Calculation of 3-pion Coulomb Scattering using Scalar QED

Dhevan Gangadharan (University of Houston)

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Outline

Central question

How do the true 3-pion Coulomb interactions compare to the commonly used asymptotic approximations in high-energy hadronic collisions?

1) Why are 3-pion Coulomb interactions important?

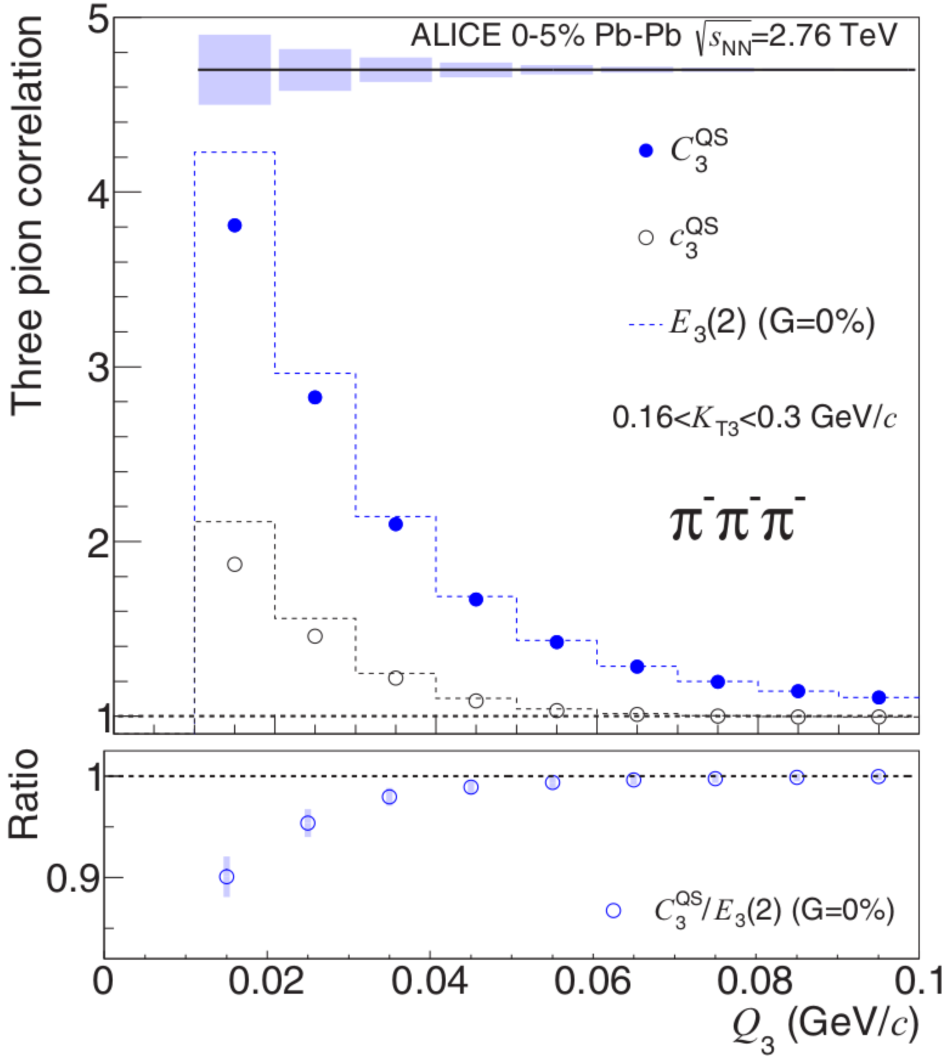
2) Feynman diagram approach to calculating Coulomb interactions.

3) Scalar QED Feynamn rules.

4) Benchmark studies using 2-pion QED calculations at NNLO.

5) 3-pion QED calculations at NNLO.

Motivation: Why are 3-pion Coulomb interactions important?



PRC 93 054908 (2016)

ALICE measurements of 3-pion Bose-Einstein correlations revealed a suppression wrt expectations (dashed lines).

3-body Coulomb interactions were taken into account using an asymptotic ansatz that is valid at sufficiently "large" triplet relative momentum Q_3.

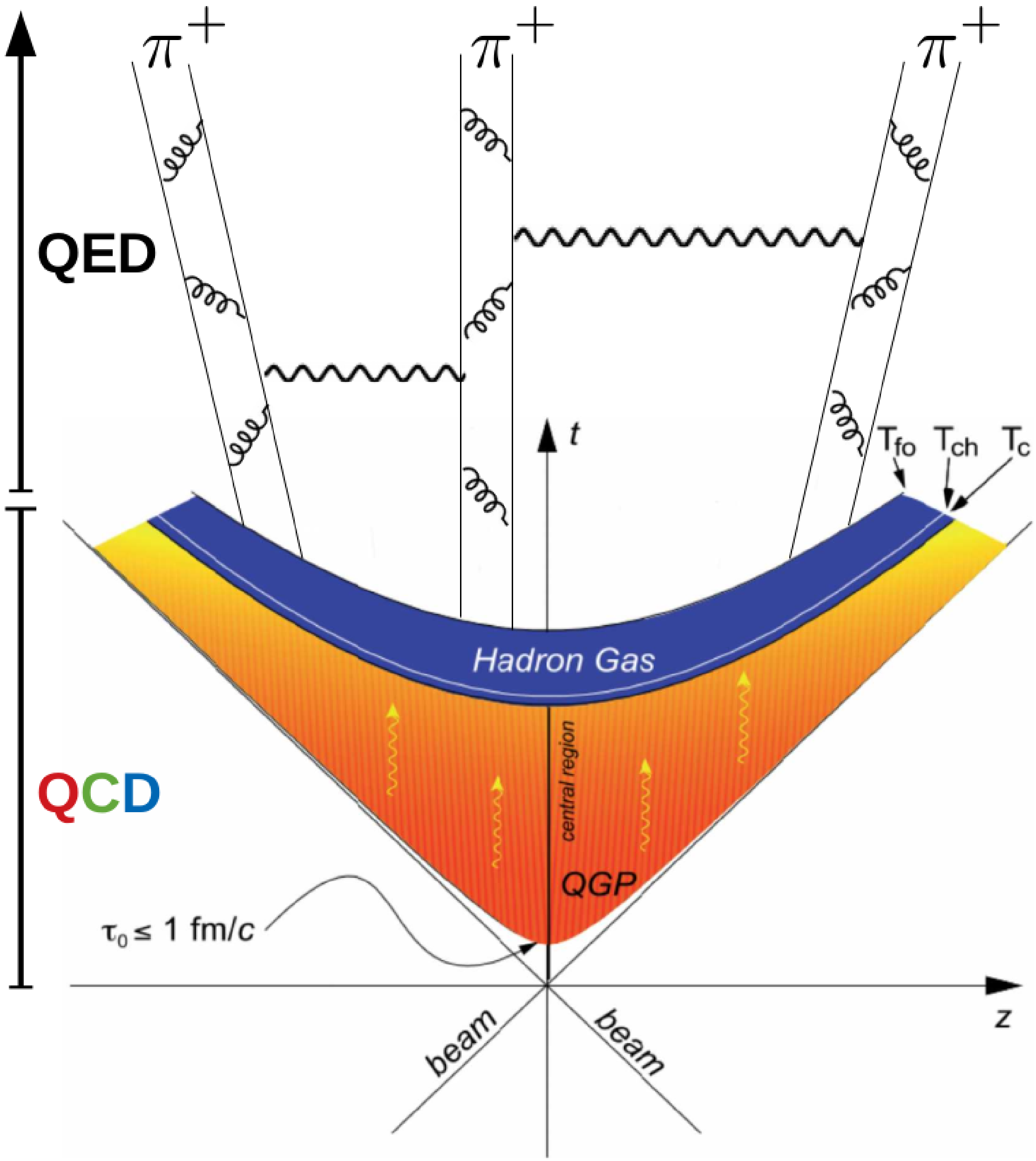
Ansatz Coulomb factor used:

$$K_3 = K_{12}\, K_{13}\, K_{23}$$

A calculation of genuine 3-body Coulomb interactions is needed to be sure about the origin of this suppression.

$$Q_3 = \sqrt{q_{12}^2 + q_{13}^2 + q_{23}^2}$$

Feynman diagram approach to calculating Coulomb interactions



Before freeze out,
QCD processes dominate.

After freeze out,
QED dominates the interaction
Between charged pions.

The production amplitude of a pair/triplet at kinetic freeze out is referred to as M_0

For simplicity,
M_0 is treated as momentum independent (point-source Gamow approximation).

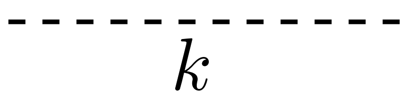
The calculation presented here pertains to the QED interactions after freeze out.

The calculation is needed to help interpret the suppression of 3-pion Bose-Einstein correlations.
Quantum coherence at freeze out or "extra" Coulomb repulsion?

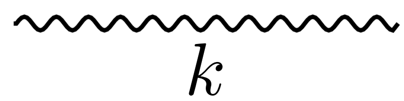
Scalar QED Feynman rules

Lagrangian: $$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + (D_\mu\phi)^*(D^\nu\phi) - m^2\phi^*\phi$$

Propagators:

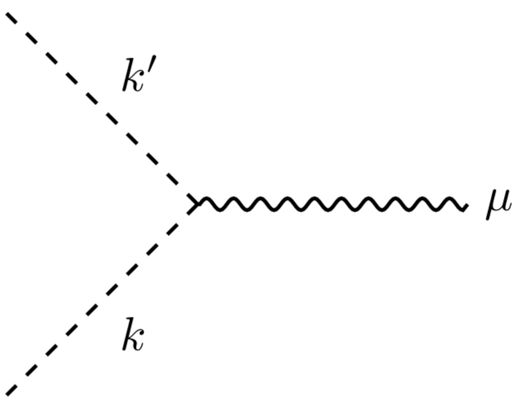


$$= \frac{i}{k^2 - m^2 + i\varepsilon}$$

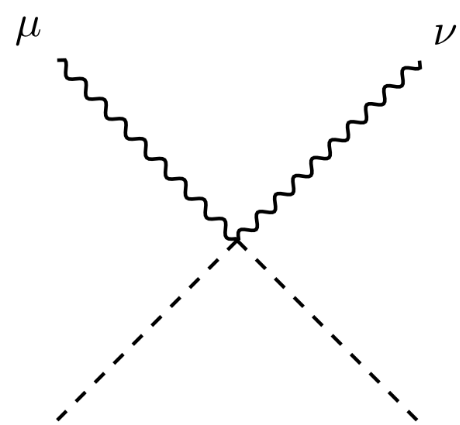


$$= \frac{-ig^{\mu\nu}}{k^2 + i\varepsilon}$$

Vertices:



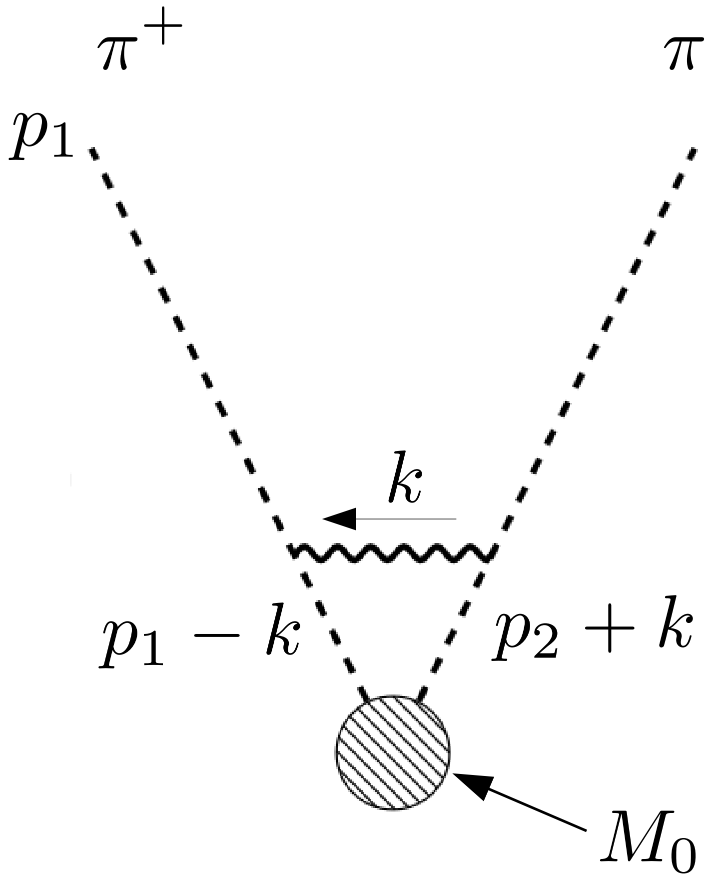
$$= -ie(k + k')^\mu$$



$$= 2ie^2 g^{\mu\nu}$$

Matthew Schwartz
Quantum field theory and the standard model

Example of 1-loop amplitude



QCD amplitude factor
(momentum independent)

Dimensionally
regulated integral
d = 4 - 2ϵ

$$I = M_0\, \mu^{2\epsilon}\, e^{\gamma\epsilon} \int_{-\infty}^{\infty} \frac{d^d k}{(2\pi)^4} \frac{i}{(p_1 - k)^2 - m^2 + i\varepsilon} [-ie(2p_1 - k)^\mu] \frac{-ig^{\mu\nu}}{k^2 + i\varepsilon} [-ie(2p_2 + k)_\nu] \frac{i}{(p_2 + k)^2 - m^2 + i\varepsilon}$$

$\mu = m_{\text{pion}}$

$\gamma = \text{Euler constant} = 0.577$

Loop momenta can be dropped from the numerator.
This obviates renormalization.

Baier & Fadin
Sov. Phys. JETP 30
127 (1970)

Non-relativistic solution to 2-pion problem: Gamow factor

Some non-relativistic simplifications to the propagator are justified after a scale transformation:

Baier & Fadin
Sov. Phys. JETP 30
127 (1970)

$\mathbf{k} \rightarrow \mathbf{p}\mathbf{k}$, $k^0 \rightarrow \mathbf{p}^2/m\, k^0$

$$\frac{1}{(p-k)^2 - m^2} = \frac{1}{k^2 - 2pk} \rightarrow \left[\frac{1}{\mathbf{p}^2}\right] \frac{1}{-\mathbf{k}^2 - 2k^0 + 2\hat{\mathbf{p}}\mathbf{k}}$$

After this, Baier and Fadin showed how to resum the entire perturbative series analytically to obtain the well-known QM result of 2-body Coulomb from George Gamow

$$|\psi|^2 \equiv C_2 = \frac{\frac{4\pi}{qa}}{e^{\frac{4\pi}{qa}} - 1}$$

Gamow 2-body Coulomb factor
Valid for point-source emission

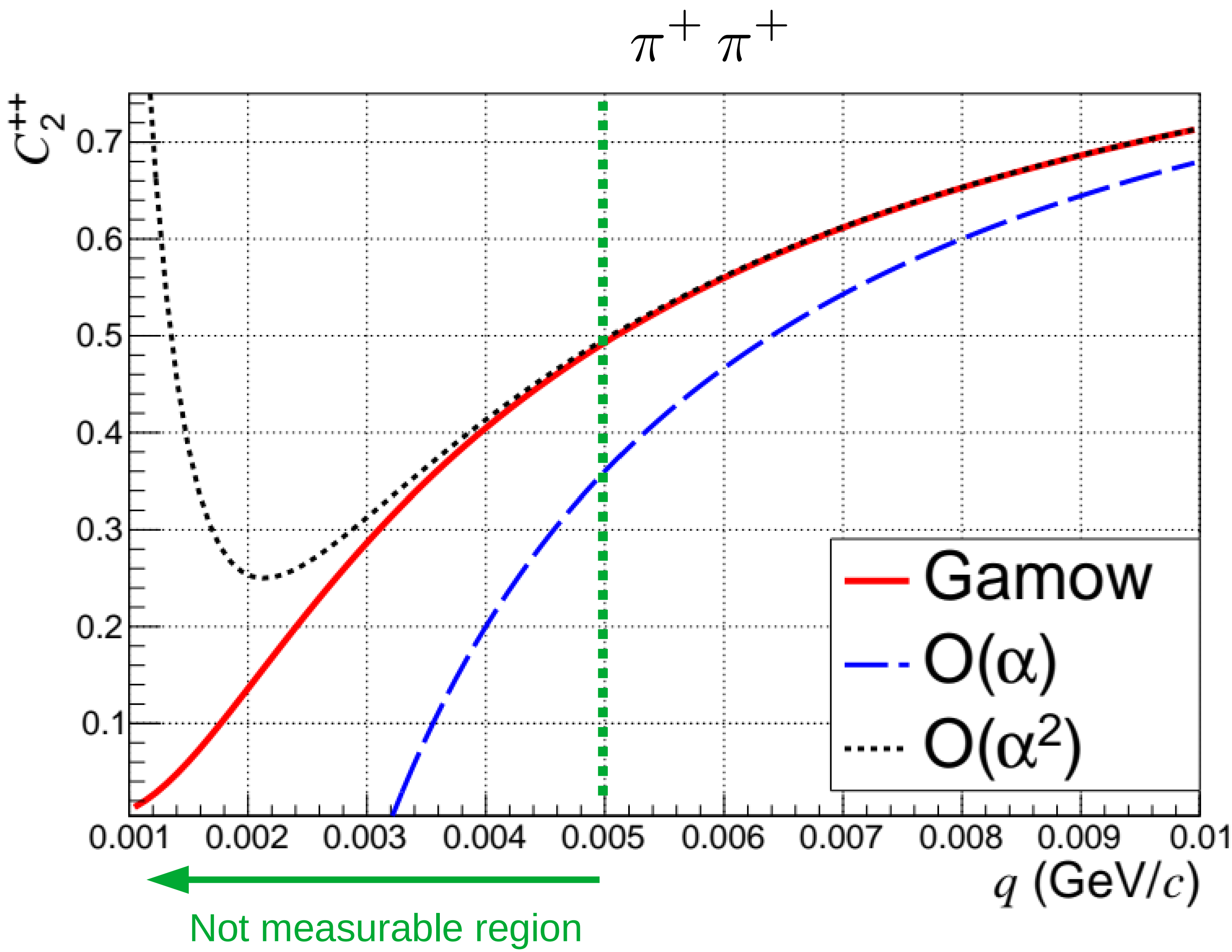
$$q = \sqrt{-(p_1^\mu - p_2^\mu)^2}$$

Lorentz invariant relative momentum

$$a$$

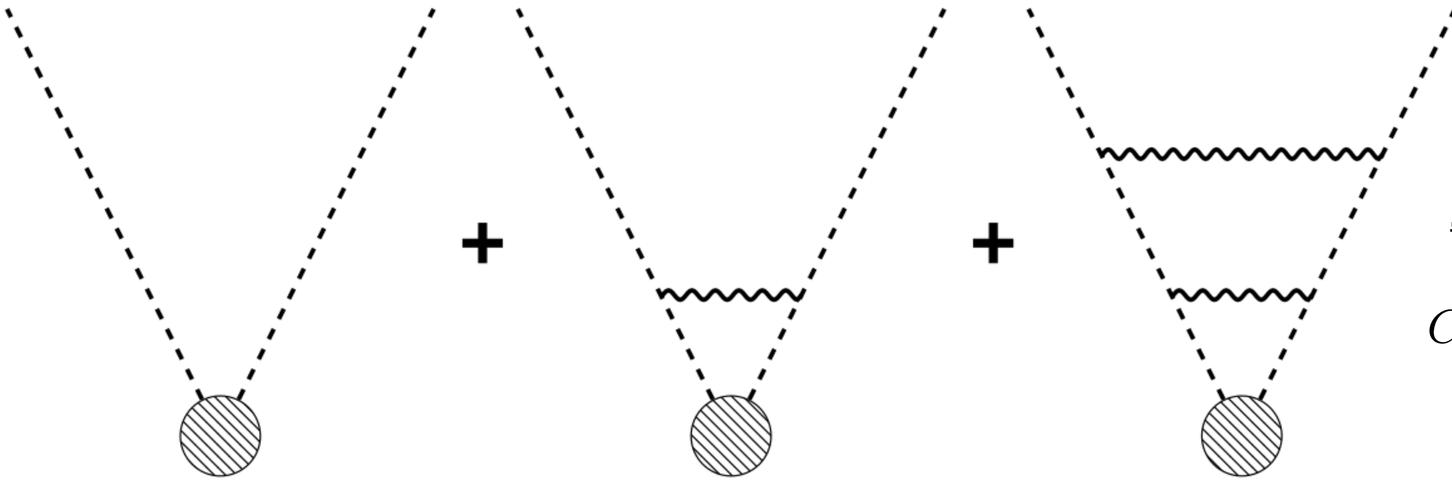
Bohr radius of the pair: 388 fm for pions

2-pion Gamow compared to O(α) and O(α²) expansion



Just 2 terms in the series is very accurate in the measurable region!!

2-pion diagrams to be calculated using scalar QED

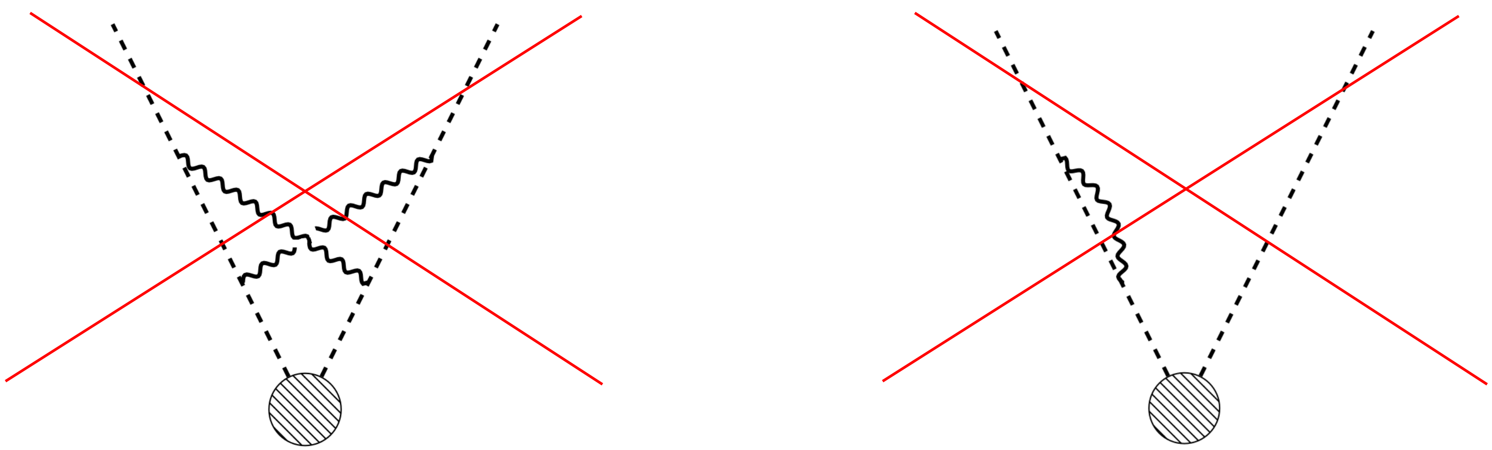


2-pion correlation function

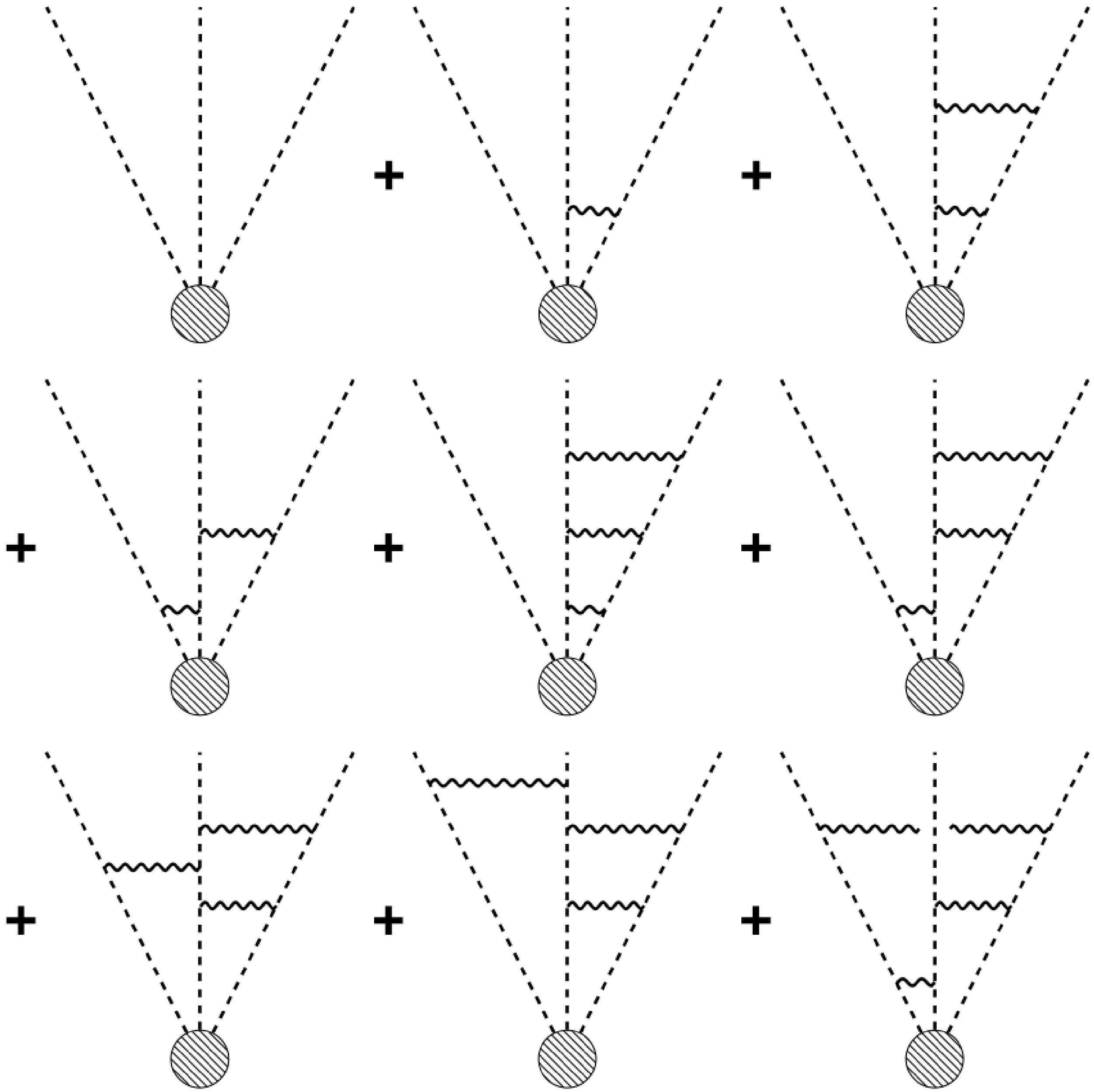
$$C_2 = \frac{|\text{sum of all terms}|^2}{|\text{no photon term}|^2}$$

Only straight “ladder” diagrams are important at low relative momentum

Baier & Fadin
Sov. Phys. JETP 30
127 (1970)



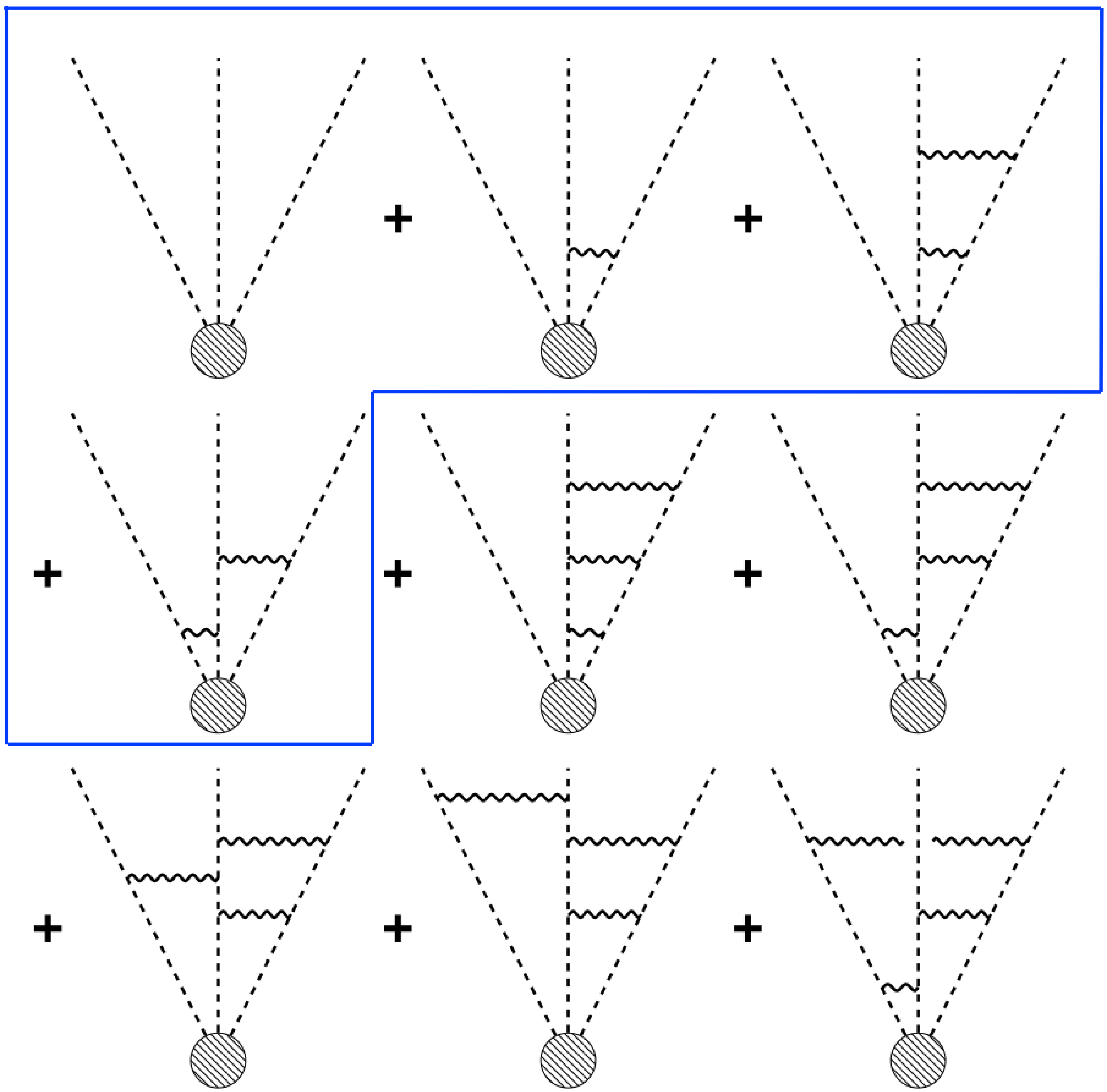
3-pion diagrams to be calculated using scalar QED



3-pion correlation function

$$C_3 = \frac{|\text{sum of all terms}|^2}{|\text{no photon term}|^2}$$

3-pion diagrams to be calculated using scalar QED



3-pion correlation function

$$C_3 = \frac{|\text{sum of all terms}|^2}{|\text{no photon term}|^2}$$

Only these terms have been calculated at the moment

The rest need special treatment on a GPU farm

Sector decomposition and dimensional regularization

Regularize by shifting exponents by a small parameter ϵ

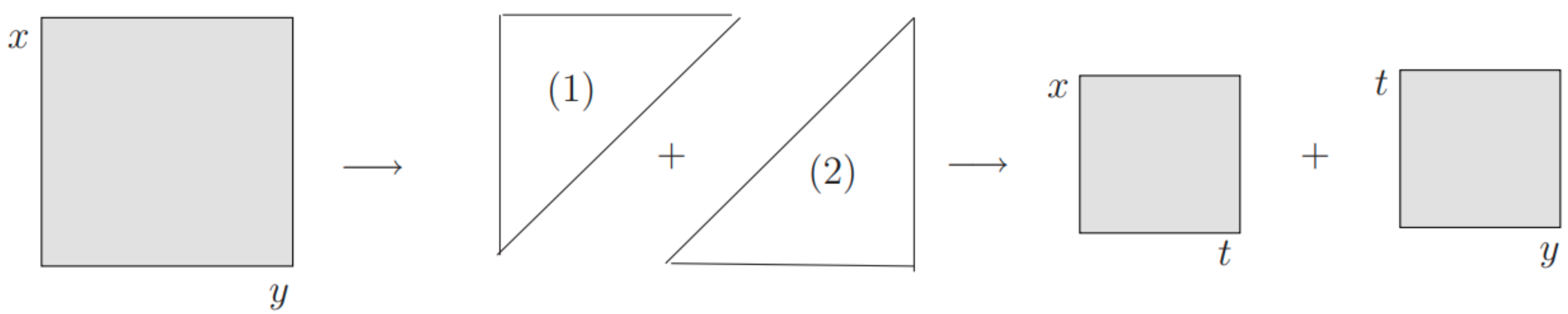
Decompose integral into 2 sectors to factor out the divergent part

$$\int_0^1 \frac{dx\,dy}{(x+y)^2} \to \int_0^1 \frac{dx\,dy}{(x+y)^{2-\epsilon}} \to \int_0^1 \frac{dx\,dy}{(x+y)^{2-\epsilon}} [\overset{(1)}{\Theta(x-y)} + \overset{(2)}{\Theta(y-x)}]$$

Change variables

$y \to xt$, $0 \leq t \leq 1$

$x \to yt$, $0 \leq t \leq 1$



G. Heinrich
arXiv:0803.4177

Sector decomposition and dimensional regularization

$$\int_0^1 \frac{dx\,dy}{(x+y)^{2-\epsilon}}\left[\Theta(x-y)+\Theta(y-x)\right] = \int_0^1 \frac{x\,dx\,dt}{(x(1+t))^{2-\epsilon}} + \int_0^1 \frac{y\,dy\,dt}{(y(1+t))^{2-\epsilon}}$$

Change variables

$y \to xt$ $\quad$ $x \to yt$

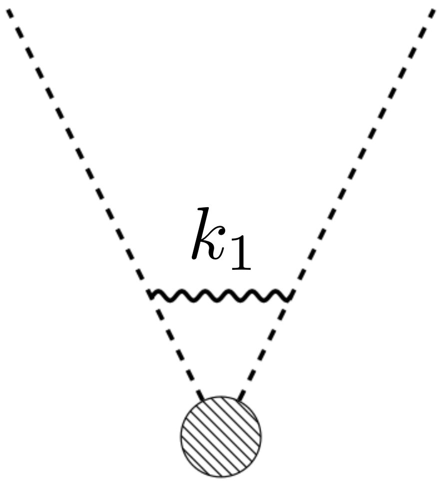
$0 \le t \le 1$ $\quad$ $0 \le t \le 1$

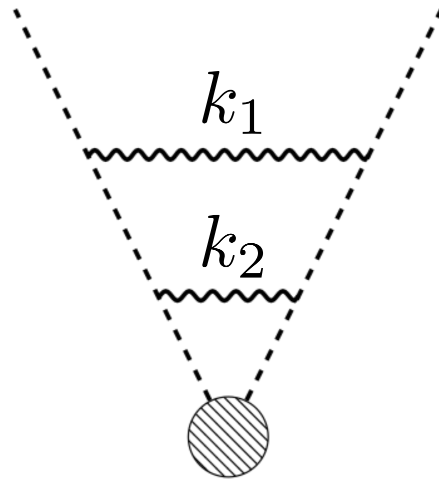
$$= 2\left[\int_0^1 \frac{dx}{x^{1-\epsilon}}\right]\left[\int_0^1 \frac{dt}{(1+t)^{2-\epsilon}}\right] = 2\left[\frac{1}{\epsilon}\right]\left[\frac{1}{2}(1+\epsilon(1-\ln 2))\right]$$

$$= \frac{1}{\epsilon} + (1-\ln 2)$$

Divergent part $\quad$ Finite part

General form of Feynman integrals with dimensional regularization





$$I = \frac{A}{\epsilon} + B + C\epsilon + \mathcal{O}(\epsilon^2)$$

$$I = \frac{A}{\epsilon^2} + \frac{B}{\epsilon} + C + D\epsilon + \mathcal{O}(\epsilon^2)$$

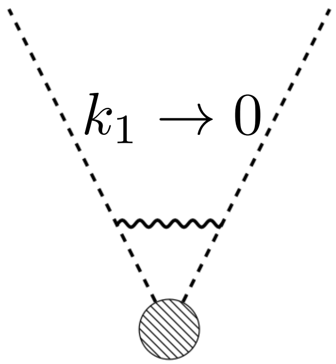
Divergence when $k_1 \rightarrow 0$

Divergence when k_1 & $k_2 \rightarrow 0$

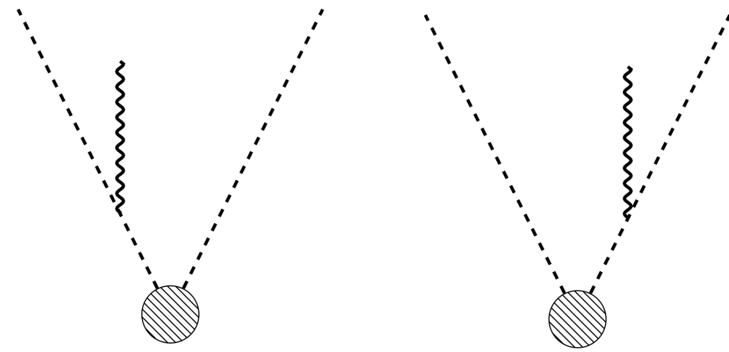
Divergence when k_1 or $k_2 \rightarrow 0$

Infrared and Ultraviolet divergencies

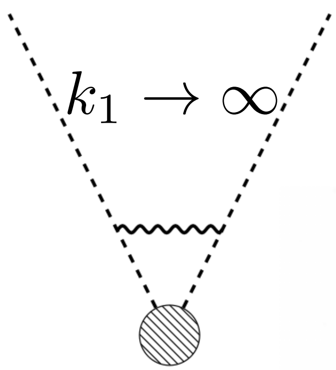
Infrared divergencies



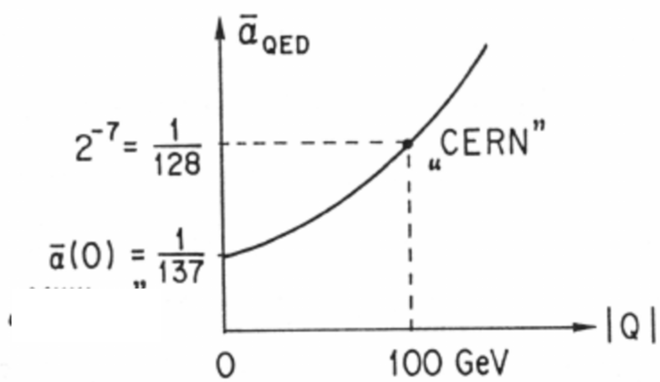
Canceled by contributions with a soft real photon emission

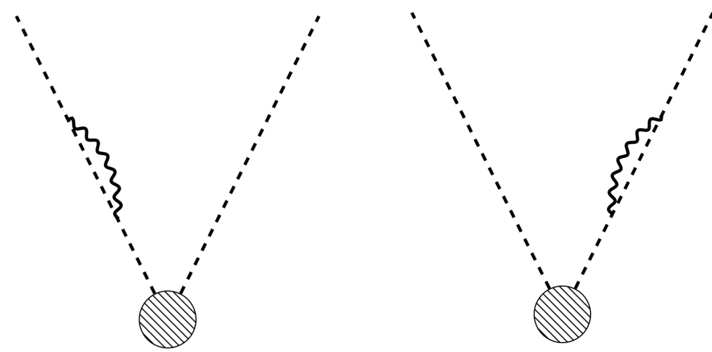


Ultraviolet divergencies



Absorbed into a modified coupling and pion mass (Renormalization)





Also need these diagrams to renormalize

Baier & Fadin
Sov. Phys. JETP 30 127 (1970)

Important simplification: For low momentum pions, real photon emission is highly suppressed and also the "standard" values of α_{QED} and m_{pion} can be used.
That means that the divergent terms can simply be discarded.

GPGPU-based computation

The Feynman integrals are evaluated numerically using the software package:

pySecDec
Gudrun Heinrich et al.
arXiv: 1703.09692

- General-Purpose GPUs (GPGPU) are used to speed up the calculation.
- 1- and 2-loop diagrams can be calculated on a single GPGPU card
- 3-loop diagrams need a farm of GPGPUs

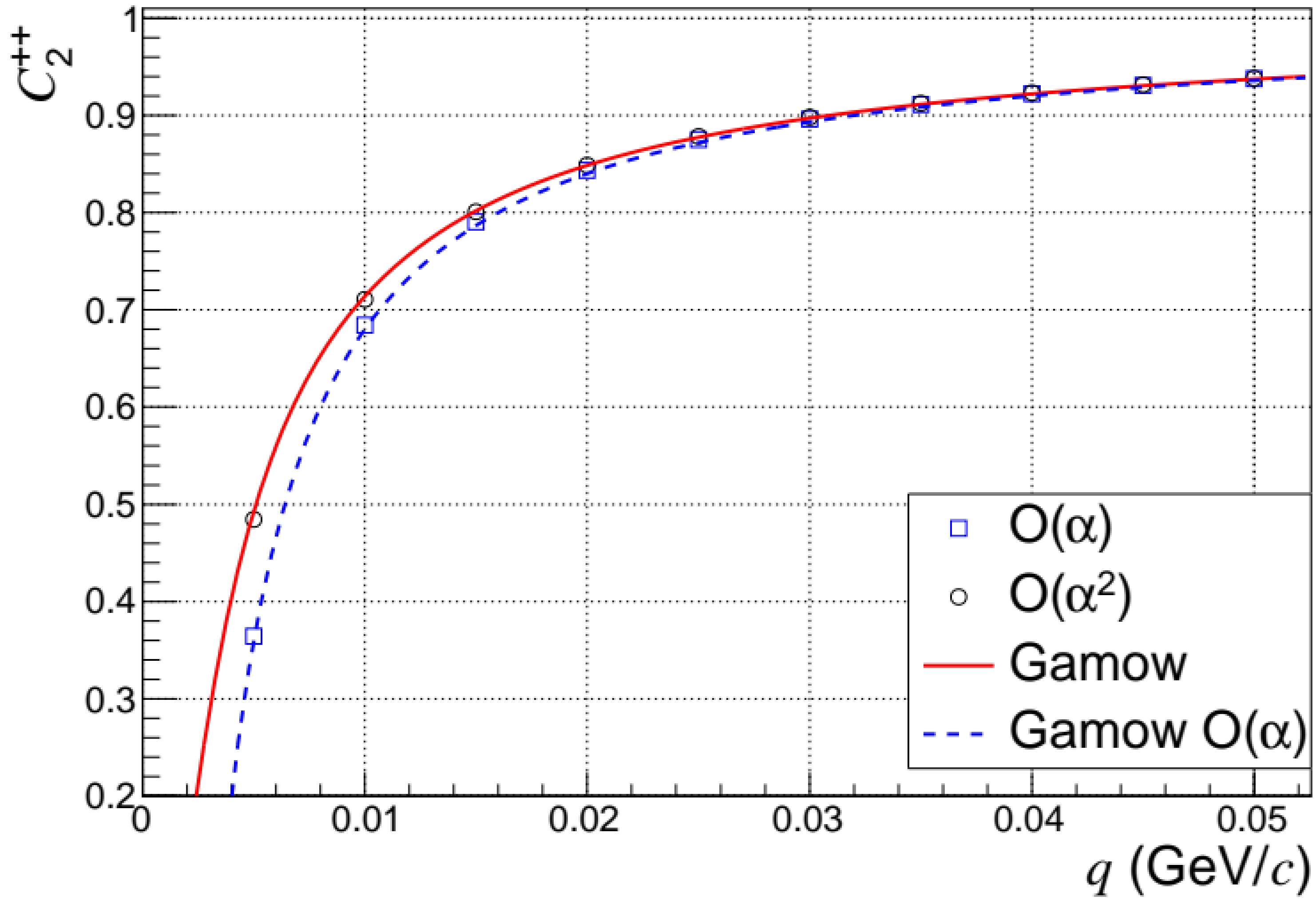
My GPGPU

Claudia Ratti's GPGPU farm at UH

Compute times with a **single** high-end GPGPU

	1-loop	2-loop	3-loop
Compute time	~1 min	~1 day	> 1 month

Benchmark results:
2-pion calculation compared to non-relativistic expectation



Calculation and Gamow are similar at the % level

The calculation should **not** be identical to Gamow but they should be close:

Quantum Field Theory → Quantum Mechanics as $q \to 0$

3-pion kinematics

Amplitudes are projected against the usual triplet Lorentz invariant relative momentum

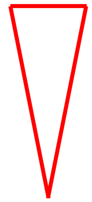
$$Q_3 = \sqrt{q_{12}^2 + q_{13}^2 + q_{23}^2}$$

For a single value of Q_3, there is a continuum of pair configurations.
For a point-source, the momentum spectrum is flat, which yields the phase-space weight:

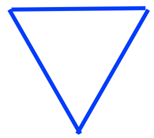
$$W \propto q_{12}\, q_{13}\, q_{23}$$

Two schemes of 3-pion kinematics are chosen:

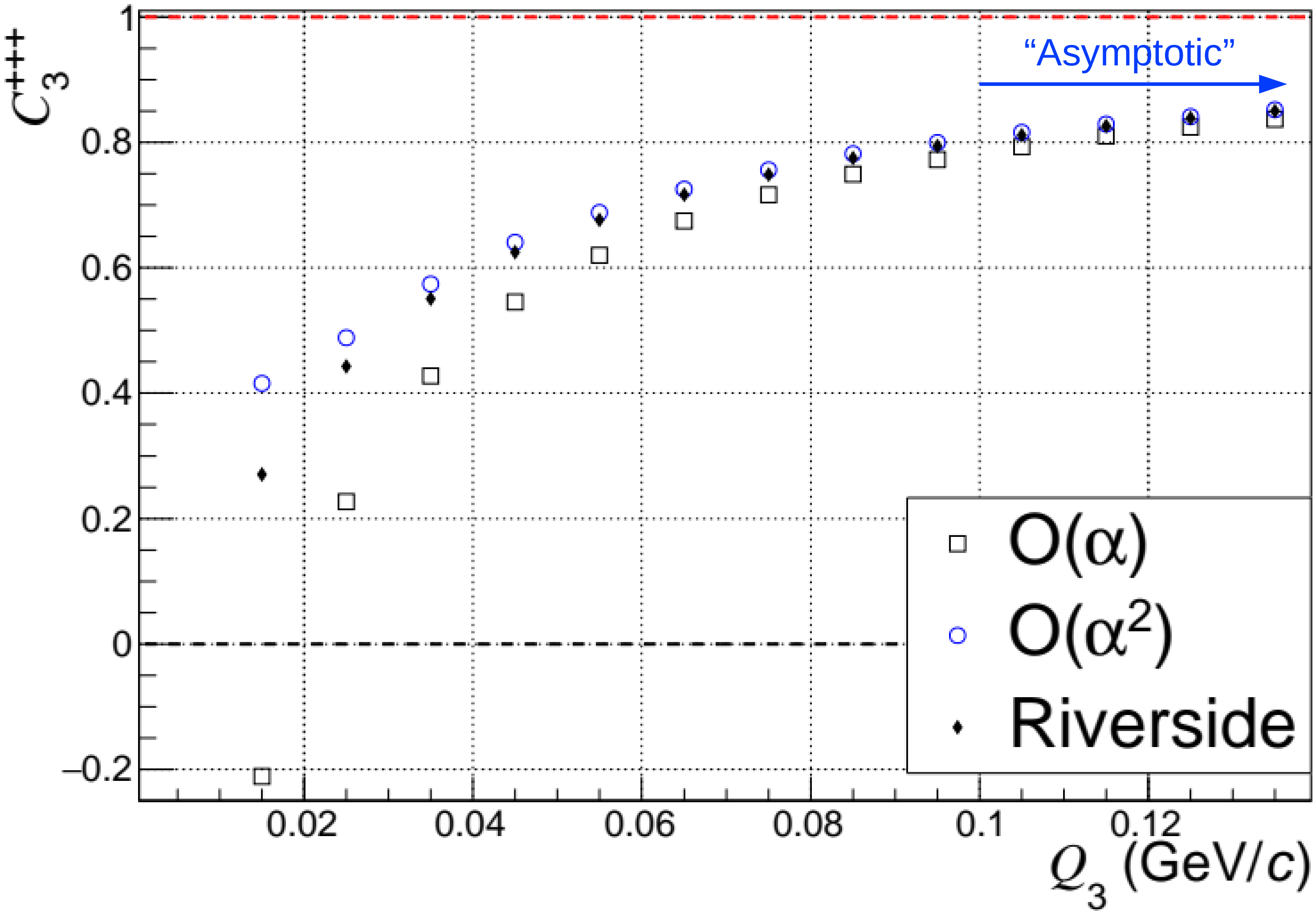
1) Weighted pair configurations (4 chosen): ranges from the most asymmetric to symmetric pair configurations. For each, 6 pair permutated diagrams need to be computed: **4*6 = 24 diagrams per Q_3 value**.



2) Symmetric configuration: $q_{12} = q_{13} = q_{23}$. **1 diagram per Q_3 value**.
This scheme might be the only way forward for 3-loop diagrams.



3-pion Result:
Weighted pair configurations

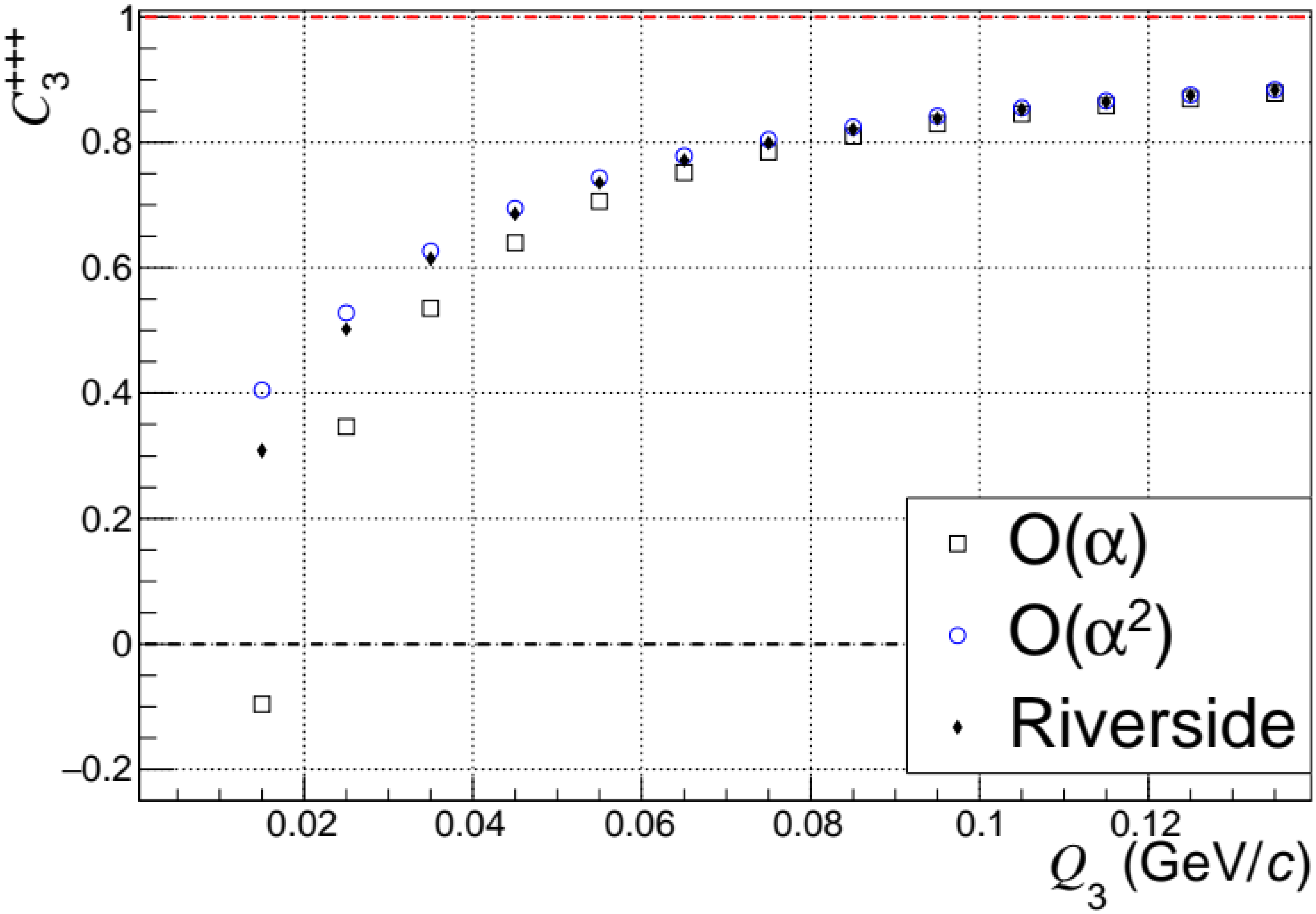


Central Question:

Is the genuine 3-body Coulomb correlation greater or less than Riverside??

- Q_3 > 0.1 GeV, O(α^2) is very similar to Riverside (asymptotic solution).
- Each order appears to contribute with sign = $(-1)^N$: O(α) negative, O(α^2) positive.
  - Need one more order to get an answer to the central question.

3-pion Result:
Symmetric pair configuration



24x fewer diagrams to calculate

- ToDo: calculate 3-loop contributions, O(α^3), at lowest and highest Q_3 points only.
- That should be decisive in answering the central question.

Summary

- Coulomb interactions between 2- and 3-pions are perturbatively calculable with scalar QED in the measurable kinematics of virtually all high-energy experiments.
- Sector decomposition is a powerful tool to numerically calculate the Feynman integrals.
- At low relative momentum (non-relativistic case), IR and UV divergencies can simply be discarded from the dimensionally regularized integrals.
- For 2-pions, 2 terms in the series is sufficient.
- For 3-pions, one may need 3 terms for Q_3 <~ 100 MeV/c (series decreases less rapidly).

ToDo

- Calculate $O(\alpha^3)$ diagrams. Should be decisive in answering the central question:

  Is the asymptotic 3-body Coulomb solution justified in high-energy hadronic collisions??