



Open Access Article
Event-by-Event Investigation of the Two-Particle Source Function in Heavy-Ion Collisions with EPOS

by [Dániel Kincses](#) ^{1,*} [Maria Stefaniak](#) ^{2,3} and [Máté Csanád](#) ¹

¹ Department of Atomic Physics, Eötvös Loránd University, H-1117 Budapest, Hungary

² GSI Helmholtzzentrum für Schwerionenforschung, Planckstr. 1, 64291 Darmstadt, Germany

³ Faculty of Physics, Warsaw University of Technology, pl. Politechniki 1, 00-661 Warsaw, Poland

* Author to whom correspondence should be addressed.

Academic Editors: Udo Von Toussaint and Edward Sarkisyan-Grinbaum

Entropy **2022**, *24*(3), 308; <https://doi.org/10.3390/e24030308>

Received: 20 January 2022 / Revised: 18 February 2022 /

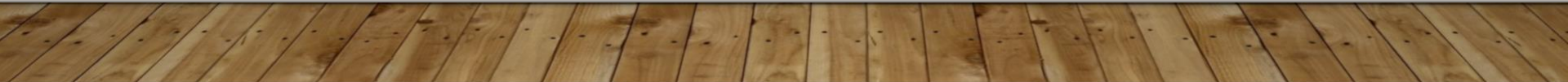
Accepted: 18 February 2022 / Published: 22 February 2022

(This article belongs to the Special Issue *Zimányi School: Hydrodynamics and Thermodynamics in High Energy Particle and Nuclear Physics*)

Entropy **2022**, *24*(3), 308
[arXiv: 2201.07962]

EVENT-BY-EVENT LEVY FEMTOSCOPY WITH EPOS

MÁTÉ CSANÁD (EOTVOS U, BUDAPEST), D KINCSES, M STEFANIAK
THE 37TH WINTER WORKSHOP ON NUCLEAR DYNAMICS, 2022





2/25

CONTENTS OF THIS TALK

- Basics of femtoscopy and Lévy sources
- EPOS event-by-event analysis
- Summary and outlook



3/25

CONTENTS OF THIS TALK

- Basics of femtoscopy and Lévy sources
- EPOS event-by-event analysis
- Summary and outlook

FEMTOSCOPY IN HIGH ENERGY PHYSICS

- R. Hanbury Brown, R. Q. Twiss - observing Sirius with radio telescopes

- Intensity correlations vs detector distance \Rightarrow source size
- Measure the sizes of apparently point-like sources!

- Goldhaber et al: applicable in high energy physics

- Understanding: Glauber, Fano, Baym, ...

Phys. Rev. Lett. 10, 84; Rev. Mod. Phys. 78 1267, ...

- Momentum correlation $C(q)$ related to source $S(r)$

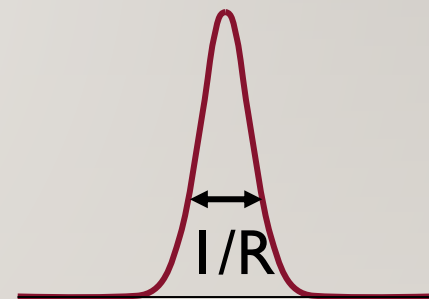
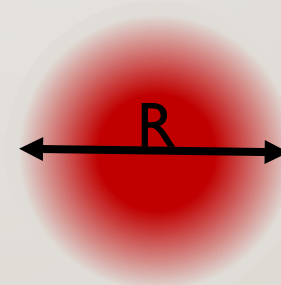
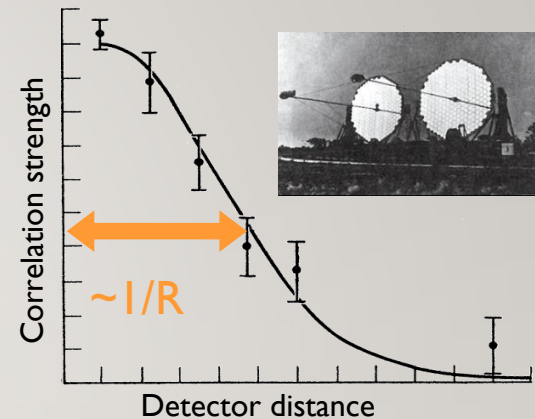
$$C(q) \cong 1 + \left| \int S(r) e^{iqr} dr \right|^2$$

(under some assumptions)

or the distance distribution $D(r)$

$$C(q) \cong 1 + \int D(r) e^{iqr} dr$$

- Neglected: pair reco., final state int., N-particle correlations, coherence, ...



source function $S(r)$ correlation funct. $C(q)$

- Measure $C(q)$: map out source space-time geometry on femtometer scale!



5/25 SOURCE OR PAIR DISTRIBUTION?

- Under some circumstances (thermal emission, no interactions, ...):

$$C_2(q, K) = \int S\left(r_1, K + \frac{q}{2}\right) S\left(r_2, K - \frac{q}{2}\right) |\Psi_2(r_1, r_2)|^2 dr_1 dr_2$$

$$\cong 1 + \left| \int S(r, K) e^{iqr} dr \right|^2$$

- Let us introduce the spatial pair distribution:

$$D(r, K) = \int S\left(\rho + \frac{r}{2}, K\right) S\left(\rho - \frac{r}{2}, K\right) d\rho$$

- Then the Bose-Einstein correlation function becomes:

$$C_2(q, K) \cong \int D(r, K) |\Psi_2(r)|^2 dr = 1 + \int D(r, K) e^{iqr} dr$$

- **Bose-Einstein correlations measure spatial pair distributions!**
- Coulomb and strong Final State Interactions? Under control for Lévy sources

Csanad, Lökös, Nagy, Phys. Part. Nuclei 51 (2020) 238 [arXiv:1910.02231]

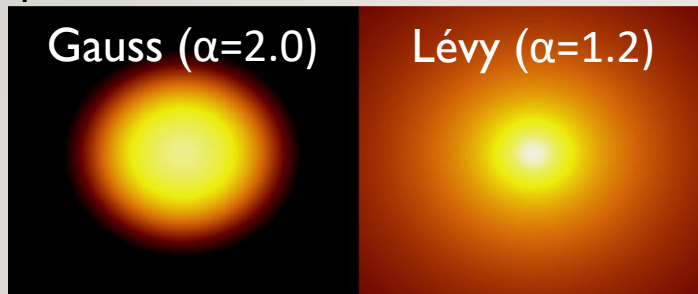
Kincses, Nagy, Csanad Phys. Rev. C102, 064912 (2020) [arXiv:1912.01381]

LÉVY DISTRIBUTIONS IN HEAVY ION PHYSICS

- Measurements suggest phenomena beyond Gaussian distribution

- Lévy-stable distribution: $\mathcal{L}(\alpha, R; r) = (2\pi)^{-3} \int d^3q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$

- From generalized central limit theorem, power-law tail $\sim r^{-(1+\alpha)}$
- Special cases: $\alpha = 2$ Gaussian, $\alpha = 1$ Cauchy



- Shape of the correlation functions with Levy source:

- $C_2(q) = 1 + \lambda \cdot e^{-|qR|^\alpha}$; $\alpha = 2$: Gaussian; $\alpha = 1$: exponential
Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67-78

- Reasons for Levy source:

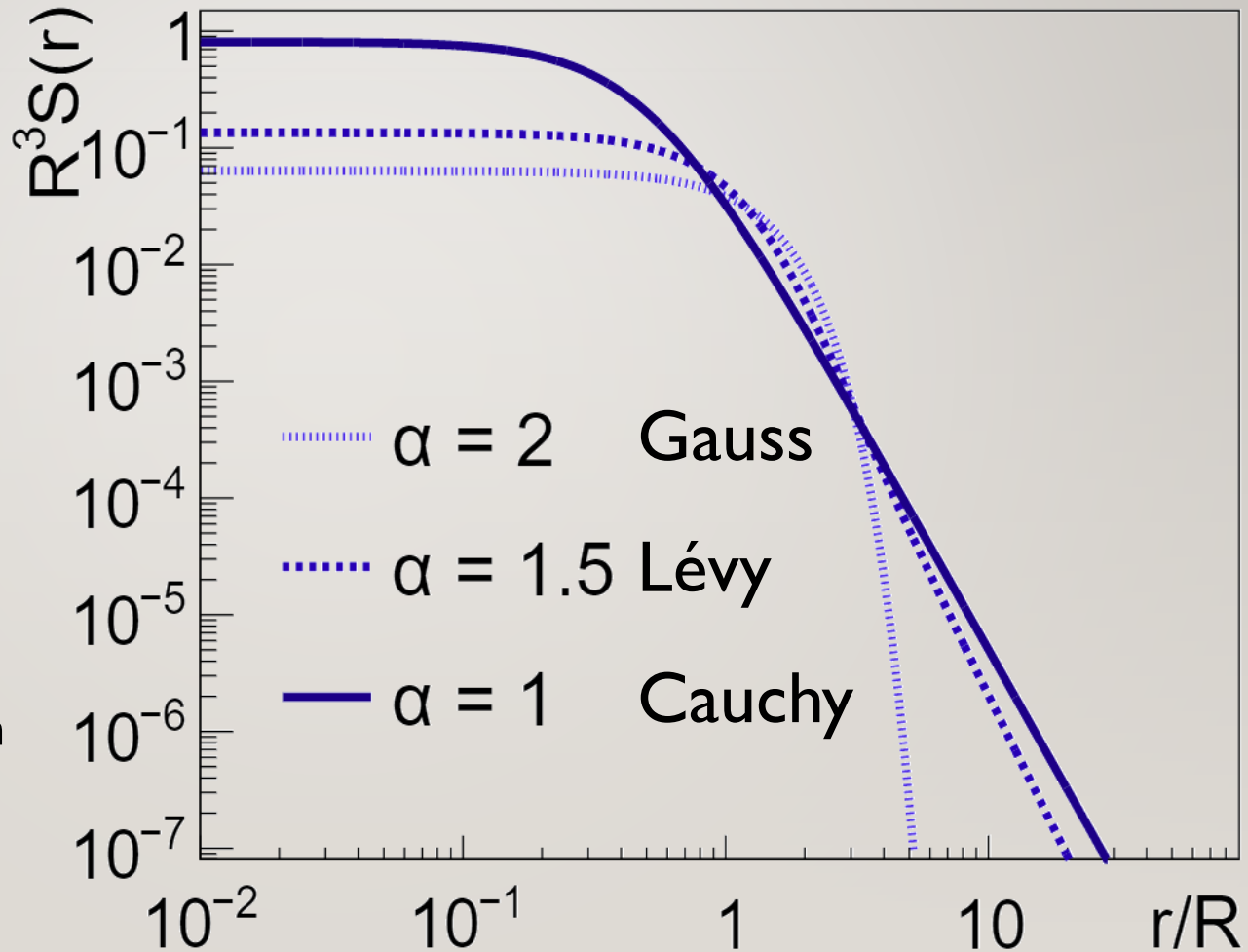
- Critical phenomena; QCD jets; Anomalous diffusion; what else?
Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828 (2006) no.1, 525-532;
Csörgő, Hegyi, Novák, Zajc, Acta Phys.Polon. B36 (2005) 329-337
Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002; Metzler, Klafter, Physics Reports 339 (2000) 1-77





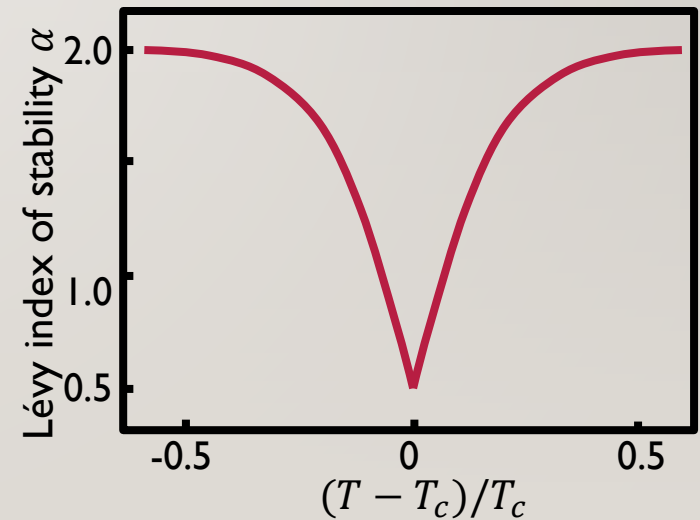
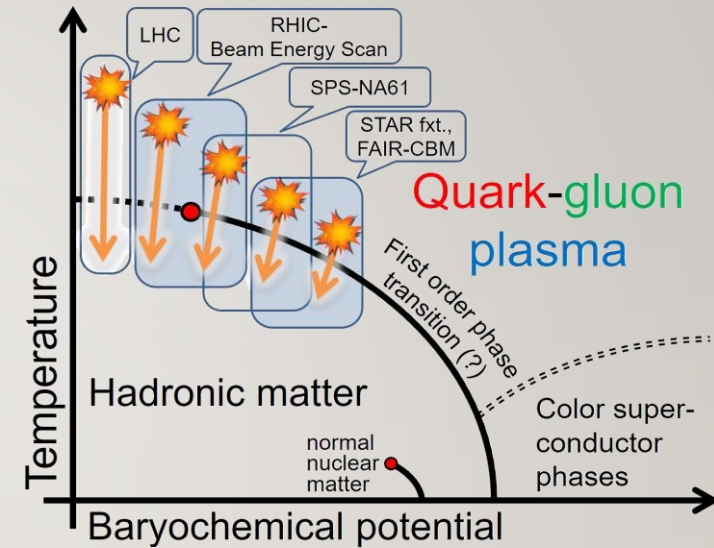
7 /25 LÉVY VERSUS GAUSS VERSUS EXPONENTIAL

- No tail if $\alpha = 2$, power law if $\alpha < 2$; tail strength depends on α
- If $S(r)$ Lévy, $D(r)$ also Lévy with same α and $R \rightarrow 2^{1/\alpha} R$
- In principle, $RMS = \infty$ if $\alpha < 2$
- In practice, RMS depends on cutoff
- What do Gaussian HBT radii mean?



8/25 LÉVY INDEX AS A CRITICAL EXPONENT?

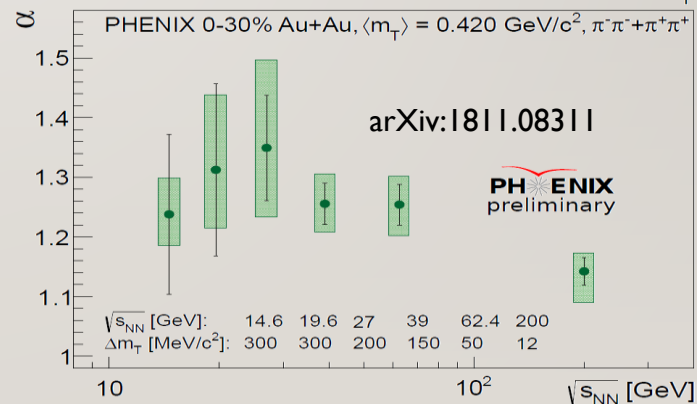
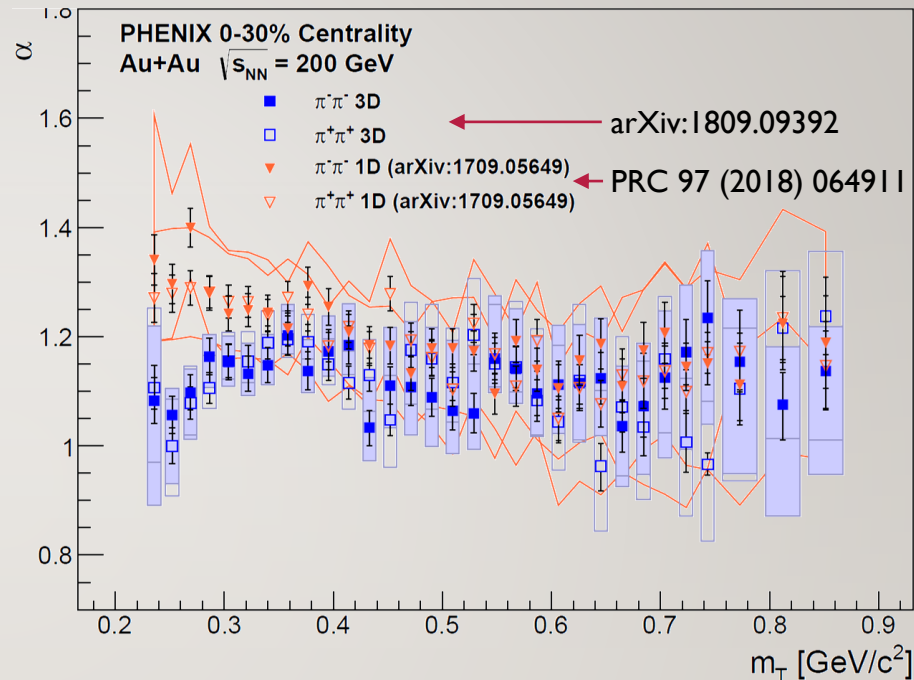
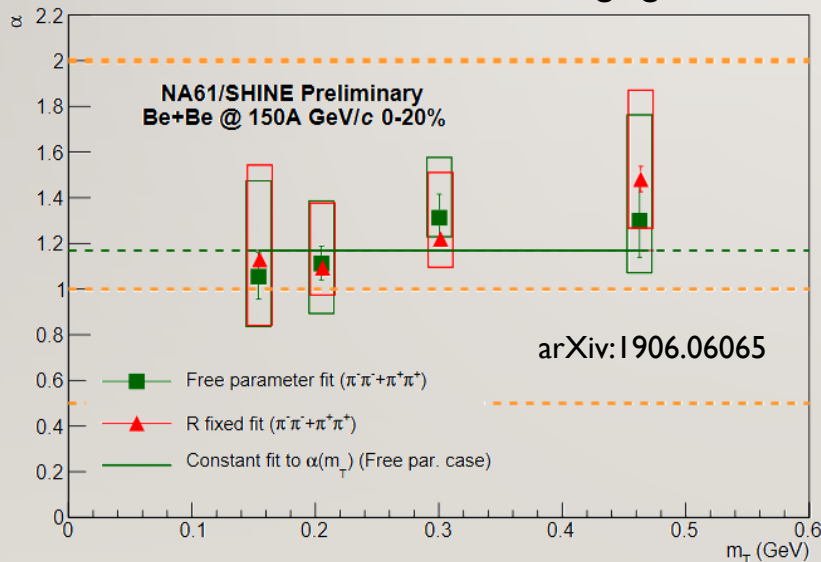
- Critical spatial correlation: $\sim r^{-(d-2+\eta)}$;
Lévy source: $\sim r^{-(1+\alpha)}$; $\alpha \Leftrightarrow \eta$
Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67,
- QCD universality class \leftrightarrow 3D Ising
Halasz et al., Phys.Rev.D58 (1998) 096007
Stephanov et al., Phys.Rev.Lett.81 (1998) 4816
- At the critical point:
 - Random field 3D Ising: $\eta = 0.50 \pm 0.05$
Rieger, Phys.Rev.B52 (1995) 6659
 - 3D Ising: $\eta = 0.03631(3)$
El-Showk et al., J.Stat.Phys. 157 (4-5): 869
- Motivation for precise Lévy HBT!
- Change in $\alpha_{\text{Lévy}}$ proximity of CEP?
- Finite size/time & non-equilibrium effects
→ what does power law mean?





LÉVY HBT MEASUREMENTS

- Many experimental results
 - PHENIX Au+Au: $\alpha \approx 1 - 1.5$
 - STAR Au+Au: ongoing
 - NA6I Be+Be: $\alpha \approx 1 - 1.5$
 - CMS Pb+Pb: $\alpha = 1$ fixed
- Where does this Lévy shape come from? What does it mean?
 - Role of event class averaging?





10/25

CONTENTS OF THIS TALK

- Basics of femtoscopy and Lévy sources
- EPOS event-by-event analysis
- Summary and outlook



TWO-PARTICLE SPATIAL CORRELATIONS

- Object to be investigated: two-particle source

$$D(r, K) = \int d^4\rho S\left(\rho + \frac{r}{2}, K\right) S\left(\rho - \frac{r}{2}, K\right)$$

- Experimental results measure power-law tails, Lévy shapes
 - Measure momentum-space correlations, reconstruct $D(r)$ or fit its parameters
- Why do these Lévy shapes appear?
 - What physics does contribute to it? Rescattering, decays?
 - What role does event averaging have in it?
Cimerman, Plumberg, Tomasik, Phys.Part.Nucl. 51 (2020) 282, PoS ICHEP2020 538
 - What do specific α values mean?
- Event generator models (like EPOS) – direct access to pair-source!
 - Phenomenological investigations of $D(r)$ possible
 - Effects can be turned off or on, investigated separately



12/25

THE EPOS MODEL

- **E**nergy conserving quantum-mechanical multiple scattering approach, based on **P**artons ladders, **O**ff-shell remnants, and **S**plitting of parton ladders
 - K. Werner et al., PRC82 (2010) 044904, PRC89 (2014) 064903, ...
- Based on Monte-Carlo simulation
- Theoretical framework: parton-based Gribov-Regge theory (PBGRT)
- Three main parts of the model:
 - Core-Corona division (based on dE/dx of string segments)
 - Hydrodynamical evolution (vHLLE 3D+1 viscous hydro)
 - Hadronic cascades (UrQMD afterburner)
- Effects/components to be turned on or off (on top of Core):
 - Corona
 - Rescattering
 - Decays



13_{/25} DETAILS OF THE ANALYSIS

- $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions generated by EPOS359

- Observable: $D(\mathbf{r}_{LCMS}) = \int d\Omega dt D(t, \mathbf{r}_x, \mathbf{r}_y, \mathbf{r}_z)$

angle-averaged radial source distribution of like-sign pion pairs

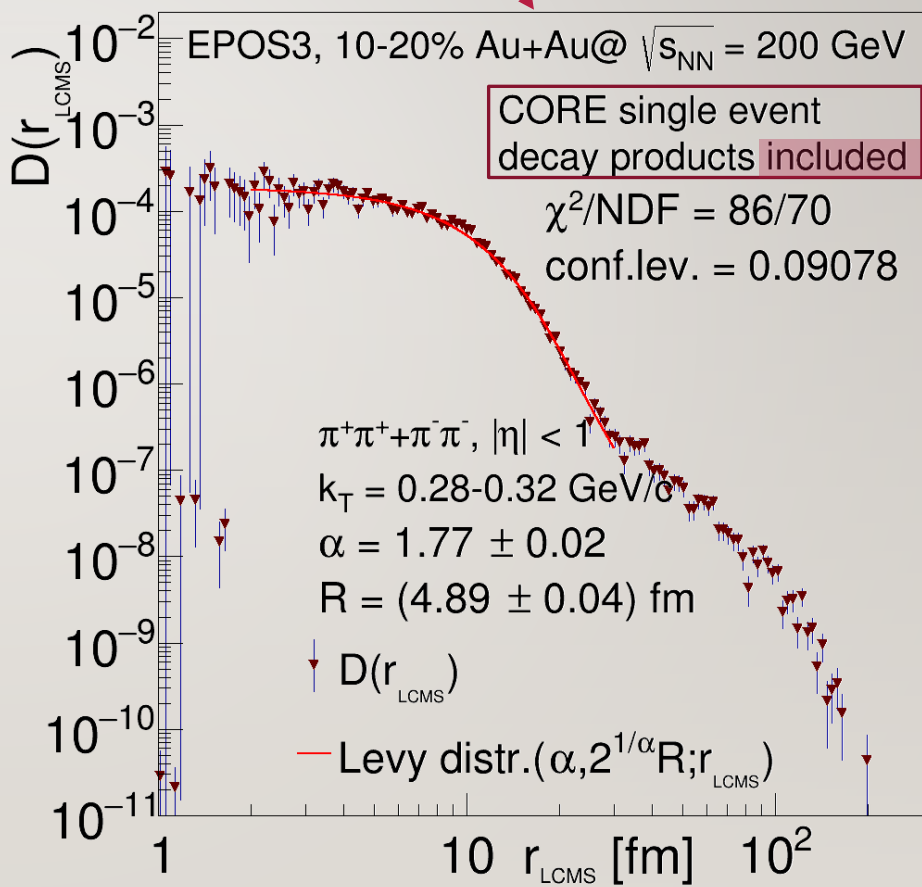
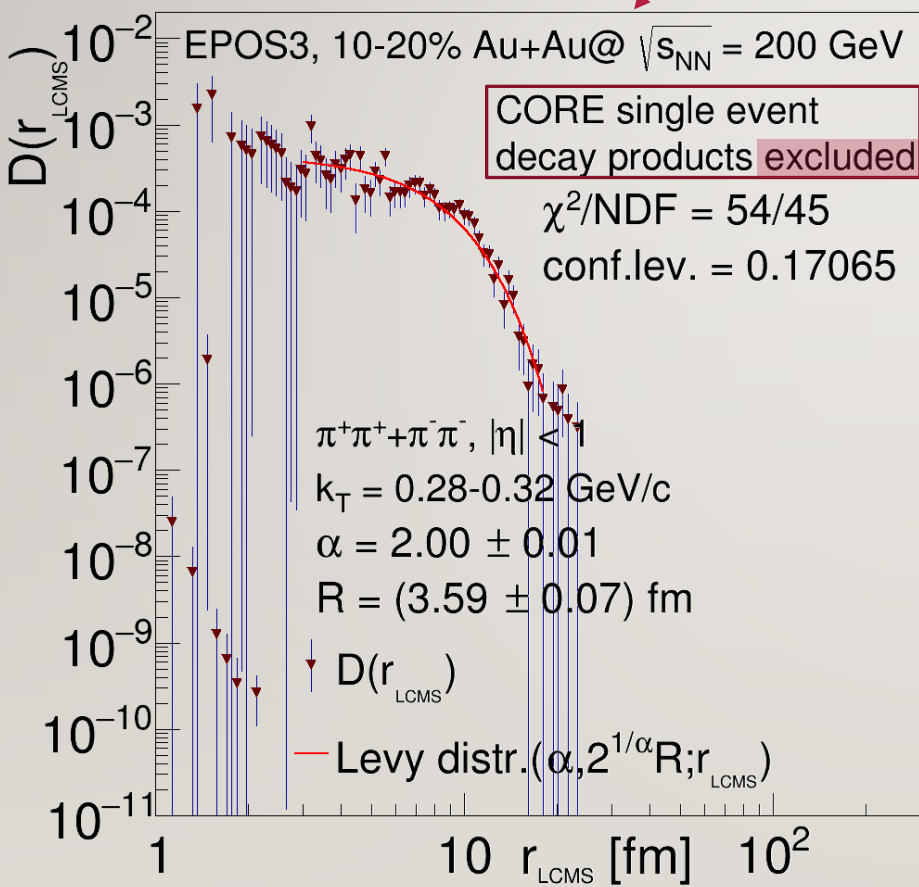
$$r_{LCMS} = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z_{LCMS})^2}; \Delta z_{LCMS} = \Delta z - \frac{\beta(\Delta t)}{\sqrt{1 - \beta^2}}; \beta = \frac{p_{z,1} + p_{z,2}}{E_1 + E_2}$$

- Investigated cases:
 - CORE, primordial pions only
 - CORE, decay products included
 - CORE+CORONA+UrQMD, primordial pions only
 - CORE+CORONA+UrQMD, decay products included



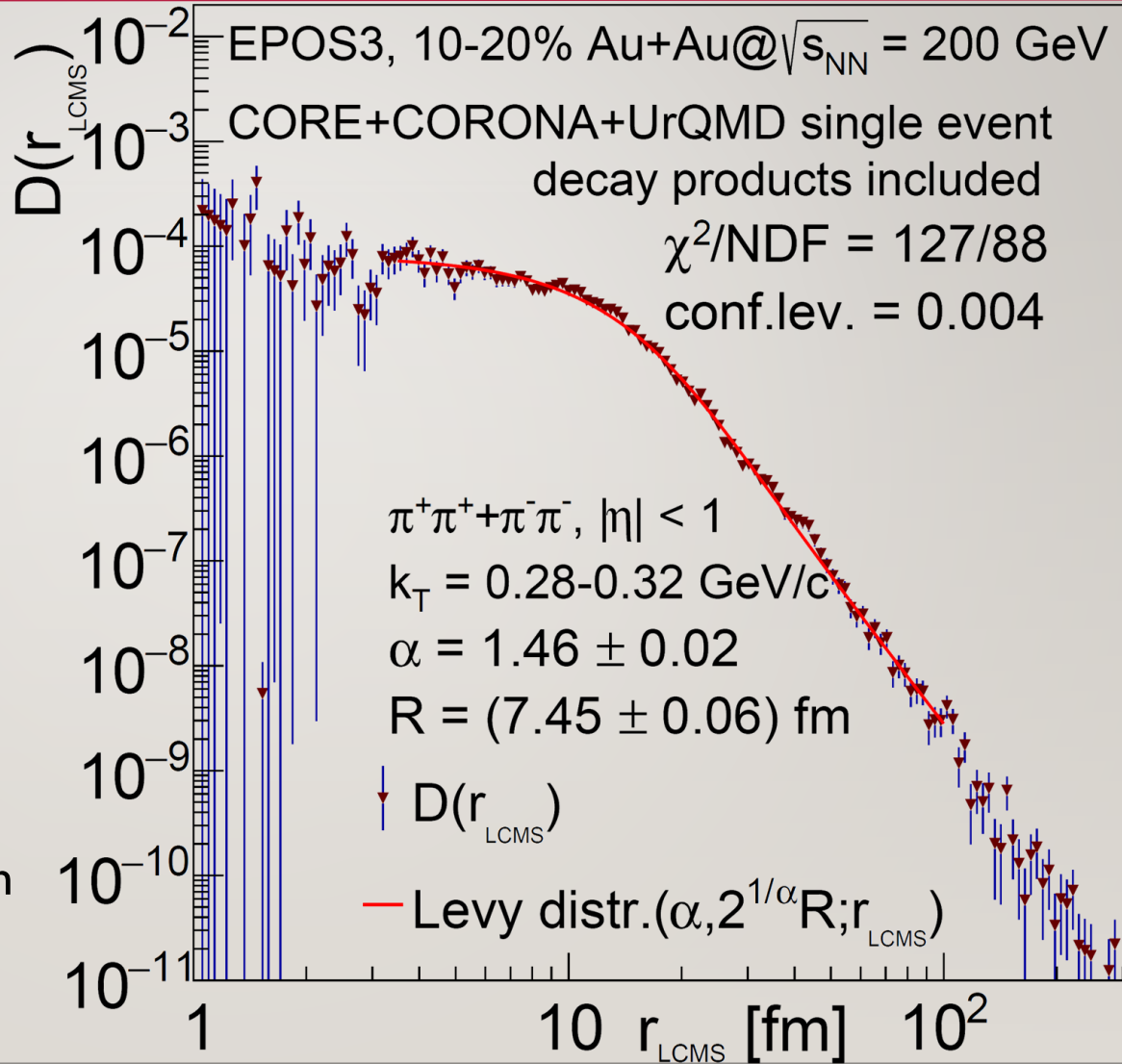
4/25 EXAMPLE SINGLE EVENT, CORE ONLY

- Gaussian shape without decays, additional structure with decays



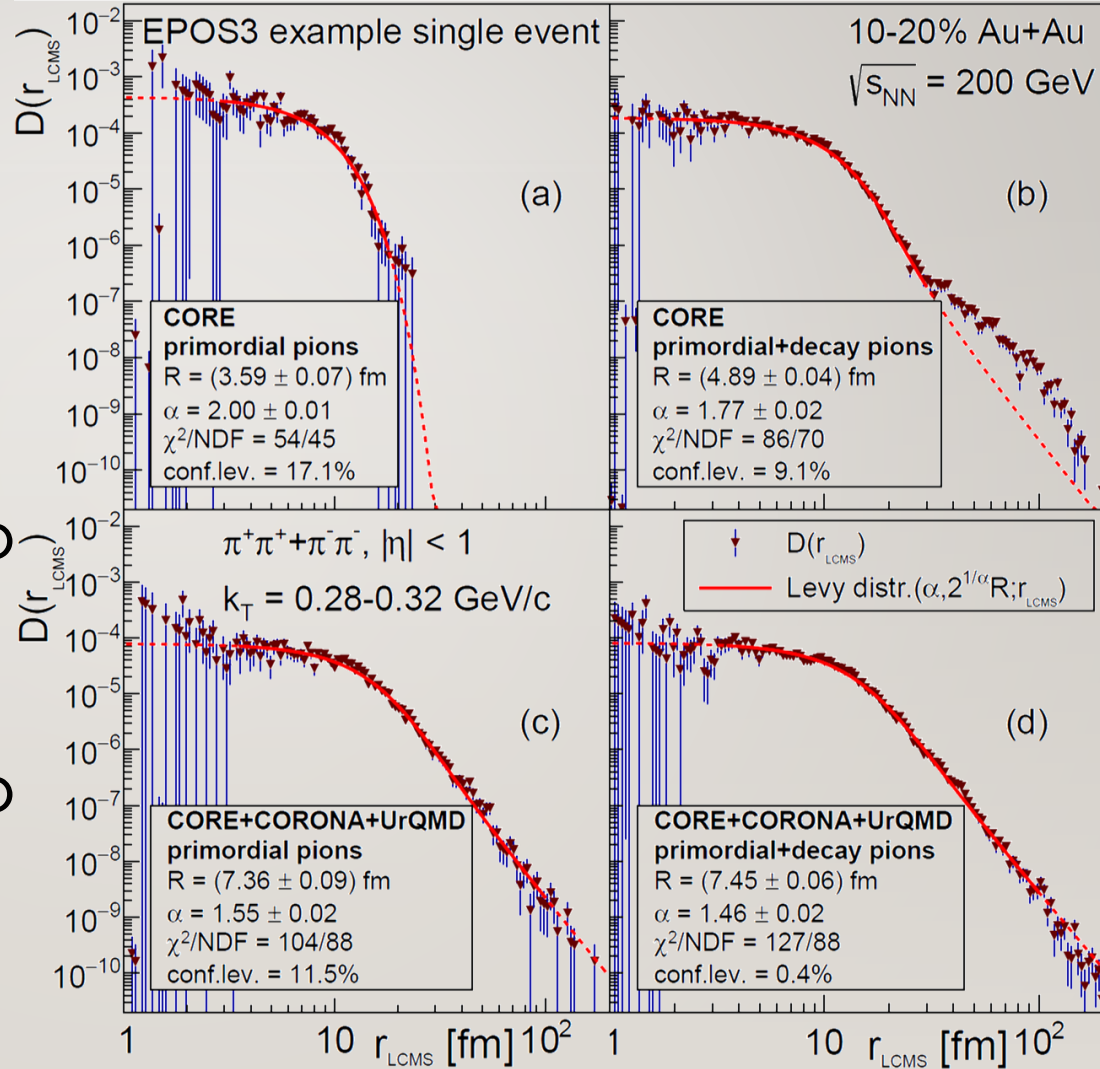
15_{/25} EXAMPLE EVENT, CORE+CORONA+URQMD

- Investigating $D(r)$ event-by-event
- Lévy-fits provide good description (2-100 fm range)
- Repeat such fits for thousands of events
- Extract α, R distribution



16/25 VARIOUS EVENT TYPES COMPARED

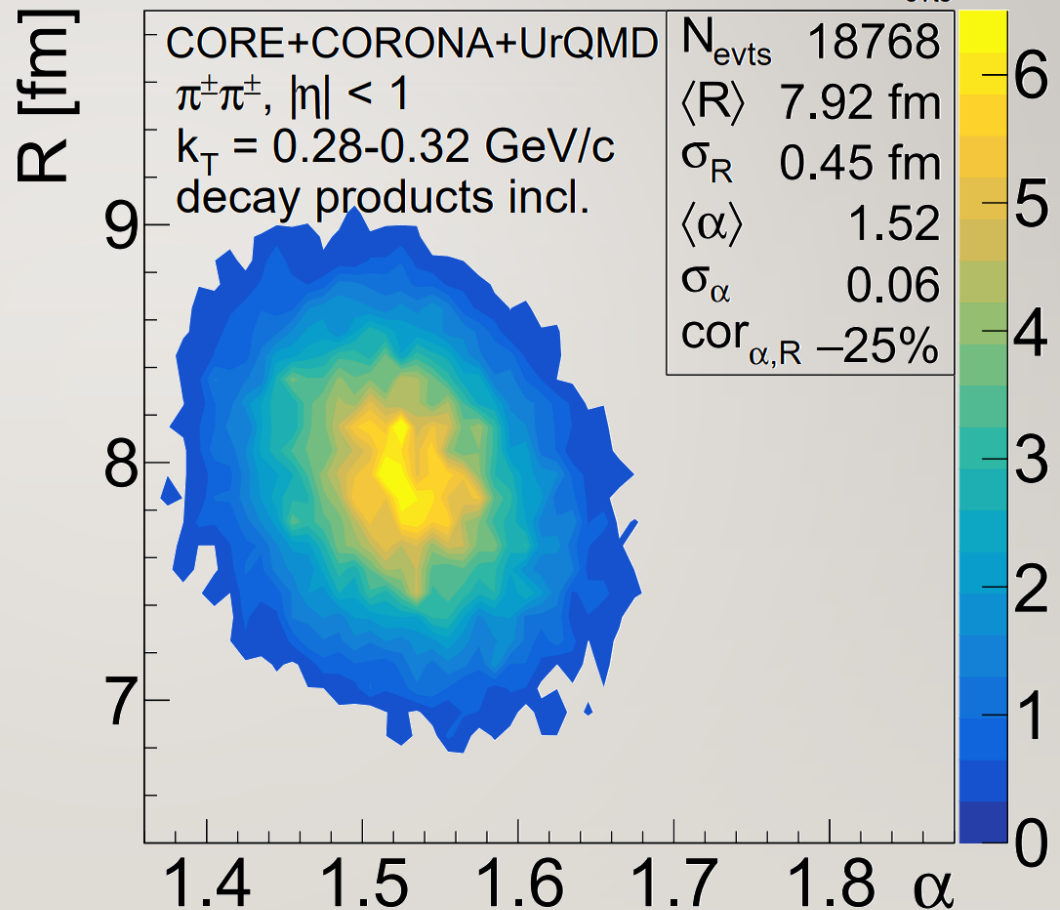
- CORE, primordial pions
 - Gaussian source
- CORE + decays
 - power-law structures
- CORE+CORONA+UrQMD
 - Lévy-shape
- CORE+CORONA+UrQMD +decays
 - Lévy-shape



17_{/25} DISTRIBUTION OF α , R PARAMETERS

- Normal distribution of α , R for given centrality & kT
- Extract mean and std.dev,
- Investigate centrality & kT dependence
- kT dependence investigated around the peak of the pair- kT distr. to have adequate stat.

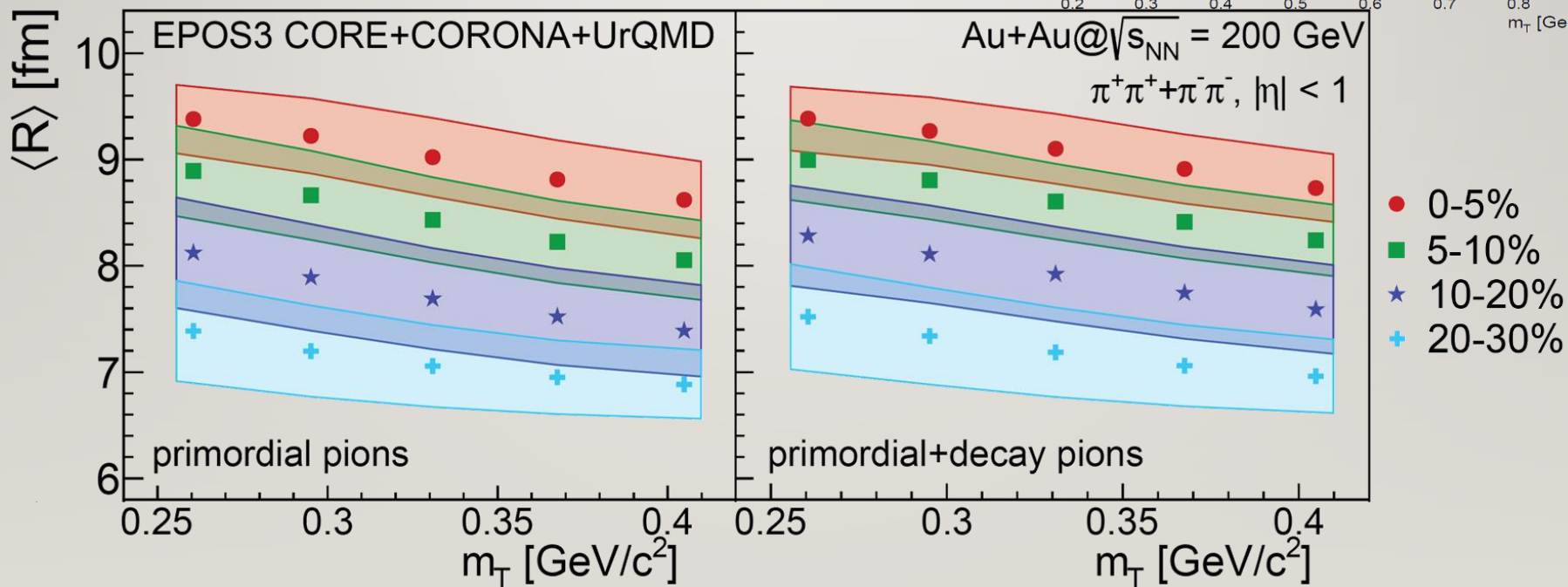
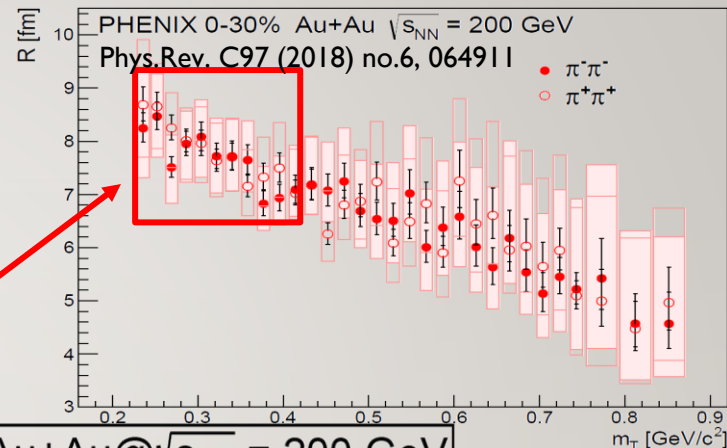
EPOS3 10-20% Au+Au@ $\sqrt{s_{NN}}=200$ GeV $\frac{1}{N_{\text{evts}}} \frac{d^2N}{dRd\alpha}$





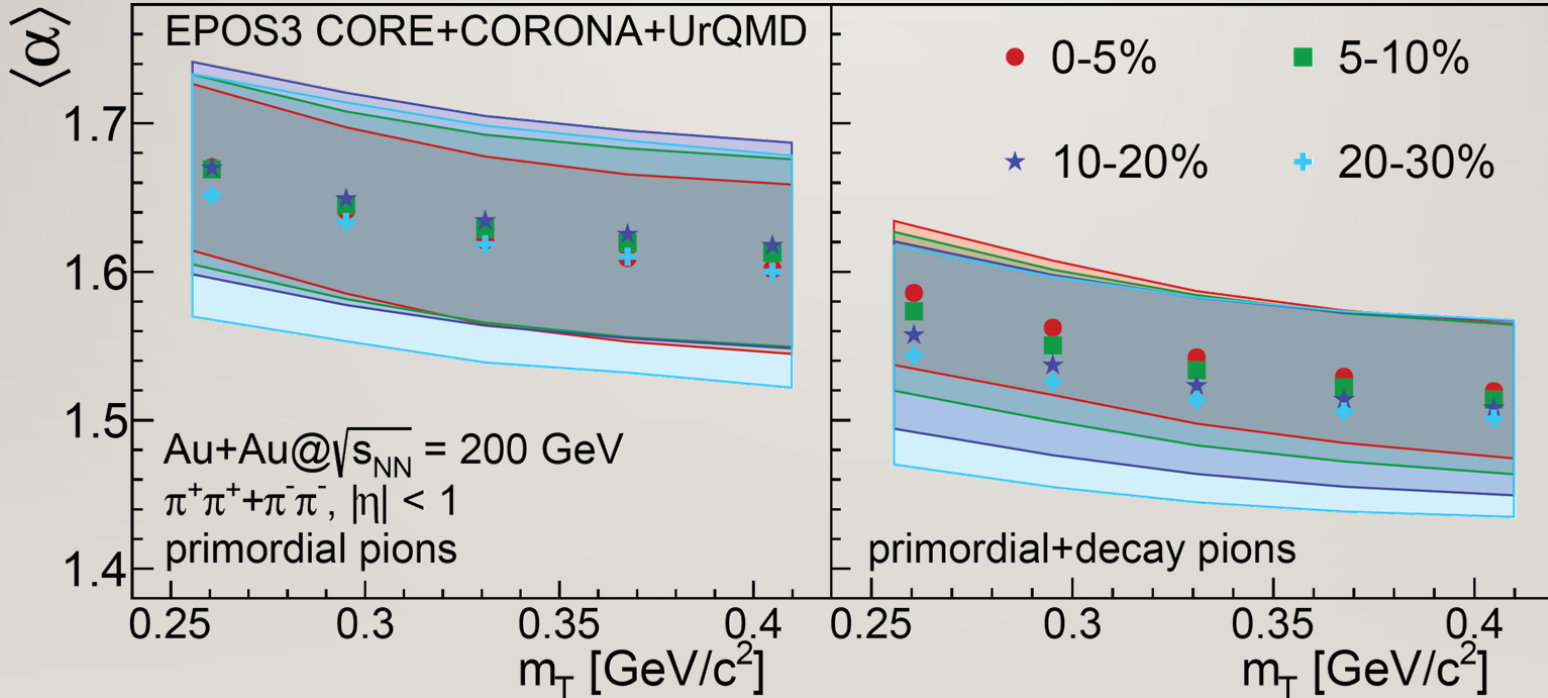
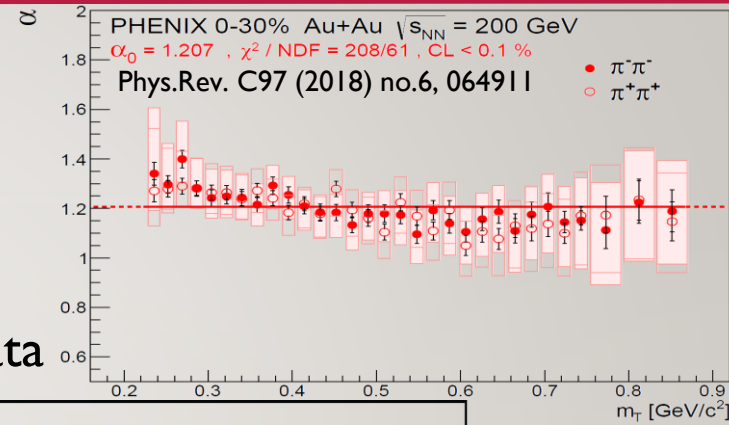
8/25 AVERAGE LÉVY SCALE R VS TRANSVERSE MASS

- $\langle R \rangle$ as a function of m_T and centrality
 - Clear dependence on both
 - Distribution width displayed as uncertainty band
- Trends, magnitudes like data
- With decays: slightly higher $\langle R \rangle$ values



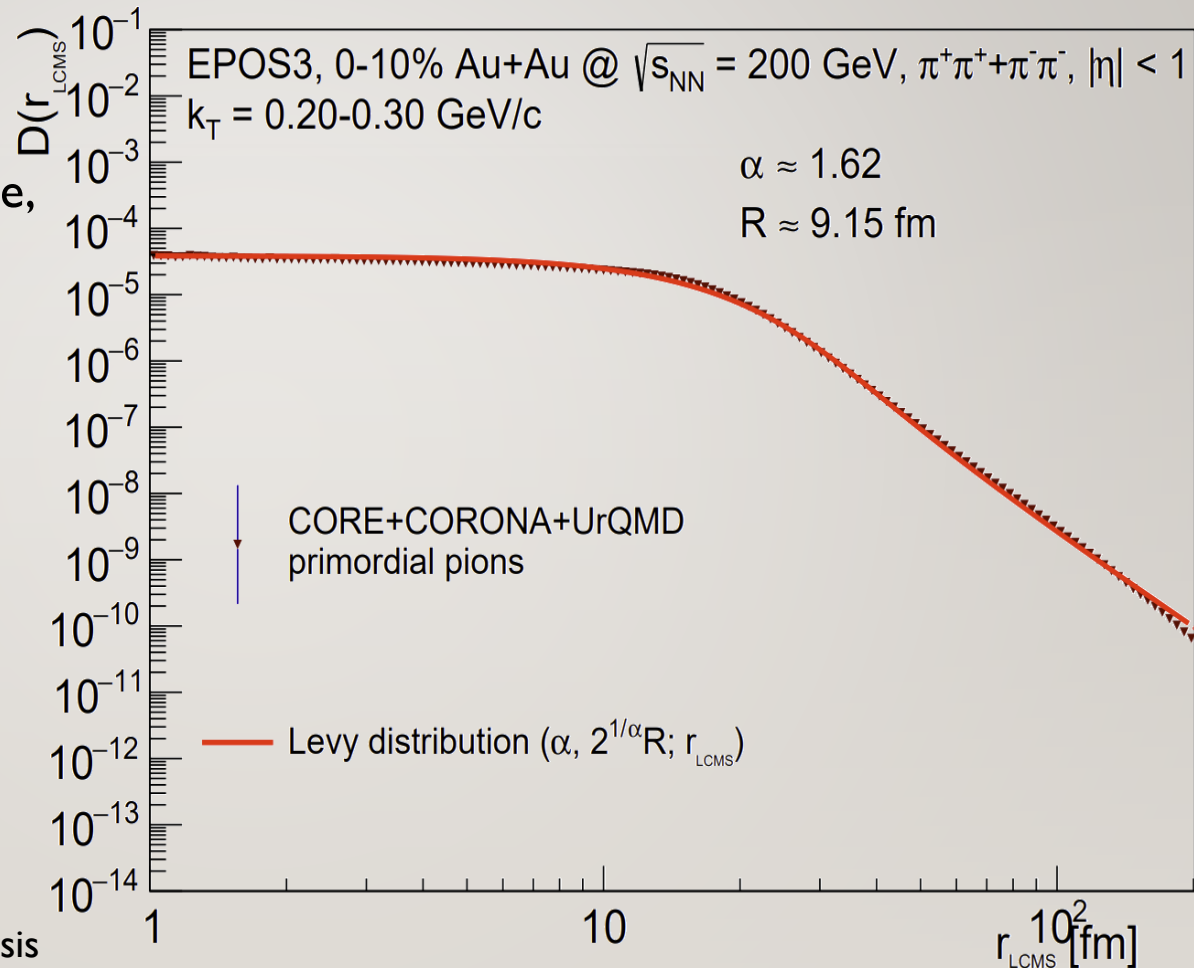
AVERAGE LÉVY EXPONENT VS TRANSVERSE MASS

- $\langle \alpha \rangle$ as a function of m_T and centrality
 - Small m_T and centrality dependence
- With decays: clearly smaller values
- Both with and without decays: larger than data



ROLE OF EVENT AVERAGING?

- Event-averaged source also analyzed
- Not perfectly Lévy shape, very large χ^2
- Nevertheless: similar parameters achieved
 - Event averaged:
 $\alpha \approx 1.62, R \approx 9.15$ fm
 - Event-by-event:
 $\alpha \approx 1.66, R \approx 8.96$ fm
- More reasonable approach for kaons
 - No event-by-event analysis possible for kaons



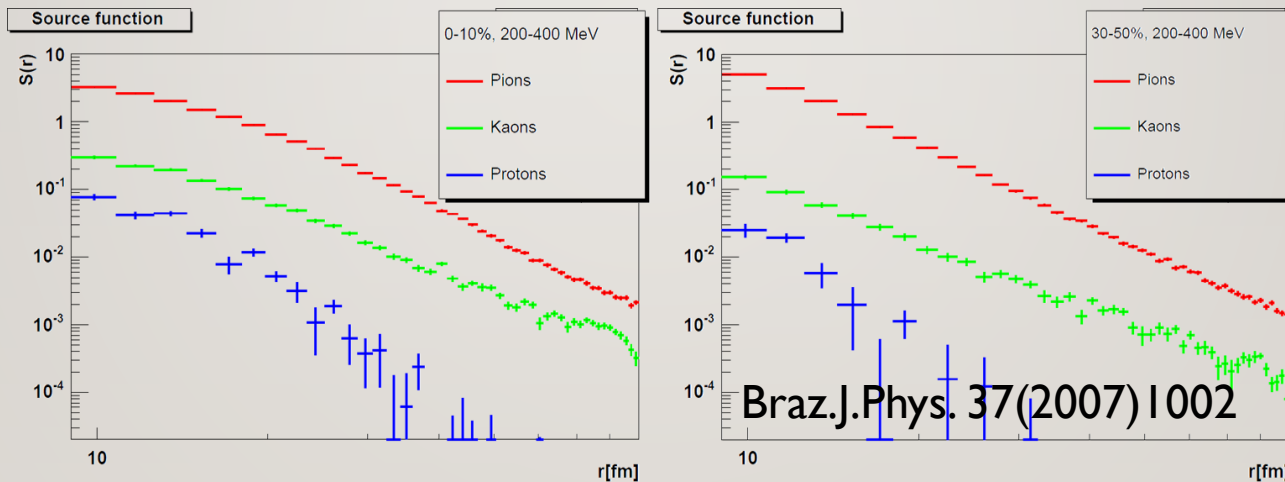


THE IMPORTANCE OF A KAON ANALYSIS

- Kaons: smaller cross-section, larger mean free path
- Heavier power-law tail?
- Prediction for π, K, p based on Humanic's Resonance Model (HRM): anomalous diffusion due to rescattering

Humanic, Int.J.Mod.Phys. E15 (2006) 197 [nucl-th/0510049]

Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002 [hep-ph/0702032]



- $R_{HBT}(Kaon)$ mT-scaling or its violation for Lévy scale R ?



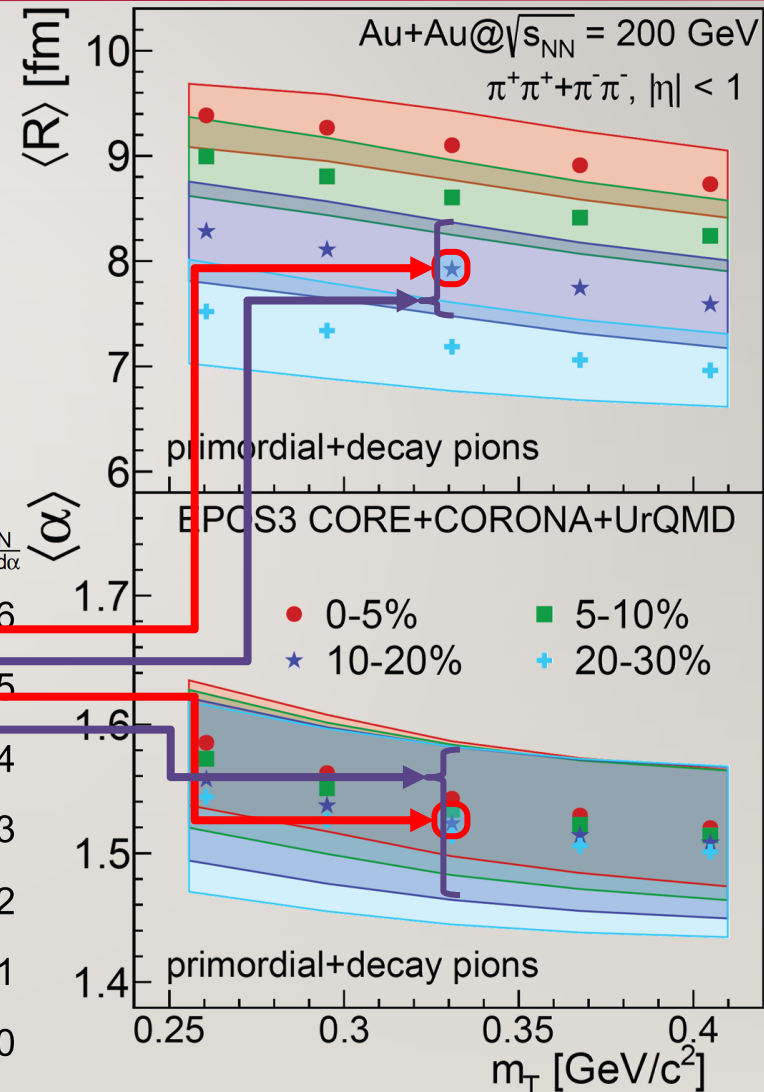
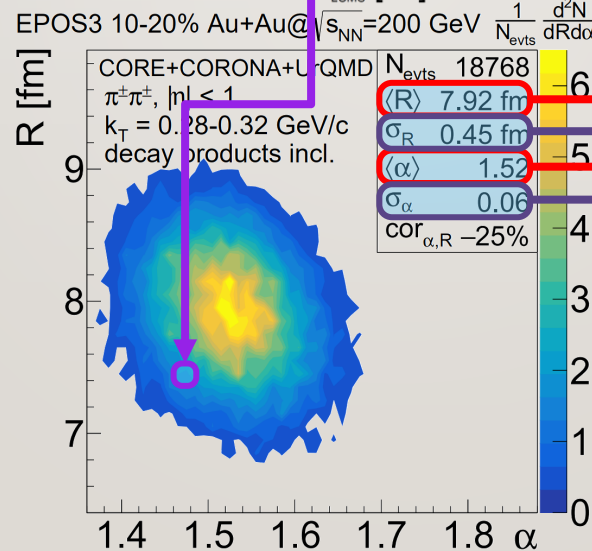
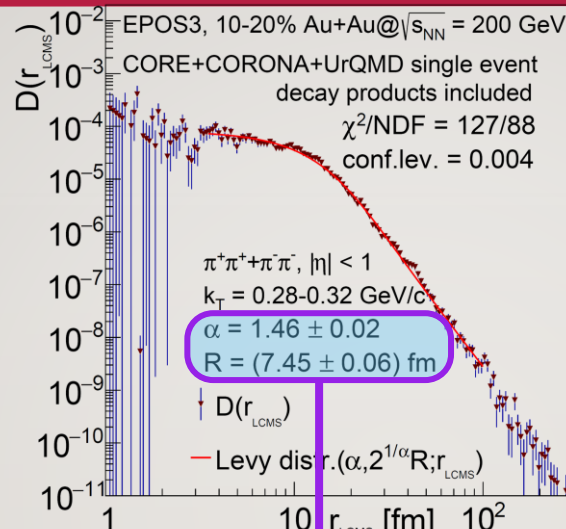
22/25

CONTENTS OF THIS TALK

- Basics of femtoscopy and Lévy sources
- EPOS event-by-event analysis
- Summary and outlook

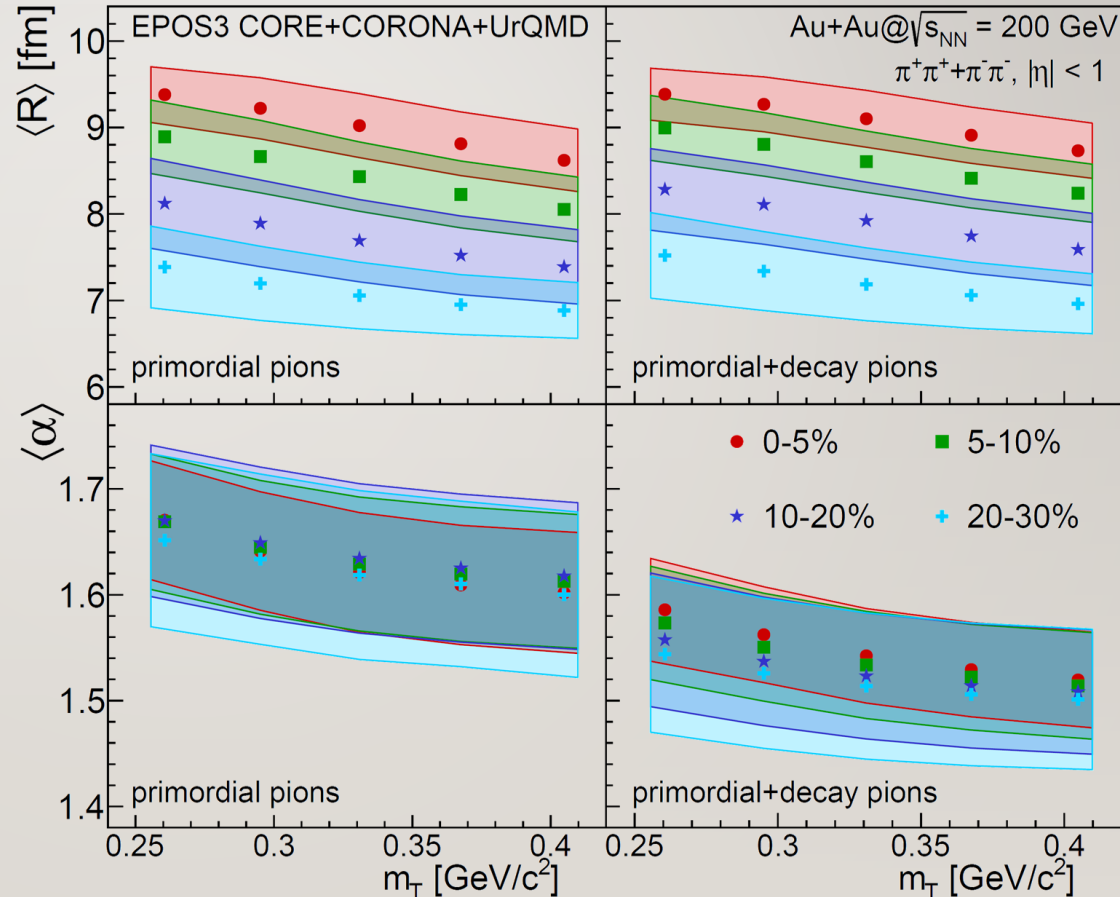
23_{/25} SUMMARY

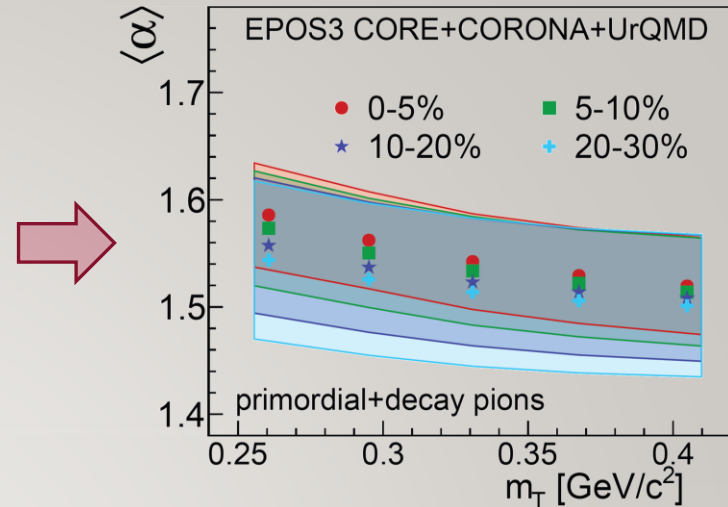
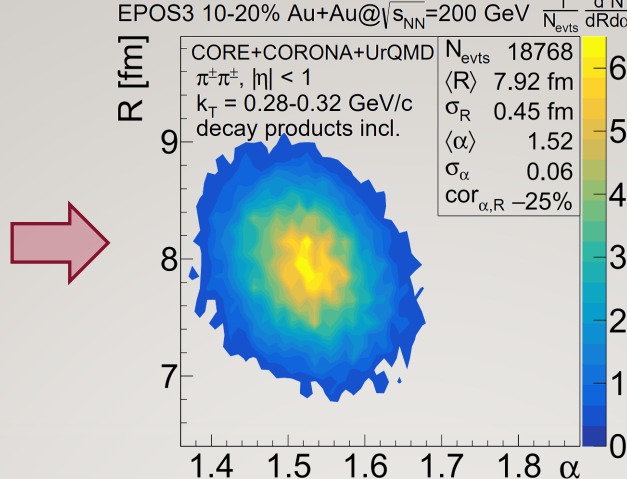
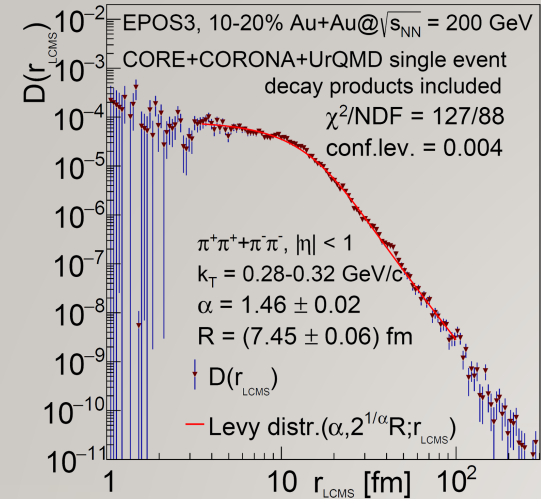
- $D(r)$ calculated in EPOS evt-by-evt
- Lévy fits done evt-by-evt
- Non-Gaussianity in single events
- Extracting mean, & std.dev. of R, α
- m_T & centrality dependence



CONCLUSIONS AND OUTLOOK

- Lévy fits done to **event-by-event** EPOS spatial distributions, good description
- Power-law tail strongly affected by rescattering and decays
- Lévy R in EPOS:
similar to data
- Lévy α in EPOS:
larger than data
- Details in:
 - Entropy 24 (2022) 308 [arXiv:2201.07962]
- Next steps:
 - Multiple dimensions
 - Different particle species
 - Correlation function





THANK YOU FOR YOUR ATTENTION

If you are interested in these subjects:

ZIMÁNYI SCHOOL 2022



Andrea Katalin Gulyás: Error 2

22nd ZIMÁNYI SCHOOL
WINTER WORKSHOP
ON HEAVY ION PHYSICS

December 5-9, 2022
Budapest, Hungary



József Zimányi (1931 - 2006)

<http://zimanyischool.kfki.hu/22>

26 BACKUP

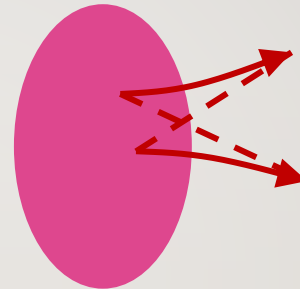
27 / 25 INTERACTIONS: THE COULOMB-EFFECT

- Plane-wave result, based on $|\Psi_2^{(0)}(r)|^2 = 1 + e^{iqr}$:

$$C_2(q, K) \cong \int D(r, K) |\Psi_2^{(0)}(r)|^2 dr = 1 + \int D(r, K) e^{iqr} dr$$

- If there is interaction:

$$\Psi_2^{(0)}(r) \rightarrow \Psi_2^{(int)}(r_1, r_2)$$



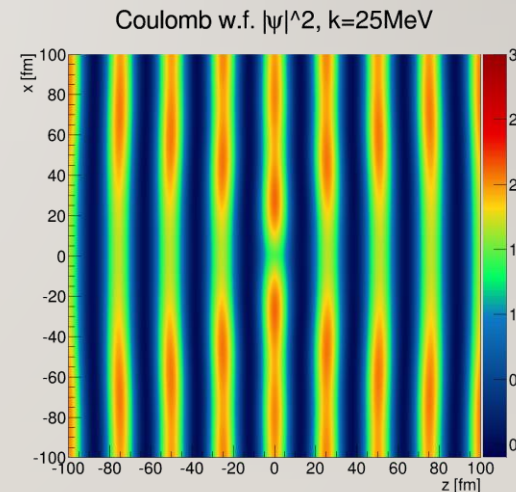
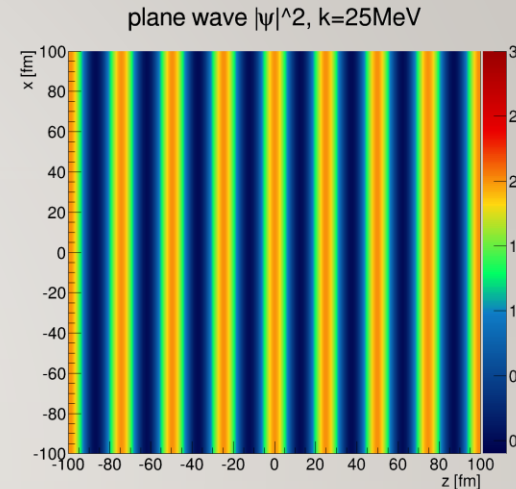
- For Coulomb:

$$|\Psi_2^{(C)}(r)|^2 = \frac{\pi\eta}{e^{2\pi\eta} - 1} \cdot (\text{complicated hypergeometric expression})$$

- Direct fit with this, or the usual iterative Coulomb-correction:

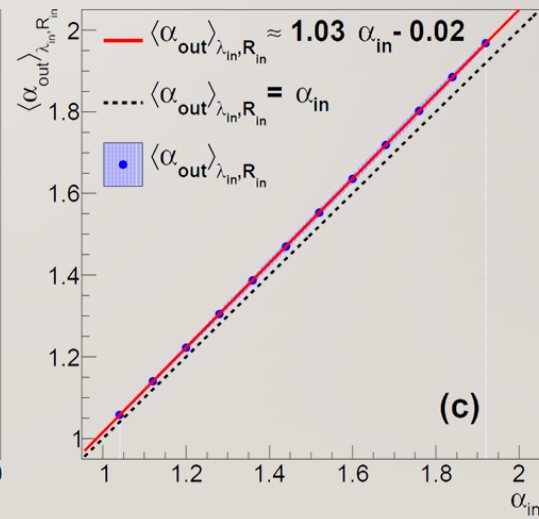
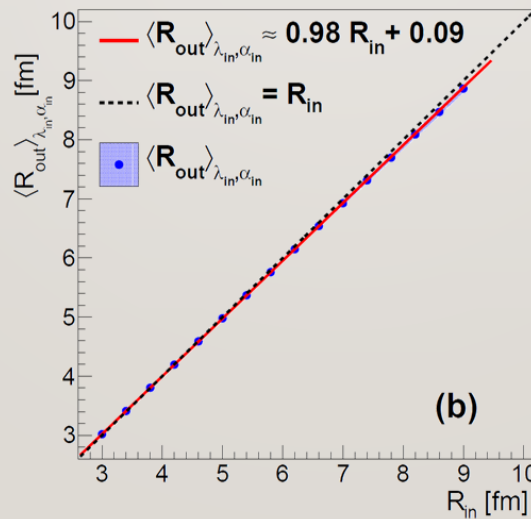
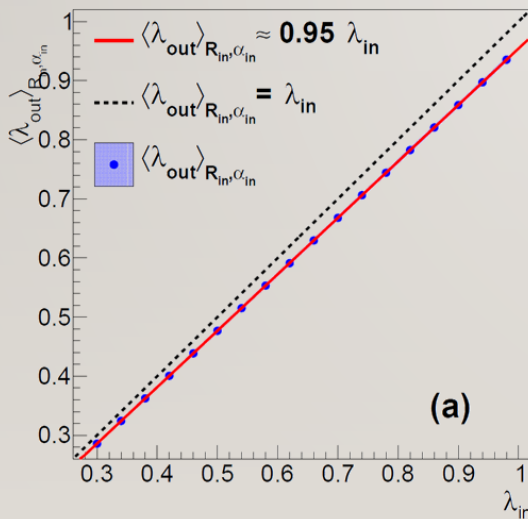
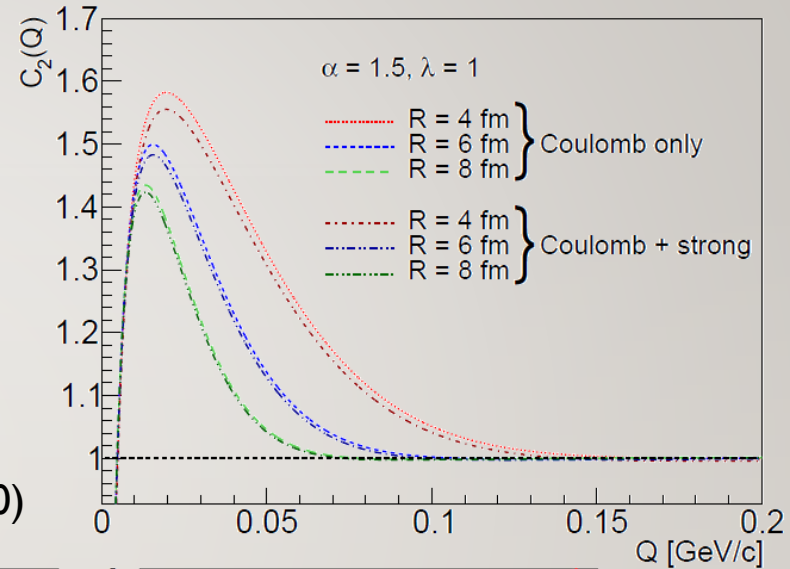
$$C_{\text{Bose-Einstein}}(q)K(q), \text{ where } K(q) = \frac{\int D(r, K) |\Psi_2^{(C)}(r)|^2 dr}{\int D(r, K) |\Psi_2^{(0)}(r)|^2 dr}$$

- Complication: need for integrating power-law tails
- Parametrization possible
 - MCs, S. Lökös, M. Nagy, Phys. Part. Nuclei 51 (2020) 238
 - MCs, S. Lökös, M. Nagy, Universe 5 (2019) 6, 133



28_{/25} STRONG INTERACTION FOR PION PAIRS

- Additional potential appearing
- Possible handling: strong phase shift, modify s-wave component in wave func.
R. Lednicky, Phys. Part. Nucl.40, 307 (2009)
- Small difference in case of pions
- Few percent modification in λ, α
D. Kincses, M. I. Nagy, M. Cs. PRC 102, 064912 (2020)



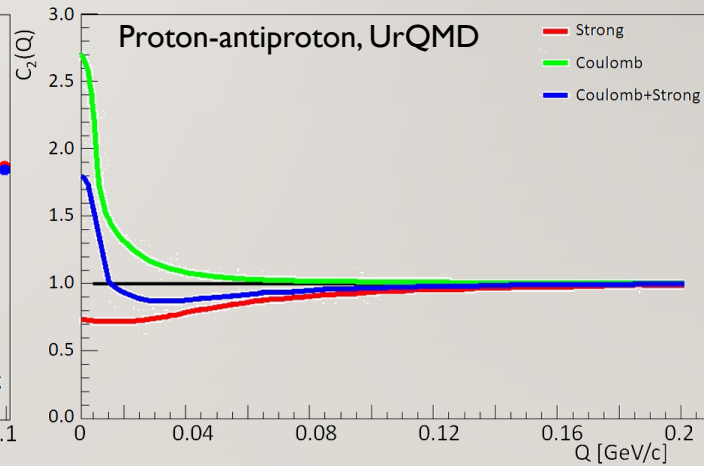
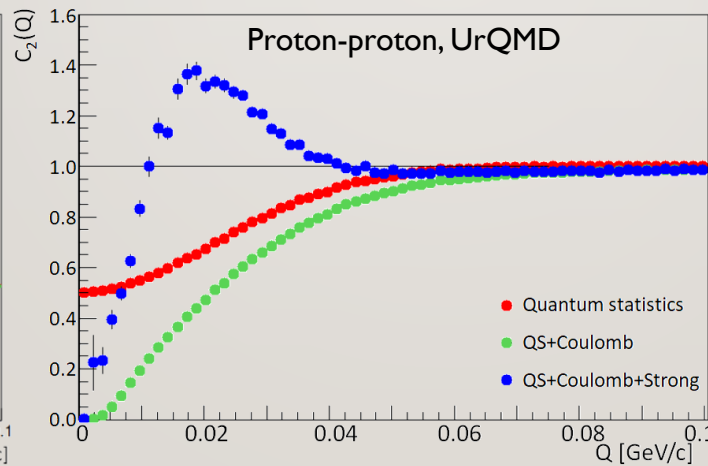
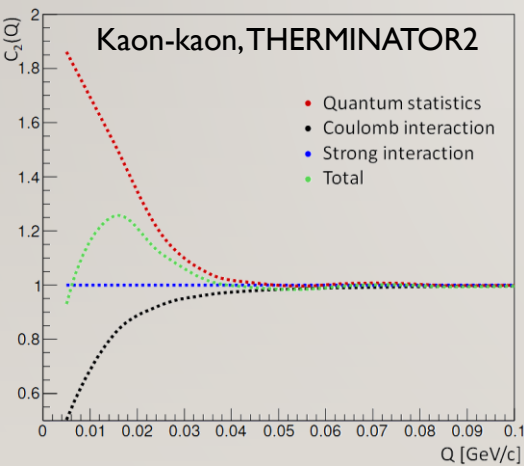


29/25 ROLE OF THE STRONG INTERACTION

- In case of other interactions or not identical bosons, the formula still works:

$$C_2(q, K) \cong \int D(r, K) |\Psi_2(r)|^2 dr$$

- Pair wave function determines $D \leftrightarrow C_2$ connection
- Mesons, baryons: strong interaction; fermions: anticorrelation
- Non-identical pairs: interaction modifies wave function

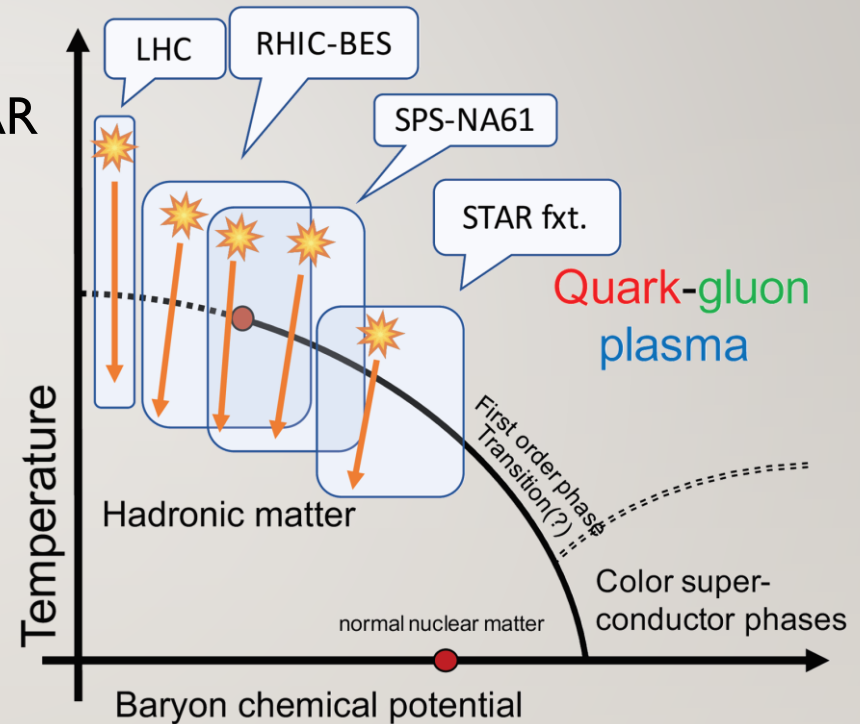


From e.g. H. Zbroszczyk's talk at Zimányi School 2019



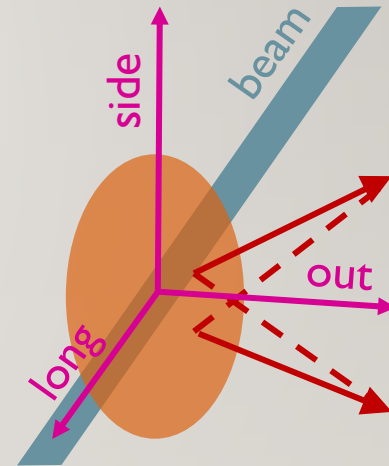
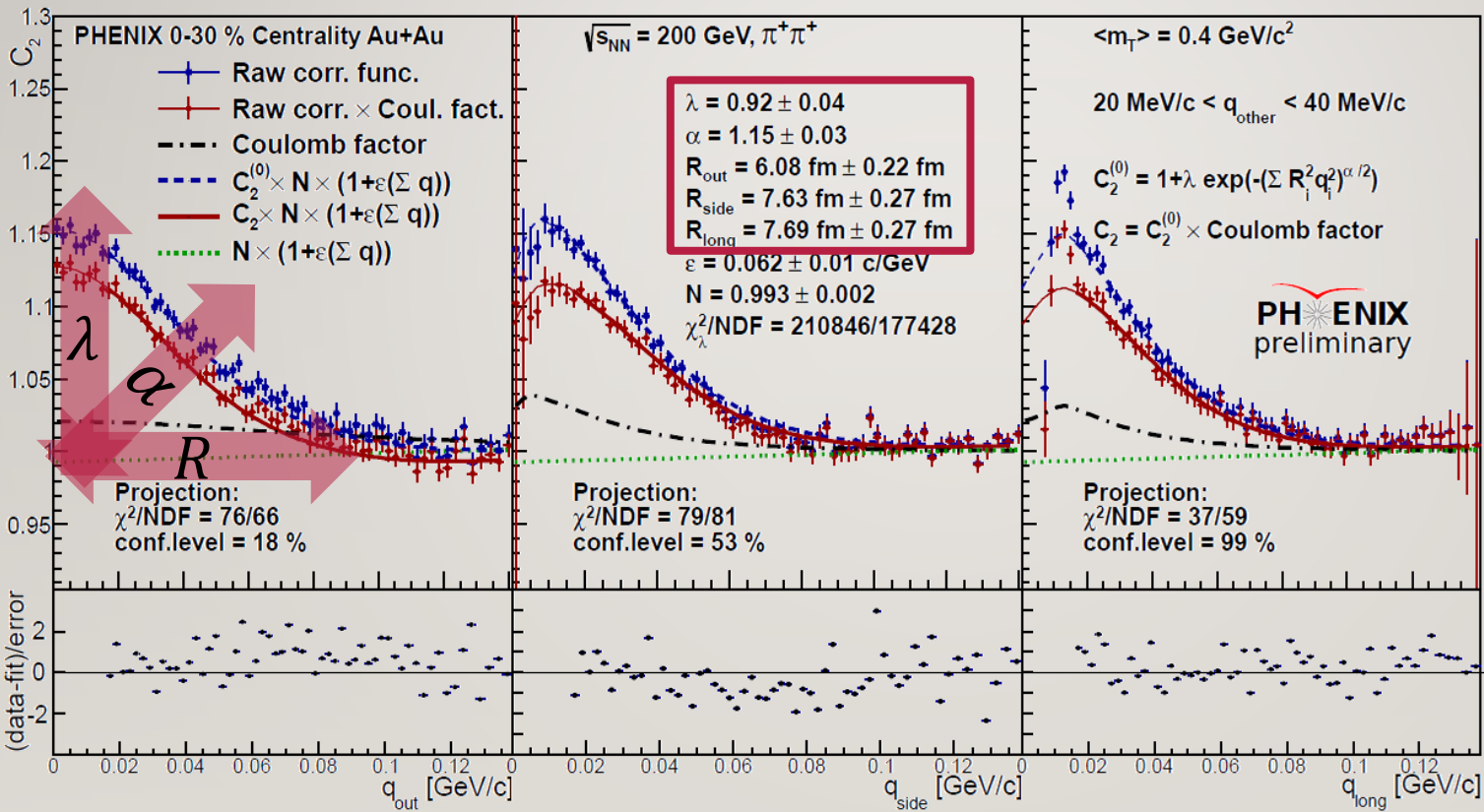
HBT MEASUREMENTS AND THE PHASE DIAGRAM

- LHC: measurement at CMS
 - 2-5 ATeV energy, p+p & Pb+Pb
- RHIC: measurement at PHENIX+STAR
 - 10-200 AGeV energy, Au+Au
- SPS: measurement at NA61
 - 17 AGeV energy, Be+Be
- Phase diagram can be investigated



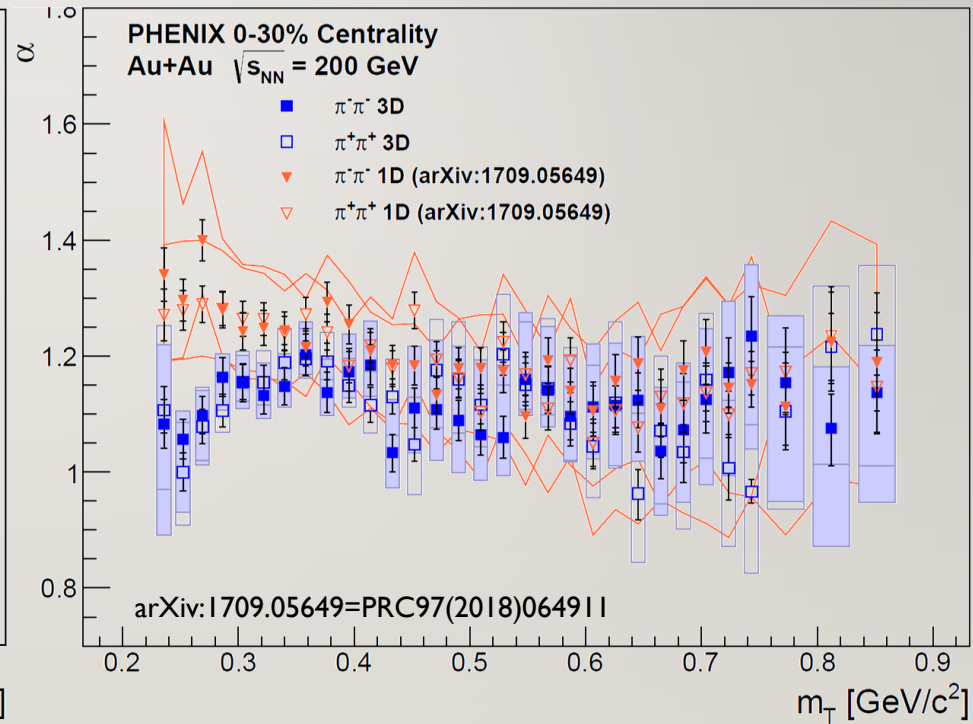
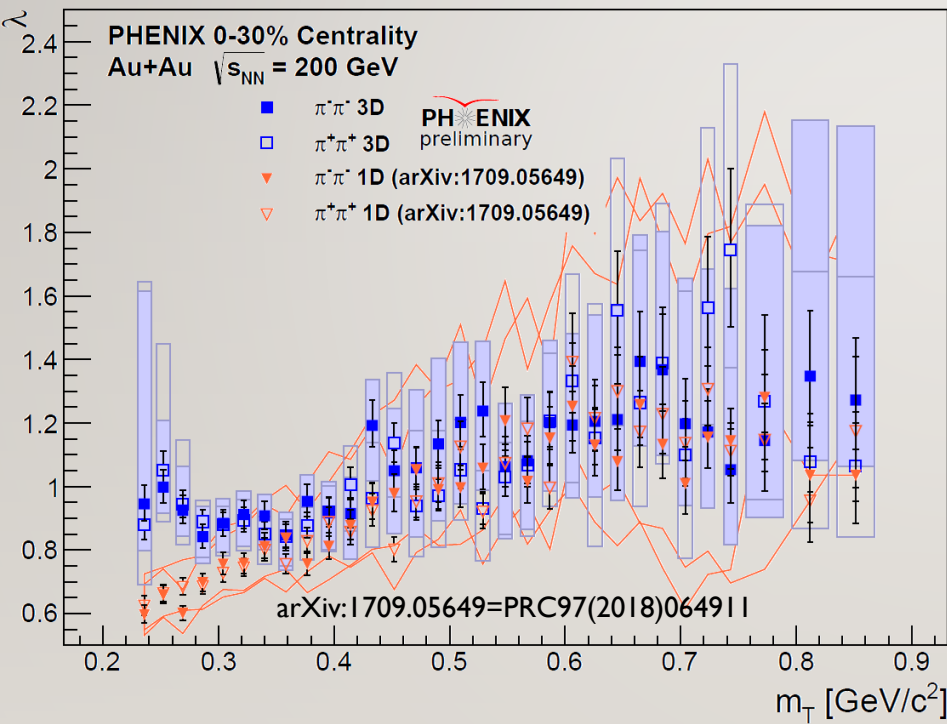
31 /25 A CROSS-CHECK: 3D LÉVY FEMTOSCOPY

- Femtoscopy done in 3D: Bertsch-Pratt pair frame (out/side/long coordinates)
- Physical parameters: $R_{\text{out/side/long}}$, λ , α measured versus pair m_T
- Fit in this case: modified log-likelihood (small statistics in peak range)



32_{/25} 3D VERSUS 1D: STRENGTH λ AND SHAPE α

- Compatible with 1D (Q_{LCMS}) measurement of PRC97(2018)0649 I I
- Small discrepancy at small m_T : due to large R_{long} at small m_T ?





OPEN QUESTIONS

- Collision energy and centrality dependence of Lévy parameters?
 - Non-monotonicity in $\alpha(\sqrt{s_{NN}})$ or $\alpha(\text{centrality})$?
 - Hole in $\lambda(m_T)$ at low $\sqrt{s_{NN}}$? Really due to η' ?
- Reason for the appearance of Lévy distributions for pions?
 - What is the Lévy exponent for kaons?
 - Kaons have smaller total cross-section thus larger mean free path, heavier tail?
 - Does m_T scaling hold for Lévy scale R ?
- Correlation strength versus core-halo picture: are there other effects?
 - Three-particle correlations may show if coherence or other effects play a role
 - Other effects may also play a role (finite meson sizes, random field phase shift, etc)