Standard Model Effective Field Theory (SMEFT) Basis

Buchmuller and Weiler, NPB 268 (1986) 377
Grzadkowski et al., JHEP 1020 (2010) 085

The Bosonic Sector (HISZ basis) – CP-Even
Hagiwara et al. PRD 48 (1993) 2128-2203; NPB 496 (1997) 66-102

$$\mathcal{O}_{GG} = \Phi^\dagger\Phi\, G^a_{\mu\nu}G^{a\mu\nu} \qquad \mathcal{O}_{WW} = \Phi^\dagger \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Phi$$
$$\mathcal{O}_{BB} = \Phi^\dagger \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\Phi \qquad \mathcal{O}_{BW} = \Phi^\dagger \hat{B}_{\mu\nu}\hat{W}^{\mu\nu}\Phi$$
$$\mathcal{O}_{W} = (D_\mu\Phi)^\dagger \hat{W}^{\mu\nu}(D_\nu\Phi) \qquad \mathcal{O}_{B} = (D_\mu\Phi)^\dagger \hat{B}^{\mu\nu}(D_\nu\Phi)$$
$$\mathcal{O}_{\Phi,1} = (D_\mu\Phi)^\dagger\, \Phi\ \Phi^\dagger\,(D^\mu\Phi) \qquad \mathcal{O}_{\Phi,2} = \frac{1}{2}\partial^\mu\left(\Phi^\dagger\Phi\right)\partial_\mu\left(\Phi^\dagger\Phi\right)$$
$$\mathcal{O}_{\Phi,3} = \frac{1}{3}\left(\Phi^\dagger\Phi\right)^3 \qquad \mathcal{O}_{\Phi,4} = (D_\mu\Phi)^\dagger\,(D^\mu\Phi)\left(\Phi^\dagger\Phi\right)$$

$$\mathcal{O}_{\Box\phi} = (D_\mu D^\mu\Phi)^\dagger\,(D_\nu D^\nu\Phi)$$
$$\mathcal{O}_{WWW} = i\epsilon_{ijk}\hat{W}^{i\,\nu}_{\mu}\hat{W}^{j\,\rho}_{\nu}\hat{W}^{k\,\mu}_{\rho} \qquad \mathcal{O}_{GGG} = if_{abc}G^{a\,\nu}_{\mu}G^{b\,\rho}_{\nu}G^{c\,\mu}_{\rho}$$
$$\mathcal{O}_{DW} = \left(\mathcal{D}^\mu\,\hat{W}_{\mu\nu}\right)^i\left(\mathcal{D}_\rho\hat{W}^{\rho\nu}\right)^i \qquad \mathcal{O}_{DB} = \left(\partial^\mu\hat{B}_{\mu\nu}\right)\left(\partial_\rho\hat{B}^{\rho\nu}\right)$$
$$\mathcal{O}_{DG} = (\mathcal{D}^\mu\,G_{\mu\nu})^a\,(\mathcal{D}_\rho G^{\rho\nu})^a$$

where $D_\mu\phi = \partial_\mu\phi + \frac{i}{2}g\sigma_i W^i_\mu + \frac{i}{2}g' B_\mu$, and $\hat{W}_{\mu\nu} = \frac{i}{2}\sigma_i W^i_{\mu\nu}$ and $\hat{B}_{\mu\nu} = \frac{i}{2}B_{\mu\nu}$.

The Fermionic Sector

$$\mathcal{O}_{u\Phi,ij} = (\Phi^\dagger\Phi)(\bar{Q}_i\tilde{\Phi}u_{R_j})$$
$$\mathcal{O}_{d\Phi,ij} = (\Phi^\dagger\Phi)(\bar{Q}_i\Phi d_{Rj})$$
$$\mathcal{O}_{e\Phi,ij} = (\Phi^\dagger\Phi)(\bar{L}_i\Phi e_{R_j})$$

$$\mathcal{O}^{(1)}_{\Phi Q,ij} = \Phi^\dagger(i\overleftrightarrow{D}_\mu\Phi)(\bar{Q}_i\gamma^\mu Q_j) \qquad \mathcal{O}^{(3)}_{\Phi Q,ij} = \Phi^\dagger(i\overleftrightarrow{D}^a_\mu\Phi)(\bar{Q}_i\gamma^\mu\sigma_a Q_j)$$
$$\mathcal{O}^{(1)}_{\Phi L,ij} = \Phi^\dagger(i\overleftrightarrow{D}_\mu\Phi)(\bar{L}_i\gamma^\mu L_j) \qquad \mathcal{O}^{(3)}_{\Phi L,ij} = \Phi^\dagger(i\overleftrightarrow{D}^a_\mu\Phi)(\bar{L}_i\gamma^\mu\sigma_a L_j)$$
$$\mathcal{O}^{(1)}_{\Phi e,ij} = \Phi^\dagger(i\overleftrightarrow{D}_\mu\Phi)(\bar{e}_{R_i}\gamma^\mu e_{R_j})$$
$$\mathcal{O}^{(1)}_{\Phi u,ij} = \Phi^\dagger(i\overleftrightarrow{D}_\mu\Phi)(\bar{u}_{R_i}\gamma^\mu u_{R_j})$$
$$\mathcal{O}^{(1)}_{\Phi d,ij} = \Phi^\dagger(i\overleftrightarrow{D}_\mu\Phi)(\bar{d}_{R_i}\gamma^\mu d_{R_j})$$
$$\mathcal{O}^{(1)}_{\Phi ud,ij} = \tilde{\Phi}^\dagger(i\overleftrightarrow{D}_\mu\Phi)(\bar{u}_{R_i}\gamma^\mu d_{R_j})$$

where $\Phi^\dagger\overleftrightarrow{D}_\mu\Phi \equiv \Phi^\dagger D_\mu\Phi - D_\mu\Phi^\dagger\Phi$ and $\Phi^\dagger\sigma^a\overleftrightarrow{D}_\mu\Phi \equiv \Phi^\dagger\sigma^a D_\mu\Phi - D_\mu\Phi^\dagger\sigma^a\Phi$.
To this list, we should add the 4-fermion operators and the dipole operators.

Example of fit in the SMEFT case, from Almeida et al., arXiv: 2108.04828 (see the reference for further details).

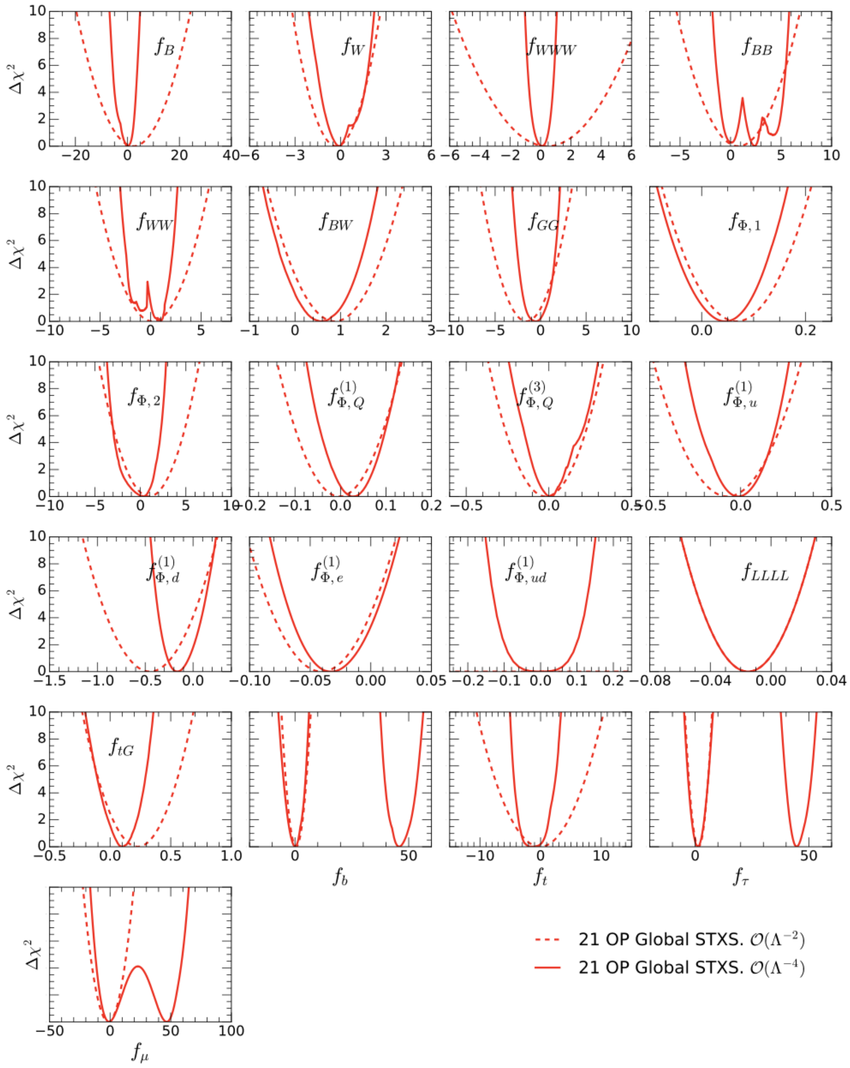

FIG. 10: Marginalized one-dimensional $\Delta\chi^2$ distributions for the 21 parameters appearing in our global fit including the STXS Higgs data sets. The dashed (solid) line stands for the results obtained with the theoretical predictions for the observables expanded at $\mathcal{O}(\Lambda^{-2})$, ($\mathcal{O}(\Lambda^{-4})$,) order in the Wilson coefficients.

Appelquist-Longhitano-Feruglio (ALF) Basis

Appelquist and Bernard, PRD 22 (1980) 22
Longhitano, PRD 22 (1980) 1166; NPB 188 (1981) 118
Feruglio, Int.J.Mod.Phys. A8 (1993) 4937

$$
\begin{aligned}
\mathcal{A}_1 &= B_{\mu\nu}\mathrm{Tr}(\mathbf{T}W^{\mu\nu}) \\
\mathcal{A}_2 &= iB_{\mu\nu}\mathrm{Tr}(\mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu]) \\
\mathcal{A}_3 &= i\mathrm{Tr}(W_{\mu\nu}[\mathbf{V}^\mu, \mathbf{V}^\nu]) \\
\mathcal{A}_4 &= (\mathrm{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu))^2 \\
\mathcal{A}_5 &= \mathrm{Tr}((\mathcal{D}_\mu \mathbf{V}^\mu)^2) \\
\mathcal{A}_6 &= (\mathrm{Tr}(\mathbf{V}_\mu \mathbf{V}_\nu))^2 \\
\mathcal{A}_7 &= (\mathrm{Tr}(\mathbf{T}W_{\mu\nu}))^2 \\
\mathcal{A}_8 &= i\mathrm{Tr}(\mathbf{T}W_{\mu\nu})\mathrm{Tr}(\mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu]) \\
\mathcal{A}_9 &= \varepsilon^{\mu\nu\rho\lambda}\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)\mathrm{Tr}(\mathbf{V}_\nu W_{\rho\lambda}) \\
\mathcal{A}_{10} &= \mathrm{Tr}(\mathbf{T}\mathcal{D}_\mu \mathbf{V}^\mu)\mathrm{Tr}(\mathbf{T}\mathcal{D}_\nu \mathbf{V}^\nu) \\
\mathcal{A}_{11} &= \mathrm{Tr}([\mathbf{T}, \mathbf{V}_\nu]\mathcal{D}_\mu \mathbf{V}^\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}^\nu) \\
\mathcal{A}_{12} &= \mathrm{Tr}(\mathbf{V}_\mu \mathbf{V}^\mu)(\mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu))^2 \\
\mathcal{A}_{13} &= \mathrm{Tr}(\mathbf{V}_\mu \mathbf{V}_\nu)\mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}^\nu) \\
\mathcal{A}_{14} &= (\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu))^2
\end{aligned}
$$

Higgs Effective Field Theory (HEFT) Basis

Feruglio, Int.J.Mod.Phys. A8 (1993) 4937
Contino at al., JHEP 1005 (2010) 089
Alonso et al., PLB 722 (2013) 330
Brivio et al., JHEP 1403 (2014) 024
Buchalla et al., NPB 880 (2014) 552-573
Brivio et al., EPJC 76 (2016) 416

The Bosonic Sector – CP-Even

$$\mathcal{P}_T(h) = \frac{v^2}{4}\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\mathcal{F}_T(h) \qquad \mathcal{P}_G(h) = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}\mathcal{F}_G(h)$$

$$\mathcal{P}_B(h) = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu}\mathcal{F}_B(h) \qquad \mathcal{P}_W(h) = -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu}\mathcal{F}_W(h)$$

$$\mathcal{P}_1(h) = B_{\mu\nu}\mathrm{Tr}(\mathbf{T}W^{\mu\nu})\mathcal{F}_1(h) \qquad \mathcal{P}_2(h) = iB_{\mu\nu}\mathrm{Tr}(\mathbf{T}[\mathbf{V}^\mu,\mathbf{V}^\nu])\mathcal{F}_2(h)$$

$$\mathcal{P}_3(h) = i\mathrm{Tr}(W_{\mu\nu}[\mathbf{V}^\mu,\mathbf{V}^\nu])\mathcal{F}_3(h) \qquad \mathcal{P}_4(h) = iB_{\mu\nu}\mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\partial^\nu\mathcal{F}_4(h)$$

$$\mathcal{P}_5(h) = i\mathrm{Tr}(W_{\mu\nu}\mathbf{V}^\mu)\partial^\nu\mathcal{F}_5(h) \qquad \mathcal{P}_6(h) = (\mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}^\mu))^2\mathcal{F}_6(h)$$

$$\mathcal{P}_7(h) = \mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}^\mu)\partial_\nu\partial^\nu\mathcal{F}_7(h) \qquad \mathcal{P}_8(h) = \mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}_\nu)\partial^\mu\mathcal{F}_8(h)\partial^\nu\mathcal{F}'_8(h)$$

$$\mathcal{P}_9(h) = \mathrm{Tr}((\mathcal{D}_\mu\mathbf{V}^\mu)^2)\mathcal{F}_9(h) \qquad \mathcal{P}_{10}(h) = \mathrm{Tr}(\mathbf{V}_\nu\mathcal{D}_\mu\mathbf{V}^\mu)\partial^\nu\mathcal{F}_{10}(h)$$

$$\mathcal{P}_{11}(h) = (\mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}_\nu))^2\mathcal{F}_{11}(h) \qquad \mathcal{P}_{12}(h) = (\mathrm{Tr}(\mathbf{T}W_{\mu\nu}))^2\mathcal{F}_{12}(h)$$

$$\mathcal{P}_{13}(h) = i\mathrm{Tr}(\mathbf{T}W_{\mu\nu})\mathrm{Tr}(\mathbf{T}[\mathbf{V}^\mu,\mathbf{V}^\nu])\mathcal{F}_{13}(h) \qquad \mathcal{P}_{14}(h) = \varepsilon^{\mu\nu\rho\lambda}\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)\mathrm{Tr}(\mathbf{V}_\nu W_{\rho\lambda})\mathcal{F}_{14}(h)$$

$$\mathcal{P}_{15}(h) = \mathrm{Tr}(\mathbf{T}\mathcal{D}_\mu\mathbf{V}^\mu)\mathrm{Tr}(\mathbf{T}\mathcal{D}_\nu\mathbf{V}^\nu)\mathcal{F}_{15}(h) \qquad \mathcal{P}_{16}(h) = \mathrm{Tr}([\mathbf{T},\mathbf{V}_\nu]\mathcal{D}_\mu\mathbf{V}^\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}^\nu)\mathcal{F}_{16}(h)$$

$$\mathcal{P}_{17}(h) = i\mathrm{Tr}(\mathbf{T}W_{\mu\nu})\mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\partial^\nu\mathcal{F}_{17}(h) \qquad \mathcal{P}_{18}(h) = \mathrm{Tr}(\mathbf{T}[\mathbf{V}_\mu,\mathbf{V}_\nu])\mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\partial^\nu\mathcal{F}_{18}(h)$$

$$\mathcal{P}_{19}(h) = \mathrm{Tr}(\mathbf{T}\mathcal{D}_\mu\mathbf{V}^\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu)\partial^\nu\mathcal{F}_{19}(h) \qquad \mathcal{P}_{20}(h) = \mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}^\mu)\partial_\nu\mathcal{F}_{20}(h)\partial^\nu\mathcal{F}'_{20}(h)$$

$$\mathcal{P}_{21}(h) = (\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu))^2\partial_\nu\mathcal{F}_{21}(h)\partial^\nu\mathcal{F}'_{21}(h) \qquad \mathcal{P}_{22}(h) = \mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu)\partial^\mu\mathcal{F}_{22}(h)\partial^\nu\mathcal{F}'_{22}(h)$$

$$\mathcal{P}_{23}(h) = \mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}^\mu)(\mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu))^2\mathcal{F}_{23}(h) \qquad \mathcal{P}_{24}(h) = \mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}_\nu)\mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}^\nu)\mathcal{F}_{24}(h)$$

$$\mathcal{P}_{25}(h) = (\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu))^2\partial_\nu\partial^\nu\mathcal{F}_{25}(h) \qquad \mathcal{P}_{26}(h) = (\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu))^2\mathcal{F}_{26}(h)$$

$$\mathcal{P}_{\Box H}(h) = \frac{1}{v^2}\left(\partial_\mu\partial^\mu h\right)^2\mathcal{F}_{\Box H}(h) \qquad \mathcal{P}_{DH}(h) = \frac{1}{v^4}\left((\partial_\mu h)(\partial^\mu h)\right)^2\mathcal{F}_{DH}(h)$$

The Fermionic Sector

$$N_1(h) \equiv \bar{Q}_L\,\gamma_\mu\mathbf{V}^\mu\,Q_L\,\mathcal{F} \qquad N_2(h) \equiv \bar{Q}_R\,\gamma_\mu\mathbf{U}^\dagger\mathbf{V}^\mu\mathbf{U}\,Q_R\,\mathcal{F}$$

$$N_3(h) \equiv \bar{Q}_L\,\gamma_\mu[\mathbf{V}^\mu,\mathbf{T}]\,Q_L\,\mathcal{F} \qquad N_4(h) \equiv \bar{Q}_R\,\gamma_\mu\mathbf{U}^\dagger[\mathbf{V}^\mu,\mathbf{T}]\mathbf{U}\,Q_R\,\mathcal{F}$$

$$N_5(h) \equiv \bar{Q}_L\,\gamma_\mu\{\mathbf{V}^\mu,\mathbf{T}\}\,Q_L\,\mathcal{F} \qquad N_6(h) \equiv \bar{Q}_R\,\gamma_\mu\mathbf{U}^\dagger\{\mathbf{V}^\mu,\mathbf{T}\}\mathbf{U}\,Q_R\,\mathcal{F}$$

$$N_7(h) \equiv \bar{Q}_L\,\gamma_\mu\mathbf{T}\mathbf{V}^\mu\mathbf{T}\,Q_L\,\mathcal{F} \qquad N_8(h) \equiv \bar{Q}_R\,\gamma_\mu\mathbf{U}^\dagger\mathbf{T}\mathbf{V}^\mu\mathbf{T}\mathbf{U}\,Q_R\,\mathcal{F}$$

$\dots$

Example of fit in the SMEFT case, from Brivio et al., EPJC 76 (2016) 416 (see the reference for further details).

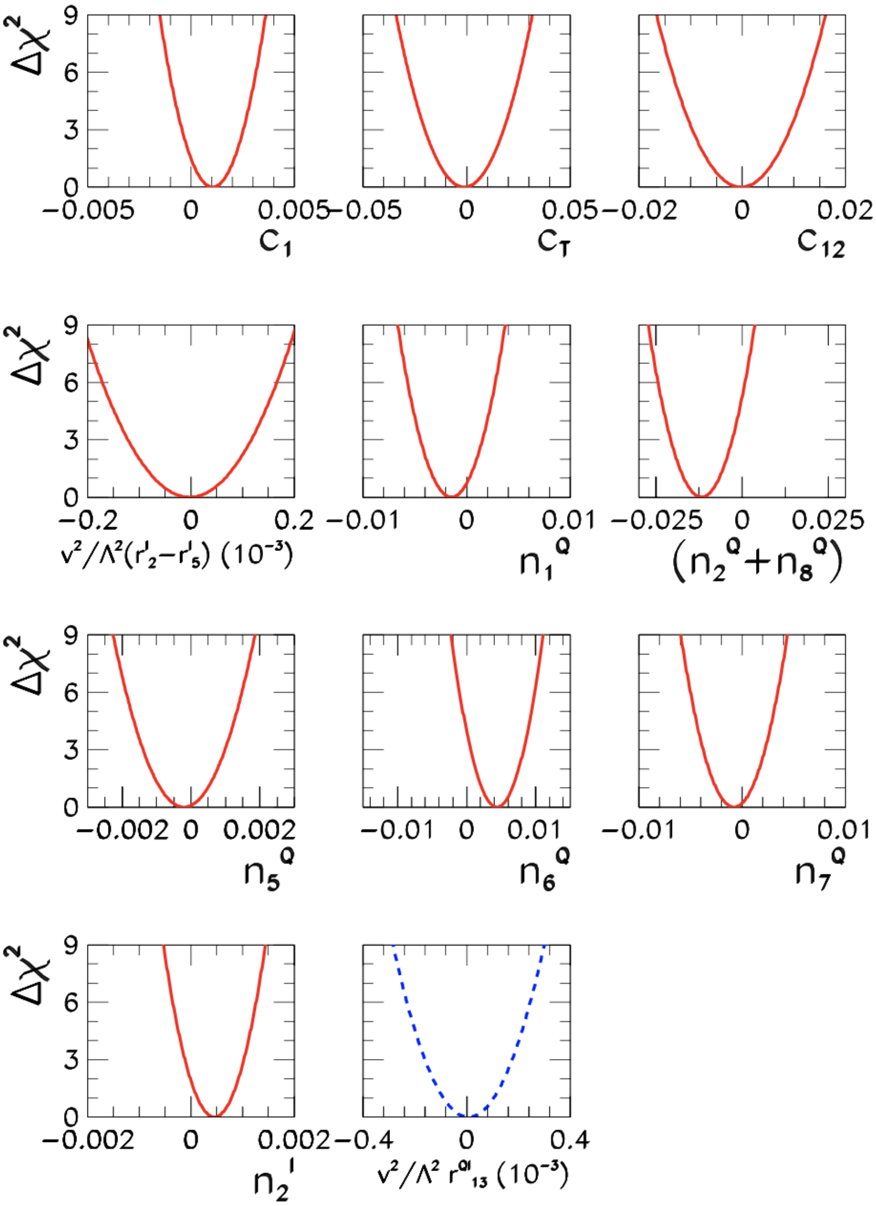

Figure 1: *Dependence of* $\Delta\chi^2_{\rm EWPD+CKM}$ *(*$= \Delta\chi^2_{\rm EWPD}$ *for all, but last panel) on the 11 independent operator coefficients as labeled in the figure. In each panel* $\Delta\chi^2_{\rm EWPD+CKM}$ *is shown after marginalising over the other undisplayed parameters.*

Minimal Linear σ Model

When considering the MLσM, integrating out σ, the contributions to the low-energy operators can be read in the following table taken by Feruglio et al., JHEP 1606 (2016) 038

	Operator	$\mathcal{F}_k(\varphi)$	$1/\lambda^n$
$\mathcal{P}_H$	$\frac{1}{2}(\partial_\mu h)^2$	$1-\frac{1}{4\lambda}(\alpha c_\varphi - 2\beta s_\varphi^2)$	0
$\mathcal{P}_C$	$-\frac{v^2}{4}\langle V_\mu V^\mu\rangle$	$\frac{1}{\xi}\left[1-\frac{1}{4\lambda}\left(\alpha c_\varphi - 2\beta s_\varphi^2\right)\right] s_\varphi^2$	0
$\mathcal{P}_{\text{Yuk}}$	$v\,\bar{q}_{iL} U P_\pm \mathbf{q}_{iR} + \text{h.c.}$	$-\frac{y_i^0}{\sqrt{2}\,\xi} c_\varphi^n s_\varphi^{2m+1}\left(1-\frac{n+2m+1}{8\lambda}(\alpha c_\varphi - 2\beta s_\varphi^2)\right)$	0
$\mathcal{P}_{DH}$	$\frac{1}{v^4}(\partial_\mu h)^4$	$\frac{\xi^2}{16\lambda}$	1
$\mathcal{P}_6$	$\langle V_\mu V^\mu\rangle^2$	$\frac{s_\varphi^4}{64\lambda}$	1
$\mathcal{P}_{20}$	$\frac{1}{v^2}\langle V_\mu V^\mu\rangle(\partial_\nu h)^2$	$-\frac{\xi}{16\lambda} s_\varphi^2$	1
$\mathcal{P}_{qH}$	$\frac{1}{v^3}(\partial_\mu h)^2 \bar{q}_{iL} U P_\pm \mathbf{q}_{iR} + \text{h.c.}$	$-\frac{y_i^0}{\sqrt{2}}\xi^{3/2}\left(\frac{n+2m+1}{8\lambda}\right) c_\varphi^n s_\varphi^{2m+1}$	1
$\mathcal{P}_{qV}$	$\frac{1}{v}\langle V_\mu V^\mu\rangle \bar{q}_{iL} U P_\pm \mathbf{q}_{iR} + \text{h.c.}$	$\frac{y_i^0}{\sqrt{2}}\sqrt{\xi}\left(\frac{n+2m+1}{16\lambda}\right) c_\varphi^n s_\varphi^{2m+3}$	1
$\mathcal{P}_{4q}$	$\frac{1}{v^2}(\bar{q}_{iL} U P_\pm \mathbf{q}_{iR})(\bar{q}_{jL} U P_\pm \mathbf{q}_{jR}) + \text{h.c.}$	$(2-\delta_{ij}) y_i^0 y_j^0 \xi \frac{(n+2m+1)^2}{32\lambda} c_\varphi^{2n} s_\varphi^{4m+2}$	1
$\mathcal{P}_{4q'}$	$\frac{1}{v^2}(\bar{q}_{iL} U P_\pm \mathbf{q}_{iR})(\bar{\mathbf{q}}_{jR} P_\pm U^\dagger q_{jL}) + \text{h.c.}$	$(2-\delta_{ij}) y_i^0 y_j^0 \xi \frac{(n+2m+1)^2}{32\lambda} c_\varphi^{2n} s_\varphi^{4m+2}$	1
$\mathcal{P}_7$	$\frac{1}{v}\langle V_\mu V^\mu\rangle(\Box h)$	$\sqrt{\xi}\left[\frac{1}{128\lambda^2}\left(\alpha + 4\beta c_\varphi\right) s_\varphi^3\right]$	2
$\mathcal{P}_{\Delta H}$	$\frac{1}{v^3}(\partial_\mu h)^2(\Box h)$	$-\xi^{3/2}\left[\frac{1}{64 f^3\lambda^2}\left(\alpha + 4\beta c_\varphi\right) s_\varphi\right]$	2
$\mathcal{P}_{\Box H}$	$\frac{1}{v^2}\left(\Box h\right)^2$	$\mathcal{O}\left(\frac{1}{\lambda^3}\right)$	3

Table 2: Effective operators before electroweak symmetry breaking, including two and four derivative couplings, together with their coefficients up to their first corrections in the $1/\lambda$ expansion. The bosonic contributions from $SO(5)$ breaking contributions ($\alpha \neq 0$ and/or $\beta \neq 0$) are also shown. The right-hand column indicates the order in $1/\lambda$ at which a given couplings first appears. The Higgs field h is defined as the excitation of the field φ, see Eq. (12).