Luminosity distance in an arbitrary space-time

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Introduction

- From the measurements of the luminosity distance to far away type Ia supernovae it has been discovered that the present universe is in an accelerated expansion phase.
- Assuming to live in an homogeneous and isotropic universe, we need to postulate the existence of the dark energy, or a cosmological constant.
- Some measurements can be influenced by local perturbations.
- In this context it is important to derive a formula for the luminosity distance in a perturbed universe.

Differential geometry

- Jacobi equation

$$\boxed{\nabla_k \nabla_k Y = R\left(k, Y\right) k}$$

- Infinitesimally thin bundle

$$B = \left\{ c^A Y_A | c^1, c^2 \in \mathbb{R}, \delta_{AB} c^A c^B \leq 1 \right\},$$

- Sachs Basis

$$Y_A = D_A^B E_B + y_A k,$$

- Jacobi Matrix

$$\ddot{D} = DT,$$

where $T = \begin{pmatrix} -R - \mathrm{Re}\left(F\right) & \mathrm{Im}\left(F\right) \\ \mathrm{Im}\left(F\right) & -R + \mathrm{Re}\left(F\right) \end{pmatrix},$

$R = \frac{1}{2} R\left(k, k\right) = \frac{1}{2} R_{\mu\nu} k^\mu k^\nu, \; F = \frac{1}{2} R_{\alpha\mu\beta\nu} \bar{\epsilon}^\alpha \bar{\epsilon}^\beta k^\mu k^\nu.$

Differential geometry

- Optical scalars: σ shear, θ expansion, ω vorticity of light bundle,

$$\dot{D} = DS,$$

where

$$S = \begin{pmatrix} \theta & 0 \\ 0 & \theta \end{pmatrix} + \begin{pmatrix} \mathrm{Re}\,(\sigma) & \mathrm{Im}\,(\sigma) \\ \mathrm{Im}\,(\sigma) & -\mathrm{Re}\,(\sigma) \end{pmatrix} + \begin{pmatrix} 0 & \omega \\ -\omega & 0 \end{pmatrix},$$

- Sachs propagation equations

$$\boxed{\dot{S} + S^2 = T}$$

Sachs focusing equation

- From the linearity of $\ddot{D} = DT$ we get $\vec{y}(s) = D^T(s)\,\alpha$
- Area distance: $D_A = \frac{L}{\alpha} \;\Rightarrow\; D_A = \sqrt{\det D}$
- From $\dot{D} = DS$ in terms of shape parameters and the Sachs equations $\dot{S} + S^2 = T$ we get the Sachs focusing equation

$$\boxed{\ddot{D}_A = -\left(|\sigma|^2 + R\right) D_A,}$$

  where $R = \frac{1}{2} R_{\mu\nu} k^\mu k^\nu$ and $\sigma = 0$ for conformally flat spacetimes.
- Luminosity distance: $D_L = (1+z)^2\, D_A$

Distances, Friedmann

- $\sigma = 0$, from the vanishing conformal curvature
- $R = \frac{1}{2} R_{\mu\nu} k^{\mu} k^{\nu} = K$
- $\ddot{D}_A = -K D_A$
-
$$D_A(\lambda) = \begin{cases} |\sin(\lambda)| & , K = 1 \\ |\lambda| & , K = 0 \\ |\sinh(\lambda)| & , K = -1 \end{cases}$$

Distances, Perturbed Friedmann

- $g = -(1+2\Psi)\, d\eta^2 + (1-2\Phi)\, \delta_{ij} dx^i dx^j$
- Luminosity distance for moving observer and source:

$$\begin{aligned} D_L &= (\eta_O - \eta_S)\left(1 - \Psi_S - 2\Phi_O + 2\Phi_S + \vec{n}\left(\vec{v}_O - 2\vec{v}_S\right)\right) \\ &\quad + (\eta_O - \eta_S) \int_{\eta_S}^{\eta_O} d\eta \nabla (\Phi + \Psi) \cdot \vec{n} \\ &\quad + \int_{\eta_S}^{\eta_O} d\eta\, (\Phi + \Psi) + \int_{\eta_S}^{\eta_O} d\eta \int_{\eta_S}^{\eta} d\eta' \nabla (\Phi + \Psi) \cdot \vec{n} \\ &\quad - \int_{\eta_S}^{\eta_O} d\eta \int_{\eta_S}^{\eta} d\eta' \left(\left(\nabla^2 \left(\frac{\Phi + \Psi}{2}\right) - n^i n^j \partial_i \partial_j \left(\frac{\Phi + \Psi}{2}\right)\right)\right. \end{aligned}$$

Distances, Gravitational waves

- $g_{\mu\nu} = \eta_{\mu\nu} + h_{ij}$, where $h_{ij} = 2H_{ij}$ describes a pure tensor perturbation, and we adopt the transverse traceless gauge,
- To first order we have $|\sigma|^2 = 0$ and $R = 0$, hence the luminosity fluctuations are due to the redshift term only.
- Considering a binary star system in the Virgo cluster, as the source of gravitational waves, we find that the maximal luminosity fluctuation is of the order of 10^{-20}, which is by far too small to be detected.