QCD Equilibrium and Dynamical Properties from Holographic Black Holes

Joaquin Grefa

UNIVERSITY of HOUSTON

DEPARTMENT OF PHYSICS

with: Claudia Ratti & Israel Portillo(UH), Romulo Rougemont (UERJ),
Jacquelyn Noronha-Hostler, Jorge Noronha & Mauricio Hippert (UIUC)

The 37th Winter Workshop on Nuclear Dynamics
February 27th- March 5th

Outline

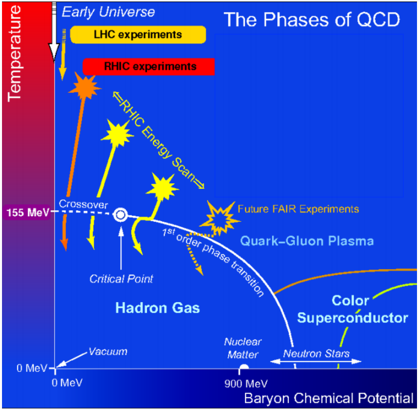
Table of Contents

QCD Phase Diagram

We can explore the QCD phase diagram by changing $\sqrt{s}$ in relativistic heavy ion collisions

Many models predict a first order phase transition line with a critical point

Lattice QCD is the most reliable theoretical tool to study the QCD phase diagram.



QCD Phase Diagram

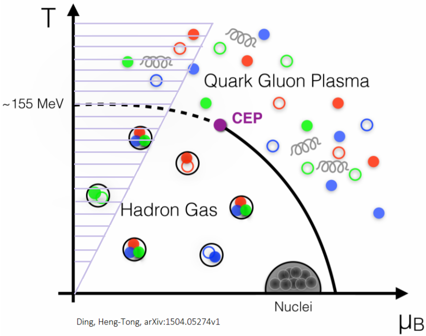
We can explore the QCD phase diagram by changing $\sqrt{s}$ in relativistic heavy ion collisions

Many models predict a first order phase transition line with a critical point

Lattice QCD is the most reliable theoretical tool to study the QCD phase diagram.

Limitation

Sign problem



Model Requirements

The model should exhibit:

-Deconfinement
-Nearly perfect fluidity
-Agreement with Lattice EoS at $\mu_B = 0$
-Agreement with baryon susceptibilities at $\mu_B = 0$

Taylor Expansion for small μ_B

$$\frac{P(T,\mu_B) - P(T,\mu_B = 0)}{T^4} = \sum_{n=1}^{\infty} \frac{1}{(2n)!} \chi_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}$$

where $\chi_n(T,\mu_B) = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n}$

- How can we fulfill these conditions?

Model Requirements

The model should exhibit:

-Deconfinement
-Nearly perfect fluidity
-Agreement with Lattice EoS at $\mu_B = 0$
-Agreement with baryon susceptibilities at $\mu_B = 0$

Taylor Expansion for small μ_B

$$\frac{P(T,\mu_B) - P(T,\mu_B = 0)}{T^4} = \sum_{n=1}^{\infty} \frac{1}{(2n)!} \chi_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}$$

where $\chi_n(T,\mu_B) = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n}$

- How can we fulfill these conditions?
  HOLOGRAPHIC BLACK HOLES!!!

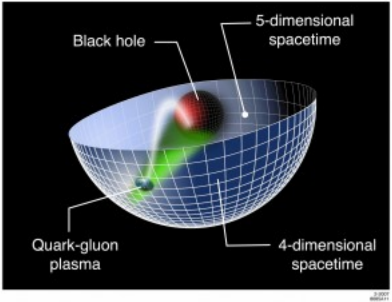
Table of Contents

Holography (Gauge/String duality)

Holographic gauge/gravity correspondence

String Theory/Classical Gravity in 5-dimensions $\Longleftrightarrow$ Quantum Field Theory in 4-dimensions

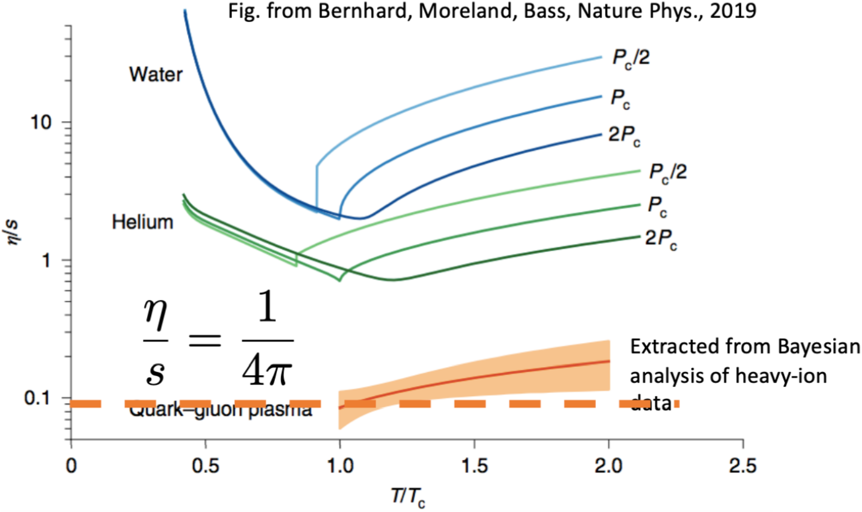
Maldacena 1997; Witten 1998; Gubser, Polyakov, Klebanov 1998



- Near Perfect fluidity
- string theory/classical gravity
  $\Updownarrow$
  strong coupling limit of QFT.
- BH solutions $\rightarrow$ T and μ_B in QFT

Nearly perfect Fluidity of the QGP

Fig. from Bernhard, Moreland, Bass, Nature Phys., 2019



Gravitational Action

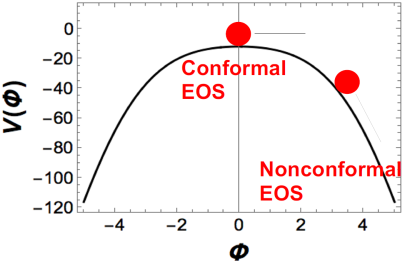
Minimal 5d holography for a non-conformal plasma at $\mu_B = 0$

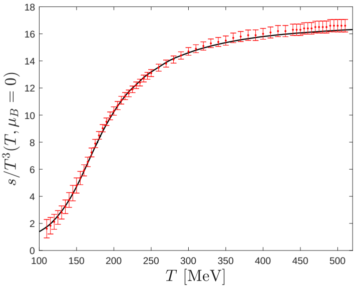
Gursoy, Kiritsis, Mazzanti, Nitti (2008)
Gubser and Nellore, (2008)

Noronha, (2009).

$$S = \frac{1}{2\kappa_5^2} \int_{M_5} d^5x \sqrt{-g} \left[R - \frac{(\partial_\mu \phi)^2}{2} - \underbrace{V(\phi)}_{\text{nonconformal}} \right]$$

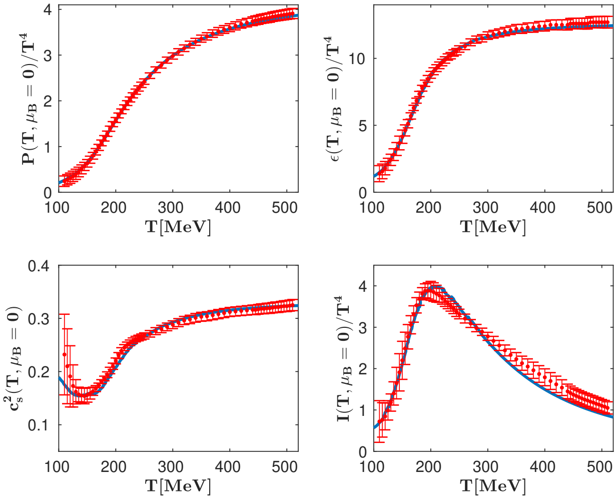
Conformal invariance is broken by $V(\phi)$.





Thermodynamics at $\mu_B = 0$

Matching to Lattice EoS around the crossover



Lattice Results: [WB] S Borsanyi et al.Phys. Lett. B730.99.

BH curves: BH curves: J. G et al. PRD.104 (2021)

what about finite μ_B?

Following DeWolfe, Gubser, Rosen (2011)

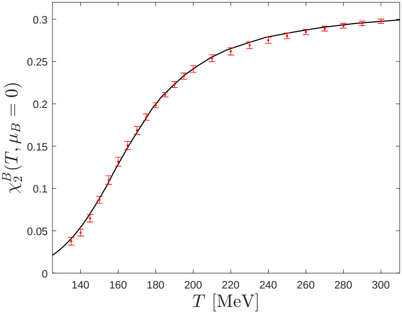
$$S = \frac{1}{2\kappa_5^2}\int_{M_5} d^5x\sqrt{-g}\left[R - \frac{(\partial_\mu\phi)^2}{2} - \underbrace{V(\phi)}_{\text{nonconformal}} - \underbrace{\frac{f(\phi)F_{\mu\nu}^2}{4}}_{\mu_B\neq 0}\right]$$

The coupling $f(\phi)$ is fixed to match $\chi_2^B(T, \mu_B = 0)$

Baryon susceptibility

$$\chi_2(T,\mu_B) = \frac{\partial^2(P/T^4)}{\partial(\mu_B/T)^2} = \frac{\partial(\rho_B/T^3)}{\partial(\mu_B/T)}$$

Any calculation at $\mu_B \neq 0$ is a prediction!!



[WB] S Borsanyi et al.Phys. Lett. B730.99.

BH curves: J. G et al. PRD.104 (2021)

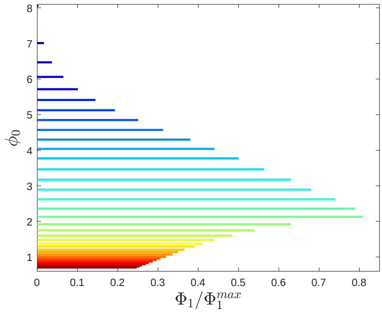
Table of Contents

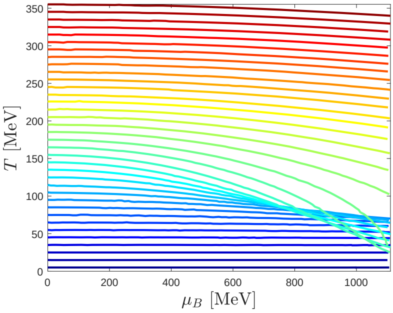
Mapping the QCD phase diagram from Black Hole solutions

The BH solutions are parametrized by (ϕ_0, Φ_1), where

$\phi_0 \rightarrow$ value of the scalar field at the horizon, and

$\Phi_1 \rightarrow$ electric field in the radial direction at the horizon





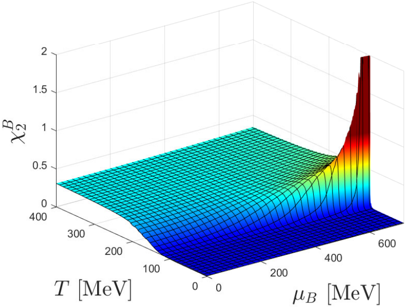
J. G et al. PRD.104 (2021)

Thermodynamics in a wide region of the phase diagram..!!

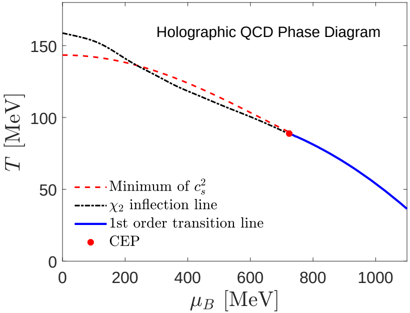
$T \in [2, 550]$ MeV, $\mu_B \in [0, 1100]$ MeV

Locating the Critical End Point (CEP)

$T_{CEP} = 89$ MeV

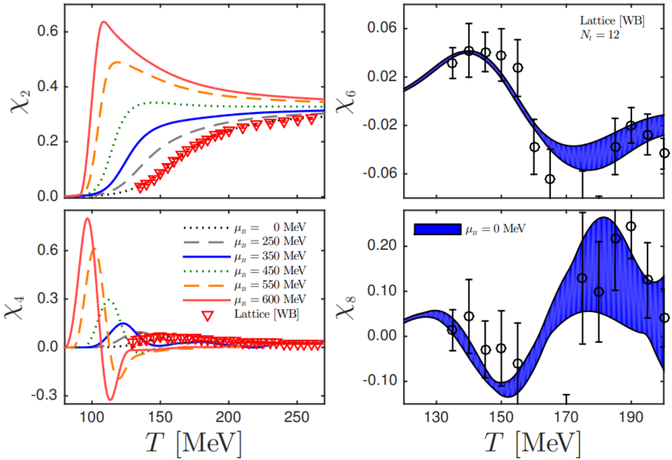


$\mu_B^{CEP} = 724$ MeV



BH curves: J. G et al. PRD.104 (2021)

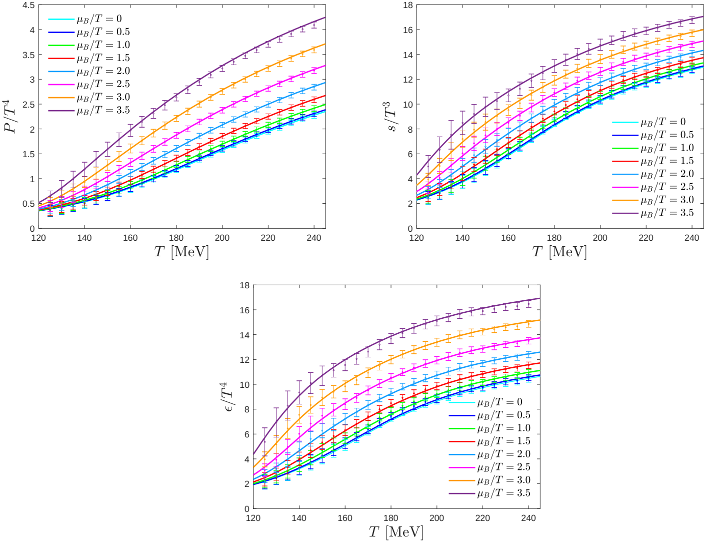
Higher order susceptibilities



Lattice Results: [WB] S Borsanyi et al. arXiv:1805.04445v1.

BH curves: R. Critelli et al., Phys.Rev.D96(2017).

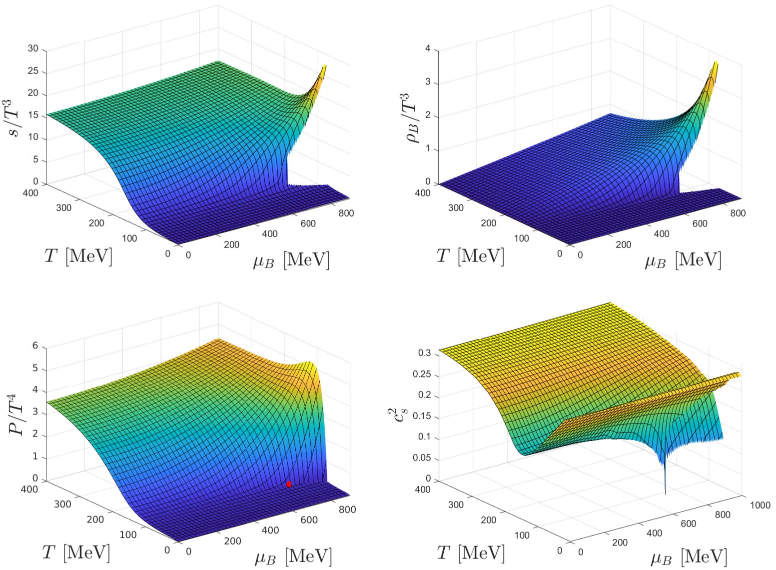
Comparison with the state-of-the-art lattice QCD thermodynamics



Lattice results: S. Borsanyi et al. 10.1103/PhysRevLett.126.232001

BH curves: J. G et al. PRD.104 (2021)

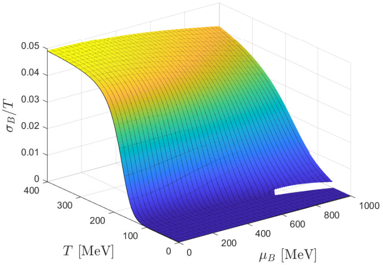
Equation of State

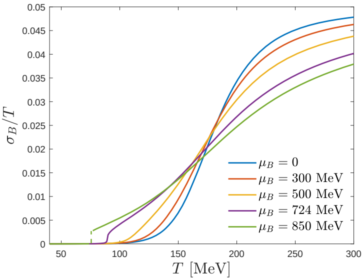


BH curves: J. G et al. PRD.104 (2021)

Table of Contents

Baryon Conductivity σ_B





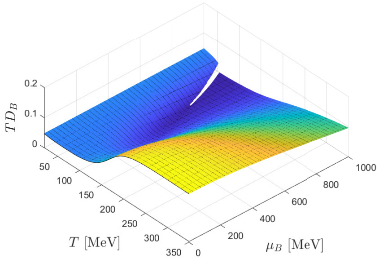
J.G. et al. arXiv:2203.00139 [nucl-th]

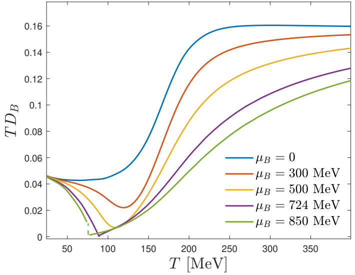
- Can be computed from linear perturbations to the black hole background fields.

- Overall dependence of σ_B/T with μ_B is relatively small.

- σ_B/T remains finite at the critical point, and exhibits a discontinuity over the line of first order phase transition.

Baryon Diffusion Coefficient





J.G. et al. arXiv:2203.00139 [nucl-th]

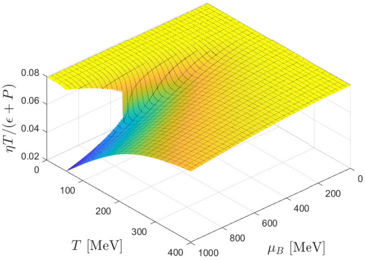
Nernst-Einstein Relation

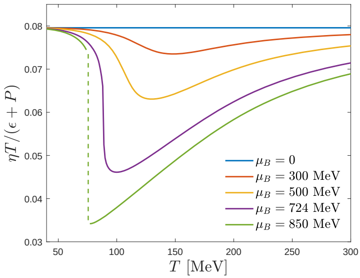
$$TD_B = \frac{\sigma_B / T}{\chi_2^B / T^2}$$

-Controls the fluid response to inhomogeneities in the baryon density

- The baryon diffusion charge is suppressed as the baryon chemical potential increases.

- Vanishes at the location of the critical point.





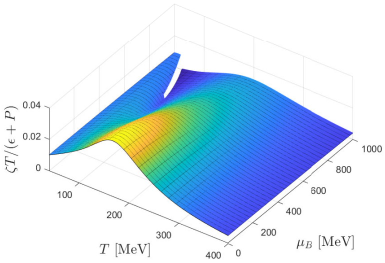
J.G. et al. arXiv:2203.00139 [nucl-th]

- measures the resistance to deformation in the presence of a velocity gradient in the layers of the fluid.

$$\frac{\eta T}{\epsilon + p} = \frac{1}{4\pi} \frac{1}{1 + \frac{\mu_B \rho_B}{Ts}}$$

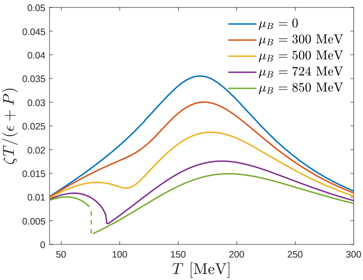
At $\mu_B = 0$, it reduces to the well known holographic prediction $1/4\pi$

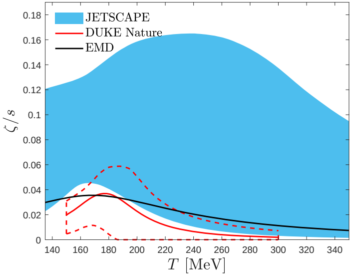
Bulk Viscosity



$$\frac{\zeta T}{\epsilon + p}(T, \mu_B) = \frac{\zeta}{s} \frac{1}{1 + \frac{\mu_B \rho_B}{Ts}}$$

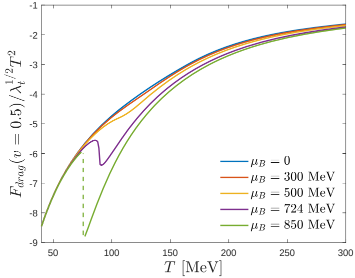
Measures the resistance to deformation of a fluid to a compression or expansion.

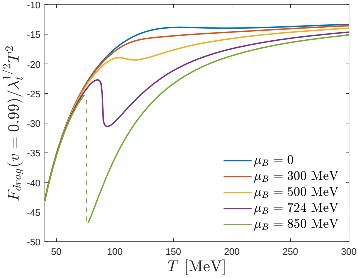




J.G. et al. arXiv:2203.00139 [nucl-th]

Heavy quark drag force



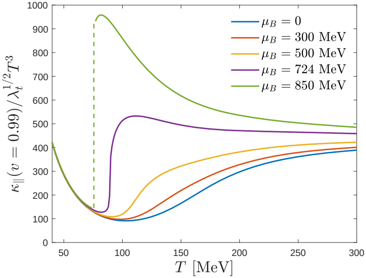


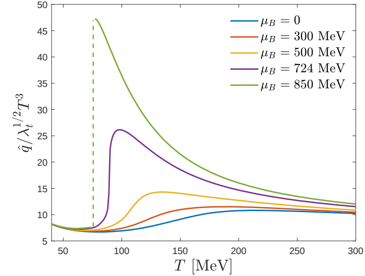
J.G. et al. arXiv:2203.00139 [nucl-th]

- rate at which the heavy quark loses momentum as it moves through the strongly coupled medium with a constant velocity.

A very heavy quark (e.g. the bottom), which might not achieve a very high velocity within the plasma, is less sensitive to the in-medium effects in comparison with a less massive quark (e.g. the charm), which could attain higher velocities within the fluid.

Heavy quark Langevin Diffusion coefficient and jet quenching parameter





J.G. et al. arXiv:2203.00139 [nucl-th]

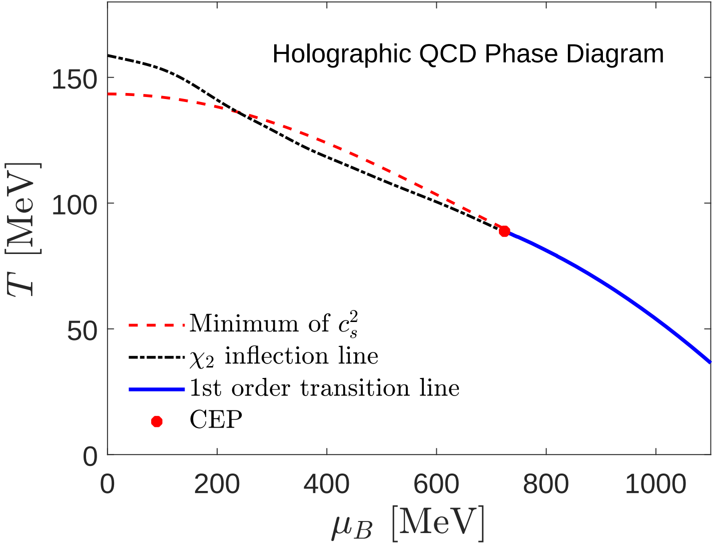
Langevin diffusion coefficients

describe the thermal fluctuations of a heavy quark trajectory with constant velocity under Brownian motion.

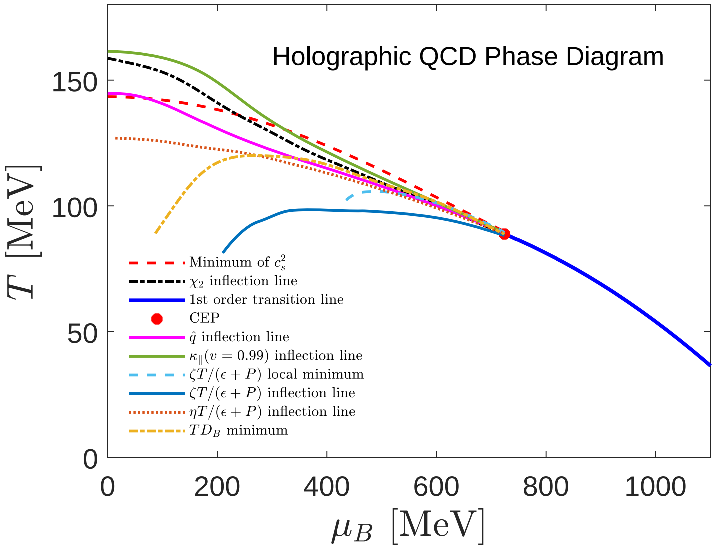
The jet quenching parameter

characterizes the energy loss from collisional and radiative processes of high energy partons produced by the interaction with the hot and dense medium they travel through.

Their inflection point provides another way to characterize the crossover region.



J. G et al. PRD.104 (2021)



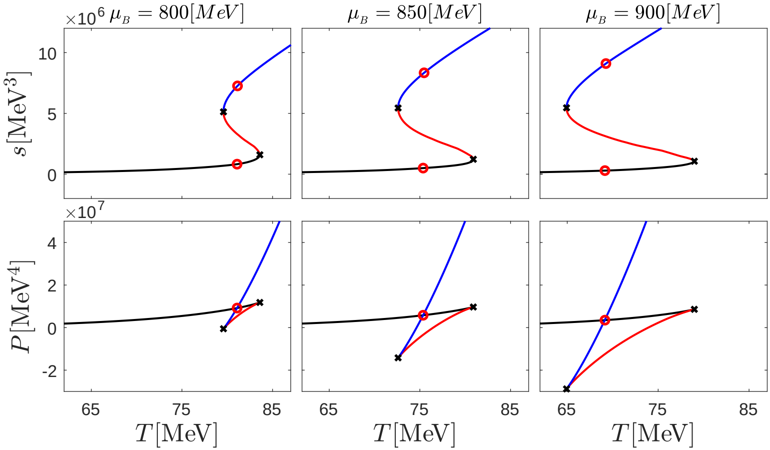
J.G. et al. arXiv:2203.00139 [nucl-th]

Table of Contents

Summary

- There is an excellent agreement between the Lattice QCD EoS and the Holographic result where there is lattice data available.
- The holographic model, which is fixed to mimic the Lattice EoS for $\mu_B = 0$, predicts a CEP:
$$T^{CEP} = 89 \text{ MeV}, \; \mu_B^{CEP} = 724 \text{ MeV}$$
- With the first order phase transition line located in the QCD phase diagram, we considerably extended the baryon chemical potential coverage of the EoS in the QCD phase diagram.
- The transport coefficients related to transport of baryon charge, viscosities, and parton energy loss (12 in total) were obtained over finite baryon chemical potential and particularly over the predicted line of first order phase transition and the critical end point.
- Future work:
  -Obtain the EoS in the strangeness and isospin chemical potential axes.

Locating the first order phase transition line



J. G et al. arXiv:2102.12042 [nucl-th].

$V(\phi)$ and $f(\phi)$

R. Critelli et al., Phys.Rev.D96(2017).

J. G et al. arXiv:2102.12042 [nucl-th].

Free Parameters for the Holographic Model

$$\kappa_5^2 = 8\pi G_5 = 8\pi(0.46), \qquad \Lambda = 1053.83 MeV,$$

$$V(\phi) = -12\cosh(0.63\phi) + 0.65\phi^2 - 0.05\phi^4 + 0.003\phi^6,$$

$$f(\phi) = \frac{\text{sech}(c_1\phi + c_2\phi^2)}{1 + c_3} + \frac{c_3}{1 + c_3}\text{sech}(c_4\phi),$$

where

$$c_1 = -0.27, \quad c_2 = 0.4, \quad c_3 = 1.7, \quad c_4 = 100$$

Equations of Motion

$$S = \frac{1}{2\kappa_5^2} \int_{M_5} d^5x \sqrt{-g} \left[R - \frac{(\partial_\mu \phi)^2}{2} - \underbrace{V(\phi)}_{\text{nonconformal}} - \underbrace{\frac{f(\phi) F_{\mu\nu}^2}{4}}_{\mu_B \neq 0} \right]$$

$$\begin{aligned} ds^2 &= e^{2A(r)}[-h(r)dt^2 + d\vec{x}^2] + \frac{e^{2B(r)}dr^2}{h(r)} \\ \phi &= \phi(r) \\ A_\mu dx^\mu &= \Phi(r)dt \end{aligned}$$

Equations of Motion

$$\phi''(r) + \left[\frac{h'(r)}{h(r)} + 4A'(r)\right]\phi'(r) - \frac{1}{h(r)}\left[\frac{\partial V(\phi)}{\partial \phi} - \frac{e^{-2A(r)}\Phi'(r)^2}{2}\frac{\partial f(\phi)}{\partial \phi}\right] = 0$$

$$\Phi''(r) + \left[2A'(r) + \frac{d[\ln f(\phi)]}{d\phi}\phi'(r)\right]\Phi'(r) = 0$$

$$A''(r) + \frac{\phi'(r)^2}{6} = 0$$

$$h''(r) + 4A'(r)h'(r) - e^{-2A(r)} f(\phi)\Phi'(r)^2 = 0$$

$$h(r)[24A'(r)^2 - \phi'(r)^2] + 6A'(r)h'(r) + 2V(\phi) + e^{-2A(r)} f(\phi)\Phi'(r)^2 = 0$$

Far-Region asymptotics:

$$A(r) = \alpha(r) + \mathcal{O}(e^{-2\nu\alpha(r)}), \qquad \textit{where } \alpha(r) = A_{-1}^{far} r + A_0^{far}$$
$$h(r) = h_0^{far} + \mathcal{O}(e^{-4\alpha(r)}),$$
$$\phi(r) = \phi_A e^{-\nu\alpha(r)} + \mathcal{O}(e^{-(2+\nu)\alpha(r)}),$$
$$\Phi(r) = \Phi_0^{far} + \Phi_2^{far} e^{-1\alpha(r)} + \mathcal{O}(e^{-(2+\nu)\alpha(r)}),$$

Thermodynamics:

$$T = \frac{1}{4\pi\phi_A^{1/\nu}\sqrt{h_0^{far}}}\Lambda \qquad s = \frac{2\pi}{\kappa_5^2\phi_A^{3/\nu}}\Lambda^3$$

$$\mu_B = \frac{\Phi_0^{far}}{\phi_A^{1/\nu}\sqrt{h_0^{far}}}\Lambda \qquad \rho_B = -\frac{\Phi_2^{far}}{\kappa_5^2\phi_A^{3/\nu}\sqrt{h_0^{far}}}\Lambda^3$$

Baryon Conductivity

The EOM for the gauge invariant linearized vector perturbation $a(r,\omega)$ associated to the baryon conductivity is given by,

$$a'' + \left(2A' + \frac{h'}{h} + \frac{f'(\phi)}{f(\phi)}\phi'\right)a' + \frac{e^{-2A}}{h}\left(\frac{\omega^2}{h} - f(\phi)\Phi'^2\right)a = 0$$

which again must be solved with infalling wave condition at the horizon and normalized to unity at the boundary, what may be done by setting,

$$a(r,\omega) = \frac{r^{-i\omega}P(r,\omega)}{r_{max}^{-i\omega}P(r_{max},\omega)}$$

The DC baryon conductivity in the EMD model is calculated by means of the following holographic Kubo formula,

$$\sigma_B(T,\mu_B) = -\frac{\Lambda}{2\kappa_5^2\phi_A^{1/\nu}}\lim_{\omega\to 0}\frac{1}{\omega}\left(e^{2A}hf(\phi)Im[a * a']\right)[MeV]$$

The equation of motion (EOM) for the gauge and diffeomorphism invariant linearized scalar perturbation $H(r, \omega)$ associated to the bulk viscosity through the holographic dictionary is,

$$H'' + \left(4A' + \frac{h'}{h} + \frac{2\phi''}{\phi} - \frac{2A''}{A'}\right) H' +$$
$$\left[\frac{e^{-2A}\omega^2}{h^2} + \frac{h'}{h}\left(\frac{A''}{A'} - \frac{\phi''}{\phi'}\right) + \frac{e^{-2A}}{h\phi'}(3A'f'(\phi) - f(\phi)\phi')\Phi'^2\right] H = 0$$

which must be solved with infalling wave condition at the black hole horizon, and normalized to unity at the boundary, what may be done be setting,

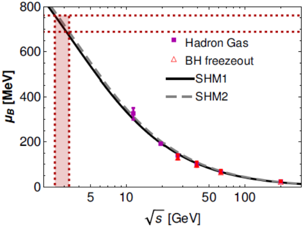
$$H(r, \omega) = \frac{r^{-i\omega} F(r, \omega)}{r_{max}^{-i\omega} F(r_{max}, \omega)}$$

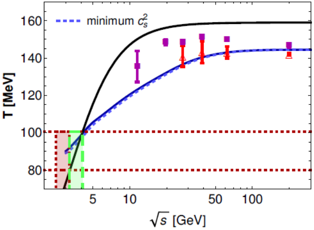
The ratio between the bulk viscosity and the entropy density in the EMD model is then calculated by making use of the following holographic Kubo formula,

$$\frac{\zeta}{s}(T, \mu_B) = -\frac{1}{36\pi} \lim_{\omega \to 0} \frac{1}{\omega} \left(\frac{e^{4A} h \phi'^2 Im[H^* H']}{A'^2}\right)$$

We estimate a collision energy needed to hit the CEP

- $\sqrt{s} = 2.5 - 4.1$ GeV

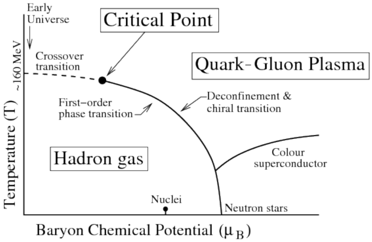


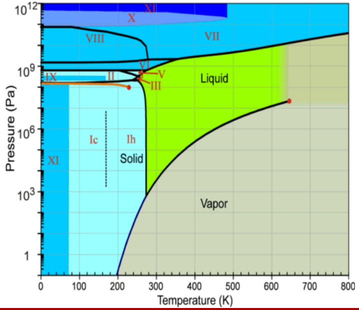


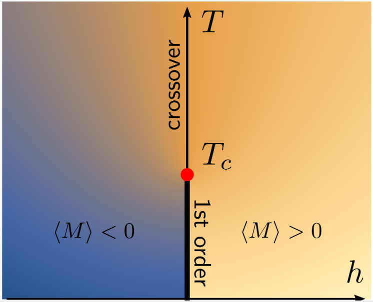
- The collision energy is reachable by the next generation of experiments.

[BH] R.Critelli, I.P. et al., Phys.Rev.D**96**(2017).
[HRG] Paolo Alba et al. Phys.Lett.B**738**(2014),
[SHM1] A. Andronic et al. Phys.Lett.B**673**(2009).
[SHM2] J. Cleymans et al. Phys.Rev.C**73**(2006).

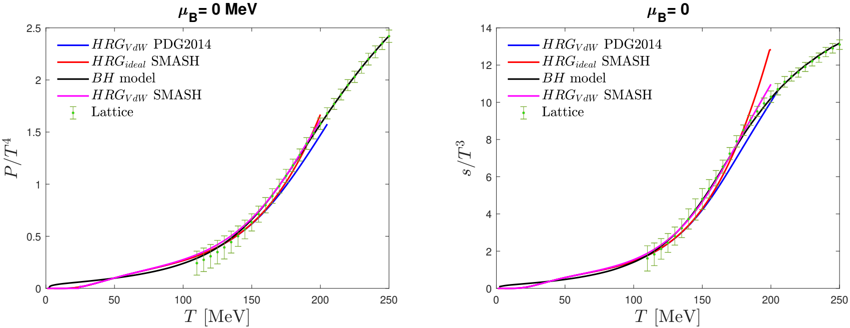
The QCD Critical Point







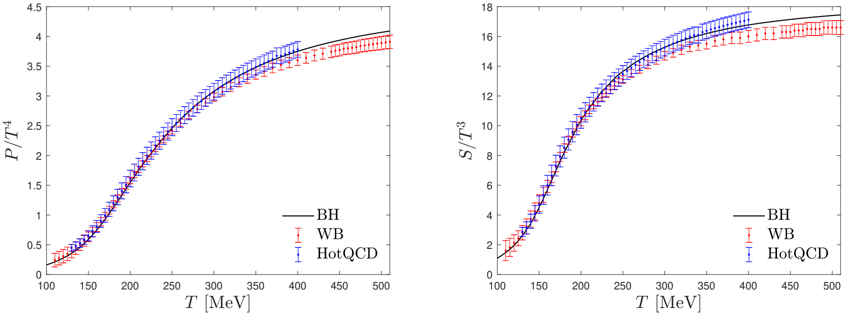
Low T EoS



μ_B= 0 MeV
HRG_{VdW} PDG2014
HRG_{ideal} SMASH
BH model
HRG_{VdW} SMASH
Lattice
P/T^4
T [MeV]
μ_B= 0
s/T^3

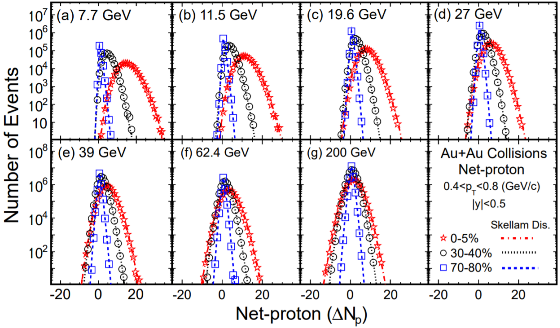
BH from arXiv:2201.02004v1

Critical point at $T = 105$ MeV and $\mu_B = 558$ MeV.



Fluctuations in The Theory and Experiment

- Susceptibilities $\chi_n^B = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n}$
- Susceptibilities χ_n^B are directly related to the moments of the distribution
- Volume independent ratios are useful to compare experimental data



L. Adamczyk et al. Phys. Rev. Lett. 112 (2014), p. 032302.

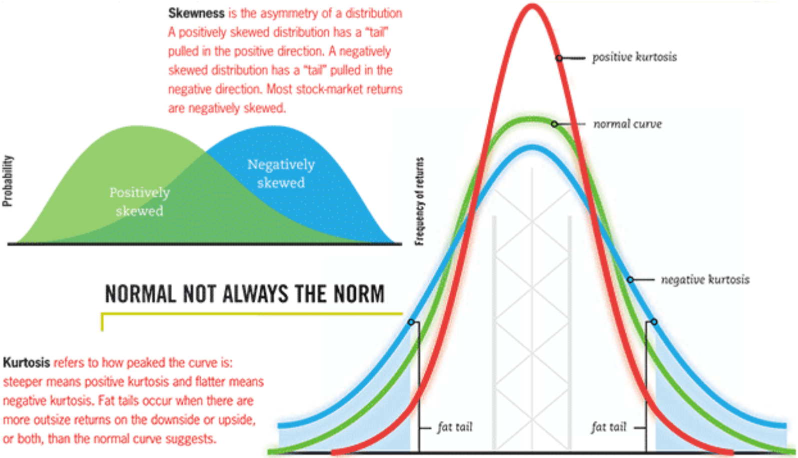
mean: $M \sim \chi_1$

variance: $\sigma^2 \sim \chi_2$

skewness: $S \sim \chi_3/\chi_2^{3/2}$

kurtosis: $\kappa \sim \chi_4/\chi_2^2$

$$M/\sigma^2 = \chi_1/\chi_2$$
$$S\sigma = \chi_3/\chi_2$$
$$\kappa\sigma^2 = \chi_4/\chi_2$$
$$S\sigma^3/M = \chi_3/\chi_1$$



Skewness is the asymmetry of a distribution A positively skewed distribution has a "tail" pulled in the positive direction. A negatively skewed distribution has a "tail" pulled in the negative direction. Most stock-market returns are negatively skewed.
Probability
Positively skewed
Negatively skewed
NORMAL NOT ALWAYS THE NORM
Kurtosis refers to how peaked the curve is: steeper means positive kurtosis and flatter means negative kurtosis. Fat tails occur when there are more outsize returns on the downside or upside, or both, than the normal curve suggests.
Frequency of returns
positive kurtosis
normal curve
negative kurtosis
fat tail
fat tail