

# DETERMINING THE PHYSICAL IMPACT OF OPERATORS IN EFFECTIVE FIELD THEORIES

Luca Merlo

based on:

B. Gavela, E. Jenkins, A. Manohar & LM,  
Analysis of General Power Counting Rules in Effective Field Theory,  
arXiv: 1601.07551

Invisibles' Webinar, February 23rd, 2016



Cincuenta  
Aniversario

**UAM** Universidad Autónoma  
de Madrid



Instituto de  
Física  
Teórica  
UAM-CSIC

elusives-invisiblesPlus  
neutrinos, dark matter & dark energy physics

# DETERMINING THE PHYSICAL IMPACT OF OPERATORS IN EFFECTIVE FIELD THEORIES

Luca Merlo

Milan, 21st of August 2021

Advanced VBS training school

# Motivations: which rule?

# Motivations: which rule?

*Physica* 96A (1979) 327-340 © North-Holland Publishing Co.

## PHENOMENOLOGICAL LAGRANGIANS\*

STEVEN WEINBERG

# Motivations: which rule?

*Physica* 96A (1979) 327-340 © North-Holland Publishing Co.

Nuclear Physics B234 (1984) 189-212  
© North-Holland Publishing Company

## CHIRAL QUARKS AND THE NON-RELATIVISTIC QUARK MODEL\*

Aneesh MANOHAR and Howard GEORGI

# Motivations: which rule?

Physica 96A (1979) 327-340 © North-Holland Publishing Co.

Nuclear Physics B234 (1984) 189-212  
© North-Holland Publishing Company

Physics Letters B 726 (2013) 697-702



Contents lists available at ScienceDirect

Physics Letters B

[www.elsevier.com/locate/physletb](http://www.elsevier.com/locate/physletb)

\*

Naive dimensional analysis counting of gauge theory amplitudes and anomalous dimensions

Elizabeth E. Jenkins<sup>a</sup>, Aneesh V. Manohar<sup>a</sup>, Michael Trott<sup>b,\*</sup>

# Motivations: which rule?

Physica 96A (1979) 327-340 © North-Holland Publishing Co.

Nuclear Physics B234 (1984) 189-212  
© North-Holland Publishing Company

Physics Letters B 726 (2013) 697-702



ELSEVIER

Contents lists available at ScienceDirect

Physics Letters B

Physics Letters B 731 (2014) 80-86

\*

Naive di  
anomaly



Elizabeth J

Contents lists available at ScienceDirect

Physics Letters B

[www.elsevier.com/locate/physletb](http://www.elsevier.com/locate/physletb)

On the power counting in effective field theories

Gerhard Buchalla<sup>a,\*</sup>, Oscar Catà<sup>a,b,c</sup>, Claudio Krause<sup>a</sup>

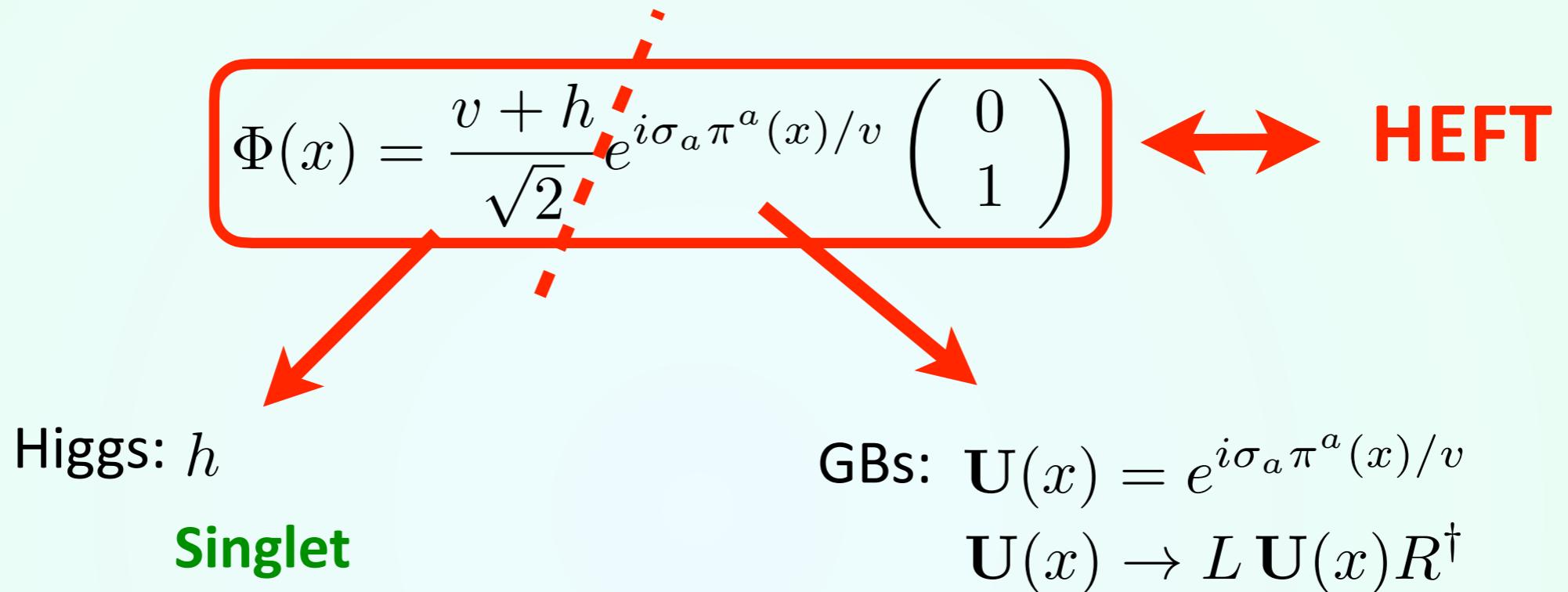
# Motivations: predictions in HEFT

SMEFT: constructed with

$$\Phi(x) = \frac{v + h}{\sqrt{2}} e^{i\sigma_a \pi^a(x)/v} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

# Motivations: predictions in HEFT

SMEFT: constructed with



**Independent!!**

# Motivations: predictions in HEFT

SMEFT: constructed with

$$\Phi(x) = \frac{v + h}{\sqrt{2}} e^{i\sigma_a \pi^a(x)/v} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{↔ HEFT}$$

Higgs:  $h$

**Singlet**

GBs:  $\mathbf{U}(x) = e^{i\sigma_a \pi^a(x)/v}$   
 $\mathbf{U}(x) \rightarrow L \mathbf{U}(x) R^\dagger$

**Independent!!**

- Being  $h$  a singlet: generic functions of  $h$

$$\mathcal{F}_i(h) = 1 + 2\alpha_i \frac{h}{v} + \beta_i \frac{h^2}{v^2} + \dots$$

# Motivations: predictions in HEFT

SMEFT: constructed with

$$\Phi(x) = \frac{v + h}{\sqrt{2}} e^{i\sigma_a \pi^a(x)/v} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{↔ HEFT}$$

Higgs:  $h$

**Singlet**

GBs:  $\mathbf{U}(x) = e^{i\sigma_a \pi^a(x)/v}$   
 $\mathbf{U}(x) \rightarrow L \mathbf{U}(x) R^\dagger$

**Independent!!**

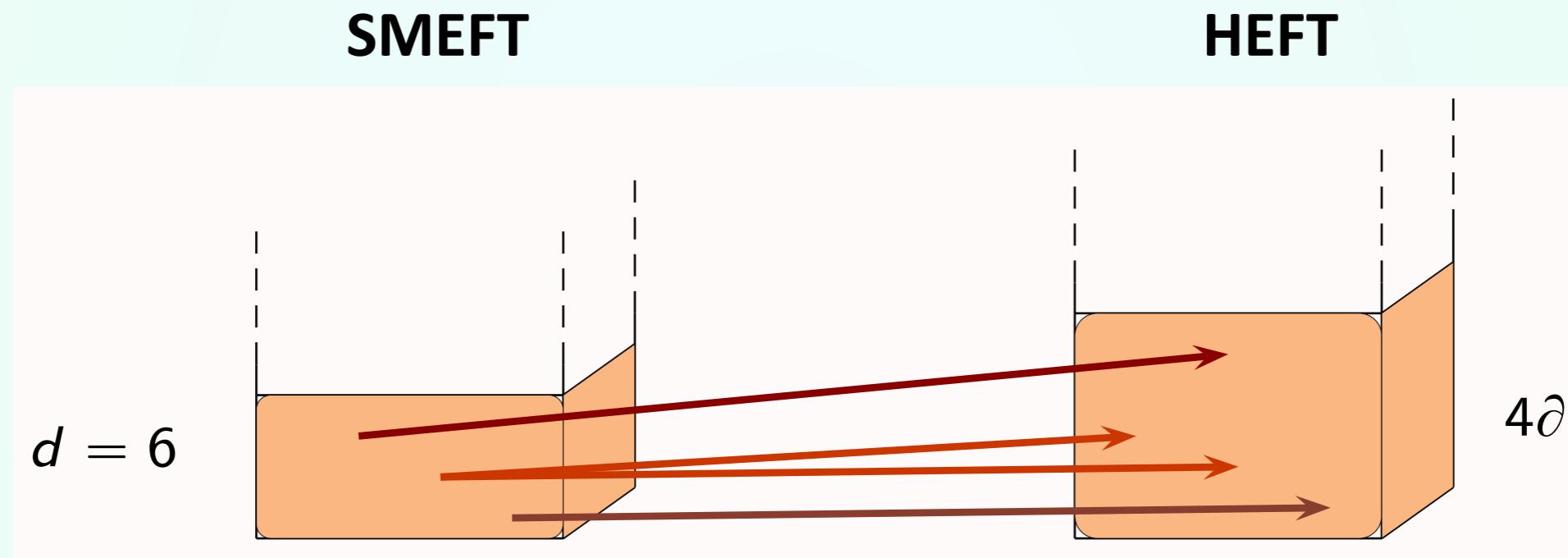
- Being  $h$  a singlet: generic functions of  $h$

$$\mathcal{F}_i(h) = 1 + 2\alpha_i \frac{h}{v} + \beta_i \frac{h^2}{v^2} + \dots$$

- Being  $\mathbf{U}(x)$  vs.  $h$  independent, many more operators can be constructed

# Decorrelations

- Investigate on the signals of decorrelations: due to the nature of the chiral expansion vs. the linear one, and due to  $\mathcal{F}_i(h) \neq \left(1 + \frac{h}{v}\right)^2$



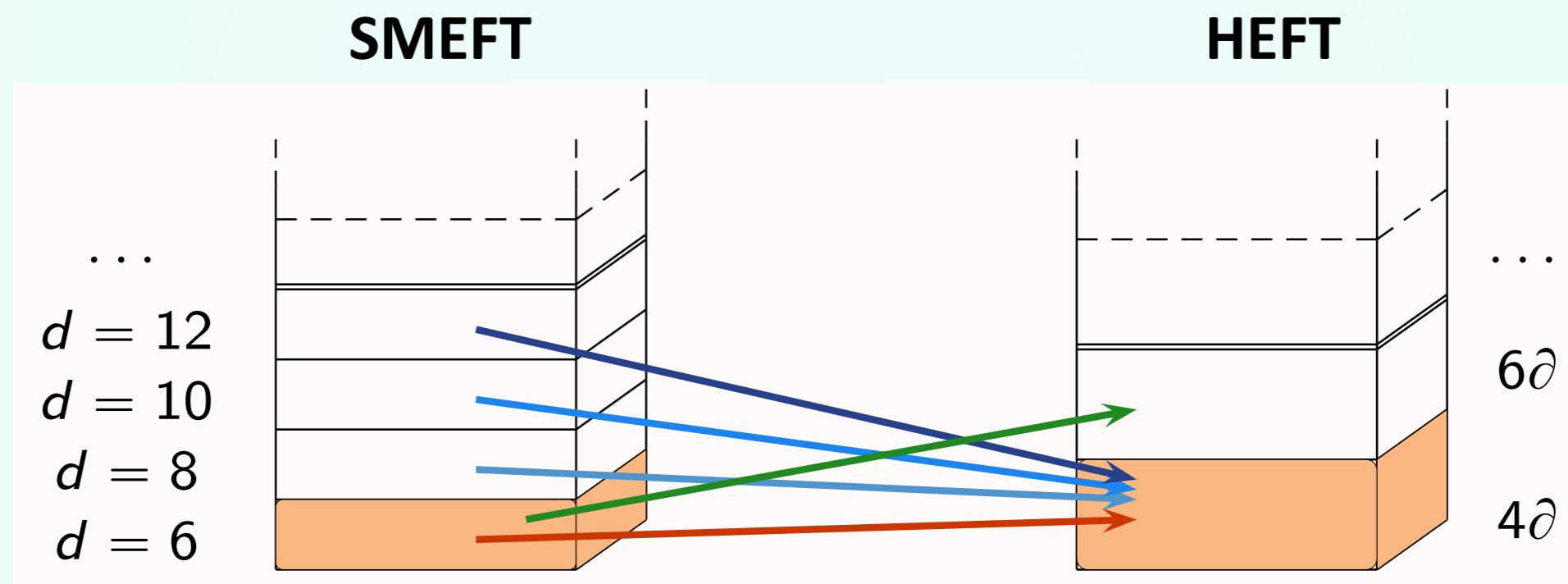
Brivio,Corbett,Eboli,Gavela,Gonzalez-Fraile, Gonzalez-Garcia,LM&Rigolin, JHEP 1403 (2014)

Brivio,Eboli,Gavela,Gonzalez-Garcia,LM&Rigolin, JHEP 1412 (2014)

Brivio,Gavela,LM,Mimasu,No,Rey&Sanz, arXiv:1511.0109

# New Signals

- Study the anomalous signal present in the chiral description, but absent in the linear one



# Content

- Why and how EFTs
- The master formulas for operator counting and cross sections
- SMEFT
- $\chi$ PT
- HEFT

based on:

B. Gavela, E. Jenkins, A. Manohar & LM,  
Analysis of General Power Counting Rules in Effective Field Theory,  
arXiv: 1601.07551

# Why to use an EFT?

**It is convenient**

# Why to use an EFT?

## It is convenient

The observables considered are measured in a determined energy range



- Only on relevant contributions at that energy
- Calculations are easier
- Benefits in the renormalisation procedure
- Accidental (approximate) symmetries

# Why to use an EFT?

## It is convenient

The observables considered are measured in a determined energy range



Only on relevant contributions at that energy  
Calculations are easier  
Benefits in the renormalisation procedure  
Accidental (approximate) symmetries

Top-down approach from the full theory to the EFT: i.e.

- EFT applied to B physics;
- QCD chiral perturbation theory for pions;
- etc...

# Why to use an EFT?

**It is necessary**

# Why to use an EFT?

**It is necessary**

The full theory is NOT known



Use of the known particles and known interactions to infer the symmetries and the nature of the full theory.

# Why to use an EFT?

## It is necessary

The full theory is NOT known



Use of the known particles and known interactions to infer the symmetries and the nature of the full theory.

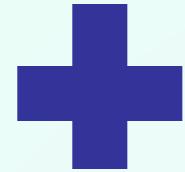
Bottom-up approach from the EFT to the full theory  
(with some luck): i.e.

- Fermi theory;
- Higgs effective theories;
- etc...

# How to construct an EFT?

# How to construct an EFT?

Spectrum



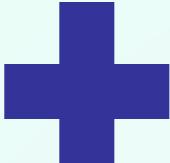
Symmetries

# How to construct an EFT?



Construct ALL possible operators with the fields of the given spectrum and invariant under the chosen symmetries.

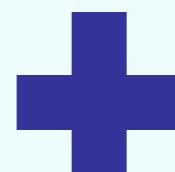
# How to construct an EFT?

**Spectrum** 



**Symmetries**

Construct ALL possible operators with the fields of the given spectrum and invariant under the chosen symmetries.



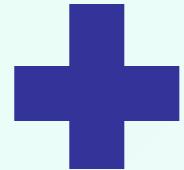
**Power counting**



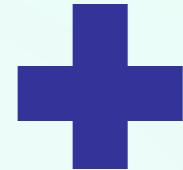
Reduces the number of operators at each order of the expansion(s), organises the hierarchy among the operators, sets the validity of the EFT.

# How to construct an EFT?

Spectrum



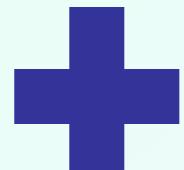
Symmetries



Power counting

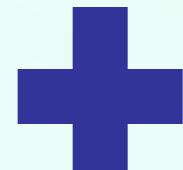
# How to construct an EFT?

**Spectrum**



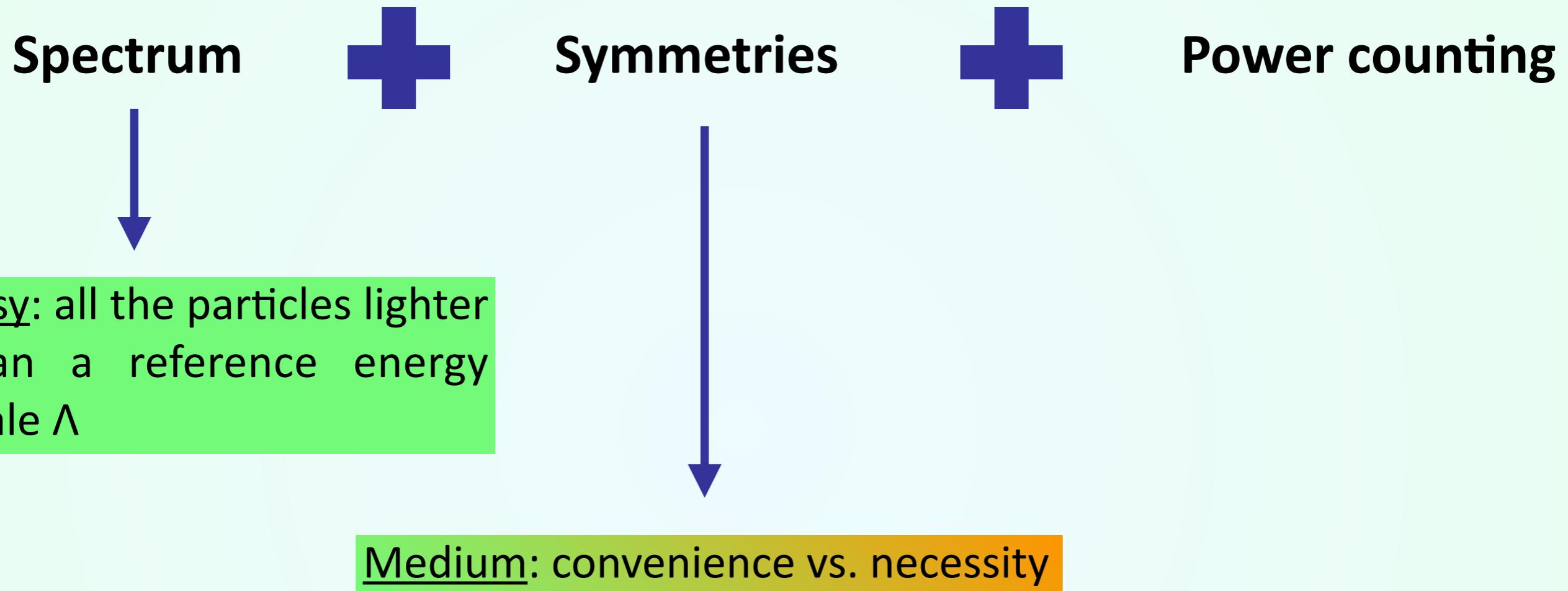
Easy: all the particles lighter than a reference energy scale  $\Lambda$

**Symmetries**

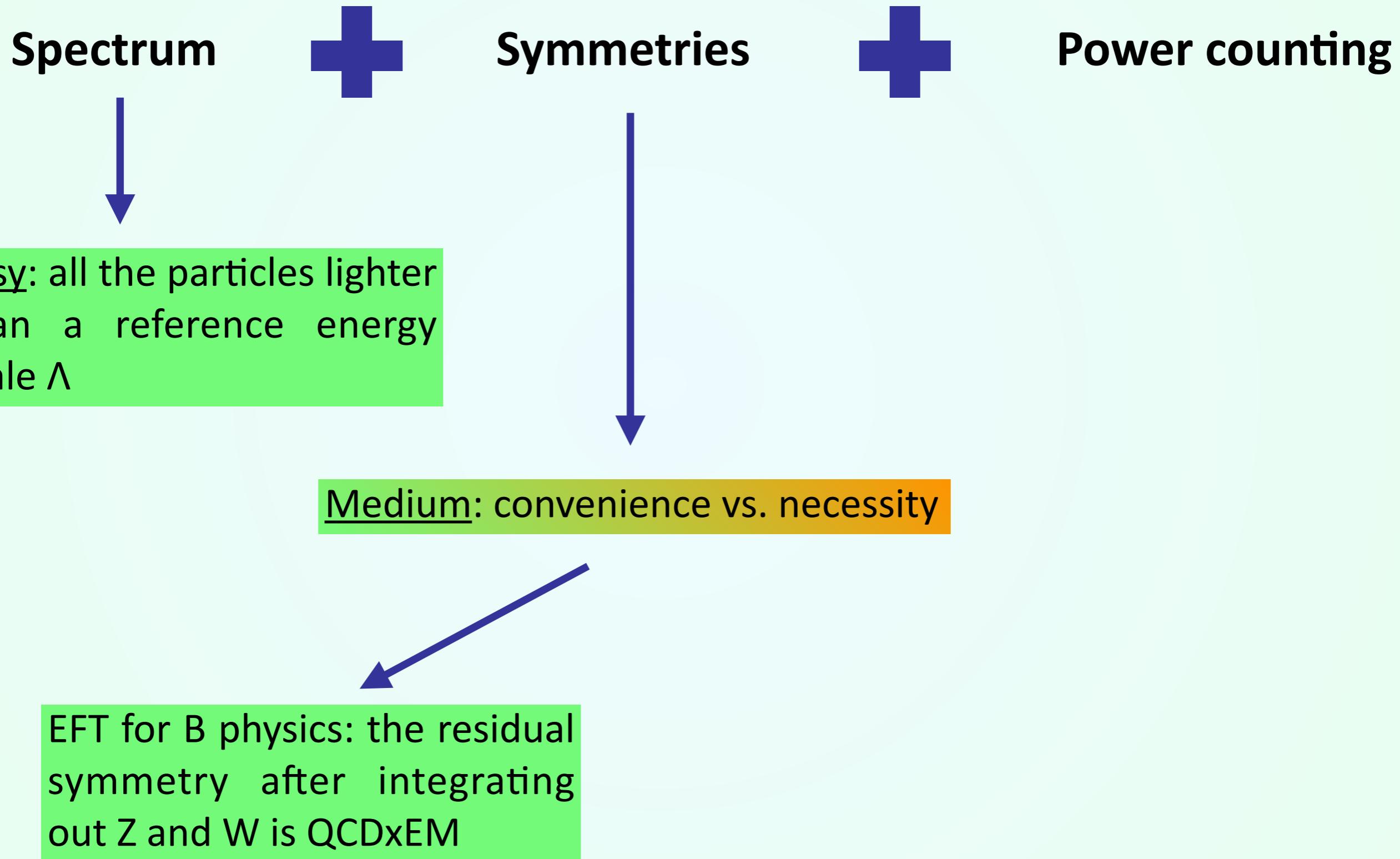


**Power counting**

# How to construct an EFT?

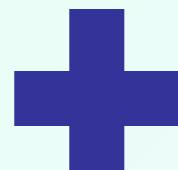


# How to construct an EFT?

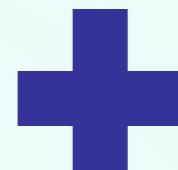


# How to construct an EFT?

Spectrum



Symmetries

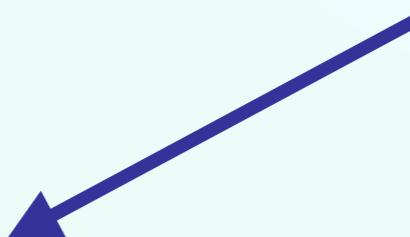


Power counting

Easy: all the particles lighter than a reference energy scale  $\Lambda$



Medium: convenience vs. necessity



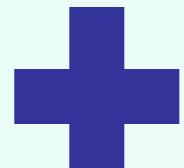
EFT for B physics: the residual symmetry after integrating out Z and W is QCDxEM



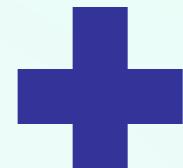
Fermi Theory: based on EM, with only V-A interactions  
DEPENDS ON THE DATA CONSIDERED

# How to construct an EFT?

Spectrum



Symmetries



Power counting

Easy: all the particles lighter than a reference energy scale  $\Lambda$

Hard: based on some rigorous method but with some freedom.

Medium: convenience vs. necessity

EFT for B physics: the residual symmetry after integrating out Z and W is QCDxEM

Fermi Theory: based on EM, with only V-A interactions  
DEPENDS ON THE DATA CONSIDERED

# Basics

Independently of the specific spectrum or symmetries, let's identify common aspects of the fundamental building blocks in EFTs.

# Basics

Independently of the specific spectrum or symmetries, let's identify common aspects of the fundamental building blocks in EFTs.

## Canonical mass dimensions in 4 space-time dimensions:

For the generalisation to d dimensions of all the formulae that follow see:

B. Gavela, E. Jenkins, A. Manohar & LM, arXiv: 1601.07551

# Basics

Independently of the specific spectrum or symmetries, let's identify common aspects of the fundamental building blocks in EFTs.

Canonical mass dimensions in 4 space-time dimensions:

$$[\partial] = 1$$

For the generalisation to d dimensions of all the formulae that follow see:  
B. Gavela, E. Jenkins, A. Manohar & LM, arXiv: 1601.07551

# Basics

Independently of the specific spectrum or symmetries, let's identify common aspects of the fundamental building blocks in EFTs.

Canonical mass dimensions in 4 space-time dimensions:

$$i\bar{\psi} D\psi \quad \text{should be } d = 4 \quad \xrightarrow{\hspace{1cm}} \quad [\partial] = 1$$
$$[\psi] = 3/2$$

For the generalisation to  $d$  dimensions of all the formulae that follow see:  
B. Gavela, E. Jenkins, A. Manohar & LM, arXiv: 1601.07551

# Basics

Independently of the specific spectrum or symmetries, let's identify common aspects of the fundamental building blocks in EFTs.

## Canonical mass dimensions in 4 space-time dimensions:

			$[\partial] = 1$
$i\bar{\psi}D\psi$	<b>should be</b>	$d = 4$	$\rightarrow [\psi] = 3/2$
$\partial_\mu \phi \partial^\mu \phi$	<b>should be</b>	$d = 4$	$\rightarrow [\phi] = 1$
$X_{\mu\nu} X^{\mu\nu}$	<b>should be</b>	$d = 4$	$\rightarrow [X_{\mu\nu}] = 2$
$D_\mu \equiv \partial_\mu + gA_\mu$			$[A] = 1$
$X_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]$			$[g] = 0$
$m\bar{\psi}\psi$	<b>should be</b>	$d = 4$	$\rightarrow [m] = 1$
$y\phi\bar{\psi}\psi$	<b>should be</b>	$d = 4$	$\rightarrow [y] = 0$
$\lambda\phi^4$	<b>should be</b>	$d = 4$	$\rightarrow [\lambda] = 0$

For the generalisation to d dimensions of all the formulae that follow see:  
B. Gavela, E. Jenkins, A. Manohar & LM, arXiv: 1601.07551

# Basics

Independently of the specific spectrum or symmetries, let's identify common aspects of the fundamental building blocks in EFTs.

## Canonical mass dimensions in 4 space-time dimensions:

<b>renormalisable terms</b>	$i\bar{\psi}D\psi$	should be	$d = 4$	$\rightarrow$	$[\partial] = 1$
	$\partial_\mu \phi \partial^\mu \phi$	should be	$d = 4$	$\rightarrow$	$[\phi] = 1$
	$X_{\mu\nu} X^{\mu\nu}$	should be	$d = 4$	$\rightarrow$	$[X_{\mu\nu}] = 2$
	$D_\mu \equiv \partial_\mu + gA_\mu$	$\left. \begin{array}{l} \text{should be } d = 4 \\ X_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu] \end{array} \right\}$	$\rightarrow$	$[A] = 1$	
	$X_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]$			$[g] = 0$	
	$m\bar{\psi}\psi$	should be	$d = 4$	$\rightarrow$	$[m] = 1$
	$y\phi\bar{\psi}\psi$	should be	$d = 4$	$\rightarrow$	$[y] = 0$
	$\lambda\phi^4$	should be	$d = 4$	$\rightarrow$	$[\lambda] = 0$

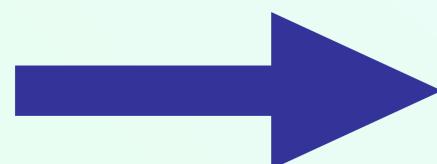
For the generalisation to  $d$  dimensions of all the formulae that follow see:  
 B. Gavela, E. Jenkins, A. Manohar & LM, arXiv: 1601.07551

# Basics

Independently of the specific spectrum or symmetries, let's identify common aspects of the fundamental building blocks in EFTs.

## Canonical mass dimensions in 4 space-time dimensions:

renormalisable terms	$i\bar{\psi}D\psi$	should be	$d = 4$	$\rightarrow$	$[\partial] = 1$
	$\partial_\mu \phi \partial^\mu \phi$	should be	$d = 4$	$\rightarrow$	$[\phi] = 1$
	$X_{\mu\nu} X^{\mu\nu}$	should be	$d = 4$	$\rightarrow$	$[X_{\mu\nu}] = 2$
	$D_\mu \equiv \partial_\mu + gA_\mu$	$\left. \begin{array}{l} \\ \\ \end{array} \right\}$		$\rightarrow$	$[A] = 1$
	$X_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]$			$\rightarrow$	$[g] = 0$
	$m\bar{\psi}\psi$	should be	$d = 4$	$\rightarrow$	$[m] = 1$
	$y\phi\bar{\psi}\psi$	should be	$d = 4$	$\rightarrow$	$[y] = 0$
	$\lambda\phi^4$	should be	$d = 4$	$\rightarrow$	$[\lambda] = 0$



$$\mathcal{L}_{\text{eff}} \sim \Lambda^4 \mathcal{L} \left( \frac{\partial}{\Lambda}, \frac{\psi}{\Lambda^{3/2}}, \frac{\phi}{\Lambda}, \frac{X_{\mu\nu}}{\Lambda^2} \right)$$

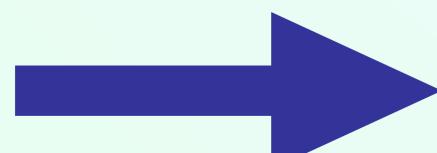
For the generalisation to d dimensions of all the formulae that follow see:  
 B. Gavela, E. Jenkins, A. Manohar & LM, arXiv: 1601.07551

# Basics

Independently of the specific spectrum or symmetries, let's identify common aspects of the fundamental building blocks in EFTs.

## Canonical mass dimensions in 4 space-time dimensions:

<b>renormalisable terms</b>	$i\bar{\psi}D\psi$	should be	$d = 4$	$\rightarrow$	$[\partial] = 1$
	$\partial_\mu \phi \partial^\mu \phi$	should be	$d = 4$	$\rightarrow$	$[\phi] = 1$
	$X_{\mu\nu} X^{\mu\nu}$	should be	$d = 4$	$\rightarrow$	$[X_{\mu\nu}] = 2$
	$D_\mu \equiv \partial_\mu + gA_\mu$	$\left. \begin{array}{l} \\ \\ \end{array} \right\}$	$d = 4$	$\rightarrow$	$[A] = 1$
	$X_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]$			$\rightarrow$	$[g] = 0$
	$m\bar{\psi}\psi$	should be	$d = 4$	$\rightarrow$	$[m] = 1$
	$y\phi\bar{\psi}\psi$	should be	$d = 4$	$\rightarrow$	$[y] = 0$
	$\lambda\phi^4$	should be	$d = 4$	$\rightarrow$	$[\lambda] = 0$



$$\mathcal{L}_{\text{eff}} \sim \Lambda^4 \mathcal{L} \left( \frac{\partial}{\Lambda}, \frac{\psi}{\Lambda^{3/2}}, \frac{\phi}{\Lambda}, \frac{X_{\mu\nu}}{\Lambda^2} \right)$$

not the end of the story!!

For the generalisation to d dimensions of all the formulae that follow see:  
 B. Gavela, E. Jenkins, A. Manohar & LM, arXiv: 1601.07551

■ A generic interaction vertex  $i$  constructed out of the building blocks has the form

$$\partial^{N_{p,i}} \phi^{N_{\phi,i}} A^{N_{A,i}} \psi^{N_{\psi,i}} \Lambda^{N_{\Lambda,i}} g^{N_{g,i}} y^{N_{y,i}} \lambda^{N_{\lambda,i}} (4\pi)^{N_{4\pi,i}}$$

$N_{a,i}$  refers to the number of such field/coupling appearing in the vertex

A generic interaction vertex  $i$  constructed out of the building blocks has the form

$$\partial^{N_{p,i}} \phi^{N_{\phi,i}} A^{N_{A,i}} \psi^{N_{\psi,i}} \Lambda^{N_{\Lambda,i}} g^{N_{g,i}} y^{N_{y,i}} \lambda^{N_{\lambda,i}} (4\pi)^{N_{4\pi,i}}$$

$N_{a,i}$  refers to the number of such field/coupling appearing in the vertex

The  $N_{a,i}$  are NOT independent, but they should give  $d = 4$ :

$$\longrightarrow 4 = N_{p,i} + N_{\phi,i} + N_{A,i} + \frac{3}{2}N_{\psi,i} + N_{\Lambda,i}$$

A generic interaction vertex  $i$  constructed out of the building blocks has the form

$$\partial^{N_{p,i}} \phi^{N_{\phi,i}} A^{N_{A,i}} \psi^{N_{\psi,i}} \Lambda^{N_{\Lambda,i}} g^{N_{g,i}} y^{N_{y,i}} \lambda^{N_{\lambda,i}} (4\pi)^{N_{4\pi,i}}$$

$N_{a,i}$  refers to the number of such field/coupling appearing in the vertex

The  $N_{a,i}$  are NOT independent, but they should give  $d = 4$ :

$$\longrightarrow 4 = N_{p,i} + N_{\phi,i} + N_{A,i} + \frac{3}{2}N_{\psi,i} + N_{\Lambda,i}$$

i.e.:

$$\partial_\mu \phi \partial^\mu \phi \longrightarrow \begin{cases} N_p = 2 \\ N_\phi = 2 \\ N_A = 0 = N_\psi \end{cases}$$

A generic interaction vertex  $i$  constructed out of the building blocks has the form

$$\partial^{N_{p,i}} \phi^{N_{\phi,i}} A^{N_{A,i}} \psi^{N_{\psi,i}} \Lambda^{N_{\Lambda,i}} g^{N_{g,i}} y^{N_{y,i}} \lambda^{N_{\lambda,i}} (4\pi)^{N_{4\pi,i}}$$

$N_{a,i}$  refers to the number of such field/coupling appearing in the vertex

The  $N_{a,i}$  are NOT independent, but they should give  $d = 4$ :

$$\rightarrow 4 = N_{p,i} + N_{\phi,i} + N_{A,i} + \frac{3}{2}N_{\psi,i} + N_{\Lambda,i}$$

i.e.:

$$\partial_\mu \phi \partial^\mu \phi \rightarrow \begin{cases} N_p = 2 \\ N_\phi = 2 \\ N_A = 0 = N_\psi \end{cases} \rightarrow N_\Lambda = 0 \rightarrow \partial_\mu \phi \partial^\mu \phi$$

A generic interaction vertex  $i$  constructed out of the building blocks has the form

$$\partial^{N_{p,i}} \phi^{N_{\phi,i}} A^{N_{A,i}} \psi^{N_{\psi,i}} \Lambda^{N_{\Lambda,i}} g^{N_{g,i}} y^{N_{y,i}} \lambda^{N_{\lambda,i}} (4\pi)^{N_{4\pi,i}}$$

$N_{a,i}$  refers to the number of such field/coupling appearing in the vertex

The  $N_{a,i}$  are NOT independent, but they should give  $d = 4$ :

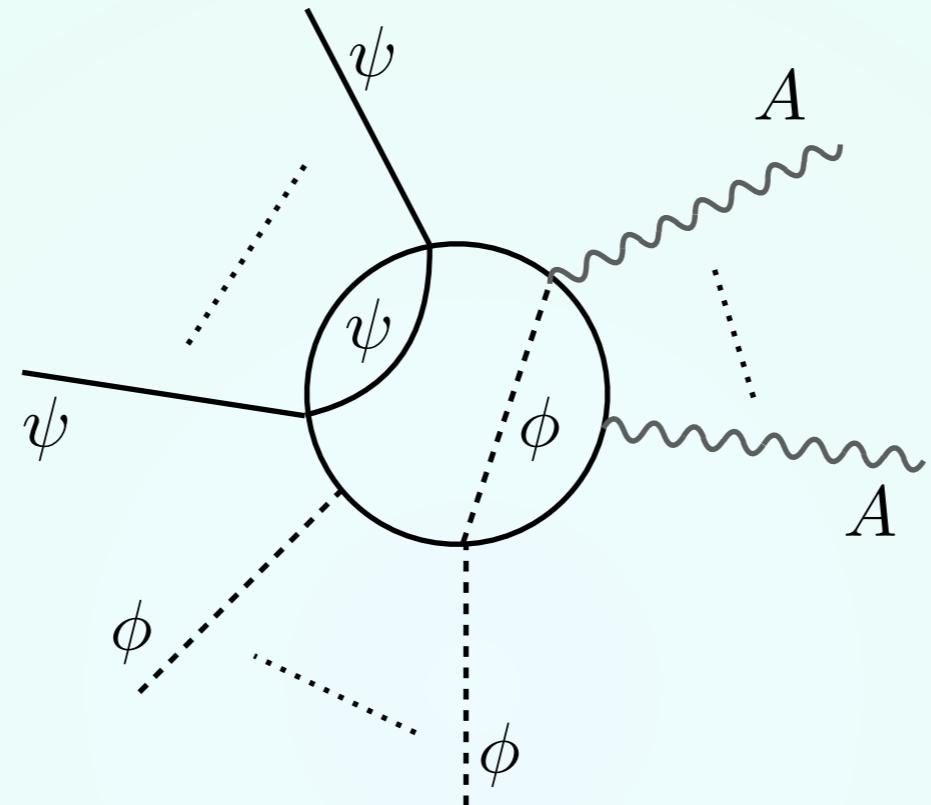
$$\rightarrow 4 = N_{p,i} + N_{\phi,i} + N_{A,i} + \frac{3}{2}N_{\psi,i} + N_{\Lambda,i}$$

i.e.:

$$\partial_\mu \phi \partial^\mu \phi \rightarrow \begin{cases} N_p = 2 \\ N_\phi = 2 \\ N_A = 0 = N_\psi \end{cases} \rightarrow N_\Lambda = 0 \rightarrow \partial_\mu \phi \partial^\mu \phi$$

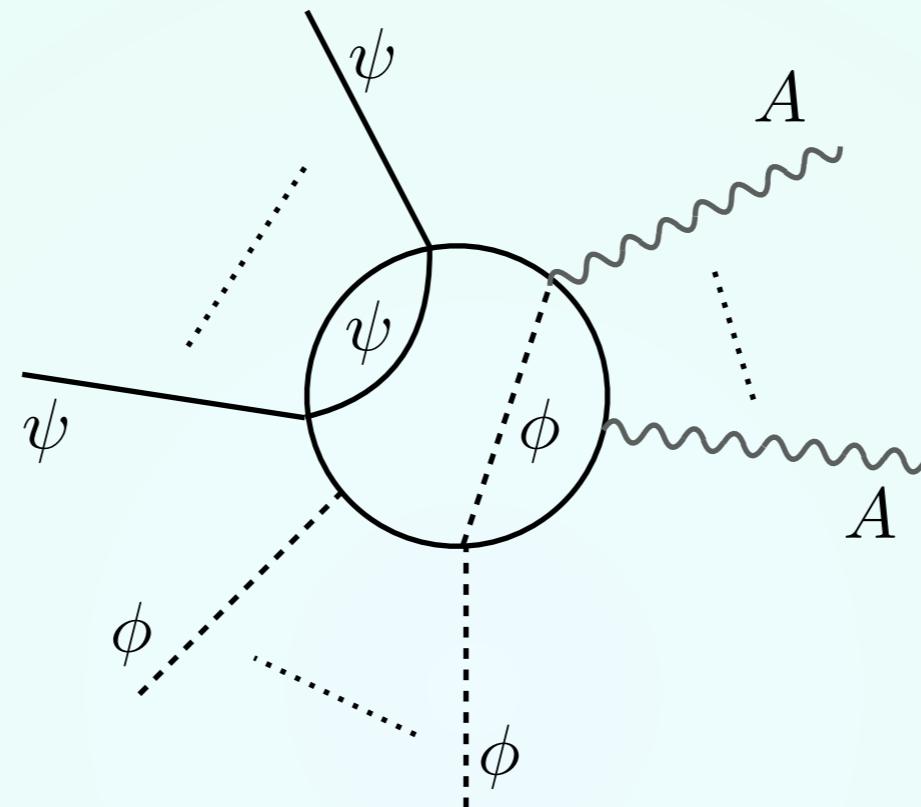
$$(\bar{\psi}\psi)^2 \rightarrow \begin{cases} N_\psi = 4 \\ N_p = 0 = N_\phi = N_A \end{cases} \rightarrow N_\Lambda = -2 \rightarrow \frac{(\bar{\psi}\psi)^2}{\Lambda^2}$$

- An arbitrary connected diagram with insertions of the generic vertex will have an amplitude of the same form of the vertex but the  $N_a$  undergo some conditions

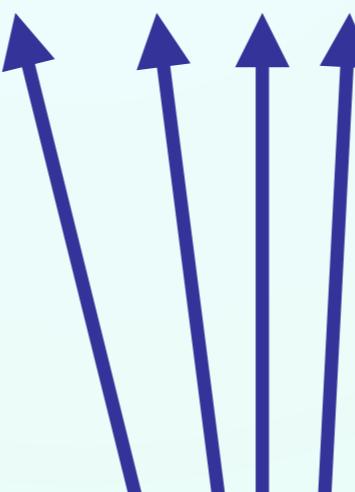


$$\partial^{N_p} \phi^{N_\phi} A^{N_A} \psi^{N_\psi} \Lambda^{N_\Lambda} g^{N_g} y^{N_y} \lambda^{N_\lambda} (4\pi)^{N_{4\pi}}$$

- An arbitrary connected diagram with insertions of the generic vertex will have an amplitude of the same form of the vertex but the  $N_a$  undergo some conditions

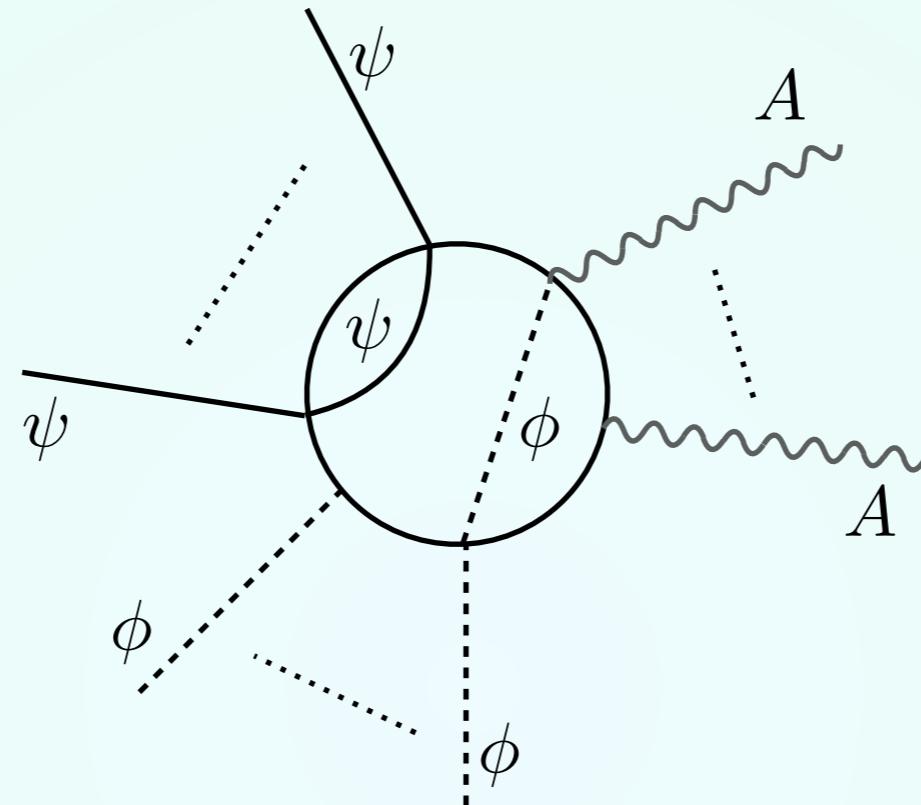


$$\partial^{N_p} \phi^{N_\phi} A^{N_A} \psi^{N_\psi} \Lambda^{N_\Lambda} g^{N_g} y^{N_y} \lambda^{N_\lambda} (4\pi)^{N_{4\pi}}$$

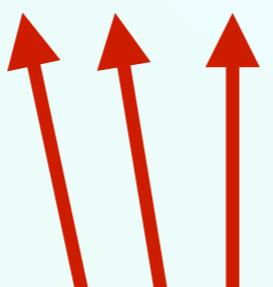
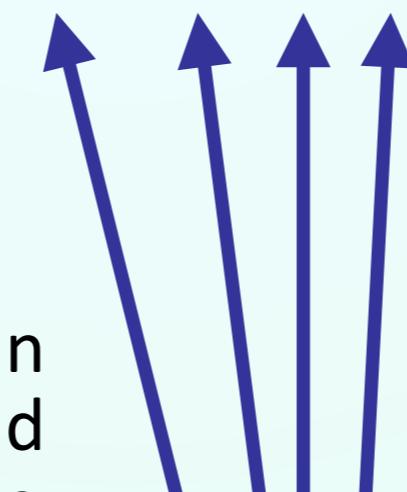


$$N_a = \sum_i N_{a,i}$$

- An arbitrary connected diagram with insertions of the generic vertex will have an amplitude of the same form of the vertex but the  $N_a$  undergo some conditions

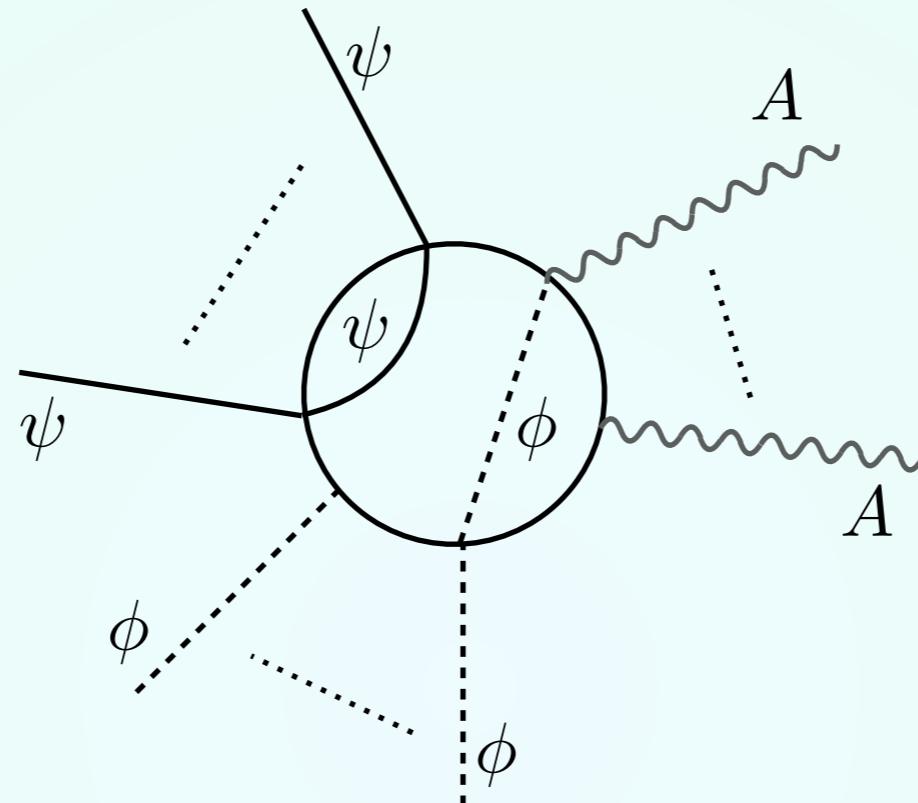


$$\partial^{N_p} \phi^{N_\phi} A^{N_A} \psi^{N_\psi} \Lambda^{N_\Lambda} g^{N_g} y^{N_y} \lambda^{N_\lambda} (4\pi)^{N_{4\pi}}$$



  
depend on  
external and  
internal legs

$$N_a = \sum_i N_{a,i}$$

- An arbitrary connected diagram with insertions of the generic vertex will have an amplitude of the same form of the vertex but the  $N_a$  undergo some conditions



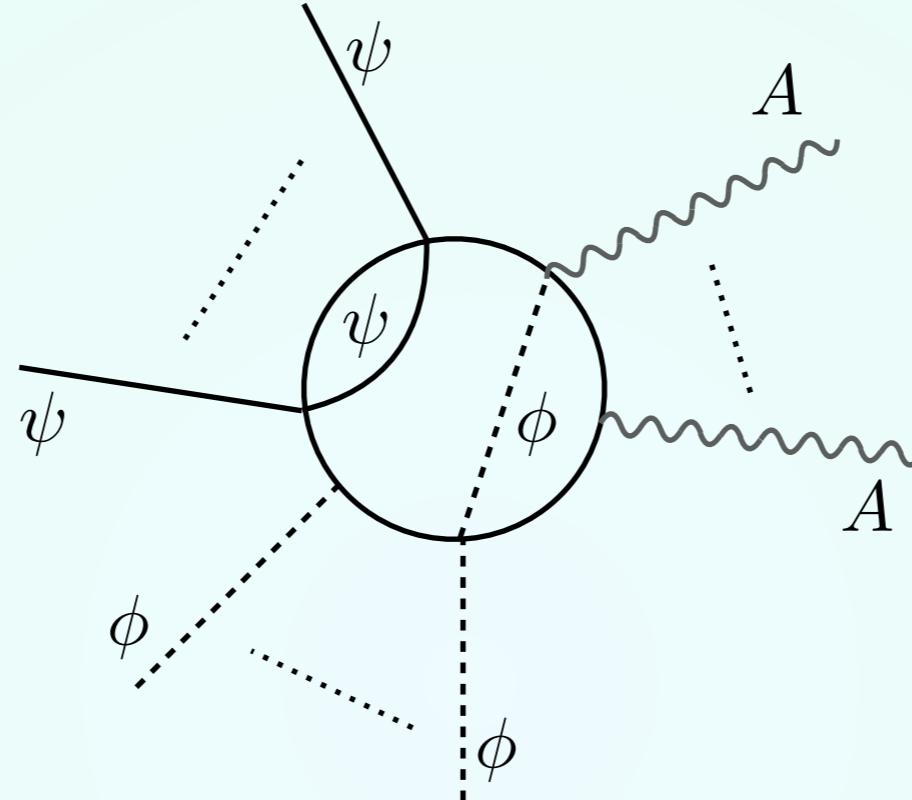
$$\partial^{N_p} \phi^{N_\phi} A^{N_A} \psi^{N_\psi} \Lambda^{N_\Lambda} g^{N_g} y^{N_y} \lambda^{N_\lambda} (4\pi)^{N_{4\pi}}$$

depends on external momenta and internal legs (propagators brings momenta)

depend on external and internal legs

$$N_a = \sum_i N_{a,i}$$

- An arbitrary connected diagram with insertions of the generic vertex will have an amplitude of the same form of the vertex but the  $N_a$  undergo some conditions



$$\partial^{N_p} \phi^{N_\phi} A^{N_A} \psi^{N_\psi} \Lambda^{N_\Lambda} g^{N_g} y^{N_y} \lambda^{N_\lambda} (4\pi)^{N_{4\pi}}$$

depends on external momenta and internal legs (propagators brings momenta)

depend on external and internal legs

depends on the “external”  $4\pi$  and on the number of loops

$$N_a = \sum_i N_{a,i}$$

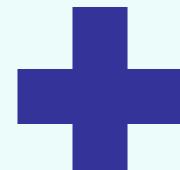
$$N_{4\pi} = \sum_i N_{4\pi,i} - 2L$$

When we have an operator, only the **external** legs, and not the internal, are relevant!

→ Eliminate the dependence from the internal legs

When we have an operator, only the **external** legs, and not the internal, are relevant!

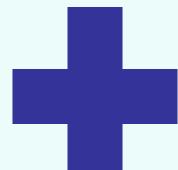
→ Eliminate the dependence from the internal legs



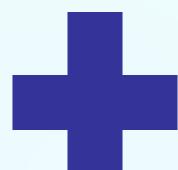
→ Imposing the theory identity  $V - I + L = 1$

When we have an operator, only the **external** legs, and not the internal, are relevant!

→ Eliminate the dependence from the internal legs



→ Imposing the theory identity  $V - I + L = 1$

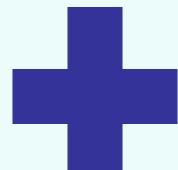


→ Imposing that the total dimension is 4

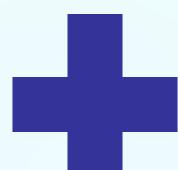
$$4 = N_{p,i} + N_{\phi,i} + N_{A,i} + \frac{3}{2}N_{\psi,i} + N_{\Lambda,i}$$

When we have an operator, only the **external** legs, and not the internal, are relevant!

→ Eliminate the dependence from the internal legs



→ Imposing the theory identity  $V - I + L = 1$



→ Imposing that the total dimension is 4

$$4 = N_{p,i} + N_{\phi,i} + N_{A,i} + \frac{3}{2}N_{\psi,i} + N_{\Lambda,i}$$



**only 6 relations are independent**

$$\partial^{N_p}\phi^{N_\phi}A^{N_A}\psi^{N_\psi}\Lambda^{N_\Lambda}g^{N_g}y^{N_y}\lambda^{N_\lambda}(4\pi)^{N_{4\pi}}$$

$$\partial^{N_p} \phi^{N_\phi} A^{N_A} \psi^{N_\psi} \Lambda^{N_\Lambda} g^{N_g} y^{N_y} \lambda^{N_\lambda} (4\pi)^{N_{4\pi}}$$

A diagram consisting of a rounded rectangular box with a dark blue border and a light blue interior. Inside the box, the mathematical expression  $N_a = \sum_i N_{a,i}$  is centered. Four dark blue arrows originate from the bottom edge of the box and point upwards towards the top edge.

$$N_a = \sum_i N_{a,i}$$

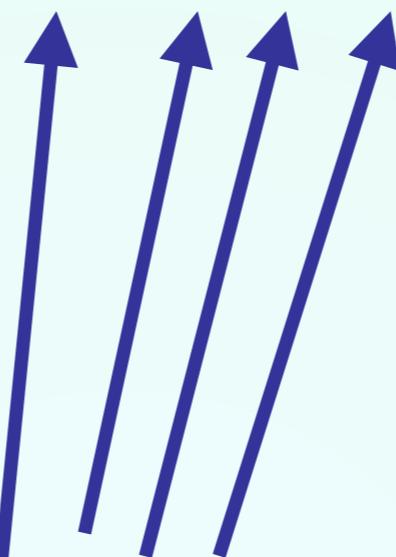
$$\partial^{N_p} \phi^{N_\phi} A^{N_A} \psi^{N_\psi} \Lambda^{N_\Lambda} g^{N_g} y^{N_y} \lambda^{N_\lambda} (4\pi)^{N_{4\pi}}$$

$$N_{\chi,i} \equiv N_{p,i} + \frac{1}{2} N_{\psi,i}$$

$$N_\chi \equiv N_p + \frac{1}{2} N_\psi$$

$$N_\chi - 2 = \sum_i (N_{\chi,i} - 2) + 2L$$

$$N_a = \sum_i N_{a,i}$$



$$\partial^{N_p} \phi^{N_\phi} A^{N_A} \psi^{N_\psi} \Lambda^{N_\Lambda} g^{N_g} y^{N_y} \lambda^{N_\lambda} (4\pi)^{N_{4\pi}}$$

$$N_{\chi,i} \equiv N_{p,i} + \frac{1}{2} N_{\psi,i}$$

$$N_\chi \equiv N_p + \frac{1}{2} N_\psi$$



$$N_\chi - 2 = \sum_i (N_{\chi,i} - 2) + 2L$$



$$N_a = \sum_i N_{a,i}$$



$$N_{F_i} \equiv N_{\phi_i} + N_{A_i} + N_{\psi_i}$$

$$N_F \equiv \sum_i (N_{F_i} - 2) - (2L - 2)$$



$$N_{4\pi} = N_F - N_g - N_y - 2N_\lambda - 2$$

$$\partial^{N_p} \phi^{N_\phi} A^{N_A} \psi^{N_\psi} \Lambda^{N_\Lambda} g^{N_g} y^{N_y} \lambda^{N_\lambda} (4\pi)^{N_{4\pi}}$$

$$N_{\chi,i} \equiv N_{p,i} + \frac{1}{2} N_{\psi,i}$$

$$N_\chi \equiv N_p + \frac{1}{2} N_\psi$$

$$N_\chi - 2 = \sum_i (N_{\chi,i} - 2) + 2L$$

$$N_a = \sum_i N_{a,i}$$

$$N_{F_i} \equiv N_{\phi_i} + N_{A_i} + N_{\psi_i}$$

$$N_F \equiv \sum_i (N_{F_i} - 2) - (2L - 2)$$

$$N_{4\pi} = N_F - N_g - N_y - 2N_\lambda - 2$$

equivalent to say that:

$$\{\phi, A, \psi\} \rightarrow 4\pi \{\phi, A, \psi\}$$

$$\{g, y, \sqrt{\lambda}\} \rightarrow \frac{1}{4\pi} \{g, y, \sqrt{\lambda}\}$$

$$\mathcal{L} \rightarrow \frac{1}{(4\pi)^2} \mathcal{L}$$

$$\partial^{N_p} \phi^{N_\phi} A^{N_A} \psi^{N_\psi} \Lambda^{N_\Lambda} g^{N_g} y^{N_y} \lambda^{N_\lambda} (4\pi)^{N_{4\pi}}$$

$$N_{\chi,i} \equiv N_{p,i} + \frac{1}{2} N_{\psi,i}$$

$$N_\chi \equiv N_p + \frac{1}{2} N_\psi$$

$$N_\chi - 2 = \sum_i (N_{\chi,i} - 2) + 2L$$

$$N_a = \sum_i N_{a,i}$$

$$N_{F_i} \equiv N_{\phi_i} + N_{A_i} + N_{\psi_i}$$

$$N_F \equiv \sum_i (N_{F_i} - 2) - (2L - 2)$$

$$N_{4\pi} = N_F - N_g - N_y - 2N_\lambda - 2$$

equivalent to say that:

$$\{\phi, A, \psi\} \rightarrow 4\pi \{\phi, A, \psi\}$$

$$\{g, y, \sqrt{\lambda}\} \rightarrow \frac{1}{4\pi} \{g, y, \sqrt{\lambda}\}$$

$$\mathcal{L} \rightarrow \frac{1}{(4\pi)^2} \mathcal{L}$$

## Master Formula

$$\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi \phi}{\Lambda} \right]^{N_\phi} \left[ \frac{4\pi A}{\Lambda} \right]^{N_A} \left[ \frac{4\pi \psi}{\Lambda^{3/2}} \right]^{N_\psi} \left[ \frac{g}{4\pi} \right]^{N_g} \left[ \frac{y}{4\pi} \right]^{N_y} \left[ \frac{\lambda}{16\pi^2} \right]^{N_\lambda}$$

**Each operator in the effective Lagrangian should be written according to this normalisation.**

Compare with:  
 Manohar&Georgi 1984  
 Luty 1997  
 Cohen,Kaplan&Nelson 1997

# Consequences

$$\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi \phi}{\Lambda} \right]^{N_\phi} \left[ \frac{4\pi A}{\Lambda} \right]^{N_A} \left[ \frac{4\pi \psi}{\Lambda^{3/2}} \right]^{N_\psi} \left[ \frac{g}{4\pi} \right]^{N_g} \left[ \frac{y}{4\pi} \right]^{N_y} \left[ \frac{\lambda}{16\pi^2} \right]^{N_\lambda}$$

■ The master formula can be generalised to masses and cubic couplings:

$$m_\phi^2 \phi^2$$
$$\downarrow$$
$$\left[ \frac{m_\phi^2}{\Lambda^2} \right]^{N_{m_\phi}}$$

$$m_\psi \bar{\psi} \psi$$
$$\downarrow$$
$$\left[ \frac{m_\psi}{\Lambda} \right]^{N_{m_\psi}}$$

$$k \phi^3$$
$$\downarrow$$
$$\left[ \frac{k}{4\pi \Lambda} \right]^{N_k}$$

# Consequences

$$\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi \phi}{\Lambda} \right]^{N_\phi} \left[ \frac{4\pi A}{\Lambda} \right]^{N_A} \left[ \frac{4\pi \psi}{\Lambda^{3/2}} \right]^{N_\psi} \left[ \frac{g}{4\pi} \right]^{N_g} \left[ \frac{y}{4\pi} \right]^{N_y} \left[ \frac{\lambda}{16\pi^2} \right]^{N_\lambda}$$

- The master formula can be generalised to masses and cubic couplings:

$$\begin{array}{ccc} m_\phi^2 \phi^2 & m_\psi \bar{\psi} \psi & k \phi^3 \\ \downarrow & \downarrow & \downarrow \\ \left[ \frac{m_\phi^2}{\Lambda^2} \right]^{N_{m_\phi}} & \left[ \frac{m_\psi}{\Lambda} \right]^{N_{m_\psi}} & \left[ \frac{k}{4\pi \Lambda} \right]^{N_k} \end{array}$$

- Covariant derivative homogeneous in power counting of  $\Lambda$  and  $4\pi$  (not of  $N_\chi$ )

$$\frac{D}{\Lambda} = \frac{\partial}{\Lambda} + i \left[ \frac{g}{4\pi} \right] \left[ \frac{4\pi A}{\Lambda} \right] = \frac{\partial + igA}{\Lambda}$$

# Consequences

$$\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi \phi}{\Lambda} \right]^{N_\phi} \left[ \frac{4\pi A}{\Lambda} \right]^{N_A} \left[ \frac{4\pi \psi}{\Lambda^{3/2}} \right]^{N_\psi} \left[ \frac{g}{4\pi} \right]^{N_g} \left[ \frac{y}{4\pi} \right]^{N_y} \left[ \frac{\lambda}{16\pi^2} \right]^{N_\lambda}$$

- The master formula can be generalised to masses and cubic couplings:

$$\begin{array}{ccc}
 m_\phi^2 \phi^2 & m_\psi \bar{\psi} \psi & k \phi^3 \\
 \downarrow & \downarrow & \downarrow \\
 \left[ \frac{m_\phi^2}{\Lambda^2} \right]^{N_{m_\phi}} & \left[ \frac{m_\psi}{\Lambda} \right]^{N_{m_\psi}} & \left[ \frac{k}{4\pi \Lambda} \right]^{N_k}
 \end{array}$$

- Covariant derivative homogeneous in power counting of  $\Lambda$  and  $4\pi$  (not of  $N_\chi$ )

$$\frac{D}{\Lambda} = \frac{\partial}{\Lambda} + i \left[ \frac{g}{4\pi} \right] \left[ \frac{4\pi A}{\Lambda} \right] = \frac{\partial + igA}{\Lambda}$$

- Equation of Motion is homogeneous in power counting of  $\Lambda$  and  $4\pi$  (not of  $N_\chi$ ):  
i.e. consider the SM Higgs doublet

$$\square H + m^2 H + \lambda(H^\dagger H)H + y\bar{\psi}\psi = 0$$

all the terms scale as  $\frac{4\pi}{\Lambda^3}$ .

# Consequences

$$\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi \phi}{\Lambda} \right]^{N_\phi} \left[ \frac{4\pi A}{\Lambda} \right]^{N_A} \left[ \frac{4\pi \psi}{\Lambda^{3/2}} \right]^{N_\psi} \left[ \frac{g}{4\pi} \right]^{N_g} \left[ \frac{y}{4\pi} \right]^{N_y} \left[ \frac{\lambda}{16\pi^2} \right]^{N_\lambda}$$

■ Gauge field strengths scale as

$$\frac{\partial_{\{\mu}}}{\Lambda} \frac{4\pi A_{\nu\}}{\Lambda} + \frac{g}{4\pi} \left[ \frac{4\pi A_\mu}{\Lambda}, \frac{4\pi A_\nu}{\Lambda} \right] = \frac{4\pi X_{\mu\nu}}{\Lambda^2}$$

# Consequences

$$\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi \phi}{\Lambda} \right]^{N_\phi} \left[ \frac{4\pi A}{\Lambda} \right]^{N_A} \left[ \frac{4\pi \psi}{\Lambda^{3/2}} \right]^{N_\psi} \left[ \frac{g}{4\pi} \right]^{N_g} \left[ \frac{y}{4\pi} \right]^{N_y} \left[ \frac{\lambda}{16\pi^2} \right]^{N_\lambda}$$

■ Gauge field strengths scale as

$$\frac{\partial_{\{\mu}}}{\Lambda} \frac{4\pi A_{\nu\}}{\Lambda} + \frac{g}{4\pi} \left[ \frac{4\pi A_\mu}{\Lambda}, \frac{4\pi A_\nu}{\Lambda} \right] = \frac{4\pi X_{\mu\nu}}{\Lambda^2}$$

■ Kinetic terms are all canonically normalised

# Consequences

$$\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi \phi}{\Lambda} \right]^{N_\phi} \left[ \frac{4\pi A}{\Lambda} \right]^{N_A} \left[ \frac{4\pi \psi}{\Lambda^{3/2}} \right]^{N_\psi} \left[ \frac{g}{4\pi} \right]^{N_g} \left[ \frac{y}{4\pi} \right]^{N_y} \left[ \frac{\lambda}{16\pi^2} \right]^{N_\lambda}$$

■ Gauge field strengths scale as

$$\frac{\partial_{\{\mu}}}{\Lambda} \frac{4\pi A_{\nu\}}{\Lambda} + \frac{g}{4\pi} \left[ \frac{4\pi A_\mu}{\Lambda}, \frac{4\pi A_\nu}{\Lambda} \right] = \frac{4\pi X_{\mu\nu}}{\Lambda^2}$$

■ Kinetic terms are all canonically normalised

$$\frac{\Lambda^4}{(4\pi)^2} \times i \frac{4\pi \bar{\psi}}{\Lambda^{3/2}} \frac{\not{\partial}}{\Lambda} \frac{4\pi \psi}{\Lambda^{3/2}} \quad \rightarrow \quad i \bar{\psi} \not{\partial} \psi$$

# Consequences

$$\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi \phi}{\Lambda} \right]^{N_\phi} \left[ \frac{4\pi A}{\Lambda} \right]^{N_A} \left[ \frac{4\pi \psi}{\Lambda^{3/2}} \right]^{N_\psi} \left[ \frac{g}{4\pi} \right]^{N_g} \left[ \frac{y}{4\pi} \right]^{N_y} \left[ \frac{\lambda}{16\pi^2} \right]^{N_\lambda}$$

■ Gauge field strengths scale as

$$\frac{\partial_{\{\mu}}}{\Lambda} \frac{4\pi A_{\nu\}}{\Lambda} + \frac{g}{4\pi} \left[ \frac{4\pi A_\mu}{\Lambda}, \frac{4\pi A_\nu}{\Lambda} \right] = \frac{4\pi X_{\mu\nu}}{\Lambda^2}$$

■ Kinetic terms are all canonically normalised

$$\begin{aligned} \frac{\Lambda^4}{(4\pi)^2} \times i \frac{4\pi \bar{\psi}}{\Lambda^{3/2}} \frac{\not{\partial}}{\Lambda} \frac{4\pi \psi}{\Lambda^{3/2}} &\rightarrow & i \bar{\psi} \not{\partial} \psi \\ \frac{\Lambda^4}{(4\pi)^2} \times \frac{\partial_\mu}{\Lambda} \frac{4\pi \phi}{\Lambda} \frac{\partial^\mu}{\Lambda} \frac{4\pi \phi}{\Lambda} &\rightarrow & \partial_\mu \phi \partial^\mu \phi \end{aligned}$$

# Consequences

$$\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi \phi}{\Lambda} \right]^{N_\phi} \left[ \frac{4\pi A}{\Lambda} \right]^{N_A} \left[ \frac{4\pi \psi}{\Lambda^{3/2}} \right]^{N_\psi} \left[ \frac{g}{4\pi} \right]^{N_g} \left[ \frac{y}{4\pi} \right]^{N_y} \left[ \frac{\lambda}{16\pi^2} \right]^{N_\lambda}$$

■ Gauge field strengths scale as

$$\frac{\partial_{\{\mu}, \frac{4\pi A_{\nu}\}}}{\Lambda} + \frac{g}{4\pi} \left[ \frac{4\pi A_\mu}{\Lambda}, \frac{4\pi A_\nu}{\Lambda} \right] = \frac{4\pi X_{\mu\nu}}{\Lambda^2}$$

■ Kinetic terms are all canonically normalised

$$\begin{aligned} \frac{\Lambda^4}{(4\pi)^2} \times i \frac{4\pi \bar{\psi}}{\Lambda^{3/2}} \frac{\not{\partial}}{\Lambda} \frac{4\pi \psi}{\Lambda^{3/2}} &\rightarrow & i \bar{\psi} \not{\partial} \psi \\ \frac{\Lambda^4}{(4\pi)^2} \times \frac{\partial_\mu}{\Lambda} \frac{4\pi \phi}{\Lambda} \frac{\partial^\mu}{\Lambda} \frac{4\pi \phi}{\Lambda} &\rightarrow & \partial_\mu \phi \partial^\mu \phi \\ \frac{\Lambda^4}{(4\pi)^2} \times \frac{4\pi X_{\mu\nu}}{\Lambda^2} \frac{4\pi X^{\mu\nu}}{\Lambda^2} &\rightarrow & X_{\mu\nu} X^{\mu\nu} \end{aligned}$$

# Comparison with the old NDA

**What is different wrt the well-known NDA master formula?**

# Comparison with the old NDA

What is different wrt the well-known NDA master formula?

$$\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi \phi}{\Lambda} \right]^{N_\phi} \left[ \frac{4\pi A}{\Lambda} \right]^{N_A} \left[ \frac{4\pi \psi}{\Lambda^{3/2}} \right]^{N_\psi} \left[ \frac{g}{4\pi} \right]^{N_g} \left[ \frac{y}{4\pi} \right]^{N_y} \left[ \frac{\lambda}{16\pi^2} \right]^{N_\lambda}$$

# Comparison with the old NDA

What is different wrt the well-known NDA master formula?

$$\Lambda = 4\pi f \quad \frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi \phi}{\Lambda} \right]^{N_\phi} \left[ \frac{4\pi A}{\Lambda} \right]^{N_A} \left[ \frac{4\pi \psi}{\Lambda^{3/2}} \right]^{N_\psi} \left[ \frac{g}{4\pi} \right]^{N_g} \left[ \frac{y}{4\pi} \right]^{N_y} \left[ \frac{\lambda}{16\pi^2} \right]^{N_\lambda}$$
$$f^2 \Lambda^2 \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{\phi}{f} \right]^{N_\phi} \left[ \frac{A}{f} \right]^{N_A} \left[ \frac{\psi}{f\sqrt{\Lambda}} \right]^{N_\psi} \left[ \frac{fg}{\Lambda} \right]^{N_g} \left[ \frac{fy}{\Lambda} \right]^{N_y} \left[ \frac{f^2 \lambda}{\Lambda^2} \right]^{N_\lambda}$$


# Comparison with the old NDA

What is different wrt the well-known NDA master formula?

$$\Lambda = 4\pi f \quad \frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi \phi}{\Lambda} \right]^{N_\phi} \left[ \frac{4\pi A}{\Lambda} \right]^{N_A} \left[ \frac{4\pi \psi}{\Lambda^{3/2}} \right]^{N_\psi} \left[ \frac{g}{4\pi} \right]^{N_g} \left[ \frac{y}{4\pi} \right]^{N_y} \left[ \frac{\lambda}{16\pi^2} \right]^{N_\lambda}$$
$$f^2 \Lambda^2 \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{\phi}{f} \right]^{N_\phi} \left[ \frac{A}{f} \right]^{N_A} \left[ \frac{\psi}{f\sqrt{\Lambda}} \right]^{N_\psi} \left[ \frac{fg}{\Lambda} \right]^{N_g} \left[ \frac{fy}{\Lambda} \right]^{N_y} \left[ \frac{f^2 \lambda}{\Lambda^2} \right]^{N_\lambda}$$
$$\left[ \frac{X_{\mu\nu}}{f\Lambda} \right]^{N_X} \longrightarrow f^2 \Lambda^2 \times \frac{X_{\mu\nu}}{f\Lambda} \frac{X^{\mu\nu}}{f\Lambda}$$

# Comparison with the old NDA

What is different wrt the well-known NDA master formula?

$$\Lambda = 4\pi f \quad \frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi \phi}{\Lambda} \right]^{N_\phi} \left[ \frac{4\pi A}{\Lambda} \right]^{N_A} \left[ \frac{4\pi \psi}{\Lambda^{3/2}} \right]^{N_\psi} \left[ \frac{g}{4\pi} \right]^{N_g} \left[ \frac{y}{4\pi} \right]^{N_y} \left[ \frac{\lambda}{16\pi^2} \right]^{N_\lambda}$$

Original NDA master formula:

$$f^2 \Lambda^2 \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{\phi}{f} \right]^{N_\phi} \left[ \frac{A}{f} \right]^{N_A} \left[ \frac{\psi}{f\sqrt{\Lambda}} \right]^{N_\psi} \left[ \frac{fg}{\Lambda} \right]^{N_g} \left[ \frac{fy}{\Lambda} \right]^{N_y} \left[ \frac{f^2 \lambda}{\Lambda^2} \right]^{N_\lambda}$$

$\left[ \frac{X_{\mu\nu}}{f\Lambda} \right]^{N_X} \xrightarrow{\quad} f^2 \Lambda^2 \times \frac{X_{\mu\nu}}{f\Lambda} \frac{X^{\mu\nu}}{f\Lambda}$

$$f^2 \Lambda^2 \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{\phi}{f} \right]^{N_\phi} \left[ \frac{\psi}{f\sqrt{\Lambda}} \right]^{N_\psi} \left[ \frac{gX_{\mu\nu}}{\Lambda^2} \right]^{N_{gX}} \left[ \frac{fy}{\Lambda} \right]^{N_y} \left[ \frac{f^2 \lambda}{\Lambda^2} \right]^{N_\lambda}$$

# Comparison with the old NDA

What is different wrt the well-known NDA master formula?

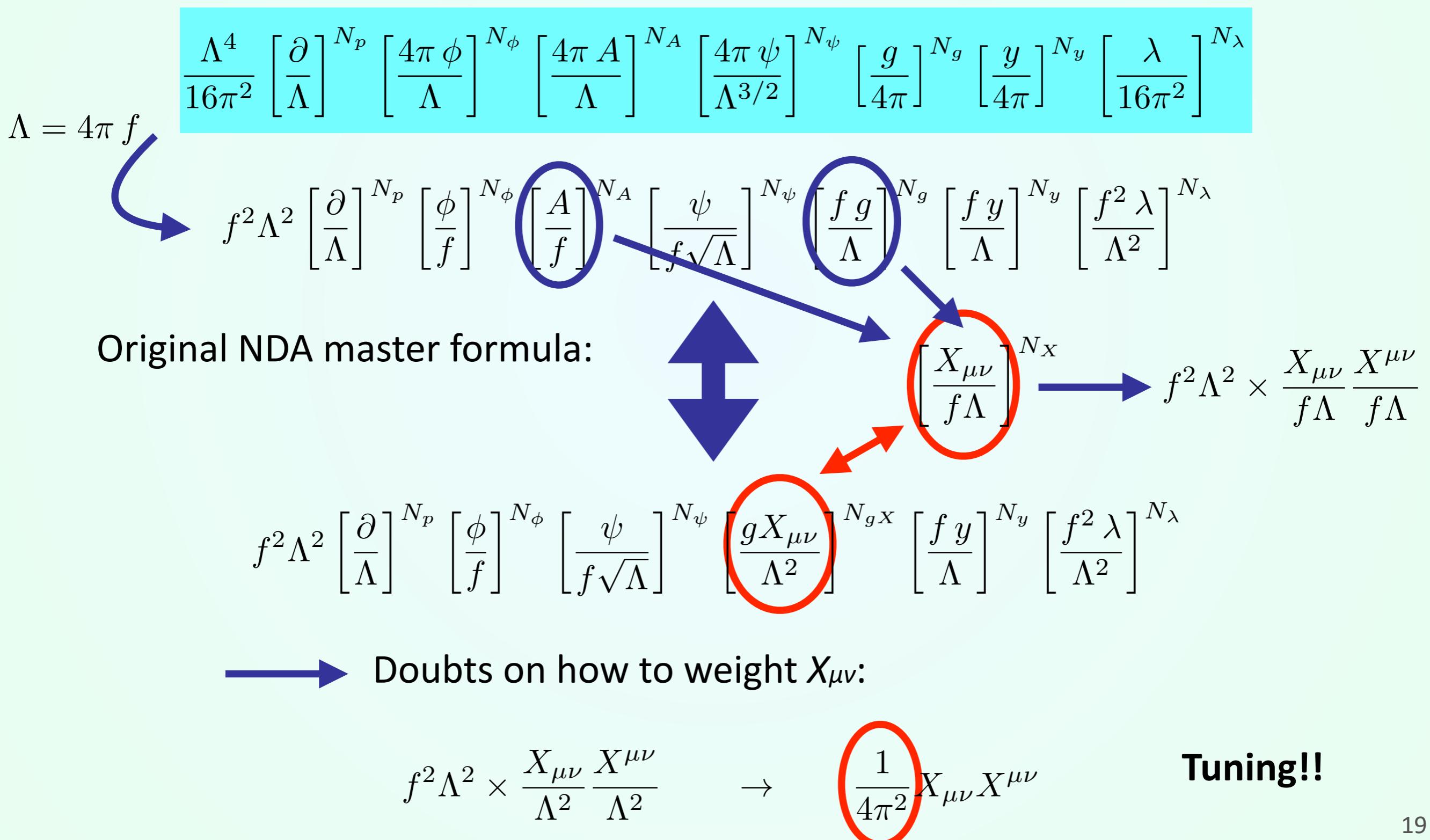
$$\Lambda = 4\pi f \quad \frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi \phi}{\Lambda} \right]^{N_\phi} \left[ \frac{4\pi A}{\Lambda} \right]^{N_A} \left[ \frac{4\pi \psi}{\Lambda^{3/2}} \right]^{N_\psi} \left[ \frac{g}{4\pi} \right]^{N_g} \left[ \frac{y}{4\pi} \right]^{N_y} \left[ \frac{\lambda}{16\pi^2} \right]^{N_\lambda}$$

Original NDA master formula:

$$f^2 \Lambda^2 \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{\phi}{f} \right]^{N_\phi} \left[ \frac{A}{f} \right]^{N_A} \left[ \frac{\psi}{f\sqrt{\Lambda}} \right]^{N_\psi} \left[ \frac{fg}{\Lambda} \right]^{N_g} \left[ \frac{fy}{\Lambda} \right]^{N_y} \left[ \frac{f^2 \lambda}{\Lambda^2} \right]^{N_\lambda}$$
$$f^2 \Lambda^2 \times \frac{X_{\mu\nu}}{f\Lambda} \frac{X^{\mu\nu}}{f\Lambda}$$
$$f^2 \Lambda^2 \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{\phi}{f} \right]^{N_\phi} \left[ \frac{\psi}{f\sqrt{\Lambda}} \right]^{N_\psi} \left[ \frac{gX_{\mu\nu}}{\Lambda^2} \right]^{N_{gX}} \left[ \frac{fy}{\Lambda} \right]^{N_y} \left[ \frac{f^2 \lambda}{\Lambda^2} \right]^{N_\lambda}$$

# Comparison with the old NDA

What is different wrt the well-known NDA master formula?



# Why to use the Master Formula

- It is a convenient tool to avoid computing loop amplitudes explicitly:

# Why to use the Master Formula

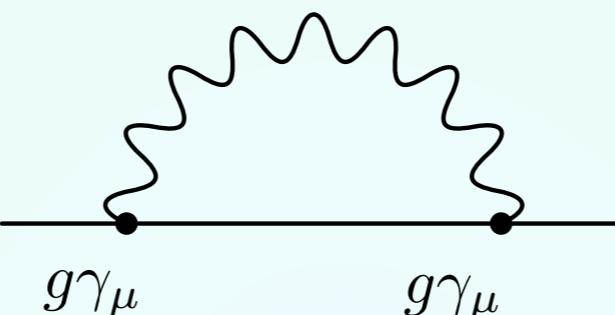
- It is a convenient tool to avoid computing loop amplitudes explicitly:

$$\mathcal{L}_{\text{LO}} \supset g \bar{\psi} A \psi$$

# Why to use the Master Formula

- It is a convenient tool to avoid computing loop amplitudes explicitly:

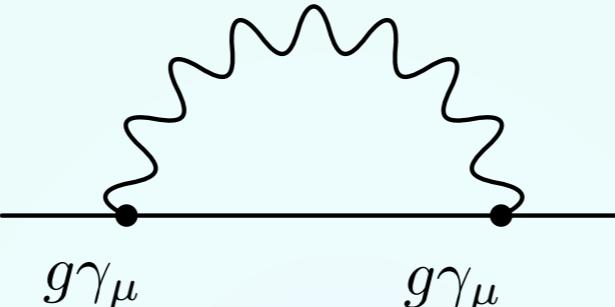
$$\mathcal{L}_{\text{LO}} \supset g\bar{\psi}A\psi \quad \longrightarrow \quad \begin{array}{c} \text{---} \\ | \qquad | \\ \text{---} \end{array} \quad \longrightarrow \quad \frac{g^2}{(4\pi)^2} i\bar{\psi}\partial\psi$$

  
explicit loop computation

# Why to use the Master Formula

- It is a convenient tool to avoid computing loop amplitudes explicitly:

$$\mathcal{L}_{\text{LO}} \supset g\bar{\psi}A\psi \quad \xrightarrow{\hspace{2cm}} \quad \begin{array}{c} \text{---} \\ | \quad | \\ \text{---} \end{array} \quad \xrightarrow{\hspace{2cm}} \quad \frac{g^2}{(4\pi)^2} i\bar{\psi}\partial\psi$$

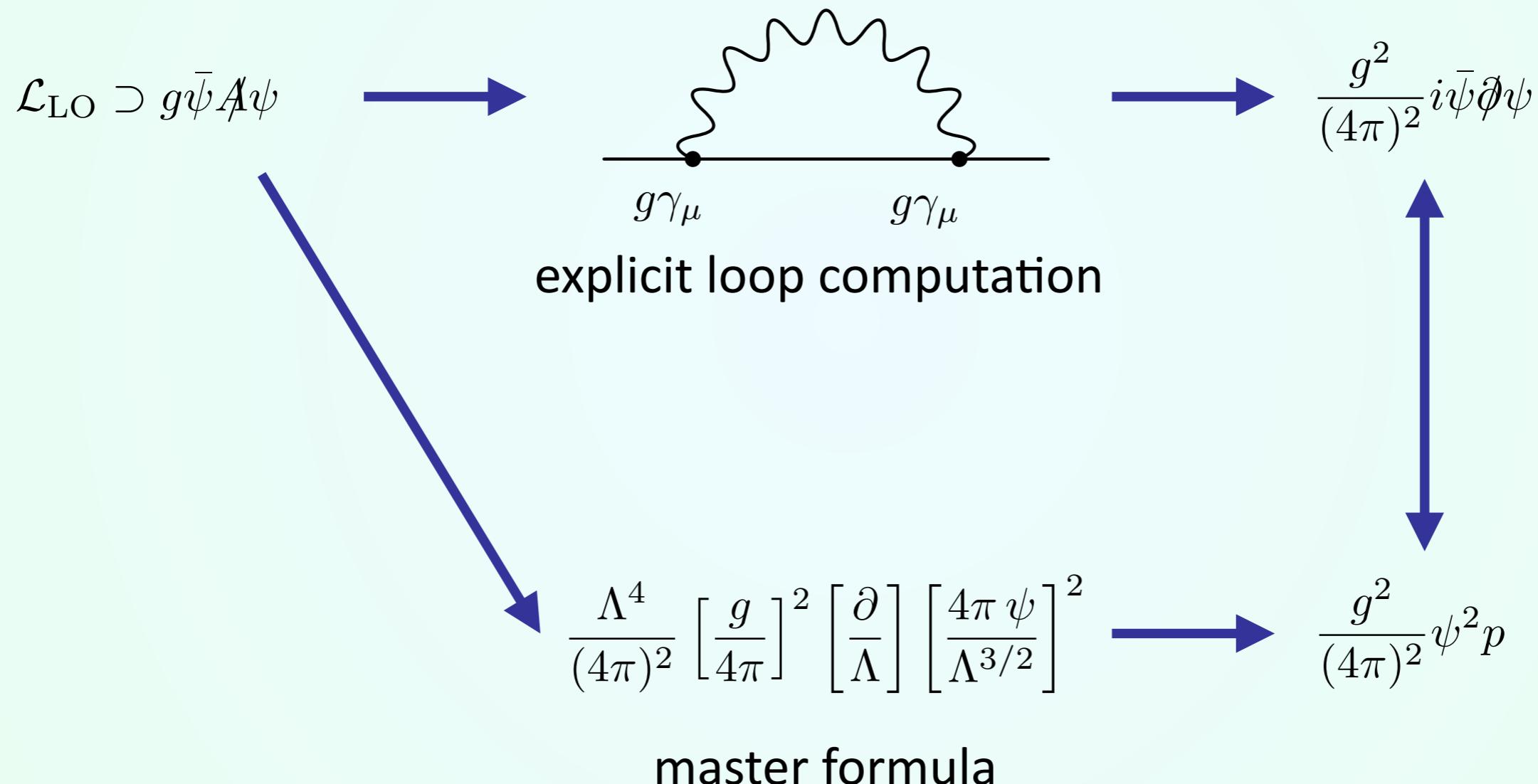
  
explicit loop computation

$$\frac{\Lambda^4}{(4\pi)^2} \left[ \frac{g}{4\pi} \right]^2 \left[ \frac{\partial}{\Lambda} \right] \left[ \frac{4\pi \psi}{\Lambda^{3/2}} \right]^2$$

master formula

# Why to use the Master Formula

- It is a convenient tool to avoid computing loop amplitudes explicitly:



# Why to use the Master Formula

- It gives the main dependence of physical observables:

# Why to use the Master Formula

- It gives the main dependence of physical observables:

$2 \rightarrow n$   
scattering  
 $(m = 0)$

$$\sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^{-N_\Lambda} \left[\frac{g}{4\pi}\right]^{2N_g} \left[\frac{y}{4\pi}\right]^{2N_y} \left[\frac{\lambda}{(4\pi)^2}\right]^{2N_\lambda}$$

**Master Formula for Cross Sections**

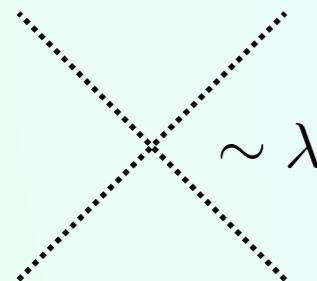
# Why to use the Master Formula

- It gives the main dependence of physical observables:

$2 \rightarrow n$   
scattering  
( $m = 0$ )

$$\sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^{-N_\Lambda} \left[\frac{g}{4\pi}\right]^{2N_g} \left[\frac{y}{4\pi}\right]^{2N_y} \left[\frac{\lambda}{(4\pi)^2}\right]^{2N_\lambda}$$

## Master Formula for Cross Sections



$$\begin{aligned}\sigma &\sim \frac{1}{E^2} \rho |\mathcal{M}|^2 \\ &\sim \frac{1}{E^2} \frac{1}{16\pi} \lambda^2 \\ &\sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{\lambda}{(4\pi)^2}\right)^2\end{aligned}$$



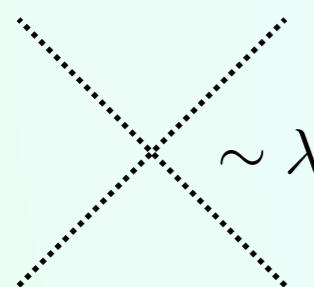
# Why to use the Master Formula

- It gives the main dependence of physical observables:

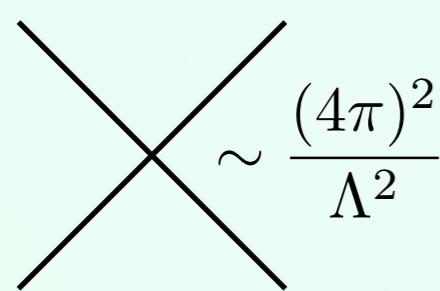
$2 \rightarrow n$   
scattering  
( $m = 0$ )

$$\sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^{-N_\Lambda} \left[\frac{g}{4\pi}\right]^{2N_g} \left[\frac{y}{4\pi}\right]^{2N_y} \left[\frac{\lambda}{(4\pi)^2}\right]^{2N_\lambda}$$

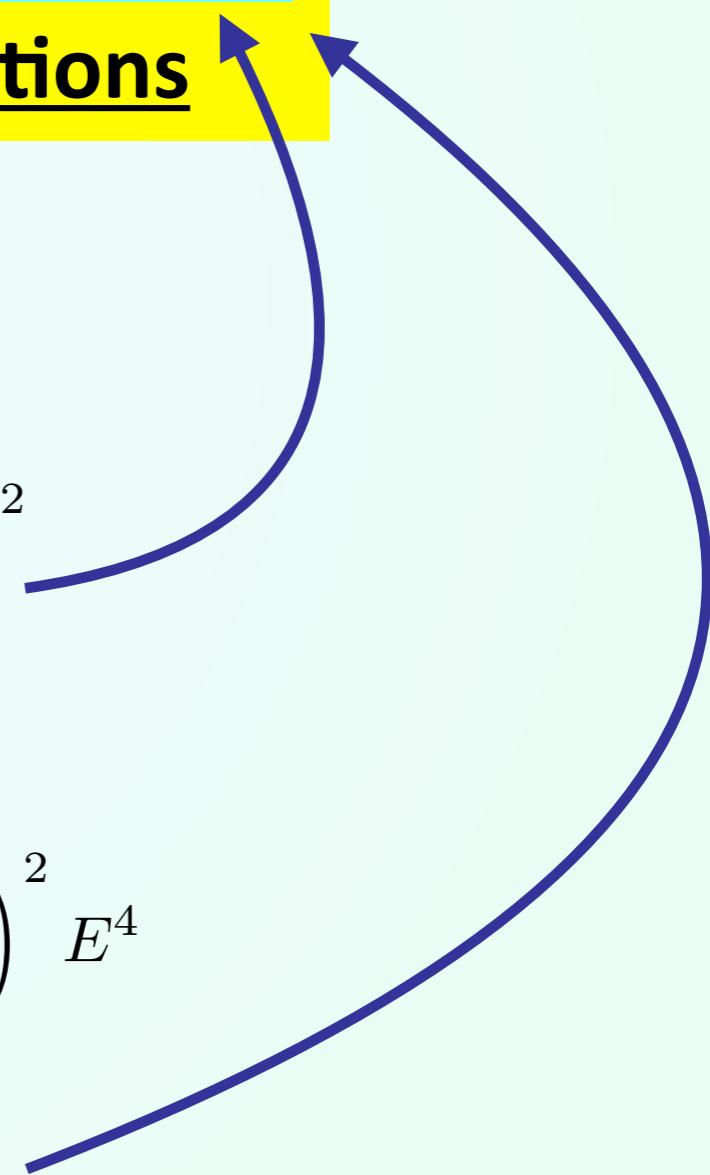
## Master Formula for Cross Sections



$$\begin{aligned}\sigma &\sim \frac{1}{E^2} \rho |\mathcal{M}|^2 \\ &\sim \frac{1}{E^2} \frac{1}{16\pi} \lambda^2 \\ &\sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{\lambda}{(4\pi)^2}\right)^2\end{aligned}$$



$$\begin{aligned}\sigma &\sim \frac{1}{E^2} \rho |\mathcal{M}|^2 \\ &\sim \frac{1}{E^2} \frac{1}{16\pi} \left(\frac{(4\pi)^2}{\Lambda^2}\right)^2 E^4 \\ &\sim \frac{\pi(4\pi)^4}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^4\end{aligned}$$



# The Physical Impact

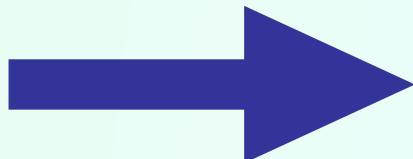
$$\sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^{-N_\Lambda} \left[\frac{g}{4\pi}\right]^{2N_g} \left[\frac{y}{4\pi}\right]^{2N_y} \left[\frac{\lambda}{(4\pi)^2}\right]^{2N_\lambda}$$

**There ONLY dependence is on  $\Lambda$**

# The Physical Impact

$$\sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^{-N_\Lambda} \left[\frac{g}{4\pi}\right]^{2N_g} \left[\frac{y}{4\pi}\right]^{2N_y} \left[\frac{\lambda}{(4\pi)^2}\right]^{2N_\lambda}$$

**There ONLY dependence is on  $\Lambda$**

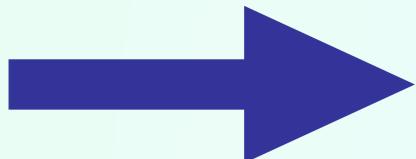


The hierarchy between the size of cross sections only depends on  $\Lambda$ , and not on the number of derivates!!

# The Physical Impact

$$\sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^{-N_\Lambda} \left[\frac{g}{4\pi}\right]^{2N_g} \left[\frac{y}{4\pi}\right]^{2N_y} \left[\frac{\lambda}{(4\pi)^2}\right]^{2N_\lambda}$$

**There ONLY dependence is on  $\Lambda$**



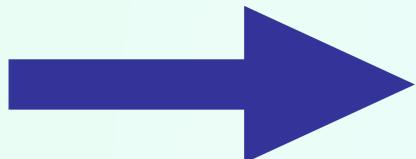
The hierarchy between the size of cross sections only depends on  $\Lambda$ , and not on the number of derivates!!

$$d = 6 \begin{cases} \frac{(4\pi)^4}{\Lambda^2} \phi^6 \\ \frac{(4\pi)^2}{\Lambda^2} (\phi \partial_\mu \phi)^2 \end{cases}$$

# The Physical Impact

$$\sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^{-N_\Lambda} \left[\frac{g}{4\pi}\right]^{2N_g} \left[\frac{y}{4\pi}\right]^{2N_y} \left[\frac{\lambda}{(4\pi)^2}\right]^{2N_\lambda}$$

**There ONLY dependence is on  $\Lambda$**



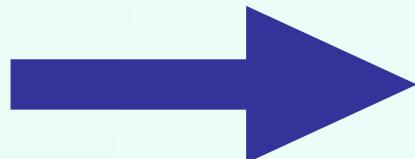
The hierarchy between the size of cross sections only depends on  $\Lambda$ , and not on the number of derivatives!!

$$d = 6 \begin{cases} \frac{(4\pi)^4}{\Lambda^2} \phi^6 & \longrightarrow 2\phi \rightarrow 4\phi \\ \frac{(4\pi)^2}{\Lambda^2} (\phi \partial_\mu \phi)^2 & \longrightarrow 2\phi \rightarrow 2\phi \end{cases}$$

# The Physical Impact

$$\sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^{-N_\Lambda} \left[\frac{g}{4\pi}\right]^{2N_g} \left[\frac{y}{4\pi}\right]^{2N_y} \left[\frac{\lambda}{(4\pi)^2}\right]^{2N_\lambda}$$

**There ONLY dependence is on  $\Lambda$**



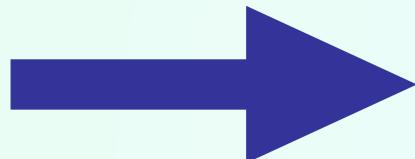
The hierarchy between the size of cross sections only depends on  $\Lambda$ , and not on the number of derivatives!!

$$d = 6 \begin{cases} \frac{(4\pi)^4}{\Lambda^2} \phi^6 \\ \frac{(4\pi)^2}{\Lambda^2} (\phi \partial_\mu \phi)^2 \end{cases} \rightarrow 2\phi \rightarrow 4\phi \rightarrow \sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^2$$

# The Physical Impact

$$\sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^{-N_\Lambda} \left[\frac{g}{4\pi}\right]^{2N_g} \left[\frac{y}{4\pi}\right]^{2N_y} \left[\frac{\lambda}{(4\pi)^2}\right]^{2N_\lambda}$$

**There ONLY dependence is on  $\Lambda$**



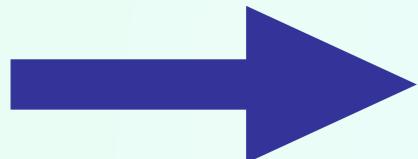
The hierarchy between the size of cross sections only depends on  $\Lambda$ , and not on the number of derivatives!!

$$d = 6 \begin{cases} \frac{(4\pi)^4}{\Lambda^2} \phi^6 \\ \frac{(4\pi)^2}{\Lambda^2} (\phi \partial_\mu \phi)^2 \end{cases} \rightarrow 2\phi \rightarrow 4\phi \rightarrow \sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^2$$
$$d = 8 \begin{cases} \frac{(4\pi)^6}{\Lambda^4} \phi^8 \\ \frac{(4\pi)^2}{\Lambda^4} (\phi \square \phi)^2 \end{cases} \rightarrow 2\phi \rightarrow 2\phi$$

# The Physical Impact

$$\sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^{-N_\Lambda} \left[\frac{g}{4\pi}\right]^{2N_g} \left[\frac{y}{4\pi}\right]^{2N_y} \left[\frac{\lambda}{(4\pi)^2}\right]^{2N_\lambda}$$

**There ONLY dependence is on  $\Lambda$**



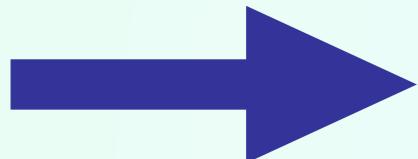
The hierarchy between the size of cross sections only depends on  $\Lambda$ , and not on the number of derivatives!!

$$d = 6 \begin{cases} \frac{(4\pi)^4}{\Lambda^2} \phi^6 \\ \frac{(4\pi)^2}{\Lambda^2} (\phi \partial_\mu \phi)^2 \end{cases} \rightarrow 2\phi \rightarrow 4\phi \rightarrow \sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^2$$
$$d = 8 \begin{cases} \frac{(4\pi)^6}{\Lambda^4} \phi^8 \\ \frac{(4\pi)^2}{\Lambda^4} (\phi \square \phi)^2 \end{cases} \rightarrow 2\phi \rightarrow 6\phi \rightarrow 2\phi$$

# The Physical Impact

$$\sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^{-N_\Lambda} \left[\frac{g}{4\pi}\right]^{2N_g} \left[\frac{y}{4\pi}\right]^{2N_y} \left[\frac{\lambda}{(4\pi)^2}\right]^{2N_\lambda}$$

**There ONLY dependence is on  $\Lambda$**



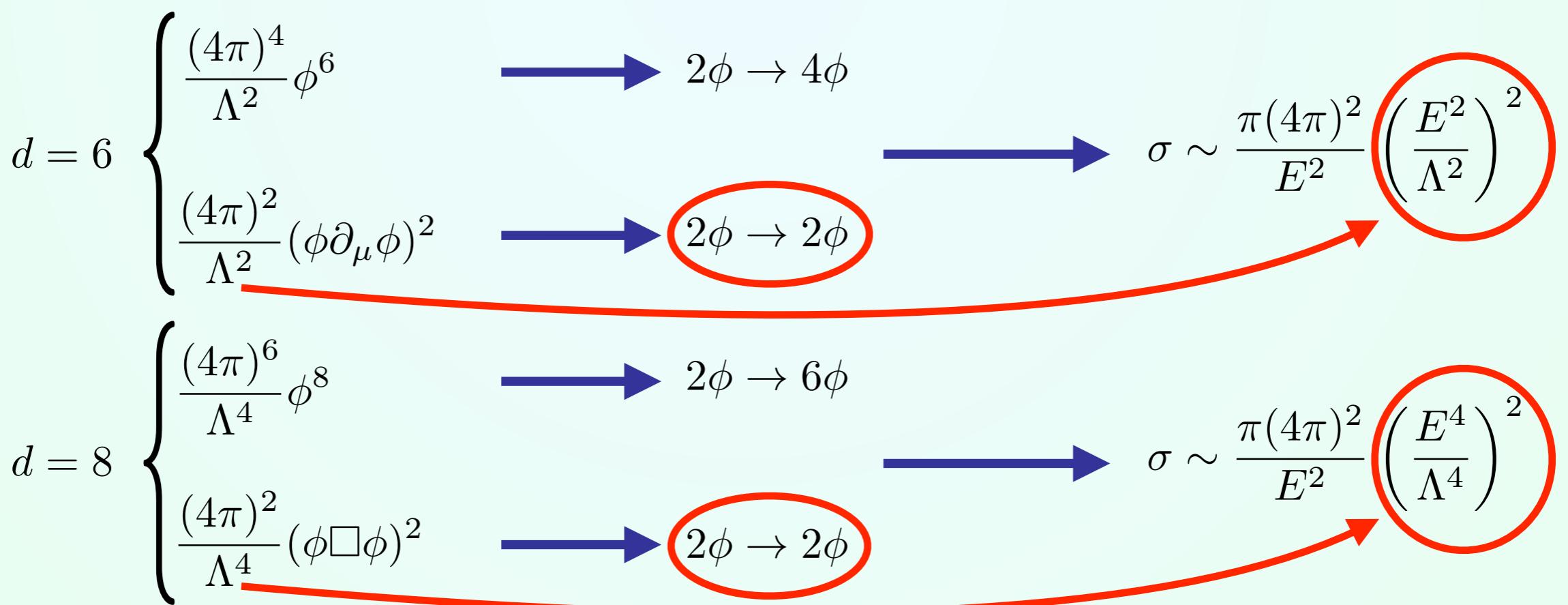
The hierarchy between the size of cross sections only depends on  $\Lambda$ , and not on the number of derivatives!!

$$d = 6 \begin{cases} \frac{(4\pi)^4}{\Lambda^2} \phi^6 \\ \frac{(4\pi)^2}{\Lambda^2} (\phi \partial_\mu \phi)^2 \end{cases} \rightarrow 2\phi \rightarrow 4\phi \rightarrow \sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^2$$
$$d = 8 \begin{cases} \frac{(4\pi)^6}{\Lambda^4} \phi^8 \\ \frac{(4\pi)^2}{\Lambda^4} (\phi \square \phi)^2 \end{cases} \rightarrow 2\phi \rightarrow 6\phi \rightarrow \sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^4}{\Lambda^4}\right)^2$$

# The Physical Impact

$$\sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^{-N_\Lambda} \left[\frac{g}{4\pi}\right]^{2N_g} \left[\frac{y}{4\pi}\right]^{2N_y} \left[\frac{\lambda}{(4\pi)^2}\right]^{2N_\lambda}$$

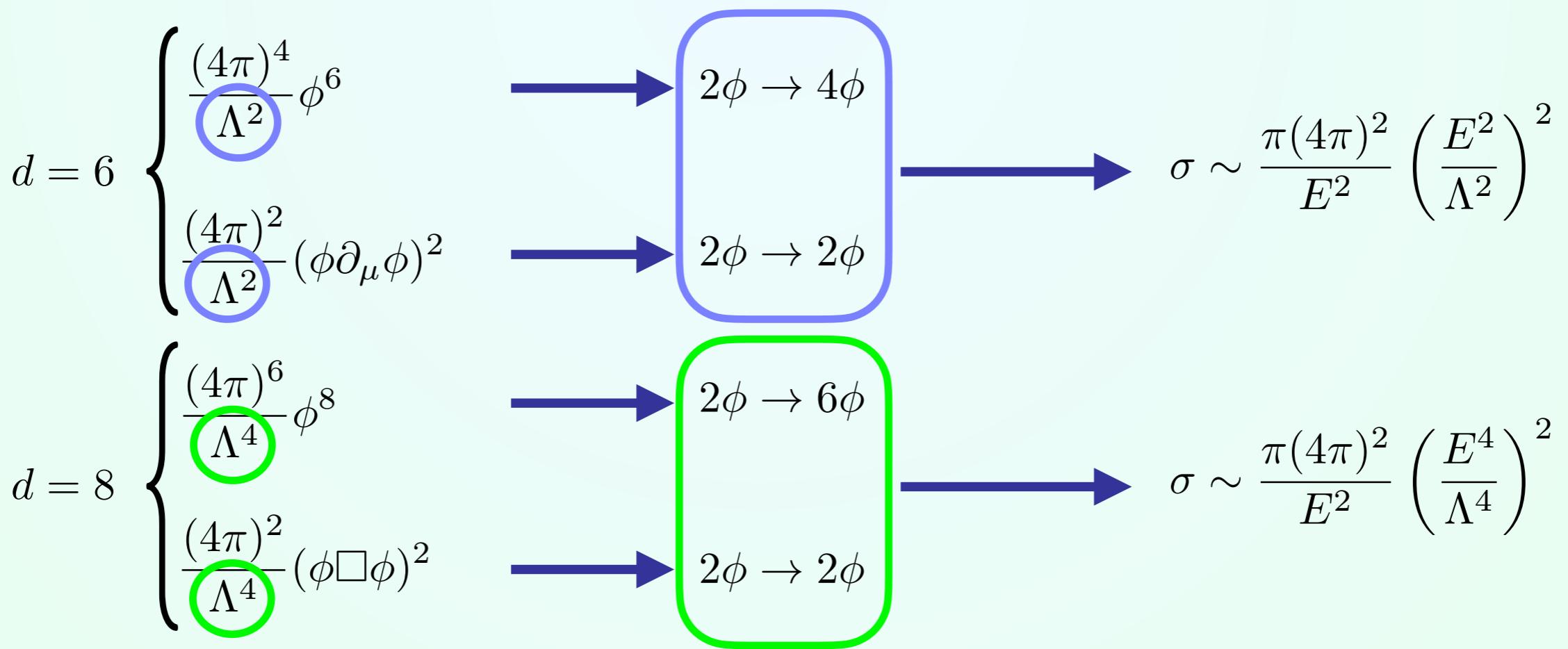
- Processes with *same* number of particles can have *different* cross sections: the difference is ruled by  $\Lambda$



# The Physical Impact

$$\sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^{-N_\Lambda} \left[\frac{g}{4\pi}\right]^{2N_g} \left[\frac{y}{4\pi}\right]^{2N_y} \left[\frac{\lambda}{(4\pi)^2}\right]^{2N_\lambda}$$

- Processes with *same* number of particles can have *different* cross sections: the difference is ruled by  $\Lambda$
- Processes with *different* number of particles/derivatives can have *same* cross sections: same number of  $\Lambda$



# SMEFT

The Standard Model Effective Field Theory (SMEFT): assuming B and L conservation

Operator	$d$	$N_\chi$	NDA Form
$H^2$	2	0	$\Lambda^2 H^2$
$\psi^2$	3	1	$\Lambda \psi^2$
$H^4$	4	0	$(4\pi)^2 H^4$
$\psi^2 H$	4	1	$(4\pi) \psi^2 H$
$\psi^2 D$	4	2	$\psi^2 D$
$H^2 D^2$	4	2	$H^2 D^2$
$X^2$	4	2	$X^2$
<hr/>			
$H^6$	6	0	$\frac{(4\pi)^4}{\Lambda^2} H^6$
$\psi^2 H^3$	6	1	$\frac{(4\pi)^3}{\Lambda^2} \psi^2 H^3 2$
$H^4 D^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} H^4 D^2$
$X^2 H^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} X^2 H^2$
$\psi^2 X H$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 X H$
$\psi^2 H^2 D$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 H^2 D$
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3$

[Buchmuller&Wyler 1984]

[Gradkoski,Iskrzynski,Misiak&Rosiek 2010]

# SMEFT

The Standard Model Effective Field Theory (SMEFT): assuming B and L conservation

Operator	$d$	$N_\chi$	NDA Form
$H^2$	2	0	$\Lambda^2 H^2$
$\psi^2$	3	1	$\Lambda \psi^2$
$H^4$	4	0	$(4\pi)^2 H^4$
$\psi^2 H$	4	1	$(4\pi) \psi^2 H$
$\psi^2 D$	4	2	$\psi^2 D$
$H^2 D^2$	4	2	$H^2 D^2$
$X^2$	4	2	$X^2$
<hr/>			
$H^6$	6	0	$\frac{(4\pi)^4}{\Lambda^2} H^6$
$\psi^2 H^3$	6	1	$\frac{(4\pi)^3}{\Lambda^2} \psi^2 H^3 2$
$H^4 D^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} H^4 D^2$
$X^2 H^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} X^2 H^2$
$\psi^2 X H$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 X H$
$\psi^2 H^2 D$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 H^2 D$
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3$

$$\begin{array}{ccc}
 & & i\bar{\psi} \not{D} \psi \\
 \longleftrightarrow & & \longleftrightarrow \\
 & & D_\mu H^\dagger D^\mu H \\
 \longleftrightarrow & & \text{Tr} (W_{\mu\nu} W^{\mu\nu})
 \end{array}$$

[Buchmuller&Wyler 1984]  
[Gradkoski,Iskrzynski,Misiak&Rosiek 2010]

# SMEFT

The Standard Model Effective Field Theory (SMEFT): assuming B and L conservation

Operator	$d$	$N_\chi$	NDA Form
$H^2$	2	0	$\Lambda^2 H^2$
$\psi^2$	3	1	$\Lambda \psi^2$
$H^4$	4	0	$(4\pi)^2 H^4$
$\psi^2 H$	4	1	$(4\pi) \psi^2 H$
$\psi^2 D$	4	2	$\psi^2 D$
$H^2 D^2$	4	2	$H^2 D^2$
$X^2$	4	2	$X^2$
$H^6$	6	0	$\frac{(4\pi)^4}{\Lambda^2} H^6$
$\psi^2 H^3$	6	1	$\frac{(4\pi)^3}{\Lambda^2} \psi^2 H^3 2$
$H^4 D^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} H^4 D^2$
$X^2 H^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} X^2 H^2$
$\psi^2 X H$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 X H$
$\psi^2 H^2 D$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 H^2 D$
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3$

[Buchmuller&Wyler 1984]  
 [Gradkoski,Iskrzynski,Misiak&Rosiek 2010]

$$\begin{array}{ccc}
 & & i\bar{\psi} \not{D} \psi \\
 \longleftrightarrow & & D_\mu H^\dagger D^\mu H \\
 \longleftrightarrow & & \text{Tr} (W_{\mu\nu} W^{\mu\nu}) \\
 & & \\
 \longleftrightarrow & & \frac{(4\pi)^2}{\Lambda^2} \text{Tr} (W_{\mu\nu} W^{\mu\nu}) (H^\dagger H) \\
 \longleftrightarrow & & \frac{(4\pi)^2}{\Lambda^2} \bar{\psi} \gamma_\mu \psi (H^\dagger D^\mu H)
 \end{array}$$

# SMEFT

The Standard Model Effective Field Theory (SMEFT): assuming B and L conservation

# SMEFT

The Standard Model Effective Field Theory (SMEFT): assuming B and L conservation

$$\mathcal{L} = \mathcal{L}^{d \leq 4} + \mathcal{L}^{d=6} + \mathcal{L}^{d=8} + \dots$$

Operator	$d$	$N_\chi$	NDA Form
$H^2$	2	0	$\Lambda^2 H^2$
$\psi^2$	3	1	$\Lambda \psi^2$
$H^4$	4	0	$(4\pi)^2 H^4$
$\psi^2 H$	4	1	$(4\pi) \psi^2 H$
$\psi^2 D$	4	2	$\psi^2 D$
$H^2 D^2$	4	2	$H^2 D^2$
$X^2$	4	2	$X^2$
<hr/>			
$H^6$	6	0	$\frac{(4\pi)^4}{\Lambda^2} H^6$
$\psi^2 H^3$	6	1	$\frac{(4\pi)^3}{\Lambda^2} \psi^2 H^3 2$
$H^4 D^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} H^4 D^2$
$X^2 H^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} X^2 H^2$
$\psi^2 X H$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 X H$
$\psi^2 H^2 D$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 H^2 D$
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3$

# SMEFT

The Standard Model Effective Field Theory (SMEFT): assuming B and L conservation

$$\mathcal{L} = \mathcal{L}^{d \leq 4} + \mathcal{L}^{d=6} + \mathcal{L}^{d=8} + \dots$$

The  $\Lambda$  expansion determines the physical impact of the operators and it coincides with the ordering of the operators due to renormalisation:

Operator	$d$	$N_\chi$	NDA Form
$H^2$	2	0	$\Lambda^2 H^2$
$\psi^2$	3	1	$\Lambda \psi^2$
$H^4$	4	0	$(4\pi)^2 H^4$
$\psi^2 H$	4	1	$(4\pi) \psi^2 H$
$\psi^2 D$	4	2	$\psi^2 D$
$H^2 D^2$	4	2	$H^2 D^2$
$X^2$	4	2	$X^2$
<hr/>			
$H^6$	6	0	$\frac{(4\pi)^4}{\Lambda^2} H^6$
$\psi^2 H^3$	6	1	$\frac{(4\pi)^3}{\Lambda^2} \psi^2 H^3$
$H^4 D^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} H^4 D^2$
$X^2 H^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} X^2 H^2$
$\psi^2 X H$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 X H$
$\psi^2 H^2 D$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 H^2 D$
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3$

# SMEFT

The Standard Model Effective Field Theory (SMEFT): assuming B and L conservation

$$\mathcal{L} = \mathcal{L}^{d \leq 4} + \mathcal{L}^{d=6} + \mathcal{L}^{d=8} + \dots$$

The  $\Lambda$  expansion determines the physical impact of the operators and it coincides with the ordering of the operators due to renormalisation:

LO  $\left\{ \begin{array}{l} \mathcal{L}^{d \leq 4} \\ \text{n-loop with } \mathcal{L}^{d \leq 4} \text{ vertices} \end{array} \right.$

Operator	$d$	$N_\chi$	NDA Form
$H^2$	2	0	$\Lambda^2 H^2$
$\psi^2$	3	1	$\Lambda \psi^2$
$H^4$	4	0	$(4\pi)^2 H^4$
$\psi^2 H$	4	1	$(4\pi) \psi^2 H$
$\psi^2 D$	4	2	$\psi^2 D$
$H^2 D^2$	4	2	$H^2 D^2$
$X^2$	4	2	$X^2$
<hr/>			
$H^6$	6	0	$\frac{(4\pi)^4}{\Lambda^2} H^6$
$\psi^2 H^3$	6	1	$\frac{(4\pi)^3}{\Lambda^2} \psi^2 H^3 2$
$H^4 D^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} H^4 D^2$
$X^2 H^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} X^2 H^2$
$\psi^2 X H$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 X H$
$\psi^2 H^2 D$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 H^2 D$
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3$

# SMEFT

The Standard Model Effective Field Theory (SMEFT): assuming B and L conservation

$$\mathcal{L} = \mathcal{L}^{d \leq 4} + \mathcal{L}^{d=6} + \mathcal{L}^{d=8} + \dots$$

The  $\Lambda$  expansion determines the physical impact of the operators and it coincides with the ordering of the operators due to renormalisation:

$$\begin{aligned} \text{LO } & \left\{ \begin{array}{l} \mathcal{L}^{d \leq 4} \\ \text{n-loop with } \mathcal{L}^{d \leq 4} \text{ vertices} \end{array} \right. \\ \text{NLO } & \left\{ \begin{array}{l} \mathcal{L}^{d=6} \\ \text{n-loop with 1 } \mathcal{L}^{d=6} \text{ vertex} \end{array} \right. \end{aligned}$$

Operator	$d$	$N_\chi$	NDA Form
$H^2$	2	0	$\Lambda^2 H^2$
$\psi^2$	3	1	$\Lambda \psi^2$
$H^4$	4	0	$(4\pi)^2 H^4$
$\psi^2 H$	4	1	$(4\pi) \psi^2 H$
$\psi^2 D$	4	2	$\psi^2 D$
$H^2 D^2$	4	2	$H^2 D^2$
$X^2$	4	2	$X^2$
<hr/>			
$H^6$	6	0	$\frac{(4\pi)^4}{\Lambda^2} H^6$
$\psi^2 H^3$	6	1	$\frac{(4\pi)^3}{\Lambda^2} \psi^2 H^3$
$H^4 D^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} H^4 D^2$
$X^2 H^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} X^2 H^2$
$\psi^2 X H$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 X H$
$\psi^2 H^2 D$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 H^2 D$
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3$

# SMEFT

The Standard Model Effective Field Theory (SMEFT): assuming B and L conservation

$$\mathcal{L} = \mathcal{L}^{d \leq 4} + \mathcal{L}^{d=6} + \mathcal{L}^{d=8} + \dots$$

The  $\Lambda$  expansion determines the physical impact of the operators and it coincides with the ordering of the operators due to renormalisation:

- LO     $\left\{ \begin{array}{l} \mathcal{L}^{d \leq 4} \\ \text{n-loop with } \mathcal{L}^{d \leq 4} \text{ vertices} \end{array} \right.$
- NLO     $\left\{ \begin{array}{l} \mathcal{L}^{d=6} \\ \text{n-loop with 1 } \mathcal{L}^{d=6} \text{ vertex} \end{array} \right.$
- NNLO     $\left\{ \begin{array}{l} \mathcal{L}^{d=8} \\ \text{n-loop with 1 } \mathcal{L}^{d=8} \text{ vertex} \\ \text{n-loop with 2 } \mathcal{L}^{d=6} \text{ vertices} \end{array} \right.$

Operator	$d$	$N_\chi$	NDA Form
$H^2$	2	0	$\Lambda^2 H^2$
$\psi^2$	3	1	$\Lambda \psi^2$
$H^4$	4	0	$(4\pi)^2 H^4$
$\psi^2 H$	4	1	$(4\pi) \psi^2 H$
$\psi^2 D$	4	2	$\psi^2 D$
$H^2 D^2$	4	2	$H^2 D^2$
$X^2$	4	2	$X^2$
<hr/>			
$H^6$	6	0	$\frac{(4\pi)^4}{\Lambda^2} H^6$
$\psi^2 H^3$	6	1	$\frac{(4\pi)^3}{\Lambda^2} \psi^2 H^3$
$H^4 D^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} H^4 D^2$
$X^2 H^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} X^2 H^2$
$\psi^2 X H$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 X H$
$\psi^2 H^2 D$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 H^2 D$
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3$

# SMEFT

The Standard Model Effective Field Theory (SMEFT): assuming B and L conservation

$$\mathcal{L} = \mathcal{L}^{d \leq 4} + \mathcal{L}^{d=6} + \mathcal{L}^{d=8} + \dots$$

The  $\Lambda$  expansion determines the physical impact of the operators and it coincides with the ordering of the operators due to renormalisation:

LO	$\left\{ \begin{array}{l} \mathcal{L}^{d \leq 4} \\ \text{n-loop with } \mathcal{L}^{d \leq 4} \text{ vertices} \end{array} \right.$
NLO	$\left\{ \begin{array}{l} \mathcal{L}^{d=6} \\ \text{n-loop with 1 } \mathcal{L}^{d=6} \text{ vertex} \end{array} \right.$
NNLO	$\left\{ \begin{array}{l} \mathcal{L}^{d=8} \\ \text{n-loop with 1 } \mathcal{L}^{d=8} \text{ vertex} \\ \text{n-loop with 2 } \mathcal{L}^{d=6} \text{ vertices} \end{array} \right.$
<u>independently of number of loops!!</u>	

Operator	$d$	$N_\chi$	NDA Form
$H^2$	2	0	$\Lambda^2 H^2$
$\psi^2$	3	1	$\Lambda \psi^2$
$H^4$	4	0	$(4\pi)^2 H^4$
$\psi^2 H$	4	1	$(4\pi) \psi^2 H$
$\psi^2 D$	4	2	$\psi^2 D$
$H^2 D^2$	4	2	$H^2 D^2$
$X^2$	4	2	$X^2$
<hr/>			
$H^6$	6	0	$\frac{(4\pi)^4}{\Lambda^2} H^6$
$\psi^2 H^3$	6	1	$\frac{(4\pi)^3}{\Lambda^2} \psi^2 H^3 2$
$H^4 D^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} H^4 D^2$
$X^2 H^2$	6	2	$\frac{(4\pi)^2}{\Lambda^2} X^2 H^2$
$\psi^2 X H$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 X H$
$\psi^2 H^2 D$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^2 H^2 D$
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3$

# $\chi$ PT

- Chiral Perturbation Theory ( $\chi$ PT) has been used for low-energy QCD: considering only  $u$  and  $d$  quarks and neglecting their mass

Chiral Symmetry  $SU(2)_L \times SU(2)_R$  :

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \rightarrow \Omega_L \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} u_R \\ d_R \end{pmatrix} \rightarrow \Omega_R \begin{pmatrix} u_R \\ d_R \end{pmatrix} \quad \Omega_{L,R} \in SU(2)_{L,R}$$

- Chiral Perturbation Theory ( $\chi$ PT) has been used for low-energy QCD: considering only  $u$  and  $d$  quarks and neglecting their mass

Chiral Symmetry  $SU(2)_L \times SU(2)_R$ :

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \rightarrow \Omega_L \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} u_R \\ d_R \end{pmatrix} \rightarrow \Omega_R \begin{pmatrix} u_R \\ d_R \end{pmatrix} \quad \Omega_{L,R} \in SU(2)_{L,R}$$

- As  $q - \bar{q}$  pairs are energetically cheap, the QCD vacuum will contain condensates:

# $\chi$ PT

- Chiral Perturbation Theory ( $\chi$ PT) has been used for low-energy QCD: considering only  $u$  and  $d$  quarks and neglecting their mass

Chiral Symmetry  $SU(2)_L \times SU(2)_R$ :

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \rightarrow \Omega_L \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} u_R \\ d_R \end{pmatrix} \rightarrow \Omega_R \begin{pmatrix} u_R \\ d_R \end{pmatrix} \quad \Omega_{L,R} \in SU(2)_{L,R}$$

- As  $q - \bar{q}$  pairs are energetically cheap, the QCD vacuum will contain condensates:

Chiral Symmetry spontaneous breaking  $\langle \bar{q}q \rangle \neq 0$ :

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_{\text{diag}} \longrightarrow 3 \text{ Goldstone bosons } \vec{\pi}$$

$$U \equiv e^{2i\vec{\pi} \cdot \vec{\sigma}/f} \quad U \rightarrow \Omega_L^\dagger U \Omega_R$$

The pion Lagrangian is written as

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \dots \text{ (+ soft breaking part)}$$

The pion Lagrangian is written as

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \dots \text{ (+ soft breaking part)}$$

$$\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi\phi}{\Lambda} \right]^{N_p}$$

$$\Lambda = 4\pi f$$

f scale of the pions

$$\mathcal{L}_2 = \frac{f^2}{4} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger)$$

$$\mathcal{L}_4 = \frac{c_4}{16\pi^2} [\text{Tr} (\partial_\mu U \partial^\mu U^\dagger)]^2$$

The pion Lagrangian is written as

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \dots \text{ (+ soft breaking part)}$$

$$\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi\phi}{\Lambda} \right]^{N_p}$$

$$\Lambda = 4\pi f$$

f scale of the pions

$$\mathcal{L}_2 = \frac{f^2}{4} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) \quad N_\chi \equiv N_p + \frac{1}{2} N_\psi = 2$$

$$\mathcal{L}_4 = \frac{c_4}{16\pi^2} [\text{Tr} (\partial_\mu U \partial^\mu U^\dagger)]^2 \quad N_\chi = 4$$

→ The ordering of the operators due to renormalisation,  $\mathcal{L}_2$ ,  $\mathcal{L}_4$ , etc... coincides with the ordering in  $N_\chi$

The pion Lagrangian is written as

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \dots \text{ (+ soft breaking part)}$$

$$\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi\phi}{\Lambda} \right]^{N_p}$$

$$\Lambda = 4\pi f$$

f scale of the pions

$$\mathcal{L}_2 = \frac{f^2}{4} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) \quad N_\chi \equiv N_p + \frac{1}{2} N_\psi = 2$$

$$\mathcal{L}_4 = \frac{c_4}{16\pi^2} [\text{Tr} (\partial_\mu U \partial^\mu U^\dagger)]^2 \quad N_\chi = 4$$

→ The ordering of the operators due to renormalisation,  $\mathcal{L}_2$ ,  $\mathcal{L}_4$ , etc... coincides with the ordering in  $N_\chi$

LO     $\left\{ \begin{array}{l} N_\chi = 2 \end{array} \right.$

The pion Lagrangian is written as

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \dots \text{ (+ soft breaking part)}$$

$$\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi\phi}{\Lambda} \right]^{N_p}$$

$$\Lambda = 4\pi f$$

f scale of the pions

$$\mathcal{L}_2 = \frac{f^2}{4} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) \quad N_\chi \equiv N_p + \frac{1}{2} N_\psi = 2$$

$$\mathcal{L}_4 = \frac{c_4}{16\pi^2} [\text{Tr} (\partial_\mu U \partial^\mu U^\dagger)]^2 \quad N_\chi = 4$$

→ The ordering of the operators due to renormalisation,  $\mathcal{L}_2$ ,  $\mathcal{L}_4$ , etc... coincides with the ordering in  $N_\chi$

$$\text{LO } \left\{ N_\chi = 2 \right.$$

$$\text{NLO } \left\{ \text{1-loop with an arbitrary number of } N_\chi = 2 \text{ vertices} \right\} \equiv N_\chi = 4$$

The pion Lagrangian is written as

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \dots \text{ (+ soft breaking part)}$$

$$\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi\phi}{\Lambda} \right]^{N_p}$$

$$\Lambda = 4\pi f$$

f scale of the pions

$$\mathcal{L}_2 = \frac{f^2}{4} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) \quad N_\chi \equiv N_p + \frac{1}{2} N_\psi = 2$$

$$\mathcal{L}_4 = \frac{c_4}{16\pi^2} [\text{Tr} (\partial_\mu U \partial^\mu U^\dagger)]^2 \quad N_\chi = 4$$

→ The ordering of the operators due to renormalisation,  $\mathcal{L}_2$ ,  $\mathcal{L}_4$ , etc... coincides with the ordering in  $N_\chi$

$$\text{LO } \left\{ N_\chi = 2 \right.$$

$$\text{NLO } \left\{ \text{1-loop with an arbitrary number of } N_\chi = 2 \text{ vertices} \right\} \equiv N_\chi = 4$$

$$\text{NNLO } \left\{ \begin{array}{l} \text{1-loop with an arbitrary number of NLO vertices} \\ \text{2-loop with an arbitrary number of } N_\chi = 2 \text{ vertices} \end{array} \right\} \equiv N_\chi = 6$$

$$\mathcal{L}_2 = \frac{f^2}{4} \text{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) \qquad \qquad N_\chi \equiv N_p + \frac{1}{2}N_\psi = 2$$

$$\mathcal{L}_4 = \frac{c_4}{16\pi^2}\left[\text{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right)\right]^2 \qquad \qquad N_\chi = 4$$

$$\mathcal{L}_2 = \frac{f^2}{4} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) \quad N_\chi \equiv N_p + \frac{1}{2} N_\psi = 2 \quad d = 2$$

$$\mathcal{L}_4 = \frac{c_4}{16\pi^2} [\text{Tr} (\partial_\mu U \partial^\mu U^\dagger)]^2 \quad N_\chi = 4 \quad d = 4$$

**APPARENTLY:**

→ Counting dimensions is equivalent of counting derivatives

$$N_\chi \equiv N_p + \frac{1}{2} \cancel{N_\psi} = d$$

$$\mathcal{L}_2 = \frac{f^2}{4} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) \quad N_\chi \equiv N_p + \frac{1}{2} N_\psi = 2 \quad d = 2$$

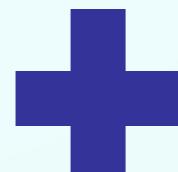
$$\mathcal{L}_4 = \frac{c_4}{16\pi^2} [\text{Tr} (\partial_\mu U \partial^\mu U^\dagger)]^2 \quad N_\chi = 4 \quad d = 4$$

**APPARENTLY:**

→ Counting dimensions is equivalent of counting derivatives

$$N_\chi \equiv N_p + \frac{1}{2} \cancel{N_\psi} = d$$

→ Counting  $d$  is equivalent to counting  $\Lambda$



$$\sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^{-N_\Lambda}$$

Does  $N_\chi$  determine the physical impact of operators in  $\chi$ PT?

→ Convenient to expand in pion fields:

→ Convenient to expand in pion fields:

$$\mathcal{L}_2 \sim \partial^2 \Pi^2 + \frac{\partial^2 \Pi^4}{f^2} + \frac{\partial^2 \Pi^6}{f^4} + \dots$$

→ Convenient to expand in pion fields:

$$\mathcal{L}_2 \sim \partial^2 \mathbf{\Pi}^2 + \frac{\partial^2 \mathbf{\Pi}^4}{f^2} + \frac{\partial^2 \mathbf{\Pi}^6}{f^4} + \dots \quad d = 4, 6, 8, \dots \quad N_\chi = 2$$

→ Convenient to expand in pion fields:

$$\mathcal{L}_2 \sim \partial^2 \boldsymbol{\Pi}^2 + \frac{\partial^2 \boldsymbol{\Pi}^4}{f^2} + \frac{\partial^2 \boldsymbol{\Pi}^6}{f^4} + \dots \quad d = 4, 6, 8, \dots \quad N_\chi = 2$$

$$\mathcal{L}_4 \sim c_4 \left( \frac{\partial^4 \boldsymbol{\Pi}^4}{\Lambda^2 f^2} + \frac{\partial^4 \boldsymbol{\Pi}^6}{\Lambda^2 f^4} + \dots \right)$$

→ Convenient to expand in pion fields:

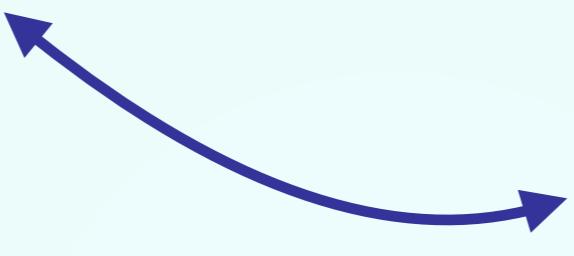
$$\mathcal{L}_2 \sim \partial^2 \mathbf{\Pi}^2 + \frac{\partial^2 \mathbf{\Pi}^4}{f^2} + \frac{\partial^2 \mathbf{\Pi}^6}{f^4} + \dots \quad d = 4, 6, 8, \dots \quad N_\chi = 2$$

$$\mathcal{L}_4 \sim c_4 \left( \frac{\partial^4 \mathbf{\Pi}^4}{\Lambda^2 f^2} + \frac{\partial^4 \mathbf{\Pi}^6}{\Lambda^2 f^4} + \dots \right) \quad d = 8, 10, 12, \dots \quad N_\chi = 4$$

→ Convenient to expand in pion fields:

$$\mathcal{L}_2 \sim \partial^2 \boldsymbol{\Pi}^2 + \frac{\partial^2 \boldsymbol{\Pi}^4}{f^2} + \frac{\partial^2 \boldsymbol{\Pi}^6}{f^4} + \dots \quad d = 4, 6, 8, \dots \quad N_\chi = 2$$

$$\mathcal{L}_4 \sim c_4 \left( \frac{\partial^4 \boldsymbol{\Pi}^4}{\Lambda^2 f^2} + \frac{\partial^4 \boldsymbol{\Pi}^6}{\Lambda^2 f^4} + \dots \right) \quad d = 8, 10, 12, \dots \quad N_\chi = 4$$



$$N_\chi \equiv N_p + \frac{1}{2} \cancel{N_\psi} \cancel{- d}$$

→ Convenient to expand in pion fields:

$$\mathcal{L}_2 \sim \partial^2 \Pi^2 + \frac{\partial^2 \Pi^4}{f^2} + \frac{\partial^2 \Pi^6}{f^4} + \dots \quad d = 4, 6, 8, \dots \quad N_\chi = 2$$

$$\mathcal{L}_4 \sim c_4 \left( \frac{\partial^4 \Pi^4}{\Lambda^2 f^2} + \frac{\partial^4 \Pi^6}{\Lambda^2 f^4} + \dots \right) \quad d = 8, 10, 12, \dots \quad N_\chi = 4$$

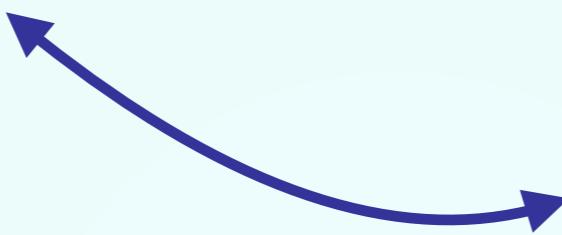

$$N_\chi \equiv N_p + \frac{1}{2} \cancel{N_\psi} \cancel{-} d$$

→ Distinct terms of the expansion gives different effects:

→ Convenient to expand in pion fields:

$$\mathcal{L}_2 \sim \partial^2 \Pi^2 + \frac{\partial^2 \Pi^4}{f^2} + \frac{\partial^2 \Pi^6}{f^4} + \dots \quad d = 4, 6, 8, \dots \quad N_\chi = 2$$

$$\mathcal{L}_4 \sim c_4 \left( \frac{\partial^4 \Pi^4}{\Lambda^2 f^2} + \frac{\partial^4 \Pi^6}{\Lambda^2 f^4} + \dots \right) \quad d = 8, 10, 12, \dots \quad N_\chi = 4$$


$$N_\chi \equiv N_p + \frac{1}{2} \cancel{N_\psi} \cancel{+ d}$$

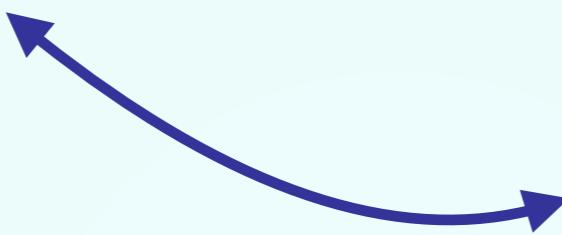
→ Distinct terms of the expansion gives different effects:

$$\mathcal{L}_2 \sim \partial^2 \Pi^2 + \frac{\partial^2 \Pi^4}{f^2} + \frac{\partial^2 \Pi^6}{f^4} + \dots$$

→ Convenient to expand in pion fields:

$$\mathcal{L}_2 \sim \partial^2 \Pi^2 + \frac{\partial^2 \Pi^4}{f^2} + \frac{\partial^2 \Pi^6}{f^4} + \dots \quad d = 4, 6, 8, \dots \quad N_\chi = 2$$

$$\mathcal{L}_4 \sim c_4 \left( \frac{\partial^4 \Pi^4}{\Lambda^2 f^2} + \frac{\partial^4 \Pi^6}{\Lambda^2 f^4} + \dots \right) \quad d = 8, 10, 12, \dots \quad N_\chi = 4$$



$$N_\chi \equiv N_p + \frac{1}{2} \cancel{N_\psi} \cancel{+ d}$$

→ Distinct terms of the expansion gives different effects:

$$\mathcal{L}_2 \sim \partial^2 \Pi^2 + \frac{\partial^2 \Pi^4}{f^2} + \frac{\partial^2 \Pi^6}{f^4} + \dots$$



$$\sigma \sim \frac{\pi(4\pi)^2}{E^2} \left( \frac{E^2}{\Lambda^2} \right)^{-N_\Lambda}$$



$$\sigma(\pi\pi \rightarrow 4\pi)_k \sim \frac{\pi(4\pi)^2}{E^2} \left( \frac{E^4}{\Lambda^4} \right)^2$$



$$\sigma(\pi\pi \rightarrow \pi\pi)_k \sim \frac{\pi(4\pi)^2}{E^2} \left( \frac{E^2}{\Lambda^2} \right)^2$$

→ Convenient to expand in pion fields:

$$\mathcal{L}_2 \sim \partial^2 \Pi^2 + \frac{\partial^2 \Pi^4}{f^2} + \frac{\partial^2 \Pi^6}{f^4} + \dots \quad d = 4, 6, 8, \dots \quad N_\chi = 2$$

$$\mathcal{L}_4 \sim c_4 \left( \frac{\partial^4 \Pi^4}{\Lambda^2 f^2} + \frac{\partial^4 \Pi^6}{\Lambda^2 f^4} + \dots \right) \quad d = 8, 10, 12, \dots \quad N_\chi = 4$$

$$N_\chi \equiv N_p + \frac{1}{2} \cancel{N_\psi} \cancel{+ d}$$

→ Distinct terms of the expansion gives different effects:

$$\mathcal{L}_2 \sim \partial^2 \Pi^2 + \frac{\partial^2 \Pi^4}{f^2} + \frac{\partial^2 \Pi^6}{f^4} + \dots$$

$$\sigma \sim \frac{\pi(4\pi)^2}{E^2} \left( \frac{E^2}{\Lambda^2} \right)^{-N_\Lambda}$$

$$\sigma(\pi\pi \rightarrow 4\pi)_k \sim \frac{\pi(4\pi)^2}{E^2} \left( \frac{E^4}{\Lambda^4} \right)^2$$

$$\sigma(\pi\pi \rightarrow \pi\pi)_k \sim \frac{\pi(4\pi)^2}{E^2} \left( \frac{E^2}{\Lambda^2} \right)^2$$

The same operator gives two cross sections with different suppression and therefore  $N_\chi$  cannot be useful to determine the ordering of the physical impact

→ However, considering a given process, it receives contributions from different operators and ...

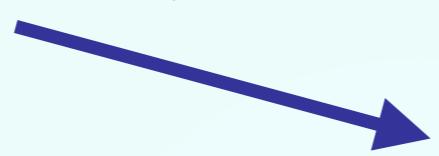
→ However, considering a given process, it receives contributions from different operators and ...

$$\mathcal{L}_2 \sim \partial^2 \mathbf{\Pi}^2 + \frac{\partial^2 \mathbf{\Pi}^4}{f^2} + \frac{\partial^2 \mathbf{\Pi}^6}{f^4} + \dots$$


$$\sigma(\pi\pi \rightarrow \pi\pi)_k \sim \frac{\pi(4\pi)^2}{E^2} \left( \frac{E^2}{\Lambda^2} \right)^2$$

→ However, considering a given process, it receives contributions from different operators and ...

$$\mathcal{L}_2 \sim \partial^2 \mathbf{\Pi}^2 + \frac{\partial^2 \mathbf{\Pi}^4}{f^2} + \frac{\partial^2 \mathbf{\Pi}^6}{f^4} + \dots$$



$$\sigma(\pi\pi \rightarrow \pi\pi)_k \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^2$$

$$\mathcal{L}_4 \sim c_4 \left( \frac{\partial^4 \mathbf{\Pi}^4}{\Lambda^2 f^2} + \frac{\partial^4 \mathbf{\Pi}^6}{\Lambda^2 f^4} + \dots \right)$$



$$\sigma(\pi\pi \rightarrow \pi\pi)_4 \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^4}{\Lambda^4}\right)^2$$

→ However, considering a given process, it receives contributions from different operators and ...

$$\mathcal{L}_2 \sim \partial^2 \mathbf{\Pi}^2 + \frac{\partial^2 \mathbf{\Pi}^4}{f^2} + \frac{\partial^2 \mathbf{\Pi}^6}{f^4} + \dots$$

$$\mathcal{L}_4 \sim c_4 \left( \frac{\partial^4 \mathbf{\Pi}^4}{\Lambda^2 f^2} + \frac{\partial^4 \mathbf{\Pi}^6}{\Lambda^2 f^4} + \dots \right)$$

$$\frac{\sigma(\pi\pi \rightarrow \pi\pi)_4}{\sigma(\pi\pi \rightarrow \pi\pi)_2} \sim \left( \frac{E^2}{\Lambda^2} \right)^2$$

$$\sigma(\pi\pi \rightarrow \pi\pi)_k \sim \frac{\pi(4\pi)^2}{E^2} \left( \frac{E^2}{\Lambda^2} \right)^2$$

$$\sigma(\pi\pi \rightarrow \pi\pi)_4 \sim \frac{\pi(4\pi)^2}{E^2} \left( \frac{E^4}{\Lambda^4} \right)^2$$

→ However, considering a given process, it receives contributions from different operators and ...

$$\mathcal{L}_2 \sim \partial^2 \mathbf{\Pi}^2 + \frac{\partial^2 \mathbf{\Pi}^4}{f^2} + \frac{\partial^2 \mathbf{\Pi}^6}{f^4} + \dots$$

$$\mathcal{L}_4 \sim c_4 \left( \frac{\partial^4 \mathbf{\Pi}^4}{\Lambda^2 f^2} + \frac{\partial^4 \mathbf{\Pi}^6}{\Lambda^2 f^4} + \dots \right)$$

$$\sigma(\pi\pi \rightarrow \pi\pi)_k \sim \frac{\pi(4\pi)^2}{E^2} \left( \frac{E^2}{\Lambda^2} \right)^2$$

$$\sigma(\pi\pi \rightarrow \pi\pi)_4 \sim \frac{\pi(4\pi)^2}{E^2} \left( \frac{E^4}{\Lambda^4} \right)^2$$

$$\frac{\sigma(\pi\pi \rightarrow \pi\pi)_4}{\sigma(\pi\pi \rightarrow \pi\pi)_2} \sim \left( \frac{E^2}{\Lambda^2} \right)^2$$

according to  $N_\chi$  ordering:  $\begin{cases} \mathcal{L}_2 \sim \mathcal{O}(p^2) \\ \mathcal{L}_4 \sim \mathcal{O}(p^4) \end{cases}$

→ However, considering a given process, it receives contributions from different operators and ...

$$\mathcal{L}_2 \sim \partial^2 \mathbf{\Pi}^2 + \frac{\partial^2 \mathbf{\Pi}^4}{f^2} + \frac{\partial^2 \mathbf{\Pi}^6}{f^4} + \dots$$

$$\mathcal{L}_4 \sim c_4 \left( \frac{\partial^4 \mathbf{\Pi}^4}{\Lambda^2 f^2} + \frac{\partial^4 \mathbf{\Pi}^6}{\Lambda^2 f^4} + \dots \right)$$

$$\sigma(\pi\pi \rightarrow \pi\pi)_k \sim \frac{\pi(4\pi)^2}{E^2} \left( \frac{E^2}{\Lambda^2} \right)^2$$

$$\sigma(\pi\pi \rightarrow \pi\pi)_4 \sim \frac{\pi(4\pi)^2}{E^2} \left( \frac{E^4}{\Lambda^4} \right)^2$$

$$\frac{\sigma(\pi\pi \rightarrow \pi\pi)_4}{\sigma(\pi\pi \rightarrow \pi\pi)_2} \sim \left( \frac{E^2}{\Lambda^2} \right)^2$$

according to  $N_\chi$  ordering:  $\begin{cases} \mathcal{L}_2 \sim \mathcal{O}(p^2) \\ \mathcal{L}_4 \sim \mathcal{O}(p^4) \end{cases}$

→ The physical ordering is again determined only by  $\Lambda$ , although it can be hidden in the GB matrix.

→ However, considering a given process, it receives contributions from different operators and ...

$$\mathcal{L}_2 \sim \partial^2 \mathbf{\Pi}^2 + \frac{\partial^2 \mathbf{\Pi}^4}{f^2} + \frac{\partial^2 \mathbf{\Pi}^6}{f^4} + \dots$$

$$\mathcal{L}_4 \sim c_4 \left( \frac{\partial^4 \mathbf{\Pi}^4}{\Lambda^2 f^2} + \frac{\partial^4 \mathbf{\Pi}^6}{\Lambda^2 f^4} + \dots \right)$$

$$\sigma(\pi\pi \rightarrow \pi\pi)_k \sim \frac{\pi(4\pi)^2}{E^2} \left( \frac{E^2}{\Lambda^2} \right)^2$$

$$\sigma(\pi\pi \rightarrow \pi\pi)_4 \sim \frac{\pi(4\pi)^2}{E^2} \left( \frac{E^4}{\Lambda^4} \right)^2$$

$$\frac{\sigma(\pi\pi \rightarrow \pi\pi)_4}{\sigma(\pi\pi \rightarrow \pi\pi)_2} \sim \left( \frac{E^2}{\Lambda^2} \right)^2$$

according to  $N_\chi$  ordering:  $\begin{cases} \mathcal{L}_2 \sim \mathcal{O}(p^2) \\ \mathcal{L}_4 \sim \mathcal{O}(p^4) \end{cases}$

→ The physical ordering is again determined only by  $\Lambda$ , although it can be hidden in the GB matrix.

→ The ordering in  $\Lambda$  coincides with  $N_\chi$  only for processes with same number of external fields.

# HEFT

Grinstein & Trott, PRD 76 (2007)

Azatov, Contino & Galloway JHEP 1204 (2012)

Alonso, Gavela, LM, Rigolin & Yepes, JHEP 1206 (2012)

Alonso, Gavela, LM, Rigolin & Yepes, PLB 722 (2013)

Alonso, Gavela, LM, Rigolin & Yepes, PRD 87 (2013)

Buchalla, Catà & Krause, NPB 880 (2014)

Gavela, Gonzalez-Fraile, Gonzalez-Garcia, LM, Rigolin & Yepes, JHEP 1410 (2014)

$$\Phi(x) = \frac{v + h}{\sqrt{2}} e^{i\sigma_a \pi^a(x)/v} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Higgs:  $h$

**Singlet**

GBs:  $\mathbf{U}(x) = e^{i\sigma_a \pi^a(x)/v}$   
 $\mathbf{U}(x) \rightarrow L \mathbf{U}(x) R^\dagger$

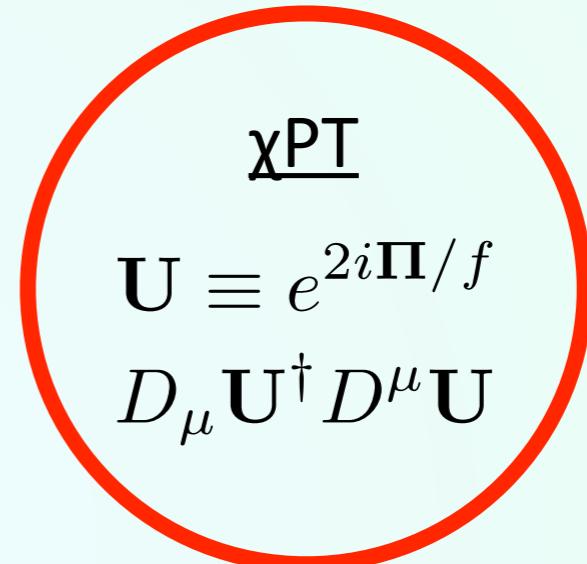
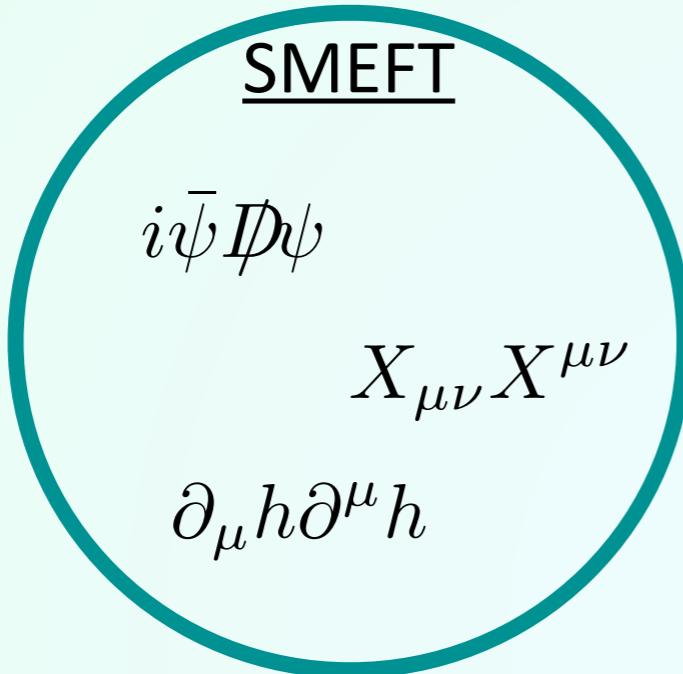
**Independent!!**

# HEFT

The Higgs Effective Field Theory (HEFT) is a fusion of SMEFT and  $\chi$ PT

# HEFT

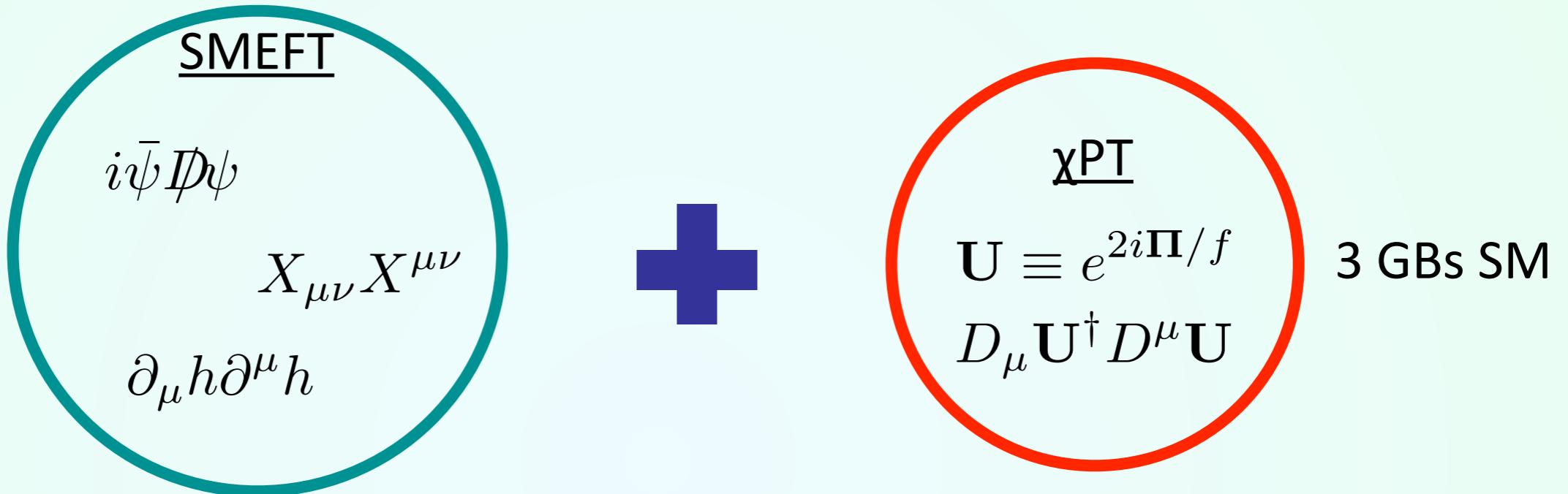
The Higgs Effective Field Theory (HEFT) is a fusion of SMEFT and  $\chi$ PT



3 GBs SM

# HEFT

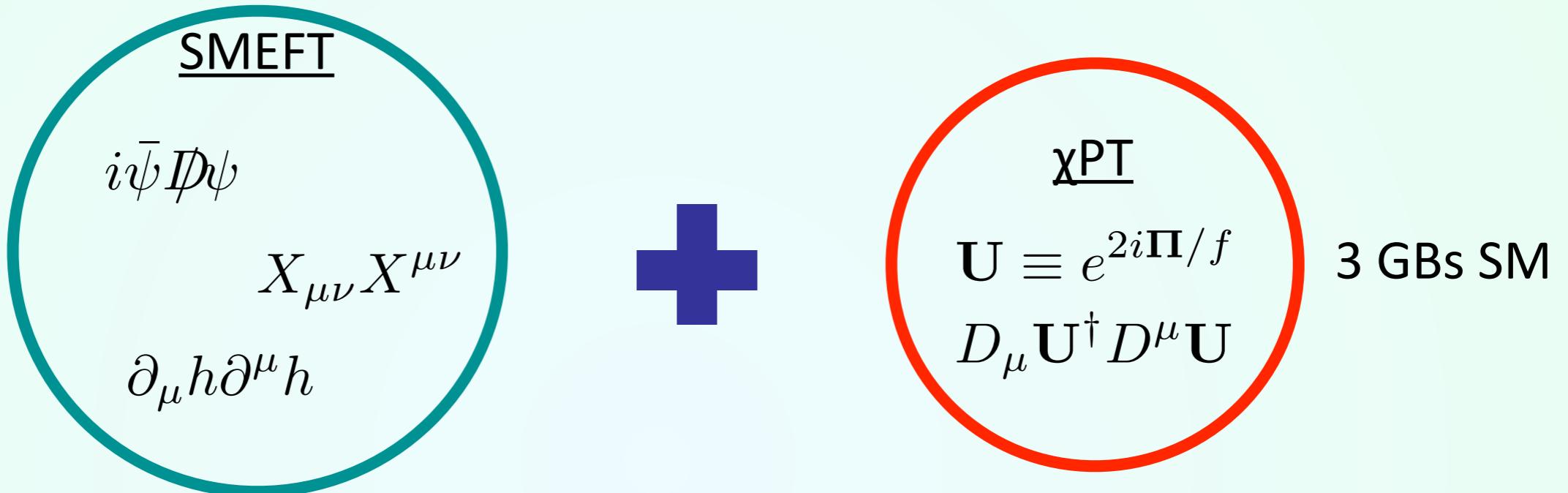
The Higgs Effective Field Theory (HEFT) is a fusion of SMEFT and  $\chi$ PT



Renormalisation is different between SMEFT and  $\chi$ PT:

# HEFT

The Higgs Effective Field Theory (HEFT) is a fusion of SMEFT and  $\chi$ PT

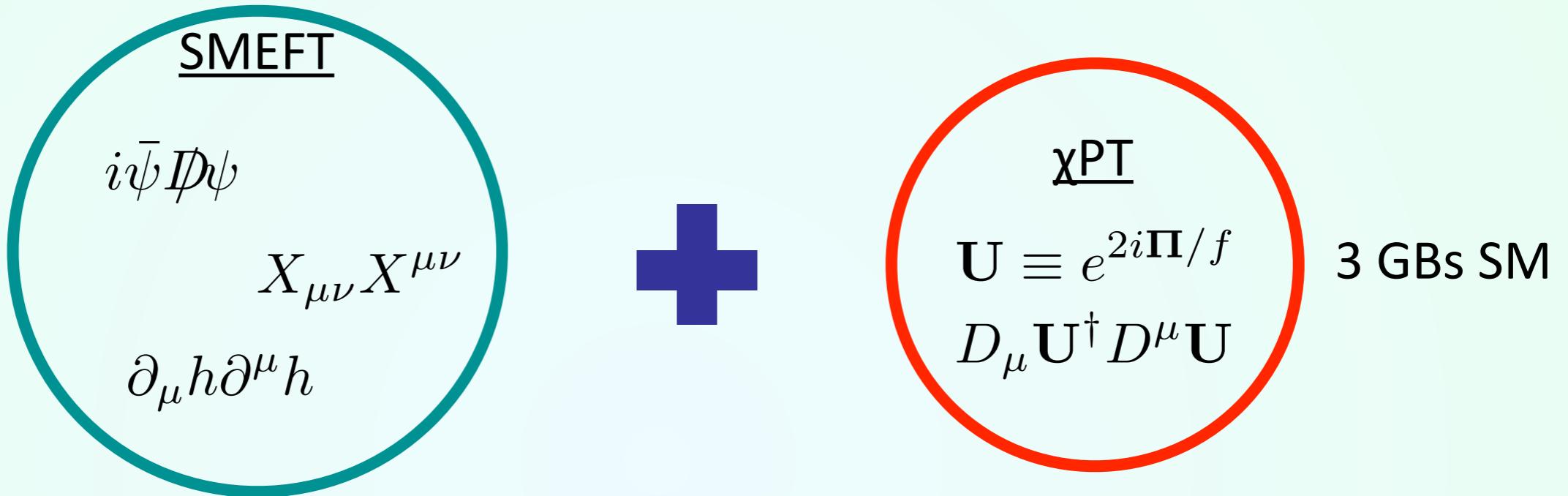


Renormalisation is different between SMEFT and  $\chi$ PT:

SMEFT	Tree	1 loop	2 loop
LO	LO	LO	LO
NLO	NLO $\frac{1}{\Lambda^2}$	NLO $\frac{1}{\Lambda^2}$	NLO $\frac{1}{\Lambda^2}$
NNLO	NNLO $\frac{1}{\Lambda^4}$	NNLO $\frac{1}{\Lambda^4}$	NNLO $\frac{1}{\Lambda^4}$

# HEFT

The Higgs Effective Field Theory (HEFT) is a fusion of SMEFT and  $\chi$ PT



Renormalisation is different between SMEFT and  $\chi$ PT:

SMEFT	Tree	1 loop	2 loop	$\chi$ PT	Tree	1 loop	2 loop
LO	LO	LO	LO	LO	LO	NLO $\frac{1}{\Lambda^2}$	NNLO $\frac{1}{\Lambda^4}$
NLO	NLO $\frac{1}{\Lambda^2}$	NNLO $\frac{1}{\Lambda^4}$	...				
NNLO	NNLO $\frac{1}{\Lambda^4}$	...	...				

# How to merge the two theories??

# How to merge the two theories??

■ Building blocks:

$$\mathbf{V}_\mu \equiv (\mathbf{D}_\mu \mathbf{U}) \mathbf{U}^\dagger$$

$$\mathbf{V} \rightarrow L \mathbf{V} L^\dagger$$

$$L \in SU(2)_L$$

$$\mathbf{T} \equiv \mathbf{U} \sigma_3 \mathbf{U}^\dagger$$

$$\mathbf{T} \rightarrow L \mathbf{T} L^\dagger$$

$$\psi_{L,R}$$

$$A_\mu \quad X_{\mu\nu}$$

$$h \quad \text{singlet of SM syms: arbitrary} \quad \mathcal{F}(h) = \sum_{i=0} a_i \left( \frac{h}{f} \right)^i$$

# How to merge the two theories??

## ■ Building blocks:

$$\mathbf{V}_\mu \equiv (\mathbf{D}_\mu \mathbf{U}) \mathbf{U}^\dagger$$

$$\mathbf{V} \rightarrow L \mathbf{V} L^\dagger$$

$$L \in SU(2)_L$$

$$\mathbf{T} \equiv \mathbf{U} \sigma_3 \mathbf{U}^\dagger$$

$$\mathbf{T} \rightarrow L \mathbf{T} L^\dagger$$

$$\psi_{L,R}$$

$$A_\mu \quad X_{\mu\nu}$$

$$h \quad \text{singlet of SM syms: arbitrary} \quad \mathcal{F}(h) = \sum_{i=0} \left(\frac{h}{f}\right)^i$$

## ■ Leading interacting terms: expanding the fields

$$\mathbf{U} = 1 + \dots$$

$$\mathbf{T} = \sigma_3$$

$$\mathbf{V}_\mu = \frac{2i}{f} \partial_\mu \boldsymbol{\Pi} + \frac{2i}{f} [\boldsymbol{\Pi}, g A_\mu] + \frac{gv}{f} B_\mu$$

# How to merge the two theories??

## ■ Building blocks:

$$\mathbf{V}_\mu \equiv (\mathbf{D}_\mu \mathbf{U}) \mathbf{U}^\dagger$$

$$\mathbf{V} \rightarrow L \mathbf{V} L^\dagger$$

$$L \in SU(2)_L$$

$$\mathbf{T} \equiv \mathbf{U} \sigma_3 \mathbf{U}^\dagger$$

$$\mathbf{T} \rightarrow L \mathbf{T} L^\dagger$$

$$\psi_{L,R}$$

$$A_\mu \quad X_{\mu\nu}$$

$$h \quad \text{singlet of SM syms: arbitrary} \quad \mathcal{F}(h) = \sum_{i=0} \left(\frac{h}{f}\right)^i$$

## ■ Leading interacting terms: expanding the fields

$$\mathbf{U} = 1 + \dots$$

$$N_\chi = 0$$

$$\mathbf{T} = \sigma_3$$

$$N_\chi = 0$$

$$\mathbf{V}_\mu = \frac{2i}{f} \partial_\mu \boldsymbol{\Pi} + \frac{2i}{f} [\boldsymbol{\Pi}, g A_\mu] + \frac{gv}{f} B_\mu \quad N_\chi = 1$$

# How to merge the two theories??

## ■ Building blocks:

$$\mathbf{V}_\mu \equiv (\mathbf{D}_\mu \mathbf{U}) \mathbf{U}^\dagger$$

$$\mathbf{V} \rightarrow L \mathbf{V} L^\dagger$$

$$L \in SU(2)_L$$

$$\mathbf{T} \equiv \mathbf{U} \sigma_3 \mathbf{U}^\dagger$$

$$\mathbf{T} \rightarrow L \mathbf{T} L^\dagger$$

$$\psi_{L,R}$$

$$A_\mu \quad X_{\mu\nu}$$

$$h \quad \text{singlet of SM syms: arbitrary} \quad \mathcal{F}(h) = \sum_{i=0} \left(\frac{h}{f}\right)^i$$

## ■ Leading interacting terms: expanding the fields

$$\mathbf{U} = 1 + \dots$$

$$N_\chi = 0 \quad d = 0, 1, 2, \dots$$

$$\mathbf{T} = \sigma_3$$

$$N_\chi = 0 \quad d = 0, 1, 2, \dots$$

$$\mathbf{V}_\mu = \frac{2i}{f} \partial_\mu \boldsymbol{\Pi} + \frac{2i}{f} [\boldsymbol{\Pi}, g A_\mu] + \frac{gv}{f} B_\mu$$

$$N_\chi = 1 \quad d = 1, 2, 3, \dots$$

# How to merge the two theories??

## ■ Building blocks:

$$\mathbf{V}_\mu \equiv (\mathbf{D}_\mu \mathbf{U}) \mathbf{U}^\dagger$$

$$\mathbf{V} \rightarrow L \mathbf{V} L^\dagger$$

$$L \in SU(2)_L$$

$$\mathbf{T} \equiv \mathbf{U} \sigma_3 \mathbf{U}^\dagger$$

$$\mathbf{T} \rightarrow L \mathbf{T} L^\dagger$$

$$\psi_{L,R}$$

$$A_\mu \quad X_{\mu\nu}$$

$$h \quad \text{singlet of SM syms: arbitrary} \quad \mathcal{F}(h) = \sum_{i=0} a_i \left(\frac{h}{f}\right)^i$$

## ■ Leading interacting terms: expanding the fields

### Primary Dimension

$$\mathbf{U} = 1 + \dots$$

$$N_\chi = 0 \quad d_p = 0$$

$$\mathbf{T} = \sigma_3$$

$$N_\chi = 0 \quad d_p = 0$$

$$\mathbf{V}_\mu = \frac{2i}{f} \partial_\mu \boldsymbol{\Pi} + \frac{2i}{f} [\boldsymbol{\Pi}, g A_\mu] + \frac{gv}{f} B_\mu$$

$$N_\chi = 1 \quad d_p = 2$$

# HEFT basis

Assuming B and L conservation, and no BSM custodial breaking

Operator	$d_p$	$N_\chi$	NDA form
$\psi^2 \mathbf{U}$	3	1	$\Lambda \psi^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{U}}(h)$
$X^2$	4	2	$X^2 \mathcal{F}_{X^2}(h)$
$\psi^2 D$	4	2	$\psi^2 D$
$(\partial h)^2$	4	2	$(\partial h)^2$
$\mathbf{V}^2$	4	2	$\frac{\Lambda^2}{(4\pi)^2} \mathbf{V}^2 \mathcal{F}_{\mathbf{V}^2}(h)$
$\psi^2 \mathbf{V}$	5	2	$\psi^2 \mathbf{V} \mathcal{F}_{\psi^2 \mathbf{V}}(h)$
$\psi^2 X \mathbf{U}$	5	2	$\frac{4\pi}{\Lambda} \psi^2 X \mathbf{U} \mathcal{F}_{\psi^2 X \mathbf{U}}(h)$
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4 \mathcal{F}_{\psi^4}(h)$
$X \mathbf{V}^2$	6	3	$\frac{1}{4\pi} X \mathbf{V}^2 \mathcal{F}_{X \mathbf{V}^2}(h)$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3 \mathcal{F}_{X^3}(h)$
$X \mathbf{V} \partial$	6	3	$\frac{1}{4\pi} X \mathbf{V} \partial \mathcal{F}_{X \mathbf{V} \partial}(h)$
$\psi^2 \mathbf{V} \mathbf{U} \partial$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V} \mathbf{U} \partial \mathcal{F}_{\psi^2 \mathbf{V} \mathbf{U} \partial}(h)$
$\psi^2 \mathbf{V}^2 \mathbf{U}$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V}^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{V}^2 \mathbf{U}}(h)$
$\psi^2 \mathbf{U} \partial^2$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{U} \partial^2 \mathcal{F}_{\psi^2 \mathbf{U} \partial^2}(h)$
$\mathbf{V}^2 \partial^2$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^2 \partial^2 \mathcal{F}_{\mathbf{V}^2 \partial^2}(h)$
$\mathbf{V}^4$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^4 \mathcal{F}_{\mathbf{V}^4}(h)$

# HEFT basis

Assuming B and L conservation, and no BSM custodial breaking

Operator	$d_p$	$N_\chi$	NDA form	
$\psi^2 \mathbf{U}$	3	1	$\Lambda \psi^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{U}}(h)$	$\leftrightarrow$ $\Lambda \bar{\psi}_L \mathbf{U} \psi_R \mathcal{F}_{\psi^2 \mathbf{U}}(h)$
$X^2$	4	2	$X^2 \mathcal{F}_{X^2}(h)$	
$\psi^2 D$	4	2	$\psi^2 D$	$\leftrightarrow$ $i \bar{\psi} \not{D} \psi$
$(\partial h)^2$	4	2	$(\partial h)^2$	
$\mathbf{V}^2$	4	2	$\frac{\Lambda^2}{(4\pi)^2} \mathbf{V}^2 \mathcal{F}_{\mathbf{V}^2}(h)$	$\leftrightarrow$ $\frac{\Lambda^2}{(4\pi)^2} \text{Tr} (\mathbf{V}_\mu \mathbf{V}^\mu) \mathcal{F}_{\mathbf{V}^2}(h)$
$\psi^2 \mathbf{V}$	5	2	$\psi^2 \mathbf{V} \mathcal{F}_{\psi^2 \mathbf{V}}(h)$	
$\psi^2 X \mathbf{U}$	5	2	$\frac{4\pi}{\Lambda} \psi^2 X \mathbf{U} \mathcal{F}_{\psi^2 X \mathbf{U}}(h)$	
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4 \mathcal{F}_{\psi^4}(h)$	
$X \mathbf{V}^2$	6	3	$\frac{1}{4\pi} X \mathbf{V}^2 \mathcal{F}_{X \mathbf{V}^2}(h)$	$\leftrightarrow$ $\frac{1}{4\pi} \text{Tr} (W_{\mu\nu} [\mathbf{V}^\mu, \mathbf{V}^\nu]) \mathcal{F}_{X \mathbf{V}^2}(h)$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3 \mathcal{F}_{X^3}(h)$	
$X \mathbf{V} \partial$	6	3	$\frac{1}{4\pi} X \mathbf{V} \partial \mathcal{F}_{X \mathbf{V} \partial}(h)$	
$\psi^2 \mathbf{V} \mathbf{U} \partial$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V} \mathbf{U} \partial \mathcal{F}_{\psi^2 \mathbf{V} \mathbf{U} \partial}(h)$	
$\psi^2 \mathbf{V}^2 \mathbf{U}$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V}^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{V}^2 \mathbf{U}}(h)$	
$\psi^2 \mathbf{U} \partial^2$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{U} \partial^2 \mathcal{F}_{\psi^2 \mathbf{U} \partial^2}(h)$	
$\mathbf{V}^2 \partial^2$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^2 \partial^2 \mathcal{F}_{\mathbf{V}^2 \partial^2}(h)$	$\leftrightarrow$ $\frac{1}{(4\pi)^2} \text{Tr} (\mathbf{V}^\mu \mathbf{V}^\mu)^2 \mathcal{F}_{\mathbf{V}^4}(h)$
$\mathbf{V}^4$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^4 \mathcal{F}_{\mathbf{V}^4}(h)$	

# Primary Dimension $d_p$

Operator	$d_p$	$N_\chi$	NDA form
$\psi^2 \mathbf{U}$	3	1	$\Lambda \psi^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{U}}(h)$
$X^2$	4	2	$X^2 \mathcal{F}_{X^2}(h)$
$\psi^2 D$	4	2	$\psi^2 D$
$(\partial h)^2$	4	2	$(\partial h)^2$
$\mathbf{V}^2$	4	2	$\frac{\Lambda^2}{(4\pi)^2} \mathbf{V}^2 \mathcal{F}_{\mathbf{V}^2}(h)$
$\psi^2 \mathbf{V}$	5	2	$\psi^2 \mathbf{V} \mathcal{F}_{\psi^2 \mathbf{V}}(h)$
$\psi^2 X \mathbf{U}$	5	2	$\frac{4\pi}{\Lambda} \psi^2 X \mathbf{U} \mathcal{F}_{\psi^2 X \mathbf{U}}(h)$
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4 \mathcal{F}_{\psi^4}(h)$
$X \mathbf{V}^2$	6	3	$\frac{1}{4\pi} X \mathbf{V}^2 \mathcal{F}_{X \mathbf{V}^2}(h)$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3 \mathcal{F}_{X^3}(h)$
$X \mathbf{V} \partial$	6	3	$\frac{1}{4\pi} X \mathbf{V} \partial \mathcal{F}_{X \mathbf{V} \partial}(h)$
$\psi^2 \mathbf{V} \mathbf{U} \partial$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V} \mathbf{U} \partial \mathcal{F}_{\psi^2 \mathbf{V} \mathbf{U} \partial}(h)$
$\psi^2 \mathbf{V}^2 \mathbf{U}$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V}^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{V}^2 \mathbf{U}}(h)$
$\psi^2 \mathbf{U} \partial^2$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{U} \partial^2 \mathcal{F}_{\psi^2 \mathbf{U} \partial^2}(h)$
$\mathbf{V}^2 \partial^2$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^2 \partial^2 \mathcal{F}_{\mathbf{V}^2 \partial^2}(h)$
$\mathbf{V}^4$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^4 \mathcal{F}_{\mathbf{V}^4}(h)$

# Primary Dimension $d_p$

$d_p$  counts the dimensions of the leading interacting term

Operator	$d_p$	$N_\chi$	NDA form
$\psi^2 \mathbf{U}$	3	1	$\Lambda \psi^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{U}}(h)$
$X^2$	4	2	$X^2 \mathcal{F}_{X^2}(h)$
$\psi^2 D$	4	2	$\psi^2 D$
$(\partial h)^2$	4	2	$(\partial h)^2$
$\mathbf{V}^2$	4	2	$\frac{\Lambda^2}{(4\pi)^2} \mathbf{V}^2 \mathcal{F}_{\mathbf{V}^2}(h)$
$\psi^2 \mathbf{V}$	5	2	$\psi^2 \mathbf{V} \mathcal{F}_{\psi^2 \mathbf{V}}(h)$
$\psi^2 X \mathbf{U}$	5	2	$\frac{4\pi}{\Lambda} \psi^2 X \mathbf{U} \mathcal{F}_{\psi^2 X \mathbf{U}}(h)$
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4 \mathcal{F}_{\psi^4}(h)$
$X \mathbf{V}^2$	6	3	$\frac{1}{4\pi} X \mathbf{V}^2 \mathcal{F}_{X \mathbf{V}^2}(h)$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3 \mathcal{F}_{X^3}(h)$
$X \mathbf{V} \partial$	6	3	$\frac{1}{4\pi} X \mathbf{V} \partial \mathcal{F}_{X \mathbf{V} \partial}(h)$
$\psi^2 \mathbf{V} \mathbf{U} \partial$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V} \mathbf{U} \partial \mathcal{F}_{\psi^2 \mathbf{V} \mathbf{U} \partial}(h)$
$\psi^2 \mathbf{V}^2 \mathbf{U}$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V}^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{V}^2 \mathbf{U}}(h)$
$\psi^2 \mathbf{U} \partial^2$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{U} \partial^2 \mathcal{F}_{\psi^2 \mathbf{U} \partial^2}(h)$
$\mathbf{V}^2 \partial^2$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^2 \partial^2 \mathcal{F}_{\mathbf{V}^2 \partial^2}(h)$
$\mathbf{V}^4$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^4 \mathcal{F}_{\mathbf{V}^4}(h)$

# Primary Dimension $d_p$

$d_p$  counts the dimensions of the leading interacting term



$d_p$  counts the number of scales, explicit and implicit

Operator	$d_p$	$N_\chi$	NDA form
$\psi^2 \mathbf{U}$	3	1	$\Lambda \psi^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{U}}(h)$
$X^2$	4	2	$X^2 \mathcal{F}_{X^2}(h)$
$\psi^2 D$	4	2	$\psi^2 D$
$(\partial h)^2$	4	2	$(\partial h)^2$
$\mathbf{V}^2$	4	2	$\frac{\Lambda^2}{(4\pi)^2} \mathbf{V}^2 \mathcal{F}_{\mathbf{V}^2}(h)$
$\psi^2 \mathbf{V}$	5	2	$\psi^2 \mathbf{V} \mathcal{F}_{\psi^2 \mathbf{V}}(h)$
$\psi^2 X \mathbf{U}$	5	2	$\frac{4\pi}{\Lambda} \psi^2 X \mathbf{U} \mathcal{F}_{\psi^2 X \mathbf{U}}(h)$
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4 \mathcal{F}_{\psi^4}(h)$
$X \mathbf{V}^2$	6	3	$\frac{1}{4\pi} X \mathbf{V}^2 \mathcal{F}_{X \mathbf{V}^2}(h)$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3 \mathcal{F}_{X^3}(h)$
$X \mathbf{V} \partial$	6	3	$\frac{1}{4\pi} X \mathbf{V} \partial \mathcal{F}_{X \mathbf{V} \partial}(h)$
$\psi^2 \mathbf{V} \mathbf{U} \partial$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V} \mathbf{U} \partial \mathcal{F}_{\psi^2 \mathbf{V} \mathbf{U} \partial}(h)$
$\psi^2 \mathbf{V}^2 \mathbf{U}$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V}^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{V}^2 \mathbf{U}}(h)$
$\psi^2 \mathbf{U} \partial^2$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{U} \partial^2 \mathcal{F}_{\psi^2 \mathbf{U} \partial^2}(h)$
$\mathbf{V}^2 \partial^2$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^2 \partial^2 \mathcal{F}_{\mathbf{V}^2 \partial^2}(h)$
$\mathbf{V}^4$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^4 \mathcal{F}_{\mathbf{V}^4}(h)$

# Primary Dimension $d_p$

$d_p$  counts the dimensions of the leading interacting term



$d_p$  counts the number of scales, explicit and implicit



$$\sigma \sim \frac{\pi(4\pi)^2}{E^2} \left(\frac{E^2}{\Lambda^2}\right)^{-N_\Lambda}$$

The physical impact on cross sections is ordered by  $d_p$

Operator	$d_p$	$N_\chi$	NDA form
$\psi^2 \mathbf{U}$	3	1	$\Lambda \psi^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{U}}(h)$
$X^2$	4	2	$X^2 \mathcal{F}_{X^2}(h)$
$\psi^2 D$	4	2	$\psi^2 D$
$(\partial h)^2$	4	2	$(\partial h)^2$
$\mathbf{V}^2$	4	2	$\frac{\Lambda^2}{(4\pi)^2} \mathbf{V}^2 \mathcal{F}_{\mathbf{V}^2}(h)$
<hr/>			
$\psi^2 \mathbf{V}$	5	2	$\psi^2 \mathbf{V} \mathcal{F}_{\psi^2 \mathbf{V}}(h)$
$\psi^2 X \mathbf{U}$	5	2	$\frac{4\pi}{\Lambda} \psi^2 X \mathbf{U} \mathcal{F}_{\psi^2 X \mathbf{U}}(h)$
<hr/>			
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4 \mathcal{F}_{\psi^4}(h)$
$X \mathbf{V}^2$	6	3	$\frac{1}{4\pi} X \mathbf{V}^2 \mathcal{F}_{X \mathbf{V}^2}(h)$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3 \mathcal{F}_{X^3}(h)$
$X \mathbf{V} \partial$	6	3	$\frac{1}{4\pi} X \mathbf{V} \partial \mathcal{F}_{X \mathbf{V} \partial}(h)$
<hr/>			
$\psi^2 \mathbf{V} \mathbf{U} \partial$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V} \mathbf{U} \partial \mathcal{F}_{\psi^2 \mathbf{V} \mathbf{U} \partial}(h)$
$\psi^2 \mathbf{V}^2 \mathbf{U}$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V}^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{V}^2 \mathbf{U}}(h)$
$\psi^2 \mathbf{U} \partial^2$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{U} \partial^2 \mathcal{F}_{\psi^2 \mathbf{U} \partial^2}(h)$
<hr/>			
$\mathbf{V}^2 \partial^2$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^2 \partial^2 \mathcal{F}_{\mathbf{V}^2 \partial^2}(h)$
$\mathbf{V}^4$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^4 \mathcal{F}_{\mathbf{V}^4}(h)$

$d_p$  is orthogonal to the ordering for renormalisation:

Operator	$d_p$	$N_\chi$	NDA form
$\psi^2 \mathbf{U}$	3	1	$\Lambda \psi^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{U}}(h)$
$X^2$	4	2	$X^2 \mathcal{F}_{X^2}(h)$
$\psi^2 D$	4	2	$\psi^2 D$
$(\partial h)^2$	4	2	$(\partial h)^2$
$\mathbf{V}^2$	4	2	$\frac{\Lambda^2}{(4\pi)^2} \mathbf{V}^2 \mathcal{F}_{\mathbf{V}^2}(h)$
$\psi^2 \mathbf{V}$	5	2	$\psi^2 \mathbf{V} \mathcal{F}_{\psi^2 \mathbf{V}}(h)$
$\psi^2 X \mathbf{U}$	5	2	$\frac{4\pi}{\Lambda} \psi^2 X \mathbf{U} \mathcal{F}_{\psi^2 X \mathbf{U}}(h)$
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4 \mathcal{F}_{\psi^4}(h)$
$X \mathbf{V}^2$	6	3	$\frac{1}{4\pi} X \mathbf{V}^2 \mathcal{F}_{X \mathbf{V}^2}(h)$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3 \mathcal{F}_{X^3}(h)$
$X \mathbf{V} \partial$	6	3	$\frac{1}{4\pi} X \mathbf{V} \partial \mathcal{F}_{X \mathbf{V} \partial}(h)$
$\psi^2 \mathbf{V} \mathbf{U} \partial$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V} \mathbf{U} \partial \mathcal{F}_{\psi^2 \mathbf{V} \mathbf{U} \partial}(h)$
$\psi^2 \mathbf{V}^2 \mathbf{U}$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V}^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{V}^2 \mathbf{U}}(h)$
$\psi^2 \mathbf{U} \partial^2$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{U} \partial^2 \mathcal{F}_{\psi^2 \mathbf{U} \partial^2}(h)$
$\mathbf{V}^2 \partial^2$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^2 \partial^2 \mathcal{F}_{\mathbf{V}^2 \partial^2}(h)$
$\mathbf{V}^4$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^4 \mathcal{F}_{\mathbf{V}^4}(h)$

$d_p$  is orthogonal to the ordering for renormalisation:

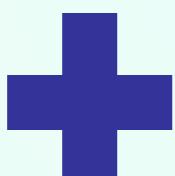
Operator	$d_p$	$N_\chi$	NDA form
LO	$\psi^2 \mathbf{U}$	3	$\Lambda \psi^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{U}}(h)$
	$X^2$	4	$X^2 \mathcal{F}_{X^2}(h)$
	$\psi^2 D$	4	$\psi^2 D$
	$(\partial h)^2$	4	$(\partial h)^2$
	$\mathbf{V}^2$	4	$\frac{\Lambda^2}{(4\pi)^2} \mathbf{V}^2 \mathcal{F}_{\mathbf{V}^2}(h)$
	$\psi^2 \mathbf{V}$	5	$\psi^2 \mathbf{V} \mathcal{F}_{\psi^2 \mathbf{V}}(h)$
	$\psi^2 X \mathbf{U}$	5	$\frac{4\pi}{\Lambda} \psi^2 X \mathbf{U} \mathcal{F}_{\psi^2 X \mathbf{U}}(h)$
	$\psi^4$	6	$\frac{(4\pi)^2}{\Lambda^2} \psi^4 \mathcal{F}_{\psi^4}(h)$
	$X \mathbf{V}^2$	6	$\frac{1}{4\pi} X \mathbf{V}^2 \mathcal{F}_{X \mathbf{V}^2}(h)$
	$X^3$	6	$\frac{(4\pi)}{\Lambda^2} X^3 \mathcal{F}_{X^3}(h)$
	$X \mathbf{V} \partial$	6	$\frac{1}{4\pi} X \mathbf{V} \partial \mathcal{F}_{X \mathbf{V} \partial}(h)$
	$\psi^2 \mathbf{V} \mathbf{U} \partial$	7	$\frac{1}{\Lambda} \psi^2 \mathbf{V} \mathbf{U} \partial \mathcal{F}_{\psi^2 \mathbf{V} \mathbf{U} \partial}(h)$
	$\psi^2 \mathbf{V}^2 \mathbf{U}$	7	$\frac{1}{\Lambda} \psi^2 \mathbf{V}^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{V}^2 \mathbf{U}}(h)$
	$\psi^2 \mathbf{U} \partial^2$	7	$\frac{1}{\Lambda} \psi^2 \mathbf{U} \partial^2 \mathcal{F}_{\psi^2 \mathbf{U} \partial^2}(h)$
	$\mathbf{V}^2 \partial^2$	8	$\frac{1}{(4\pi)^2} \mathbf{V}^2 \partial^2 \mathcal{F}_{\mathbf{V}^2 \partial^2}(h)$
	$\mathbf{V}^4$	8	$\frac{1}{(4\pi)^2} \mathbf{V}^4 \mathcal{F}_{\mathbf{V}^4}(h)$

$d_p$  is orthogonal to the ordering for renormalisation:

LO

Operator	$d_p$	$N_\chi$	NDA form
$\psi^2 \mathbf{U}$	3	1	$\Lambda \psi^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{U}}(h)$
$X^2$	4	2	$X^2 \mathcal{F}_{X^2}(h)$
$\psi^2 D$	4	2	$\psi^2 D$
$(\partial h)^2$	4	2	$(\partial h)^2$
$\mathbf{V}^2$	4	2	$\frac{\Lambda^2}{(4\pi)^2} \mathbf{V}^2 \mathcal{F}_{\mathbf{V}^2}(h)$

Operators necessary to absorb divergent contributions arising from the 1-loop renorm. of LO Lag



NLO

Operators encoding NP contributions with the same physical impact

$\psi^2 \mathbf{V}$	5	2	$\psi^2 \mathbf{V} \mathcal{F}_{\psi^2 \mathbf{V}}(h)$
$\psi^2 X \mathbf{U}$	5	2	$\frac{4\pi}{\Lambda} \psi^2 X \mathbf{U} \mathcal{F}_{\psi^2 X \mathbf{U}}(h)$
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4 \mathcal{F}_{\psi^4}(h)$
$X \mathbf{V}^2$	6	3	$\frac{1}{4\pi} X \mathbf{V}^2 \mathcal{F}_{X \mathbf{V}^2}(h)$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3 \mathcal{F}_{X^3}(h)$
$X \mathbf{V} \partial$	6	3	$\frac{1}{4\pi} X \mathbf{V} \partial \mathcal{F}_{X \mathbf{V} \partial}(h)$
$\psi^2 \mathbf{V} \mathbf{U} \partial$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V} \mathbf{U} \partial \mathcal{F}_{\psi^2 \mathbf{V} \mathbf{U} \partial}(h)$
$\psi^2 \mathbf{V}^2 \mathbf{U}$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V}^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{V}^2 \mathbf{U}}(h)$
$\psi^2 \mathbf{U} \partial^2$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{U} \partial^2 \mathcal{F}_{\psi^2 \mathbf{U} \partial^2}(h)$
$\mathbf{V}^2 \partial^2$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^2 \partial^2 \mathcal{F}_{\mathbf{V}^2 \partial^2}(h)$
$\mathbf{V}^4$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^4 \mathcal{F}_{\mathbf{V}^4}(h)$

# Disentangling the Higgs Nature

$$\Phi(x) = \frac{v + h}{\sqrt{2}} e^{i\sigma_a \pi^a(x)/v} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Higgs:  $h$

**Singlet**

GBs:  $\mathbf{U}(x) = e^{i\sigma_a \pi^a(x)/v}$

$$\mathbf{U}(x) \rightarrow L \mathbf{U}(x) R^\dagger$$

**Independent!!**

- Being  $h$  a singlet: generic functions of  $h$

$$\mathcal{F}_i(h) = 1 + 2\alpha_i \frac{h}{v} + \beta_i \frac{h^2}{v^2} + \dots$$

- Being  $\mathbf{U}(x)$  vs.  $h$  independent, many more operators can be constructed

# Disentangling the Higgs Nature

Operator	$d_p$	$N_\chi$	NDA form	SMEFT	↔	Linear sibling
$\psi^2 \mathbf{U}$	3	1	$\Lambda \psi^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{U}}(h)$	4		
$X^2$	4	2	$X^2 \mathcal{F}_{X^2}(h)$	4		
$\psi^2 D$	4	2	$\psi^2 D$	4		
$(\partial h)^2$	4	2	$(\partial h)^2$	4		
$\mathbf{V}^2$	4	2	$\frac{\Lambda^2}{(4\pi)^2} \mathbf{V}^2 \mathcal{F}_{\mathbf{V}^2}(h)$	4		
$\psi^2 \mathbf{V}$	5	2	$\psi^2 \mathbf{V} \mathcal{F}_{\psi^2 \mathbf{V}}(h)$	6, 8		
$\psi^2 X \mathbf{U}$	5	2	$\frac{4\pi}{\Lambda} \psi^2 X \mathbf{U} \mathcal{F}_{\psi^2 X \mathbf{U}}(h)$	6		
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4 \mathcal{F}_{\psi^4}(h)$	6		
$X \mathbf{V}^2$	6	3	$\frac{1}{4\pi} X \mathbf{V}^2 \mathcal{F}_{X \mathbf{V}^2}(h)$	6, 8		
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3 \mathcal{F}_{X^3}(h)$	6		
$X \mathbf{V} \partial$	6	3	$\frac{1}{4\pi} X \mathbf{V} \partial \mathcal{F}_{X \mathbf{V} \partial}(h)$	6		
$\psi^2 \mathbf{V} \mathbf{U} \partial$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V} \mathbf{U} \partial \mathcal{F}_{\psi^2 \mathbf{V} \mathbf{U} \partial}(h)$	8		
$\psi^2 \mathbf{V}^2 \mathbf{U}$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V}^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{V}^2 \mathbf{U}}(h)$	8		
$\psi^2 \mathbf{U} \partial^2$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{U} \partial^2 \mathcal{F}_{\psi^2 \mathbf{U} \partial^2}(h)$	8		
$\mathbf{V}^2 \partial^2$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^2 \partial^2 \mathcal{F}_{\mathbf{V}^2 \partial^2}(h)$	8		
$\mathbf{V}^4$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^4 \mathcal{F}_{\mathbf{V}^4}(h)$	8, 10		

# Disentangling the Higgs Nature

Operator	$d_p$	$N_\chi$	NDA form	SMEFT	↔	Linear sibling
$\psi^2 \mathbf{U}$	3	1	$\Lambda \psi^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{U}}(h)$	4		
$X^2$	4	2	$X^2 \mathcal{F}_{X^2}(h)$	4		
$\psi^2 D$	4	2	$\psi^2 D$	4		
$(\partial h)^2$	4	2	$(\partial h)^2$	4		
$\mathbf{V}^2$	4	2	$\frac{\Lambda^2}{(4\pi)^2} \mathbf{V}^2 \mathcal{F}_{\mathbf{V}^2}(h)$	4		
$\psi^2 \mathbf{V}$	5	2	$\psi^2 \mathbf{V} \mathcal{F}_{\psi^2 \mathbf{V}}(h)$	6, 8		
$\psi^2 X \mathbf{U}$	5	2	$\frac{4\pi}{\Lambda} \psi^2 X \mathbf{U} \mathcal{F}_{\psi^2 X \mathbf{U}}(h)$	6		
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4 \mathcal{F}_{\psi^4}(h)$	6		
$X \mathbf{V}^2$	6	3	$\frac{1}{4\pi} X \mathbf{V}^2 \mathcal{F}_{X \mathbf{V}^2}(h)$	6, 8		
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3 \mathcal{F}_{X^3}(h)$	6		
$X \mathbf{V} \partial$	6	3	$\frac{1}{4\pi} X \mathbf{V} \partial \mathcal{F}_{X \mathbf{V} \partial}(h)$	6		
$\psi^2 \mathbf{V} \mathbf{U} \partial$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V} \mathbf{U} \partial \mathcal{F}_{\psi^2 \mathbf{V} \mathbf{U} \partial}(h)$	8		
$\psi^2 \mathbf{V}^2 \mathbf{U}$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V}^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{V}^2 \mathbf{U}}(h)$	8		
$\psi^2 \mathbf{U} \partial^2$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{U} \partial^2 \mathcal{F}_{\psi^2 \mathbf{U} \partial^2}(h)$	8		
$\mathbf{V}^2 \partial^2$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^2 \partial^2 \mathcal{F}_{\mathbf{V}^2 \partial^2}(h)$	8		
$\mathbf{V}^4$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^4 \mathcal{F}_{\mathbf{V}^4}(h)$	8, 10		

# Disentangling the Higgs Nature

Operator	$d_p$	$N_\chi$	NDA form	SMEFT	↔	Linear sibling
$\psi^2 \mathbf{U}$	3	1	$\Lambda \psi^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{U}}(h)$	4		
$X^2$	4	2	$X^2 \mathcal{F}_{X^2}(h)$	4		
$\psi^2 D$	4	2	$\psi^2 D$	4		
$(\partial h)^2$	4	2	$(\partial h)^2$	4		
$\mathbf{V}^2$	4	2	$\frac{\Lambda^2}{(4\pi)^2} \mathbf{V}^2 \mathcal{F}_{\mathbf{V}^2}(h)$	4		
$\psi^2 \mathbf{V}$	5	2	$\psi^2 \mathbf{V} \mathcal{F}_{\psi^2 \mathbf{V}}(h)$	6, 8		
$\psi^2 X \mathbf{U}$	5	2	$\frac{4\pi}{\Lambda} \psi^2 X \mathbf{U} \mathcal{F}_{\psi^2 X \mathbf{U}}(h)$	6		
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4 \mathcal{F}_{\psi^4}(h)$	6		
$X \mathbf{V}^2$	6	3	$\frac{1}{4\pi} X \mathbf{V}^2 \mathcal{F}_{X \mathbf{V}^2}(h)$	6, 8		
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3 \mathcal{F}_{X^3}(h)$	6		
$X \mathbf{V} \partial$	6	3	$\frac{1}{4\pi} X \mathbf{V} \partial \mathcal{F}_{X \mathbf{V} \partial}(h)$	6		
$\psi^2 \mathbf{V} \mathbf{U} \partial$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V} \mathbf{U} \partial \mathcal{F}_{\psi^2 \mathbf{V} \mathbf{U} \partial}(h)$	8		
$\psi^2 \mathbf{V}^2 \mathbf{U}$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V}^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{V}^2 \mathbf{U}}(h)$	8		
$\psi^2 \mathbf{U} \partial^2$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{U} \partial^2 \mathcal{F}_{\psi^2 \mathbf{U} \partial^2}(h)$	8		
$\mathbf{V}^2 \partial^2$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^2 \partial^2 \mathcal{F}_{\mathbf{V}^2 \partial^2}(h)$	8		
$\mathbf{V}^4$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^4 \mathcal{F}_{\mathbf{V}^4}(h)$	8, 10		

# Disentangling the Higgs Nature

Operator	$d_p$	$N_\chi$	NDA form	SMEFT	↔	Linear sibling
$\psi^2 \mathbf{U}$	3	1	$\Lambda \psi^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{U}}(h)$	4		
$X^2$	4	2	$X^2 \mathcal{F}_{X^2}(h)$	4		
$\psi^2 D$	4	2	$\psi^2 D$	4		
$(\partial h)^2$	4	2	$(\partial h)^2$	4		
$\mathbf{V}^2$	4	2	$\frac{\Lambda^2}{(4\pi)^2} \mathbf{V}^2 \mathcal{F}_{\mathbf{V}^2}(h)$	4		
$\psi^2 \mathbf{V}$	5	2	$\psi^2 \mathbf{V} \mathcal{F}_{\psi^2 \mathbf{V}}(h)$	6, 8		
$\psi^2 X \mathbf{U}$	5	2	$\frac{4\pi}{\Lambda} \psi^2 X \mathbf{U} \mathcal{F}_{\psi^2 X \mathbf{U}}(h)$	6		
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4 \mathcal{F}_{\psi^4}(h)$	6		
$X \mathbf{V}^2$	6	3	$\frac{1}{4\pi} X \mathbf{V}^2 \mathcal{F}_{X \mathbf{V}^2}(h)$	6, 8		
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3 \mathcal{F}_{X^3}(h)$	6		
$\epsilon^{\mu\nu\rho\lambda} \text{Tr}(\mathbf{T} \mathbf{V}_\mu) \text{Tr}(\mathbf{V}_\nu W_{\rho\lambda}) \mathcal{F}_{14}(h)$ <span style="float: right;"><math>\varepsilon^{\mu\nu\rho\lambda} (\Phi^\dagger D_\mu \Phi) (\Phi^\dagger \sigma_i D_\nu \Phi) W_{\rho\lambda}^i</math></span>						
$\psi^2 \mathbf{U} \partial^2$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{U} \partial^2 \mathcal{F}_{\psi^2 \mathbf{U} \partial^2}(h)$	8		
$\mathbf{V}^2 \partial^2$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^2 \partial^2 \mathcal{F}_{\mathbf{V}^2 \partial^2}(h)$	8		
$\mathbf{V}^4$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^4 \mathcal{F}_{\mathbf{V}^4}(h)$	8, 10		

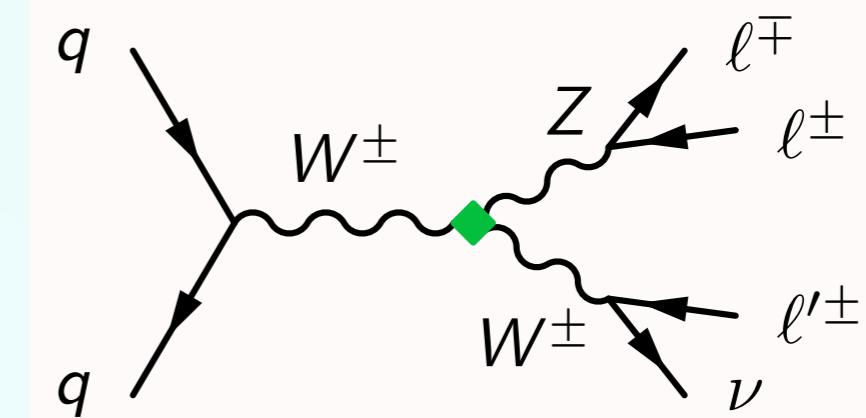
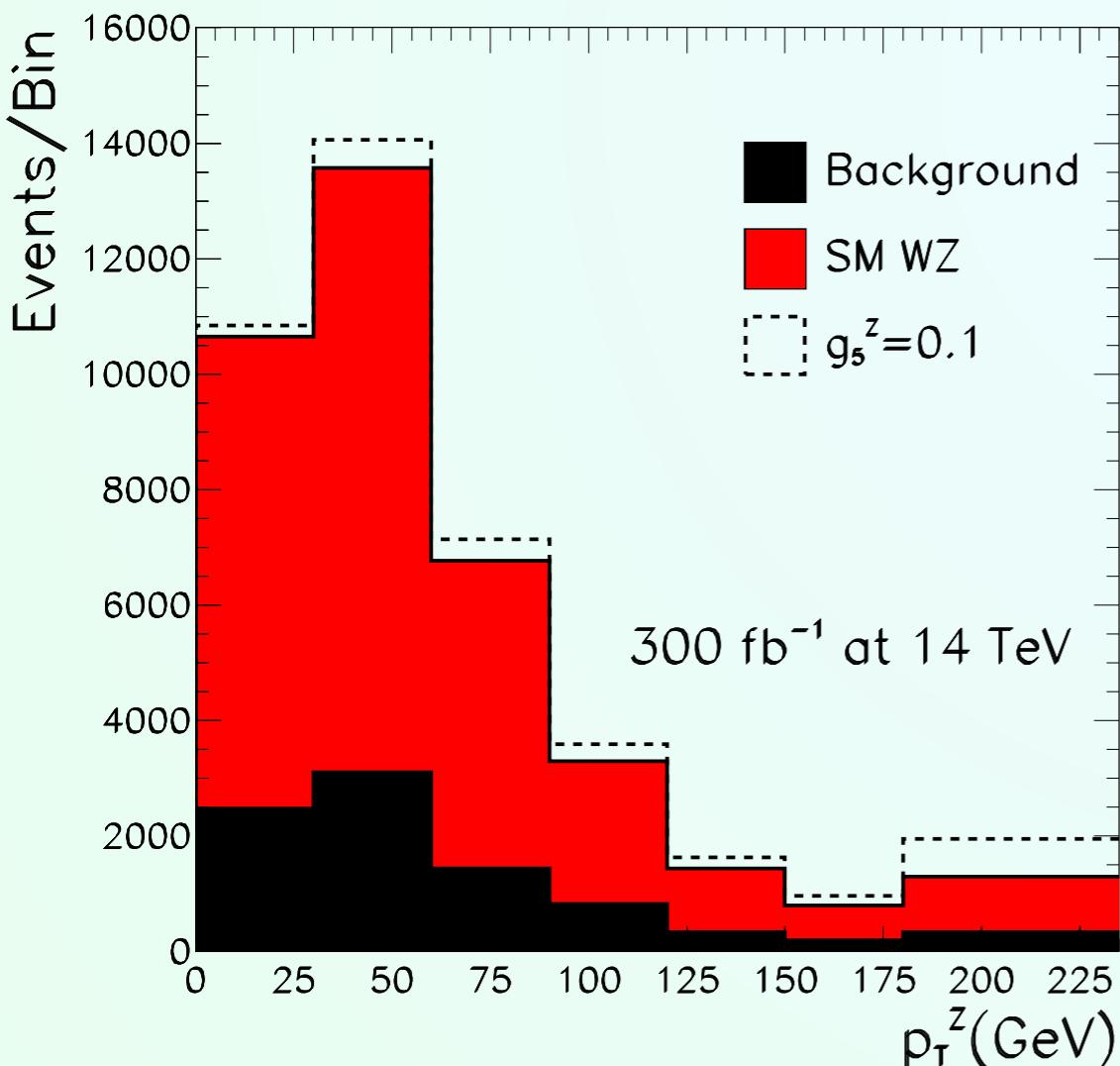
$$\epsilon^{\mu\nu\rho\lambda} \text{Tr}(\mathbf{T}\mathbf{V}_\mu) \text{Tr}(\mathbf{V}_\nu W_{\rho\lambda}) \mathcal{F}_{14}(h)$$

Brivio,Corbett,Eboli,Gavela,Gonzalez-Fraile, Gonzalez-Garcia,LM&Rigolin, JHEP 1403 (2014)

## Signals expected in the chiral basis, but not in the linear one (d=8)

$$g_5^Z \varepsilon^{\mu\nu\rho\lambda} \partial_\mu W_\nu^+ W_\rho^- Z_\lambda \mathcal{F}_{14}(h)$$

number of expected events (WZ production) with respect to the Z  $p_T$



@95% CL:

present  $g_5^Z \in [-0.08, 0.04]$   
 LHC(7+8+14)  $g_5^Z \in [-0.033, 0.028]$

# Conclusions

# Conclusions

- Use EFT is convenient and sometimes necessary. Many different counting can be defined.

# Conclusions

- Use EFT is convenient and sometimes necessary. Many different counting can be defined.
- The primary dimension counting:
  - measures the physical impact in terms of cross sections
  - is orthogonal to the renormalisation ordering(s)

# Conclusions

- Use EFT is convenient and sometimes necessary. Many different counting can be defined.
  - The primary dimension counting:
    - measures the physical impact in terms of cross sections
    - is orthogonal to the renormalisation ordering(s)
  - To disentangle the Higgs nature:
    - the presence of new signals
    - decorrelation signals (not discussed here)
    - ...
- and the primary dimension can tell which are the most promising couplings. Relevant for phenomenology!!

# Conclusions

- Use EFT is convenient and sometimes necessary. Many different counting can be defined.

■ The **dimensional analysis** is a powerful method to predict the number of independent couplings

## Thanks

- To disentangle the Higgs nature:

- the presence of new signals
- decorrelation signals (not discussed here)
- ...

and the primary dimension can tell which are the most promising couplings. Relevant for phenomenology!!

# Backup

# Comparison with Buchalla *et al.*

Buchalla, Catà & Krause, NPB 894 (2015)

## Counting based on derivatives

Naively expected at LO,  
as they are  $N_\chi = 2$

$$\tilde{N}_\chi = N_p + \frac{N_\psi}{2} + N_g + N_y + 2N_\lambda$$

$N_\chi$

Use one single parameter when  
naturally there are many!

## AD HOC ASSUMPTIONS

$$\bar{\psi}\psi \times \{g, y\}$$

$$X_{\mu\nu} \times g$$

otherwise 4 fermions at LO

LO

NLO

NNLO

NLO

Operator	$d_p$	$N_\chi$	NDA form
$\psi^2 \mathbf{U}$	3	1	$\Lambda \psi^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{U}}(h)$
$X^2$	4	2	$X^2 \mathcal{F}_{X^2}(h)$
$\psi^2 D$	4	2	$\psi^2 D$
$(\partial h)^2$	4	2	$(\partial h)^2$
$\mathbf{V}^2$	4	2	$\frac{\Lambda^2}{(4\pi)^2} \mathbf{V}^2 \mathcal{F}_{\mathbf{V}^2}(h)$
$\psi^2 \mathbf{V}$	5	2	$\psi^2 \mathbf{V} \mathcal{F}_{\psi^2 \mathbf{V}}(h)$
$\psi^2 X \mathbf{U}$	5	2	$\frac{4\pi}{\Lambda} \psi^2 X \mathbf{U} \mathcal{F}_{\psi^2 X \mathbf{U}}(h)$
$\psi^4$	6	2	$\frac{(4\pi)^2}{\Lambda^2} \psi^4 \mathcal{F}_{\psi^4}(h)$
$X \mathbf{V}^2$	6	3	$\frac{1}{4\pi} X \mathbf{V}^2 \mathcal{F}_{X \mathbf{V}^2}(h)$
$X^3$	6	3	$\frac{(4\pi)}{\Lambda^2} X^3 \mathcal{F}_{X^3}(h)$
$X \mathbf{V} \partial$	6	3	$\frac{1}{4\pi} X \mathbf{V} \partial \mathcal{F}_{X \mathbf{V} \partial}(h)$
$\psi^2 \mathbf{V} \mathbf{U} \partial$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V} \mathbf{U} \partial \mathcal{F}_{\psi^2 \mathbf{V} \mathbf{U} \partial}(h)$
$\psi^2 \mathbf{V}^2 \mathbf{U}$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{V}^2 \mathbf{U} \mathcal{F}_{\psi^2 \mathbf{V}^2 \mathbf{U}}(h)$
$\psi^2 \mathbf{U} \partial^2$	7	3	$\frac{1}{\Lambda} \psi^2 \mathbf{U} \partial^2 \mathcal{F}_{\psi^2 \mathbf{U} \partial^2}(h)$
$\mathbf{V}^2 \partial^2$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^2 \partial^2 \mathcal{F}_{\mathbf{V}^2 \partial^2}(h)$
$\mathbf{V}^4$	8	4	$\frac{1}{(4\pi)^2} \mathbf{V}^4 \mathcal{F}_{\mathbf{V}^4}(h)$

# Alternative $F_{\mu\nu}$ Normalisation

- Canonical normalisation of the gauge field strength kinetic terms as

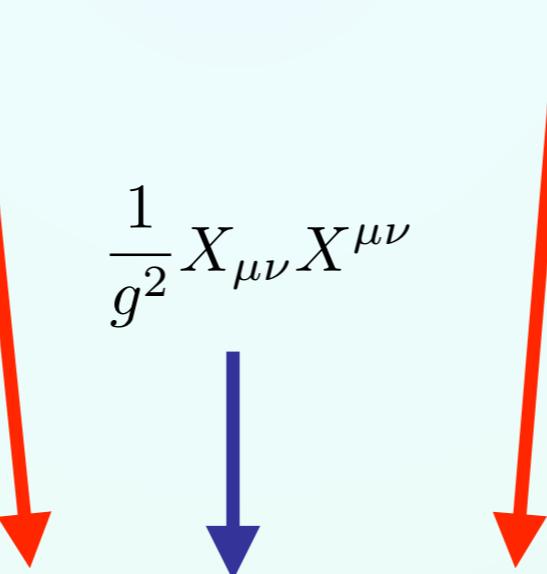
$$X_{\mu\nu} X^{\mu\nu}$$



$$\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi \phi}{\Lambda} \right]^{N_\phi} \left[ \frac{4\pi A}{\Lambda} \right]^{N_A} \left[ \frac{4\pi \psi}{\Lambda^{3/2}} \right]^{N_\psi} \left[ \frac{g}{4\pi} \right]^{N_g} \left[ \frac{y}{4\pi} \right]^{N_y} \left[ \frac{\lambda}{16\pi^2} \right]^{N_\lambda}$$

- Alternatively

$$\frac{1}{g^2} X_{\mu\nu} X^{\mu\nu}$$



$$\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{4\pi \phi}{\Lambda} \right]^{N_\phi} \left[ \frac{A}{\Lambda} \right]^{N_A} \left[ \frac{4\pi \psi}{\Lambda^{3/2}} \right]^{N_\psi} [g]^{N_g} \left[ \frac{y}{4\pi} \right]^{N_y} \left[ \frac{\lambda}{16\pi^2} \right]^{N_\lambda}$$

# $\bar{\psi}_L \gamma^\mu \mathbf{V}_\mu \psi_L$ & $\bar{\psi}_L \gamma^\mu [\mathbf{T}, \mathbf{V}_\mu] \psi_L$ in MFV

Alonso, Gavela, LM, Rigolin & Yepes, JHEP 1206 (2012)

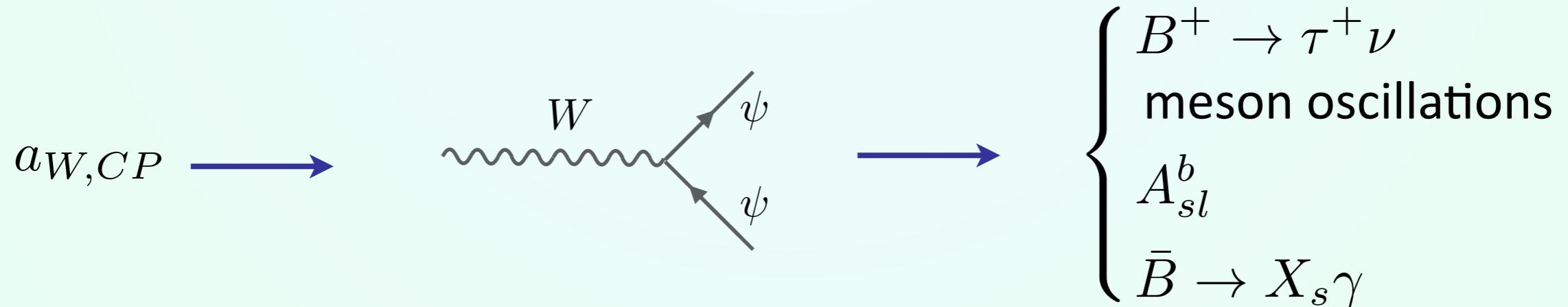
Alonso, Gavela, LM, Rigolin & Yepes, PRD 87 (2013)

It is easier to read the interaction vertices in the unitary gauge:

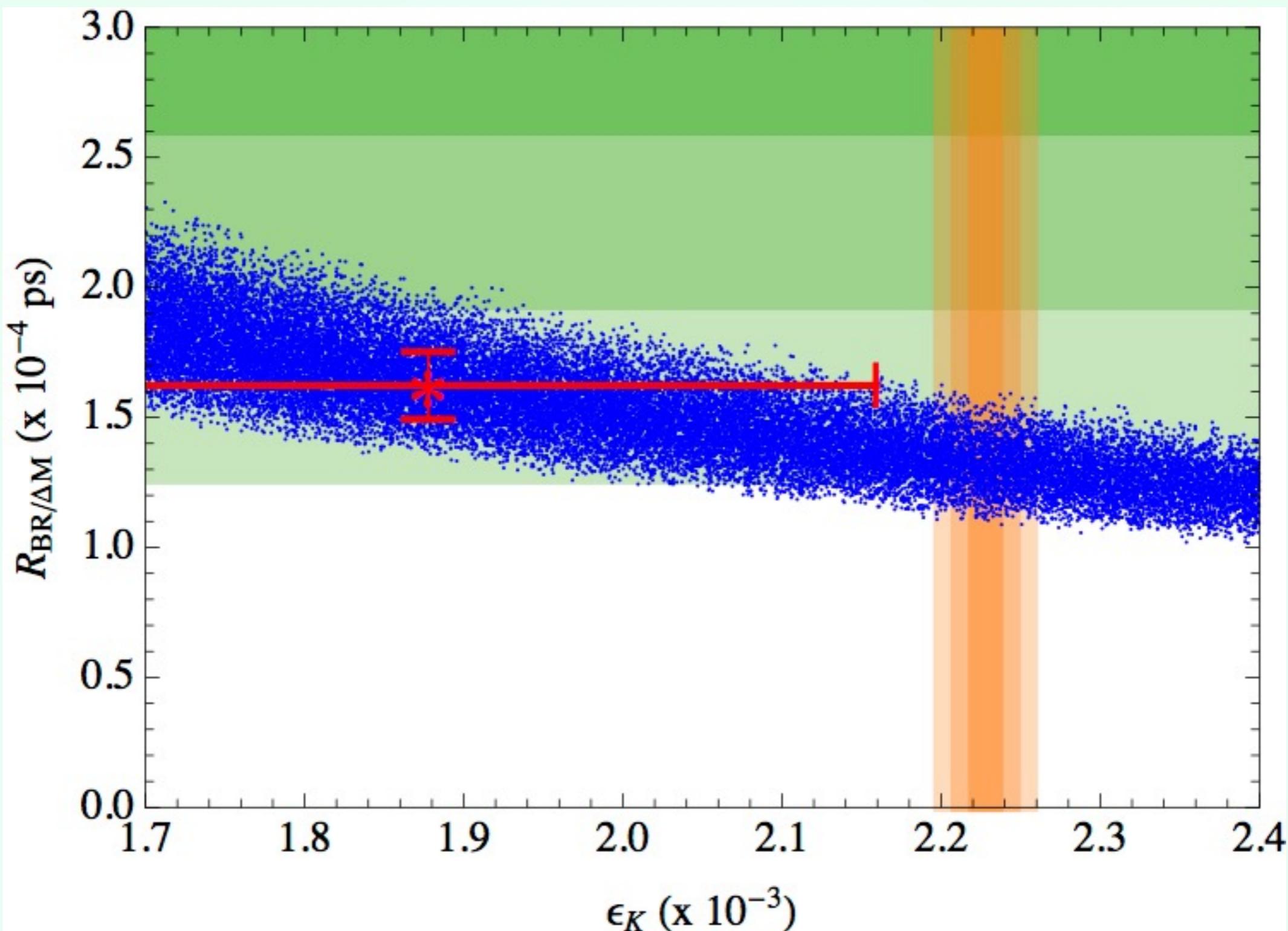
$$\mathcal{L}_{\chi=4}^f = -\frac{g}{\sqrt{2}} [W_\mu^+ \bar{U}_L \gamma^\mu [a_W(1 + \beta_W h/v) + ia_{CP}(1 + \beta_{CP} h/v)] \times \times (\mathbf{y}_U^2 V + V \mathbf{y}_D^2) D_L + \text{h.c.}] +$$

$\bar{\psi}_L \gamma^\mu \mathbf{V}_\mu \psi_L$

$\bar{\psi}_L \gamma^\mu [\mathbf{T}, \mathbf{V}_\mu] \psi_L$



$$R_{BR/\Delta M} \equiv BR(B^+ \rightarrow \tau^+ \nu) / \Delta M_{B_d}$$



SM values:

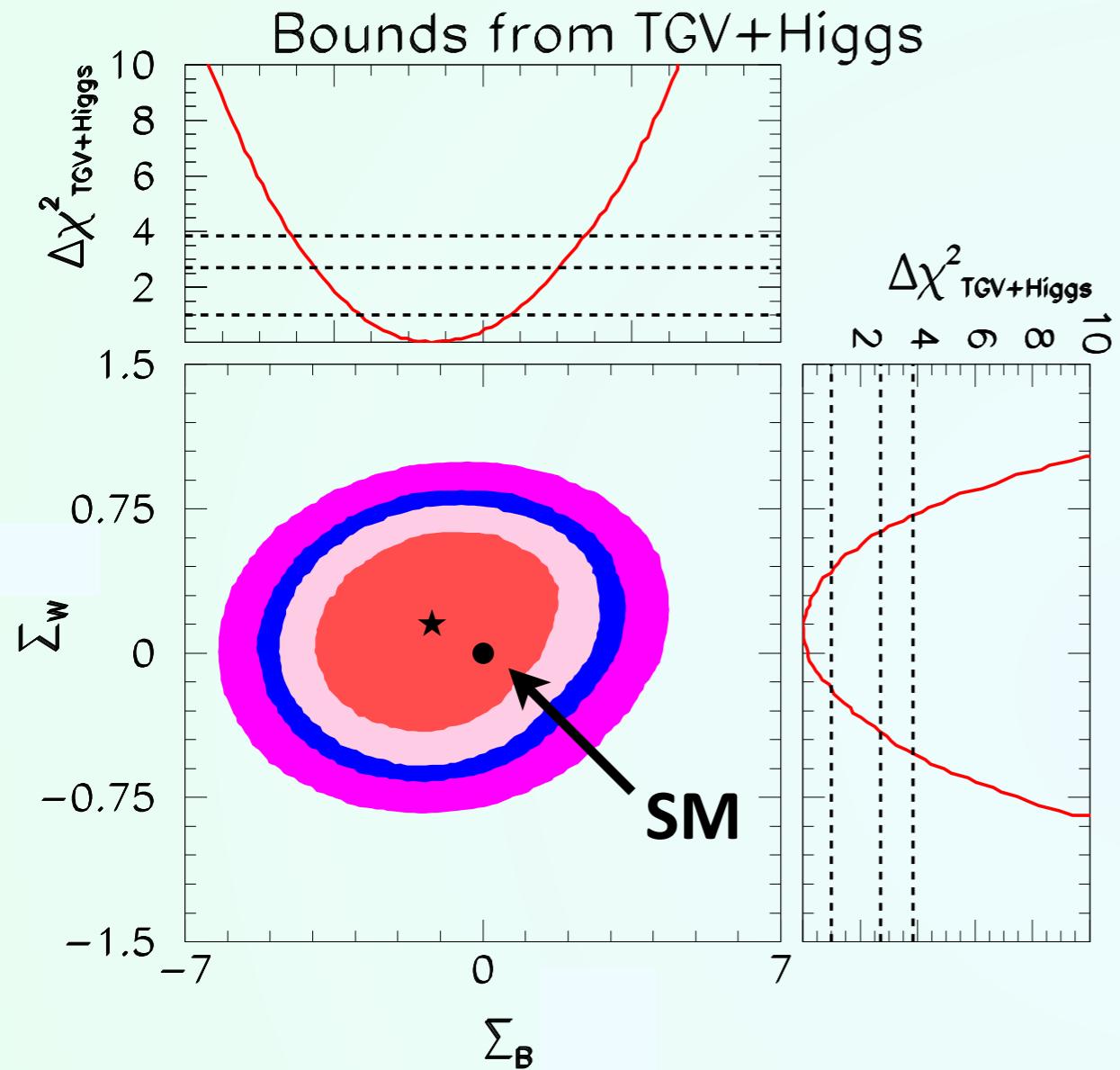
$$\epsilon_K \approx (1.88 \pm 0.3) \times 10^{-3}$$

$$R_{BR/\Delta M} \approx (1.62 \pm 0.13) \times 10^{-4}$$

$$a_W, a_{CP} \in [-1, 1]$$

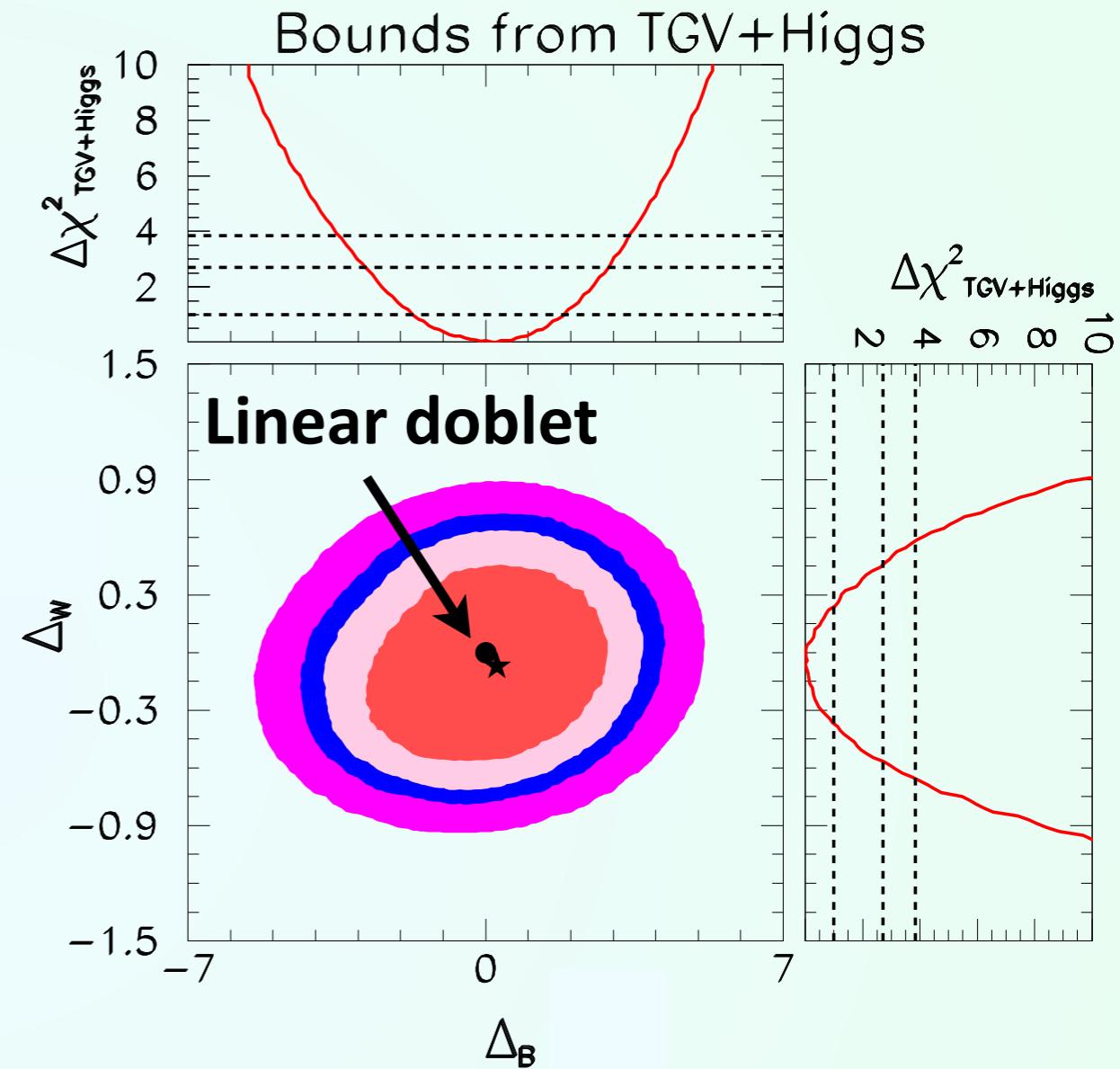
$$a_Z^d \in [-0.1, 0.1]$$

# Decorrelations



$$\Sigma_B = 4(2c_2 + a_4) \rightarrow f_B \xi$$

$$\Sigma_W = 2(2c_3 - a_5) \rightarrow f_W \xi$$



$$\Delta_B = 4(2c_2 - a_4) \rightarrow 0$$

$$\Delta_W = 2(2c_3 + a_5) \rightarrow 0$$

**Data:** Tevatron D0 and CDF Collaborations and LHC, CMS, and ATLAS Collaborations at 7 TeV and 8 TeV for final states  $\gamma\gamma$ ,  $W^+W^-$ ,  $ZZ$ ,  $Z\gamma$ ,  $b\bar{b}$ , and  $\tau\tau^-$

# Example of Decorrelation

**Correlations present in the linear basis are absent in the chiral basis**

$$\mathcal{O}_B = (D_\mu \Phi)^\dagger \hat{B}^{\mu\nu} (D_\nu \Phi)$$

# Example of Decorrelation

**Correlations present in the linear basis are absent in the chiral basis**

$$\begin{aligned}\mathcal{O}_B = & \frac{ieg^2}{8} A_{\mu\nu} W^{-\mu} W^{+\nu} (v + h)^2 - \frac{ie^2 g}{8 \cos \theta_W} Z_{\mu\nu} W^{-\mu} W^{+\nu} (v + h)^2 \\ & - \frac{eg}{4 \cos \theta_W} A_{\mu\nu} Z^\mu \partial^\nu h (v + h) + \frac{e^2}{4 \cos^2 \theta_W} Z_{\mu\nu} Z^\mu \partial^\nu h (v + h)\end{aligned}$$



# Example of Decorrelation

**Correlations present in the linear basis are absent in the chiral basis**

$$\begin{aligned}\mathcal{O}_B = & \frac{ieg^2}{8} A_{\mu\nu} W^{-\mu} W^{+\nu} (v + h)^2 - \frac{ie^2 g}{8 \cos \theta_W} Z_{\mu\nu} W^{-\mu} W^{+\nu} (v + h)^2 \\ & - \frac{eg}{4 \cos \theta_W} A_{\mu\nu} Z^\mu \partial^\nu h (v + h) + \frac{e^2}{4 \cos^2 \theta_W} Z_{\mu\nu} Z^\mu \partial^\nu h (v + h)\end{aligned}$$

→  $\mathcal{O}_B = \frac{v^2}{16} \mathcal{P}_2(h) + \frac{v^2}{8} \mathcal{P}_4(h) \quad \text{with} \quad \mathcal{F}_i(h) = \left(1 + \frac{h}{v}\right)^2$

$$\mathcal{P}_2(h) = 2ieg^2 A_{\mu\nu} W^{-\mu} W^{+\nu} \mathcal{F}_2(h) - 2 \frac{ie^2 g}{\cos \theta_W} Z_{\mu\nu} W^{-\mu} W^{+\nu} \mathcal{F}_2(h)$$

$$\mathcal{P}_4(h) = - \frac{eg}{\cos \theta_W} A_{\mu\nu} Z^\mu \partial^\nu \mathcal{F}_4(h) + \frac{e^2}{\cos^2 \theta_W} Z_{\mu\nu} Z^\mu \partial^\nu \mathcal{F}_4(h)$$



# Example of Decorrelation

**Correlations present in the linear basis are absent in the chiral basis**

$$\begin{aligned}\mathcal{O}_B = & \frac{ieg^2}{8} A_{\mu\nu} W^{-\mu} W^{+\nu} (v + h)^2 - \frac{ie^2 g}{8 \cos \theta_W} Z_{\mu\nu} W^{-\mu} W^{+\nu} (v + h)^2 \\ & - \frac{eg}{4 \cos \theta_W} A_{\mu\nu} Z^\mu \partial^\nu h (v + h) + \frac{e^2}{4 \cos^2 \theta_W} Z_{\mu\nu} Z^\mu \partial^\nu h (v + h)\end{aligned}$$

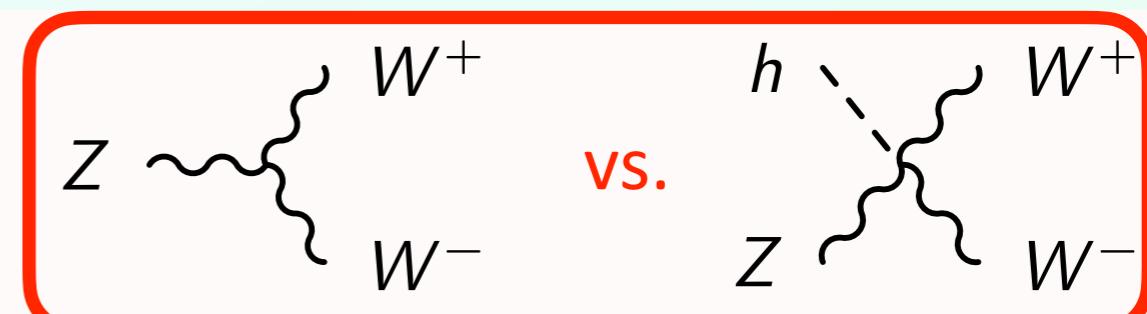
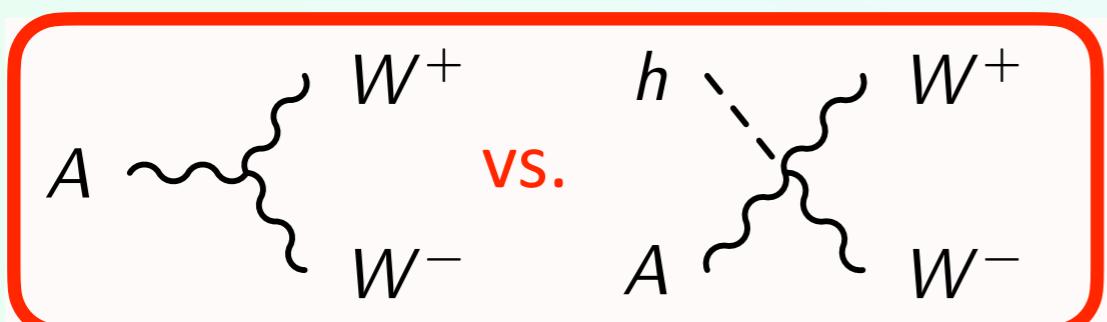
→  $\mathcal{O}_B = \frac{v^2}{16} \mathcal{P}_2(h) + \frac{v^2}{8} \mathcal{P}_4(h) \quad \text{with} \quad \mathcal{F}_i(h) = \left(1 + \frac{h}{v}\right)^2$

$$\mathcal{P}_2(h) = 2ieg^2 A_{\mu\nu} W^{-\mu} W^{+\nu} \mathcal{F}_2(h) - 2 \frac{ie^2 g}{\cos \theta_W} Z_{\mu\nu} W^{-\mu} W^{+\nu} \mathcal{F}_2(h)$$

$$\mathcal{P}_4(h) = -\frac{eg}{\cos \theta_W} A_{\mu\nu} Z^\mu \partial^\nu \mathcal{F}_4(h) + \frac{e^2}{\cos^2 \theta_W} Z_{\mu\nu} Z^\mu \partial^\nu \mathcal{F}_4(h)$$

due to the decorrelation in the  $\mathcal{F}_i(h)$  functions: i.e.

[see also Isidori&Trott, 1307.4051]



# Example of Decorrelation

**Correlations present in the linear basis are absent in the chiral basis**

$$\begin{aligned}\mathcal{O}_B = & \frac{ieg^2}{8} A_{\mu\nu} W^{-\mu} W^{+\nu} (v + h)^2 - \frac{ie^2 g}{8 \cos \theta_W} Z_{\mu\nu} W^{-\mu} W^{+\nu} (v + h)^2 \\ & - \frac{eg}{4 \cos \theta_W} A_{\mu\nu} Z^\mu \partial^\nu h (v + h) + \frac{e^2}{4 \cos^2 \theta_W} Z_{\mu\nu} Z^\mu \partial^\nu h (v + h)\end{aligned}$$

→  $\mathcal{O}_B = \frac{v^2}{16} \mathcal{P}_2(h) + \frac{v^2}{8} \mathcal{P}_4(h) \quad \text{with} \quad \mathcal{F}_i(h) = \left(1 + \frac{h}{v}\right)^2$

$$\mathcal{P}_2(h) = 2ieg^2 A_{\mu\nu} W^{-\mu} W^{+\nu} \mathcal{F}_2(h) - 2 \frac{ie^2 g}{\cos \theta_W} Z_{\mu\nu} W^{-\mu} W^{+\nu} \mathcal{F}_2(h)$$

$$\mathcal{P}_4(h) = -\frac{eg}{\cos \theta_W} A_{\mu\nu} Z^\mu \partial^\nu \mathcal{F}_4(h) + \frac{e^2}{\cos^2 \theta_W} Z_{\mu\nu} Z^\mu \partial^\nu \mathcal{F}_4(h)$$

due to the nature of the chiral operators (different  $c_i$  coefficients): i.e.

