# Polarization theory - part II: vector-boson scattering and the PHANTOM Monte Carlo 

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Part I: polarized scattering with MG5 [Diogo Buarque Franzosi]

## Outline

1......... Polarizations and the Higgs mechanism 2.......... Polarized VBS with PHANTOM

## 1. Polarizations and the Higgs mechanism

## Vector-boson scattering

Scattering of on-shell electroweak bosons

E.g. in the Standard Model (SM), at tree level $\left[\mathcal{O}\left(\alpha^{2}\right)\right], \mathrm{W}^{+} \mathrm{W}^{-} \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}$amplitude


Possible channels: $\mathrm{W}^{ \pm} \mathrm{W}^{ \pm} \rightarrow \mathrm{W}^{ \pm} \mathrm{W}^{ \pm}, \mathrm{W}^{+} \mathrm{W}^{-} \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}, \mathrm{W}^{ \pm} \mathrm{Z} \rightarrow \mathrm{W}^{ \pm} \mathrm{Z}$,

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Equations of motion [EOM] for the vector field $A_{\mu}: k^{2} A_{\mu}-k_{\mu}(k \cdot A)=0$ ].

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- if $\mu^{2}>0, \phi$ acquires a $\operatorname{VEV}\left(v=\sqrt{\mu^{2} / \lambda}\right) \rightarrow U(1)$ gauge sym. broken parametrise $\phi$ with two real scalars $\phi \rightarrow \frac{1}{\sqrt{2}}(v+h$

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The Higgs (h), the vector $\left(A_{\mu}\right)$ and the Goldstone fields $\left(\phi_{2}\right)$ all feature a mass term.

(k)

$$
\frac{i}{k^{2}-M_{A}^{2}}\left(-g^{\mu \nu}+k^{\mu} k^{\nu} \frac{1-\xi}{k^{2}-\xi M_{A}^{2}}\right), \text { with } M_{A}=2 e^{2} \mu^{2} / \lambda
$$

$$
\frac{i}{k^{2}-M_{h}^{2}}, \text { with } M_{h}=\sqrt{2 \mu^{2}}
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$$
\frac{i}{k^{2}-M_{\phi_{2}}^{2}}, \text { with } M_{\phi_{2}}=\sqrt{\xi} M_{A} \rightarrow \text { unphysical! }
$$

## Higgs mechanism in the Standard Model

Slightly more involved but same procedure can be carried out for the SM, the (spontaneously-broken) gauge symmetry is $S U(2)_{w} \times U(1)_{Y} \rightarrow U(1)_{\mathrm{em}}$.
$\rightarrow \mathrm{W}^{ \pm}$and Z acquire mass, and three Goldstone bosons are generated ( $\phi^{ \pm}, \phi^{\mathrm{Z}}$ ).

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The first term is a sum over polarizations: for an on-shell boson it understands the projector over 3 spatial directions $(k \cdot \varepsilon(k)=0) \rightarrow 3$ physical polarizations.

With any gauge choice, the additional terms $\left(\propto k^{\mu} k^{\nu}\right)$ are cancelled by Goldstone-boson contributions.
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Rationale: the Higgs mechanism is the conversion of Goldstone modes into longitudinal polarization mode of massive weak bosons.

## Goldstone-boson equivalence theorem

For a boosted $\mathrm{W}^{ \pm} / \mathrm{Z}$ boson, there is a clear difference between transverse and longitudinal modes.
the amplitude for an external longitudinal vector boson $V$ is equivalent to the amplitude for an external corresponding Goldstone boson $\phi_{V}$.


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This holds also for $\geq 1$ external longitudinal bosons: we can compute the high-energy limit of $V_{L} V_{L} \rightarrow V_{L} V_{L}$ simply computing $\phi_{V} \phi_{V} \rightarrow \phi_{V} \phi_{V}$.

## Longitudinal VBS and unitarity in the SM (1)

$\mathrm{W}_{\mathrm{L}}^{+} \mathrm{W}_{\mathrm{L}}^{-} \rightarrow \mathrm{W}_{\mathrm{L}}^{+} \mathrm{W}_{\mathrm{L}}^{-}$in the high-energy limit $\left(s \gg 4 M_{\mathrm{W}}^{2}\right)$, in the SM .
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\begin{align*}
i \mathcal{A}_{\gamma, \mathrm{z}} / g^{2} & =\left(\frac{s}{4 M_{\mathrm{w}^{2}}}\right)^{2}\left(\cos ^{2} \theta+6 \cos \theta-3\right)+\left(\frac{s}{4 M_{\mathrm{w}}{ }^{2}}\right)\left(-\frac{13}{2} \cos \theta+\frac{3}{2}\right)+\mathcal{O}  \tag{1}\\
i \mathcal{A}_{4} / g^{2} & =\left(\frac{s}{4 M_{\mathrm{w}^{2}}}\right)^{2}\left(-\cos ^{2} \theta-6 \cos \theta+3\right)+\left(\frac{s}{4 M_{\mathrm{w}}{ }^{2}}\right)(6 \cos \theta-2)+\mathcal{O}(1) \\
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## Longitudinal VBS and unitarity in the SM (2)

$S$-matrix is unitary $\longrightarrow \quad \frac{\operatorname{Im}\left[\mathcal{A}_{2 \rightarrow 2}(s, \theta=0)\right]}{s}=\sigma_{\mathrm{tot}}>\sigma_{2 \rightarrow 2}(s, \theta) \quad$ [optical theorem]

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Remark: the leading high-energy behaviour can be obtained from $\phi^{+} \phi^{-} \rightarrow \phi^{+} \phi^{-}$ [equivalence theorem].

## 2. Polarized VBS with PHANTOM

## VBS at the LHC

VBS (on-shell):


## VBS at the LHC

VBS (in LHC collisions):


The two final-state quarks become tagging jets.
Cross-section enhanced if $M_{\mathrm{jj}}$ and $\Delta \eta_{\mathrm{jj}}$ are large (e.g. $M_{\mathrm{jj}}>500 \mathrm{GeV},\left|\Delta \eta_{\mathrm{jj}}\right|>2.5$ ).
How do we compute longitudinal VBS in the realistic environment of the LHC?

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bosons can be studied in term of their polarization state.


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- Angular observables of decay products reflect the polarization mode of the decayed boson.
neutrinos in the final state
identical particles
jet substructure (hadronic decays)


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- Angular observables of decay products reflect the polarization mode of the decayed boson.
- "Initial-state" bosons cannot be accessed: badly-defined. Only "final-state" bosons can be studied in term of their polarization state.
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- Angular observables of decay products reflect the polarization mode of the decayed boson.
- "Initial-state" bosons cannot be accessed: badly-defined. Only "final-state" bosons can be studied in term of their polarization state.
- Not always easy to fully reconstruct decay products:
- neutrinos in the final state
- identical particles
- jet substructure (hadronic decays)
- ...


## PHANTOM Monte Carlo in a nutshell

PHANTOM is a parton-level generator for $2 \rightarrow 6$ processes at $e^{+} e^{-}, p \bar{p}$ and $p p$ colliders at tree level orders $\mathcal{O}\left(\alpha^{6}\right), \mathcal{O}\left(\alpha_{s}^{2} \alpha^{4}\right)$ (+interference). It enables parton-shower matching via the Les Houches Event format. SM and few istances of BSM theories available as underlying dynamics. Meant for VBS, $t \bar{t}$ and Higgs physics.
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Dedicated MC, faster and more efficient than general-purpose MC (e.g. MadGraph5).

## PHANTOM: installation and dependences

Before installing PHANTOM:

- a gfortran compiler (tested also for gcc 9.3.0)
- LHAPDF interface installed (follow instructions at this link)
- make sure that the LHAPDF library is set as an environmental variable, e.g. sourcing your home .bashrc file with the LHAPDF path included

Download the PHANTOM-1.7 tarball at this link
Enter the PHANTOM directory and edit the makefile
according to the absolute address of LHAPDF library (where libLHAPDF. so is)
then run make "compilation takes some time]

In the PHANTOM/tools folder you can find useful files, e.g. for the generation of all partonic processes that contribute to a LHC process (setupdir*.pl).

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The r.in is PHANTOM input card.

## PHANTOM: the input card (1)

The r.in (input card) contains all the information needed to generate events in a specific partonic process. Let us read it line-by-line.

```
****** IF (IONESH.EQ.0) THEN
* CALL iread('iproc',iproc,8) ! process
iproc }\begin{array}{lllllllll}{2}&{4}&{1}&{3}&{12}&{-11}&{14}&{-13}
****** ENDIF
* CALL iread('idum',idum,1) ! idum=initialization random number seed
*
                                    ! must be a large negative number
idum -123456789
* CALL cread('PDFname',PDFname)
PDFname NNPDF31_lo_as_0118
```

iproc is the partonic process, written with PDG numbers: means $u c \rightarrow d$ s $e^{+} \nu_{e} u^{+} \nu_{\mu}$
idum is the random seed for unweighted-event generation

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## PHANTOM: the input card (2)

```
* ! (for processes with two outoging leptons)
* ! =5 Q= sqrt(ptj1*ptj2) square-root of the product of pt's of the 2 jets
* ! with largest pt (only for processes with at least 2 final statejets)
i_PDFscale 5
* Here give the numerical value for Q if i_PDFscale=3 (for all other choices
* the value is irrelevant)
* if (i_PDFscale.eq.3) then
* CALL rread('fixed_PDFscale',fixed_PDFscale,1)
* endif
fixed_PDFscale 300.d0
*1_7
* Fix the numerical constant pdfconst by which the PDFscale (any of the above)
* is multiplied
* CALL rread('pdfconst',pdfconst,1)
pdfconst 1.do
*1_7end
* CALL iread('i_coll',i_coll,1) ! determines the type of accelerator:
* ! i_coll=1 => p-p
* ! i_coll=2 => p-pbar
* ! i_coll=3 => e+e-
i_coll 1
```

PDFscale is the fixed or dynamical choice for factorization and $\alpha_{\mathrm{s}}$ scale: 5 options. i coll selects the type of collider we want to consider $[1=1$ HC $]$

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* if (i_PDFscale.eq.3) then
* CALL rread('fixed_PDFscale',fixed_PDFscale,1)
* endif
fixed_PDFscale 300.d0
*1_7
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* is multiplied
* CALL rread('pdfconst',pdfconst,1)
pdfconst 1.d0
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* ! i_coll=1 => p-p
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* if (i_PDFscale.eq.3) then
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fixed_PDFscale 300.d0
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* CALL rread('pdfconst',pdfconst,1)
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PDFscale is the fixed or dynamical choice for factorization and $\alpha_{\mathrm{s}}$ scale: 5 options.
i_coll selects the type of collider we want to consider [ $1=$ LHC]
Remark: r.in flags are self-explanatory, thanks to very detailed description.

## PHANTOM: the input card (3)

```
    CALL iread('perturbativeorder',i_pertorder,1 )
            !i_pertorder = 1 alpha_em^6 with dedicated amp
            = 2 alpha_s^2alpha_em^4
                            = 3 alpha_em^6 + alpha_s^2alpha_em^4
                            = 0 alpha_em^6 with amp8fqced (for test only)
perturbativeorder 1
            CALL iread('i_massive',i_massive,1 )
            !i_massive = 0 use faster massless amplitudes unless there is
                    at least a b quark
            = 1 always use massive amplitudes (massive Z}\mathrm{ -lines)
i_massive 0
* CALL iread('ionesh',ionesh,1)
* ! 0= normal run of one process
* ! 1= one shot unweighted event generation of all processes
* ! corresponding to phavegas files indicated at the end after nfiles
ionesh 0
* CALL rread('ecoll',ecoll,1) ! collider energy
ecoll 13000.do
* CALL rread('rmh',rmh,1) ! Higgs mass (GeV)
* ! <0 means no Higgs
rmh 125.do
```

perturbativeorder is the tree-level order of the calculation polarization selection is available for pure EW calculations $(-1)$

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## PHANTOM: the input card (5)

## Here comes polarization: resonant-diagram selection

```
* WARNING:
* RESONANT COMPUTATIONS, ON SHELL PROJECTIONS AND POLARIZATIONS
    CAN BE USED ONLY FOR EW (i_pertorder = 1 alpha_em^6)
    IF ONE WANTS TO COMPUTE THEM ONLY FOR PROCESSES WITHOUT EXTERNAL b QUARKS
            (e.g. to avoid top ew resonances with final W's)
            ONE MUST USE setupdir2_nob.pl or setupdirall_nob.pl
WHEN COMPUTING RESONANT CONTRIBUTIONS THE PARTICLES DECAYNG FROM THE
RESONANCE CANNOT HAVE ANOTHER IDENTICAL PARTICLE IN THE FINAL STATE
CALL iread('i_ww',i_ww,1) ! i_ww= 0 full computation
                        ! i_ww= 1 only 1 resonant w diagrams
    ! i_ww= 2 only 2 resonant w diagrams
i_ww 0
* if (i_ww.ge.1) then
* CALL iread('idw',idw,4)!(four numbers must be given,
    CALL iread('idw',idw,4)!(four numbers must be given,
                            ! if i_ww=1, all of them if i_ww=2)
                            ! the first two correspond to the decay of the
                            ! first w, the second two eventually
                    ! to the decay of the second w
                            ! The first number of any couple must
                            ! correspond to the particle, the second to
                            ! the antiparticle (negative) of the decay
idw }\quad1
```

In the SM many diagram topologies contribute to a given $2 \rightarrow 6$ process

## PHANTOM: the input card (5)

## Here comes polarization: resonant-diagram selection

```
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    CAN BE USED ONLY FOR EW (i_pertorder = 1 alpha_em^6)
    IF ONE WANTS TO COMPUTE THEM ONLY FOR PROCESSES WITHOUT EXTERNAL b QUARKS
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WHEN COMPUTING RESONANT CONTRIBUTIONS THE PARTICLES DECAYNG FROM THE
RESONANCE CANNOT HAVE ANOTHER IDENTICAL PARTICLE IN THE FINAL STATE
CALL iread('i_ww',i_ww,1) ! i_ww= 0 full computation
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i_Ww O
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    ! correspond to the particle, the second to
    ! the antiparticle (negative) of the decay
idw }\quad12~11 14 14 -13 
```

In the SM many diagram topologies contribute to a given $2 \rightarrow 6$ process ...

## Resonant and non-resonant diagrams

Not all diagrams can be interpreted as production $\times$ decay of weak bosons.
The full calculation includes all of them: that is the truth.


No resonant-diagram selection needed:


Other related flags are ignored.
Here we are computing the full process e.g. $u c \rightarrow d s e^{+} \nu_{e} \mu^{+} \nu_{\mu}$, including all spin-correlation and non-resonant effects.

## Resonant and non-resonant diagrams

To compute polarized $\mathrm{W}^{ \pm} \mathrm{W}^{ \pm}$scattering we discard non-doubly-resonant diagrams.
But we need a prescription to recover EW gauge invariance.


```
i_ww 2
    if (i_ww.ge.1) then
        CALL iread('idw',idw,4)!(four numbers must be given,
            ! but only the first two are considered
            ! if i_ww=1, all of them if i_ww=2)
            ! the first two correspond to the decay of the
                            ! first w , the second two eventually
                            ! to the decay of the second w
                            ! The first number of any couple must
                            ! correspond to the particle, the second to
                            ! the antiparticle (negative) of the decay
idw 12 11 11 14 -13
```

IDs (idw) are particles we want to be decay products of the two Ws (no identical!). Here we are computing $u c \rightarrow d s \mathrm{~W}^{+}\left(e^{+} \nu_{e}\right) \mathrm{W}^{+}\left(\mu^{+} \nu_{\mu}\right)$.

Watch out, order matters: first and second indices for the first resonance, third and fourth ones for the second resonance (if present)!

## Pole approximation

Once (doubly- or singly-) resonant diagrams have been selected, we project on-shell (OSP) the amplitude numerator, keeping the off-shell kinematics in the Breit-Wigner of the resonant boson(s): pole-approximation (gauge invariance ok).

```
* if (i_ww.ge.1.or.i_zz.ge.1) then
* CALL iread('i_osp',i_osp,1) ! i_osp = 0 no kinematics change
                                    i_osp = 1 on shell projection scheme for
                            _ }1\mathrm{ boson decaying
i_osp = 2 on shell projection scheme for
    endif
i_osp 0
* if (i_osp.gt.0) then
* CALL iread('idosp',idosp,4) ! identity of the particles which must
    ! be projected. Only the first couple
    ! counts if i_osp.eq.1.
    ! For every couple the first is the
    ! particle, the second the antiparticle
idosp }\quad1
```

i_osp 2 for doubly-resonant diagrams, double-pole approximation (DPA) i_osp 1 for singly-resonant diagrams, single-pole approximation (SPA) i osn 0 no kinematic modification for full calculations idosp are PDG IDs for decay products of on-shell-projected resonant bosons

## Pole approximation

Once (doubly- or singly-) resonant diagrams have been selected, we project on-shell (OSP) the amplitude numerator, keeping the off-shell kinematics in the Breit-Wigner of the resonant boson(s): pole-approximation (gauge invariance ok).

```
* if (i_ww.ge.1.or.i_zz.ge.1) then
* CALL iread('i_osp',i_osp,1) ! i_osp = 0 no kinematics change
    CALL iread('i_osp',i_osp,1) : i_osp = 0 no kinematics change 
                                    1 boson decaying
                                    i_osp = 2 on shell projection scheme for
    endif
i_osp 0
* if (i_osp.gt.0) then
* CALL iread('idosp',idosp,4) ! identity of the particles which must
                        ! be projected. Only the first couple
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idosp 12 -11 14 -13
```

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Once (doubly- or singly-) resonant diagrams have been selected, we project on-shell (OSP) the amplitude numerator, keeping the off-shell kinematics in the Breit-Wigner of the resonant boson(s): pole-approximation (gauge invariance ok).

```
* if (i_ww.ge.1.or.i_zz.ge.1) then
    * CM, Cl_ww.ge.1.or.i_zz.ge.1 (tread('i_osp',i_osp,1) ! i_osp = 0 no kinematics change
                                    i_osp = 1 on shell projection scheme for
                                    1 boson decaying
                                    i_osp = 2 on shell projection scheme for
    endif
i_osp 0
* if (i_osp.gt.0) then
* CALL iread('idosp',idosp,4) ! identity of the particles which must
                        ! be projected. Only the first couple
                            ! counts if i_osp.eq.1.
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Order of idosp matters: use the same as idw/idz ones.

## Pole approximation: some more details

Single (OSP1), for diagrams that are at least single- $V$ resonant: projecting on shell the numerator of $V$ resonant amplitude, leaving the Breit Wigner modulation untouched.
$\qquad$

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$$
\begin{aligned}
\mathcal{A} & =\mathcal{A}_{\text {res }}+\mathcal{A}_{\text {nonres }} \\
& =\sum_{\lambda}\left[\frac{\mathcal{A}_{\mu}^{\mathcal{P}}\left(q_{1}, q_{2} ; k,\left\{p_{i}\right\}\right) \varepsilon_{\lambda}^{\mu}(k) \varepsilon_{\lambda}^{* \nu}(k) \mathcal{A}_{\nu}^{\mathcal{D}}\left(k,\left\{I_{1}, I_{2}\right\}\right)}{k^{2}-M_{V}^{2}+i \Gamma_{V} M_{V}}\right]+\mathcal{A}_{\text {nonres }} \\
& \rightarrow \sum_{\lambda}\left[\frac{\mathcal{A}_{\mu}^{\mathcal{P}}\left(\bar{q}_{1}, \bar{q}_{2} ; \bar{k},\left\{p_{i}\right\}\right) \varepsilon_{\lambda}^{\mu}(\bar{k}) \varepsilon_{\lambda}^{* \nu}(\bar{k}) \mathcal{A}_{\nu}^{\mathcal{D}}\left(\bar{k},\left\{\bar{I}_{1}, I_{2}\right\}\right)}{k^{2}-M_{V}^{2}+i \Gamma_{V} M_{V}}\right]=\mathcal{A}_{\mathrm{OSP} 1},
\end{aligned}
$$

where $q_{1}, q_{2}$ are the initial partons momenta, $k, l_{1}, l_{2}$ are the momenta of the $V$ boson and its decay products, $\left\{p_{i}\right\}$ are other final state particles momenta. Note: $\bar{k}^{2}=M_{V}^{2}$.
$\qquad$ other final-state particles $(\Delta k=k-k$ absorbed by initial state)

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The definition is not unique. PHANTOM preserves: (1) space components of $V$ boson in the LAB frame, decay-product directions in the $V$ rest frame, momenta of other final-state particles ( $\Delta k=k-\bar{k}$ absorbed by initial state).

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OSP2, employed for doubly-resonant diagrams, defined in a similar fashion.

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\end{aligned}
$$

where $q_{1}, q_{2}$ are the initial partons momenta, $k, l_{1}, l_{2}$ are the momenta of the $V$ boson and its decay products, $\left\{p_{i}\right\}$ are other final state particles momenta. Note: $\bar{k}^{2}=M_{V}^{2}$.

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OSP2, employed for doubly-resonant diagrams, defined in a similar fashion.
Watch out: OSP2 only defined for $M_{4 f}>M_{V_{1}}+M_{V_{2}}$, where $M_{4 f}$ is the invariant mass of the four decay products of the two resonances, and $M_{V_{1,2}}$ are the resonant-boson masses. Impose a cut to satisfy this condition!

## Separating polarizations

One outgoing $W$ decaying leptonically (unitary gauge):


$$
\begin{aligned}
\mathcal{A} & =\mathcal{A}_{\rho}^{\mu} \frac{-g_{\mu \nu}+k_{\mu} k_{\nu} / M_{w}^{2}}{k^{2}-M_{w}^{2}+i \Gamma_{w} M_{w}} \mathcal{A}_{d}^{\nu} \\
& =\mathcal{A}_{\rho}^{\mu} \frac{\sum_{\lambda} \varepsilon_{\mu}^{\lambda} \varepsilon_{\nu}^{\lambda *}}{k^{2}-M_{w}^{2}+i \Gamma_{w} M_{w}} \mathcal{A}_{d}^{\nu}=\sum_{\lambda} \mathcal{A}_{\lambda} \\
|\mathcal{A}|^{2} & =\underbrace{\sum_{\lambda}\left|\mathcal{A}_{\lambda}\right|^{2}}_{\text {incoherent sum }}+\underbrace{\sum_{\lambda^{\prime}} \mathcal{A}_{\lambda} \mathcal{A}_{\lambda^{\prime}}^{*}}_{\text {interferences }}
\end{aligned}
$$

$\sum_{\lambda} \varepsilon_{\mu}^{\lambda} \varepsilon_{\nu}^{\lambda *} \rightarrow \varepsilon_{\mu}^{\lambda} \varepsilon_{\nu}^{\lambda *}$ selects a single polarization term

ipolw selects a polarization state for the resonant boson(s) defined by the decay products (idw). If ipolw 0 unpolarized calculation (but with resonant diagrams only).

## Z bosons, polarization definition

Same treatment and analogous flags for Z bosons:

```
i_zz 2
```



```
ipolz 41
```

Recall: polarization states are defined in a specific reference frame. Two frames available in PHANTOM (for Ws and/or Zs ):

```
*4cmpol
* IF THERE IS AT LEAST ONE POLARIZED BOSON, CHOOSE TO DEFINE POLARIZATION
    IN LAB OR IN CM OF 4 GIVEN PARTICLES.
            If (1_ww.ge.1.and.ipolw.gt.0.or.1_zz.ge.1.and.lpolz.gt.0) then
            if (ipolw(1).gt.0.or.ipolw(2).gt.0.
            &
                    or.ipolz(1).gt.0.or.ipolz(2).gt.0) then
                        CALL iread('i_4cmpol',i_4cmpol,1)
                            ! i
                            L_4cmpol = 1 polarizations defined in cm of
                four particles to be indicted below
1_4cmpol 0
            If (l_4cmpol.gt.0) then
            CALL iread('id4cmpol',id4cmpol,4)
                    ! identity of the particles which form
                    ! the cm in which the polarizations
                            ! are defined
            endif
        endtf
    endif
1d4cmpol 12 -11 13-13
*4cmpolend
```


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```
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    IN LAB OR IN CM OF 4 GIVEN PARTICLES.
        If (L_ww.ge.1.and.ipolw.gt.0.or.L_zz.ge.1.and.1polz.gt.0) then
            if (ipolw(1).gt.0.or.ipolw(2).gt.0.
        &
            or.ipolz(1).gt.0.or.ipolz(2).gt.0) then
                CALL iread('i_4cmpol',i_4cmpol,1)
                        ! i_4cmpol = 0 polarizations defined in the lab
                            i_4cmpol = 1 polarizations defined in cm of
                four particles to be indicted below
1_4cmpol 0
            If (L_4cmpol.ggt.0) then
            CALL iread('id4cmpol',id4cmpol,4)
                    ! identity of the particles which form
                    ! the cm in which the polarizations
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        endtf
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        if (1_ww.ge.1.and.ipolw.gt.0.or.1_zz.ge.1.and.1polz.gt.0) then
            if (ipolw(1).gt.0.or.ipolw(2).gt.0.
        &
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                    ! the cm in which the polarizations
                            ! are defined
        endif
        endif
    endif
1d4cmpol 12 -11 13-13
*4cmpolend
```

Default is the LAB, otherwise the CM frame of two bosons (better motivated from theory p.v.). The IDs are typically the same as those of idw.

## SM and SESM parameters

PHANTOM allows you to set the Higgs mass,

```
* CALL rread('rmh',rmh,1) ! Higgs mass (GeV)
rmh
125.d0
```

the Higgs width and the coupling strenght to vector bosons [see r.in for details].

VBF-produced Higgs that then decays into four leptons, flag i_signal

```
i_signal 0
* heavh
* SINGLET MODEL OPTION
* singlet model implementation (see e.g. Pruna Robens arXiv:1303.1150)
* CALL iread('i_singlet',i_singlet,1) ! yes/no singlet implementation
i_singlet 0
```

and the parameters for the Singlet extension of the Standard Model (SESM), that
features an additional (typically heavier) scalar Higgs boson and modified
Higgs-to-gauge couplings: flag i_singlet and following.
$\square$ but are defined in the source file coupling.f (if needed modify it, but with much care!)

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You can also select a certain class of signal diagrams, e.g. diagrams with a VBF-produced Higgs that then decays into four leptons, flag i_signal

```
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Remark: the SM parameters ( $M_{\mathrm{t}, \mathrm{W}, \mathrm{Z}}, \Gamma_{\mathrm{t}, \mathrm{W}, \mathrm{Z}}, G_{F} \ldots$ ) cannot be modified in the r.in, but are defined in the source file coupling.f (if needed modify it, but with much care!).

## Process merging and selection cuts

If a full calculation is performed, you can ask PHANTOM to compute at the same time partonic processes that only differ by family- and/or charge-conjugation:

## WARNING

i_ccfam must be 0 if i_ww greater than 0 and/or $i_{\text {_ }} z z$ greater than 0 and one computes polarized cross sections
if one sets i_ccfam=0 one must use setupdirall.pl or setupdirall_nob.pl

* CALL iread('i_ccfam',i_ccfam,1) ! family+CC conjugate
i_ccfam 0
_ccfam relevant for process generation, must b

For any calculation we need kinematic cuts: many available

```
* READ INPUT FOR CUTS
* CALL iread('i_e_min_lep',i_e_min_lep,1)
! yes/no lepton energy lower cuts (GeV)
i_e_min_lep 0
* IF(i_e_min_lep.EQ.1) CALL rread('e_min_lep',e_min_lep,1)
e_min_lep 20.do
* CALL Lread('t_pt_min_lep',i_pt_mtn_lep,1)
! yes/no lepton pt lower cuts (GeV)
i_pt_min_lep 0
* IF(i_pt_min_lep.EQ.1) CALL rread('pt_min_lep',pt_min_lep,1)
pt_min_lep -20.d0
```


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i_e_min_lep 0
* IF(i_e_min_lep.EQ.1) CALL rread('e_min_lep',e_min_lep,1)
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i_e_min_lep 0
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e_min_lep 20.d0
* CALL Lread('t_pt_mtn_lep',i_pt_mtn_lep,1)
! yes/no lepton pt lower cuts (Gev)
i_pt_min_lep 0
* pt_min_lep 
```

If $\mathrm{i}_{-}<$name_of_variable $>$set to 0 , cut not applied. If 1 , then set cut value (<name_of_variable $>$ ).

Detailed description and self-explanatory names in the r.in.

## General remarks on cuts

Cuts are needed to define fiducial regions for a certain process.
Typically, in experimental analyses it is needed to generate parton-level events
very inclusive cuts, then more exclusive cuts are imposed after parton-shower.
However, using very inclusive setups is delicate, due to singular configurations (present already at LO) that may worsen (or disrupt) the convergence of MC integration. E.g


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t-channel
$s$-channel
$\square$
(resonable) cuts on jet transverse-momentum (for $t$-ch.,pt_min_j) and invariant-mass

Read carefully the details in the r.in before setting a cut.

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EW amplitude diverges if the $t$ or $s$ virtuality becomes too small: cured by imposing (resonable) cuts on jet transverse-momentum (for t-ch.,pt_min_j) and invariant-mass cut on the lepton pair (for s-ch., rm_min_ll).

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EW amplitude diverges if the $t$ or $s$ virtuality becomes too small: cured by imposing (resonable) cuts on jet transverse-momentum (for t-ch.,pt_min_j) and invariant-mass cut on the lepton pair (for s-ch., rm_min_ll).

Read carefully the details in the r.in before setting a cut.

## Accuracy and event generation

Following flags concern the accuracy and number of MC calls in the thermalization and integration during the first step (ionesh $=0$ ):

```
****** IF (IONESH.EQ.0) THEN
* CALL rread('acc_therm',acc_therm,1) ! thermalization accuracy
acc_therm
* CALL iread('ncall_therm',ncall_therm,2)
* ! thermalization calls per iteration
    ! The first component refers to the
    ! number of calls for the first 3
    ! iterations, the second one to the
    ! calls for the remaining iterations.
ncall_therm 2000000 2000000
* CALL iread('itmx_therm',itmx_therm,1) ! thermalization iterations
itmx_therm 5
* CALL rread('acc',acc,1) ! integration accuracy
acc 0.005d0
```

Present values are already optimal. If needed better accuracy, adapt values accordingly (the error from MC integration decreases like $N^{-1 / 2}$, where $N$ is the number of sampled points $\rightarrow$ increase the number of calls!)

## Accuracy and event generation

Then, details for the second step (ionesh $=1$ ), including number of unweighted events required the details to be written on the LHE file (resonances)

```
****** ELSEIF (IONESH.EQ.1) THEN
* CALL rread('scalemax',scalemax,1)
* Iscale factor for the maximum
scalemax 1.1do
* CALL iread ('nunwevts',nunwevts,1)
* ! number of unweighted events to be produced
nunwevts 10000
* CALL iread('iwrite_event',iwrite_event,1)
* ! yes/no momenta of flät events written in .dat files
iwrite_event 1
* CALL iread('iwrite_mothers',iwrite_mothers,1)
* ! yes/no information about intermediate particles (mothers)
* ! in .dat files
iwrite_mothers 1
* CALL iread('ihadronize',ihadronize,1)
! yes/no call to hadronization
ihadronize 0
* CALL iread('i_exchincoming',i_exchincoming,1)
i_exchincoming 1
```

Everything is already set properly, just set the number of events you want to generate. i_exchincoming should be set to 1 (exchange of initial states in pp collisions), to have a realistic simulation of LHC events.

## Generating LHC events: step 1

[^0]
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## Exercise:

- create a directory for your run, prepare the r.in (deleting the lines concerning iproc definition!)
- run ../tools/setupdirall_6.pl -d . -t ./r.in -s SGE -q 4 -i "e_ ve mu_ vm" where -d requires the run directory path, -t the r.in to be used, -i a string with the leptons (and gluons) that are wanted in the final state, $-s$ the system for cluster submission, $-q$ the number of external quarks (initial + final)
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Remark: do different PHANTOM runs (with ionesh $=0$ ) in different directories, as the generated phavegas*.dat are not replaced automatically (if a phavegas*.dat file is present where you run PHANTOM, you'll get a fortran runtime error.)

## Generating LHC events: step 2

In unweighted-event generation a new r.in is needed, with ionesh set to 1 , and including (at the end of the r.in) the path to all VEGAS grids (phavegas*.dat) that have been generated in the first step:

```
* nfiles= number of files from which take the input for oneshot=1 generation
* it corresponds to the number of phavegas.. .dat files generated by running
* all single processes in the inoheshot=0 calculations.
* Immediately after the line nfiles, the full address of all files
    phavegas...dat must be written, one per line
*nel file setp2 le righe che seguono sono da cancellare
nfiles
        3
....//phavegas01.dat
..../phavegas02.dat
..../phavegas03.dat
```

nfiles is the number of phavegas*.dat files that are listed afterwards.

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Remark: if you want to perform more runs (ionesh $=1$ ) for the same LHC process (therefore using the same phavegas*.dat files) make sure you use different random seeds (idum).

## Exercise 1: $\mathrm{W}^{+} \mathrm{W}^{+}$scattering in the uc channel

Prepare the r.in (step 1) for the DPA Standard-Model calculation of $\mathrm{pp} \rightarrow \mathrm{W}^{+}\left(\mathrm{e}^{+} \nu_{\mathrm{e}}\right) \mathrm{W}^{+}\left(\mu^{+} \nu_{\mu}\right) \mathrm{jj}$ at $\mathcal{O}\left(\alpha^{6}\right)$, with $\sqrt{s}=14 \mathrm{TeV}$, in the $u c$ partonic channel, with both bosons unpolarized.

Use the typical VBS factorization scale $\mu_{\mathrm{F}}=\sqrt{p_{T}, \mathrm{j} 1} p_{\mathrm{T}, \mathrm{j} 2}$. Set the SM Higgs mass and width to 125 GeV and 4 MeV , respectively. Use the NNPDF31_lo_as_0118 PDF set.

Impose the following selection cuts:

- $M_{j j}>500 \mathrm{GeV},\left|\Delta \eta_{j j}\right|>3,\left|\eta_{j}\right|<4.5, p_{\mathrm{T}, \mathrm{j}}>25 \mathrm{GeV}$

Impose a suitable cut to enable the DPA functioning ( $M_{\mathrm{e}^{+} \nu_{\mathrm{e}} \mu^{+} \nu_{\mu}}>2 M_{\mathrm{W}}$ ).

## Exercise 2: $\mathrm{W}^{+} \mathrm{W}^{+}$scattering in the uc channel, polarized boson(s)

Repeat the same exercise as the previous one, selecting the longitudinal mode, defined both in the LAB and in the WW-CM frame, for the $\mathrm{W}^{+}$boson decaying into $\mathrm{e}^{+} \nu_{\mathrm{e}}$.

| integrated cross-sections $\sigma[\mathrm{fb}]$ |  |  |
| :---: | :---: | :---: |
|  | LAB | WW-CM |
| full | $0.1671(2)$ |  |
| unpol | $0.1663(2)$ |  |
| 0-unpol | $0.04624(6)$ | $0.04341(7)$ |
| T-unpol | $0.1198(1)$ | $0.1225(1)$ |

Repeat the same exercise, but selecting the longitudinal mode for both W bosons, with polarizations defined in the WW-CM frame and in the LAB frame. How large is the difference between the two definitions?

## Exercise 3: $\mathrm{W}^{+} \mathrm{W}^{+}$scattering in the uc channel, lepton cuts

Considering the same process as before, and the same cuts on jets, calculate the DPA (OSP2) cross-section (step-1) for

- $\mathrm{W}^{+}\left(\mathrm{e}^{+} \nu_{\mathrm{e}}\right)$ unpolarized, longitudinal and transverse, $\mathrm{W}^{+}\left(\mu^{+} \nu_{\mu}\right)$ unpolarized,
- polarizations defined in the LAB frame,
- with the same cuts on jet kinematics as before, plus the following cuts on leptons: $p_{\mathrm{T}, \ell}>20 \mathrm{GeV},\left|\eta_{\ell}\right|<2.5, p_{\mathrm{T}}$, miss $>40 \mathrm{GeV}$.

Compare the sum of longitudinal and transverse cross-section with the unpolarized one, do the same with results in the absence of lepton cuts: differences?

## Exercise 4: $\mathrm{W}^{+} \mathrm{Z}$ scattering

Prepare the r.in (step 1) for the Standard-Model calculation of $u c \rightarrow W^{+}\left(e^{+} \nu_{e}\right) Z\left(\mu^{+} \mu^{-}\right) d c$ at $\mathcal{O}\left(\alpha^{6}\right)$, with $\sqrt{s}=14 \mathrm{TeV}$,

1. with full matrix elements
2. with only Z-resonant diagrams (use OSP1), with the $Z$ unpolarized
3. with only W -resonant diagrams (use OSP1), with the Z unpolarized
4. with doubly-resonant diagrams (use OSP2), with both bosons unpolarized

Use the typical VBS factorization scale $\mu_{\mathrm{F}}=\sqrt{p_{\mathrm{T}, \mathrm{j} 1} P_{\mathrm{T}, \mathrm{j} 2}}$. Set the SM Higgs mass and width to 125 GeV and 4 MeV , respectively. Use the NNPDF31_lo_as_0118 PDF set.

Impose the following selection cuts:

- $M_{i j}>600 \mathrm{GeV},\left|\Delta \eta_{j j}\right|>3.6,\left|\eta_{j}\right|<5, p_{\mathrm{T}, \mathrm{j}}>20 \mathrm{GeV}$
- $p_{\mathrm{T}, \ell}>20 \mathrm{GeV}$, $\left|y_{\ell}\right|<2.5, p_{\mathrm{T}}^{\text {miss }}>20 \mathrm{GeV}$, with $\ell=\mathrm{e}, \mu$
- $\left|M_{\mu^{+} \mu^{-}}-M_{Z}\right|<10 \mathrm{GeV}$

In the case of DPA, impose a suitable cut to enable OSP2 ( $\left.M_{4 \ell}>M_{\mathrm{W}}+M_{\mathrm{Z}}\right)$.

## Exercise 5: longitudinal ZZ scattering in the SESM

Prepare the r.in (step 1) for the calculation of $u c \rightarrow Z\left(e^{+} e^{-}\right) Z\left(\mu^{+} \mu^{-}\right) u c$ at $\mathcal{O}\left(\alpha^{6}\right)$, with $\sqrt{s}=14 \mathrm{TeV}$.

Use VBS factorization scale $\mu_{\mathrm{F}}=\sqrt{P_{\mathrm{T}, \mathrm{j} 1} P_{\mathrm{T}, \mathrm{j} 2}}$. Use the NNPDF31_lo_as_0118 PDF set.
Impose the following selection cuts:

- $M_{j j}>500 \mathrm{GeV},\left|\Delta \eta_{j j}\right|>2.5,\left|\eta_{j}\right|<4.5, p_{\mathrm{T}, \mathrm{j}}>25 \mathrm{GeV}$
- $p_{\mathrm{T}, \ell}>5 \mathrm{GeV},\left|y_{\ell}\right|<2.5$, with $\ell=\mathrm{e}, \mu$
- $\left|M_{\ell^{+} \ell^{-}}-M_{Z}\right|<10 \mathrm{GeV}, M_{4 \ell}>400 \mathrm{GeV}$

Set the SM Higgs mass and width to 125 GeV and 4 MeV , respectively.
Compute doubly-longitudinal scattering (ZZ-resonant, OSP2, polarizations defined in the ZZ-CM frame):

1. in the SM,
2. in the Singlet Extension (SESM), setting the heavy-Higgs mass to 600 GeV , $\cos \alpha=0.98, \tan \beta=0.3$, and letting PHANTOM compute heavy-Higgs width.

Remark: $\alpha$ is the mixing angle to construct the two Higgs mass eigenstates, $\tan \beta$ is the ratio betwee the two VEVs. For simple description of SESM, see [1506.02257].

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