Semi-analytical Method of Calculating Collision Trajectory in the QCD Phase Diagram

Zi-Wei Lin
East Carolina University (ECU)

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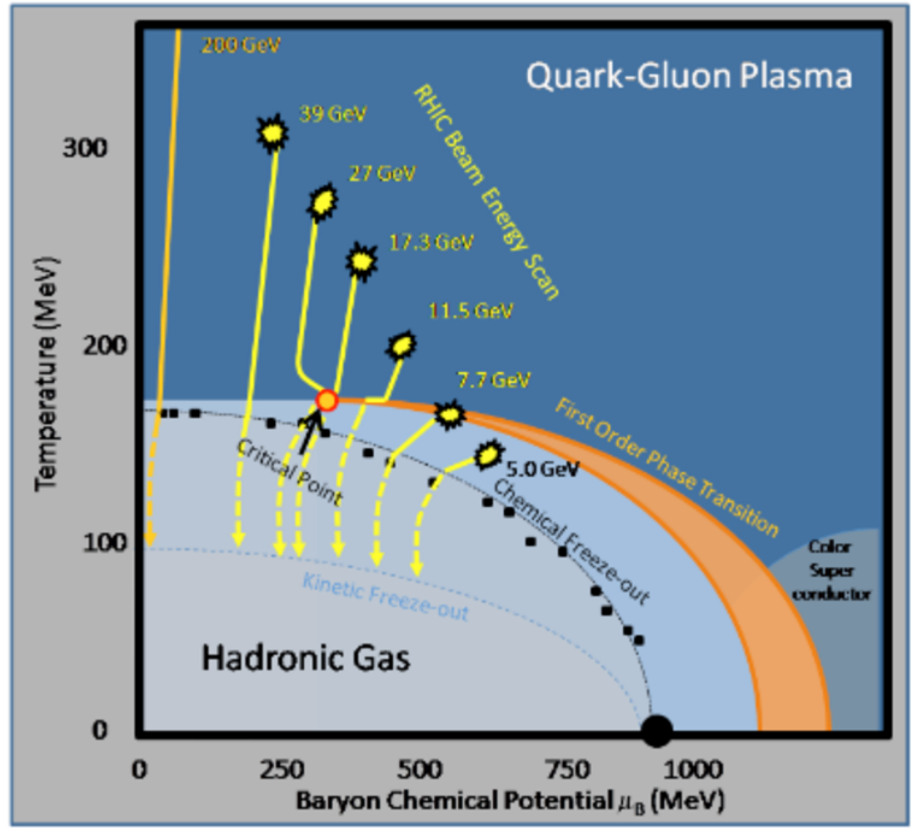
Outline

- Importance of nuclear thickness at lower energies
- Calculation of densities ε & n (n_B, n_S, n_Q)
- Extractions of T & μ (μ_B, μ_S, μ_Q)
- Collision trajectory in the T-μ_B diagram
- Conclusions

Based on
ZWL, Phys. Rev. C 98, 034908 (2018)
Todd Mendenhall & ZWL, Phys. Rev. C 103, 024907 (2021)
Todd Mendenhall & ZWL, arXiv:2111.13932
Han-Sheng Wang, Guo-Liang Ma, ZWL & Wei-Jie Fu, arXiv:2102.06937v2

Importance of nuclear thickness at lower energies

from STAR arXiv:1007.2613



- For lower energies such as BES/FAIR, particular interests are in high baryon density physics including the QCD critical end point (CEP).

- Before addressing effects of CEP, we need to know the collision trajectory in the QCD phase diagram, *including time evolutions of energy density* ε *& net-baryon density* n_B *(or* T *&* μ_B*)*

- The Bjorken energy density formula provides a semi-analytical method:

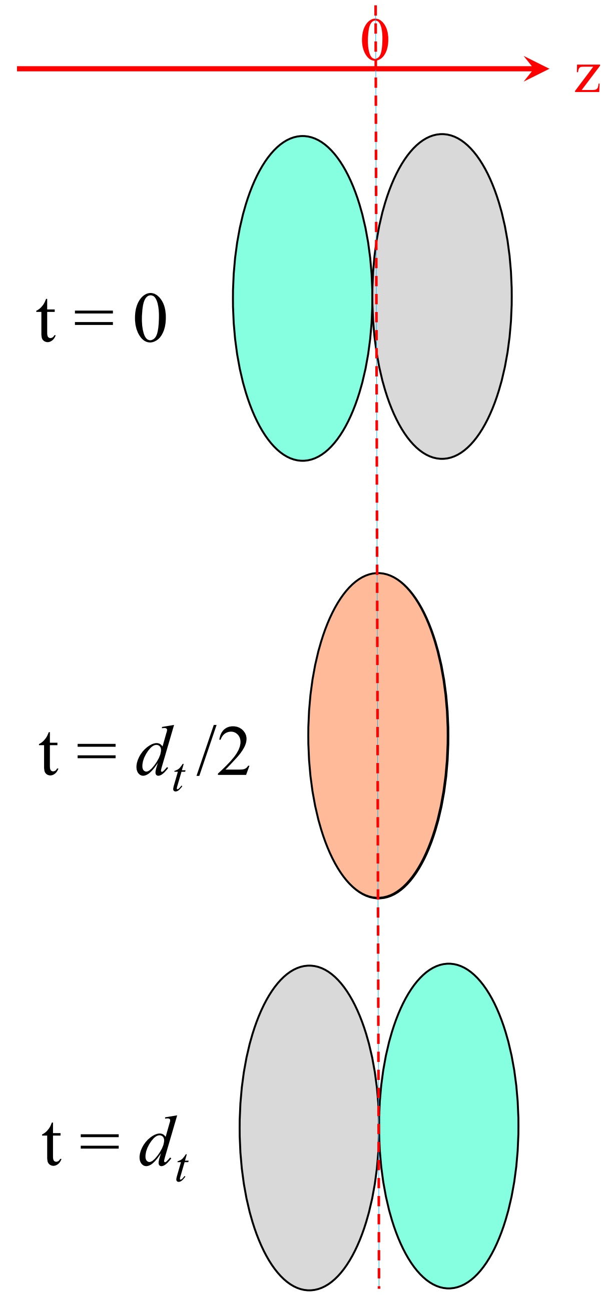
$$\epsilon(\tau) = \frac{1}{\tau\, A_T} \frac{dE_T(\tau)}{dy}$$

Importance of nuclear thickness at lower energies

The nuclear crossing time is

$$d_t = \frac{2R_A}{\sinh y_{CM}} = \frac{2R_A}{\gamma\,\beta}$$

for central A+A collisions in CM frame.



For central Au+Au collisions:

$\sqrt{s_{NN}}$ (GeV)	3	5	11.5	27	50	200
$d_t\ (fm/c)$	10.5	5.3	2.2	0.91	0.49	0.12

→ the Bjorken formula $\epsilon(\tau) = \frac{1}{\tau\, A_T}\frac{dE_T(\tau)}{dy}$

is only valid when $d_t \ll \tau_F$

or $\sqrt{s_{NN}} > \sim 50$ GeV *for* $\tau_F = 0.5\ fm/c$

Extension of Bjorken ε formula with nuclear thickness

Kajantie et al.
NPB (1983)

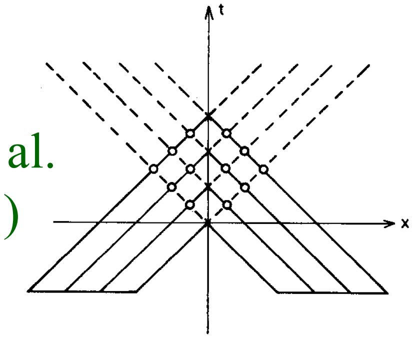
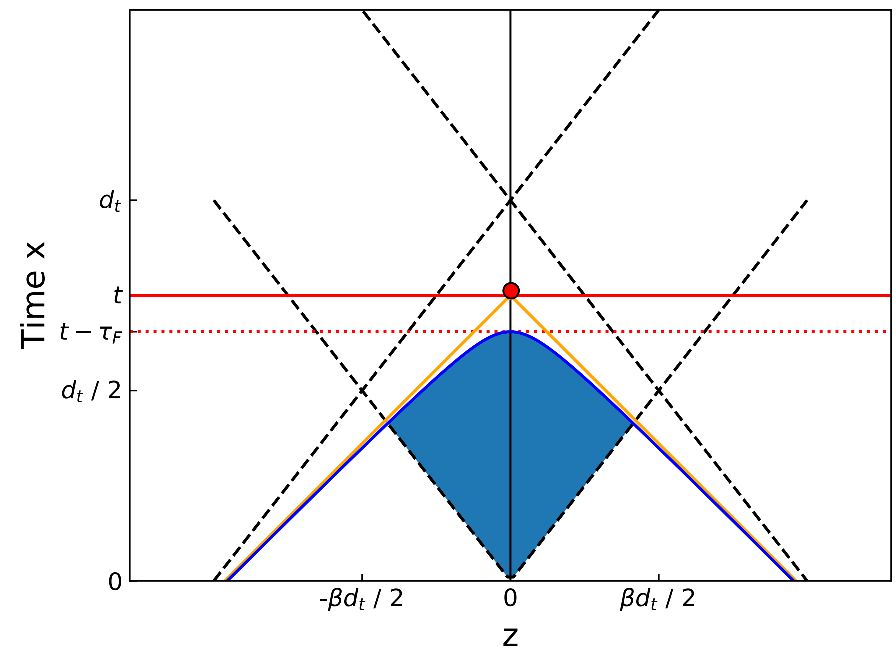


Fig. 5. An alternative description of the A + A collision. In addition to the pairwise N + N collisions on the time axis (crosses), the secondaries may further interact with the incoming nucleons (circles). This would enhance the energy density in the central region.

A schematic picture:
the shaded area
is the primary collision region
that can contribute to ***ε(t)***,
after considering formation time
$t_F = \tau_F \cosh(y)$.

At late t (> $d_t + \tau_F$),
ε(t) comes from the full primary
collision region (*the big diamond area*).

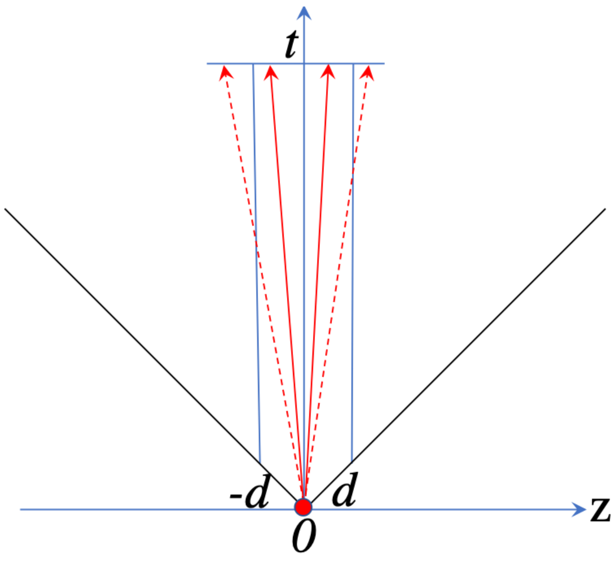
Mendenhall & ZWL, PRC (2021)

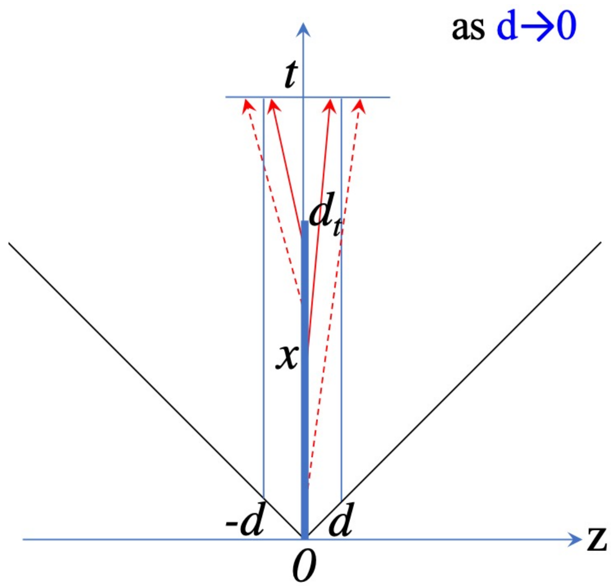


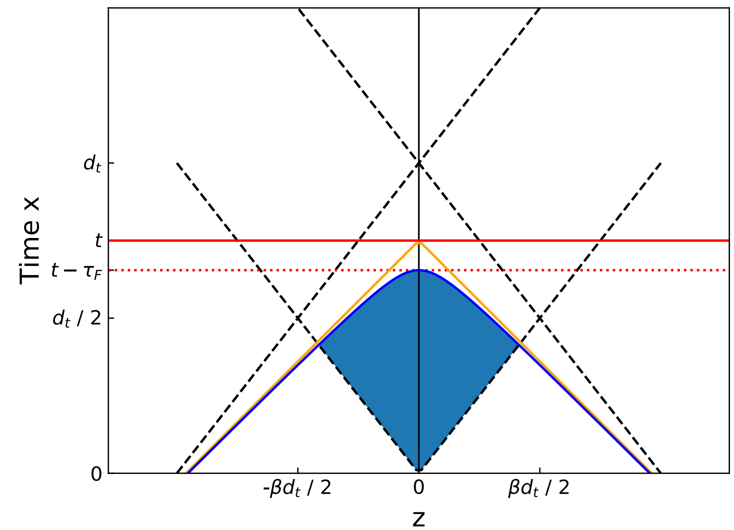
x: *production time*, $\in [0, d_t]$

To do the semi-analytical study,
we only consider central region (η_s~0)
of central A+A collisions (Au+Au in this talk)
& neglect secondary scatterings
or transverse expansion.

Extension of Bjorken ε formula with nuclear thickness







Without finite t or z:
the Bjorken ε formula

Bjorken, PRD (1983)

1) With finite t
(but not finite z-width)

ZWL, PRC (2018)

2) With both finite t & z

Mendenhall & ZWL,
PRC (2021)

We first use the simpler method **1)**
to illustrate the qualitative effect of
nuclear thickness on ***ε(t)***
(energy density at mid-pseudorapidity averaged over the transverse area)

Extension of Bjorken ε formula with nuclear thickness: 1)

$$\rightarrow \quad \varepsilon(t) = \frac{1}{A_T}\int_0^{t-\tau_F} \frac{dx}{(t-x)} \frac{d^2E_T}{dy_0 dx}$$

1) With finite t
(but not finite z-width)

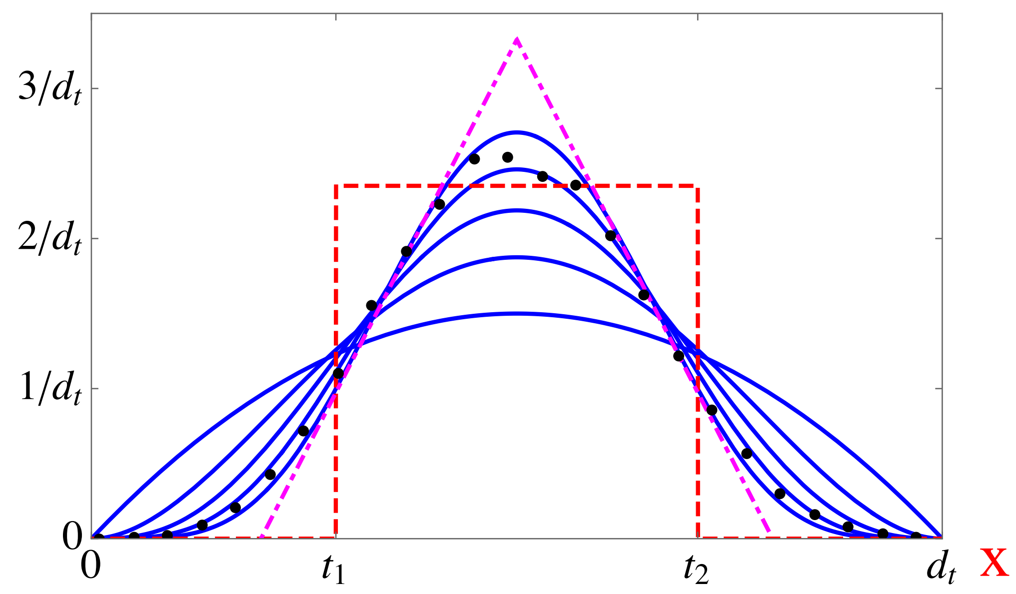
ZWL, PRC (2018)

For the simplest uniform time profile:

initial energy (at $\eta_s \sim y_0 \sim 0$)
is produced uniformly
in time ***x*** from t_1 to t_2 :

$$\frac{d^2E_T}{dy_0 dx} = \frac{1}{t_{21}} \frac{dE_T}{dy_0}$$

for $x \in [t_1, t_2]$



Circles: AMPT results

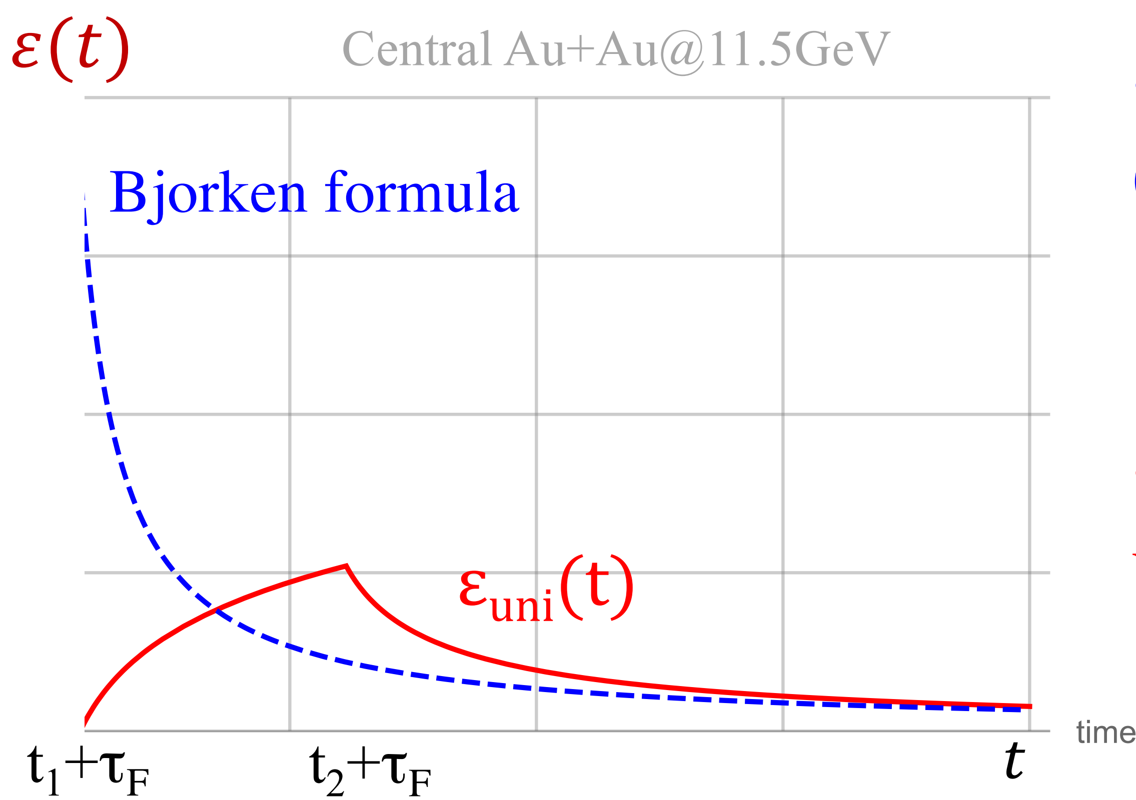
Extension of Bjorken ε formula with nuclear thickness: 1)

→ solution:

$$\epsilon_{\rm uni}(t) = \frac{1}{A_{\rm T} t_{21}} \frac{dE_{\rm T}}{dy} \ln\left(\frac{t-t_1}{\tau_{\rm F}}\right), \text{if } t \in [t_1+\tau_{\rm F}, t_2+\tau_{\rm F}];$$

$$= \frac{1}{A_{\rm T} t_{21}} \frac{dE_{\rm T}}{dy} \ln\left(\frac{t-t_1}{t-t_2}\right), \text{if } t \geq t_2+\tau_{\rm F}.$$

$t_{21} \equiv t_2 - t_1$

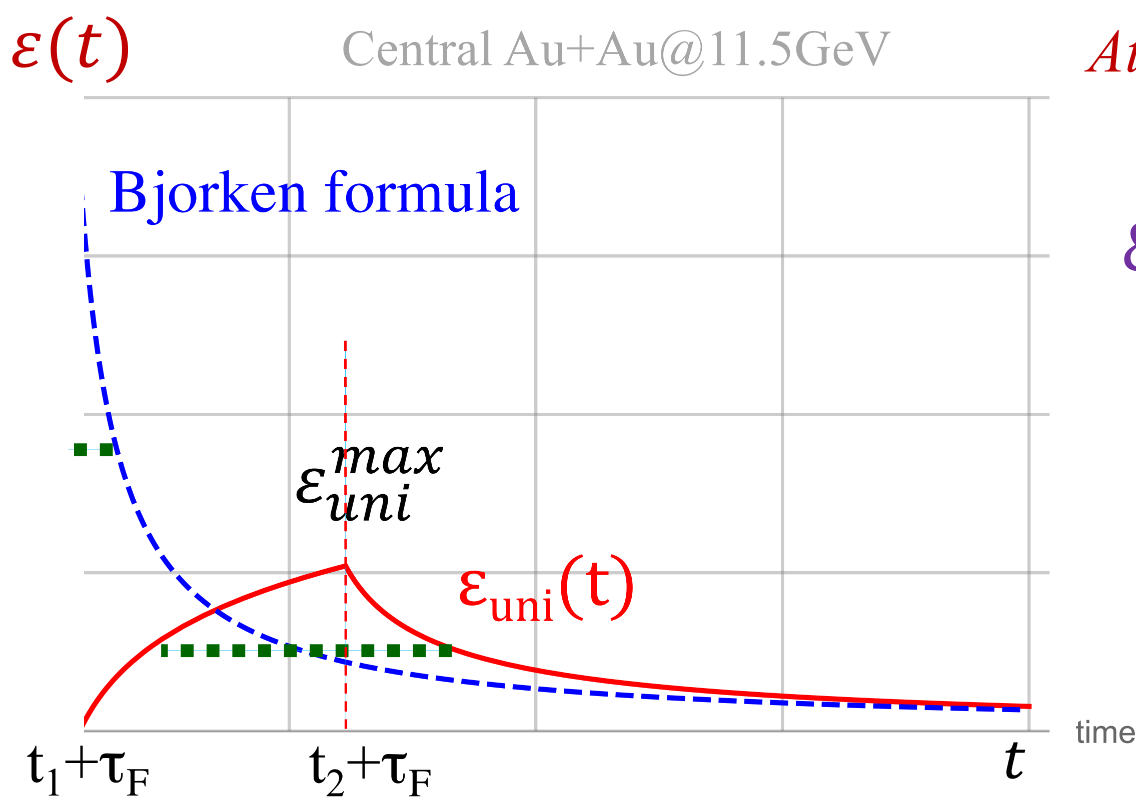


- *At high energies* (thin nuclei, or $t_{21}/\tau_F \to 0$): $\varepsilon_{uni}(t) \to \varepsilon_{Bj}(t)$ *analytically.*
- *At lower energies:* very different from Bjorken.

Extension of Bjorken ε formula with nuclear thickness: 1)

Peak energy density $\epsilon_{\rm uni}^{max} = \epsilon_{\rm uni}(t_2 + \tau_{\rm F}) = \frac{1}{A_{\rm T} t_{21}} \frac{dE_{\rm T}}{dy} \ln\left(1 + \frac{t_{21}}{\tau_{\rm F}}\right)$

→ ratio over Bjorken: $\frac{\epsilon_{\rm uni}^{max}}{\epsilon_{\rm Bj}(\tau_{\rm F})} = \frac{\tau_{\rm F}}{t_{21}} \ln\left(1 + \frac{t_{21}}{\tau_{\rm F}}\right). \quad \leq 1$



At very low energies ($t_{21}/\tau_{\rm F} >> 1$):
ratio over Bjorken → 0;

& $\varepsilon_{uni}^{max} \propto \ln\left(\frac{1}{\tau_F}\right)$, not $\frac{1}{\tau_F}$.

So the peak energy density

- << Bjorken value
- much less sensitive to $\tau_{\rm F}$

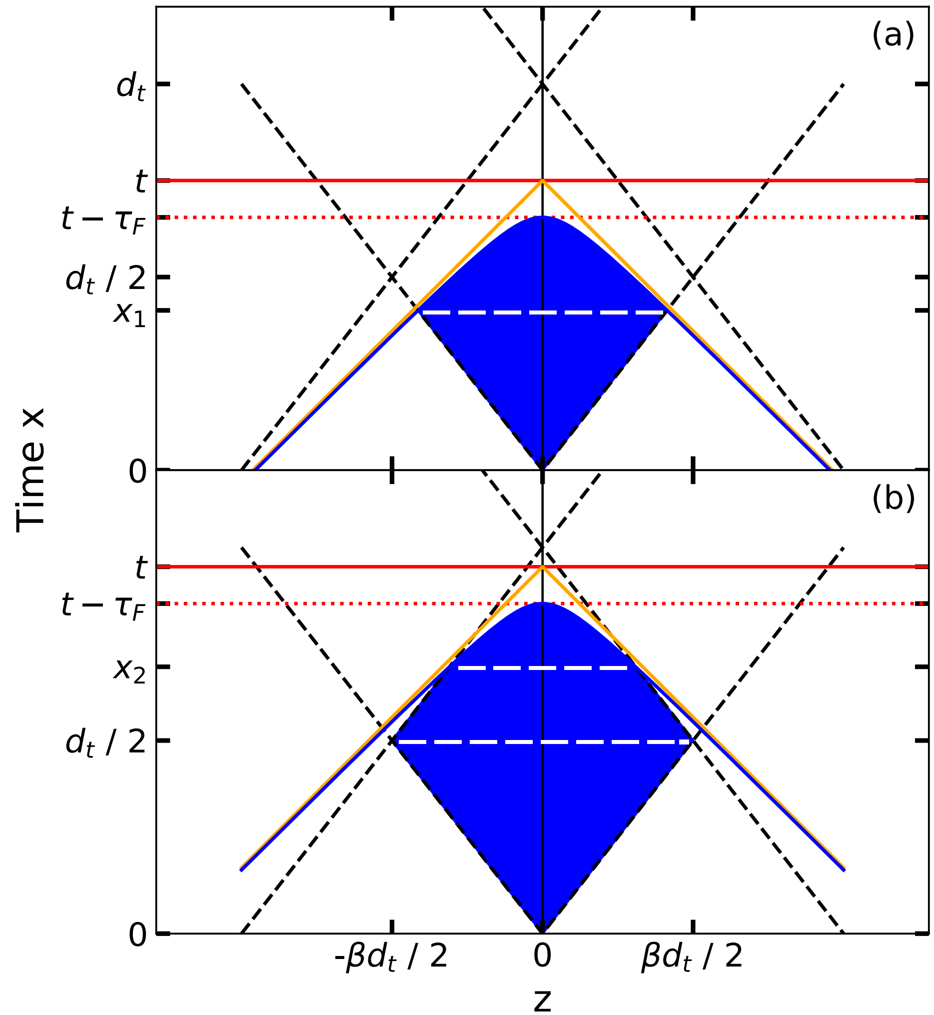
- FWHM width in t >> Bjorken

Extension of Bjorken ε formula with nuclear thickness: 2)

2) With both finite t & z

Mendenhall & ZWL, PRC (2021)

$$\rightarrow\ \varepsilon(t) = \frac{1}{A_T} \int\!\!\int_S \frac{dxdz}{(t-x)} \frac{d^3 m_T}{dy_0\, dxdz} ch^3 y_0$$

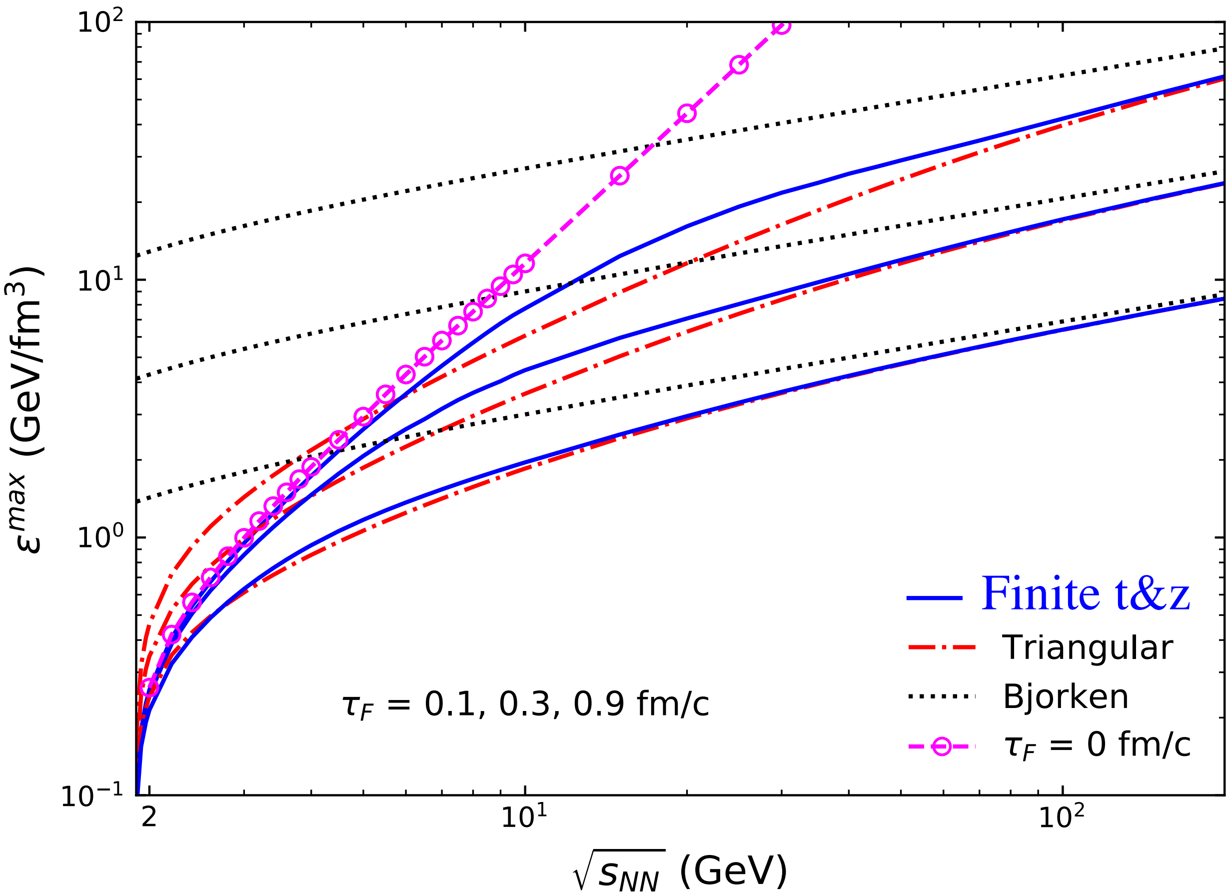


***S:** integration area (**shaded**),*
*has 2 or 3 pieces depending on **t**:*

$\frac{d^3 m_T}{dy_0\, dxdz}$: m_T production density in the primary collision region,

assumed to be uniform in the x-z plane.

Extension of Bjorken ε formula with nuclear thickness: 2)



2) With both finite t & z

Mendenhall & ZWL, PRC (2021)

- Qualitatively similar to earlier study (Triangular) ZWL, PRC (2018)

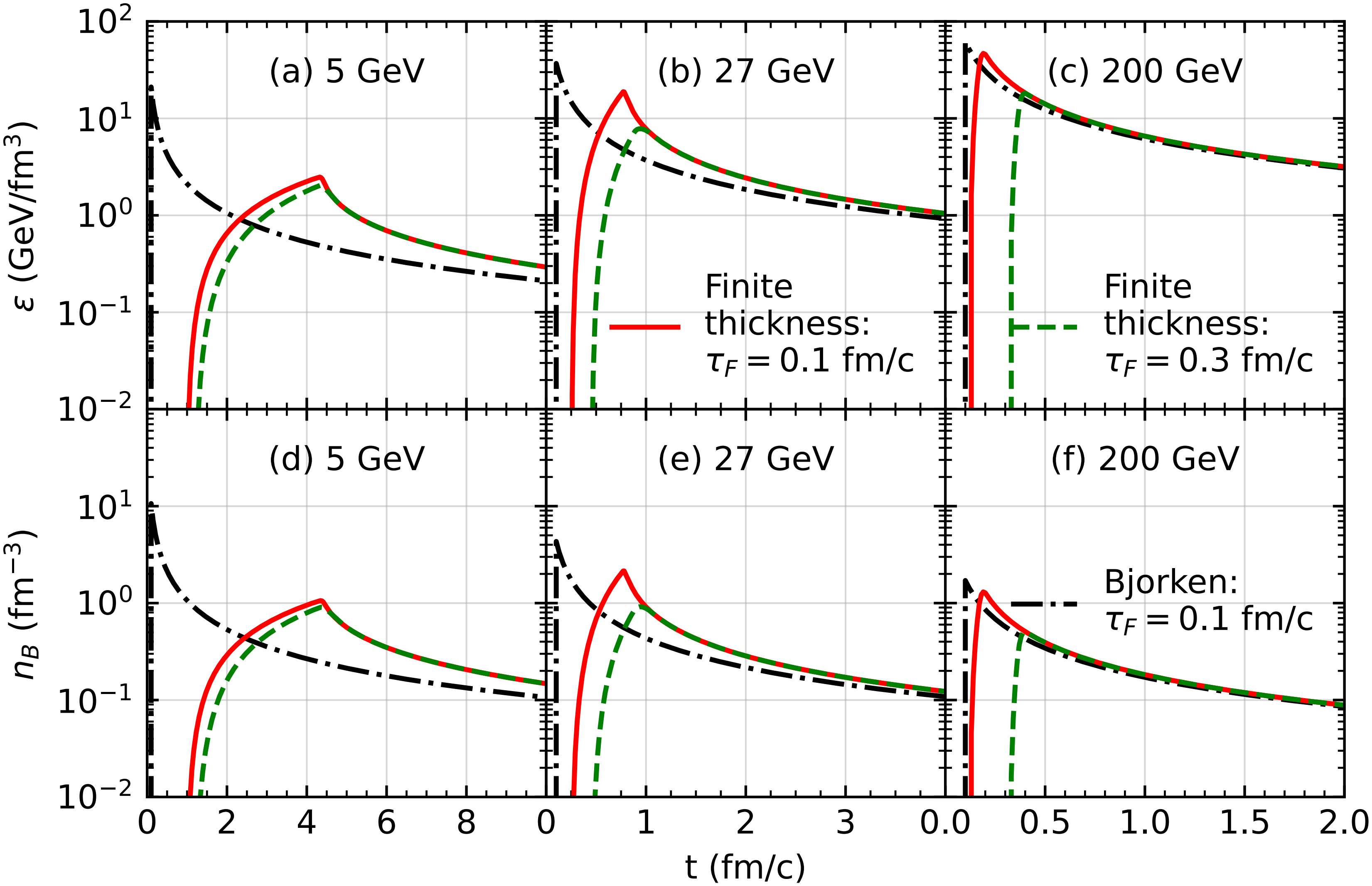
ε^{max} << Bjorken value at low energies, ≈Bjorken value at high energies;
ε^{max} & ***ε(t)*** depend on τ_F more weakly than Bjorken at lower energies.

- ε^{max} is **finite** at $\tau_F = 0$ at any colliding energy (*no divergence*).

Calculation of Densities ε & n (n_B, n_S, n_Q)

We extend the method to calculate conserved-charge (B,S,Q) densities:

Mendenhall & ZWL, arXiv:2111.13932



$$n_Q(t) = \frac{Z}{A} n_B(t), \qquad n_S(t) = 0$$

Extractions of T & μ (μ_B, μ_S, μ_Q)

If we consider QGP as non-interacting gluon+3-flavor massless quarks (*quantum stats here*):

$$\epsilon = \frac{19\pi^2}{12}T^4 + \frac{(\mu_B+2\mu_Q)^2+(\mu_B-\mu_Q)^2+(\mu_B-\mu_Q-3\mu_S)^2}{6}T^2 + \frac{(\mu_B+2\mu_Q)^4+(\mu_B-\mu_Q)^4+(\mu_B-\mu_Q-3\mu_S)^4}{108\pi^2},$$

$$n_B = \frac{\mu_B-\mu_S}{3}T^2 + \frac{(\mu_B+2\mu_Q)^3+(\mu_B-\mu_Q)^3+(\mu_B-\mu_Q-3\mu_S)^3}{81\pi^2},$$

$$n_Q = \frac{2\mu_Q+\mu_S}{3}T^2 + \frac{2(\mu_B+2\mu_Q)^3-(\mu_B-\mu_Q)^3-(\mu_B-\mu_Q-3\mu_S)^3}{81\pi^2},$$

$$n_S = -\frac{\mu_B-\mu_Q-3\mu_S}{3}T^2 - \frac{(\mu_B-\mu_Q-3\mu_S)^3}{27\pi^2}.$$

$n_S(t) = 0 \quad \rightarrow \mu_B - \mu_Q - 3\mu_S = 0$

$\rightarrow$

$$\epsilon = \frac{19\pi^2}{12}T^4 + 3\frac{(\mu_B-2\mu_S)^2+\mu_S^2}{2}T^2 + 3\frac{(\mu_B-2\mu_S)^4+\mu_S^4}{4\pi^2},$$

$$n_B = \frac{\mu_B-\mu_S}{3}T^2 + \frac{(\mu_B-2\mu_S)^3+\mu_S^3}{3\pi^2},$$

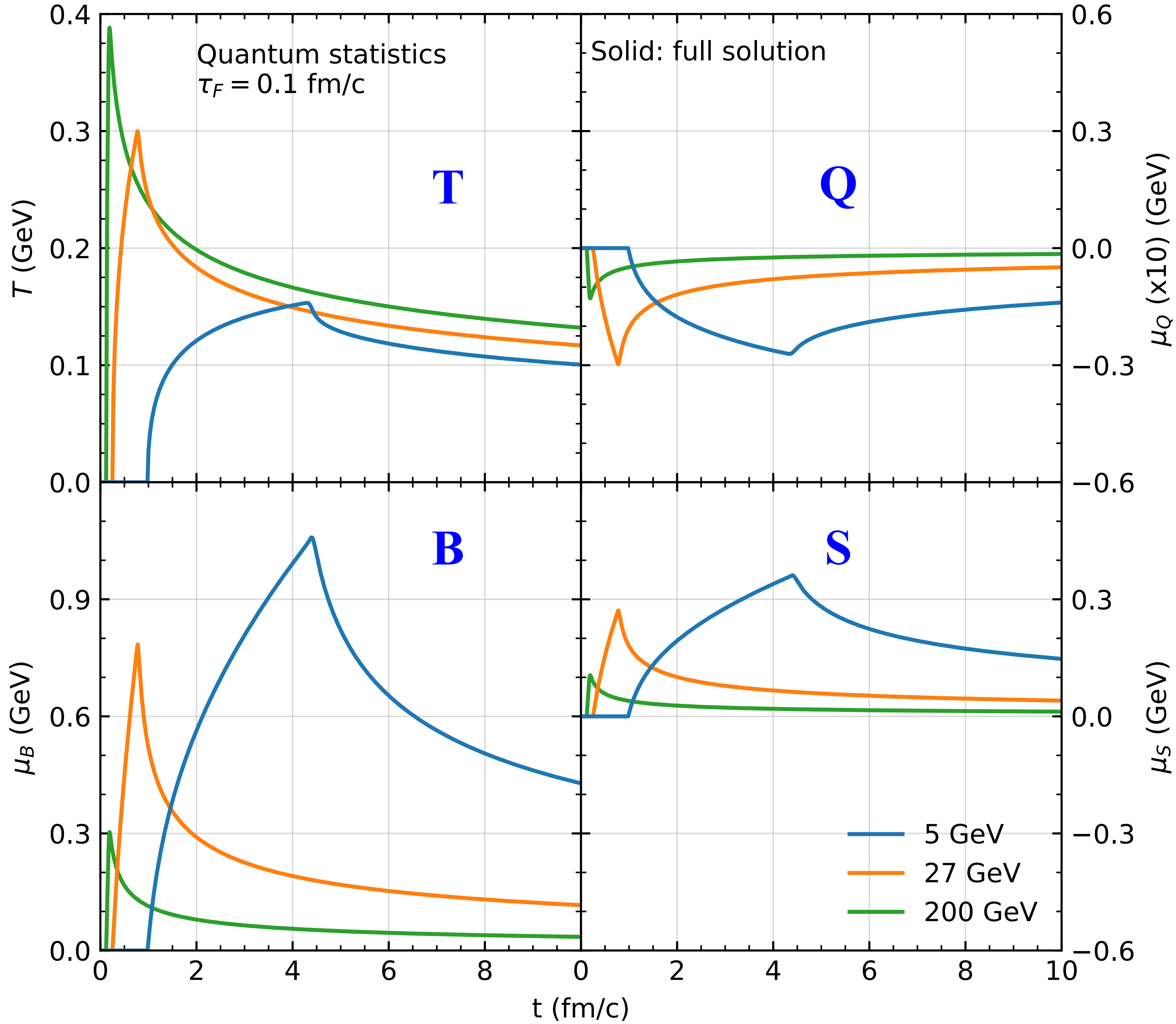
$$n_Q = \frac{2\mu_B-5\mu_S}{3}T^2 + \frac{2(\mu_B-2\mu_S)^3-\mu_S^3}{3\pi^2}.$$

used for extraction of T&μ

Extractions of T & μ (μ_B, μ_S, μ_Q)

ε & n_B, n_S, n_Q
$\rightarrow$ T & μ_B, μ_S, μ_Q

collision trajectory
in 4-d T-μ space



Extractions of T & μ (μ_B, μ_S, μ_Q)

Under strangeness neutrality

$$n_S(t) = 0:$$

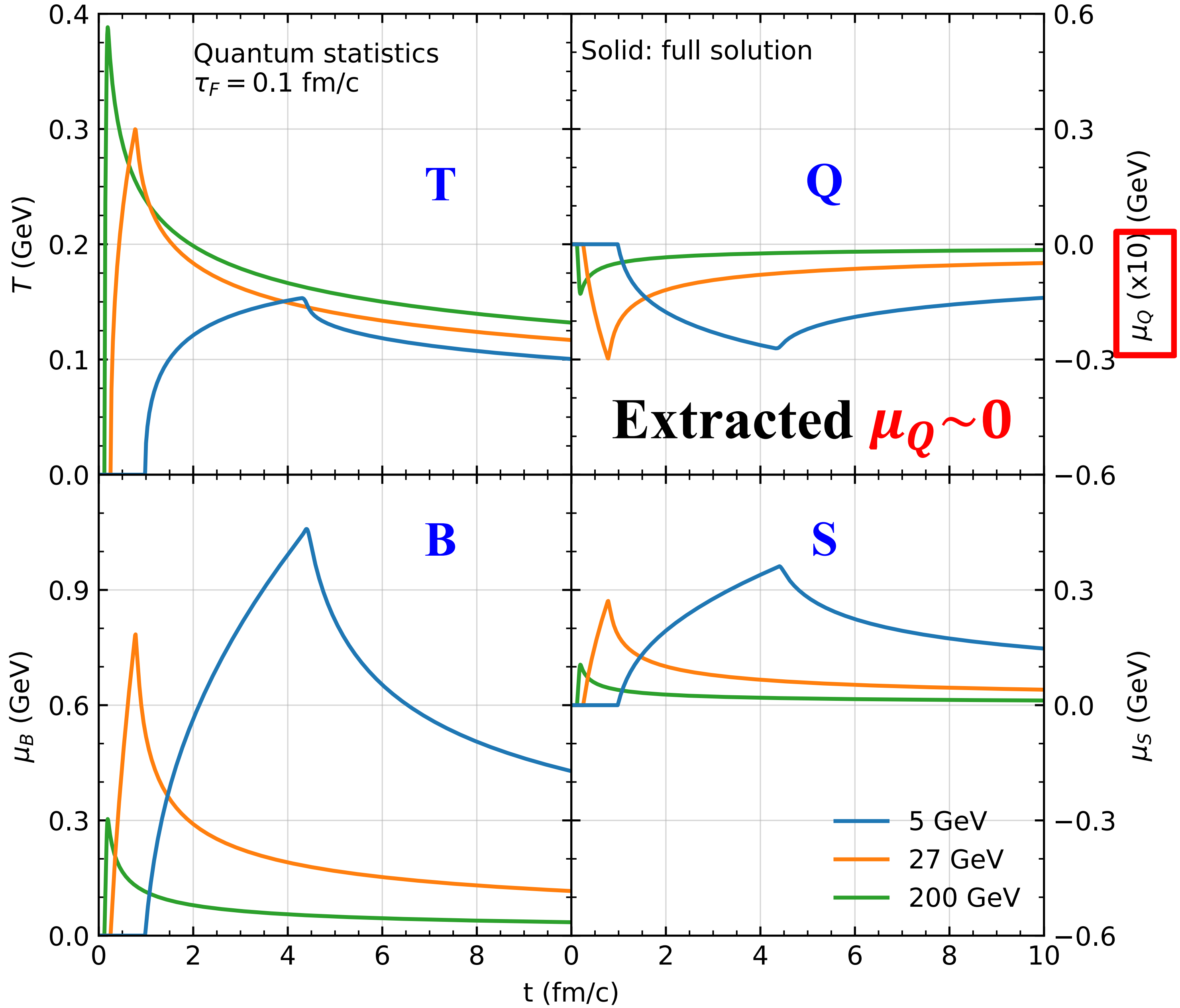
if $Z = \frac{A}{2}$

$$\rightarrow n_Q(t) = \frac{n_B(t)}{2}$$

$$\rightarrow \mu_Q\ (t) = 0$$

So $\mu_Q \sim 0$ is a result of $Z \sim \frac{A}{2}$

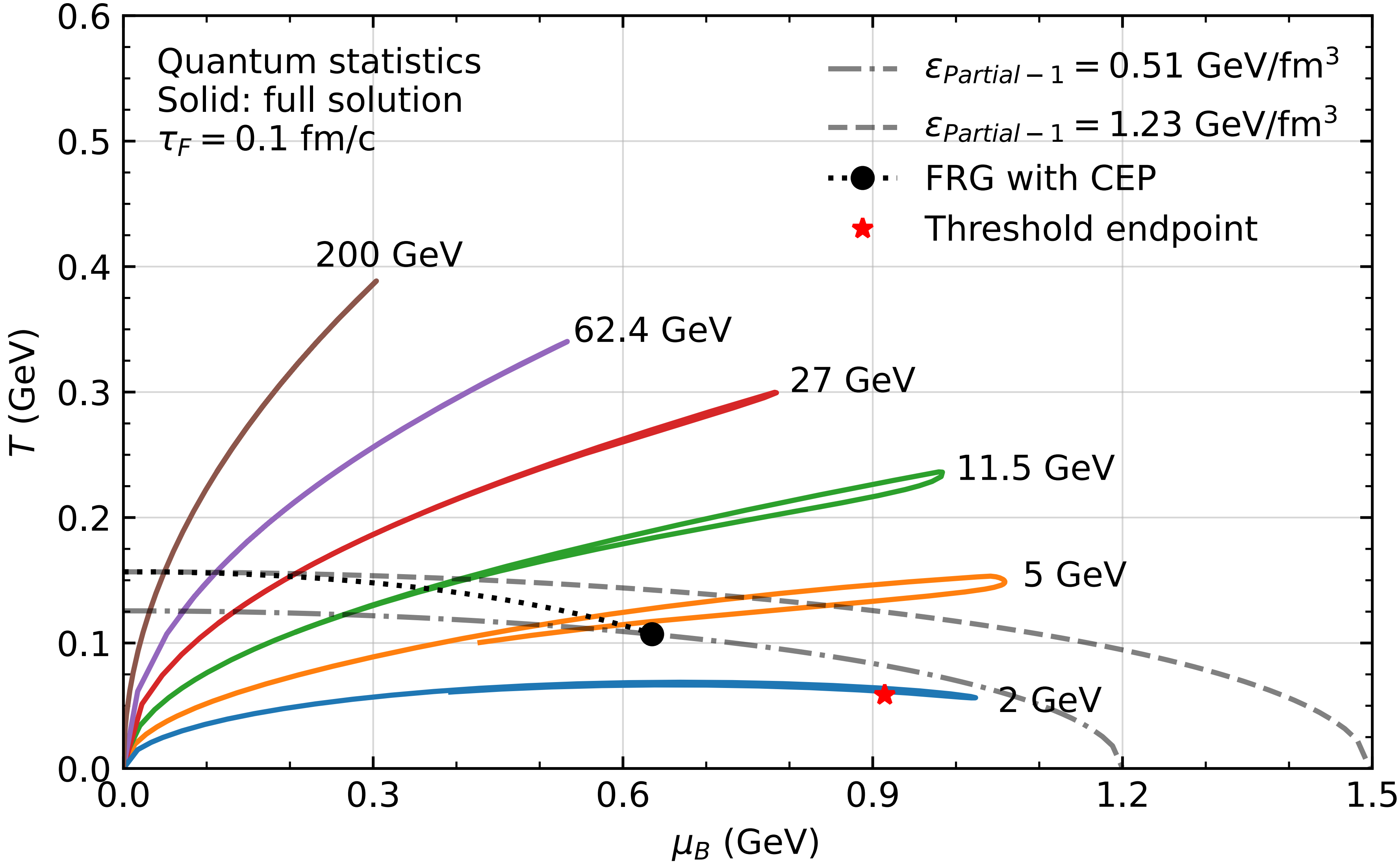
(valid for most nuclei)



Collision Trajectory in the T-μ_B Diagram

For central Au+Au collisions:

Mendenhall & ZWL, arXiv:2111.13932

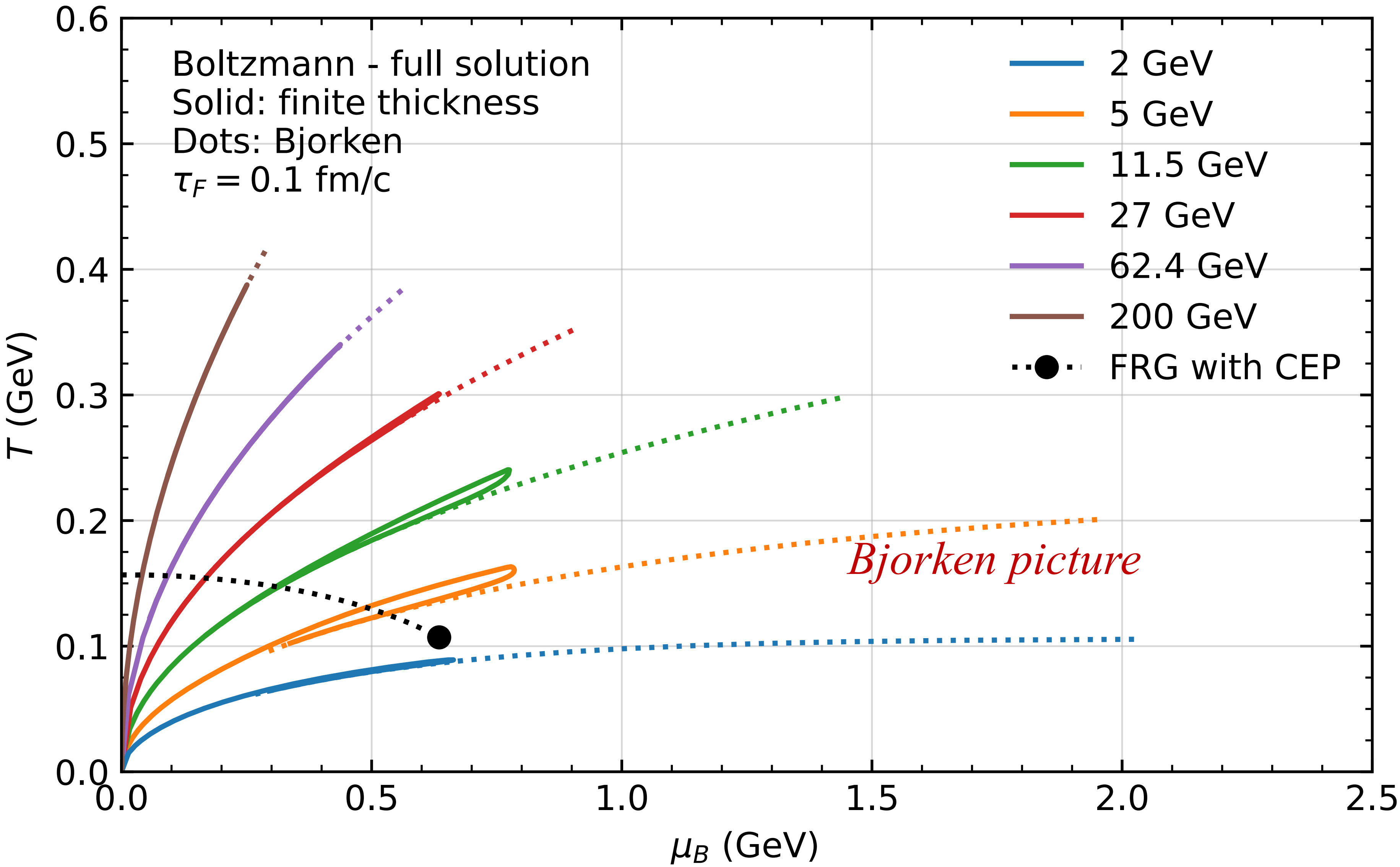


FRG crossover curve and CEP: from Functional Renormalization Group

Fu, Pawlowski & Rennecke, PRD (2020)

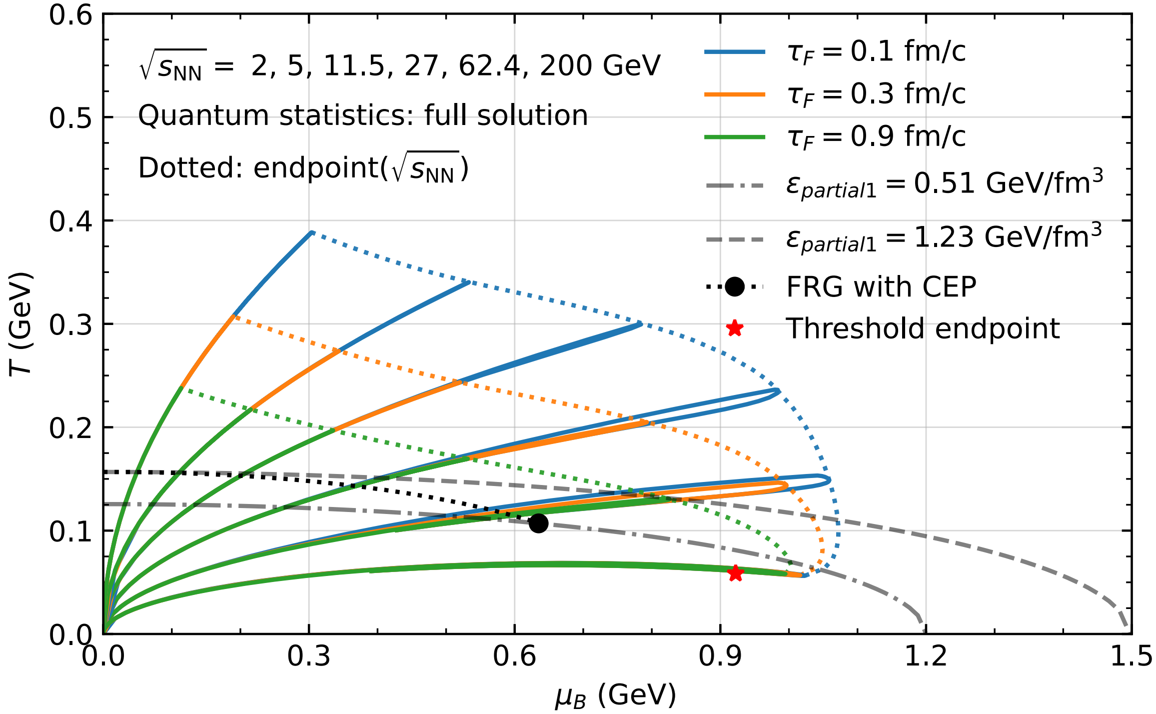
Collision Trajectory in the T-μ_B Diagram

Mendenhall & ZWL, arXiv:2111.13932



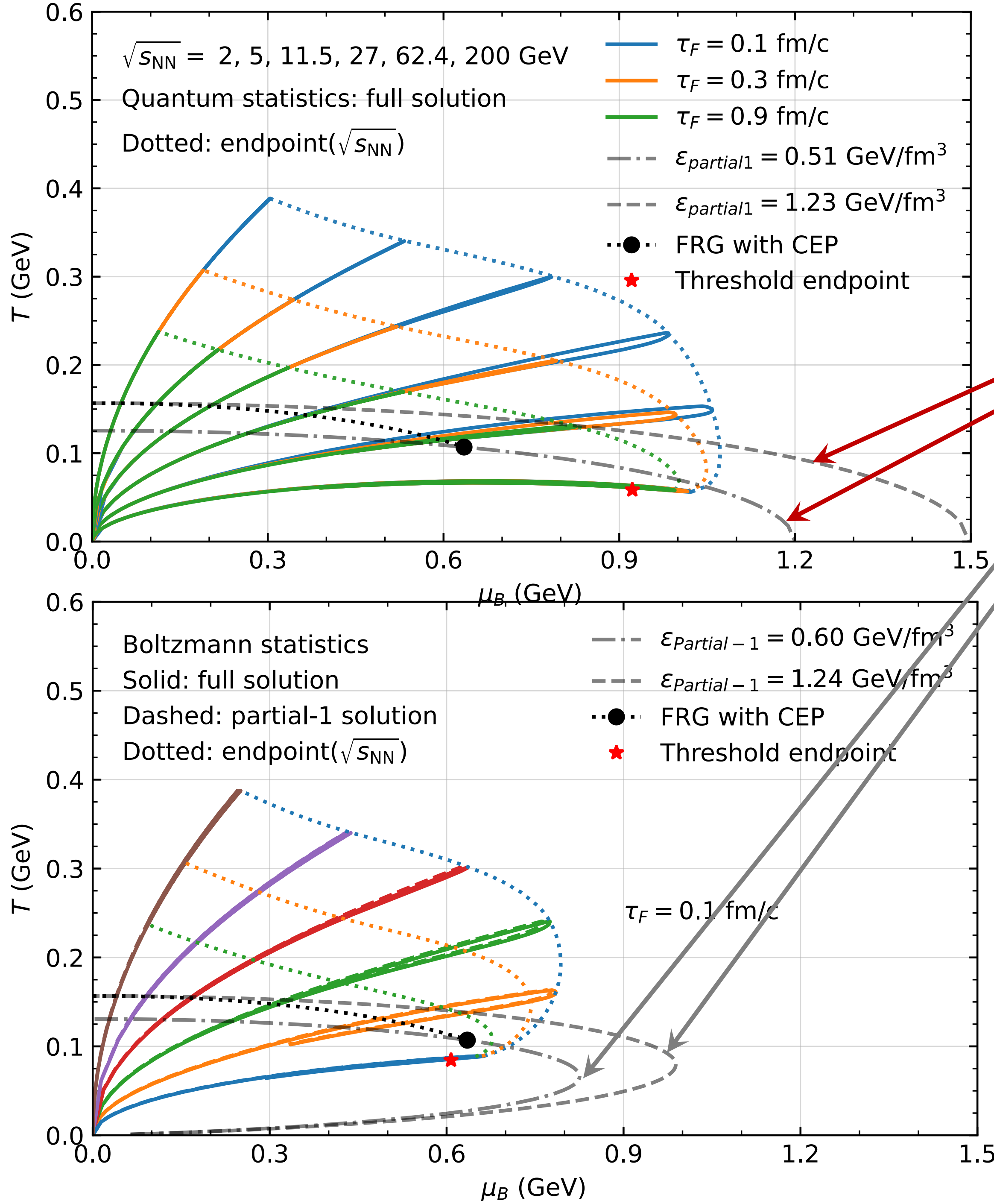
Large effect of finite nuclear thickness on T-μ_B trajectory at lower energies

Collision Trajectory in the T-μ_B Diagram



τ_F affects T&μ_B peak values, but not much the hadronization point.

Collision Trajectory in the T-μ_B Diagram



$\boldsymbol{\tau_F}$ affects T&μ_B peak values, but not much the hadronization point.

$\varepsilon_{partial1}$ given by

$$\epsilon_1 = \frac{19\pi^2}{12}T^4 + \frac{\mu_B^2}{3}T^2 + \frac{\mu_B^4}{54\pi^2}$$

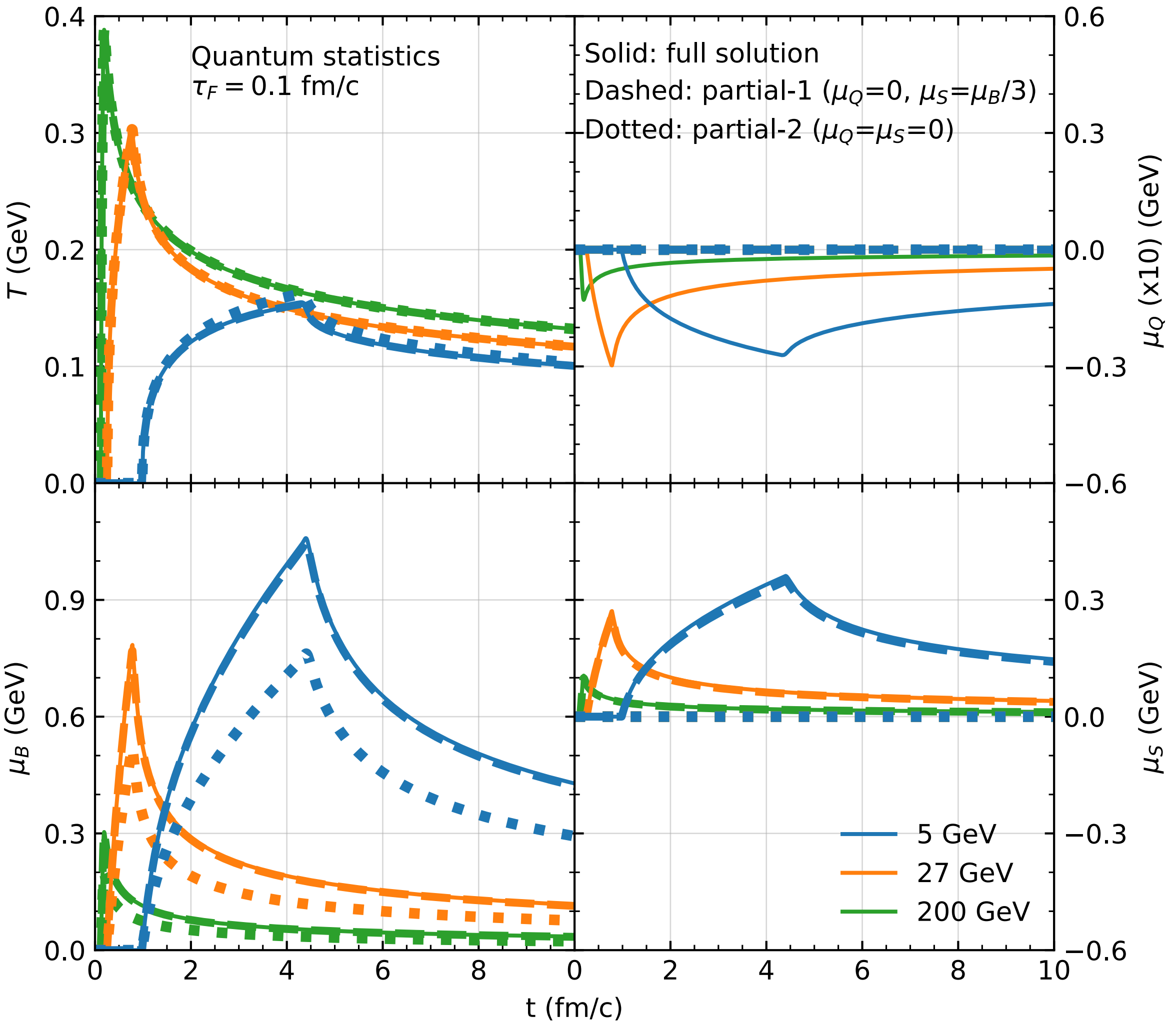
for QGP with quantum stats (*higher μ_B than Boltzmann*);

$$\epsilon_1 = \frac{12}{\pi^2}T^4\left[7 + 6\cosh\left(\frac{\mu_B}{3T}\right)\right]$$

for QGP with Boltzmann stats.

Extractions of T & μ (μ_B, μ_S, μ_Q)

For QGP with quantum stats



See C. Ratti's Tuesday talk on μ_Q & μ_S used in lattice QCD

Partial-1 solution

assumes $\mu_Q = 0$ & $\mu_S = \mu_B/3$ to simplify the problem

$\boldsymbol{\varepsilon}$ **&** $\boldsymbol{n_B, n_S, n_Q} \rightarrow$ **T &** $\boldsymbol{\mu_B, \mu_S, \mu_Q}$

to $\boldsymbol{\varepsilon}$ **&** $\boldsymbol{n_B} \rightarrow$ **T &** $\boldsymbol{\mu_B}$:

$$\epsilon_1 = \frac{19\pi^2}{12}T^4 + \frac{\mu_B^2}{3}T^2 + \frac{\mu_B^4}{54\pi^2},$$

$$n_{B,1} = \frac{2\mu_B}{9}T^2 + \frac{2\mu_B^3}{81\pi^2}.$$

Partial-2 solution:

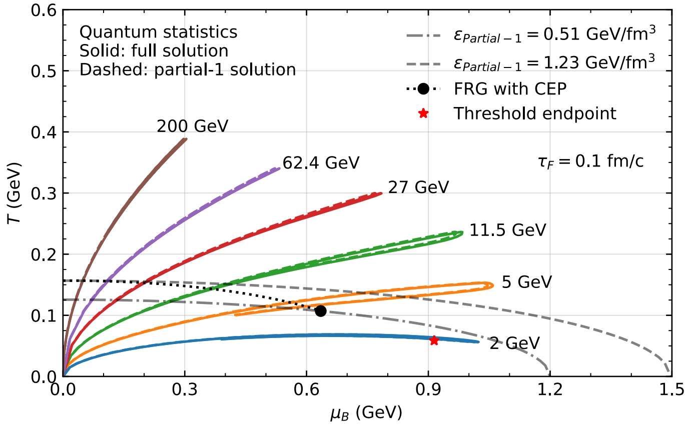
neglects μ_Q & μ_S terms in densities, equivalent to assuming $\mu_Q = 0$ & $\mu_S = 0$:

$$\epsilon_2 = \frac{19\pi^2}{12}T^4 + \frac{\mu_B^2}{2}T^2 + \frac{\mu_B^4}{36\pi^2},$$

$$n_{B,2} = \frac{\mu_B}{3}T^2 + \frac{\mu_B^3}{27\pi^2}.$$

This violates $\boldsymbol{\mu_B - \mu_Q - 3\mu_S = 0}$ (or strangeness neutrality) and gives bad results on $\boldsymbol{\mu_B}$.

Collision Trajectory in the T-μ_B Diagram: QGP Lifetime



Mendenhall & ZWL, arXiv:2111.13932

start time: time the trajectory first crosses the crossover curve

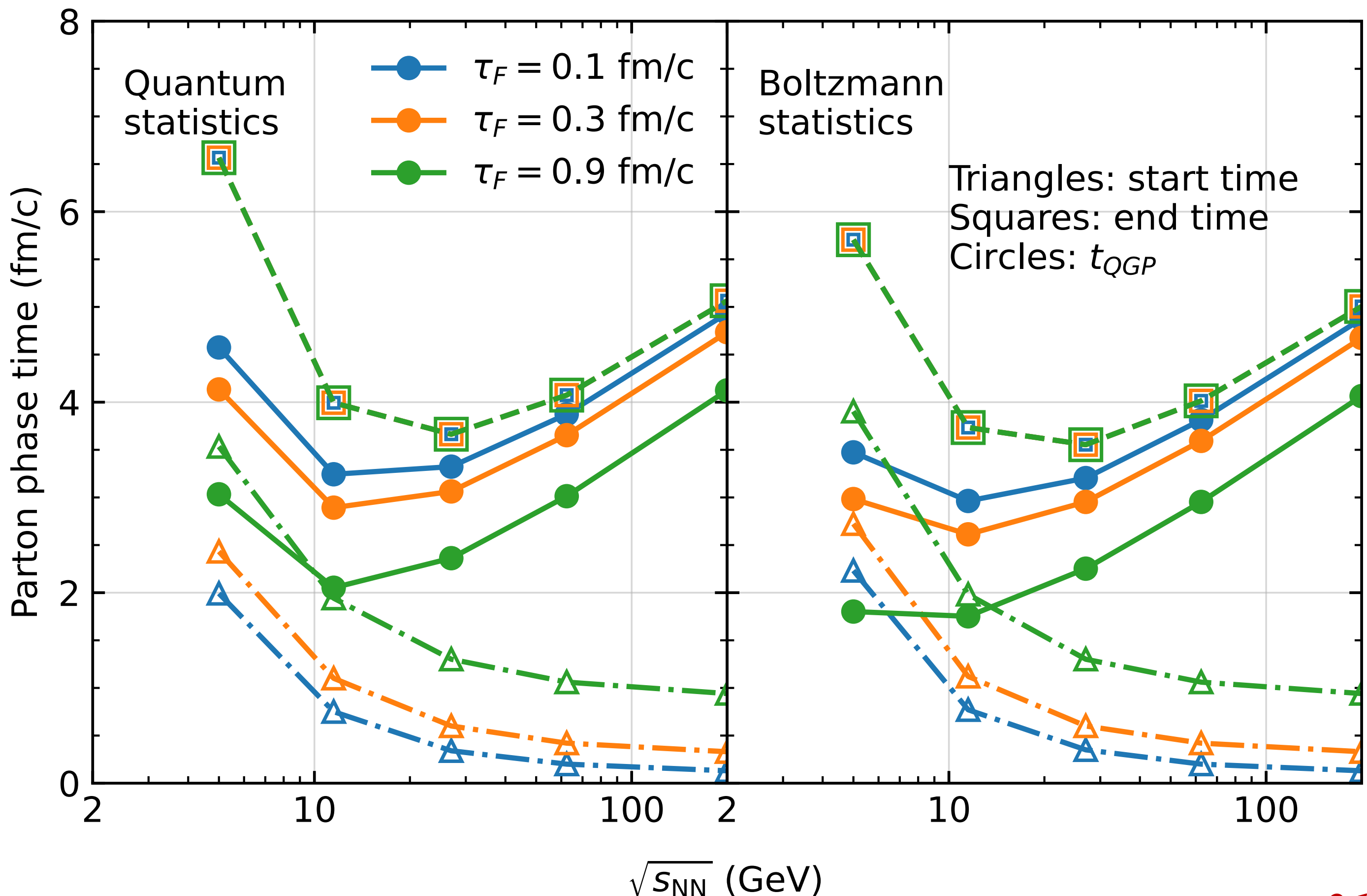
end time: time the trajectory crosses the crossover curve again

t_{QGP}=(end time) - (start time)

t_{QGP} at low energies is still big (~*2-4 fm/c*),

even shows a rise towards threshold energy.

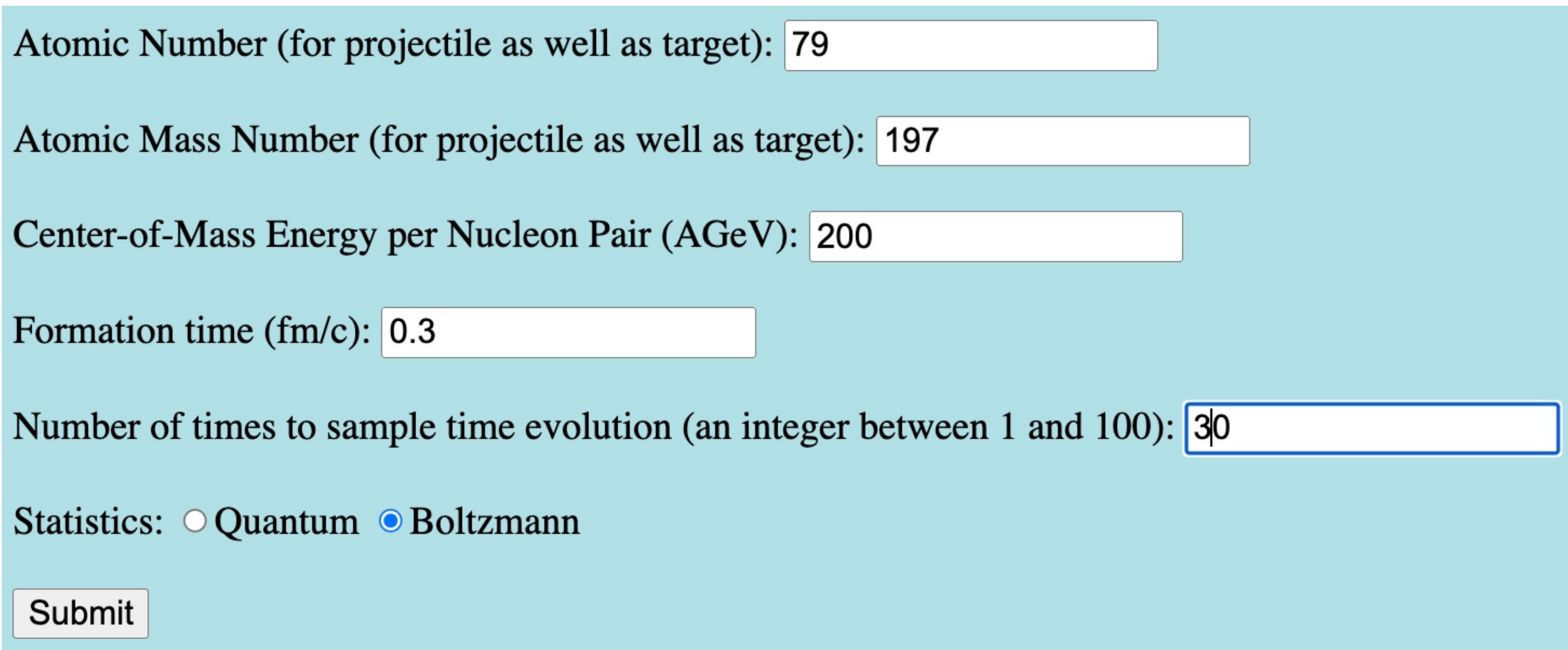
These are also seen in AMPT model results: Wang, Ma, ZWL & Fu, arXiv:2102.06937v2



Collision Trajectory in the T-μ_B Diagram: webpage

Todd Mendenhall has written a web interface
to perform these calculations of ***ε(t) and* T & *μ* (*μ_B, μ_S, μ_Q*)**

- Link is also available via http://myweb.ecu.edu/linz/densities/
  or bottom of the AMPT webpage http://myweb.ecu.edu/linz/ampt
- **Takes input from user:**

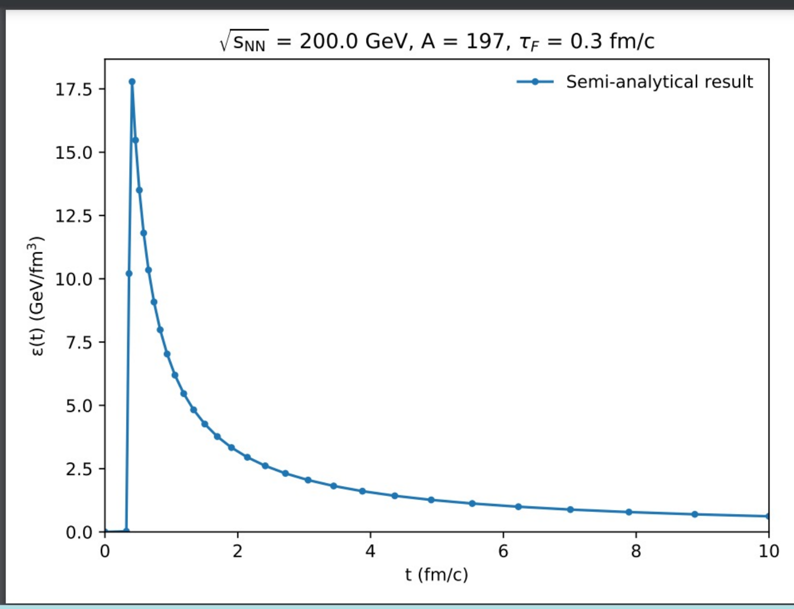


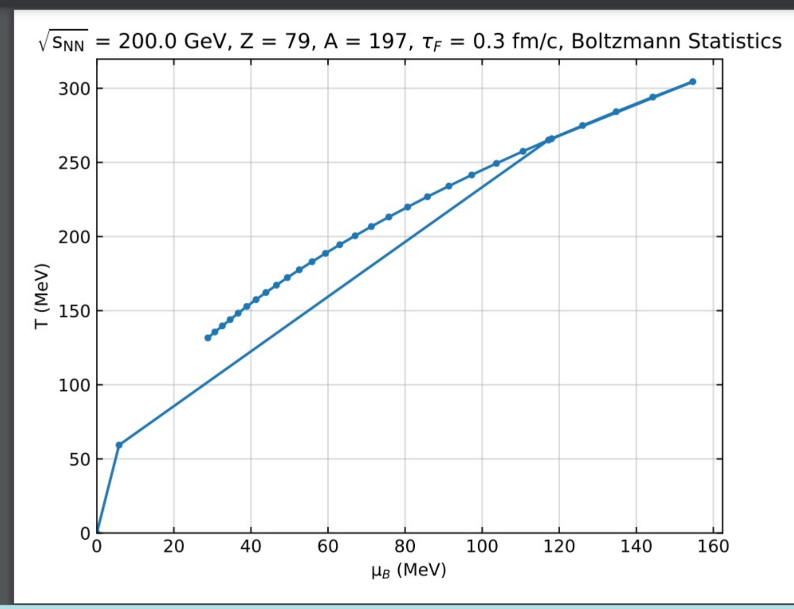
Collision Trajectory in the T-μ_B Diagram: webpage

- Plots ***ε(t) & T-μ_B*** trajectory, user can download full data file

Semi-analytical calculation of the average ϵ and $T-\mu_B$ trajectory of central A+A collisions

(Updated March 1, 2022)



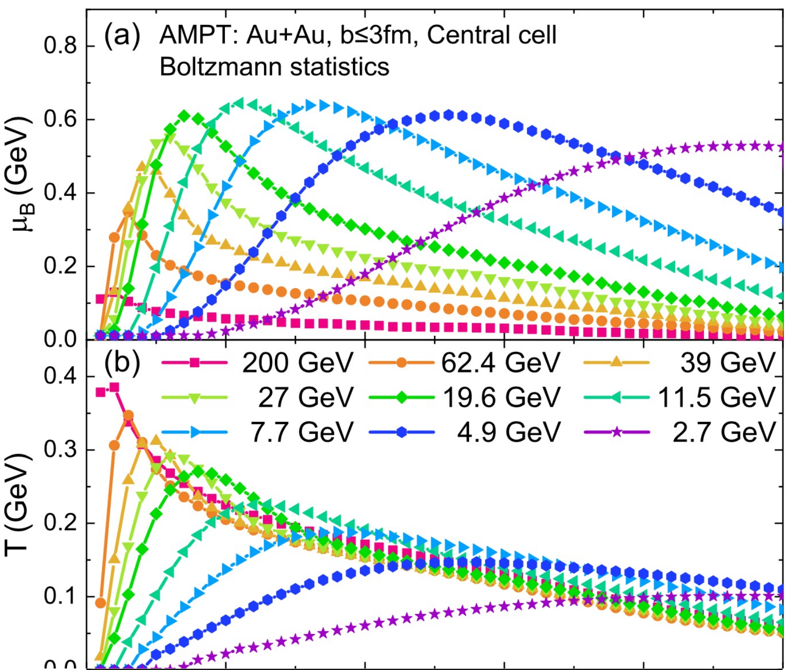


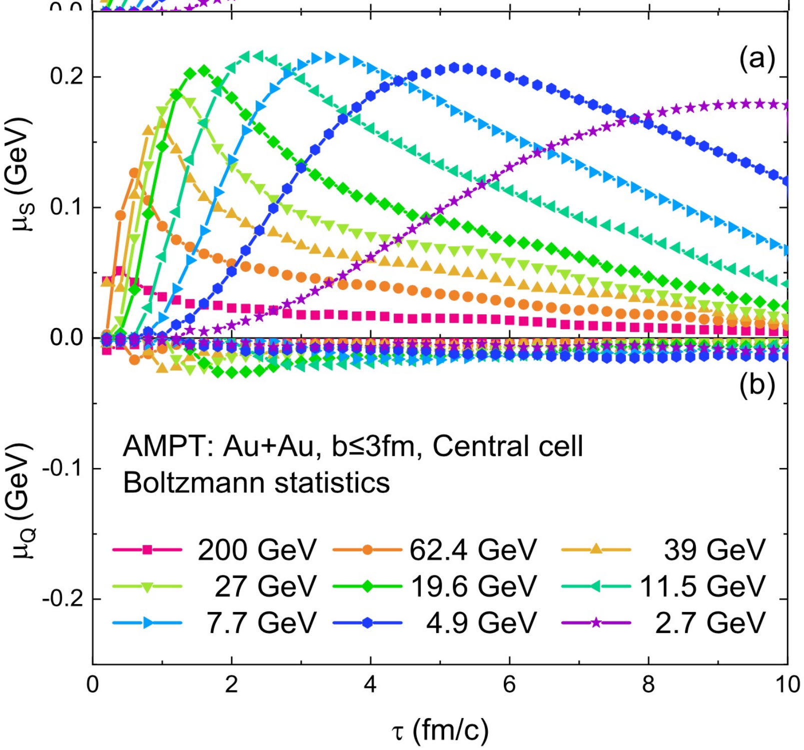
The data file contains the time evolution of the energy density, temperature, and three chemical potentials.

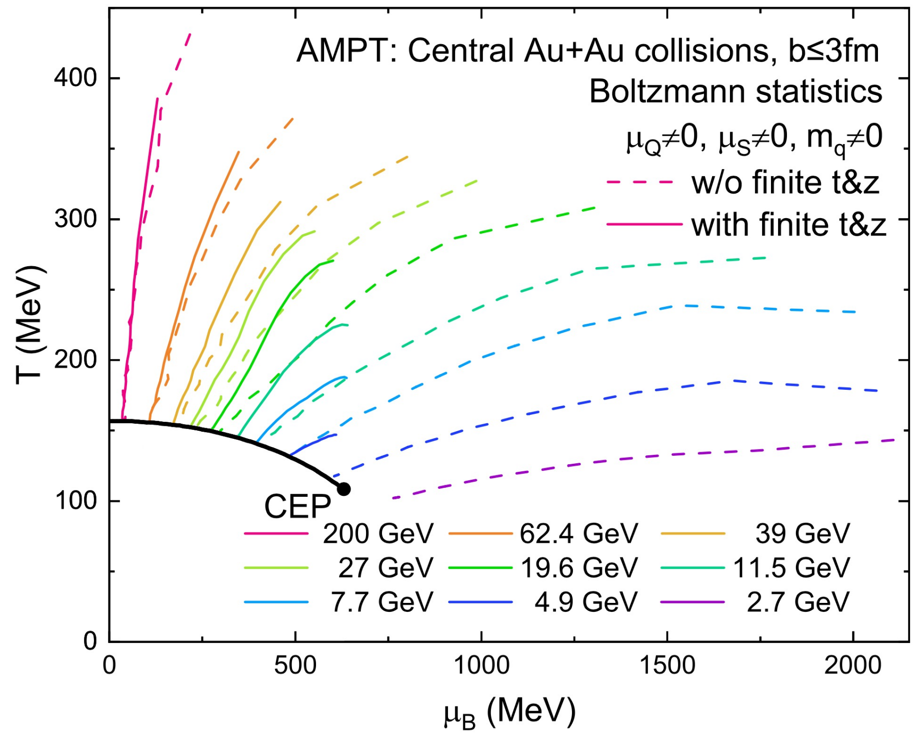
Download Data File

Collision Trajectory in the T-μ_B Diagram: from AMPT

Wang, Ma, ZWL & Fu, arXiv:2102.06937v2







Qualitatively the same as semi-analytical results:

- $\mu_Q \sim 0$ & $\mu_S \sim \mu_B/3$
- Big effect of nuclear thickness at lower energies.

Conclusions

- We have developed a semi-analytical method to calculate densities **ε & n** from the initial/primary collisions;
  the **ε** part = extension of the Bjorken **ε** formula
  to lower energies by including the finite nuclear thickness.

- At low energies like the BES, finite nuclear thickness has big effects on **ε & n** and consequently on event trajectories.

- The method can be used to calculate collision trajectory in the 4-dimensional T-μ (μ_B, μ_S, μ_Q) space including the T-μ_B plane; a webpage is written to perform these calculations.

- Partial solution ($\mu_Q = 0$ & $\mu_S = \mu_B/3$) satisfies strangeness neutrality & simplifies the 4d problem to 2d:
  ε & $n_B, n_S, n_Q \leftrightarrow$ T & μ_B, μ_S, μ_Q to ε & $n_B \leftrightarrow$ T & μ_B