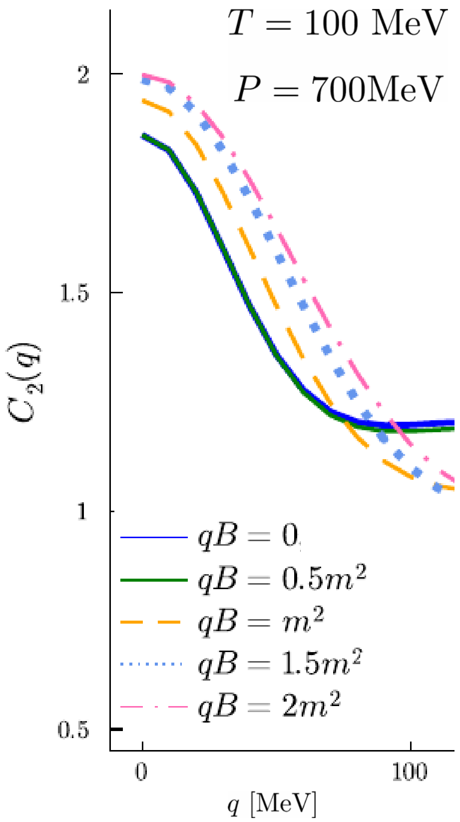

$T = 100$ MeV, $\quad P = 300$MeV

# Results

$$p_{z1} = p_{z2} = 0, \quad \theta_2 = \theta_1, \quad R = 5 \text{ fm}, \ L = 10 \text{ fm}, \quad N = 320 \quad \text{(fixed pion multiplicity)}$$

Tempeatrure and magnetic field reduce ground state contribution.
For qB > 0.5 m² it looks like a discrete transition, but it is not.



## Conclusions

- Magnetic field affects mainly slow pions (low $P$)
- $C_2$ is non-monotonic function of $P$ for large $B$ and $R > L$ in dilute gas
- $C_2$ is non-monotonic function of $B$ for low P and $R < L$ in dense gas
- Ground state diminishes $C_2(0)$
- Ground state distorts $C_2$ for low $B$
- Strong magnetic field tends to destroy the condensate

## Outlook

- Charged pions imbalance (isospin chemical potential)
- Back-reaction pion currents in response of magnetic field
- Fireball rotation

## Conclusions

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***THANKS !***

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