Conformally and Disformally Coupled Vector field Models of Dark Energy

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CoCo 2021: Cosmología en Colombia

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-Different representations of the same theory:

Jordan frame: fields and curvature couple directly, but matter only involves the metric.

Einstein frame: the gravitational Lagrangian has the Einstein-Hilbert form but the matter sector is affected by fields, which mediates an additional force.

-Coupled Multi-Proca (Gómez et. al. PDU (2020)):

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\frac{M_p^2}{2} R - \frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} - V(\tilde{X}) + f(X) \tilde{\mathcal{L}}_m \right),$$

$$\bar{g}_{\mu\nu} = f(X) g_{\mu\nu}.$$

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-Anisotropic Scalar Field Dark Energy with a Disformally Coupled Yang-Mills Field (Gómez et. al. arXiv:2103.11826 [gr-qc]):

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R + p(\phi, X) + \mathcal{L}_M(g_{\mu\nu}, \Psi_i) \right] - \int d^4x \sqrt{-\hat{g}} \left[\frac{1}{4} \hat{g}^{\alpha\beta} \hat{g}^{\gamma\delta} F^a_{\alpha\gamma} F_{\beta\delta a} \right].$$

$$\hat{g}_{\mu\nu} = C(\phi) g_{\mu\nu} + D(\phi) \phi_{,\mu} \phi_{,\nu}.$$

-Vector-tensor sector coupling to dark matter:

$$\mathcal{S} = \int d^4x \left[\sqrt{-g}\left(\frac{M_p^2}{2}R + \mathcal{L}_A(X,Y)\right) + \sqrt{-\bar{g}}\bar{\mathcal{L}}_c[\bar{g}_{\mu\nu},\psi_c]\right],$$

$$X = -\frac{1}{2}g^{\mu\nu}A_\mu A_\nu, Y = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

$$F_{\mu\nu} \equiv \nabla_\mu A_\nu - \nabla_\nu A_\nu.$$

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-Vector disformal transformation:

$$\bar{g}_{\mu\nu} = C(X) g_{\mu\nu} + B(X) A_\mu A_\nu,$$

$$\bar{g}^{\mu\nu} = \frac{1}{C} \left(g^{\mu\nu} - \frac{B}{C - 2BX} A^\mu A^\nu \right).$$

-Energy momentum tensor:

$$T^{\mu\nu}_{(A)} = \frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_A)}{\delta g_{\mu\nu}},\, T^{\mu\nu}_{(\mathrm{c})} = \frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-\bar{g}}\bar{\mathcal{L}}_{\mathrm{c}}\right)}{\delta g_{\mu\nu}},\, T^{\mu\nu}_{(i)} = \frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_i)}{\delta g_{\mu\nu}}.$$

$$\bar{T}^{\alpha\beta}_{(\mathrm{c})} = \frac{2}{\sqrt{-\bar{g}}}\frac{\delta(\sqrt{-\bar{g}}\mathcal{L}_c)}{\delta \bar{g}_{\alpha\beta}}.$$

-Cold dark matter sector:

$$T^{\mu\nu}_{(\mathrm{c})} = \sqrt{\frac{\bar{g}}{g}}\frac{\partial \bar{g}_{\alpha\beta}}{\partial g_{\mu\nu}}\bar{T}^{\alpha\beta}_{(\mathrm{c})},$$

$$T^{\mu\nu}_{(\mathrm{c})} = \sqrt{\frac{\bar{g}}{g}}\left[C\bar{T}^{\mu\nu} + \frac{1}{2}A^{\mu}A^{\nu}\left(C_{,X}g_{\alpha\beta} + B_{,X}A_{\alpha}A_{\beta}\right)\bar{T}^{\alpha\beta}_{(\mathrm{c})}\right].$$

-Vector-tensor sector:

$$\mathcal{L}_{A,Y}\nabla_{\beta}F^{\alpha\beta} + \mathcal{L}_{A,X}A^{\alpha} + \mathcal{M}_{\beta}F^{\alpha\beta} = Q^{\alpha},$$

$$\mathcal{M}_{\beta} = \mathcal{L}_{A,XX}A^{\nu}\nabla_{\beta}A_{\nu} + \mathcal{L}_{A,YY}F^{\rho\nu}\nabla_{\beta}\nabla_{\nu}A_{\rho}.$$

-Source term:

$$Q^{\alpha} = -\frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_c)}{\delta A_{\alpha}} = -\frac{1}{\sqrt{-g}} \left(\frac{\partial(\sqrt{-g}\mathcal{L}_c)}{\partial A_{\alpha}} - \nabla_{\mu} \frac{\partial(\sqrt{-g}\mathcal{L}_c)}{\partial(\nabla_{\mu} A_{\alpha})} \right).$$

-Continuity equations:

$$\begin{aligned} \nabla_{\mu} T^{\mu}_{(A)\nu} = -\nabla_{\mu} T^{\mu}_{(c)\nu} &= Q^{\mu}\nabla_{\nu}A_{\mu} - \nabla_{\mu}(Q^{\mu}A_{\nu}) \\ &= Q^{\mu}F_{\nu\mu} - A_{\nu}\nabla_{\mu}Q^{\mu}. \end{aligned}$$

$$\nabla^{\mu} T^{(A)}_{\mu\nu} + \nabla^{\mu} T^{(\mathrm{c})}_{\mu\nu} = 0.$$

-Proca theory with a potential: $\mathcal{L}_A = Y + m^2 X - V(X).$

$$\nabla_{\mu}F^{\mu\nu} + (V_{,X} - m^2)A^{\nu} = -\frac{B}{C} T^{\nu\mu}_{(\mathrm{c})} A_{\mu} + \frac{D}{2C}(C - 2BX)\left(C_{,X} T_{(\mathrm{c})} + B_{,X} T^{\alpha\beta}_{(\mathrm{c})} A_{\alpha} A_{\beta} \right) A^{\nu}$$

$$D \equiv \frac{1}{C - C_{,X}X + 2B_{,X}X^2}.$$

Background Cosmology:

-FRW Metric: $$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j\,,$$

-Temporal ansatz: $$A_\mu \equiv (A(t), 0, 0, 0).$$

-Field equations: $$3M_p^2 H^2 = \left(\frac{m^2}{2} - V_X\right) A^2 + V + \rho_c + \rho_r,$$

$$M_p^2(3H^2 + 2\dot{H}) = V - \frac{1}{2}m^2 A^2 - \frac{\rho_r}{3},$$

$$(m^2 - V_X)A = \frac{A\rho_c(C_X - B_X A^2 - 2B)}{B_X A^4 - C_X A^2 + 2C}.$$

-Effective equation of state: $$w_{\rm eff} \equiv \frac{p_{\rm T}}{\rho_{\rm T}} = -\left(1 + \frac{2\dot{H}}{3H^2}\right).$$

-Continuity equations:

$$\dot{\rho}_A + 3H(\rho_A + p_A) = -\tilde{Q},$$
$$\dot{\rho}_c + 3H\rho_c = \tilde{Q},$$

with

$$\rho_A = \left(\frac{m^2}{2} - V_X\right) A^2 + V, \qquad p_A = \frac{m^2 A^2}{2} - V,$$

Interacting term: $\tilde{Q} = \rho_c \frac{\dot{A}}{2A}\gamma = \rho_c \frac{\dot{A}}{2A}(\gamma_C + \gamma_B),$

$$\gamma_C = \frac{-2\frac{C_X}{C}A^2 + A^4\left(\frac{C_X^2}{C^2} - 2\frac{C_{XX}}{C}\right)}{\left(\frac{C_X}{C}A^2 - 2\right)\left(\frac{C_X}{C}A^2 - 1\right)}, \gamma_B = \frac{A^2\left(10B_X A^2 + 4B + A^6(B_X^2 - 2B_{XX}B) + 2A^4(B_{XX} - 3B_X B)\right)}{(2 + B_X A^4)(1 + B_X A^4 + BA^2)}.$$

Dynamical system analysis:

$$x \equiv \sqrt{\frac{-V_X A^2}{3M_p^2 H^2}}; \quad y \equiv \sqrt{\frac{V}{3M_p^2 H^2}}; \quad z \equiv \sqrt{\frac{\rho_m}{3M_p^2 H^2}}; \quad r \equiv \sqrt{\frac{\rho_r}{3M_p^2 H^2}}; \quad u \equiv \sqrt{\frac{m^2 A/2}{3M_p^2 H^2}}; \quad v \equiv \frac{A}{M_p}.$$

$$x^2 + y^2 + z^2 + u^2 = 1. \qquad V(X) = V_0 e^{-2\lambda X/M_p^2}.$$

-Autonomous system:

$$x' = x\left(-\epsilon_H + \frac{v'}{v}(1-\lambda v^2)\right),$$

$$y' = -y\left(\epsilon_H + \lambda v' v\right),$$

$$z' = \frac{z}{2}\left(-2\epsilon_H - 3 + \frac{\gamma}{2}\frac{v'}{v}\right),$$

$$r' = -r\left(2 + \epsilon_H\right),$$

$$\frac{v'}{v} = \frac{6u^2 + 3x^2}{4\lambda^2 v^4 y^2 - x^2 - 2u^2 - \frac{\gamma}{2} z^2}.$$

Conformal case:

-Model 1: $C(X) = C_0 \left(\frac{X}{M_p^2}\right)^q \quad \rightarrow \gamma_C = \frac{2q}{1-2q}.$

-Model 2: $C(X) = \tilde{C}_0\, e^{\frac{4\alpha X}{M_p^2}} \quad \rightarrow \gamma_C = -\frac{4\alpha v^2(1+2v^2\alpha)}{1-6\alpha v^2+8\alpha^2 v^4}.$

Point	r_c	y_c	z_c	u_c	v_c	Ω_A	w_A	$w_{\rm eff}$	Existence	Acceleration
$(\mathrm{A}_\pm)$	± 1	0	0	0	0	0	–	1/3	$\forall q(\alpha), \lambda$	No
$(\mathrm{B}_\pm)$	0	0	± 1	0	0	0	–	0	$\forall q(\alpha), \lambda$	No
$(\tilde{\mathrm{B}}_{1,2})$	0	0	± 1	0	$\mp\frac{1}{\sqrt{2\alpha}}$	0	–	0	$\alpha \neq 0, \forall\lambda$	No
$(\tilde{\mathrm{B}}_{3,4})$	0	0	± 1	0	$\pm\frac{1}{\sqrt{2\alpha}}$	0	–	0	$\alpha \neq 0, \forall\lambda$	No
$(\mathrm{C}_{1,2})$	0	0	$\pm\sqrt{\frac{-2+6q}{-2+5q}}$	$\mp\sqrt{\frac{q}{2-5q}}$	0	$\frac{q}{2-5q}$	1	$\frac{q}{2-5q}$	$q \neq 2/5, 0 < q < 1/3, \forall\lambda$	$2/5 < q < 1$
$(\mathrm{C}_{3,4})$	0	0	$\pm\sqrt{\frac{-2+6q}{-2+5q}}$	$\pm\sqrt{\frac{q}{2-5q}}$	0	$\frac{q}{2-5q}$	1	$\frac{q}{2-5q}$	$q \neq 2/5, 0 < q < 1/3, \forall\lambda$	$2/5 < q < 1$
$(\tilde{\mathrm{D}}_{1,2})$	0	1	$\pm\sqrt{-\frac{\lambda}{\alpha}}$	0	$\mp\frac{1}{\sqrt{2\alpha}}$	$1+\frac{\lambda}{2\alpha}$	$-\frac{1}{1+\frac{\lambda}{2\alpha}}$	-1	$\alpha \neq 0, \lambda < 0 \vert \alpha < 0$	Yes
$(\tilde{\mathrm{D}}_{3,4})$	0	1	$\pm\sqrt{-\frac{\lambda}{\alpha}}$	0	$\pm\frac{1}{\sqrt{2\alpha}}$	$1+\frac{\lambda}{2\alpha}$	$-\frac{1}{1+\frac{\lambda}{2\alpha}}$	-1	$\alpha \neq 0, \lambda < 0 \vert \alpha < 0$	Yes
$(\tilde{\mathrm{F}}_{1,2})$	0	1	$\pm\sqrt{-\frac{\lambda}{2\alpha}}$	0	$\mp\frac{1}{2\sqrt{\alpha}}$	$1+\frac{\lambda}{2\alpha}$	$-\frac{1}{1+\frac{\lambda}{2\alpha}}$	-1	$\alpha \neq 0, \lambda < 0 \vert \alpha < 0$	Yes
$(\tilde{\mathrm{F}}_{3,4})$	0	1	$\pm\sqrt{-\frac{\lambda}{2\alpha}}$	0	$\pm\frac{1}{2\sqrt{\alpha}}$	$1+\frac{\lambda}{2\alpha}$	$-\frac{1}{1+\frac{\lambda}{2\alpha}}$	-1	$\alpha \neq 0, \lambda < 0 \vert \alpha < 0$	Yes
$(\mathrm{E}_\pm)$	0	0	0	± 1	0	1	1	1	$\forall q(\alpha), \lambda$	No
$(\mathrm{D}_{1,2})$	0	$\pm\frac{1}{\sqrt{1+v^2\lambda}}$	0	$\mp v\sqrt{\frac{\lambda}{-1-v^2\lambda}}$	–	1	-1	-1	$\forall q(\alpha)$, eqn. (36)	$\forall q(\alpha), \lambda$
$(\mathrm{D}_{3,4})$	0	$\pm\frac{1}{\sqrt{1+v^2\lambda}}$	0	$\pm v\sqrt{\frac{\lambda}{-1-v^2\lambda}}$	–	1	-1	-1	$\forall q(\alpha)$, eqn. (36)	$\forall q(\alpha), \lambda$
(S)	0	1	0	0	0	1	-1	-1	$\forall q(\alpha), \lambda$	$\forall q(\alpha), \lambda$

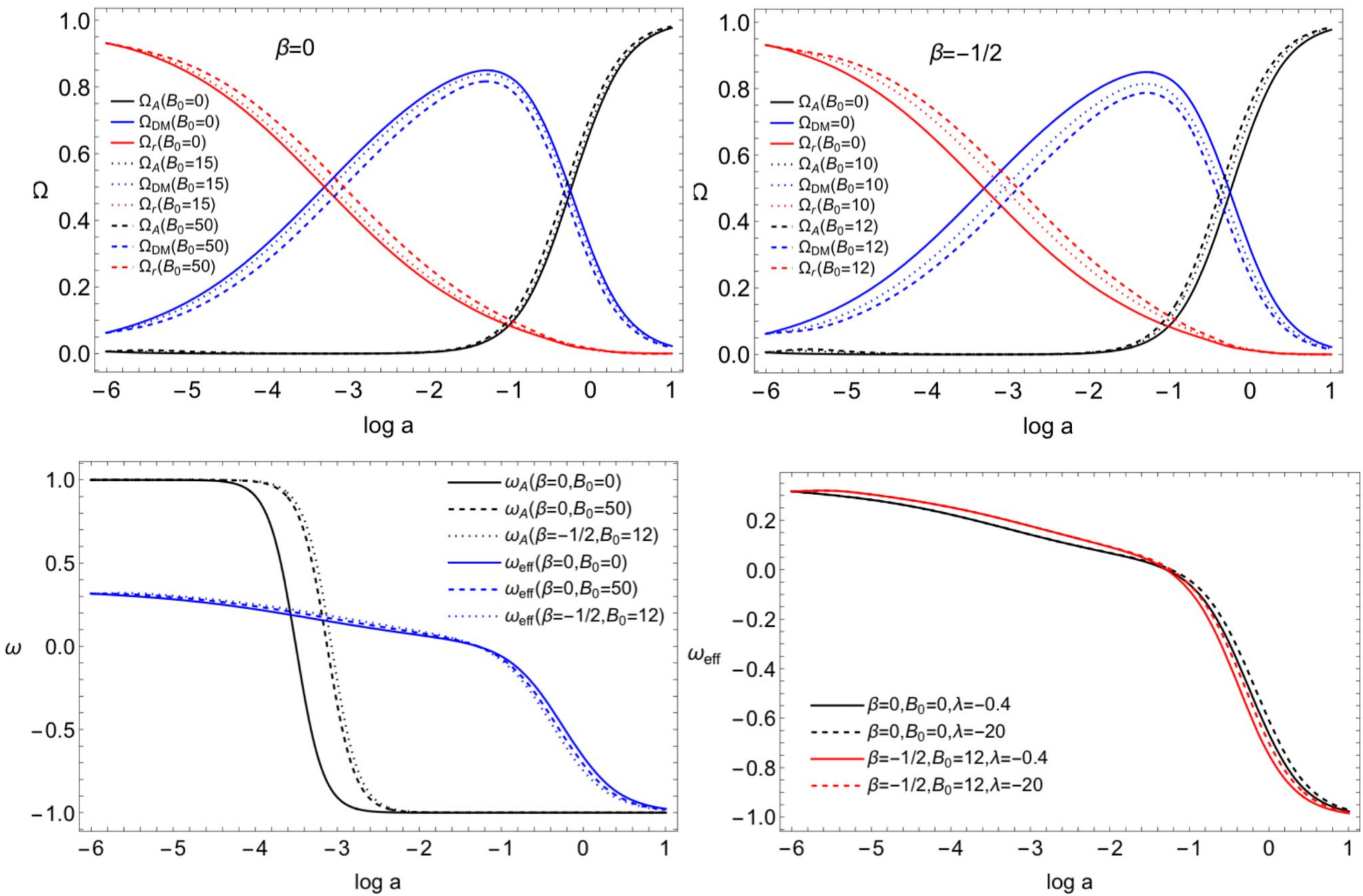
Disformal case:

-Model: $C(X)=1,\ B(X)=B_0\frac{2^{\beta}X^{\beta}}{M_p^{2+2\beta}}$

$$\rightarrow \gamma_B=-B_0\frac{2v^{2+2\beta}(1+\beta)\left(-1+\beta(-2+B_0v^{2+2\beta})\right)}{(1+\beta B_0v^{2+2\beta})(1+v^{2+2\beta}(1+2\beta)B_0)}.$$

Point	r_c	y_c	z_c	u_c	v_c	Ω_A	w_A	$w_{\rm eff}$	Existence	Acceleration
$(\mathrm{H}_{1,2})$	0	±1	$\pm2\frac{\sqrt{-2\lambda}}{B_0}$	0	$\mp\frac{2}{B_0}$	$1+\frac{8\lambda}{B_0^2}$	$-\frac{1}{1+\frac{8\lambda}{B_0^2}}$	-1	$B_0\neq0,\lambda<0$	Yes
$(\mathrm{H}_{3,4})$	0	±1	$\pm2\frac{\sqrt{-2\lambda}}{B_0}$	0	$\mp\frac{2}{B_0}$	$1+\frac{8\lambda}{B_0^2}$	$-\frac{1}{1+\frac{8\lambda}{B_0^2}}$	-1	$B_0\neq0,\lambda<0$	Yes
$(\mathrm{G}_{1,2})$	0	0	±1	0	$\pm\frac{2}{B_0}$	0	$-$	0	$B_0\neq0$	No
$(\mathrm{G}_{3,4})$	0	0	±1	0	$\mp\frac{2}{B_0}$	0	$-$	0	$B_0\neq0$	No
$(\tilde{\mathrm{H}}_{1,2})$	0	±1	$\pm\sqrt{2-2(1+cB_0\lambda}$	0	$\mp\sqrt{cB_0}$	$1+2cB_0\lambda$	$-\frac{1}{1+2cB_0\lambda}$	-1	$B_0>0,\lambda<0$	Yes
$(\tilde{\mathrm{H}}_{3,4})$	0	±1	$\pm\sqrt{2-2(1+cB_0\lambda}$	0	$\pm\sqrt{cB_0}$	$1+2cB_0\lambda$	$-\frac{1}{1+2cB_0\lambda}$	-1	$B_0>0,\lambda<0$	Yes
$(\tilde{\mathrm{G}}_{1,2})$	0	0	±1	0	$\pm\sqrt{cB_0}$	0	$-$	0	$B_0>0$	No
$(\tilde{\mathrm{G}}_{3,4})$	0	0	±1	0	$\mp\sqrt{cB_0}$	0	$-$	0	$B_0>0$	No

Cosmological background evolution:



Conclusions and perspectives:

-We have proposed a novel coupling between vector fields and CDM at the level of the action through a general disformal transformation.

-Several critical points arise in comparison to the uncoupled case, leading to interesting cosmological solutions: stable attractor solutions, scaling solutions.

-The effects of the disformal coupling can affect both the early and late-time universe, leaving some imprints that can be contrasted with observational data at different redshifts.

-Study disformal coupling for spatial vector fields: Muti-fields, non-Abelian vector fields.

-Involve the field strength in the disformal transformation or its dual in order to make the structure of the group explicit.

-Constrain the background dynamics, matter density perturbations: spherical collapse, number countst, the growth rate and the redshift-space distortion.

Thank you!