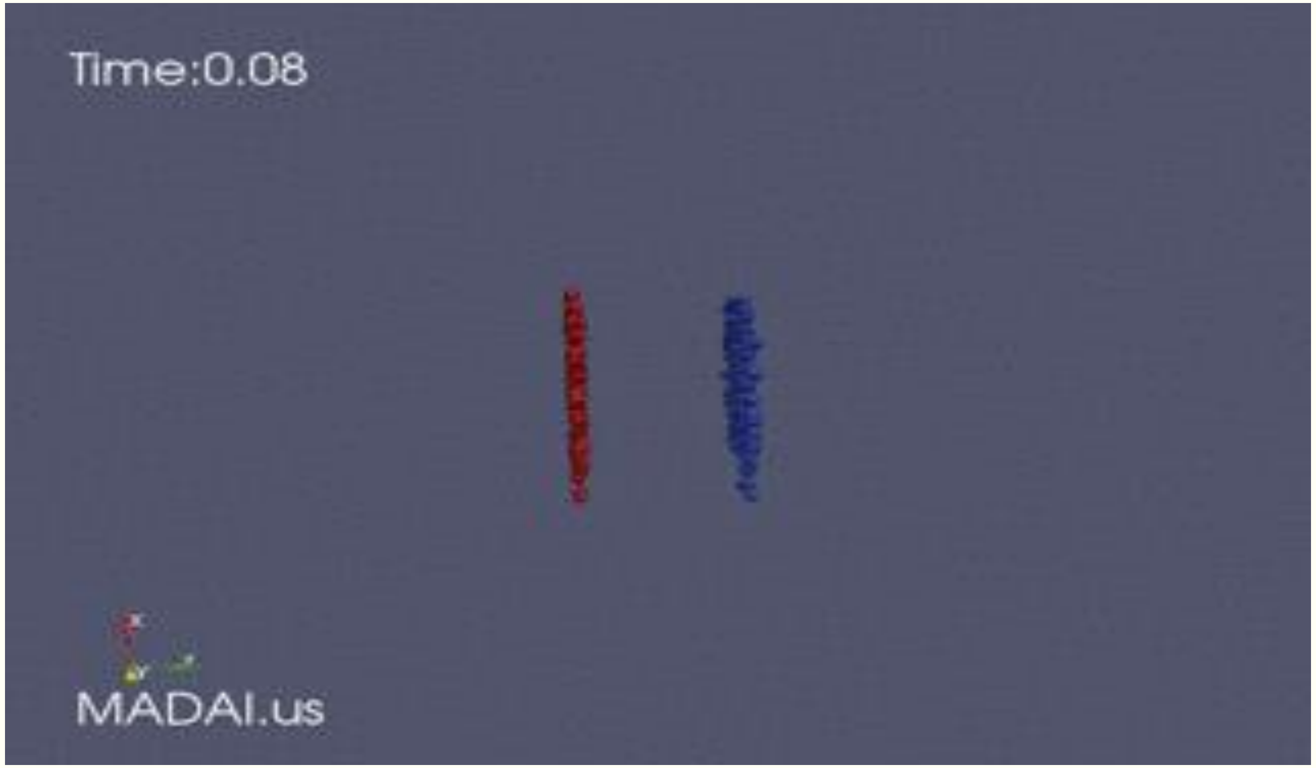
Thermodynamics of the 'Little Bang'



Tapan K. Nayak

Collaborators:

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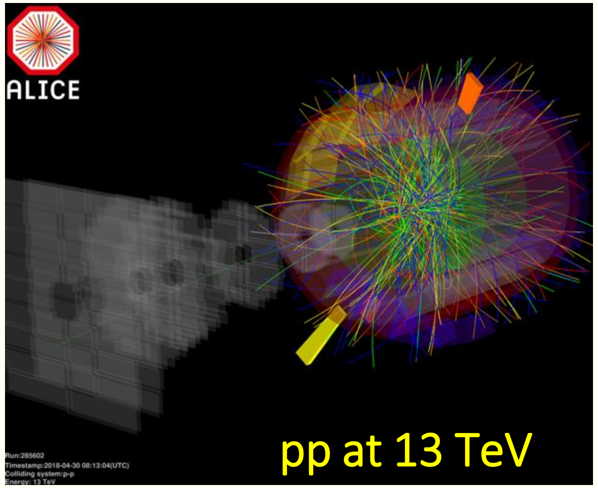
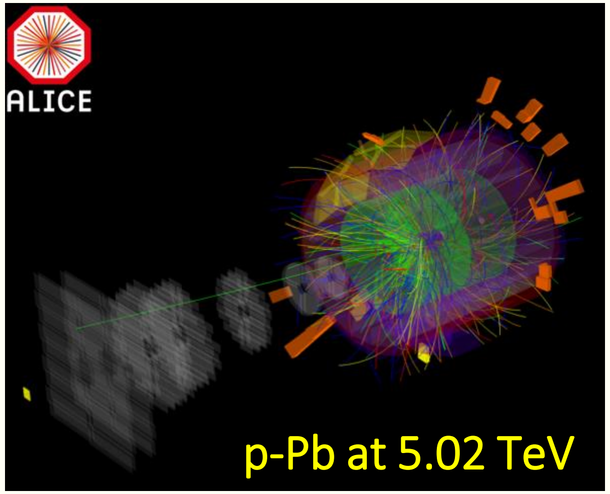
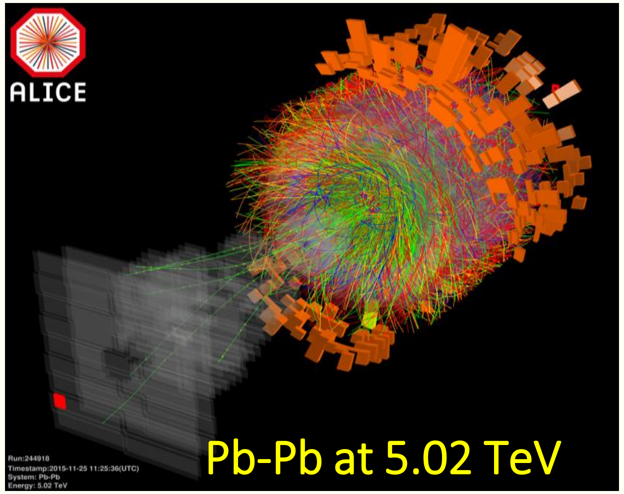
Thanks to
Claude Pruneau and
Rene Bellwied

2 March 2022

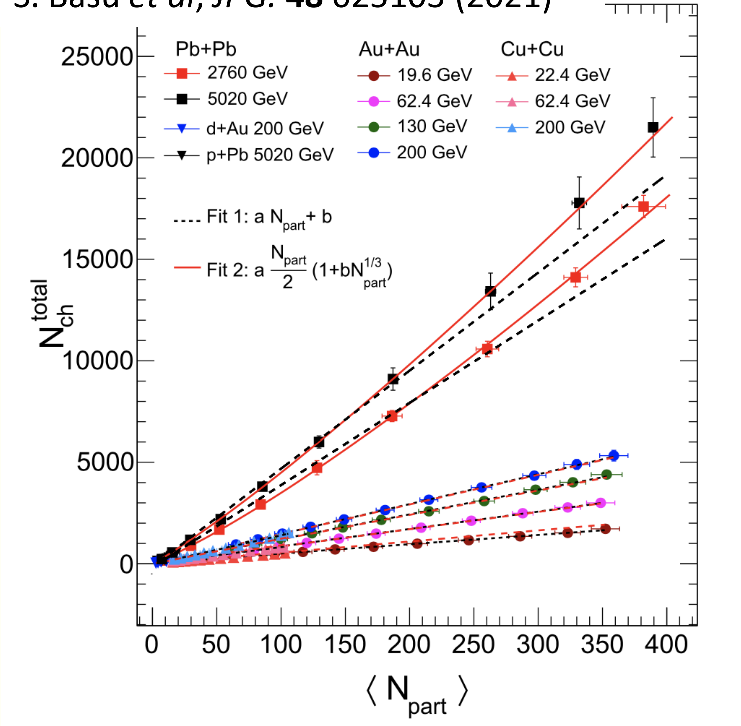
Applicability of thermodynamics



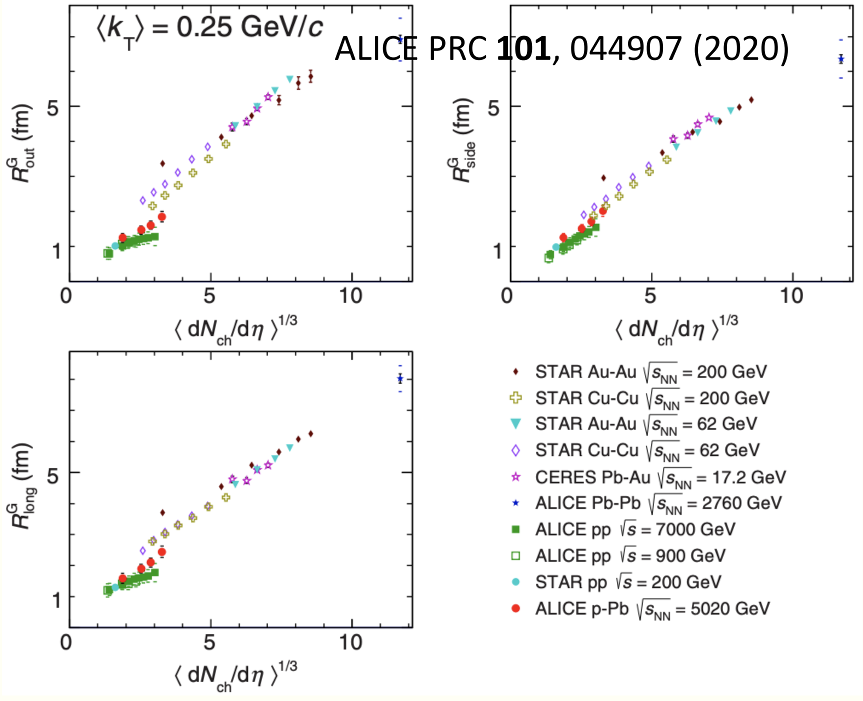




S. Basu *et al*, *JPG.* **48** 025103 (2021)



ALICE PRC **101**, 044907 (2020)



- Growth in the number of produced particles as well as volume as a function of centrality and collision energy (keeping N/V roughly constant)
- Success of hydrodynamic calculations in explaining a large number of experimental results

Properties of Nuclear Matter

Thermodynamics is applicable when dealing with *equilibrated system: equilibrated* in the sense that approximately all available phase space is equally populated.

- Basic Observables:

  Temperature, pressure, volume, entropy density, and energy density, ..

- Observables/Properties of interest (pertaining to the properties of nuclear matter)

  Properties defining/determined by the equation of state (EOS) of the QGP matter, including response functions such as the heat capacity, the isothermal compressibility, as well as the speed of sound.

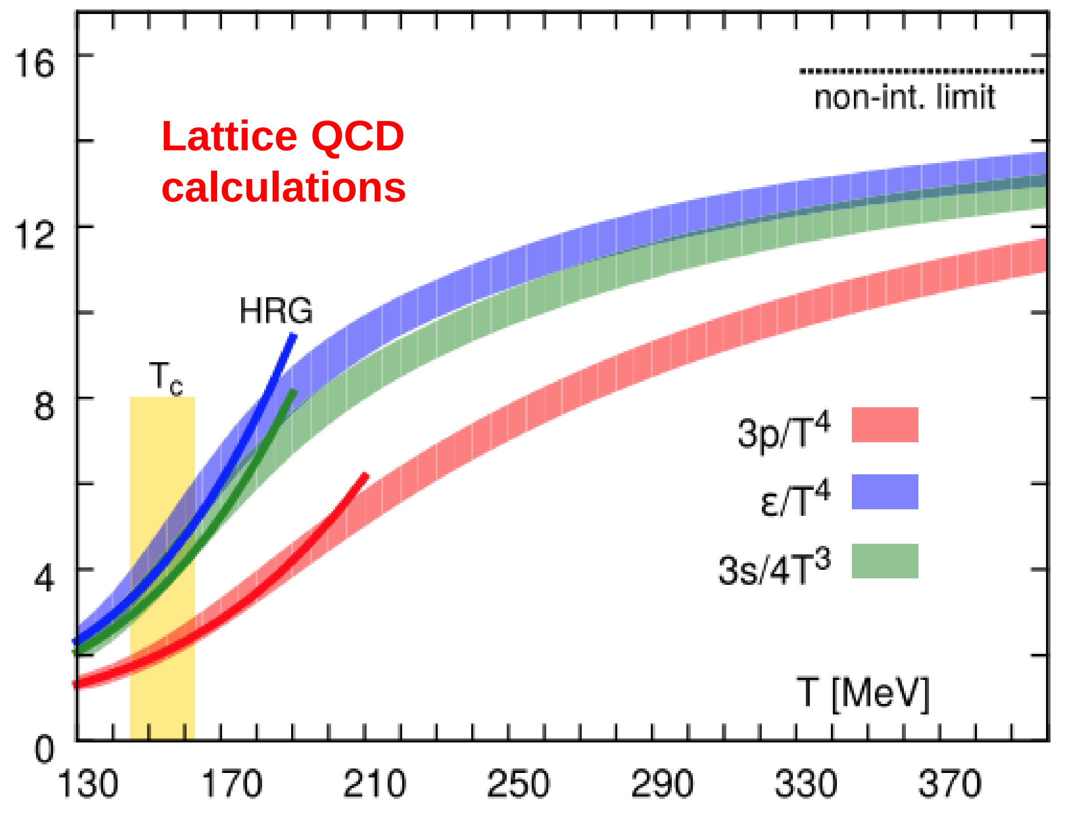
- Accessing thermodynamic properties of the system

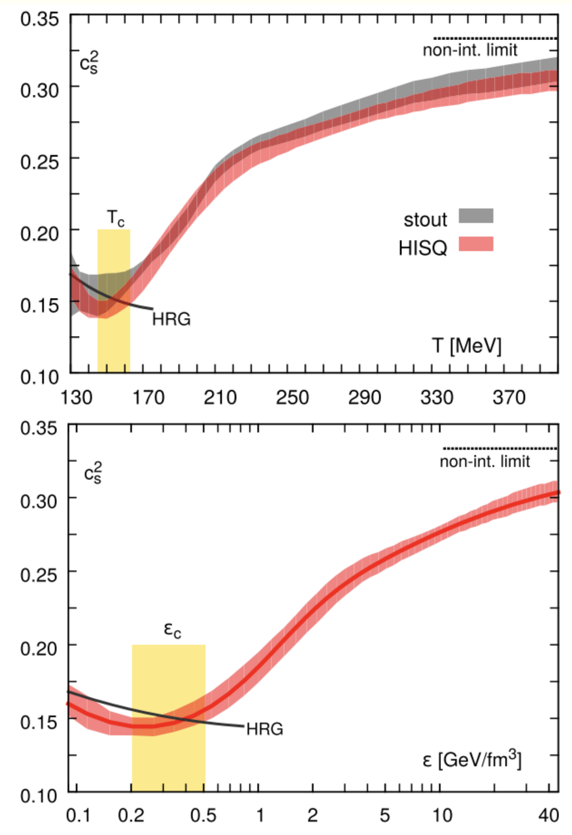
  In heavy-ion collisions after thermalization the system evolves hydrodynamically and its behavior depends on the EOS: connecting energy density, pressure, volume, temperature ….

EOS: thermodynamic equation relating variables which describe the state of matter under a given set of physical conditions.
Extensive variables: depend on the system size: energy, volume, or particle number, entropy ….
Intensive variables: do not depend on system size.

Properties of Nuclear Matter: Predictions/Expectations

HOTQCD Collaboration
Phys. Rev. D90 (2014) , 094503





Claudia Ratti's talk on Tuesday

Lattice Calculations of

- Pressure, energy density, entropy density as a function of *T*
- Speed of sound square as a function of *T* and energy density.

Pseudo-critical temperature for chiral crossover transition

- T_c = (154 ± 9) MeV
- ε_c ≈ (0.34 ± 0.16) GeV/fm3

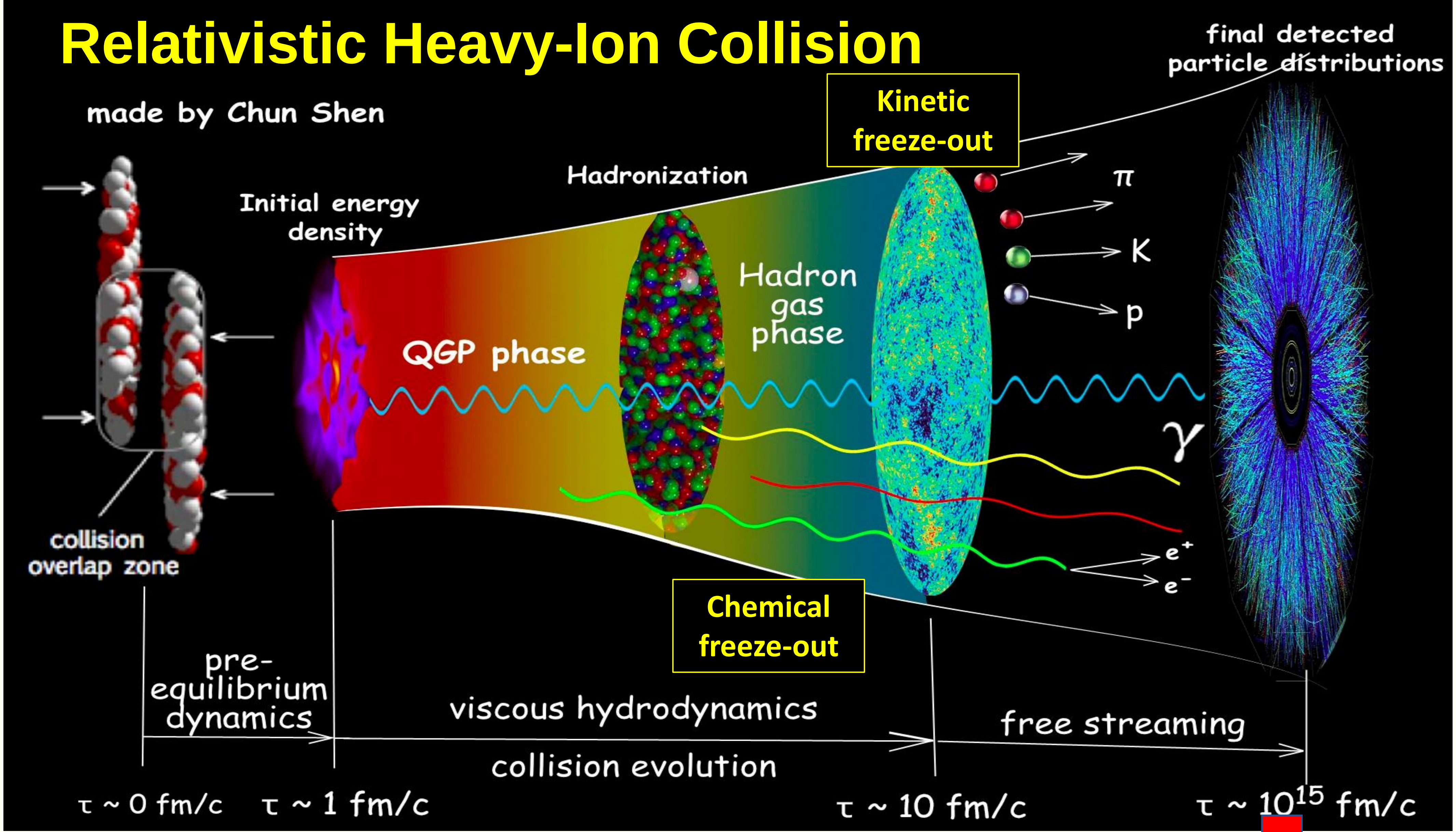
Hydro calculations use the EOS from lattice QCD.

Stefan-Boltzmann limit:

$$\varepsilon = g\frac{\pi^2}{30}T^4$$

- For hadronic matter, g=3
- For QGP: dof increases by ~10 (8 gluons, 2 quark flavours, 2 antiquarks, 2 spins, 3 colors)

Experimental measurement of temperature and other thermodynamic quantities gives access to the number of degrees of freedom …



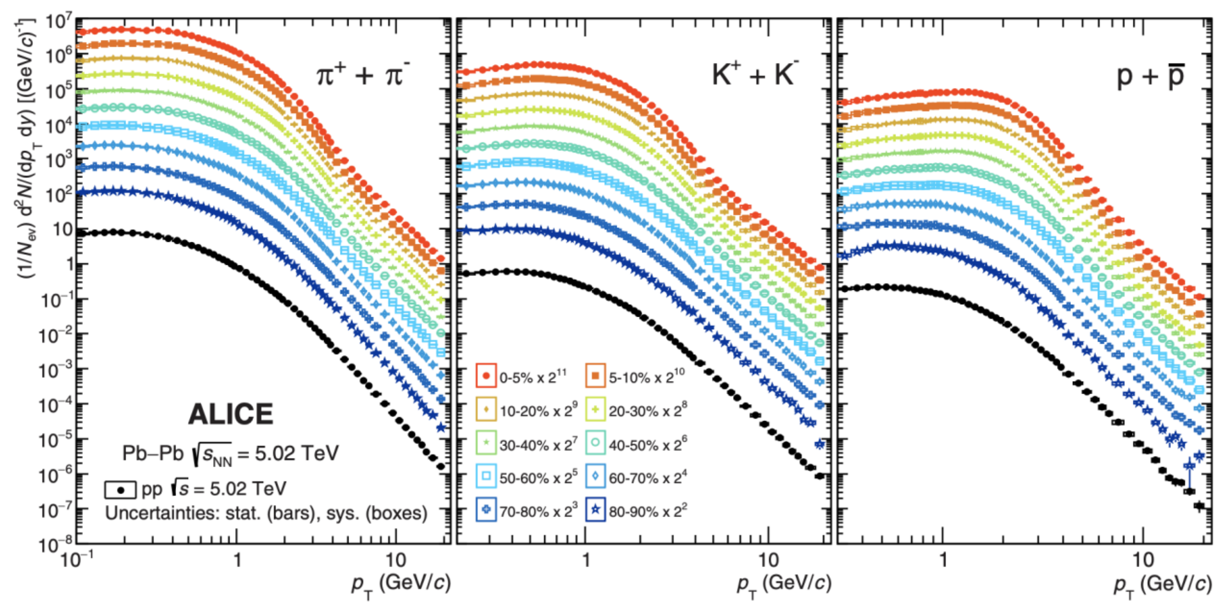
Initial State Fluctuations

Thermal Fluctuations

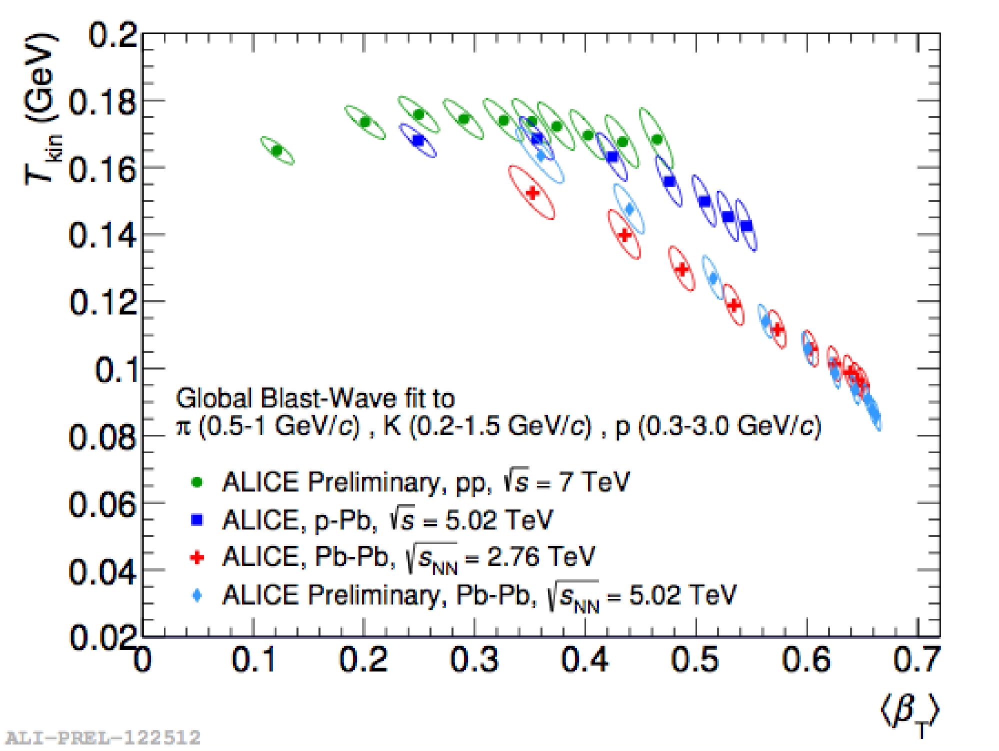
Hadronization

Measurement (Fluctuation-Correlation)

Evidence for the production of thermal systems (I)



Evolution of Kinetic freeze-out temperature T_{kin} and radial flow velocity $\langle \beta_T \rangle$



Boltzmann-Gibbs Blast-Wave model:

- Particle production from a thermalized source + a radial flow boost.
- A thermodynamic model with 3 fit parameters: T_{kin} , $\langle \beta_T \rangle$, and n (velocity profile).

$$E\frac{d^3N}{dp^3} \propto \int_0^R m_T I_0\left(\frac{p_T \sinh(\rho)}{T_{kin}}\right) K_1\left(\frac{m_T \cosh(\rho)}{T_{kin}}\right) r\,dr.$$

The velocity profile ρ is given by

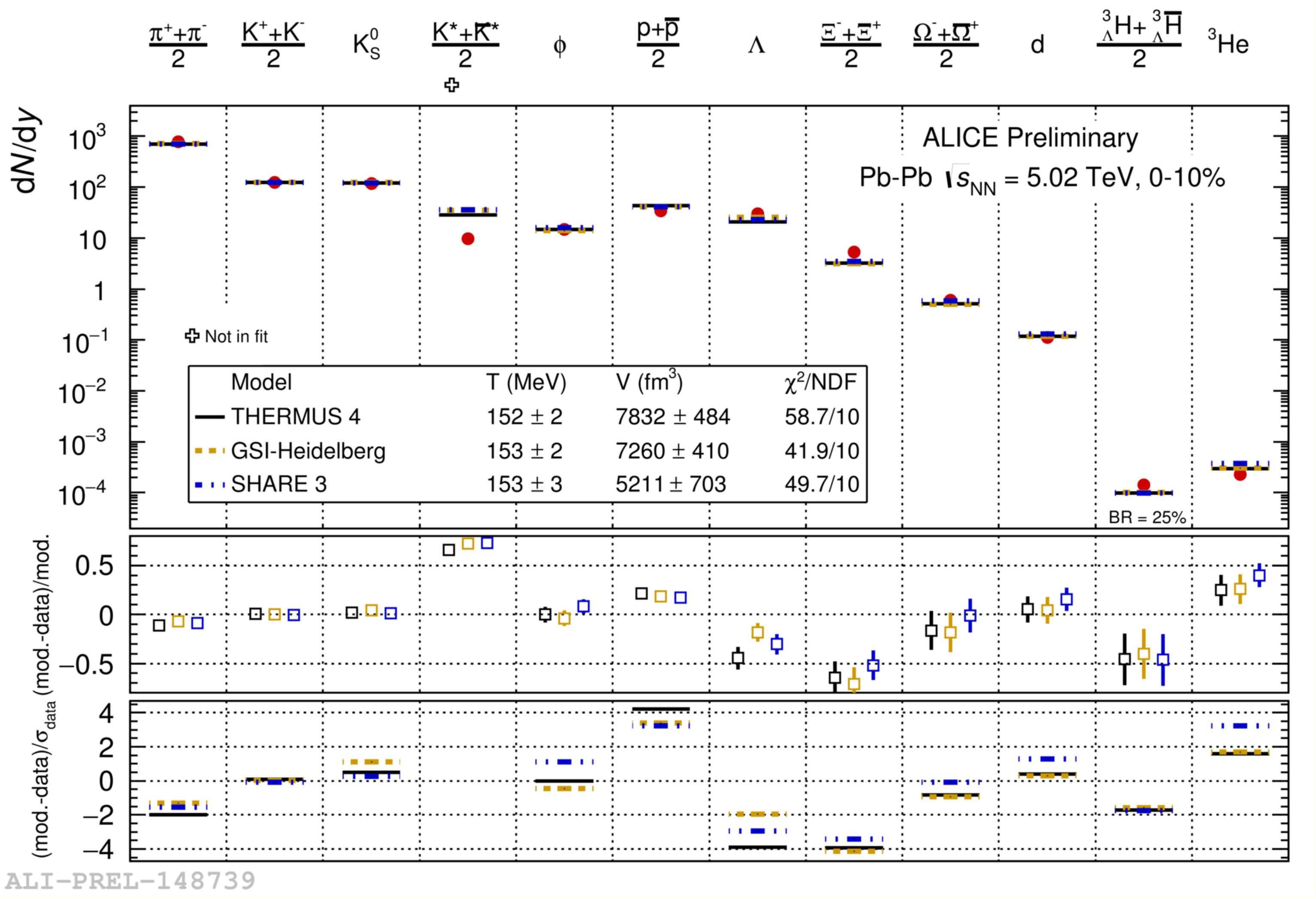
$$\rho = \tanh^{-1}\beta_T = \tanh^{-1}\left[\left(\frac{r}{R}\right)^n \beta_s\right],$$

n changes from peripheral to central (0.7 to 2.4 and is the source of radial flow fluctuation

- $\langle \beta_T \rangle$ increases with centrality
- Similar evolution of fit parameters for pp and p-Pb
- Thermalization in pp?
- At similar multiplicities, $\langle\beta_T\rangle$ is larger for smaller systems

Evidence for the production of thermal systems (II)

Particle yields in Pb-Pb at 5.02 TeV



Thermal models:

- At Chemical freeze-out => Particle yields get fixed.
- Abundance by thermodynamic equilibrium: $\frac{dN}{dy} \propto \exp\left(\frac{-m}{T_{chem}}\right)$

Particle yields are well described by statistical models

=>

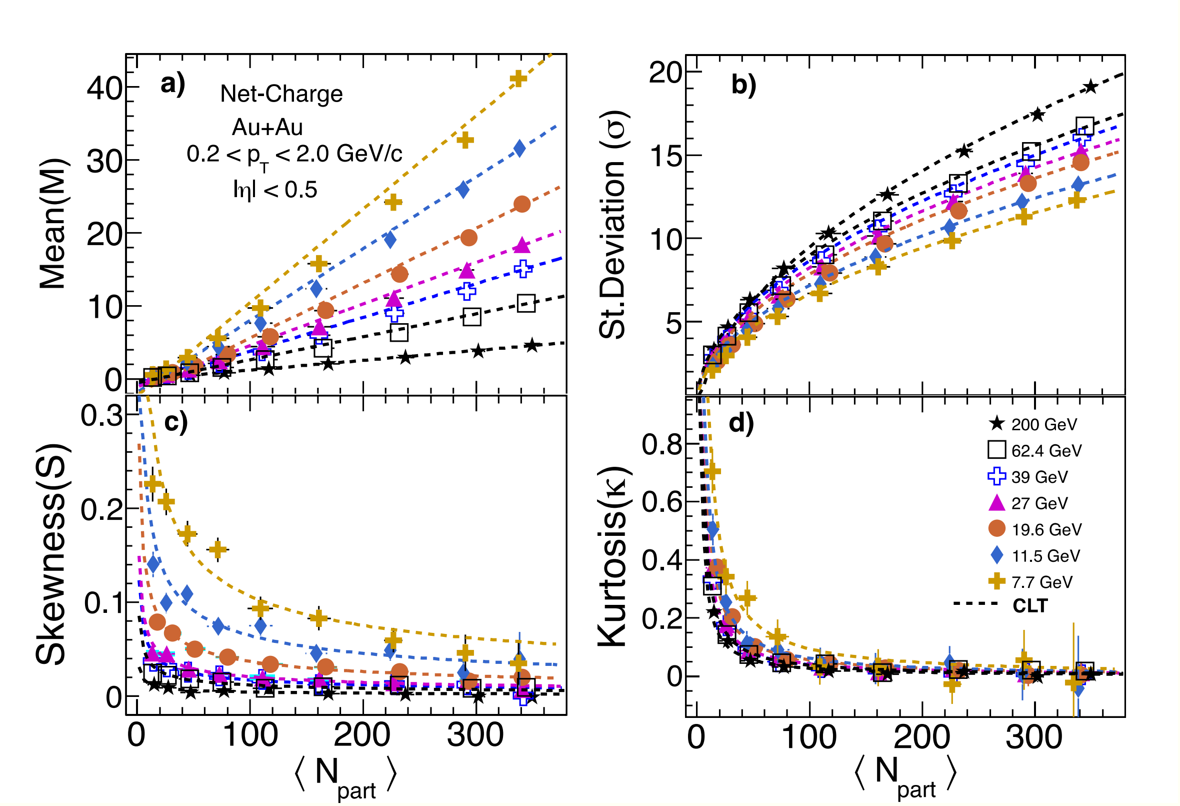
Hadrons are produced in apparent chemical equilibrium in Pb-Pb collisions at LHC.

T_{ch} (Chemical freeze-out temperature) ~153 MeV

Lattice meets experiment: fluctuation of conserved quantities

Thermodynamic Susceptibility ⟺ Moments of the conserved charge distributions

Distribution and moments of conserved charges

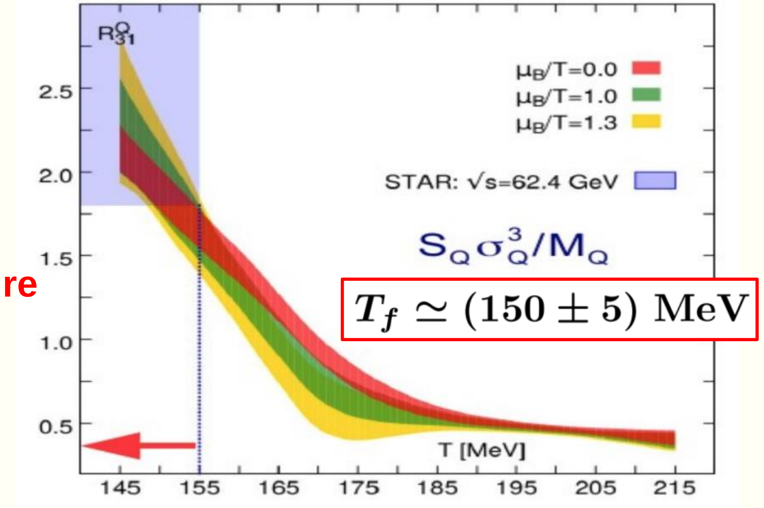


STAR: PRL 113 (2014)
Nihar R. Sahoo, TN

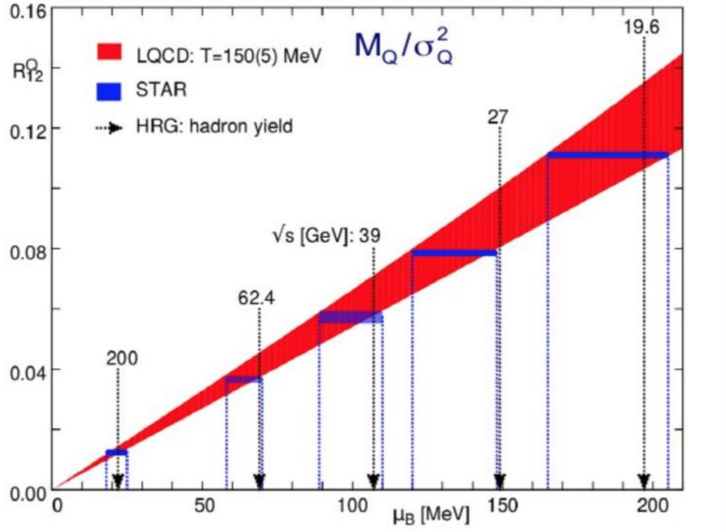
Extraction of Freezeout temperature and chemical potential

- $R_{12} = M/\sigma^2$ ⬄ μ_B/T → Baryometer
- $R_{31} = S\sigma^3/M$ ⬄ T → Thermometer

Chemical Freezeout temperature

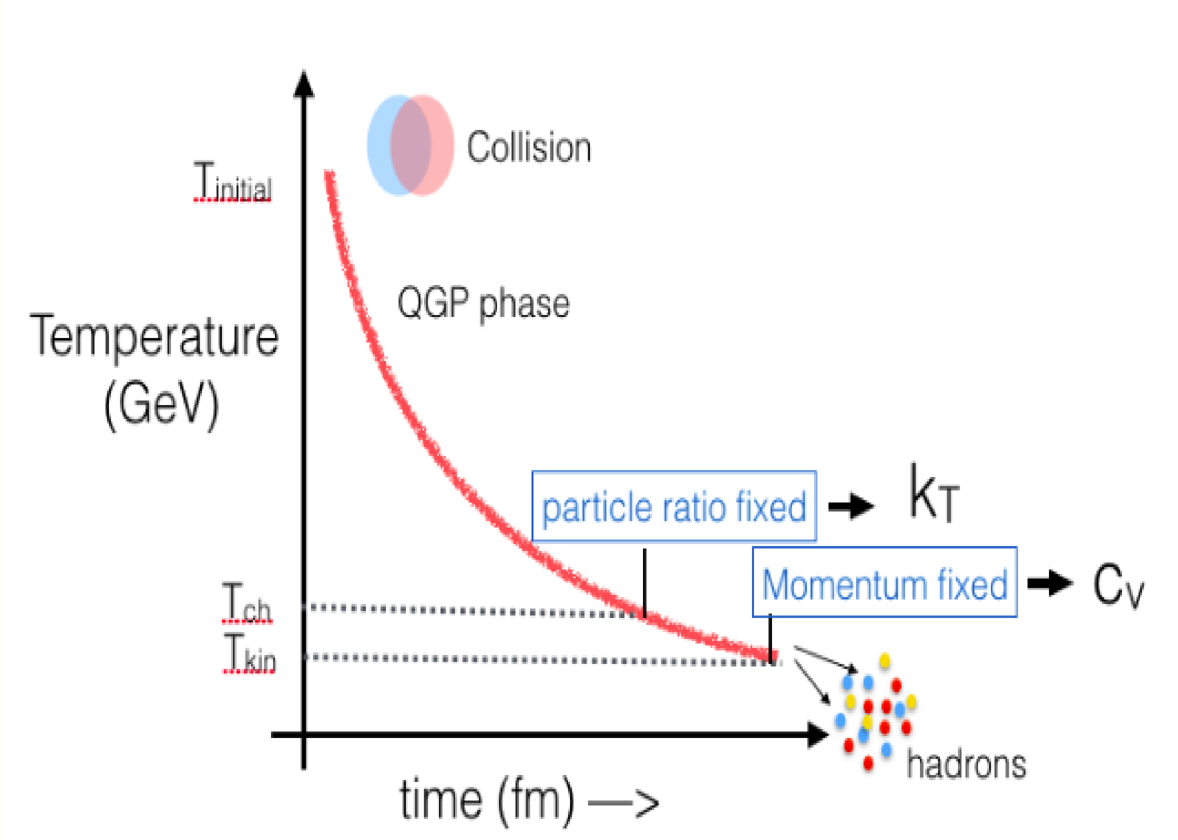


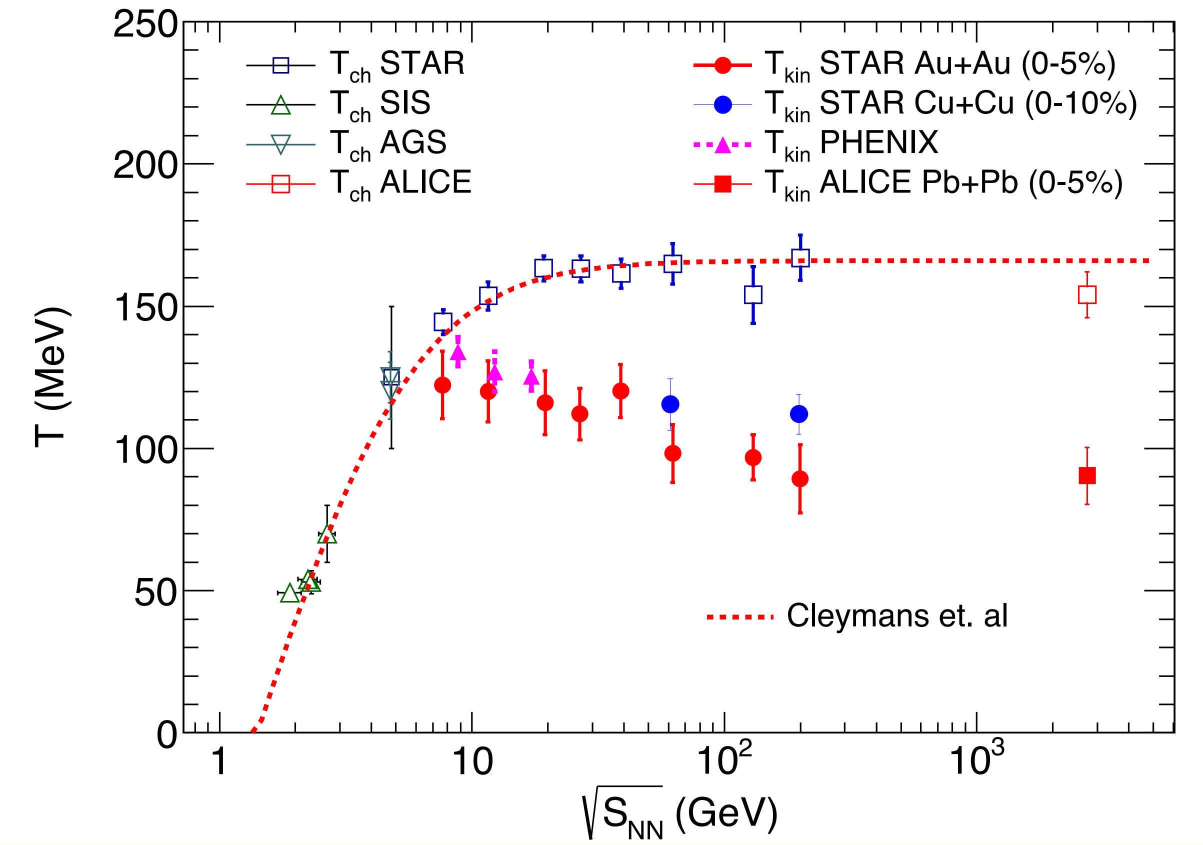
Freezeout chemical potential



Chemical freezeout parameters are similar to what had been obtained from particle ratios

Thermodynamic response functions at freeze-out



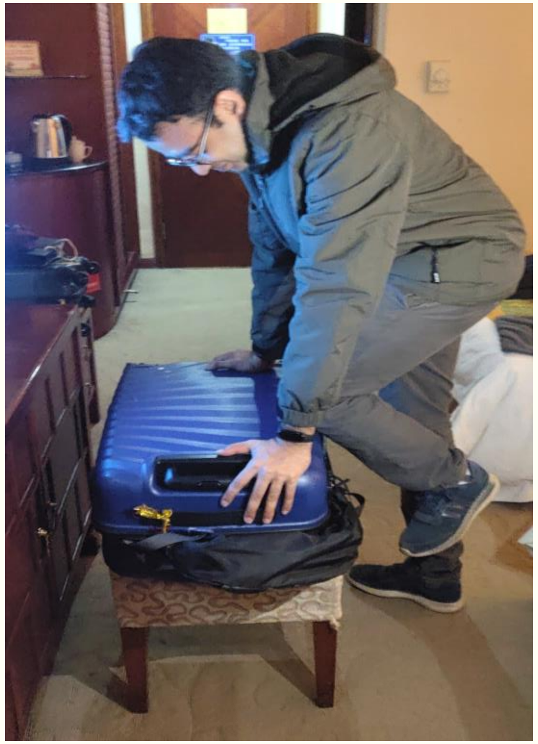


NEXT:

- Multiplicity fluctuations: (calculated at chemical freeze-out temperature) to obtain **Isothermal compressibility (k_T)**
- Temperature or <p_T> fluctuations: (calculated at kinetic freeze-out temperature) to obtain **Specific heat (c_V)**

Isothermal compressibility

Isothermal compressibility expresses how a system's volume responds to a change in the applied pressure.



$$k_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

Expected behavior of k_T at the critical point (T_c):

$$k_T \propto \left(\frac{T - T_C}{T_C}\right)^{-\gamma} \propto \epsilon^{-\gamma}$$

Critical behavior → power law scaling of isothermal compressibility (k_T) → increase by an order of magnitude close to the QCD critical point (CP).

Multiplicity Fluctuations => Isothermal compressibility (k_T)

The partition function in Grand Canonical Ensemble:

$$Z_g \equiv Z_g(T,\mu,V) = \sum_r e^{-\beta E_r - \alpha N_r}$$

(E_r :energy N_r :particle number in the r-th state of the ensemble, β= $1/k_B T$, α=-βμ

$$\frac{\sigma^2}{<N>^2} = \frac{-k_B T}{V}(\partial v/\partial \mu)_T$$

(v is specific volume, v= V/<N>)

$$k_T = \frac{\sigma^2}{<N>^2}\frac{V}{k_B T}$$

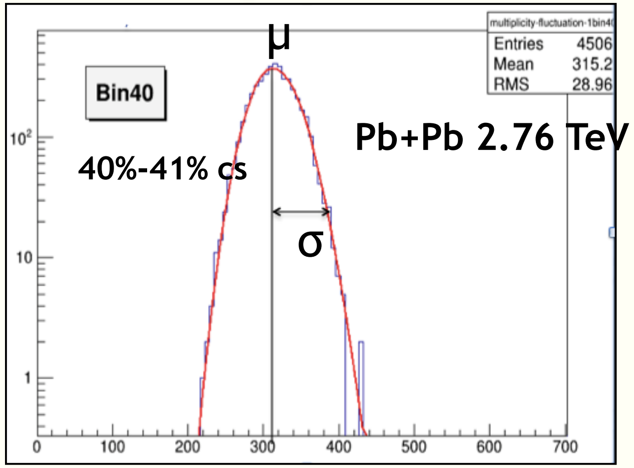
[*S. Mrowczynski, Phys. Lett. B 430 (1998) 9*]

$$\omega_{ch} = \frac{k_B T \langle N_{ch} \rangle}{V} k_T$$

- So, in principle, measurement of fluctuations of particle multiplicity should yield the isothermal compressibility of matter formed.

M. Mukherjee, S. Basu, TN et al.
Phys.Lett. B784 (2018) 1-5

Multiplicity distribution (narrow centrality):



$$\omega_{ch} = \frac{\langle N_{ch}^2 \rangle - \langle N_{ch} \rangle^2}{\langle N_{ch} \rangle} = \frac{\sigma^2}{\mu}$$

Basic Idea: Measure event-by-event fluctuations of particle multiplicity. Measure the variance of these fluctuations to assess the isothermal compressibility.

Caveat: how to address the non-physical fluctuations (background)?

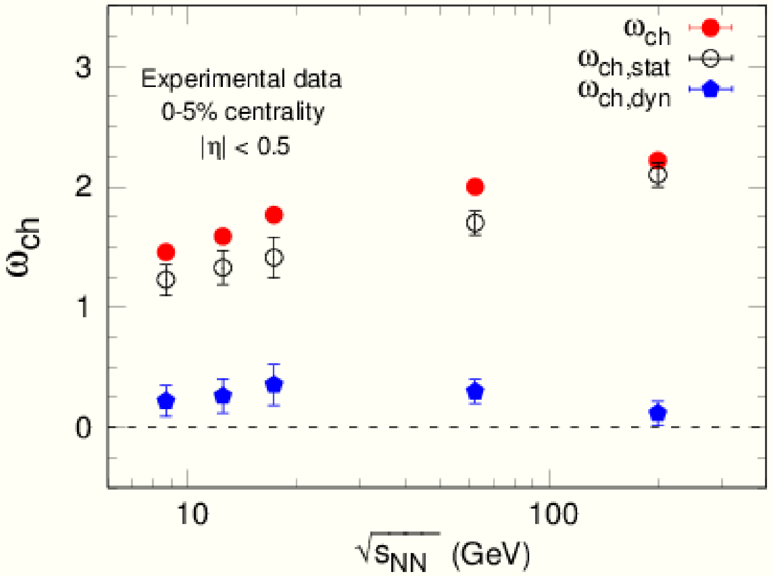
Background to the measured fluctuations:

[H. Heiselberg, Phys. Reports 2001]

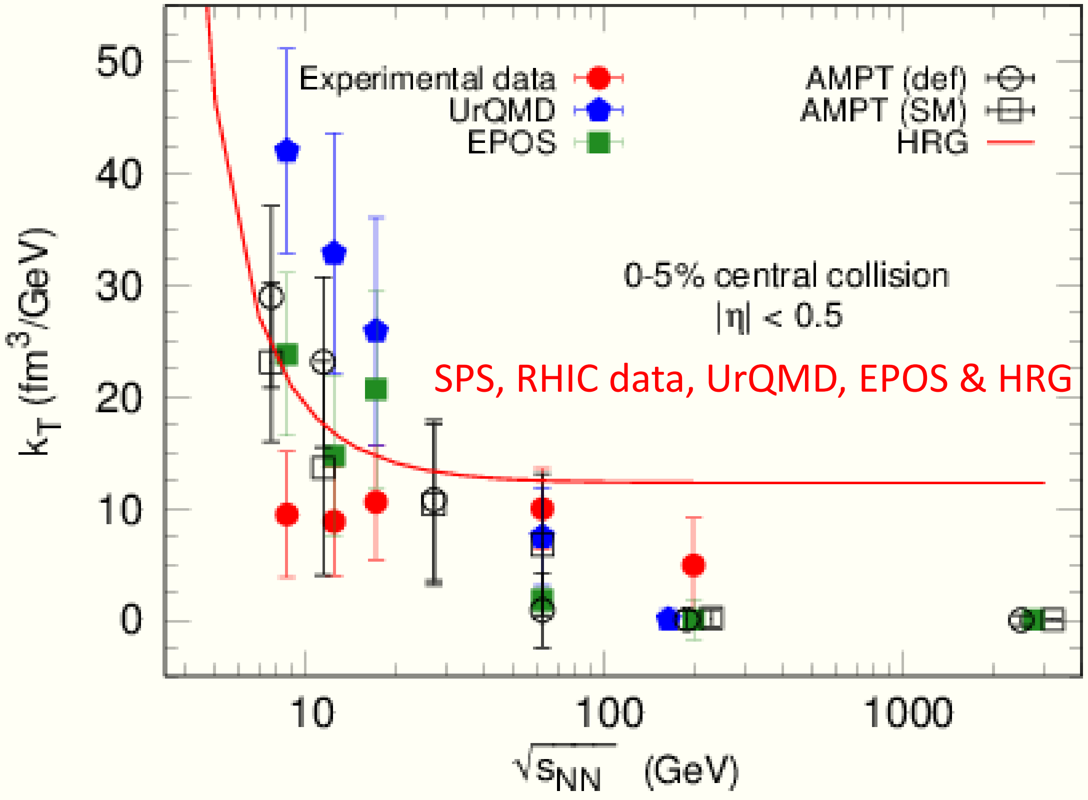
- Multiplicity fluctuations have contributions from: **statistical** and from **dynamical** origins.
- Major component of statistical fluctuations come from fluctuations in no. of participants
- **Scaled Variance from Participant Model:**

$$\omega_{\rm ch}^{\rm back} = \omega_{\rm n} + \langle n \rangle \omega_{N_{\rm part}}$$

$\langle n \rangle$: mean multiplicity from nucleon-nucleon source.



k_T vs collision energy



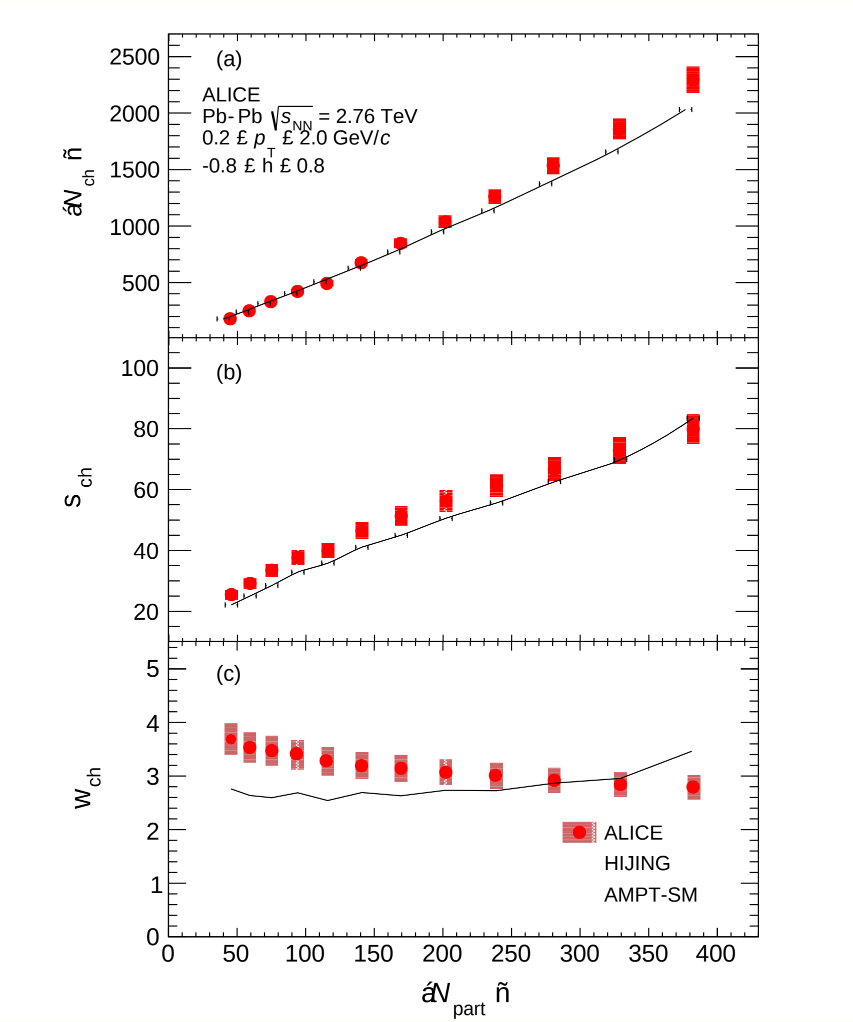
SPS & RHIC data, event generators and HRG
Extending to LHC Energies (AMPT) ….

- M. Mukherjee, S. Basu, TN et al. Phys.Lett. B784 (2018) 1-5
- A. Khuntia, R. Sahoo, TN et al. Phys. Rev. C 100, 014910 (2019

Estimate k_T at chemical freezeout temperature

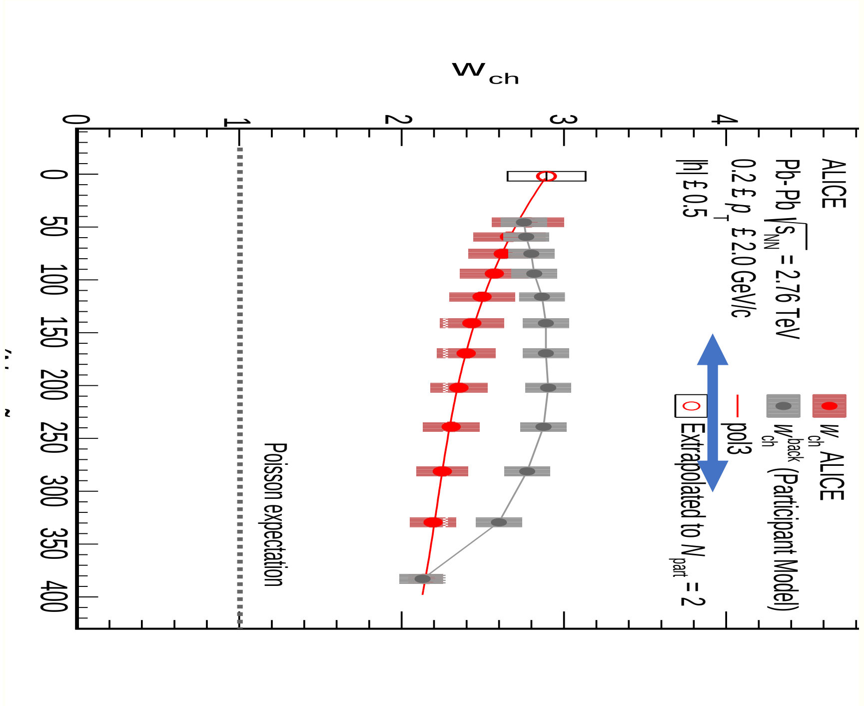
Multiplicity fluctuations in Pb–Pb@2.76 TeV

ALICE: Eur. Phys. J. C (2021) 81:1012



Background to the measured fluctuations:

- **Poisson expectation**: For an ideal gas, the number fluctuations are described by the Poisson distribution. ω_{ch} = 1, independent of multiplicity.
- **Scaled Variance from Participant Model**



For central collisions:

- T_{ch} = 0.156 ± 0.002 GeV
- Volume= 5330 ± 505 fm^3
- $\langle N_{ch} \rangle$ = 1410 ± 47 (*syst*)

- Fluctuations above the Poisson estimation gives,
  ω_{ch} = 1.15 ± 0.06

=> k_T=27.9 ± 3.18 fm^3/GeV.

This result serves as a conservative upper limit of *kT* until various contributions to the background are properly understood and evaluated.

Heat capacity

Heat capacity is a response function which expresses how much a system's temperature changes when heat is transferred to it, or equivalently how much δE is needed to obtain a given δT.

- **Heat capacity:** $$C = \left(\frac{\partial E}{\partial T}\right)_v$$

Specific Heat: the amount of energy per unit mass needed to change its temperature by one unit (for example by one deg C). This amount is directly proportional to mass, so it is expressed per unit mass.

Temperature or <p_T> Fluctuations => Specific heat

- $C_v = T\frac{\partial S}{\partial T}|N,V$ (Extensive quantity, proportional to volume, V, and/or the number of particles, N)
- For a system in equilibrium, probability of a particular state: P~exp(S), where S is the entropy of the state.
- This gives:

$$P(T) \sim \exp\left[-\frac{C}{2}\frac{(\Delta T)^2}{\langle T\rangle^2}\right]$$

$$\frac{1}{C} = \frac{(\langle T^2\rangle - \langle T\rangle^2)}{\langle T\rangle^2}$$

So, in principle, a measurement of fluctuations of *T* should yield the heat capacity of matter formed.

Landau and Lifschitz, Course of Theoretical Physics, Statistical Physics Vol. 5

L. Stodolsky, Phys. Rev. Lett. 75 ž1995. 1044.
E.V. Shuryak, Physics Letters B 423 (1998) 9.

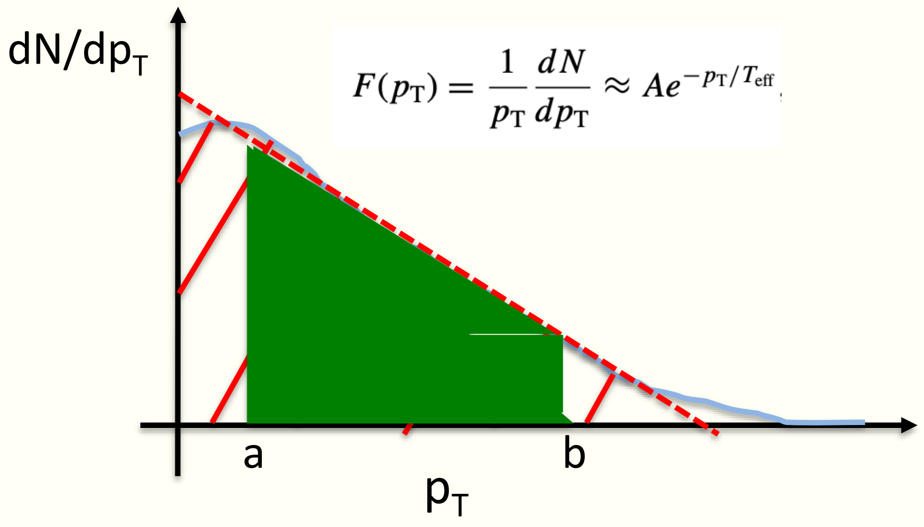
Basic Idea: Measure event-by-event fluctuations of system temperatures. Measure the variance of these fluctuations to assess the heat capacity

Caveat: Measuring the temperature of a single event based e.g., on the p_T distribution does not yield a very good precision.

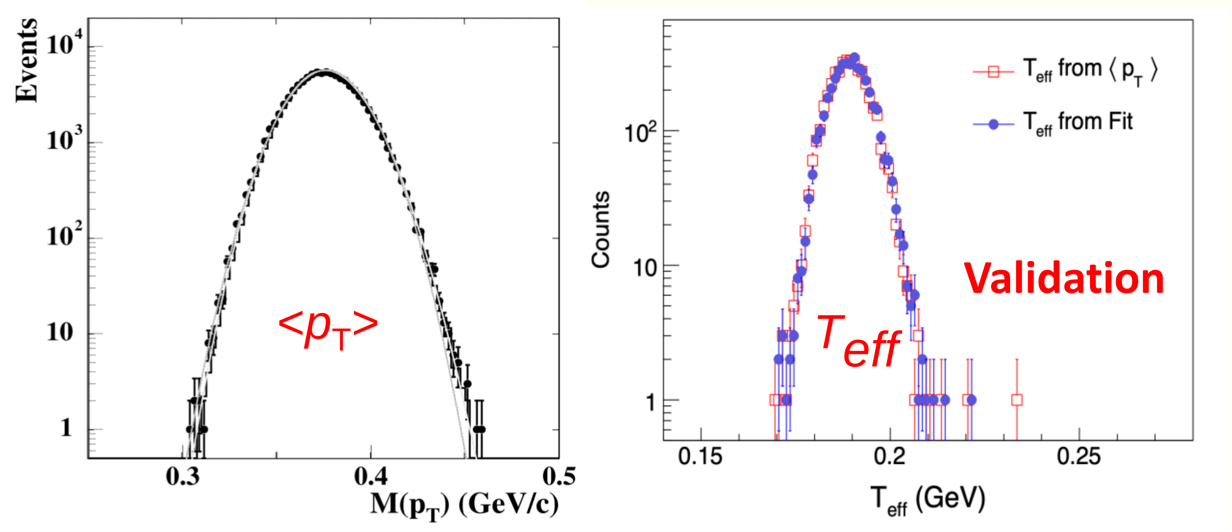
Cleymans et al. PRC 73 (2006) 034905
STAR Collaboration PRC 79 (2009) 034909
ALICE Collaboration PRD 88 (2013) 044910
Sumit Basu et al. PRC 94 (2016) 044901

Accessing the heat capacity …

Using p_T fluctuations as a proxy for temperature fluctuations



$$F(p_T) = \frac{1}{p_T}\frac{dN}{dp_T} \approx Ae^{-p_T/T_{\rm eff}}$$



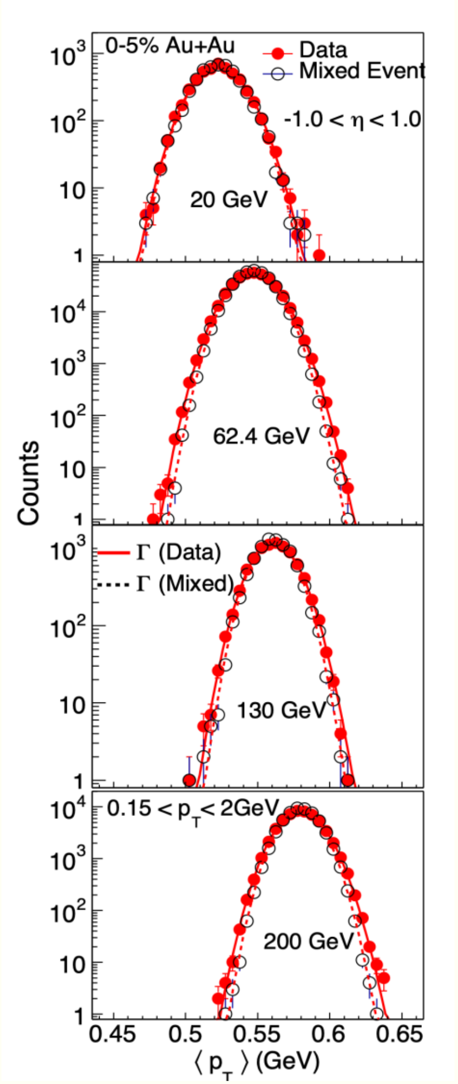
$$\langle p_T\rangle = \frac{\int_0^\infty p_T^2 F(p_T)dp_T}{\int_0^\infty p_T F(p_T)dp_T} = \frac{2T_{\rm eff}^2 + 2m_0T_{\rm eff} + m_0^2}{m_0 + T_{\rm eff}}$$

$$\langle p_T\rangle = \frac{\int_a^b p_T^2 F(p_T)dp_T}{\int_a^b p_T F(p_T)dp_T} = 2T_{\rm eff} + \frac{a^2e^{-a/T_{\rm eff}} - b^2e^{-b/T_{\rm eff}}}{(a+T_{\rm eff})e^{-a/T_{\rm eff}} - (b+T_{\rm eff})e^{-b/T_{\rm eff}}}$$

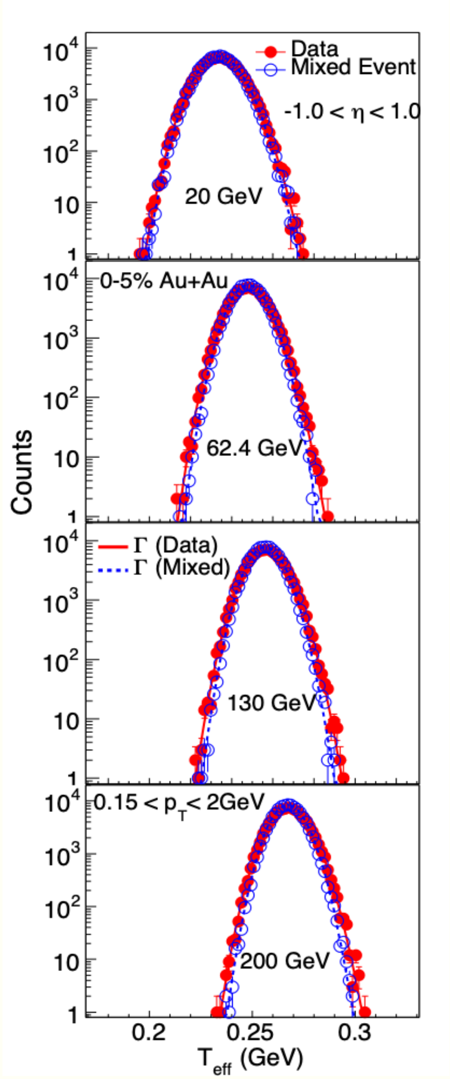
Accessing the heat capacity …

Distributions of ⟨p_T⟩ and T_{eff}

⟨p_T⟩ → T_{eff}



STAR, PRC 72 (2005)



Sumit Basu, TN et al. PRC 94 (2016)

M.J. Tannenbaum, PLB 498 (2001) 29

<p_T> distribution fitted by gamma Function:

$$f(x) = f_\Gamma(x,a,b) = \frac{b}{\Gamma(a)}(bx)^{a-1}e^{-bx}.$$

Fits of the <p_T> distribution gives *a* and *b*.

- Mean and std: $\mu = \frac{a}{b} = \langle p_\mathrm{T} \rangle; \quad \sigma = \frac{\sqrt{a}}{b}.$
- Skewness and Kurtosis $s = \frac{2}{\sqrt{a}}; \quad \kappa = \frac{6}{a}$

=> Background: obtained from <p_T> distributions of mixed events

$$(\Delta T_\mathrm{eff})^2 = (\Delta T_\mathrm{eff}^{dyn})^2 + (\Delta T_\mathrm{eff}^{stat})^2$$

Subtracting the width of mixed events,
we obtain the dynamic fluctuation.

Estimates of the specific heat (c_V) from STAR data

Heat capacity:

$$C = \left(\frac{\partial E}{\partial T}\right)_v \quad \Rightarrow \quad \frac{1}{C} = \frac{(\langle T^2\rangle - \langle T\rangle^2)}{\langle T\rangle^2}$$

T_{eff} has contributions from thermal (T_{kin}) and collective motion in the transverse direction ($\langle \beta_T \rangle$): radial flow

$$T_{\mathrm{eff}} = T_{\mathrm{kin}} + f(\beta_T)$$

Neglecting fluctuation in β_T:

$$\frac{1}{C} = \frac{(\Delta T_{\mathrm{eff}})^2}{\langle T_{\mathrm{kin}}\rangle^2}$$

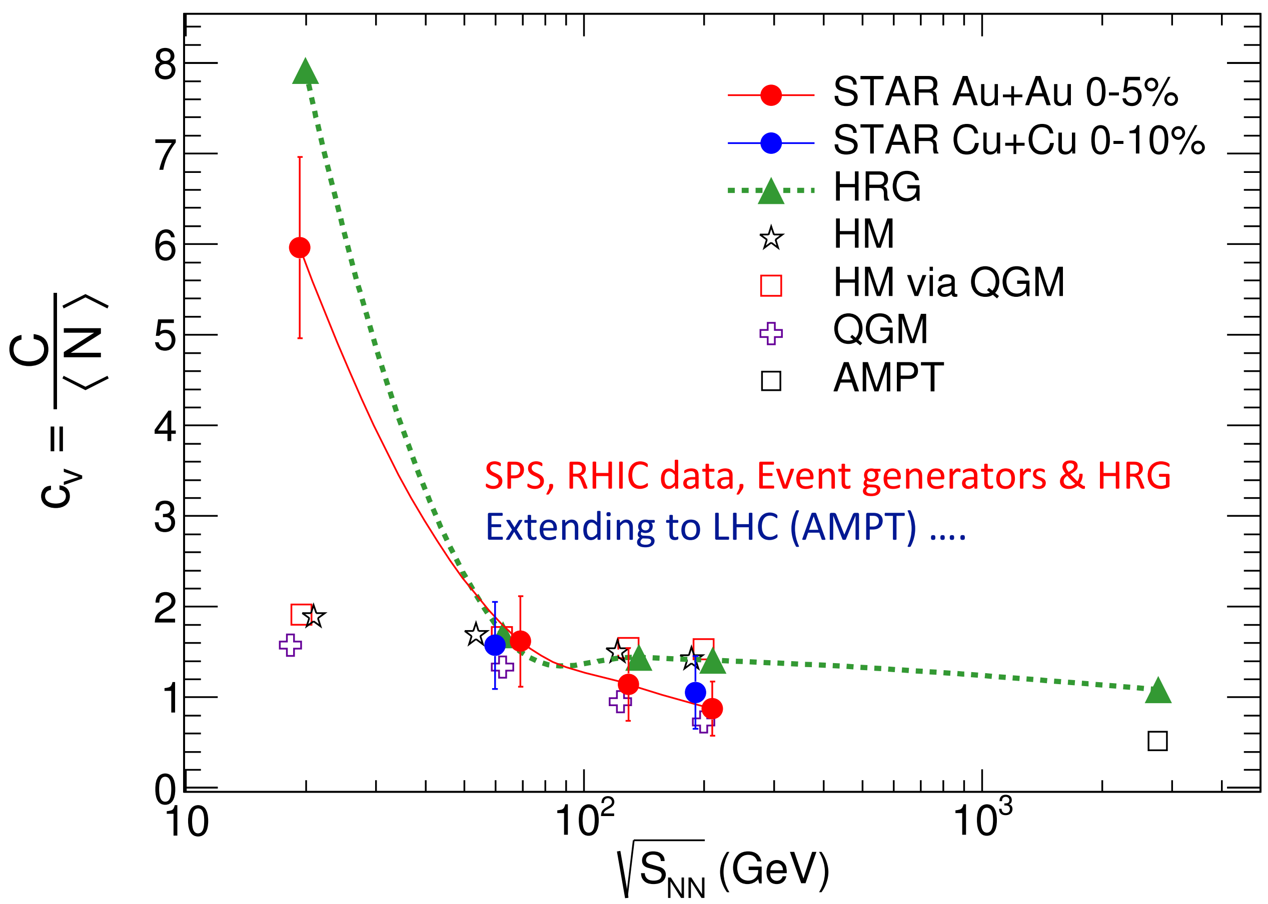
$$(\Delta T_{\mathrm{eff}})^2 = (\Delta T_{\mathrm{eff}}^{dyn})^2 + (\Delta T_{\mathrm{eff}}^{stat})^2$$

$(\Delta T_{\mathrm{eff}}^{dyn})^2$: obtained by subtraction of width of the mixed event

$$\frac{1}{C} = \frac{(\Delta T_{\mathrm{eff}}^{dyn})^2}{\langle T_{\mathrm{kin}}\rangle^2}.$$

Phys.Rev. C94 (2016)

S. Basu, S. Chatterjee, R. Chatterjee, B. Nandi TN

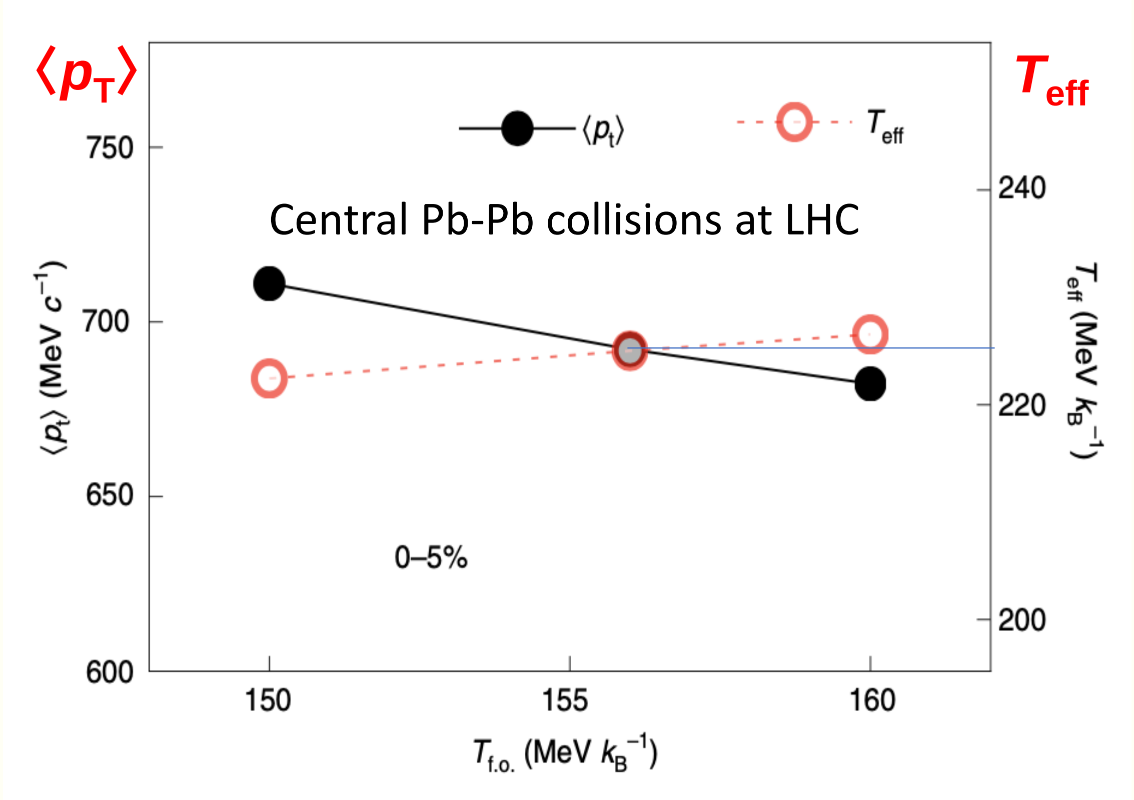


Specific heat: $c_V = \frac{C}{\langle N \rangle} = \frac{C}{VT^3}$

Estimate c_V at kinetic freezeout temperature

Thermodynamics of hot strong-interaction

Nature Physics Letters 2020
Gardim, Giacalone, Luzum and Ollitraut



Variation of $\langle p_T \rangle$ and T_{eff} as a function of the freeze-out temperature in ideal hydrodynamic simulations ….

QGP, modelled as a massless ideal gas with Boltzmann statistics, has a particle density $n = gT^3/\pi^2$

From experimental data, $g \approx 30$
=> This large number shows that the colour degrees of freedom are active.

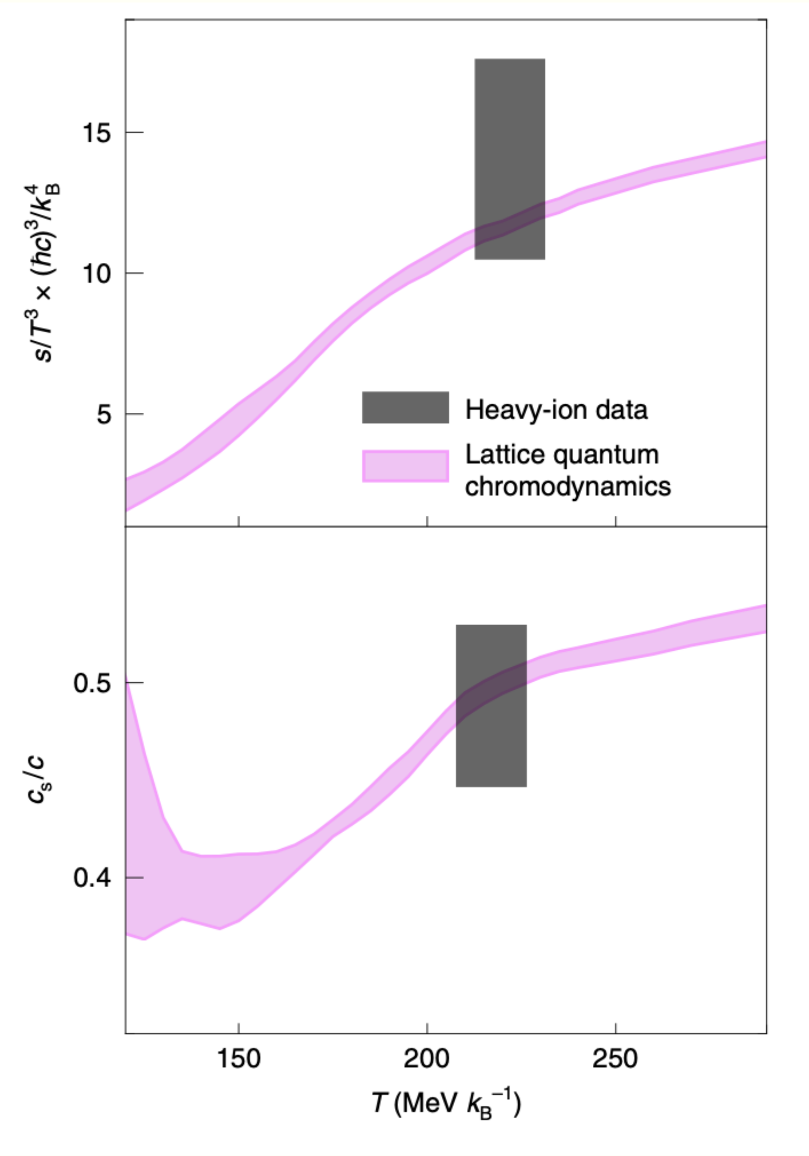
Entropy density:

$$s(T_{\text{eff}}) = \frac{1}{V_{\text{eff}}} \frac{S}{N_{\text{ch}}} \frac{dN_{\text{ch}}}{dy}$$

$$s(T_{\text{eff}}) = 20 \pm 5\ \text{fm}^{-3}$$

$$s(T_{\text{eff}})/T_{\text{eff}}^3 = 14 \pm 3.5$$

$\langle p_T \rangle$ distribution and Speed of sound



Nature Physics Letters 2020
Gardim, Giacalone, Luzum and Ollitraut

Entropy density:

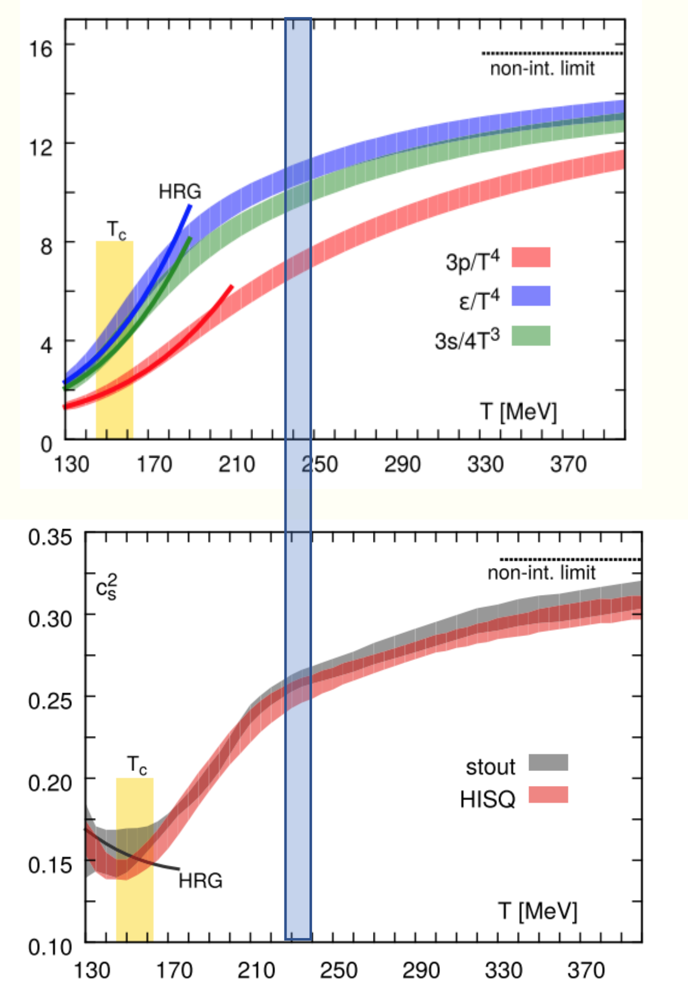
$$s(T_{\text{eff}})/T_{\text{eff}}^3 = 14 \pm 3.5$$

Speed of sound:

the velocity at which a compression wave travels in a fluid

$$c_s^2(T_{\text{eff}}) \equiv \frac{dP}{d\varepsilon} = \frac{s dT}{T ds}\bigg|_{T_{\text{eff}}} = \frac{d\ln \langle p_t \rangle}{d\ln (dN_{\text{ch}}/d\eta)}$$

$$c_s^2(T_{\text{eff}}) = 0.24 \pm 0.04$$



HOTQCD Collaboration
Phys. Rev. D90 (2014) , 094503

Transverse Momentum correlators:

Mean: $\langle\langle p_t \rangle\rangle \equiv \left\langle \frac{\sum_{i=1}^{N_{\rm ch}} p_i}{N_{\rm ch}} \right\rangle$

Variance (2-particle correlator): $\langle \Delta p_i \Delta p_j \rangle \equiv \left\langle \frac{\sum_{i,j\neq i} (p_i - \langle\langle p_t \rangle\rangle)(p_j - \langle\langle p_t \rangle\rangle)}{N_{\rm ch}(N_{\rm ch}-1)} \right\rangle$

Skewness (3-particle correlator): $\langle \Delta p_i \Delta p_j \Delta p_k \rangle \equiv \left\langle \frac{\sum_{i,j\neq i,k\neq i,j} (p_i - \langle\langle p_t \rangle\rangle)(p_j - \langle\langle p_t \rangle\rangle)(p_k - \langle\langle p_t \rangle\rangle)}{N_{\rm ch}(N_{\rm ch}-1)(N_{\rm ch}-2)} \right\rangle$

Kurtosis (4-particle correlator): $\langle \Delta p_i \Delta p_j \Delta p_k \Delta p_l \rangle \equiv \left\langle \frac{\sum_{i,i\neq j\neq k\neq l}^{N_{ch}} (p_i - \langle\langle p_{\rm T} \rangle\rangle)(p_j - \langle\langle p_{\rm T} \rangle\rangle)(p_k - \langle\langle p_{\rm T} \rangle\rangle)(p_l - \langle\langle p_{\rm T} \rangle\rangle)}{N_{ch}(N_{ch}-1)(N_{ch}-2)(N_{ch}-3)} \right\rangle$

- $\langle p_T \rangle$ fluctuations result from fluctuations of the energy of the fluid when the hydrodynamic expansion starts. Normally, mean and variances are studied ..
- Need for going to higher order cumulants: Higher-order cumulants are need for detailed probes of QCD thermodynamics at higher *T*, achieved during the early stages of the collision.

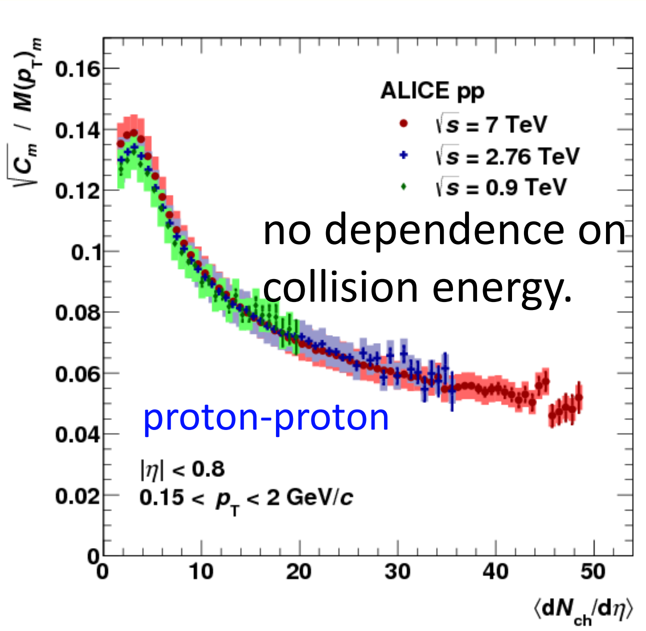
Giacalone, Gardim, Norohna-Hostler, and Ollitraut: PRC 103, 024910 (2021)

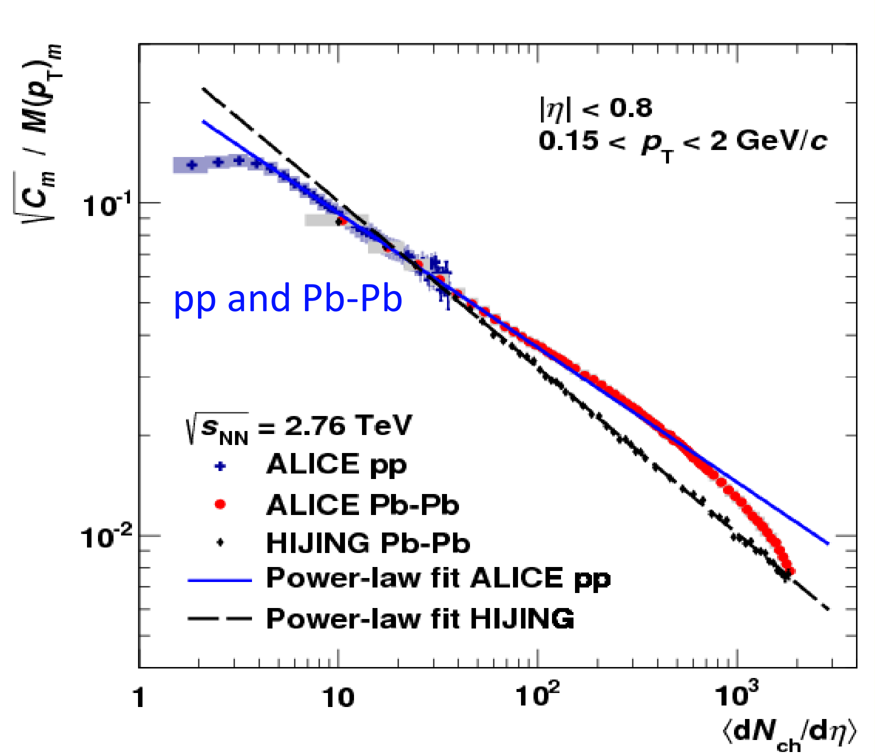
For example, Skewness serves as a fine probe of hydrodynamic behavior. Hydrodynamics predicts a positive skew.

Dynamical p_T correlations - I

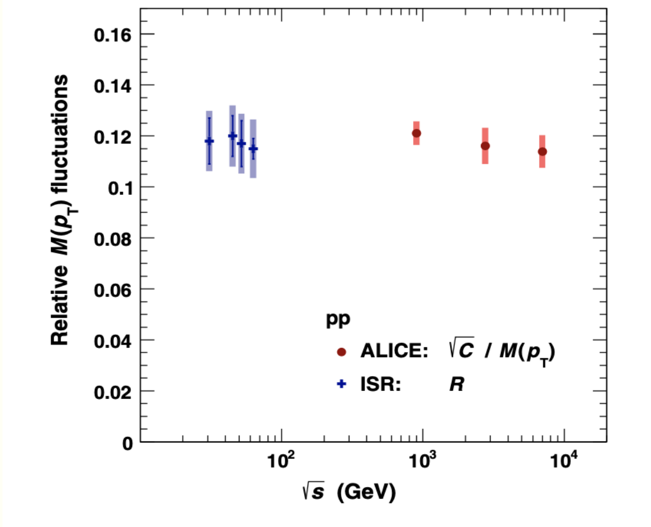
ALICE: Eur. Phys. J. C (2014) 74:3077

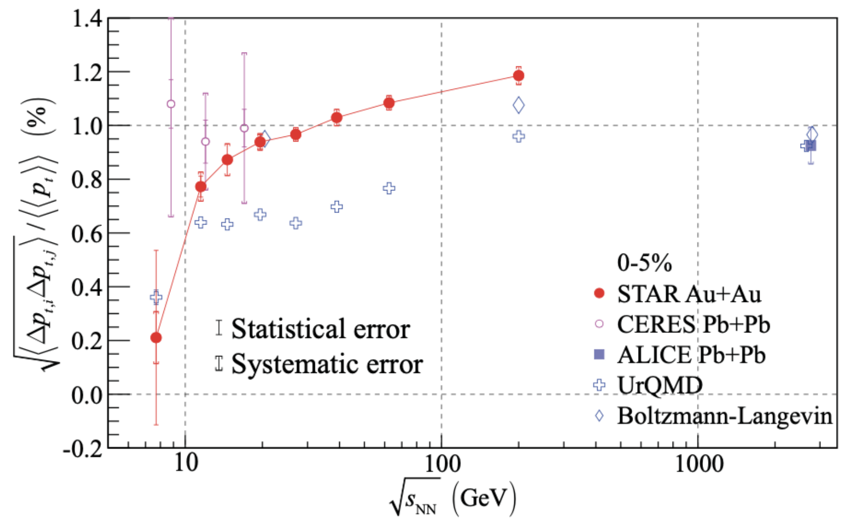
$$C_m = \frac{1}{\sum_{k=1}^{n_{\mathrm{ev},m}} N_k^{\mathrm{pairs}}} \cdot \sum_{k=1}^{n_{\mathrm{ev},m}} \sum_{i=1}^{N_{\mathrm{acc},k}} \sum_{j=i+1}^{N_{\mathrm{acc},k}} (p_{\mathrm{T},i} - M(p_{\mathrm{T}})_m) \cdot (p_{\mathrm{T},j} - M(p_{\mathrm{T}})_m)$$





Evolution of p_T correlations w/ beam energy





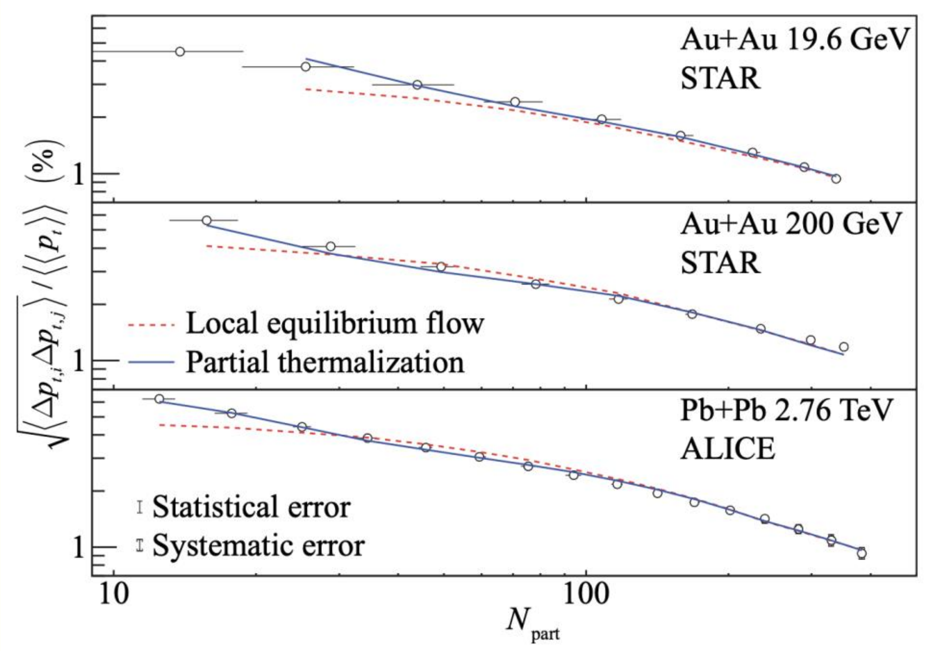
- Peripheral: similar multiplicity to pp.
- Central: **deviation from the trend.** Dynamics is different from other centralities.
- **Higher order correlators will play a role to understand this.**

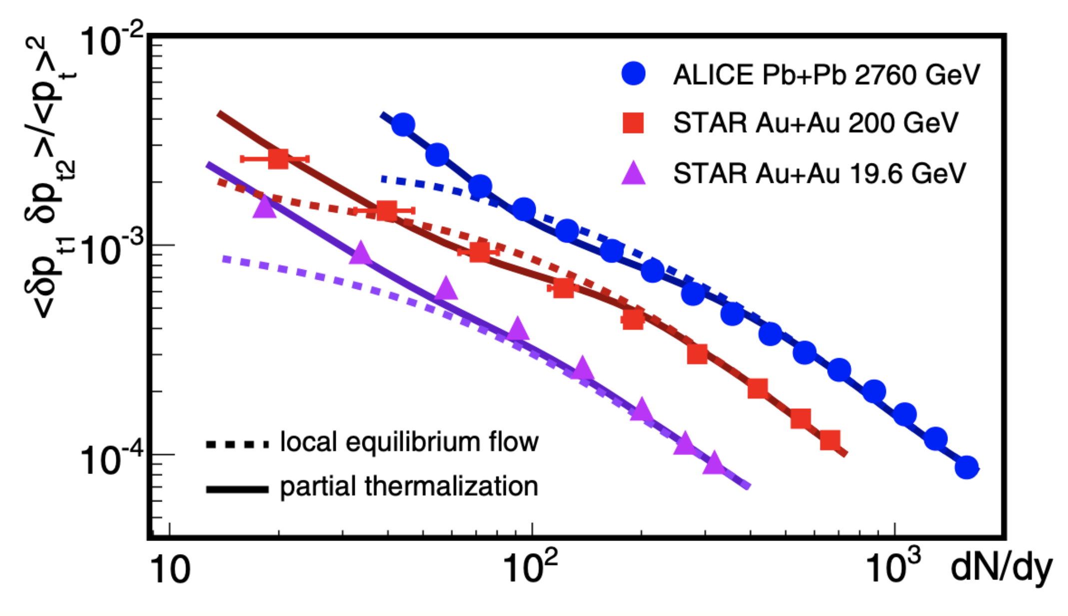
- The relative dynamical correlations increase with collision energy up to 200 GeV.
- For Pb+Pb collisions at 2.76 TeV, fluctuation is lower than that of Au+Au collisions at 200 GeV.

Dynamical p_T correlations - II

ALICE: Eur. Phys. J. C (2014) 74
STAR: Phys. Rev. C **99** (2019)

Sean Gavin et al., [PRC 95 (2017) 064901]
Boltzmann-Langevin approach to pre-equilibrium correlations



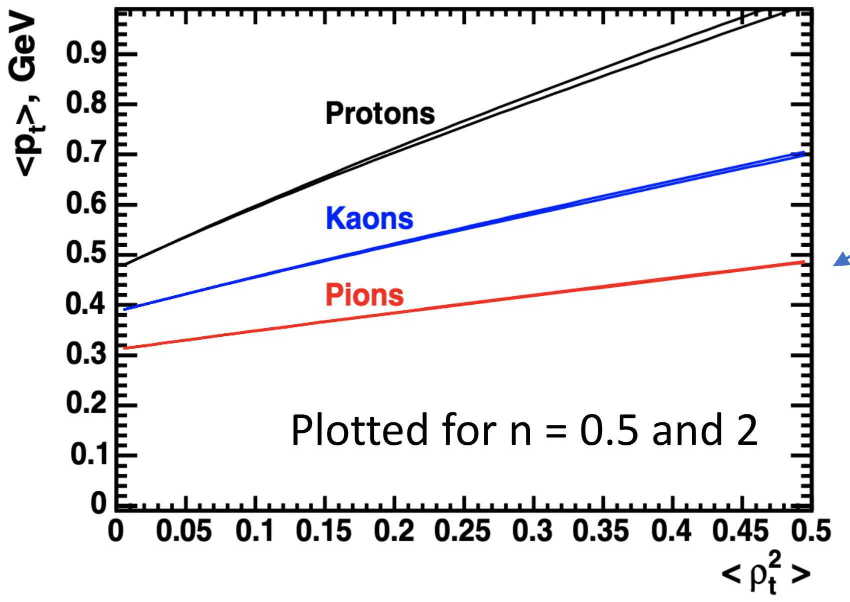


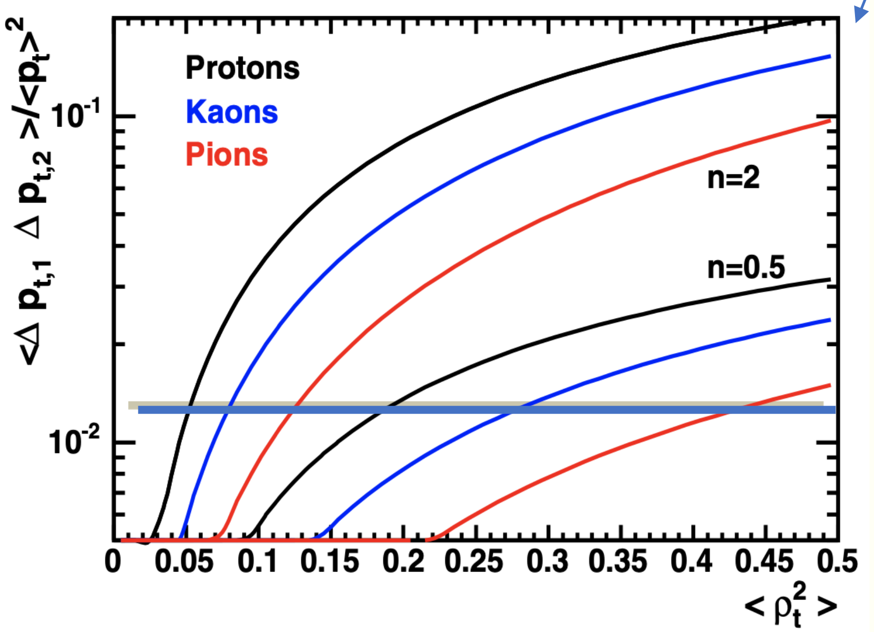
For most peripheral collisions: the two-particle $\langle p_T \rangle$ correlations show evidence of incomplete thermalization when compared with the Boltzmann-Langevin model.

- The first traces of thermalization emerges in peripheral collisions, becoming more significant with increasing centrality as the system lifetime increases.
- Peripheral collisions show a systematic discrepancy with local equilibrium flow.

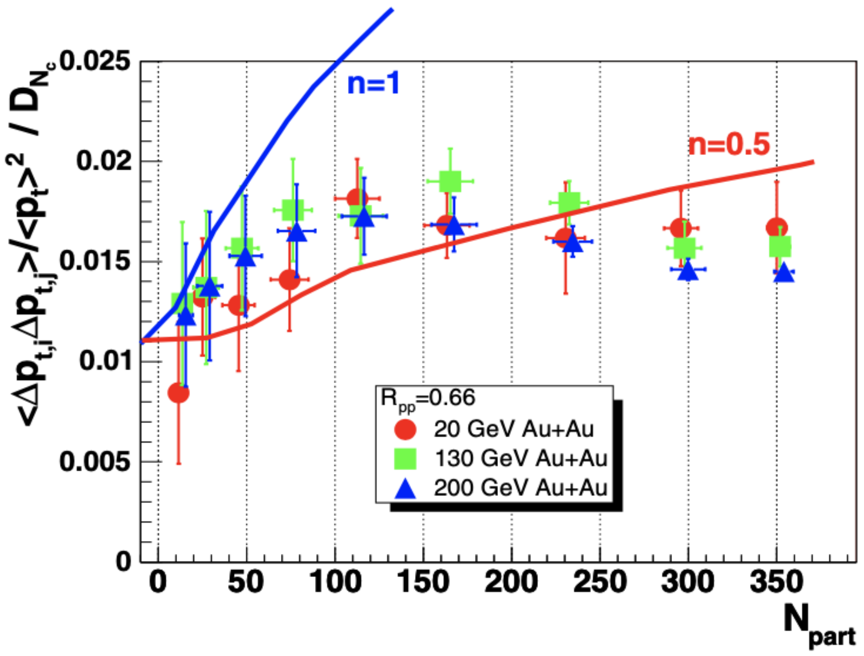
Effect of radial flow fluctuation

2006 J. Phys.: Conf. Ser. **50** 013
Sergei A Voloshin





- p_T correlations measure the variance in collective transverse expansion velocity => more sensitive to the actual velocity profile (n).
- $\langle p_T \rangle$ depends very weakly on the actual profile.
- On the other hand, the correlations are drastically different for the two velocity profiles (n) values studied.



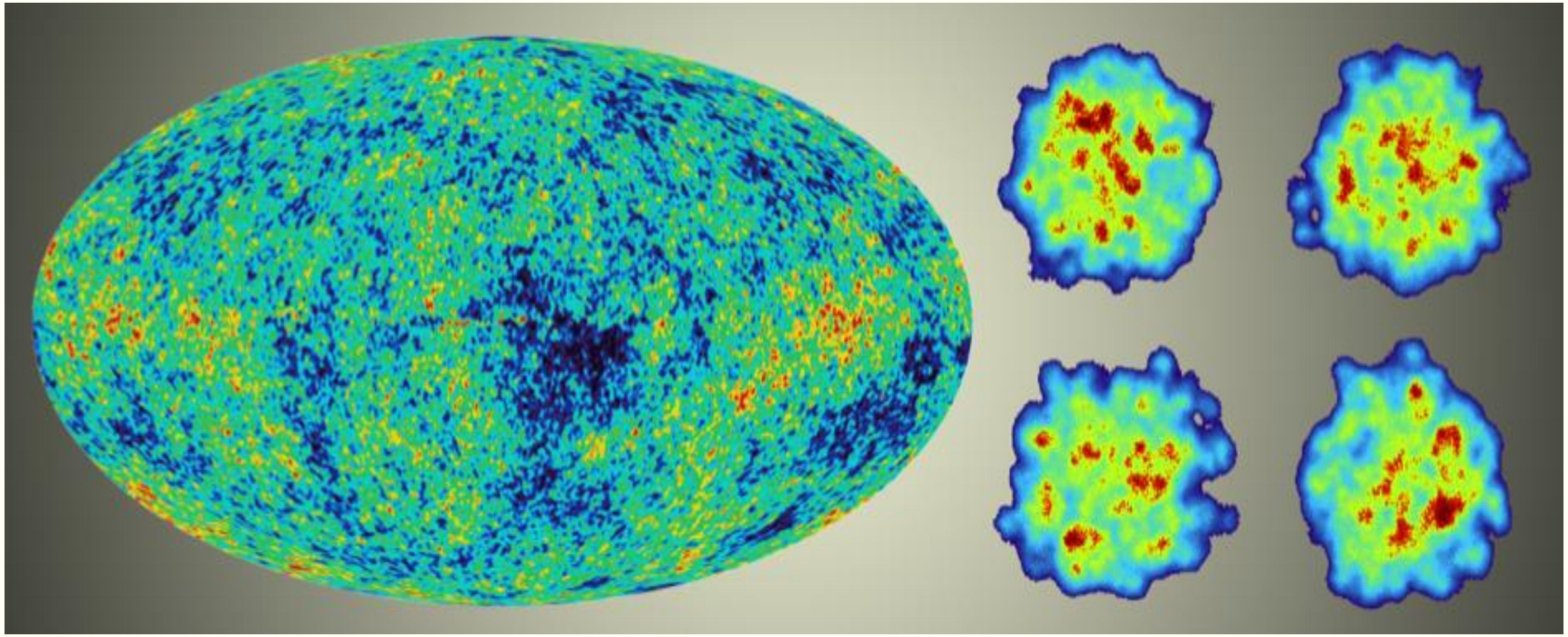
n = 0.5 describes the data. n=1 (ideal thermodynamics) does not).

Only radial flow fluctuation is not the source of this correlation.

Sensitivity to be explored in going to higher order $\Delta p_T \Delta p_T$ correlations. Analysis in progress ….

Fluctuations in the Little Bang

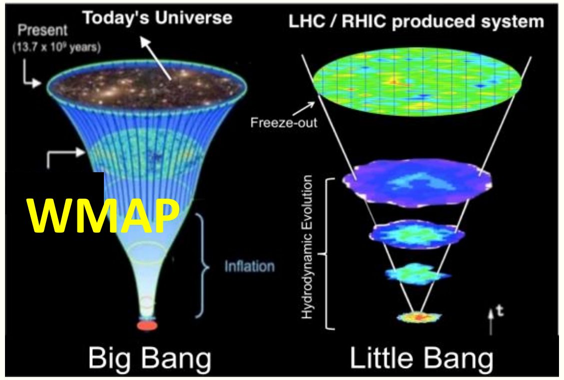
Uli Heinz, arXiv:1304.3634v1 [nucl-th] 11 Apr 2013



WMAP

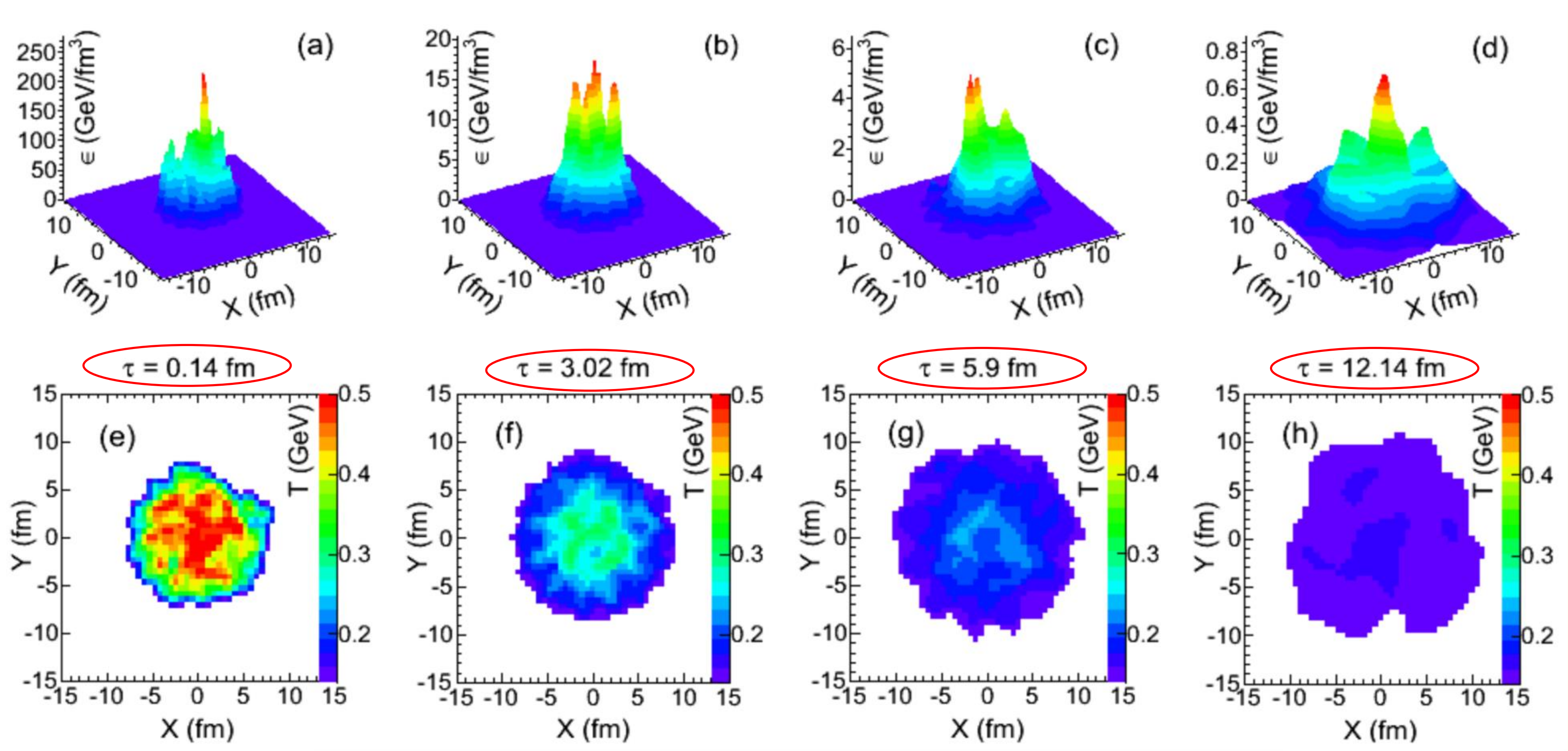
Heavy-ion Collisions

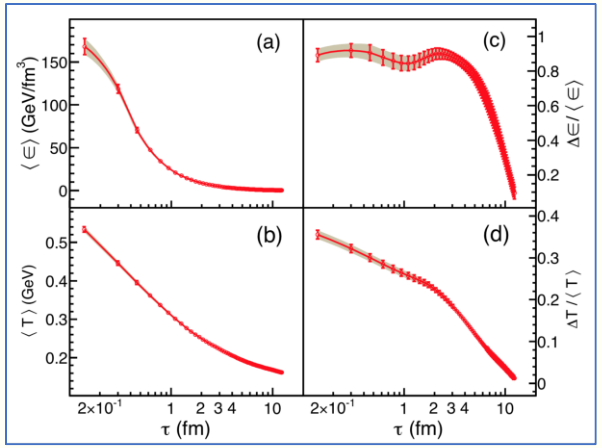
- Hadrons detected by the experiment are mostly emitted at the freeze-out
- Similar to the CMBR which carry information at the surface of last scattering in the Universe, these hadrons may provide information about the earlier stages (hadronization) of the reaction in heavy-ion collision.



Hydrodynamic simulation of central Pb-Pb at 2.76 TeV

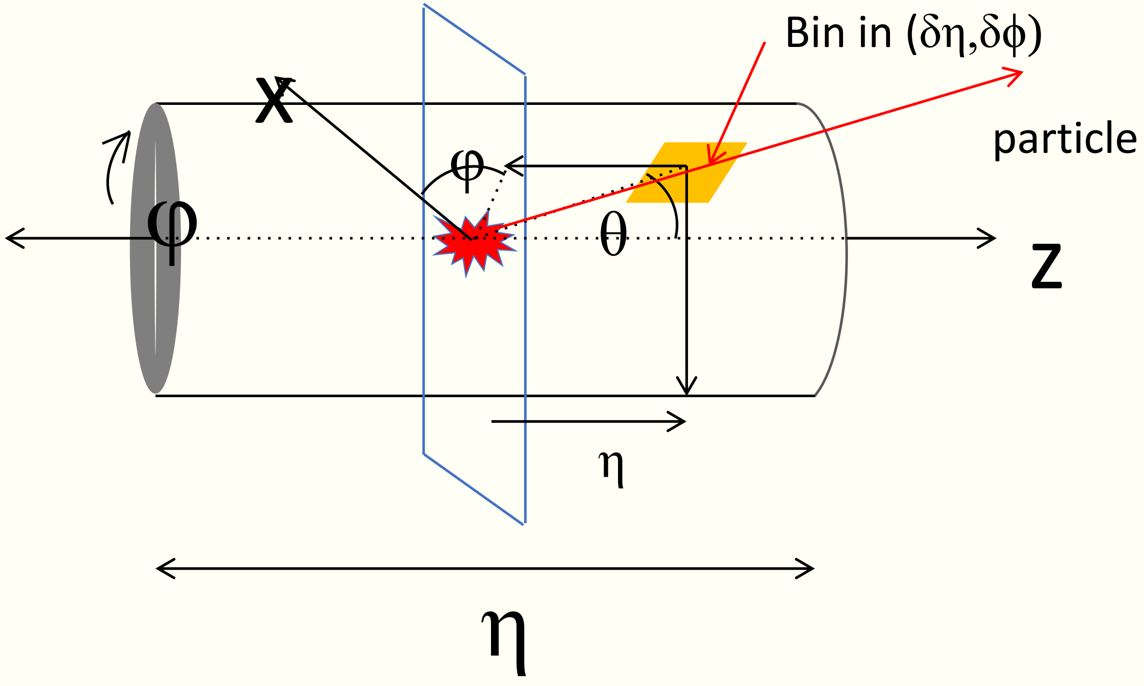
arXiv:1504.04502 [nucl-ex]
Sumit Basu, TN et al.





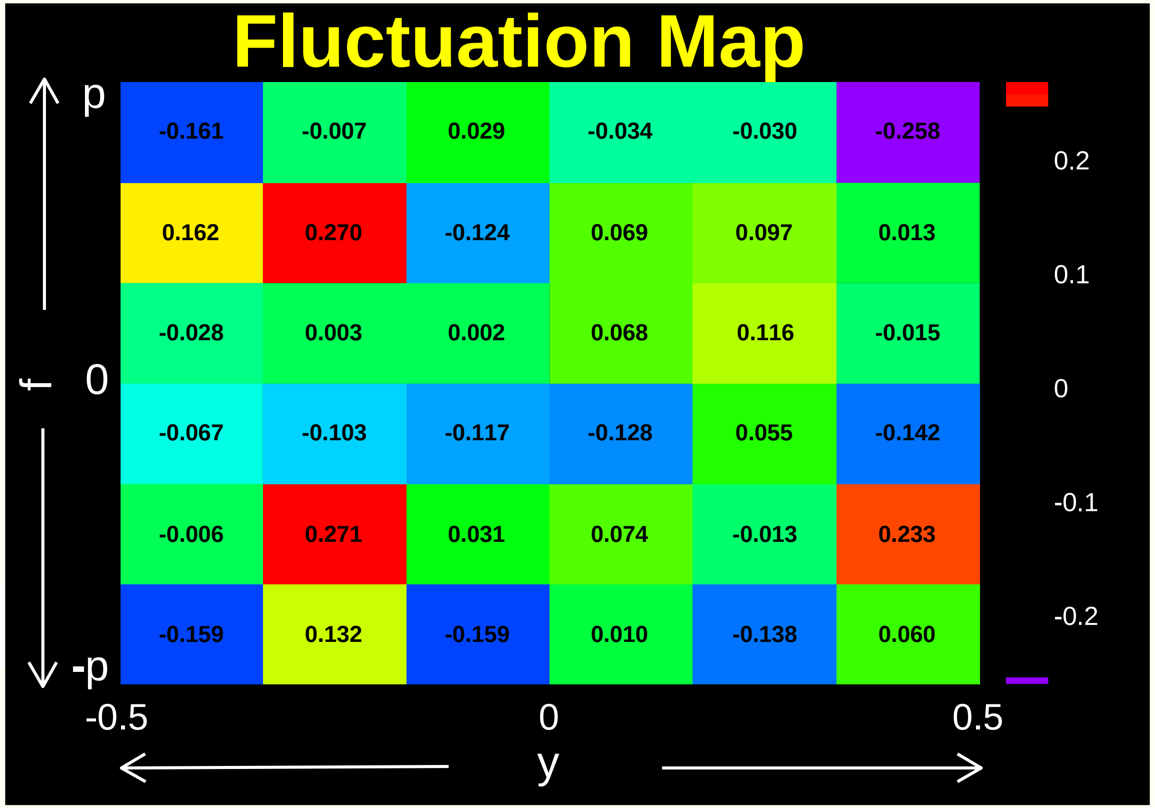
Evolution of Energy Density and temperature

Local Fluctuation



arXiv:1504.04502 [nucl-ex]
Sumit Basu, TN et al.

SINGLE Event



- Need large acceptance detectors
- Novel methods to be developed to map the Heavy-ion collisions
- Need a strong connection to theory to derive early-stage fluctuations.

To recap:

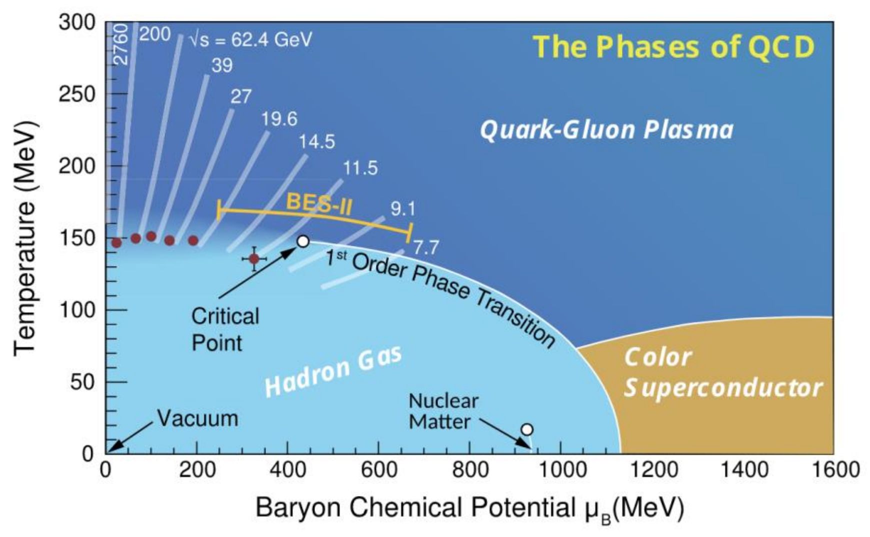
Understanding thermodynamics:

- Isothermal compressibility: multiplicity fluctuations
- Specific heat: temperature ($\langle p_T \rangle$) fluctuations
- Speed of sound: $\langle p_T \rangle$ fluctuations
- Shear viscosity/entropy ratio (η/s): Claude's talk

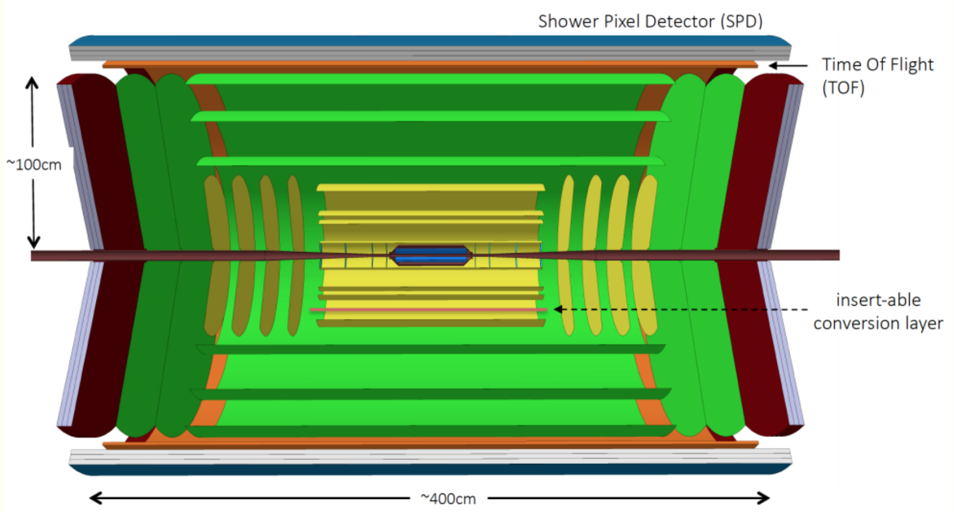
RHIC BES I and II

- Traces of early stage thermalization: $\Delta p_T \Delta p_T$ correlations
- Higher order cumulants of $\langle p_T \rangle$ distributions
- Effect of radial flow and radial flow fluctuation

- Local temperature fluctuations over small phase bins: to map the heavy-ion collision



Nucl. Phys. A 1017 (2022) 122343



ALICE 3: a new detector For LHC Run-5 (2032 ..)

https://arxiv.org/abs/1902.01211

Low p_T down to ~20 MeV/c
Extended rapidity coverage: up to 8 rapidity units