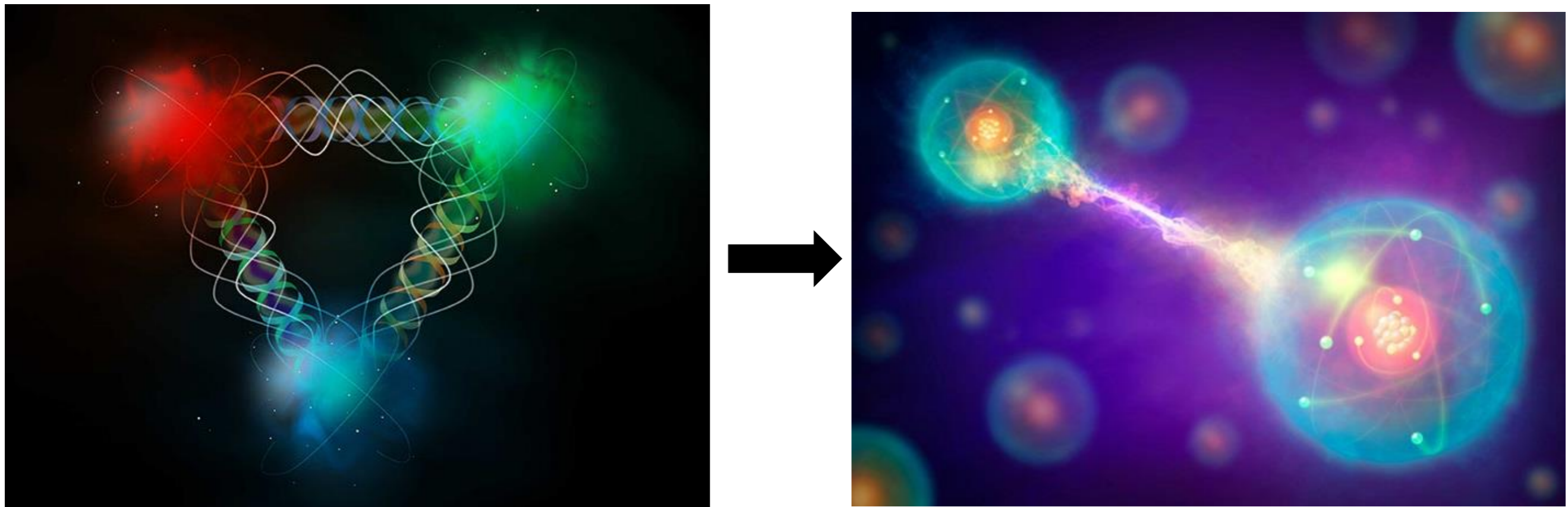


Is the nucleus produced in particle collisions a quantum entangled state ?

Rene Bellwied, Alek Hutson (University of Houston)



References: based on discussion with R. Stock: R. Bellwied (arXiv:1807.04589), Tu, Kharzeev, Ullrich (arXiv:1904.11974), and other papers by Kharzeev et al., Floerchinger, Berges, Venugopalan, (arXiv:1702.03489, arXiv:1712.04558, arXiv:1707.05338, arXiv:1712.09362)

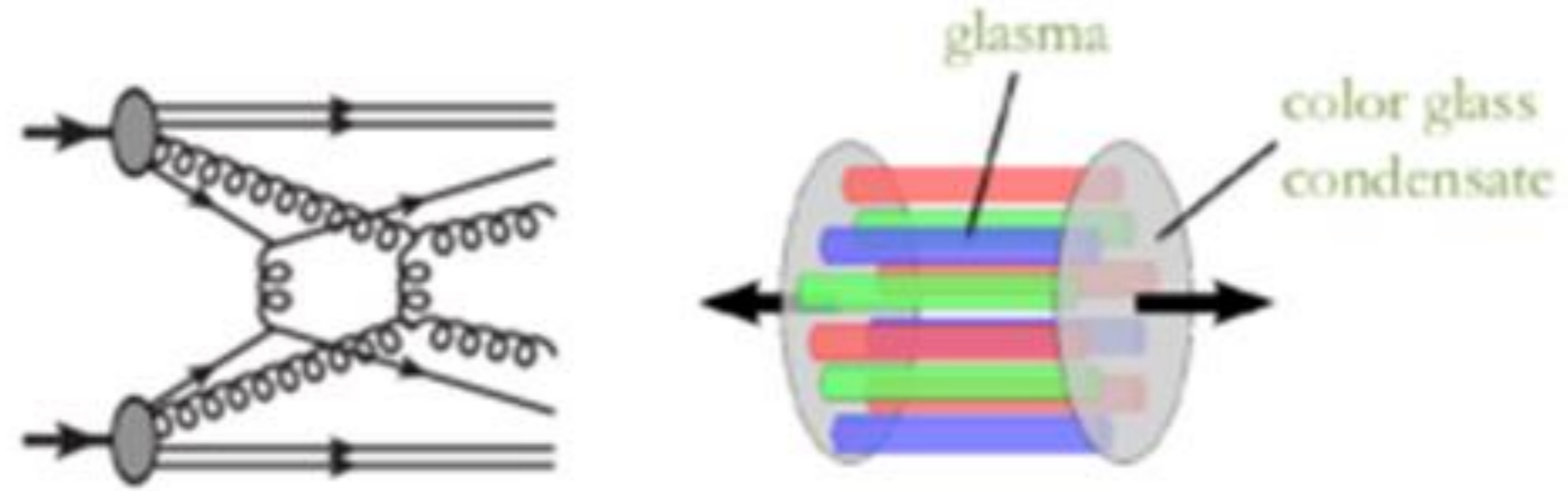
37th WWND, Puerto Vallarta,
Feb.27-Mar.5, 2022



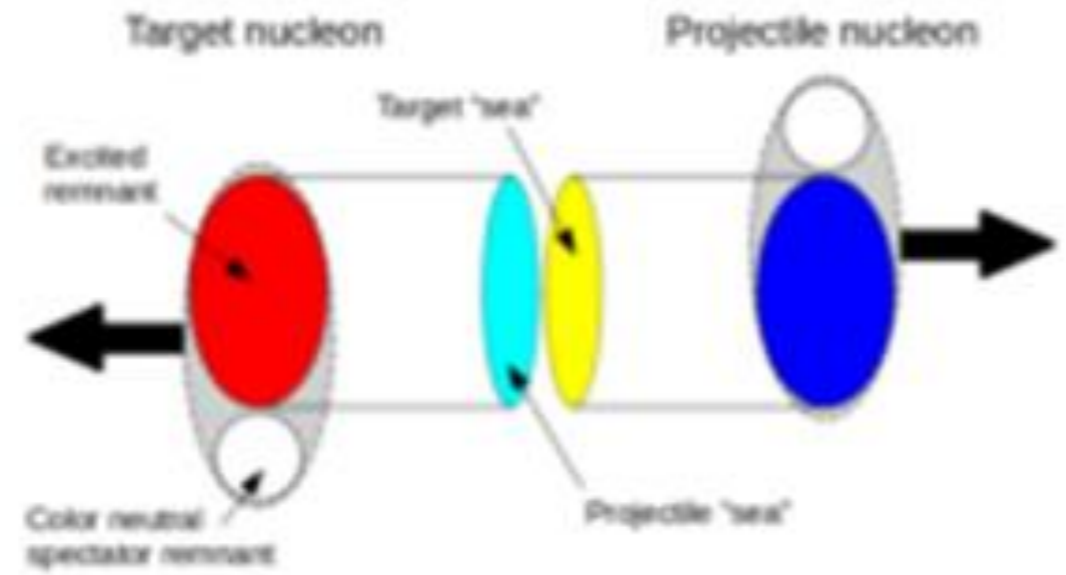
Maybe our picture of parton-parton interactions in proton-proton collisions is wrong

From Peter Christiansen talk on Monday:

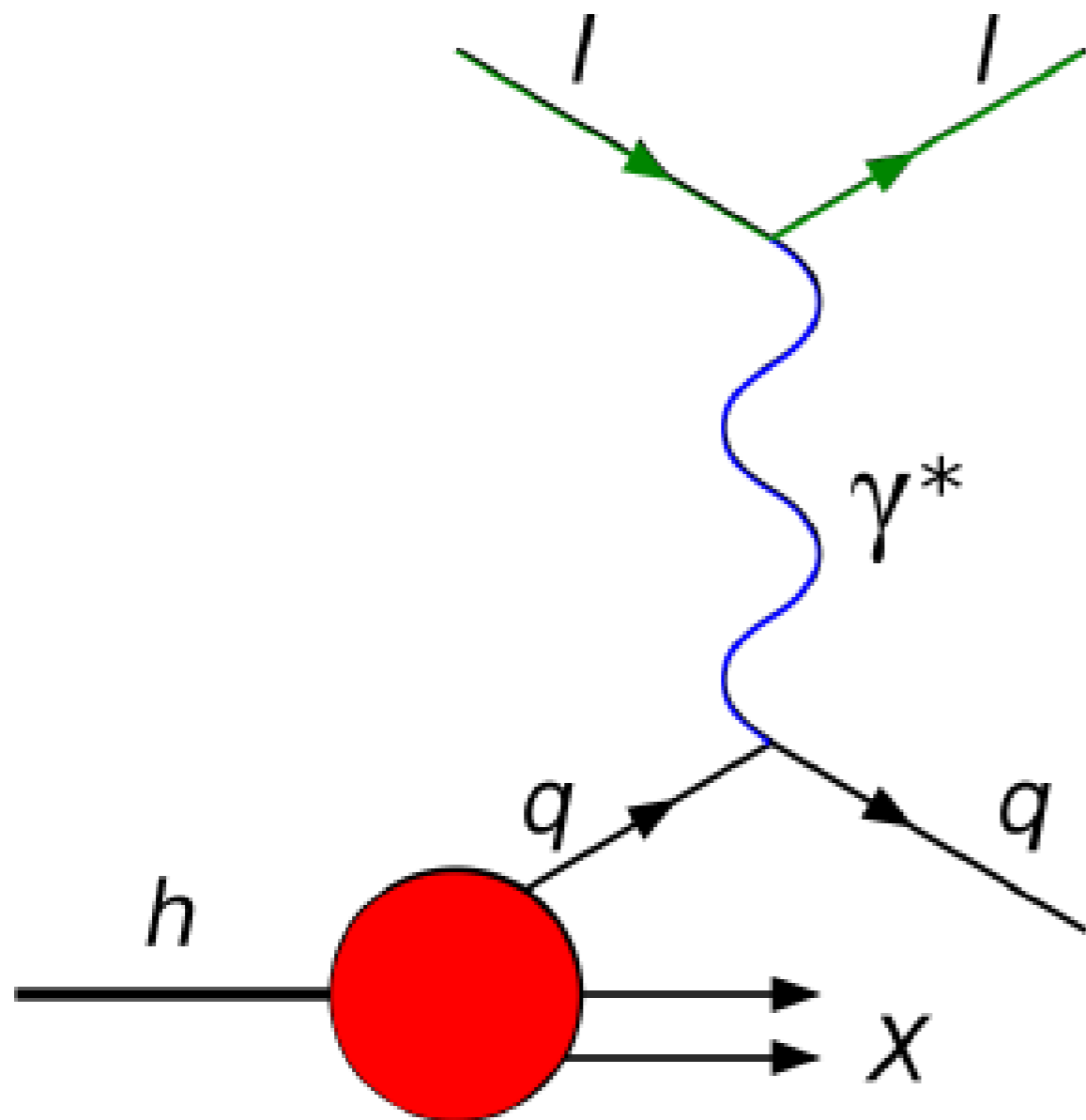
- Maybe these models of MB proton-proton collisions are wrong



- Maybe instead a coherent excitation of the protons occur



The proton in the basic parton model (PYTHIA etc.)



Any parton model describes the proton as a collection of point-like quasi-free partons frozen in the infinite momentum frame due to Lorentz dilation.

Cross-sections are given by the incoherent sum of cross sections of scattering off individual partons.

These models ignore quantum mechanics

Sometimes 'patched' through DGLAP, cluster (HERWIG), parton cascade (PCM) implementations, but e.g. DGLAP has to be applied on the energy dependent gluon saturation scale to take into account the high production of 'clusters' from soft processes in the initial state (see. T. Lappi, arXiv:1104.3725)

Quantum entanglement in transverse and longitudinal direction

Transverse:

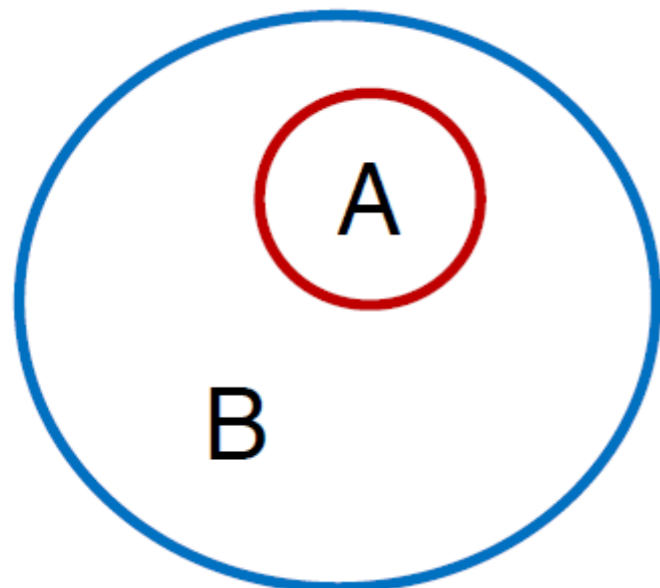
DIS probes only part of the proton's wave function (region a), but we sum over all hadronic final states, which, in QM, corresponds

to the density matrix of a mixed

state: $\hat{\rho}_A = \text{tr}_B \hat{\rho}$

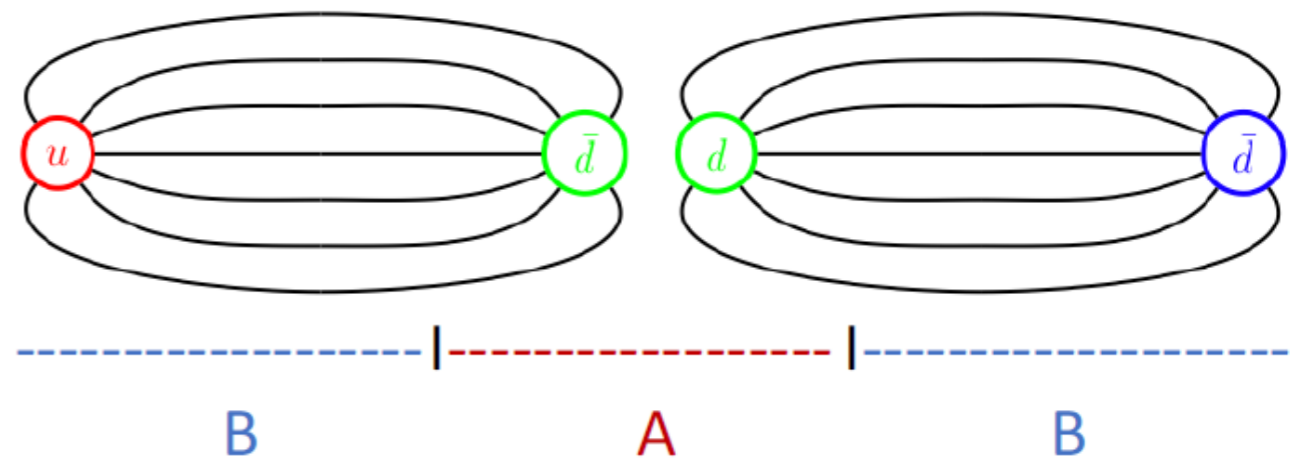
with a non-zero entanglement

entropy: $S_A = -\text{tr} [\hat{\rho}_A \ln \hat{\rho}_A]$



Longitudinal: See S.Floerchinger (QM18)

Particle production in QCD strings:



Example: PYTHIA

Different regions in a string are entangled. Again A is described by a mixed state reduced density matrix. Could this lead to thermal-like behavior in the final state particles ?

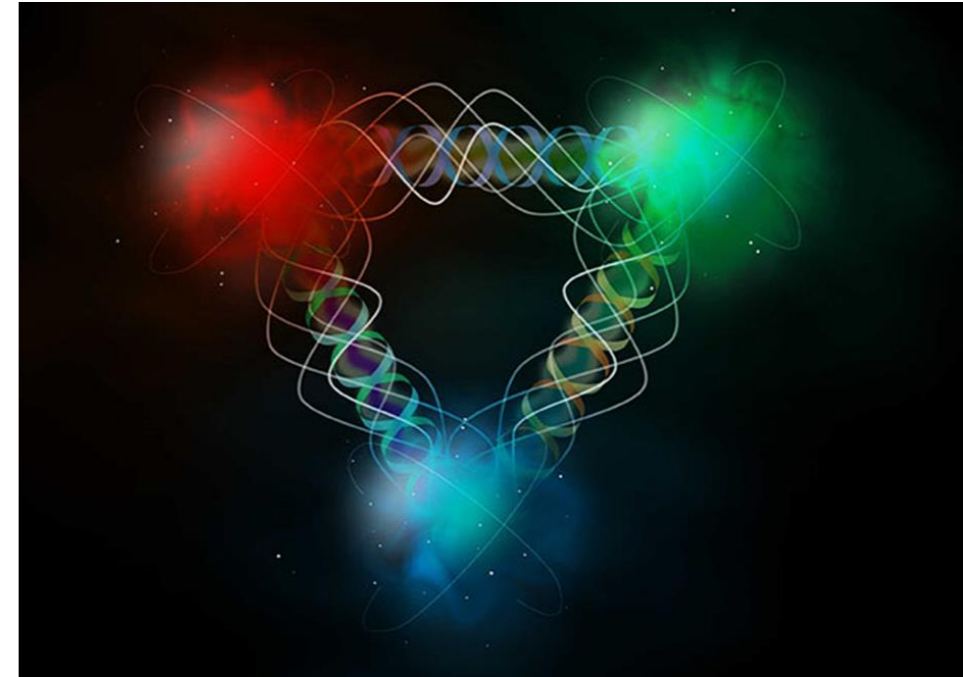
Conclusion: Entanglement entropy is an extensive quantity (depends on volume)

Why entanglement ?

“...we never experiment with just one electron or atom or (small) molecule. In thought experiments, we sometimes assume that we do; this invariably entails ridiculous consequences”



Erwin Schrödinger, 1952



Idea: initial state is entangled transversely (proton confinement) and longitudinally (string formation). Can we measure remnants of coherence ? Are final state multiplicities due to initial state entanglement (all the way out to light nuclei) ?

Entanglement entropy = thermodynamic entropy ? (parton-hadron duality). Is the system not driven by thermalization but by initial coherence, which looks thermal ?

'Thermalization' through quantum entanglement ?

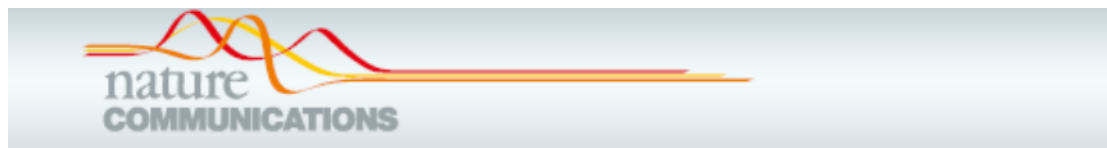
Groundbreaking paper (experimental) (published in Science):

A.M. Kaufman et al., (Harvard), arXiv:1603.04409

Quantum thermalization through entanglement in isolated many-body system, but cold and small (quantum quench in BE condensate of ^{87}Rb atoms), effective $T = 5-10 \text{ J}$, study impact on neighboring atoms

Even more groundbreaking paper (experimental) (published in Nature Comm):

J. Kong et al., May 2020



ARTICLE

<https://doi.org/10.1038/s41467-020-15899-1>

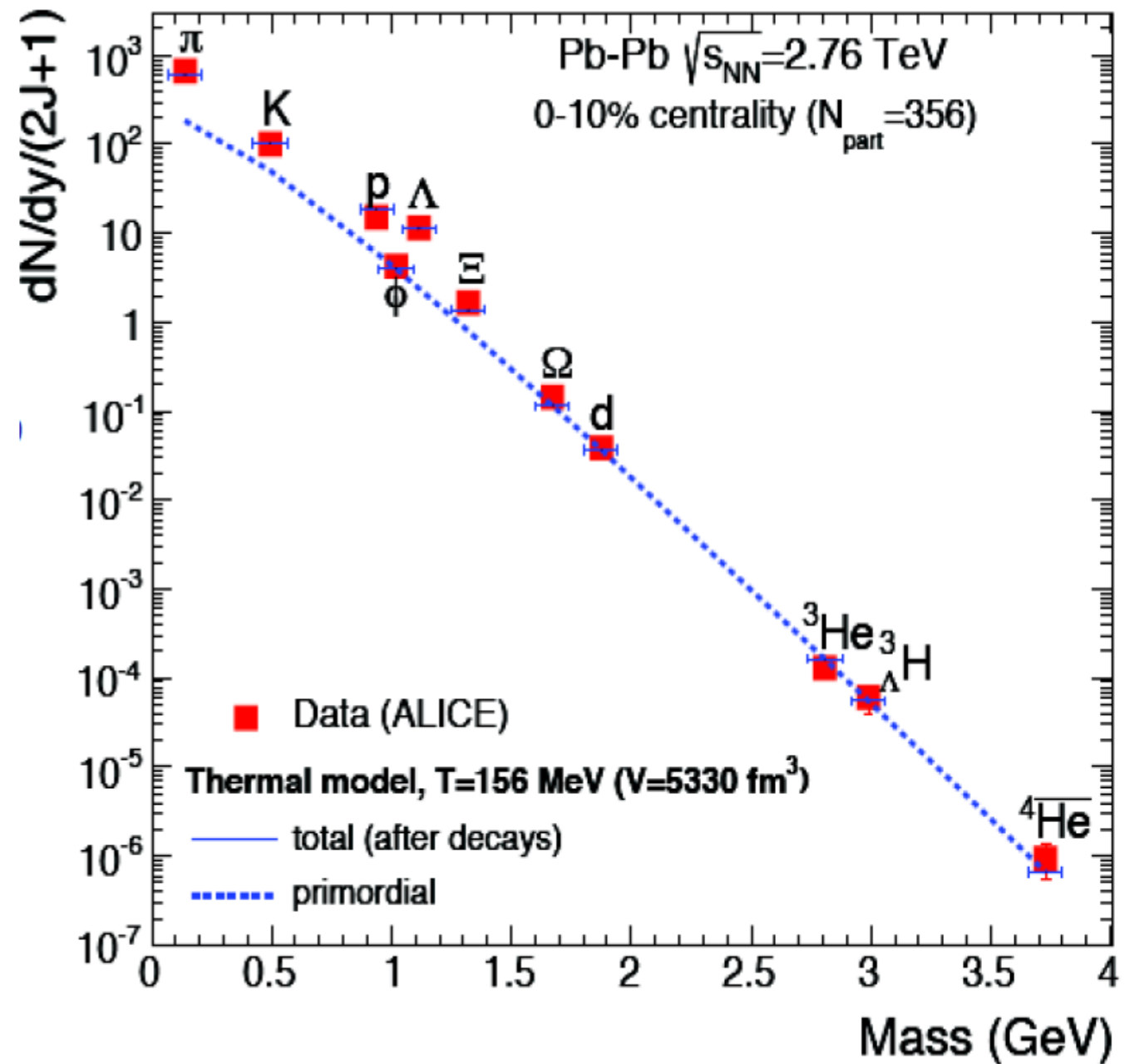
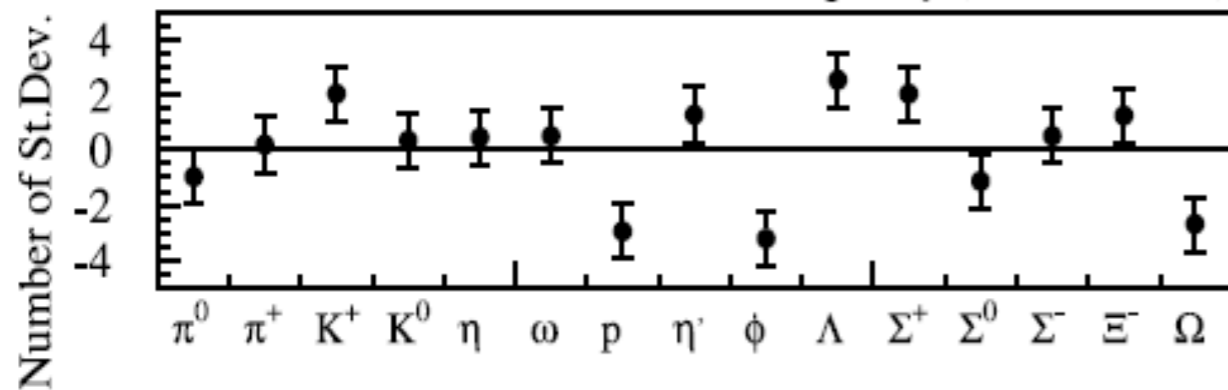
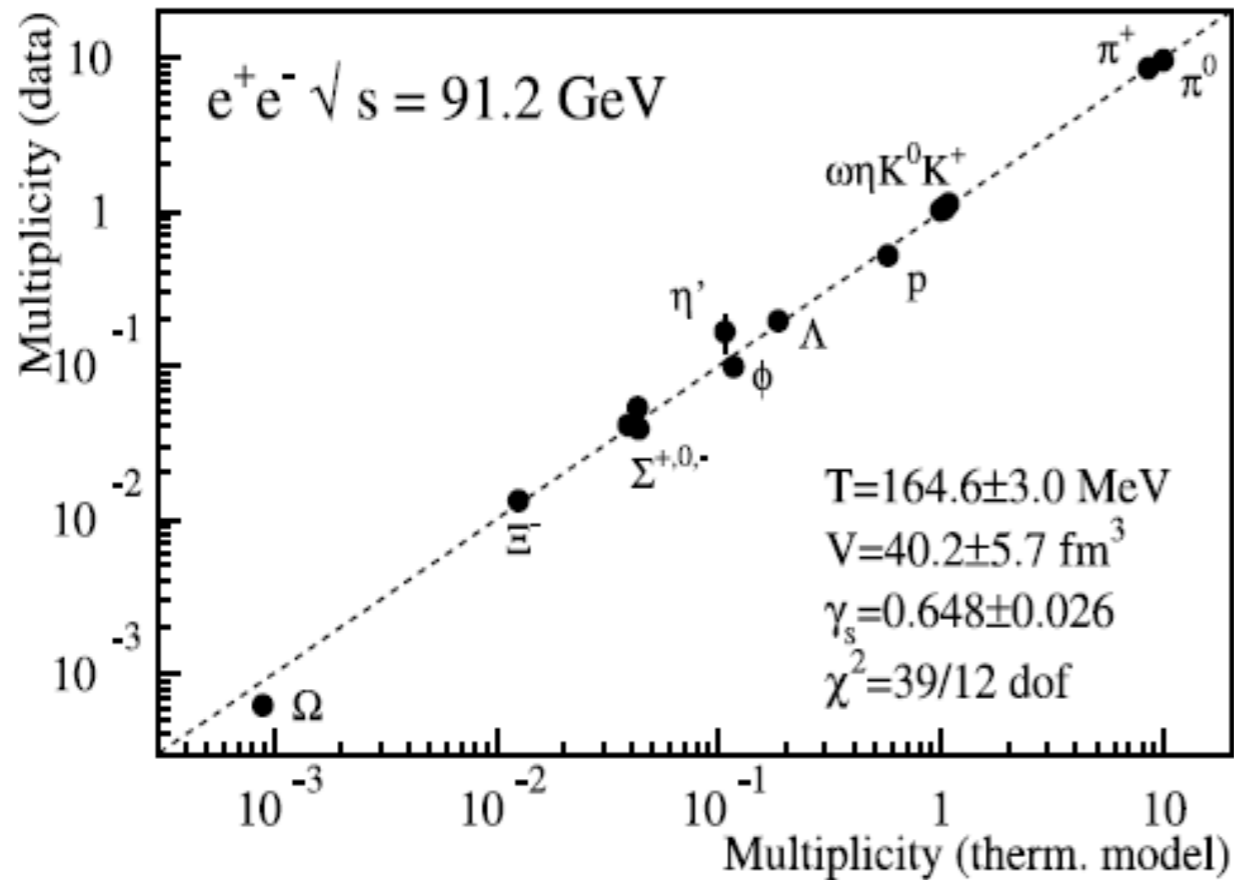
OPEN

Measurement-induced, spatially-extended entanglement in a hot, strongly-interacting atomic system

Jia Kong^{1,2}, Ricardo Jiménez-Martínez², Charikleia Troullinou², Vito Giovanni Lucivero², Géza Tóth^{3,4,5,6} & Morgan W. Mitchell^{2,7}

Quantum technologies use entanglement to outperform classical technologies, and often employ strong cooling and isolation to protect entangled entities from decoherence by random interactions. Here we show that the opposite strategy—promoting random interactions—can help generate and preserve entanglement. We use optical quantum non-demolition measurement to produce entanglement in a hot alkali vapor, in a regime dominated by random spin-exchange collisions. We use Bayesian statistics and spin-squeezing inequalities to show that at least $1.52(4) \times 10^{13}$ of the $5.32(12) \times 10^{13}$ participating atoms enter into singlet-type entangled states, which persist for tens of spin-thermalization times and span thousands of times the nearest-neighbor distance. The results show that high temperatures and strong random interactions need not destroy many-body quantum coherence, that collective measurement can produce very complex entangled states, and that the hot, strongly-interacting media now in use for extreme atomic sensing are well suited for sensing beyond the standard quantum limit.

'Thermal behavior' in elementary relativistic collisions and in light nuclei production



Becattini et al., EPJC 66, 377 (2010)

How can loosely bound objects 'survive' the fireball heat bath ?

- PBM & Stachel et al.: The 'snowball in hell' approach. (J.Phys.G21(1995) L17 and PLB 697 (2011) 203)
- Λ separation energy in hypertriton is 130 keV, i.e. a factor 1000 less than the chemical freeze-out temperature of the fireball
- Successful description of composite objects with SHM implies no entropy production after chemical freeze-out

Entanglement entropy from QCD evolution (D. Kharzeev et al.)

Basis: in an entangled proton the number of possible states is given by the parton distribution function which saturates at low x .

The entanglement entropy can then be calculated through the distribution functions. All partonic states have about equal probability, which means the entanglement entropy is maximal and the proton is a maximally entangled state.

$$S_{initial} \propto \ln(N_g)$$

$$S = \ln[xG(x)]$$

If the second law of thermodynamics applies to entanglement entropy (see black hole physics) then the entropy of the hadronic final state reflects the entanglement entropy of the initial state deduced from the structure function (*parton-hadron duality*)

$$S_{\text{hadrons}} \simeq S_{EE}(x)$$

Idea: Can we measure remnants of coherence? Are final state multiplicities due to initial state entanglement (all the way out to light nuclei)? Is the system not driven by thermalization but by initial coherence, which looks thermal?

Measurements: particle multiplicities as a function of x , particle multiplicities at hadronization trace back to initial parton entanglement (distribution of complex quark states based on string fragmentation?)

The idea applied to pp

arXiv:1904.11974

The EPR paradox and quantum entanglement at sub-nucleonic scales

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(Dated: May 27, 2019)

In 1935, in a paper [1] entitled “Can quantum-mechanical description of reality be considered complete?”, Einstein, Podolsky, and Rosen (EPR) formulated an apparent paradox of quantum theory. They considered two quantum systems that were initially allowed to interact, and were then later separated. A measurement of a physical observable performed on one system then had to have an immediate effect on the conjugate observable in the other system – even if the systems were causally disconnected! The authors viewed this as a clear indication of the inconsistency of quantum mechanics. In the parton model [2–4] of the nucleon formulated by Bjorken, Feynman, and Gribov, the partons (quarks and gluons) are viewed by an external hard probe as independent. The standard argument is that, inside the nucleon boosted to an infinite-momentum frame, the parton probed by a virtual photon with virtuality Q is causally disconnected from the rest of the nucleon during the hard interaction. Yet, the parton and the rest of the nucleon have to form a colour-singlet state due to colour confinement, and so have to be in strongly correlated quantum states – we thus encounter the EPR paradox at the sub-nucleonic scale. In this paper, we propose a resolution of this paradox based on the quantum entanglement of partons. We devise an experimental test of entanglement, and carry it out using data on proton-proton collisions from the Large Hadron Collider (LHC). Our results provide a strong direct indication of quantum entanglement at sub-nucleonic scales.

Keywords: Color confinement, Parton model, Quantum entanglement, EPR paradox

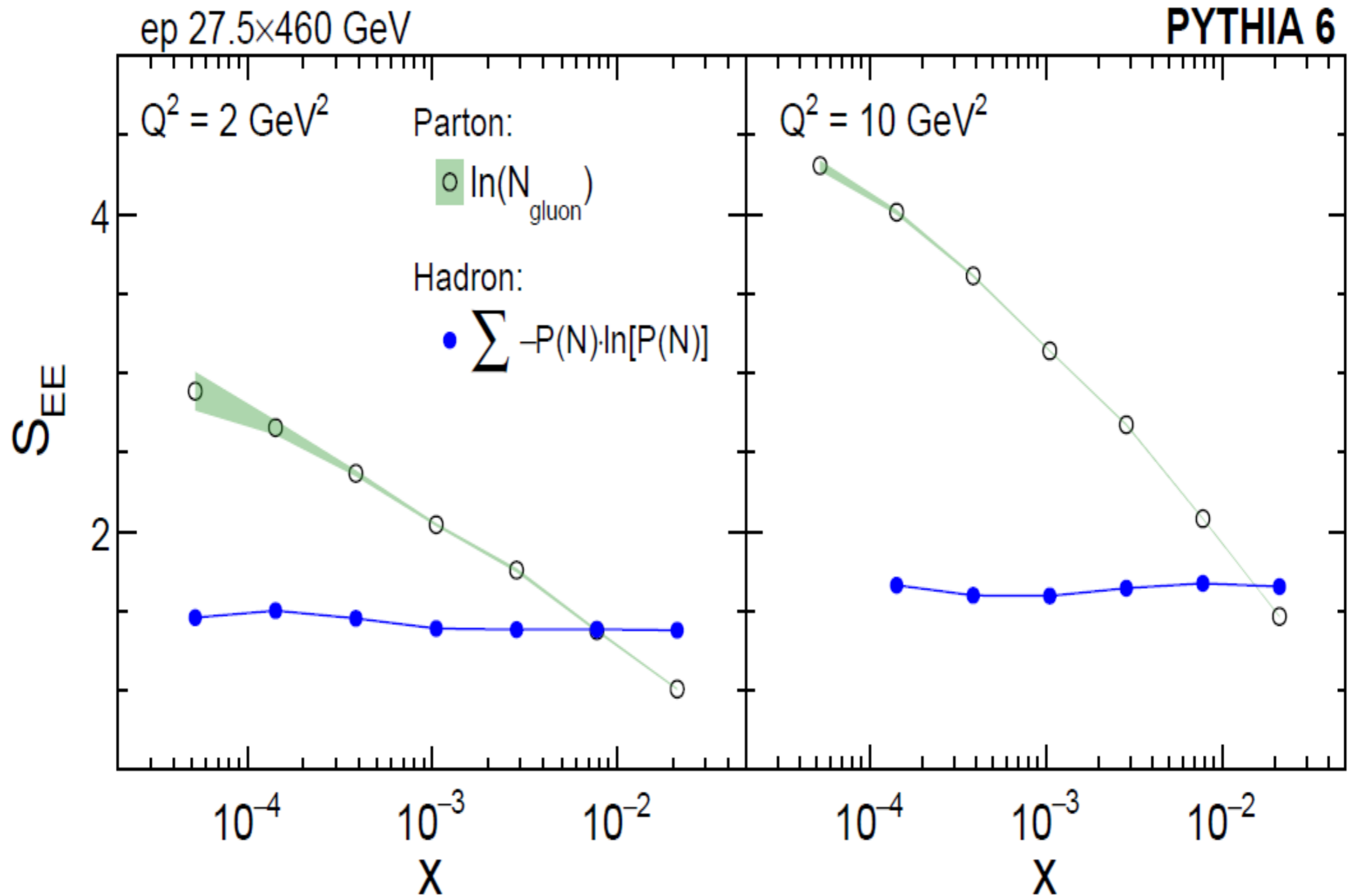
How to map multiplicity measurements to x ?

- First we obtain the the number of gluons, N_{gluon} , by integrating the gluon distribution $xG(x)$ over a given x range at a chosen scale Q^2 . We use the leading order Parton Distribution Function (PDF) set MSTW at the 90% C.L.
-> **Entanglement Entropy in green (next slide).**
- The **Boltzmann entropy of the final-state hadrons is shown as blue filled circles.** It is calculated from the multiplicity distribution, $P(N)$, in a rapidity range determined by the x range used to derive N_{gluon} . $P(N)$ is taken from ep DIS events created with the PYTHIA 6 or 8 event generator
- Since x and momentum transfer scale Q^2 are not directly available in pp collisions, an alternative way of comparing the entropy at similar x and scales are used.

$$\ln(1/x) \sim y_{\text{proton}} - y_{\text{hadron}}$$

(This might break down at large x (see Jamal's talk), but we are mostly interested in the low x , high gluon density region)

Comparing PYTHIA to PDF based calculations

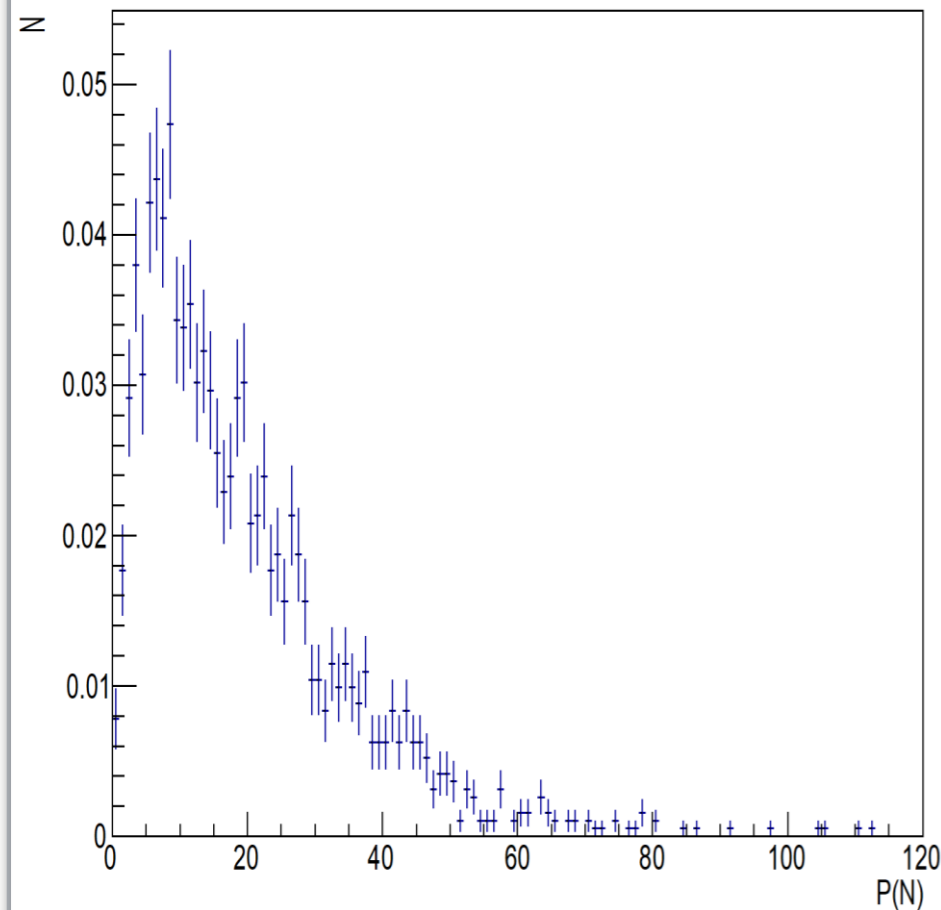


This is slightly more complicated in pp

- In ep collisions: y_{proton} is the proton beam rapidity and y_{hadron} is the final-state hadron rapidity. For example, events with 27.5 GeV electrons scattering off 460 GeV protons with x between 3×10^{-5} and 8×10^{-5} correspond to a rapidity range of $-3.5 < y < -2.5$.
- In pp collisions: two gluon distributions are involved, one from each proton, while we calculate the entanglement entropy from one distribution. Instead of altering the definition of the entanglement entropy, one can modify the $P(N)$ distributions by extrapolating the $P(N)$ distribution to reflect a single proton similar to that in ep collisions, by fitting a generalized Negative Binomial Distribution (NBD) to the $P(N)$ distributions. The final $P(N)$ is then taken as the same NBD function but with only half of the average multiplicity. This approach relies on the assumption that the final-state hadrons are produced coherently by the two colliding protons instead by incoherent and independent fragmentation.

Entropy of final state hadrons

Multiplicity Distribution P(N)



Example: Normalized multiplicity distribution for ALICE p-p collision at 13 TeV

Now that we understand how to calculate the initial state entropy we would like to compare this to the entropy of the final state hadrons.

We measure the hadron entropy using Gibbs entropy formula and summing over the probability distribution P(N).

$$S_{final} \propto \sum P(N_h) \ln(P(N_h))$$

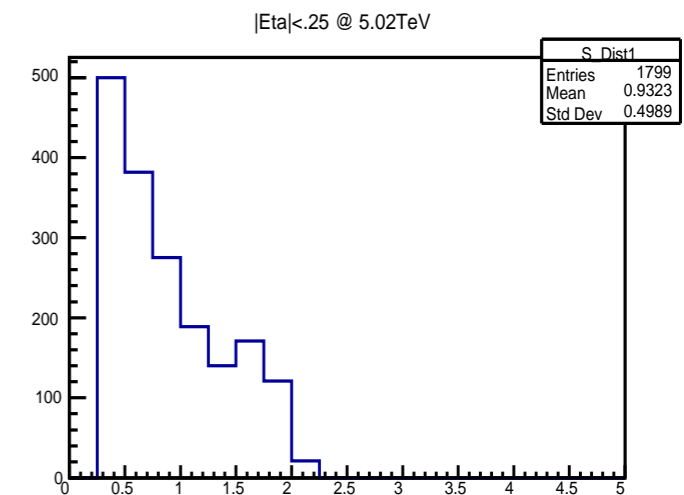
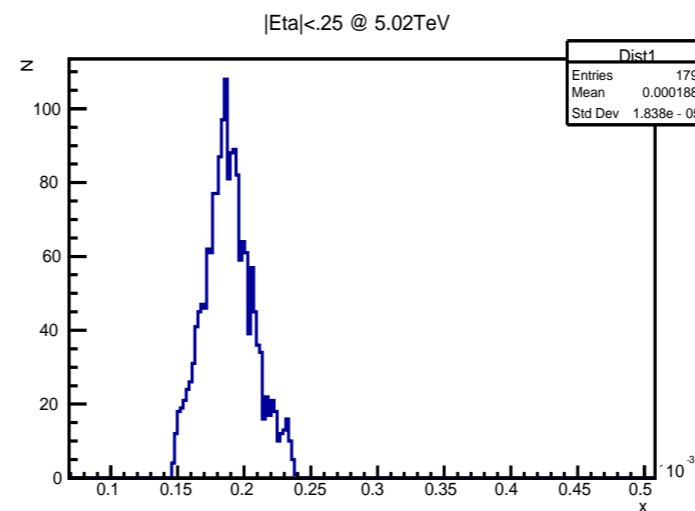
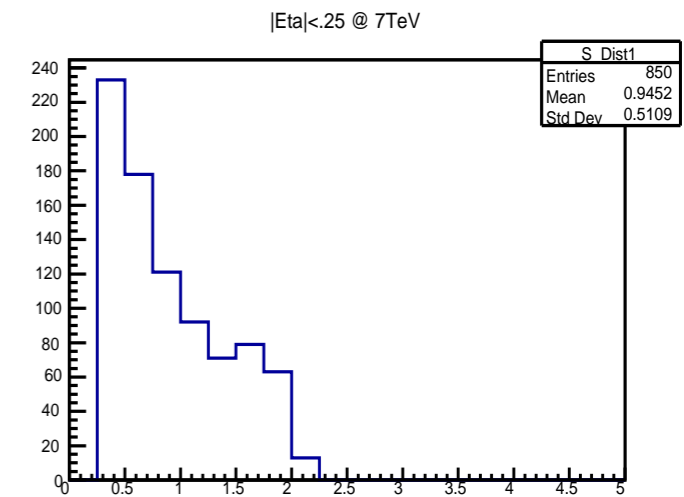
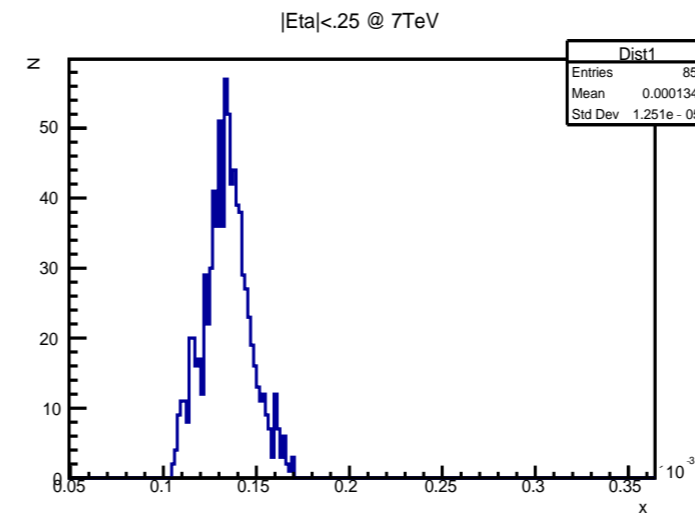
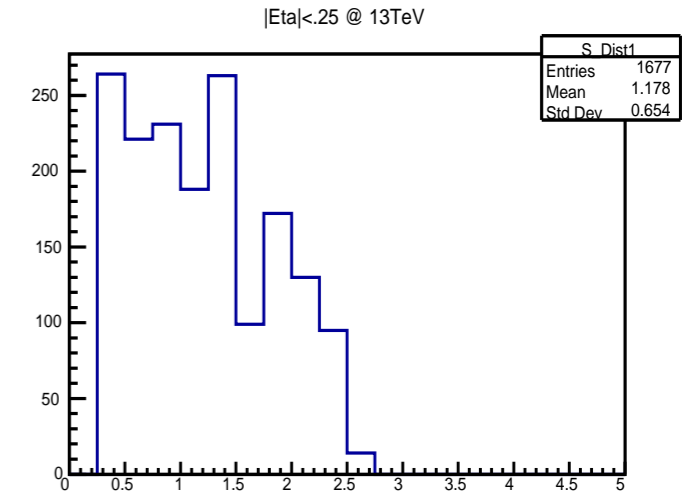
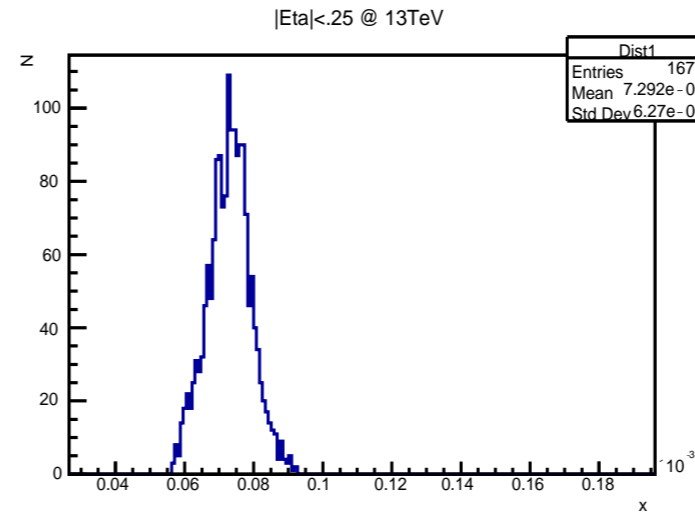
Example: ALICE 5, 7, 13 TeV pp-data: x-distribution

S-distribution

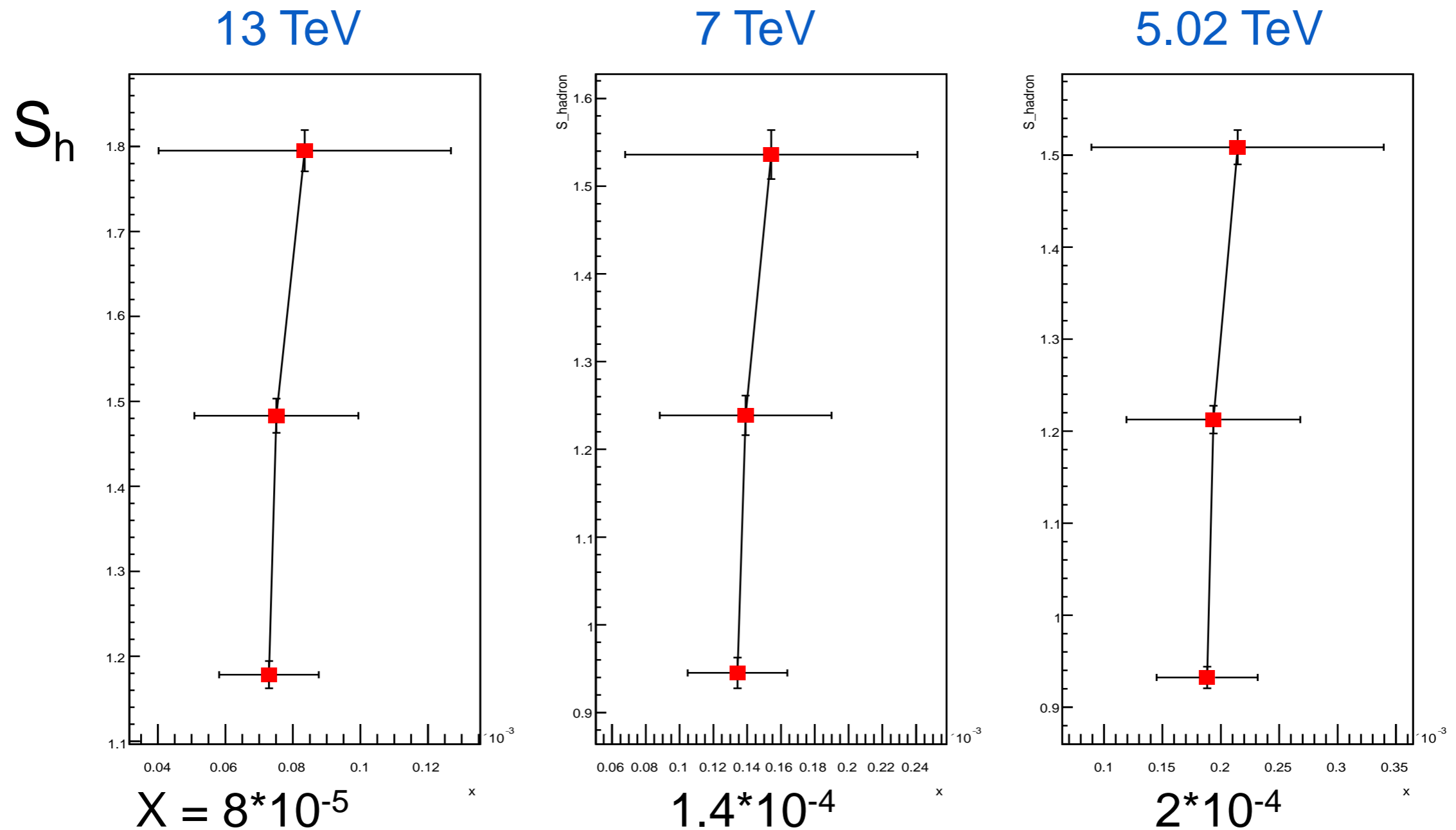
$\eta = \pm 0.25$ (preliminary)

Procedure:

- 1.) measure multiplicity distributions in a fixed rapidity range
- 2.) calculate x-value distribution
- 3.) calculate entropy distribution



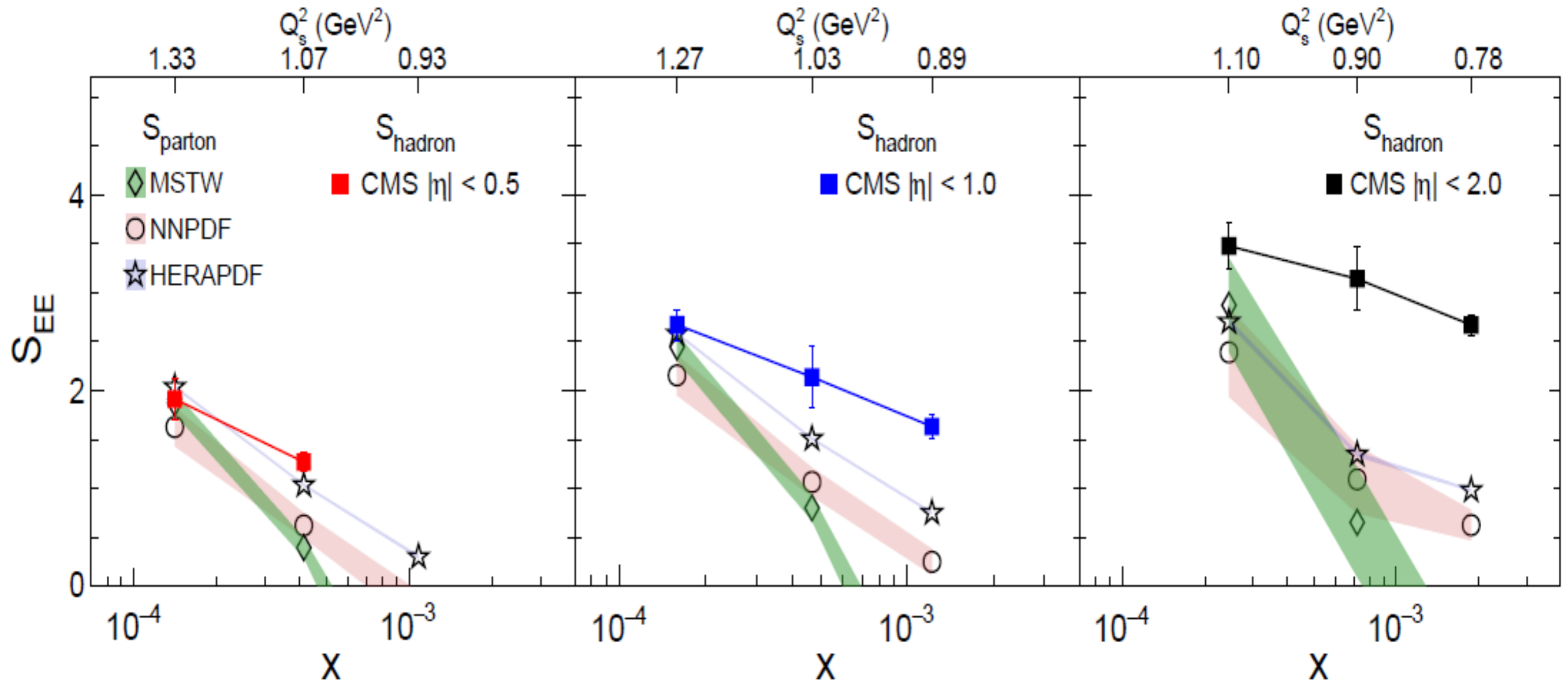
Preliminary results from ALICE data (5,7,13 TeV) on x- and S-ranges in particular pseudo-rapidity bins



You don't have to go very forward to measure low-x hadrons

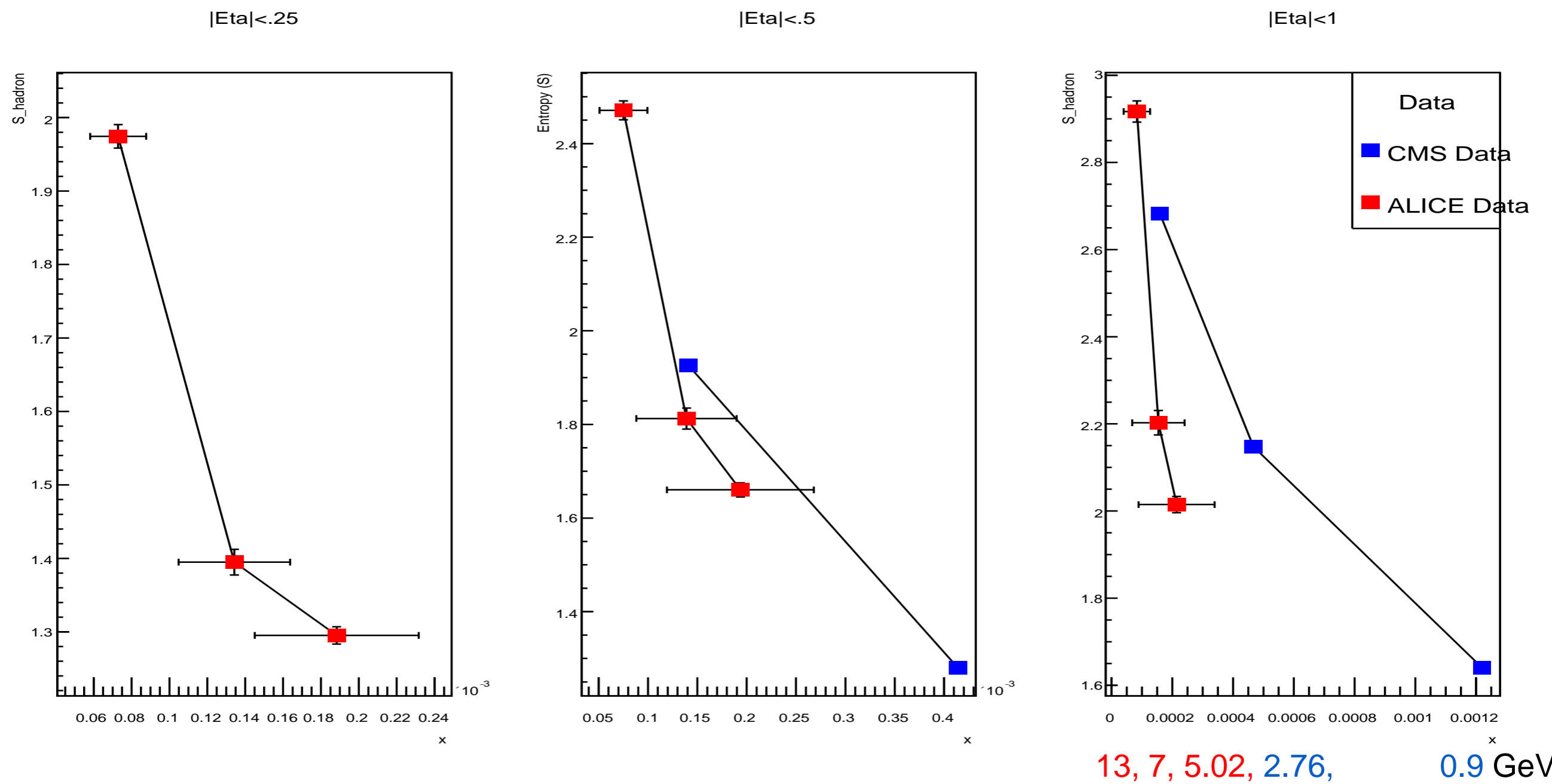
How to map multiplicity measurements to x ?

Potential early evidence (from 1904.11974 based on CMS proton-proton data)



- In an average pp collision, the Q^2 scale is set by a characteristic transverse momentum of the partons in the proton's wave function. This momentum is determined by the density of partons in the transverse plane which saturates at small x (Q_s).
- For determining the entanglement entropy from $\ln[xG(x)]$ we use the saturation scale $Q_s^2(x)$ derived in NLO BK calculations, which reasonably reproduces particle production at the LHC. For each x , the corresponding $Q_s^2(x)$ value is indicated on the top axes above

Preliminary comparison of ALICE and CMS pp data from 900 GeV to 13 TeV



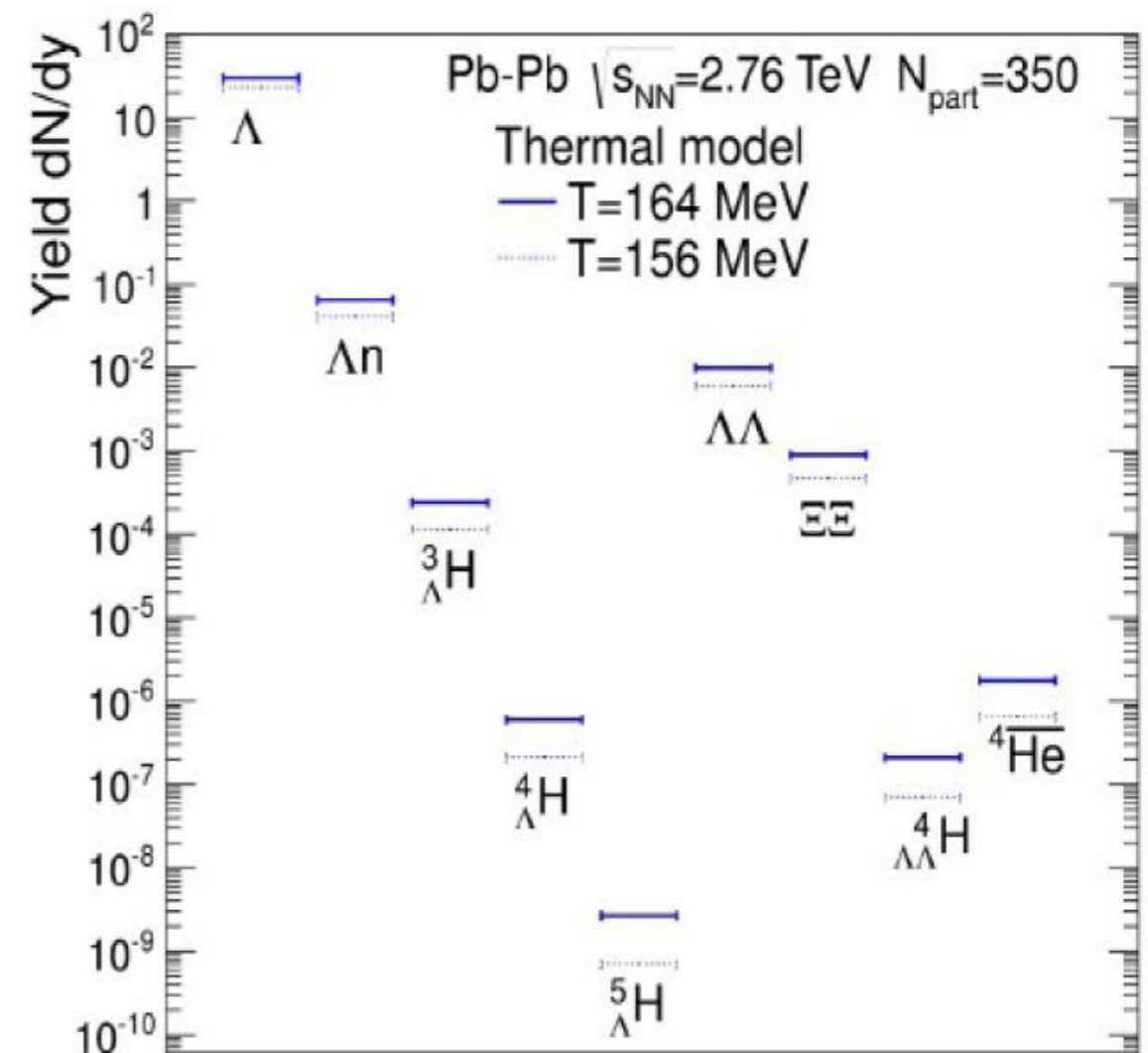
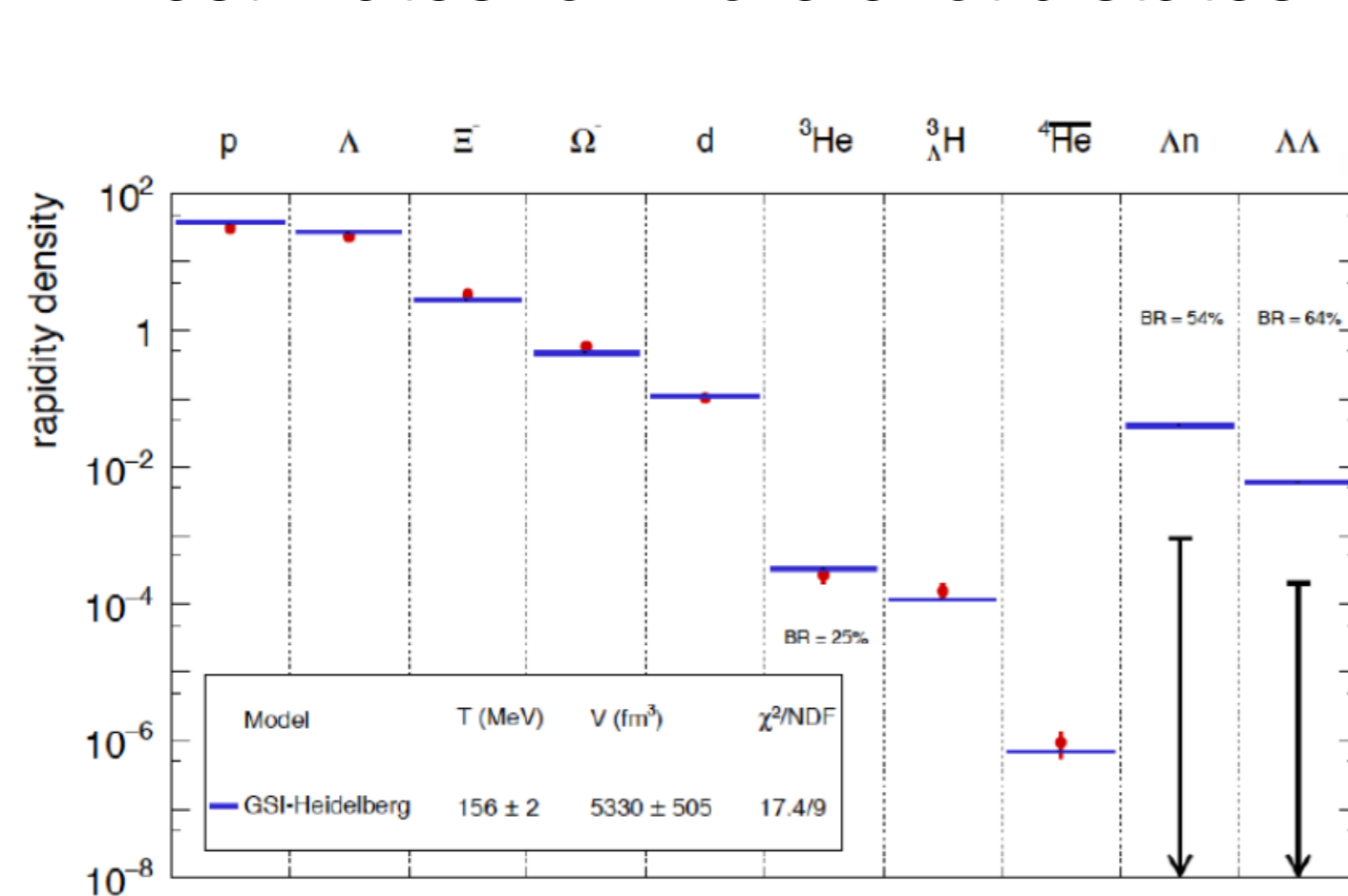
Qualitative agreement, ALICE data not yet fully corrected

Extension to heavy ion collisions

- If the system looks 'thermal' due to entanglement, but actually never thermalizes through interactions, then there is no decoherence effect and hadronic re-interaction effects are negligible.
- Particle production looks thermal, but is driven by parton-hadron duality, which also means that composite hadronic objects are formed from a single multi-quark QCD string.
- The entanglement entropy translates one to one into the final hadronic entropy and stays constant throughout the system evolution.
- All light quark hadron yields are frozen in during the initial state at a common 'temperature'. Entanglement entropy calculated over extended volume at QCD crossover. **Entropy per baryon is fixed.** Temperature should be related to Hagedorn temperature.

Coherent production of light nuclei ?

- Hadron multiplicity fluctuations in elementary collisions show already intriguing patterns that point at entanglement. Similar studies in heavy ion collisions are underway to show whether collisions in the plasma lead to decoherence (hydro, thermalization)
- If 'thermal' yields = coherent production then we can make estimates for more exotic states.



Production of ^4He from a single flux tube (arXiv:2004.14659)

QCD Hidden-Color Hexadiquark in the Core of Nuclei

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Abstract

Hidden-color configurations are a key prediction of QCD with important physical consequences. In this work we examine a QCD color-singlet configuration in nuclei formed by combining six scalar $[ud]$ diquarks in a strongly bound $SU(3)_C$ channel. The resulting hexadiquark state is a charge-2, spin-0, baryon number-4, isospin-0, color-singlet state. It contributes to alpha clustering in light nuclei and to the additional binding energy not saturated by ordinary nuclear forces in ^4He as well as the alpha-nuclei sequence of interest for nuclear astrophysics. We show that the strongly bound combination of six scalar isospin-0 $[ud]$ diquarks within the nuclear wave function - relative to free nucleons - provides a natural explanation of the EMC effect measured by the CLAS collaboration's comparison of nuclear parton distribution function ratios for a large range of nuclei. These experiments confirmed that the EMC effect; i.e., the distortion of quark distributions within nuclei, is dominantly identified with the dynamics of neutron-proton ("isophobic") short-range correlations within the nuclear wave function rather than proton-proton or neutron-neutron correlations.

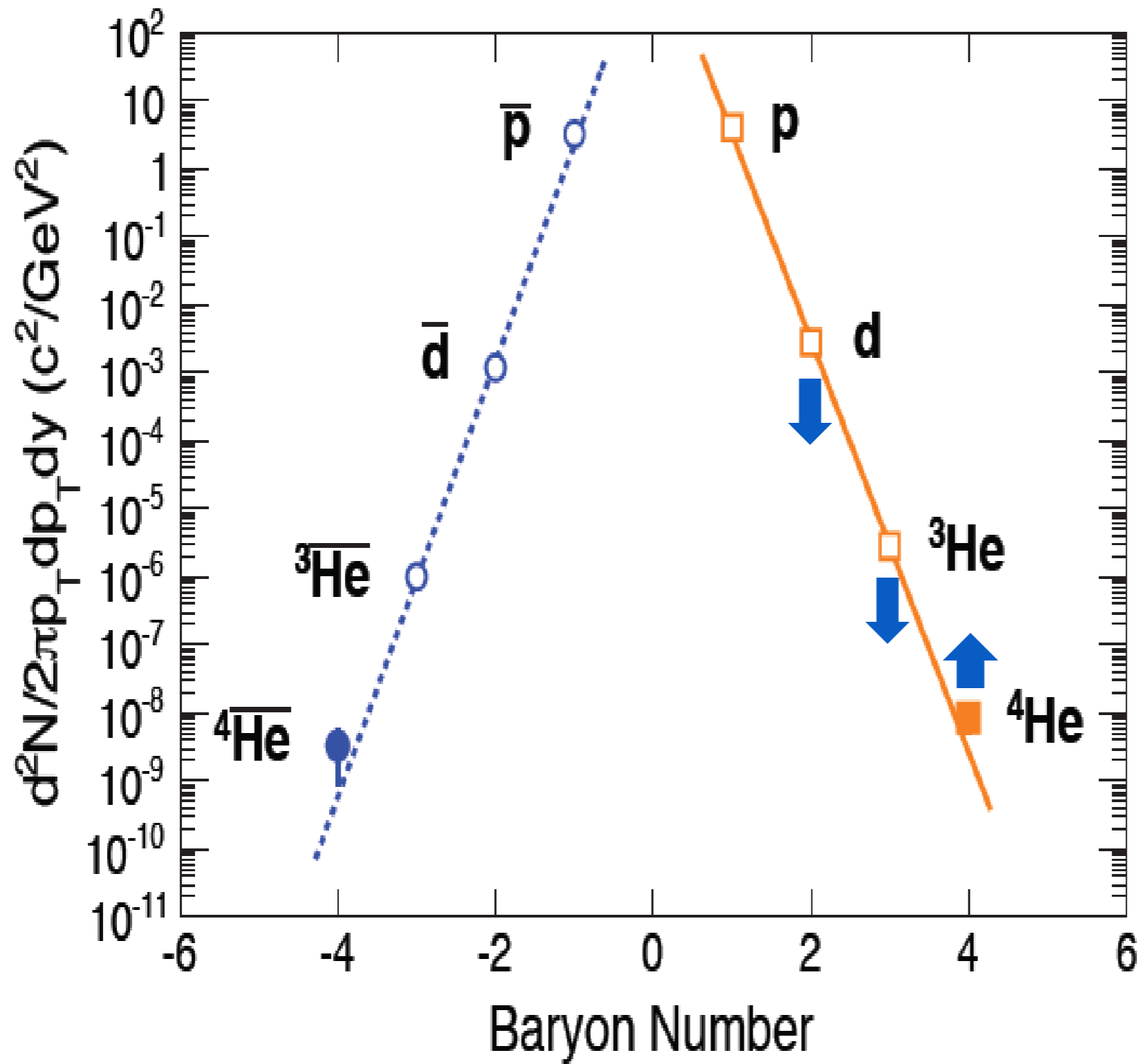
What does that mean ?

- If di-quark structures exist then the formation of light nuclei could proceed through di-quark color singlet formations.
- In this case the hexa di-quark = 12 valence quarks = ${}^4\text{He}$ is a preferred production color singlet
- Problem: 6- and 9-quark configurations are not color singlet, i.e. this process disfavors the production of deuterons and tritons.
- *BUT, this is the first calculation of light nuclei formation through string fragmentation*

The Anti-Matter Factory

AIP Science Story of 2011

Antimatter forms molecules just like matter



Discovery of Anti-Helium-4
(Nature 473, 353 (2011))

Theoretical Conclusions

- Partons in proton collisions are entangled transversely and longitudinally during the expansion of the QCD.
- Entanglement entropy is extensive (volume dependent), just like thermodynamic entropy.
- The reduced density matrix for a conformal field theory is locally thermal.

Entanglement generates 'thermalization'

- If the system looks 'thermal' due to entanglement, but actually never thermalizes through interactions, then there is no decoherence effect and hadronic re-interaction effects are negligible. The entanglement entropy translates one to one into the final hadronic entropy and stays constant throughout the system evolution.

Experimental 'Musings'

- Particle production looks thermal, but is driven by parton-hadron duality, which also means that composite hadronic objects are formed from a single multi-quark QCD string.
- All light quark hadron yields are frozen in during the initial state at a common 'temperature'. Entanglement entropy is calculated over an extended volume at QCD crossover. Temperature should then relate to Hagedorn temperature (e.g. Pajares et al., arXiv:1805.12444)

In pp: Hadron multiplicities as a function of x in elementary collisions show already intriguing patterns that point at entanglement.

In AA: *If there is no decoherence phase* (global equilibration), then the 'temperature' from the entangled phase will drive the multiplicity of all states from pion to light nuclei and even hypernuclei and rare multi quark clusters. Measure identified particles as a function of η .

Backup

Other measurements

Potentially more sophisticated measurements

Quantum tomography of lepton pairs

(see Martens, Ralston, Takaki, Eur.Phys.J. C78, 5 (2018))

- In pp measure events with two particles or two jets in order to determine the relevant reduced density matrix through dijet angular correlations (polarization and transverse momentum degrees of freedom are entangled).

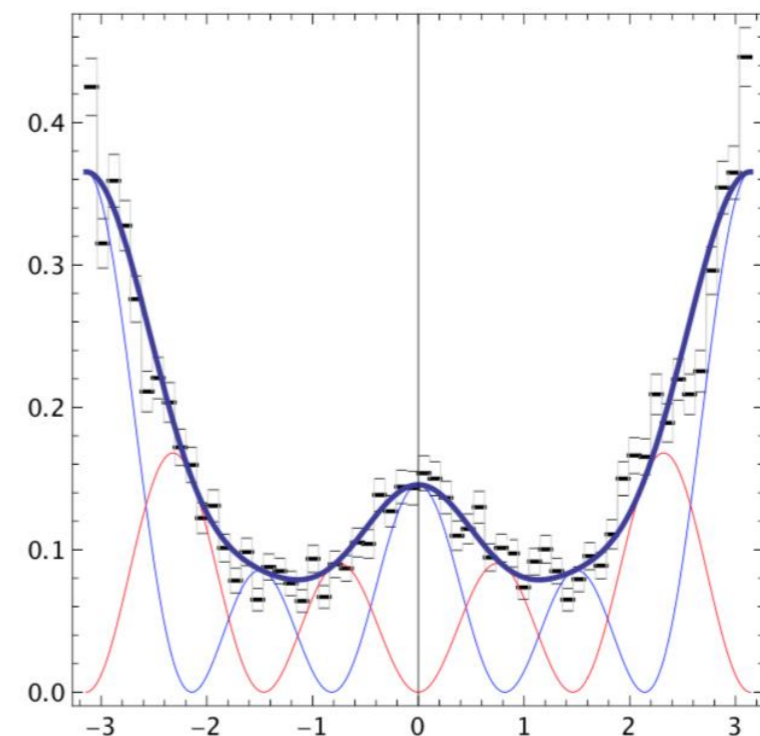
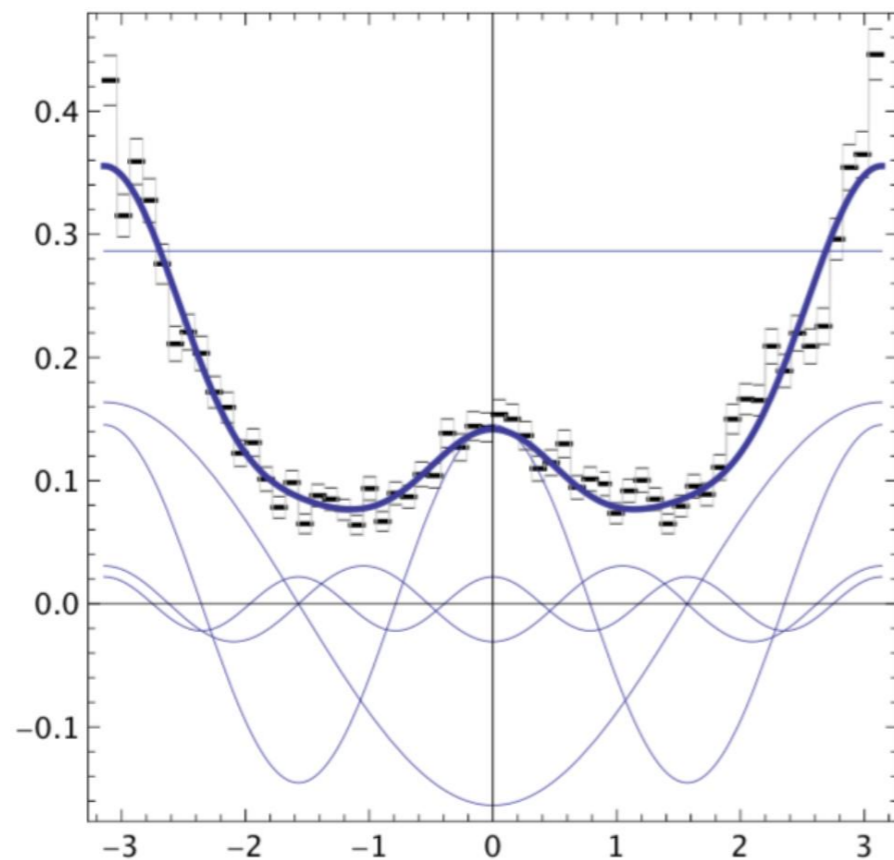
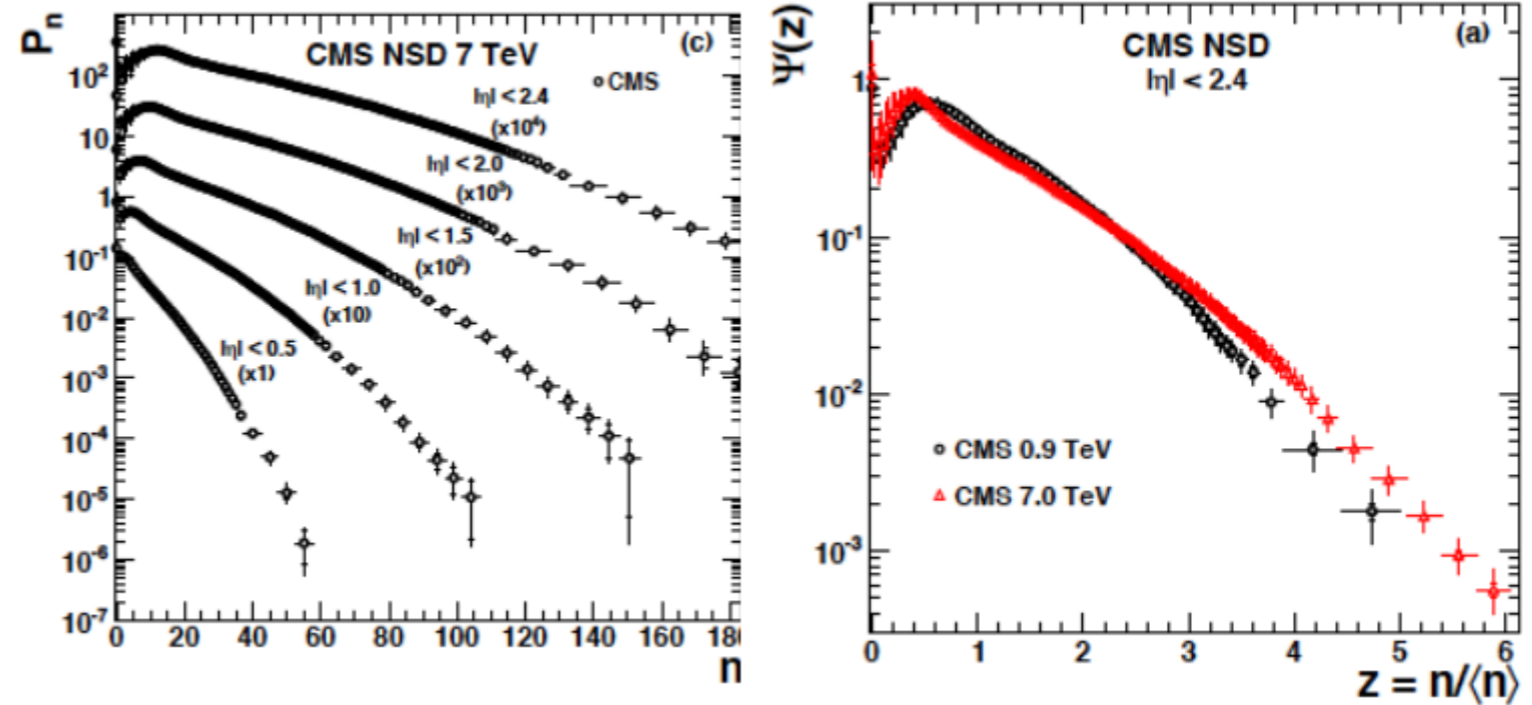
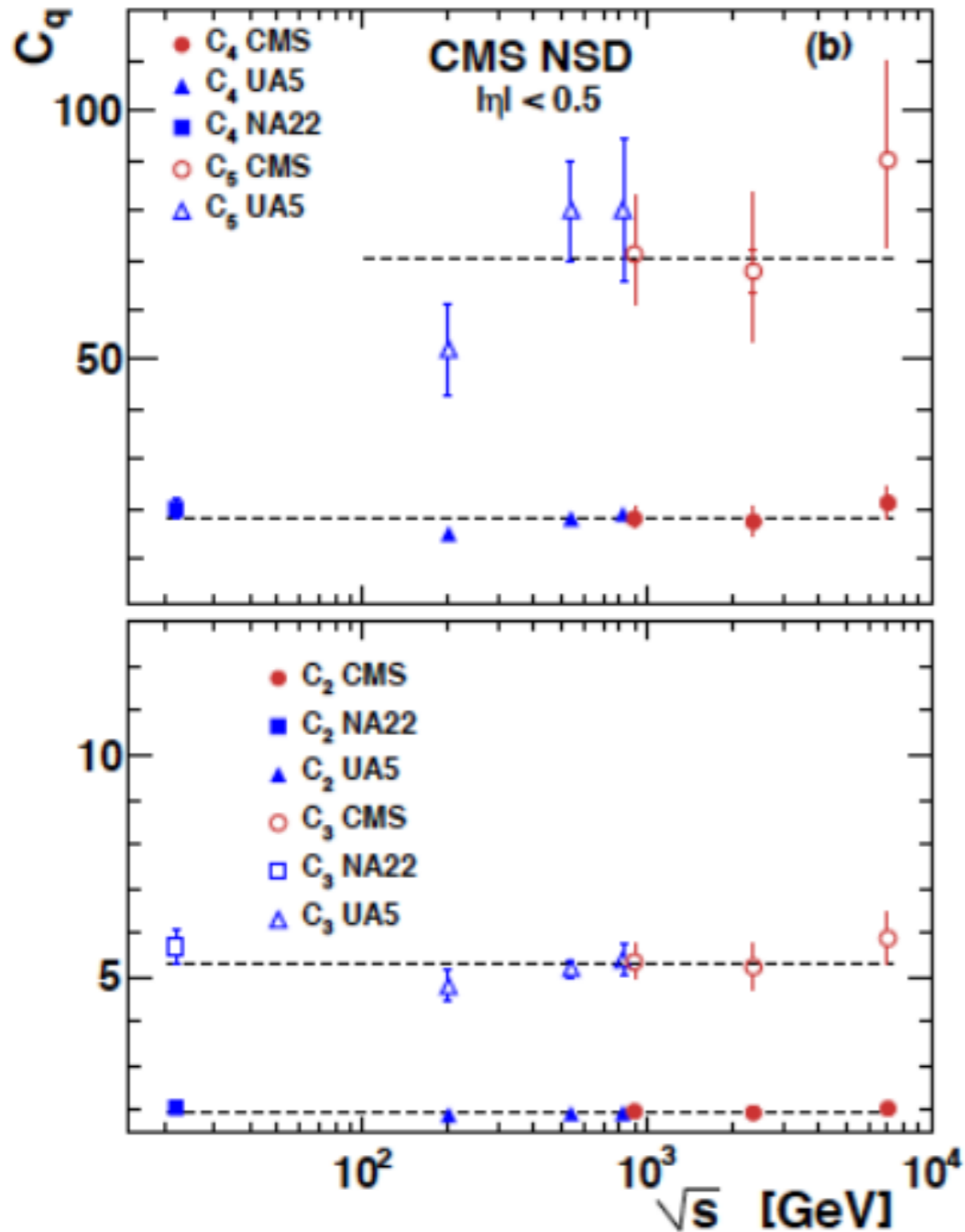


FIG. 4: Top: Maximum likelihood fit, with the contributions of $\cos m\phi$ for $m = 0 - 4$. Bottom: Two weighted distributions defined by $f_+(\phi) = \text{Re}(\psi)^2$ (blue) and $f_-(\phi) = \text{Im}(\psi)^2$ (red), coming from the eigenstates of the rank two density matrix.

Test in pp at the LHC

CMS (arXiv:1011.5531)
KNO scaling violated



The moments can be easily computed by using the generating function

$$C_q = \left(u \frac{d}{du} \right)^q Z(Y, u) \Big|_{u=1}$$

theory	exp (CMS)	theory, high energy limit
$C_2 = 1.83$	$C_2 = 2.0 \pm 0.05$	$C_2 = 2.0$
$C_3 = 5.0$	$C_3 = 5.9 \pm 0.6$	$C_3 = 6.0$
$C_4 = 18.2$	$C_4 = 21 \pm 2$	$C_4 = 24.0$
$C_5 = 83$	$C_5 = 90 \pm 19$	$C_5 = 120$

It appears that the multiplicity distributions of final state hadrons are very similar to the parton multiplicity distributions – this suggests that the entropy is close to the entanglement entropy

Parton-hadron duality could lead to specific fluctuations of the final hadron multiplicity

What is the relation between the parton and hadron multiplicity distributions?

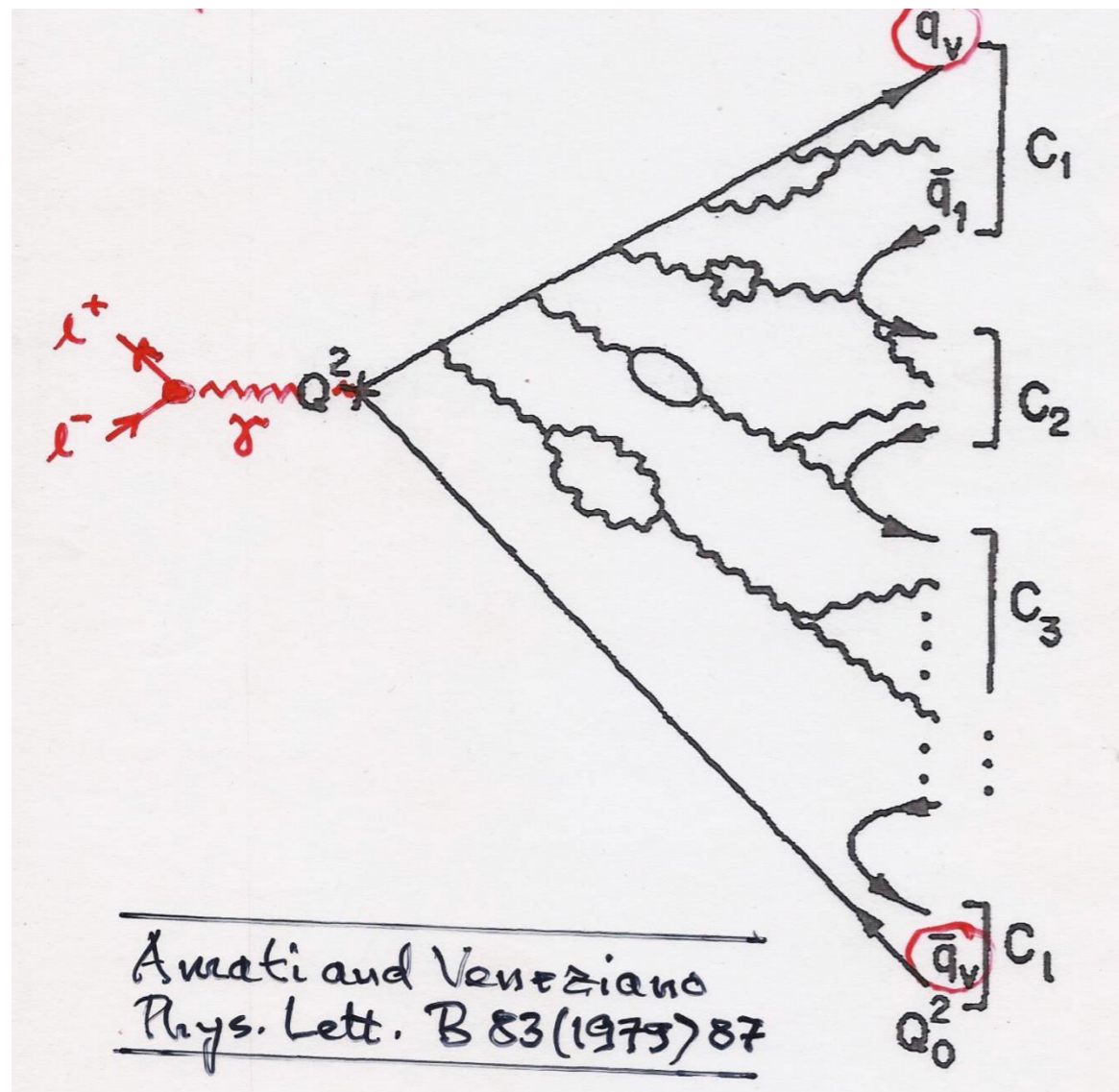
Let us assume they are the same (“EbyE parton-hadron duality”); then the hadron multiplicity distribution should be given by

$$P_n(Y) = e^{-\Delta Y} (1 - e^{-\Delta Y})^{n-1}.$$

Consider moments

$$C_q = \langle n^q \rangle / \langle n \rangle^q$$

Conceptually similar to old hadronization models (R.Stock, NeD2016, Phuket)



Veneziano-Webber model

QCD DGLAP

HERWIG Cluster

Parton Cascade (Ellis-Geiger)

But DGLAP has to be applied on the energy dependent gluon saturation scale to take into account the high production of 'clusters' from soft processes in the initial state (see. T. Lappi, arXiv:1104.3725)

Reinhard's conclusions from Phuket 2016= my conclusions from Trento 2018

- A „minimal“ model
- Hadronization occurs after a formation time (Ellis and Geiger)
- Equilibrium explained as a QM effect
- Preserves memory of Energy and Net-Charge density, and cluster size
- Maximum Entropy State: no memory to primordial QCD mechanism, other than conservation

S. Floerchinger slides

QM 2018

Relationship between entanglement and temperature (see also S. Floerchinger (QM 2018))

- For conformal fields the relationship between entanglement entropy and temperature can be derived (Calabrese, Cardy (2004)):

$$L = \tau \Delta\eta \quad (1)$$

in this case the time-dependent temperature becomes

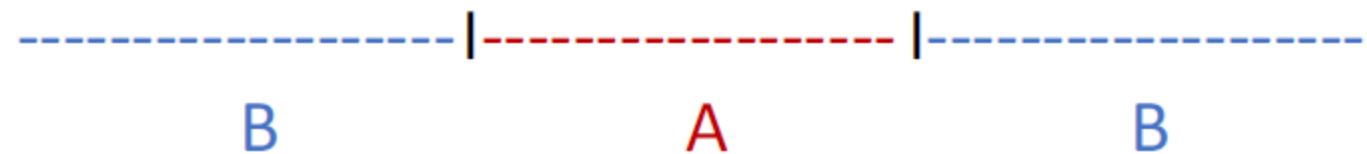
$$T = \frac{1}{2\pi\tau} \quad (2)$$

where the entropy is defined as

$$S(\tau, \Delta\eta) = \frac{c}{3} \ln\left(\frac{2\tau}{\epsilon} \sinh(\Delta\eta/2)\right) + \text{const.} \quad (3)$$

Reduced density matrix

- Consider now physical processes such as hadron formation
- Assume that these are local processes in some space region A



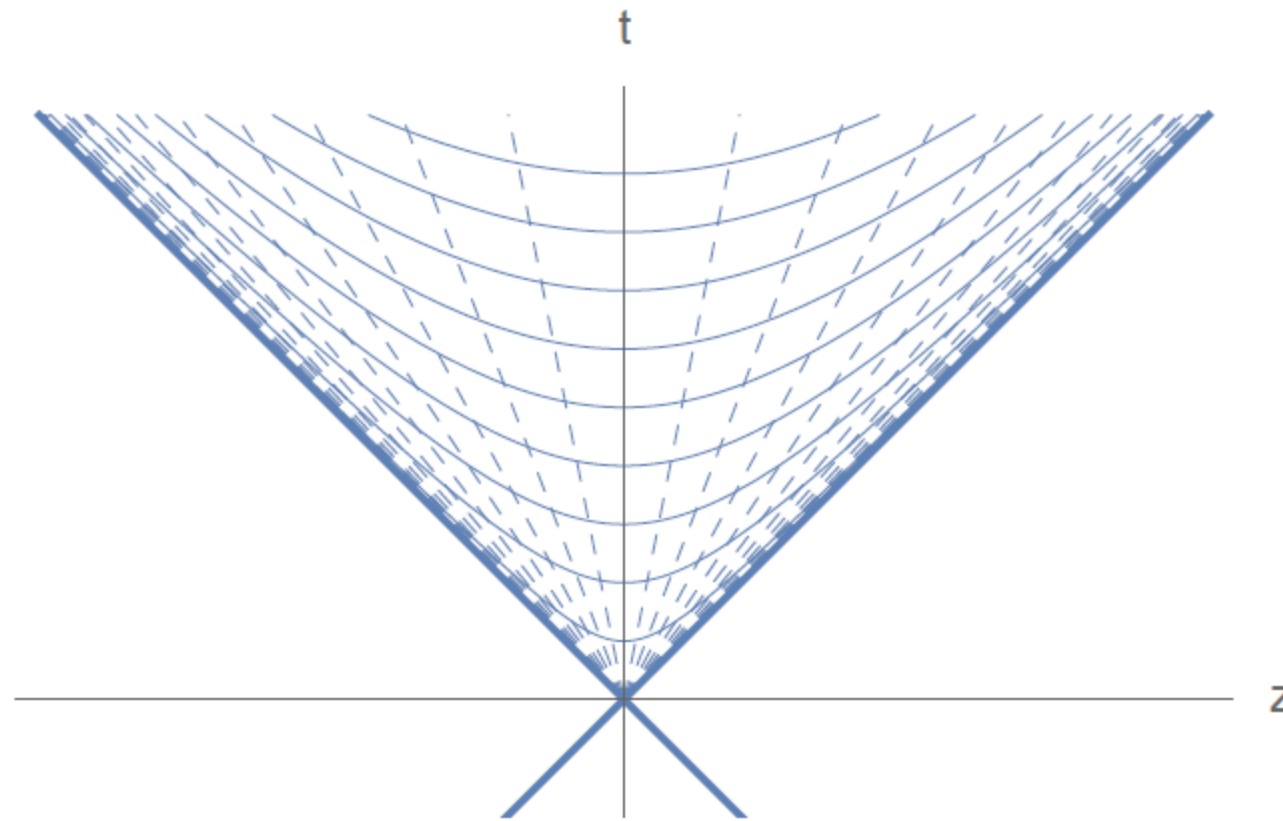
- Reduced density matrix, trace over complement region B

$$\rho_A = \text{Tr}_B \rho$$

- In general ρ_A mixed state density matrix even if ρ is pure
- Reason: entanglement between regions A and B
- Characterization by entanglement entropy

$$S_A = -\text{Tr} \{ \rho_A \ln(\rho_A) \}$$

Expanding string solution 1



- Consider string formed between (external) quark-anti-quark pair on trajectories

$$z = \pm t$$

- Coordinate system with Bjorken time $\tau = \sqrt{t^2 - z^2}$ and rapidity $\eta = \text{arctanh}(z/t)$
- Symmetry with respect to longitudinal boosts $\eta \rightarrow \eta + \Delta\eta$