Polarization theory

Part I: Polarized scattering with `MadGraph5_aMC@NLO`

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Part II: VBS with `Phantom` (Giovanni Pellicciolli)

Part I: Polarized scattering with MadGraph5_aMC@NLO

1. Introduction
2. Scattering with polarized particles
   + generating polarized samples with MG
3. Unstable particles, decays, NWA and angular correlation.
   + decaying polarized particles with MG
4. Angular variables
5. Analysis of polarization variables with MG + Pythia + Delphes
6. Final remarks

Preparations

(blue texts are links)

- Install `MadGraph5_aMC@NLO`
  Just download the latest version or pull it with Bazaar
- Install ROOT for the angular variable analysis
  Simplest way via conda
- From MG prompt install MadAnalysis5, Pythia8 and Delphes

```
>install pythia8
>install Delphes
```

- Install Jupyter
- Install some Python3 packages via pip:
  pandas, numpy, matplotlib, h5py

Motivation

The production of asymmetrically polarized fermions and weak bosons at colliders,

- is a test of the chiral nature of the SM;
- is a prediction in several promising models BSM,
  e.g. composite Higgs, extended gauge theories and SUSY.
- BSM effects parametrized in EFT operators, e.g.
  - In $t\bar{t}$ e.g. Bernreuther, Si 1305.2066
  - In Vector Boson Scattering e.g. in CH, DBF, Mattelaer, Ruiz, Shil 1912.01725, Li, Yang, Zhang 2010.13281
- A simulation tool is desired to allow accounting for kinematical cuts and higher order effects in production. Two available:
  1. MadGraph JHEP 04 (2020) 082 (1912.01725), DBF, Mattelaer, Ruiz, Sujay
  2. Phantom JHEP 03 (2018) 170 (1710.09339), Ballestrero, Maina, Pelliccioli

Unpolarized scattering

$$\left.\frac{d\sigma(pp\to\mathcal{B}+X)}{d\tilde{\mathcal{O}}}\right|_{\tilde{\mathcal{O}}=\tilde{\mathcal{O}}_0} = f\otimes f\otimes\Delta\otimes\left.\frac{d\hat{\sigma}}{d\tilde{\mathcal{O}}}\right|_{\tilde{\mathcal{O}}=\tilde{\mathcal{O}}_0} + \mathcal{O}\left(\frac{\Lambda^t_{\mathrm{NP}}}{Q^{t+2}}\right)$$

$$= \sum_{i,j=q,g,\gamma}\int_{\tau_0}^1 d\tau\int_\tau^1\frac{d\xi_1}{\xi_1}\int_{\tau/\xi_1}^1\frac{dz}{z}\frac{1}{(1+\delta_{ij})}$$

$$\times\left[f_{i/p}(\xi_1,\mu_f)f_{j/p}(\xi_2,\mu_f)+(1\leftrightarrow 2)\right]\times\Delta_{ij}(z,\mu_f,\mu_r,\mu_s)$$

$$\times\left.\frac{d\hat{\sigma}(ij\to\mathcal{B};\{Q^2,s,\mu_f,\mu_r,\mu_s\})}{d\tilde{\mathcal{O}}}\right|_{\tilde{\mathcal{O}}=\tilde{\mathcal{O}}_0} + \mathcal{O}\left(\frac{\Lambda^t_{\mathrm{NP}}}{Q^{t+2}}\right).$$

- Specific observables $\tilde{\mathcal{O}}$ can be derived from the fully differentiated scattering rate

$$\frac{d\hat{\sigma}(ij\to\mathcal{B})}{dPS_n} = \frac{1}{2Q^2}\frac{1}{(2s_i+1)(2s_j+1)N_c^iN_c^j}\sum_{\mathrm{dof}}|\mathcal{M}(ij\to\mathcal{B})|^2.$$

- With the usual final (initial) state summed (averaged) dof.

$$\overline{|\mathcal{M}(ij\to\mathcal{B})|^2} \equiv \frac{1}{(2s_i+1)(2s_j+1)N_c^iN_c^j}\sum_{\mathrm{dof}}|\mathcal{M}(ij\to\mathcal{B})|^2.$$

Helicity-polarized scattering

- We can drop the summation over helicities and define helicity-polarized matrix elements $\mathcal{M}(i_\lambda j_{\lambda'} \to \mathcal{B}_{\tilde{\lambda}})$ and the colored averaged matrix element squared

$$\overline{|\mathcal{M}(i_\lambda j_{\lambda'} \to \mathcal{B}_{\tilde{\lambda}})|^2} \equiv \frac{1}{N_c^{i_\lambda} N_c^{j_{\lambda'}}} \sum_{\text{color}} |\mathcal{M}(i_\lambda j_{\lambda'} \to \mathcal{B}_{\tilde{\lambda}})|^2.$$

  $\tilde{\lambda}$ is a collective index for all final state helicities.
- We can also do partial sums and pick specific polarizations.

MadGraph implementation

Use $\mathcal{P}\{X\}$: polarization X of particle $\mathcal{P}$,

- spin 1/2: Left ($\{X\} = \{-\}, \{L\}$), Right ($\{X\} = \{+\}, \{R\}$)
- spin 1: Left ($\{-\}, \{L\}$), Right ($\{+\}, \{R\}$), longitudinal $\{0\}$, transverse $\{T\}$
- spin 3/2: $\{-3\}, \{-1\}, \{1\}, \{3\}$
- spin 2: $\{-2\}, \{-1\}, \{0\}, \{1\}, \{2\}$

Exercise 1: $e^+e^- \to \mu^+\mu^-$ in QED helicity structure

- Compute with MG the helicity amplitudes in the relativistic limit ($m_e = m_\mu = 0$) and compare with Peskin, Schröder (5.23), (5.24).

$$\frac{d\sigma}{d\Omega}(RL \to RL) = \frac{\alpha^2}{4E_{CM}^2}(1 + \cos\theta)^2$$
$$\frac{d\sigma}{d\Omega}(RL \to LR) = \frac{\alpha^2}{4E_{CM}^2}(1 - \cos\theta)^2$$

- Run `./bin/mg5_aMC mg_ex1.cmd` (scripts can be found in Material)

```
> generate e+ e- > mu+ mu- / z ! unpolarized
> output PROC_ee-mumu
> generate e+{L} e-{R} > mu+{L} mu-{R} / z
> output PROC_eLeR-muLmuR
> generate e+{L} e-{R} > mu+{R} mu-{L} / z
> output PROC_eLeR-muLmuR
```

- What about the other polarizations? What do you expect?
- Don't forget initial state dof are averaged and final ones summed.

Exercise 1 (cont.): $e^+e^- \to \mu^+\mu^-$ in QED helicity structure

- A bit of Fortran. Modify `SubProcesses/dummy_fct.f` to get the cross section for $\theta > 0$. What do you expect?

```
      real*8 rap
      external rap

c p(mu,3) is the mu+
      if ( rap(p(0,3)) .gt.0.0d0 ) then
         dummy_cuts=.false.
         return
      endif
```

- Note: Fractional initial state polarization from -100 (left-handed) to 100 (right-handed) can be set in run_card.dat with e.g.

```
 -70  = polbeam1
```

Frame dependence

- Standard contruction of helicity amplitudes in terms of asymptotic non-interacting wave-functions QFT, Vol. I, Weinberg

$$\mathcal{M}(p_1\sigma_1 p_2\sigma_2 \ldots; q_1\bar{\sigma}_1 q_2\bar{\sigma}_2 \ldots\ldots) \propto \phi_1(p_1,\sigma_1)\phi_2(p_2,\sigma_2)\ldots\bar{\phi}_1(q_1,\bar{\sigma}_1)\bar{\phi}_2(q_2,\bar{\sigma}_2)$$

- Under homogeneous Lorentz transformation Λ^μ,

$$\phi(p,\sigma) \to \sum_{\sigma'} D_{\sigma\sigma'}\phi(\Lambda^{-1}p,\sigma')$$

  Only the $\sum_\sigma \phi^\dagger(p,\sigma)\phi(p,\sigma)$ is Lorentz invariant.
- This fact can also be seem explicitly in the spin sum completeness relations, e.g.

$$\sum_{\lambda\in\{\pm 1\}} u(q,\lambda)\overline{u}(q,\lambda) = (\not{q}+m); \quad \sum_{\lambda\in\{\pm 1\}} v(q,\lambda)\overline{v}(q,\lambda) = (\not{q}-m)$$

$$\sum_\lambda \epsilon^\mu_\lambda \epsilon^\nu_\lambda = -\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m^2}\right)$$

Exercise 2: Obtain general forms spinors and polarization vec.

- Dirac eqs: $(\not{p}-m)u(p) = 0$, $(\not{p}+m)v(p) = 0$ Peskin Sec. 3.3
- Massive spin-1 (Proca eq.): $(p^2-m^2)\epsilon^\mu - p^\mu p\cdot\epsilon = 0 \to \epsilon\cdot p = 0$
- Start from rest $p^\mu = (m,\vec{0})$ and boost in the z-direction, e.g. $\epsilon^\mu_0 = \frac{1}{m}(p_z,0,0,E)$

MG frame definition

- The *frame* where polarizations are defined can be set in run_card.dat list of particles to sum-up to define the frame, e.g.

```
[1,2] = me_frame ! partonic frame
[3,4] = me_frame ! p p > w+ w- j j (ww C.M.)
```

Exercise 3: Helicity vs. Chirality

- Using the ansatz $\Psi = \binom{\Phi_0}{\chi_0} e^{-ip\cdot x}$, deduce $\chi_0 = \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\Phi_0$.
- Choose Φ_0, χ_0 as *helicity* $\hat{\Lambda} = \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}$ eigenstates.
- Determine the relation of Φ_0, χ_0 for eigenstates of *chirality* γ_5.
- Show that for massless fermions helicity and chirality coincide.

Exercise 4: $e^+e^- \to t\bar{t}$ frame dependence

- Check with MG that the predictions for a massless left-handed e^- is frame independent

  change between partonic frame and top rest frame
- Check that the top helicity is frame dependent.
- Top-quark is unstable and manifest as a resonance. How can we assess its polarization information?
- Look at `./bin/mg_aMC mg_ex5.cmd`

Decays of Helicity-Polarized Resonances

- The helicity of final state particles are not detected at big colliders.
- Helicities of initial particles can be prepared in some specific aparatus.
- Otherwise helicities are only extracted probabilistically from the experiment-theory comparison,
- or via the decay products in case of unstable particles decaying quickly.

Spin correlated narrow-width approximation (NWA)

- (uncorrelated) NWA: $[(q^2 - M^2)^2 + (M\Gamma)^2]^{-1} \rightarrow \pi/(M\Gamma)\delta(q^2 - M^2)$. Scattering cross section decouples into on-shell production times decay.
- Spin correlated NWA Frixione, Laenen, Motylinski, Webber (0702198): keep numerator in the amplitude.
- Full resonant amplitude constructed with the propagator with a cut on the invariant mass $q^2 \sim M^2$. Spin correlation and Breit-Wigner shape.

Polarized NWA

- Define "spin-truncated" propagators with off-shell wave functions.
- Implemented in MG: massive spin-1

$$\Pi_{\mu\nu}(q, M_V, \Gamma_V) = \frac{-i\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{M_V^2}\right]}{q^2 - M_V^2 + iM_V\Gamma_V}$$

$$-i\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{M_V^2}\right] = \sum_{\lambda\in\{0,\pm1,A\}} -i\varepsilon_\mu(q,\lambda)\varepsilon^*_\nu(q,\lambda)$$

- ϵ_μ, u, v defined in HELAS conventions. $\varepsilon^\mu(q, \lambda = A) = \frac{q^\mu}{M_V}\sqrt{\frac{q^2-M_V^2}{q^2}}$.
- And spin-1/2

$$S_F(q, m_q, \Gamma_q) = \frac{i(\not{q} + m)}{q^2 - m_q^2 + im_q\Gamma_q}, \quad S_{\overline{F}}(q, m_q, \Gamma_q) = \frac{-i(\not{q} - m)}{q^2 - m_q^2 + im_q\Gamma_q}$$

$$\not{q} + m = \sum_{\lambda\in\{\pm1\}} \frac{1}{2}\left[\left(1 + \frac{m}{\sqrt{q^2}}\right) u(q,\lambda)\overline{u}(q,\lambda) + \left(1 - \frac{m}{\sqrt{q^2}}\right) v(q,\lambda)\overline{v}(q,\lambda)\right]$$

$$\not{q} - m = \sum_{\lambda\in\{\pm1\}} \frac{1}{2}\left[\left(1 - \frac{m}{\sqrt{q^2}}\right) u(q,\lambda)\overline{u}(q,\lambda) + \left(1 + \frac{m}{\sqrt{q^2}}\right) v(q,\lambda)\overline{v}(q,\lambda)\right]$$

- Note the difference w.r.t. the onshell completeness relations.

MadGraph implementation

- Use usual $\mathcal{P}\{X\}$ syntax
- Decay via decay chain syntax, e.g.

  `> generate p p > w+ w-{X} , w- > mu- vm~`
- or with MadSpin, in `madspin_card.dat` use the usual syntax

  `decay w- > mu- vm~`
- Polarized decay in MG neglects interference between different helicities

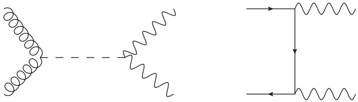
  Resonances are not asymptotic states and interferences are expected
- The supported resonances are :
  - Massive spin 1/2: Left ($\{X\} = \{-\}, \{L\}$), Right ($\{X\} = \{+\}, \{R\}$)
  - Massive spin 1: longitudinal $\{0\}$, transverse $\{T\}$
- Warning about symmetry factors (s.f)

  $\frac{1}{2}ZZ \to \frac{1}{2}(Z_L + Z_T)(Z_L + Z_T) = \frac{1}{2}(Z_L Z_L + Z_T Z_T) + Z_L Z_T$

  Correct s.f.

  $ZZ_T \to (Z_L + Z_T)(Z_T) = Z_L Z_T + Z_T Z_T$ Wrong s.f.

A practical example: $pp \to W^+ W^-_\lambda$, SM and heavy scalar



- What is the expectation for polarizations from a production of a 500 GeV scalar, compared to the SM production
- Let us choose the W^+W^- rest frame to define the polarizations (at LO this is equivalent to partonic rest frame)

Exercise 5: generate $pp \to W^+ W^-_\lambda$ samples

- You need a HEFT model to get the ggH coupling.
- We can consider a $m_H = 500$ GeV and $\Gamma_H = 50$ GeV with Higgs-like couplings.
- Decay with MadSpin in the semi-leptonic channel, $W^+ \to jj, \quad W^-_\lambda \to \ell^- \bar{\nu}$
  Modify `madspin_card.dat` accordingly

```
decay w+ > j j
decay w- > l- vl~
```

- See the script `./bin/mg5_aMC mg_ex5.cmd`

Angular variables

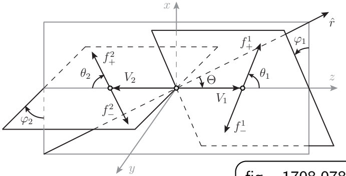


fig. 1708.07823

The optimal observables to distinguish different polarization are the so-called angular variables θ, ϕ. e.g. Collins, Soper 77, Stirling, Vryonidou 1204.6427

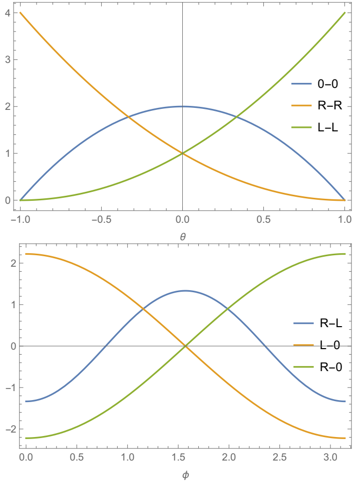
Exercise 6: $W_\lambda^- \to \ell^- \nu$ decay at LO

- Compute the helicity amplitudes $W_\lambda^- \to \ell^- \nu$ at tree-level and find

$$\mathcal{M}_0^D(\theta^*, \phi) \sim \sqrt{2} \sin\theta^* ,$$
$$\mathcal{M}_{L/R}^D(\theta^*, \phi) \sim (\cos\theta^* \pm 1) e^{\mp i\phi}$$

- In the spirit of the NWA, factorize the matrix element as a product times decay, $\mathcal{M} = \mathcal{M}_0^P \mathcal{M}_0^D + \mathcal{M}_L^P \mathcal{M}_L^D + \mathcal{M}_R^P \mathcal{M}_R^D$ and find (assume production matrix purely real or imaginary)

$$\frac{1}{\sigma} \frac{d^2\sigma}{d\cos\theta_1 d\varphi_1} = \frac{3}{16\pi} \Big[(1+\cos\theta_1)^2 f_L + (1-\cos\theta_1)^2 f_R + 2\sin^2\theta_1 f_0 - g_{RL} \sin^2\theta_1 \cos(2\varphi_1) - \sqrt{2} g_{L0} \sin\theta_1 (1+\cos\theta_1)\cos\varphi_1 + \sqrt{2} g_{R0} \sin\theta (1-\cos\theta_1)\cos\varphi_1 \Big] .$$



Integrated in ϕ

- Interference vanishes

Integrated in θ

- Diagonal part uniform

Simple analysis of angular variables in $pp \to (H \to) W^+ W^-_\lambda$

1. Generate partonic events, parton shower with Pythia8, perform fast detector simulation with Delphes3 (done in previous exercize)
2. Extract relevant observables from Delphes ROOT files with pyROOT and write them into HDF5 format
3. Read HDF5 files for ploting and further analysis

2 - Extracting observables from Delphes ROOT

Check out `root2h5.ipynb` in Material

1. Import packages
2. Get truth level information
3. Reconstruct neutrino from MET
4. Reconstruct $W^{\pm}$ bosons
5. Write down observables

3 - Analysis of observables

Check out `Analysis.ipynb` in Material

1. Compare Detector simulation with partonic level
2. What variables can we use to enhance the BSM signal?

Polarization fractions

- Integrating in ϕ we lose interference information

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta^*} = \frac{3}{8}(1+\cos\theta^*)^2 f_L + \frac{3}{8}(1-\cos\theta^*)^2 f_R + \frac{3}{4}\sin^2\theta^* f_0 .$$

- Ideally, the polarization fractions can be extracted from a fit to the θ^* distribution, e.g. using a Legendre expansion

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta} = \sum_{l=0}^{2}\alpha_l P_l(\cos\theta), \quad f_L = \frac{2}{3}(\alpha_0+\alpha_1+\alpha_2), \quad f_R = \frac{2}{3}(\alpha_0-\alpha_1+\alpha_2), \quad f_0 = \frac{2}{3}(\alpha_0-2\alpha_2).$$

- Kinematical selection (e.g. detector acceptance), NLO effects and new physics can disturb the LO expression (importance of MC simulation).

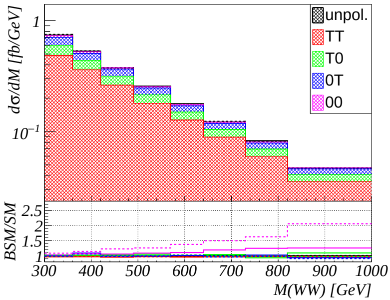
  See e.g. Ballestrero, Maina, Pelliccioli 1710.09339, DBF, Mattelaer, Ruiz, Shil 1912.01725

Exercise 7: Polarization fractions of $W^+W^-_\lambda$

- Extract f_L, f_R, f_0 from the unpolarized $\cos\theta^*$ distribution. Define and extract also f_T.
- Compare with generation.

Final remarks and applications

- Measurements of f_X are SM test and might be an import source of BSM physics, e.g. in strong longitudinal VBS



1912.01725

BSM scenarios $\kappa_V \equiv a \equiv \cos\theta$

$a = 0.8,\ a = 0.9$

Selection cuts

$p_T(j) > 20\,\text{GeV}, \quad |\eta(j)| < 5$

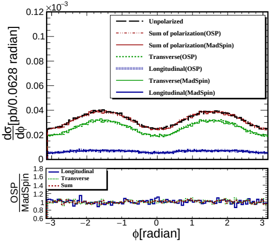
$M(jj) > 250\,\text{GeV}, \quad \Delta\eta(jj) > 2.5,$

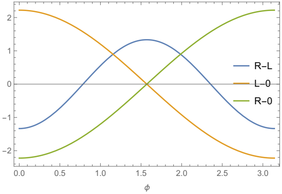
$|\eta(W^\pm)| < 2.5\,. \quad p_T(W^\pm) > 30\,\text{GeV},$

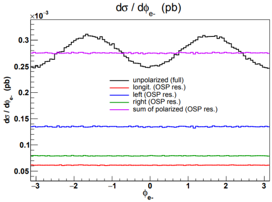
$M(W^+W^-) > 300\,\text{GeV},$

Process	WW-CM SM ($a=1$)		WW-CM CH ($a=0.8$)			WW-CM CH ($a=0.9$)		
	σ [fb]	$f_{\lambda\lambda'}$	σ [fb]	$f_{\lambda\lambda'}$	$\sigma^{\text{CH}}/\sigma^{\text{SM}}$	σ [fb]	$f_{\lambda\lambda'}$	$\sigma^{\text{CH}}/\sigma^{\text{SM}}$
jjW^+W^-	171	...	173	...	1.00	172	...	1.00
$jjW_T^+W_T^-$	118	69%	114	68%	0.96	118	69%	1.00
$jjW_0^+W_T^-$	22.2	13%	21.6	13%	0.97	21.6	12%	0.97
$jjW_T^+W_0^-$	24.1	14%	23.6	14%	0.98	24.0	14%	0.99
$jjW_0^+W_0^-$	6.93	4%	8.96	5%	1.29	7.81	5%	1.13

Interference and EFT resurrection







- Interference between weak boson polarizations manifest in the ϕ variable
- Hard to see $0-T$ interference in WW or VBS (dominated by T).
- More promising processes are Higgs strahlung and W from top decay
  ATLAS+CMS 2005.03799

- Polarization fraction might not be enough to discover new physics
- Imagine an dim-6 EFT operator $\frac{c}{\Lambda^2}\mathcal{O}$ that gives dominant contribution to W_0^+ in the $pp \to WW$ process.
- The leading new physics (NP) term, linear in c, is an interference with the SM matrix elements, but we've seen that interference vanishes for observables inclusive in ϕ.
- The NP contribution comes only from the longitudinal piece of the SM, which is suppressed.
- We must resurrect the NP contribution by looking at differential distributions, in ϕ (or other angular variables)

  see e.g. Panico, Riva, Wulzer 1708.07823, Banerjee, Gupta, Reiness, Spannowsky 1905.02728

Future MG improvements

- Implementation of spin-1 L, R decay separately
- Implementation of polarization interference
- Implementation of polarization with QCD NLO corrections (and loop-induced)