Part I: Homogeneous cosmology

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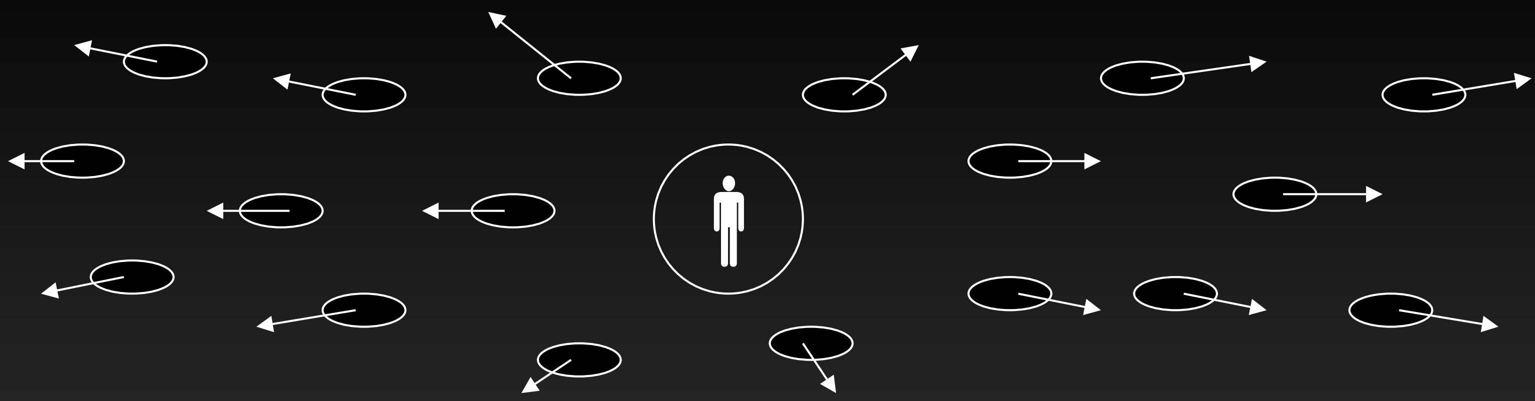
Universe Expansion
Friedmann Law
Hot Big Bang scenario

- 1923 : Edwin Hubble :
  - 2,50 m telescope at *Mount Wilson (CA)*
  - cepheids in other galaxies
  - *first probe of galactic structure* !!!
  - correlation between redshift and distance

⇩

Universe expansion ???

- IN GENERAL : expansion ⇨ center



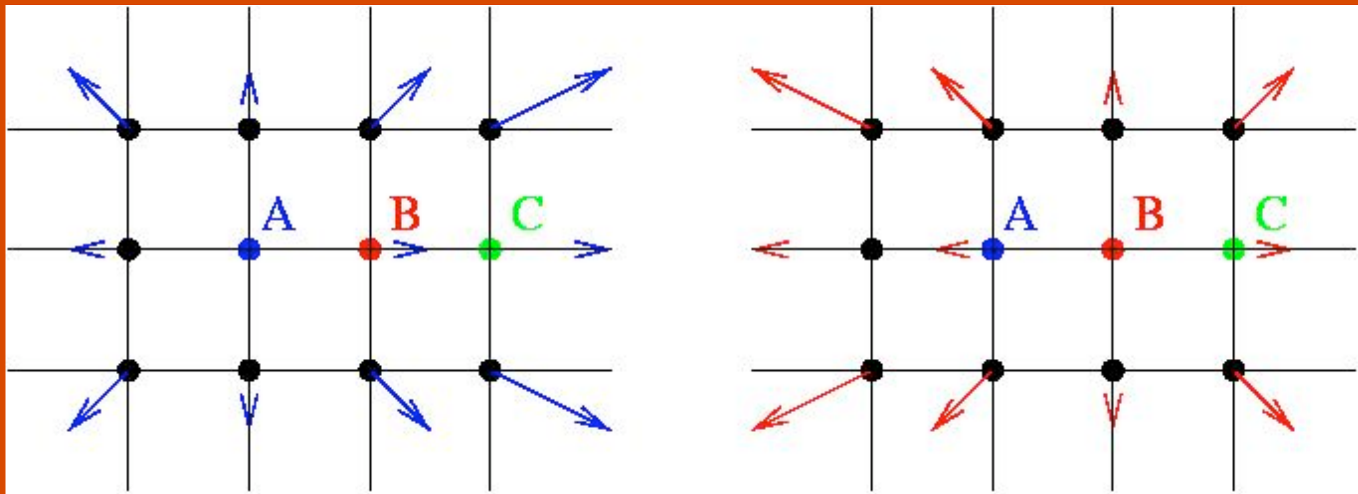
- Against « cosmological principle » (Milne):
  - Universe homogeneous …
  - no privileged point!

QUESTION : is any expansion a proof against homogeneity?

ANSWER : not if $\mathbf{v} = H\,\mathbf{r}$ ⇦ *linear expansion*

… like infinite rubber grid stretched in all directions …

Proof that *linear expansion* is the only possible homogeneous expansion :

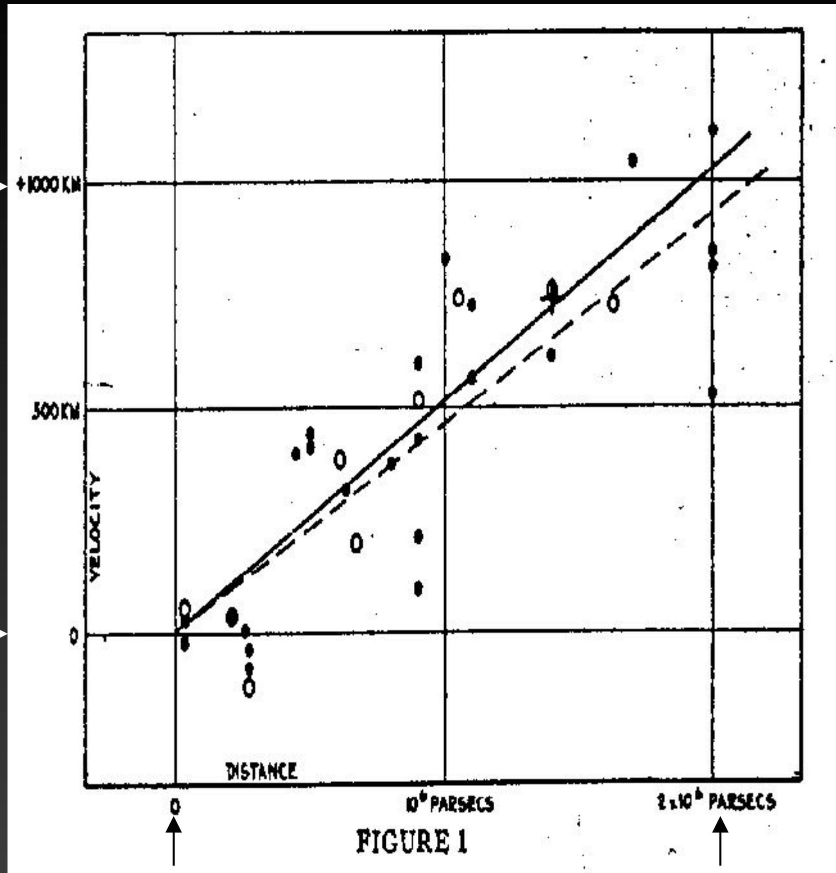


- $v_{B/A} = v_{C/B}$ ⇦ homogeneity
- $v_{C/A} = v_{C/B} + v_{B/A} = 2\, v_{B/A}$ ⇨ linearity

- 1929 : Hubble gives the first *velocity / distance* diagram :

1000 km.s^{-1}

0 km.s^{-1}



0 Mpc 2 Mpc

H = v / r = 500 km.s^{-1}.Mpc^{-1} for Hubble (≅ 70 km.s^{-1}.Mpc^{-1} for us)

1 Mpc = 3.10^{6} lyr = 3.10^{22} m

- 1929 : Hubble gives the first *velocity / distance* diagram :

1000 km.s^{-1} →

0 km.s^{-1} →

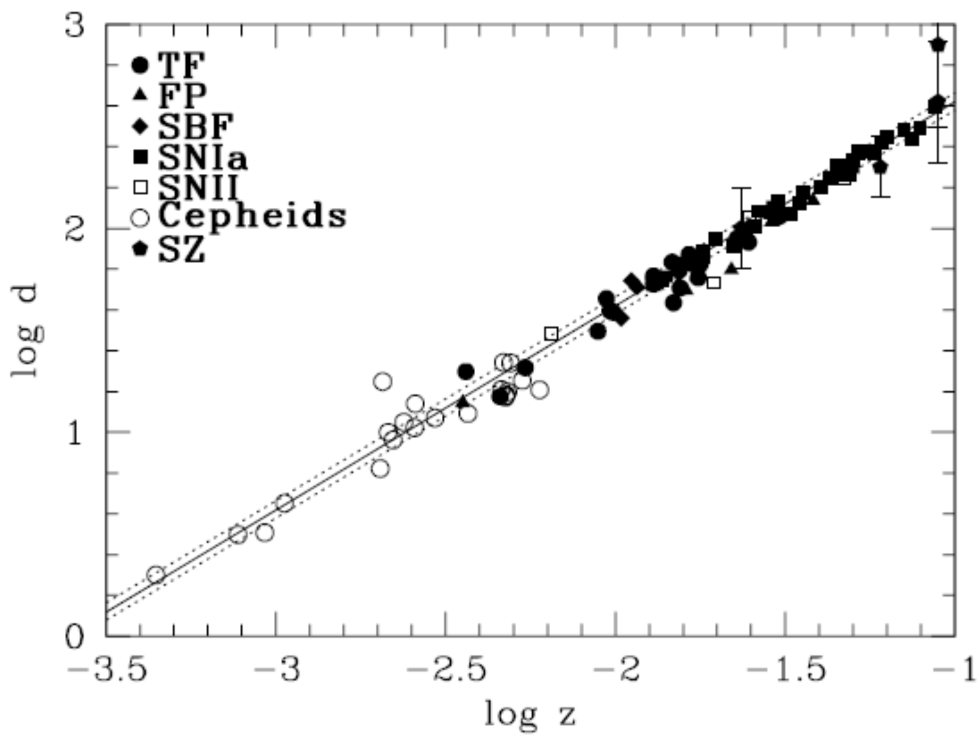


Figure 1.4: A recent Hubble diagram published by the Hubble Space Telescope Key Project in 2000 *(Astrophys.J. 553 (2001) 47-72)*, based on cepheids, supernovae and other standard candles till a distance of 400 Mpc. The horizontal axis gives the radial velocity, expressed as $\log_{10}[v/c] = \log_{10} z$ where z is redshift; the vertical axis shows the distance $\log_{10}[d/(1\mathrm{Mpc})]$.

H = v / r = 500 km.s

1 Mpc = 3.10^6 lyr = 3.10^{22} m

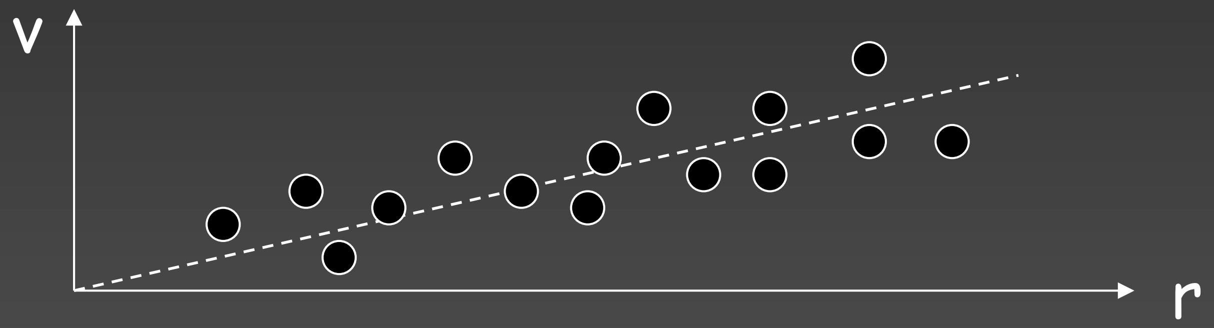
THE UNIVERSE IS IN HOMOGENEOUS EXPANSION

– 1929 : starting of modern cosmology …

Remark : what do we mean by « the Universe is homogeneous »?

– today : data on *very large scales* ⇨ confirmation of homogeneity beyond ~ 30 - 40 Mpc

– local inhomogeneities ⇨ *scattering*



- on cosmic scales, only *gravitation*

- *Newton's law* = limit of *General Relativity* (GR)

$F = G\, m_1\, m_2 / r^2$

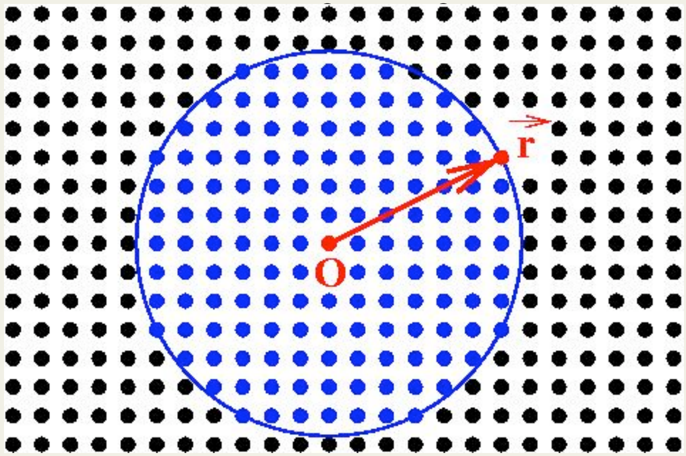
when $v \ll c$

OR | speed of object
speed of liberation

  - *Newton's law* should describe expansion at small distances with $v = H\, r \ll c$ …

  - but historically, GR proposed the first predictions / explanations !!!

- Gauss theorem :



$$\ddot{r} = - G M_r / r^2$$

$$M_r = constant = (4/3)\,\pi\, r^3\, \rho_{mass}$$

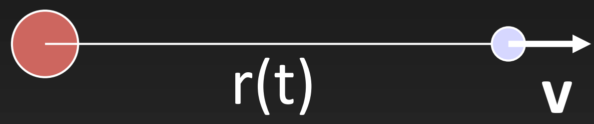
$$\Rightarrow \quad \dot{r}^2 = 2\, G M_r / r \; - \; k$$

$$= (8/3)\, p\, G\, \rho_{mass}\, r^2 \; - \; k$$

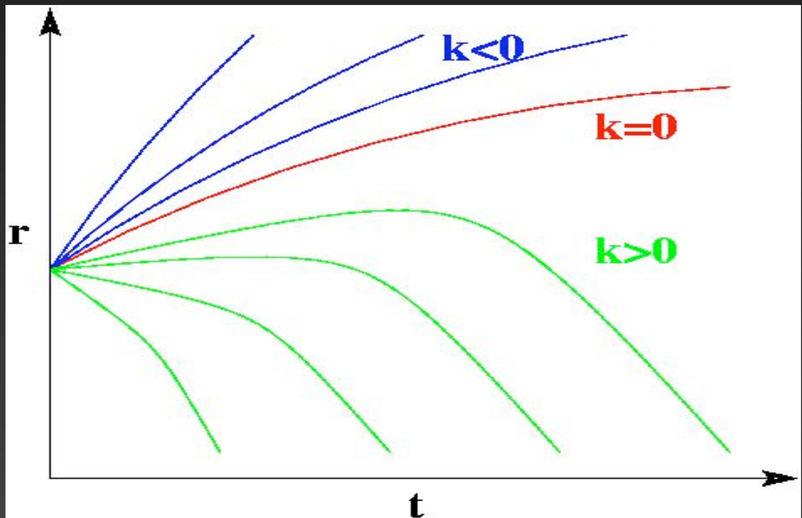
Newtonian expansion law : $(\dot{r} / r)^2 = (8\pi G/3)\, \rho_{mass} - k/r^2$

Newtonian expansion law : $(\dot{r}/r)^2 = (8\pi G/3)\,\rho_{mass} - k/r^2$

- $\rho_{mass}(t) \propto r(t)^{-3}$
- *same motion as a two-body problem* :



GRAVITY / INERTIA



$$\rho_{mass} \;<,\; =,\; > \;\; \frac{3\,(\dot{r}/r)^2}{8\pi G}$$

- $k \neq 0$ ⇨ non-homogeneous expansion ???
- $v = H\,r$ and $v \ll c$ ⇨ $r < R_H \equiv c/H$

- applying G.R. to the Universe: *some history*
  - 1916 : Einstein has formulated G.R.
  - 1917 : Einstein, De Sitter try to build the first cosmological models (PREJUDICE : STATIC / STATIONNARY UNIVERSE)

  - 1922 : A. Friedmann (Ru)
  - 1927 : G. Lemaître (B)
  - 1933 : Robertson, Walker (USA)

  } investigate most general HOMOGENEOUS, ISOTROPIC, NON-STATIONNARY solutions of G.R. equations

  - 1929 : Hubble's law (first confirmation)
  - 1930-65 : accumulation of proofs in favour of FLRW
  - 1965 : CMB discovery : full confirmation

basic principles of G.R.	FLRW solution
space-time is *curved*	2 types of curvatures : {a(t), k}
free-falling objects follow *geodesics*	ultra-relativistic matter : ↳ BENDING OF LIGHT EQUATION
curvature caused by / related to *matter*	FRIEDMANN LAW

1) the *curvature* of the FLRW Universe :

$$ds^2 = c^2 dt^2 - a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta \; d\phi^2) \right]$$

- Comoving coordinates, proper time
- 2 types of curvature, R_c, R_H

$$R_c(t) = \frac{a(t)}{\sqrt{|k|}}$$

$$R_H(t) = ca(t)/\dot{a}(t)$$

- Terminology: flat, closed, open
- All compatible with homogeneity
- with $H=\dot{a}/a$, one recovers $z=Hr/c$

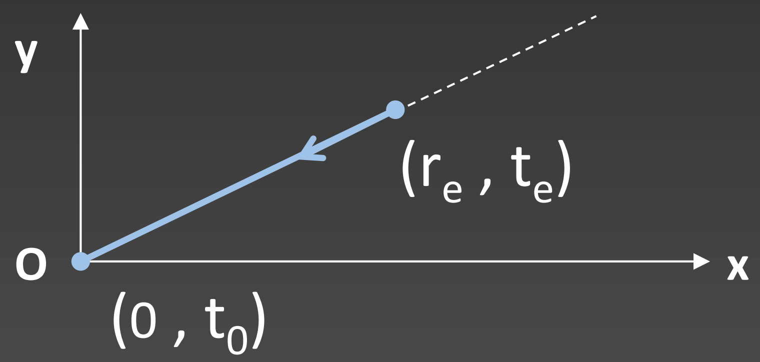
2) the *geodesics* in the FLRW Universe

a) non-relativistic limit (zero velocity):

$$(r, \theta, \phi) = \textit{constant}$$

- galaxies are still in coordinate space …
- … but all distances are proportional to a(t)

b) ultra-relativistic limit (upcoming photons):
straight line in 3-D space,



$$\int_{r_e}^{0} - \frac{dr}{\sqrt{1 - kr^2}} = \int_{t_e}^{t_0} \frac{c\,dt}{a(t)}$$

3) relation between *matter and curvature* in FLRW

EINSTEIN $G_{\mu\nu} = 8\pi G \, T_{\mu\nu}$

With FLRW metric, $G_{\mu\nu}$ diagonal with $G_{11}=G_{22}=G_{33}$

Same for $T_{\mu\nu}$ (of background cosmological fluid)

$$T^{\mu}_{\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

3) relation between *matter and curvature* in FLRW

FRIEDMANN LAW: $G_{00} = 8\pi G\, T_{00}$ gives

$$\frac{3}{R_H^2} \pm \frac{3}{R_k^2} = 8\pi\mathcal{G}\rho$$

i.e.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\mathcal{G}}{3}\frac{\rho}{c^2} - \frac{kc^2}{a^2}$$

Remark : for non-relativistic matter, $E = m\,c^2 \Rightarrow \rho = \rho_{mass}\, c^2$

⇨ then *Friedmann law* looks similar to *Newtonian expansion law* $(\dot{r} / r)^2 = (8\pi G/3)\, \rho_{mass} - k/r^2$,

but CRUCIAL DIFFERENCES :

1) $a(t) \neq r(t)$: *very different interpretation*
2) $k \neq 0$ *not in contradiction with homogeneity*
3) accounts for *non-relativistic and relativistic matter*

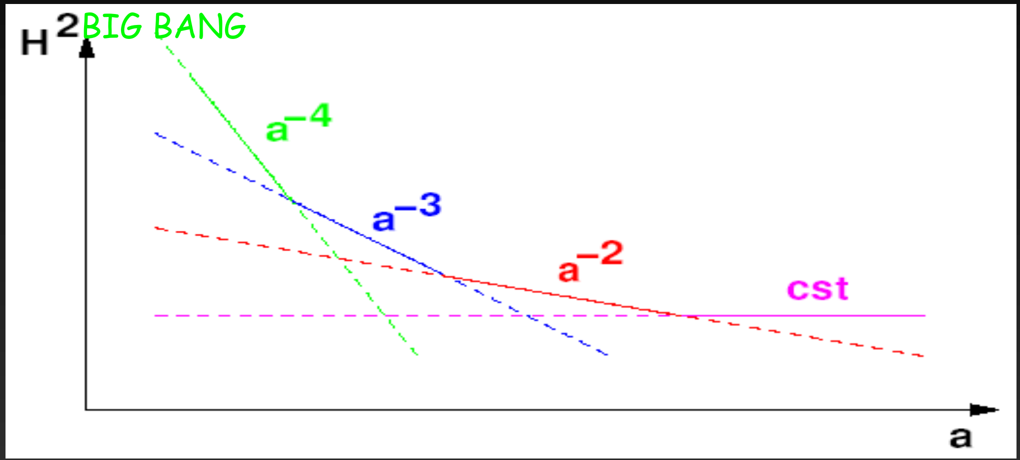
Bianchi: conservation of energy

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p)$$

Friedmann law:

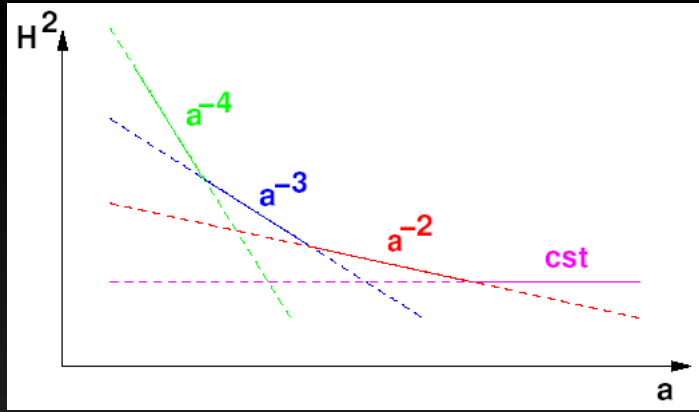
$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi\mathcal{G}}{3c^2}\rho_{\rm R} + \frac{8\pi\mathcal{G}}{3c^2}\rho_{\rm M} - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

- most « complete » scenario :
- phases can be skipped, but order cannot change



- RADIATION DOMINATION : $a \propto t^{1/2}$ $H = 1 / (2\,t)$
- MATTER DOMINATION : $a \propto t^{2/3}$ $H = 2 / (3\,t)$
- CURVATURE DOMINATION :
  - • $k < 0$ (open) : $a \propto t$ $H = 1 / t$
  - • $k > 0$ (closed) : $a \rightarrow 0$, then $a < 0$ or ➪
- VACUUM DOMINATION : $a \propto \exp(\Lambda t/3)^{1/2}$ $H \rightarrow$ *constant*

- the matter budget :
  - if we can measure $\{\rho_R, \rho_M, k, \Lambda\}$ today, we can extrapolate back …



  - today :

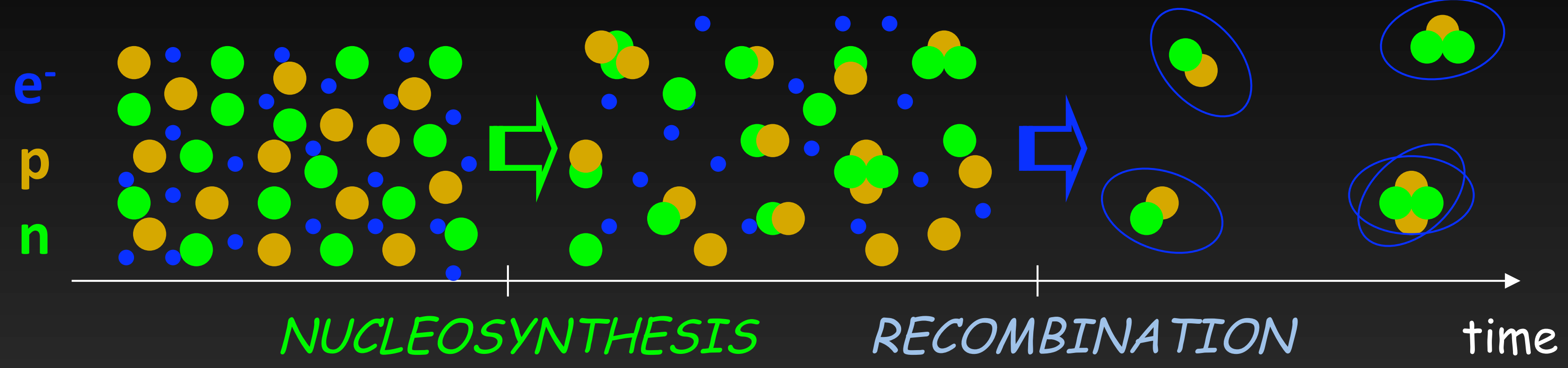
$$1 = \frac{8\pi\mathcal{G}}{3H_0^2c^2}(\rho_{R0} + \rho_{M0}) - \frac{kc^2}{a_0^2H_0^2} + \frac{\Lambda}{3H_0^2} = \Omega_R + \Omega_M - \Omega_k + \Omega_\Lambda$$

  - flatness condition : $\Omega_0 \equiv \Omega_R + \Omega_M + \Omega_\Lambda = 1$
  - $\Omega_X = \rho_X / \rho_c^0$ with $\rho_c^0 = 3\, H_0^2\, c^2 / 8\pi G$
  - $\omega_X = \Omega_x h^2$ is just ρ_x in some units ($h = H_0/100$ km s^{-1}Mpc^{-1})

- COLD or HOT BIG BANG ???
  - studies based on the *most simple* possible scenario:
    - Universe contains only non-relativistic matter
    - evolution under the laws of nuclear physics between Big Bang and today

    ↳ COLD BIG BANG SCENARIO

- **COLD BIG BANG :**
  - $H^2 = (8\pi G/3c^2)\,\rho_M \quad \Rightarrow \quad \rho \propto a^{-3} \propto t^{-2}$



  - *NUCLEOSYNTHESIS :*
    - ensemble of *nuclear reactions*

$$\left.\begin{matrix} p \\ n \\ e^- \\ \nu \end{matrix}\right\} \longrightarrow \left\{\begin{matrix} H \\ D \\ {}^3He\ ,\ {}^4He \\ Li\ ,\ etc. \end{matrix}\right.$$

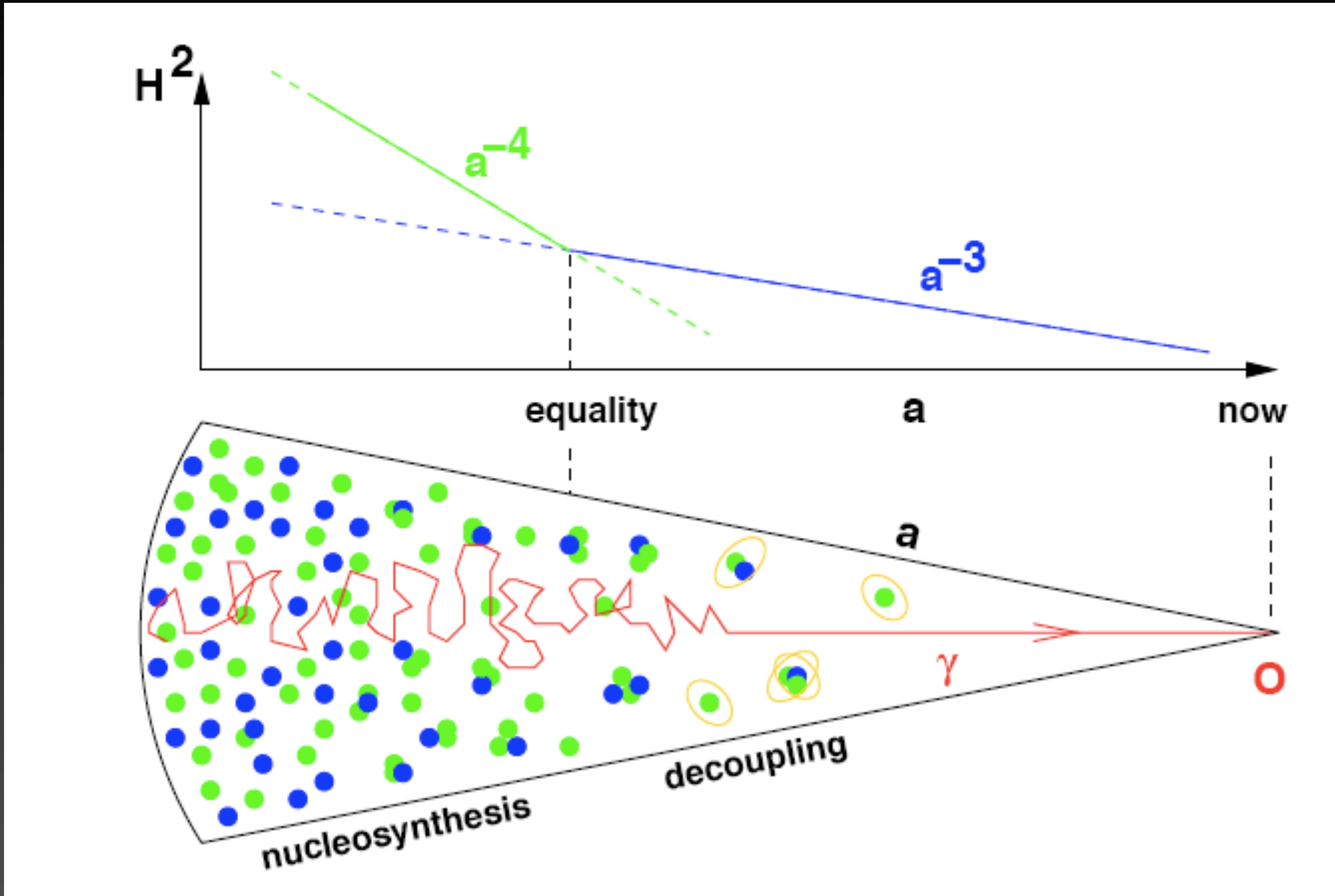
    - freeze-out due to *expansion*

- pioneering works on nucleosynthesis :

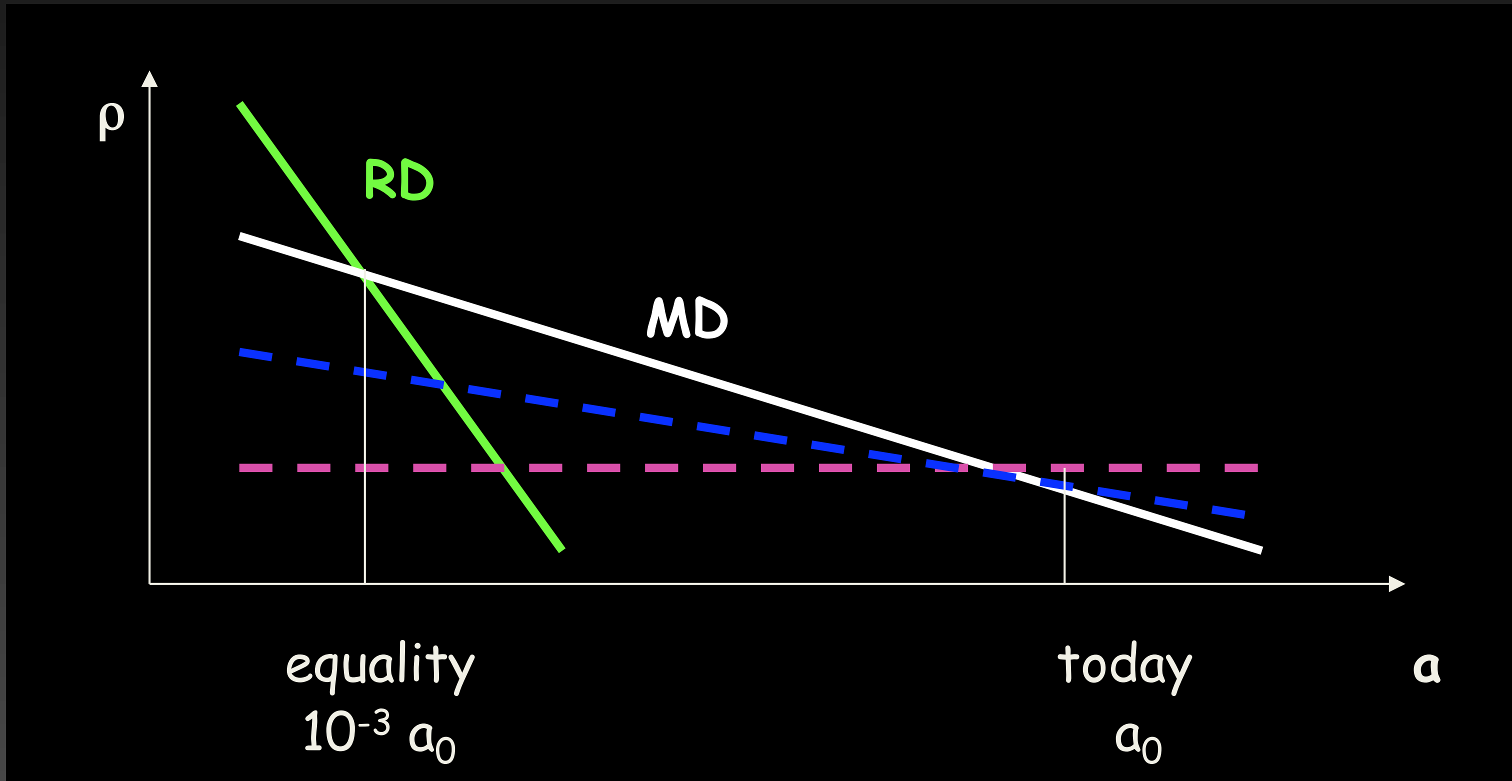
  - 1940 : Gamow et al. (USSR → USA)
    1964 : Zel'dovitch et al. (USSR)
    1965 : Hoyle & Taylor (UK)
    1965 : Peebles et al. (USA)

  - COLD BIG BANG
    - ↳ no hydrogen
      - ↳ need to change $H(t_{nucleo})$
        - ↳ add relativistic matter (photons) with $\rho_R \gg \rho_M$
          - ↳ HOT BIG BANG !!!

• HOT BIG BANG :

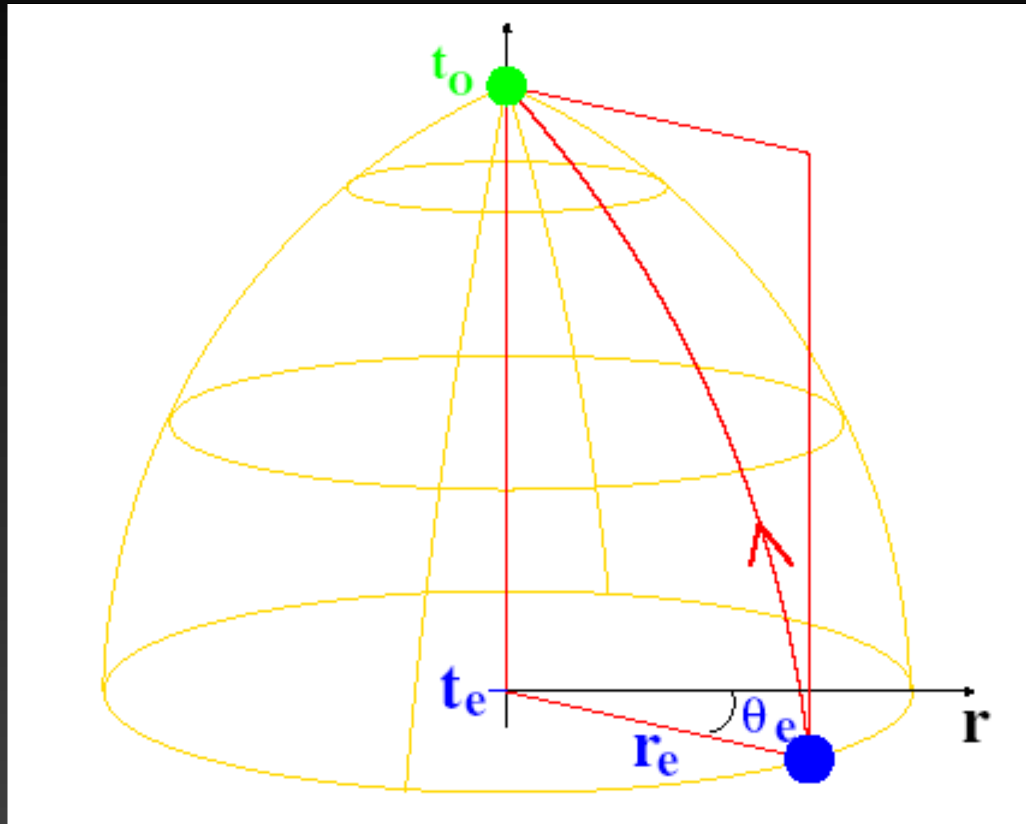


- is there curvature / Λ domination today ?
  - structure formation $\Rightarrow$ long-enough M.D. $\Rightarrow$ $\Omega_m \geq 0.2$
  - if $\Omega_k \sim 1$ or $\Omega_\Lambda \sim 1$, curvature / Λ domination *started recently :*



Distances
Horizons
Need for inflation

- past light-cone :
  - *in Euclidian space :*
    - $r_e = c\,(t_0 - t_e)$

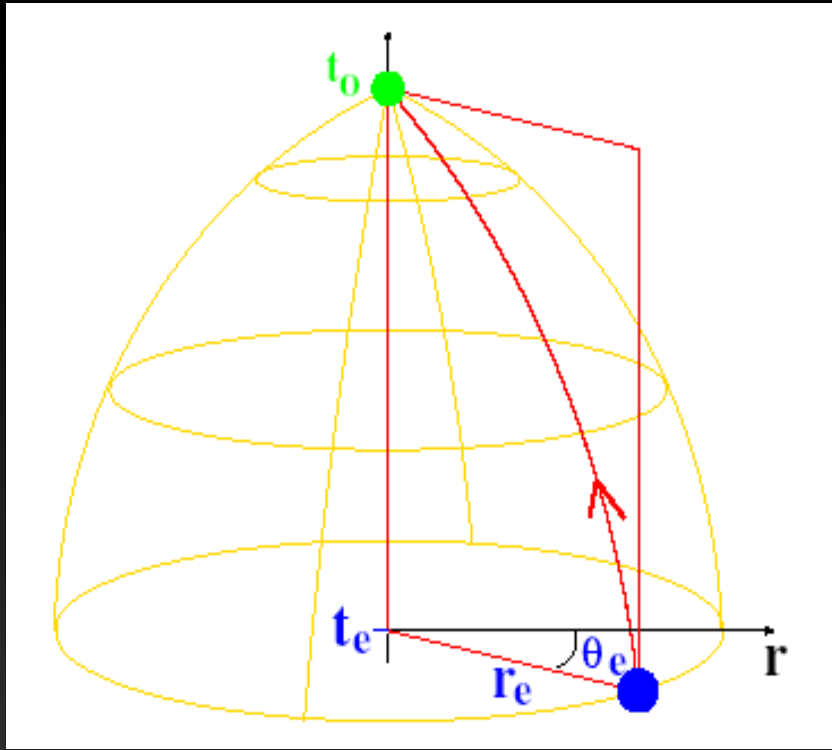


  - *in Friedmann :*
    - $$\int_{r_e}^{0} \frac{dr}{\sqrt{1-kr^2}} = \int_{t_e}^{t_0} \frac{c}{a(t)}\,dt$$

- the redshift :

$z = \Delta\lambda / \lambda = \lambda_0 / \lambda_e - 1$

$z = a(t_0) / a(t_e) - 1$



- *Newtonian* : $z = v / c \leq 1$; *G.R.* : no limit, as observed …
- at short distance, we can recover *Hubble law* ($z = v / c = H r / c$)

$$z = \frac{a(t_0)}{a(t_0 - dt)} - 1 = \frac{1}{1 - \frac{\dot{a}(t_0)}{a(t_0)} dt} - 1 = \frac{\dot{a}(t_0)}{a(t_0)} dt = \frac{\dot{a}(t_0)}{a(t_0)} \frac{dl}{c}$$

- in reality, we observe sum of gravitational + Doppler redshifts

- distances :
  - distance in coordinate space χ (comoving distance)

    χ = r in flat space, $\int_0^{r_e} \frac{dr}{\sqrt{1-kr^2}}$ otherwise

  - same expressed in physical units of today [$a(t_0)\,\chi$]
  - angular distance (for standard rulers)

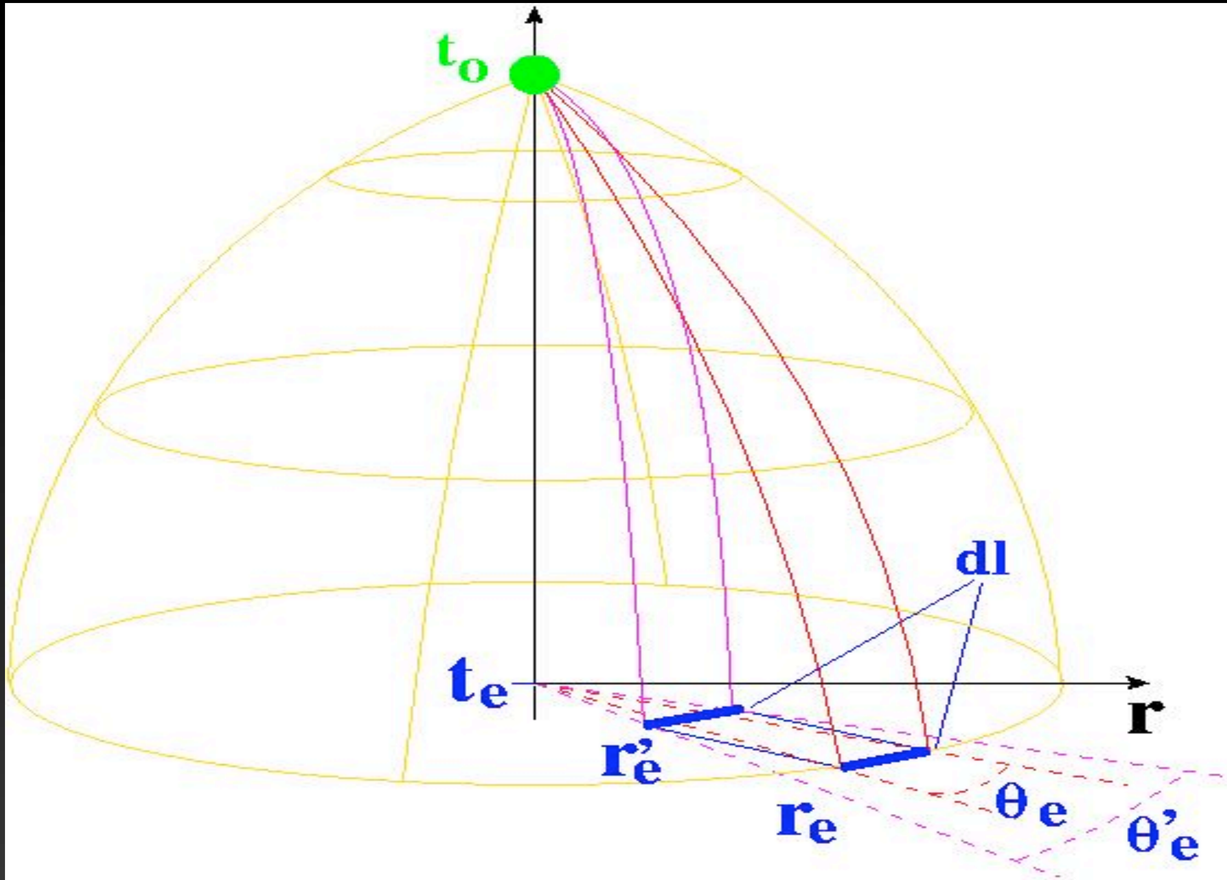
$$d_A \equiv \frac{dl}{d\theta} = a(t_e)\; r_e = a(t_0)\frac{r_e}{1+z_e}$$

    $r_e(z_e)$ and $d_A(z_e)$ non-linear!

  - luminosity distance (for standard candles)

$$d_L \equiv \sqrt{\frac{L}{4\pi l}} = a(t_0)\, r_e (1+z_e)$$

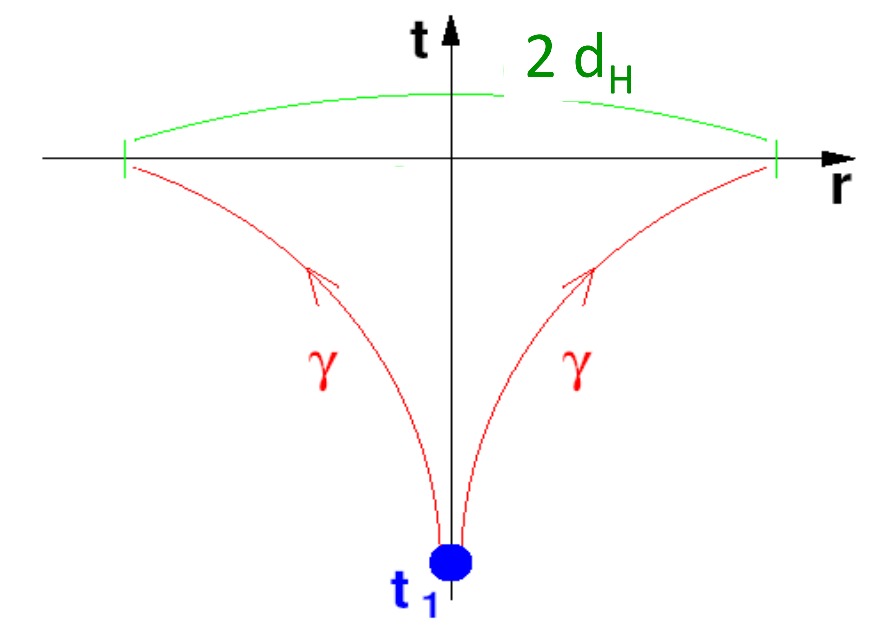
    $r_e(z_e)$ and $d_L(z_e)$ non-linear!



$$\frac{dt}{dr} = \frac{a(t)}{c\ \ (1-kr)^{1/2}}$$

- CLOSED UNIVERSE : objects seen under larger angle
- OPEN UNIVERSE : objects seen under smaller angle

- causal horizon :

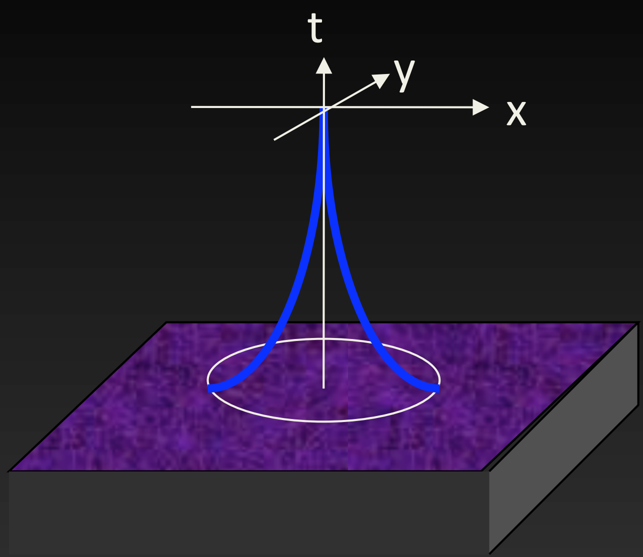


$$d_H = a(t)\int_0^r \frac{dr}{\sqrt{1-kr^2}} = a(t)\int_{t_1}^t \frac{dt'}{a(t')}$$

– if $a \sim t^n$ (n=1/2 for radiation, 2/3 for matter) and $t \gg t_1$:

$$d_H(t) = \frac{t}{1-n} = \frac{n}{1-n} R_H(t)$$

- size of observable universe :

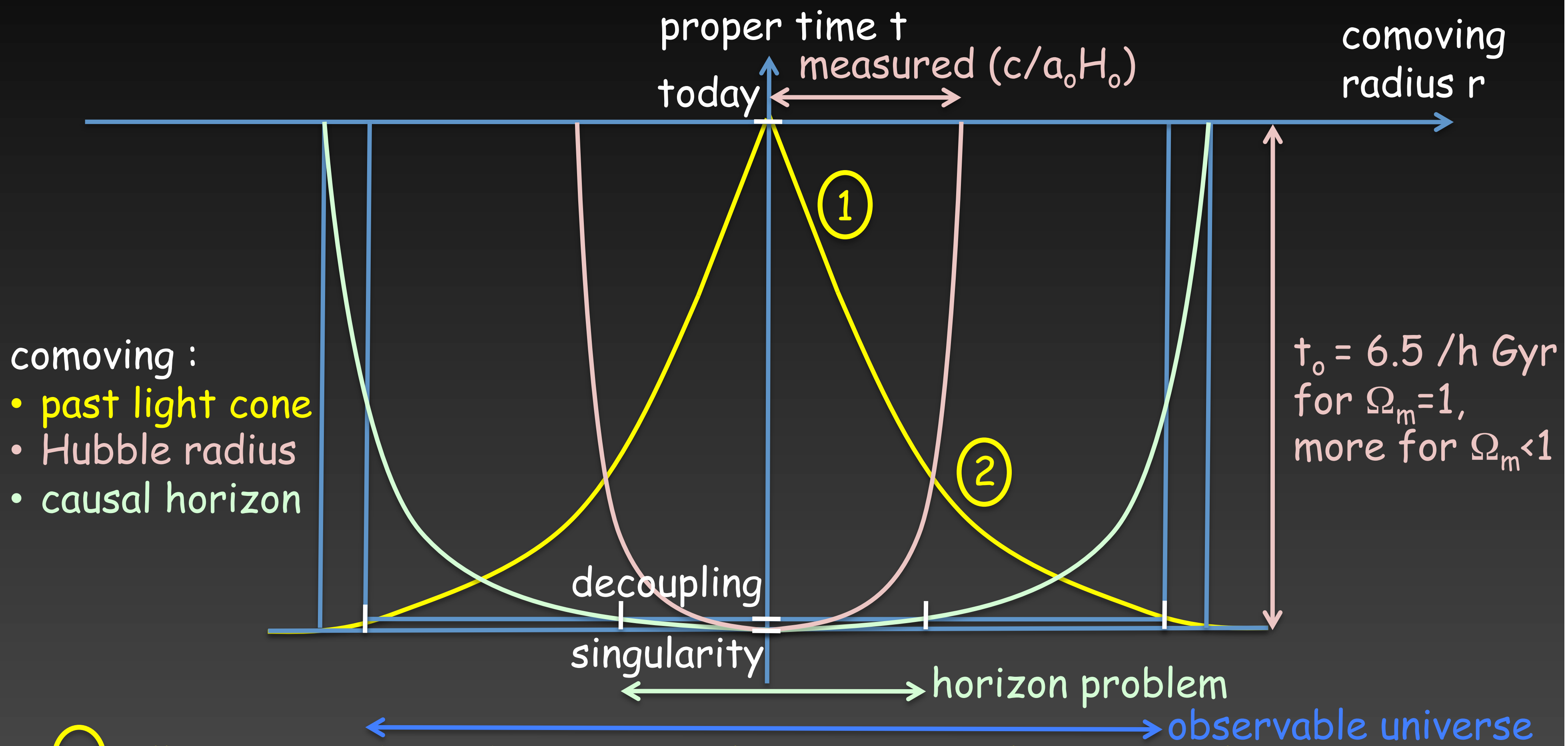


  - Nearly equal to :

$$d_H = a(t) \int_0^r \frac{dr}{\sqrt{1-kr^2}} = a(t) \int_{t_{dec}}^t \frac{dt'}{a(t')}$$

  - $R_H(t_0)$=3000/h Mpc, so radius of observable universe close to 6000/h Mpc

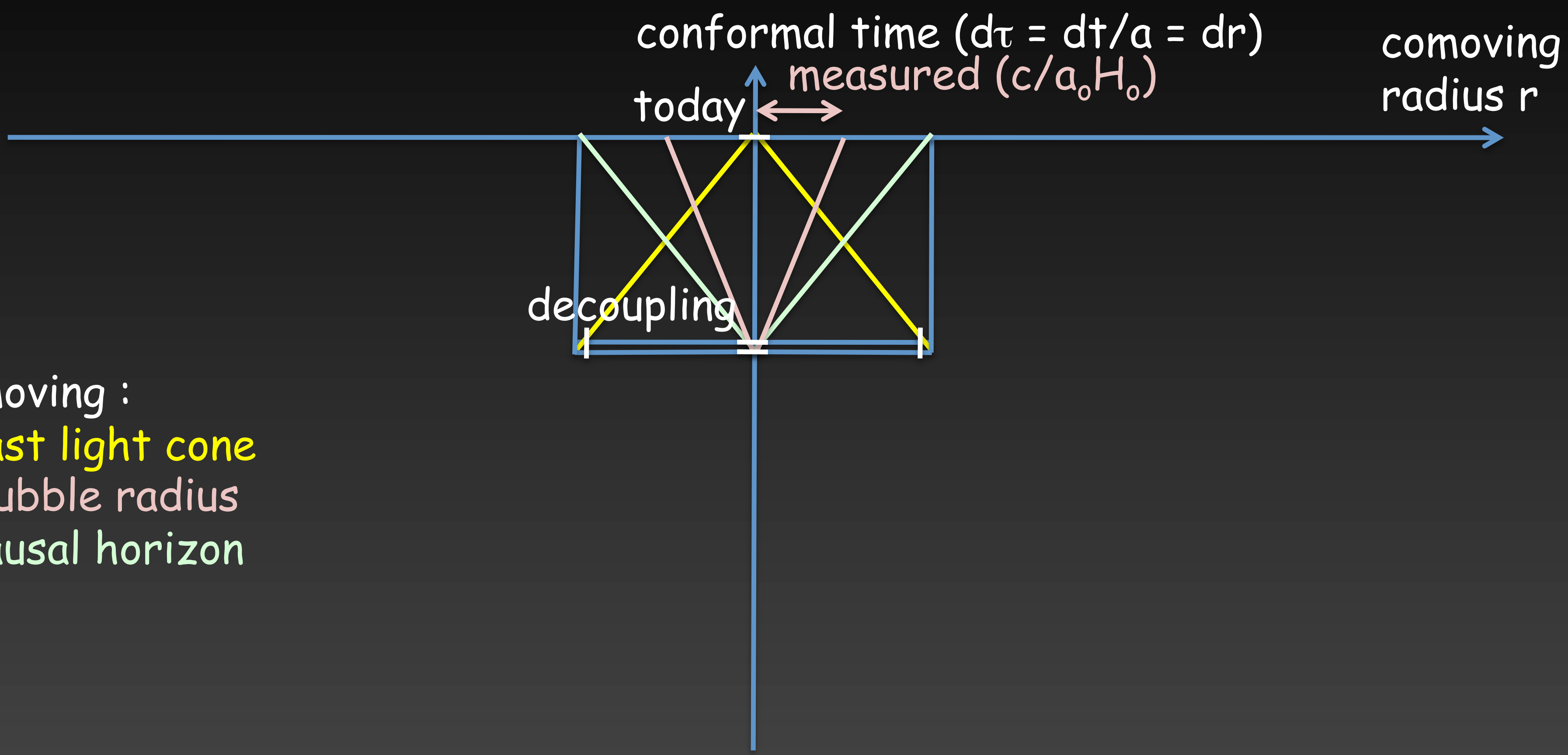
Horizon issues



(1) all distances (d_L, d_A) proportional to r and to z: linear Hubble diagram

(2) non-trivial corrections induced by spacetime curvature: R_c, a(t)

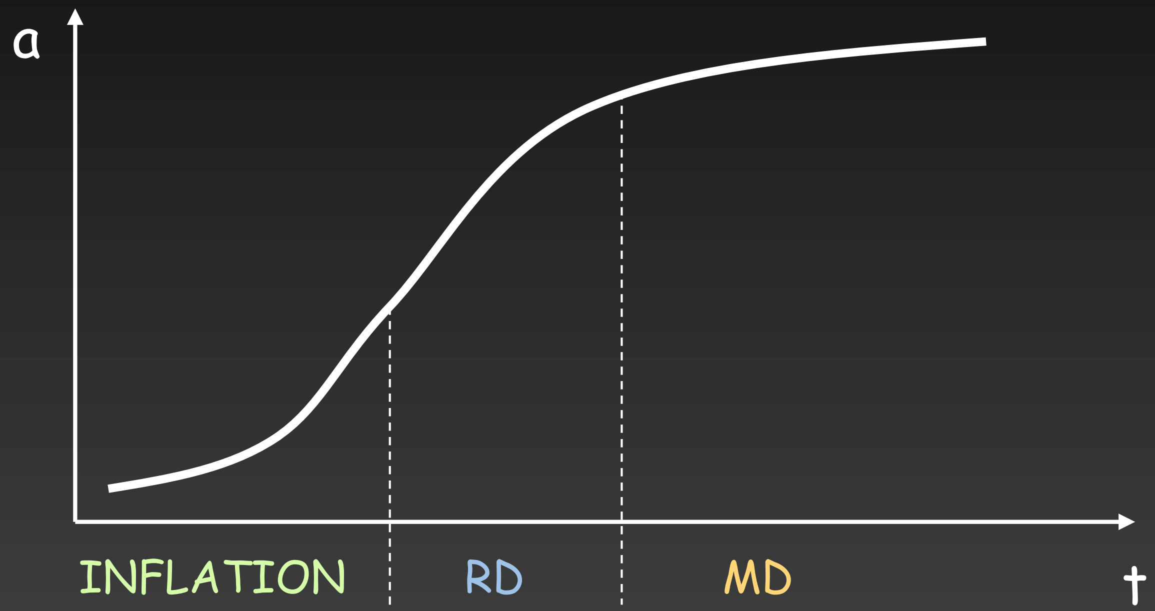
Horizon issues



comoving :

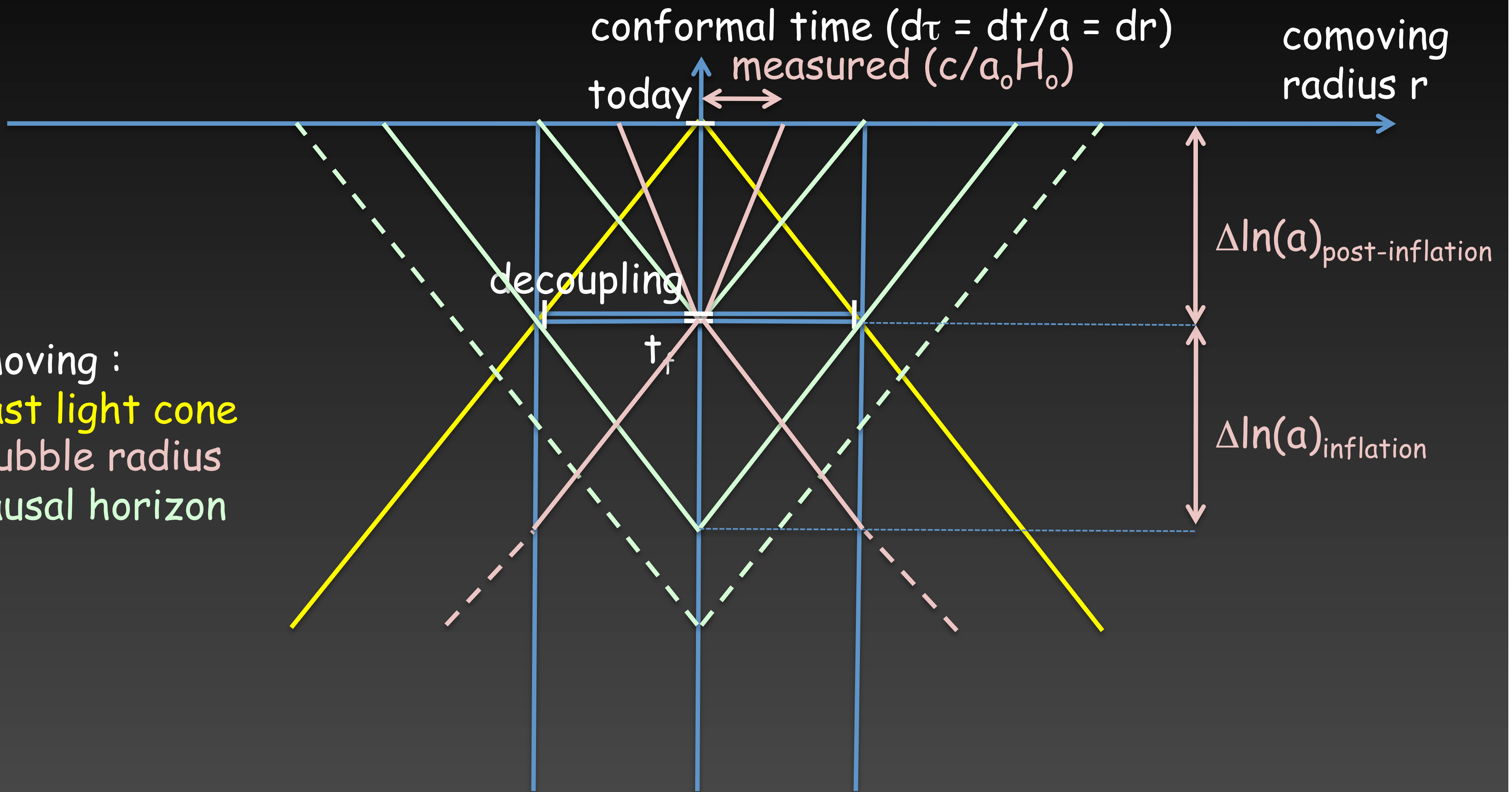
- past light cone
- Hubble radius
- causal horizon

- inflation: (Guth 79; Starobinsky 79)
  - defined as an initial *accelerated expansion* stage :



$$d_H(t_i, t) = d_H(t_i, t_f) + 2R_H(t)$$

Horizon issues



comoving :

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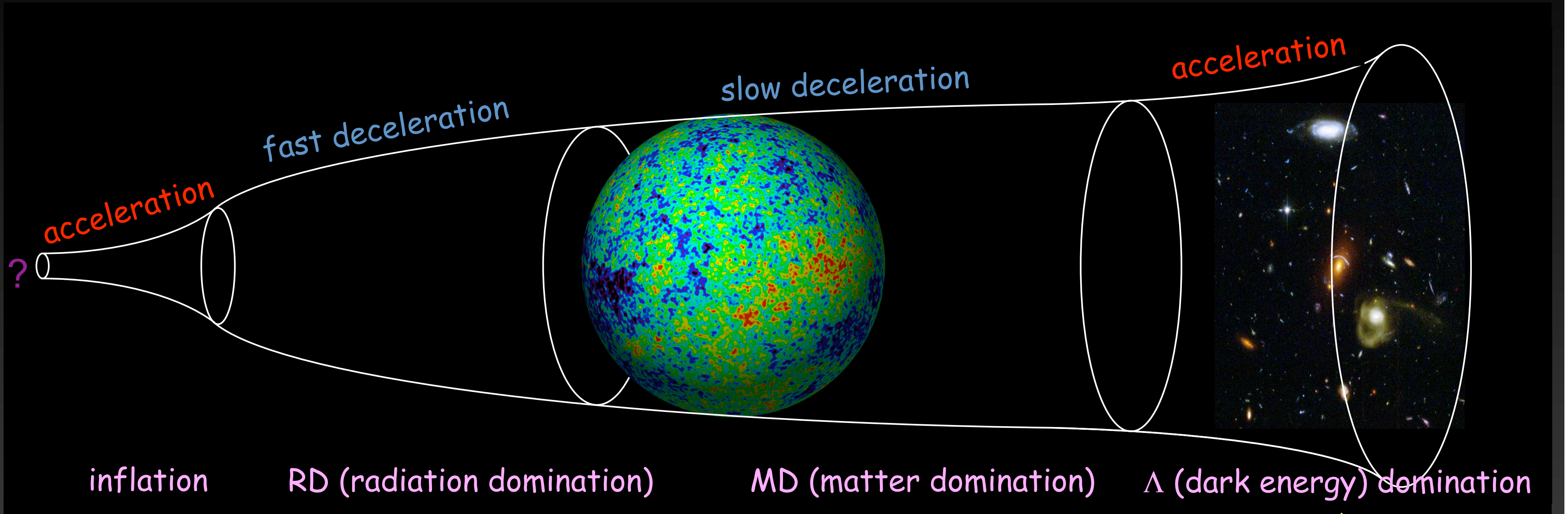
Homogeneous evolution



flatness & horizon problems

BBN predictions, CMB observation

structures form!

distance to standard candles/rulers (SNIa, CMB, BAO) + CMB, A.S.P.S.

Horizon issues

- Cosmo Quizz !!!
  1. Adding a stage of inflation to sCDM implies that the universe expands quicker in the early universe
  2. Adding a stage of inflation to sCDM implies that our observable universe was larger than expected at early times
  3. In case of everlasting Λ domination, future observers in the MW will ultimately see no other comoving objects (galaxies, …) because they will all be out of causal contact