Part II: Cosmological Perturbations, Matter Power Spectrum

CHIPP PhD Winter School 2011, Leukerbad

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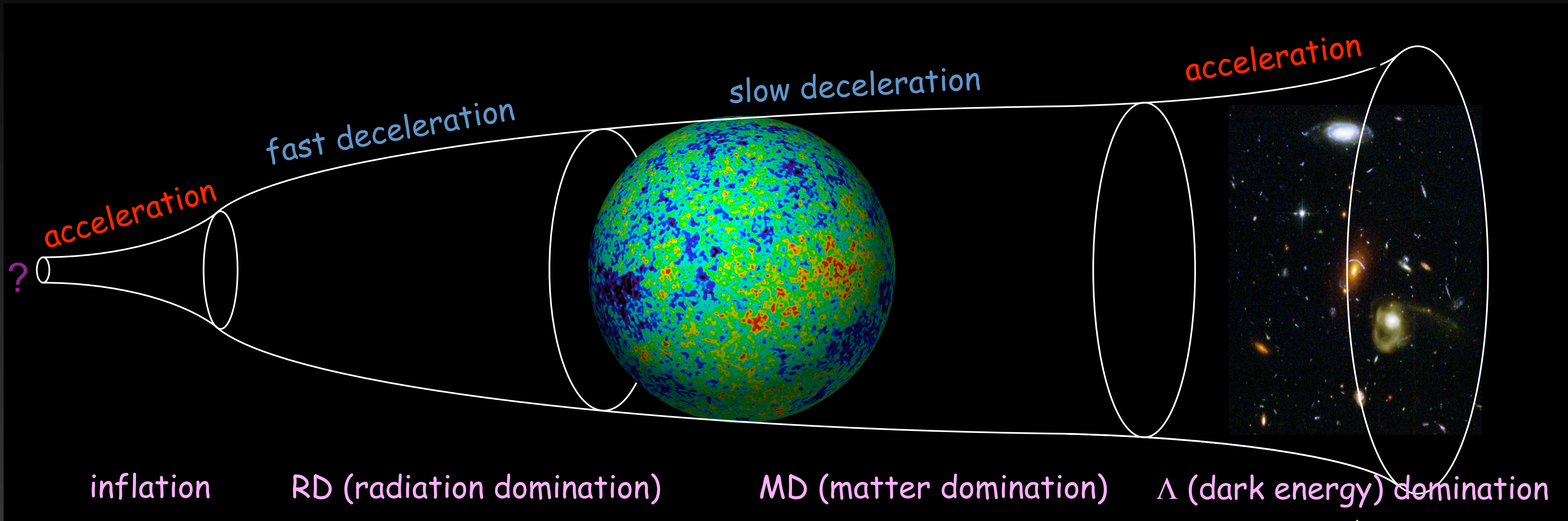
Homogeneous evolution



flatness & horizon problems

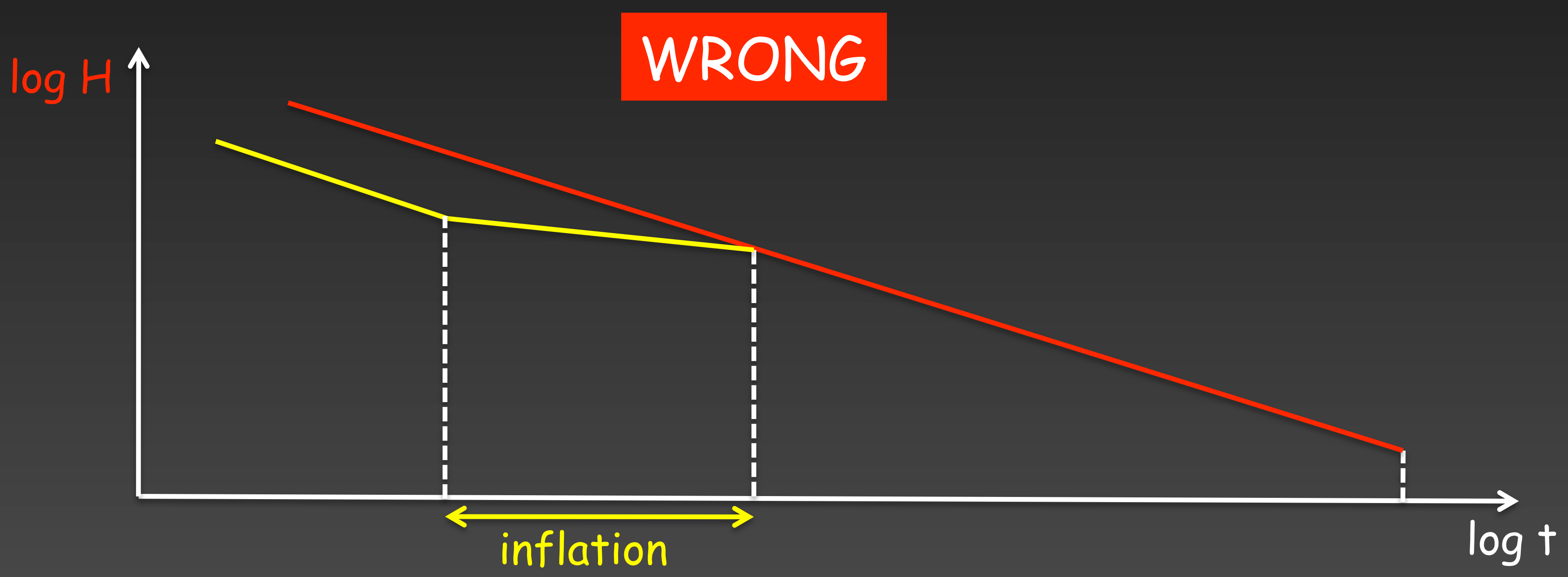
BBN predictions, CMB observation

structures form!

distance to standard candles/rulers (SNIa, CMB, BAO) + CMB, A.S.P.S.

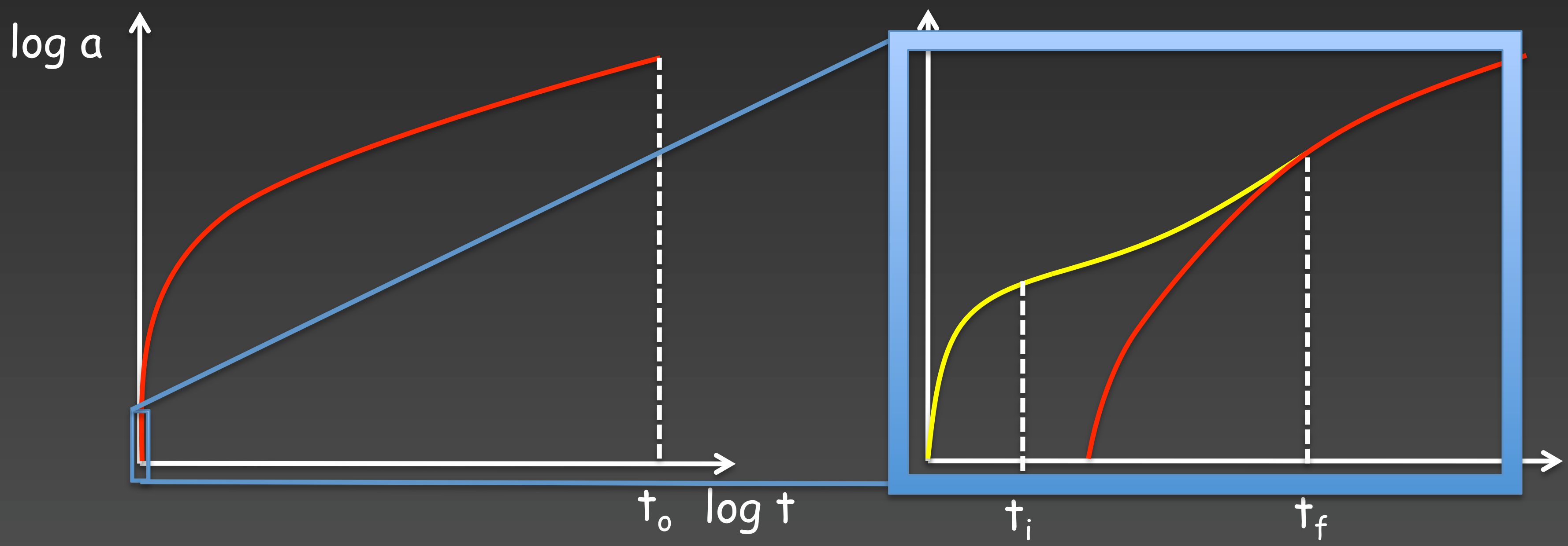
- Answer to Cosmo Quizz:
  1. Adding a stage of inflation to sCDM implies that the universe expands quicker in the early universe
  2. Adding a stage of inflation to sCDM implies that our observable universe was larger than expected at early times
  3. In case of everlasting Λ domination, future observers in the MW will ultimately see no other comoving objects (galaxies, …) because they will all be out of causal contact

- Answer to Cosmo Quizz !!!
  1. Adding a stage of inflation to sCDM implies that the universe expands quicker in the early universe



- Answer to Cosmo Quizz !!!
  2. Adding a stage of inflation to sCDM implies that our observable universe was larger than expected at early times

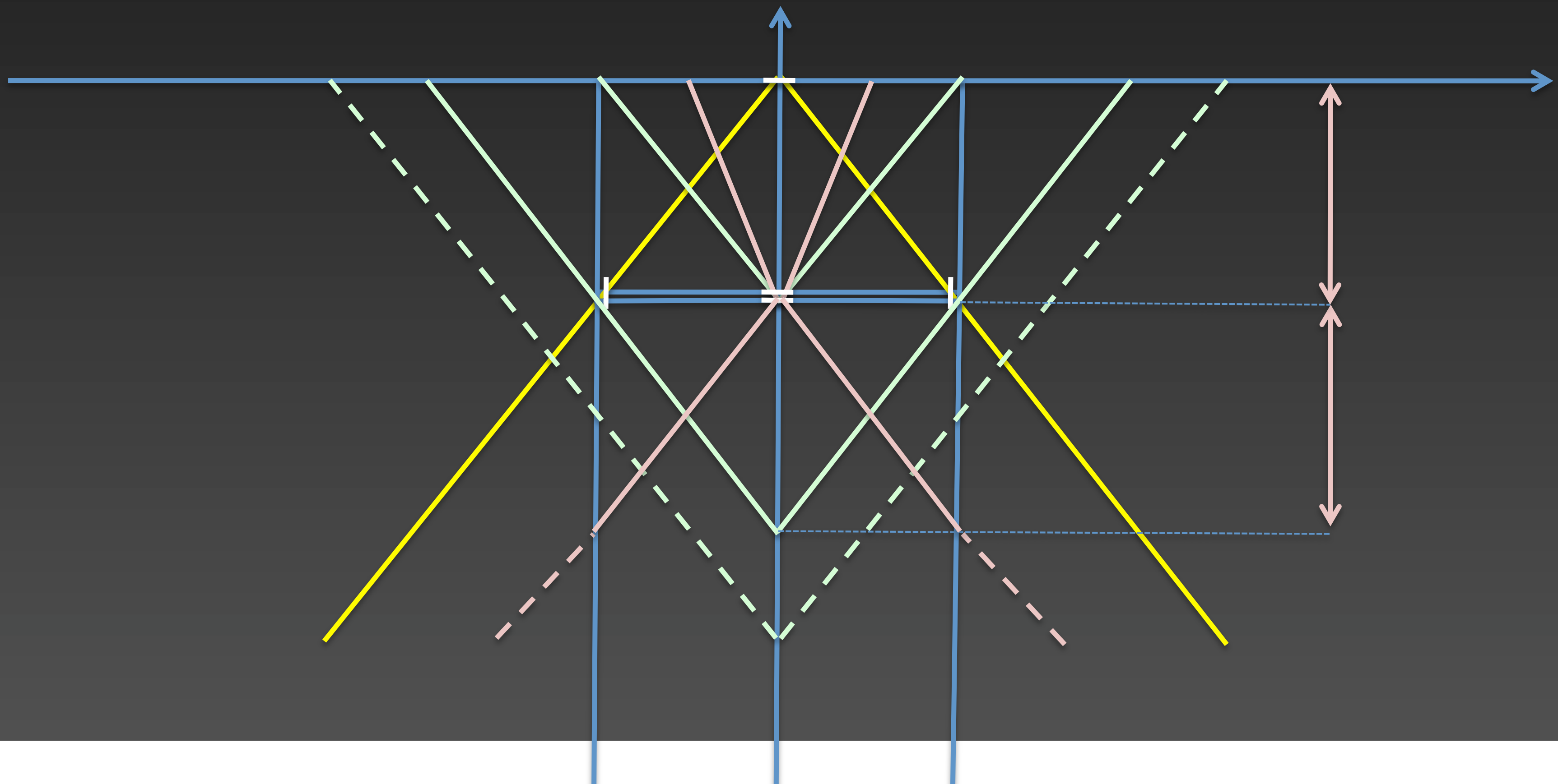
TRUE



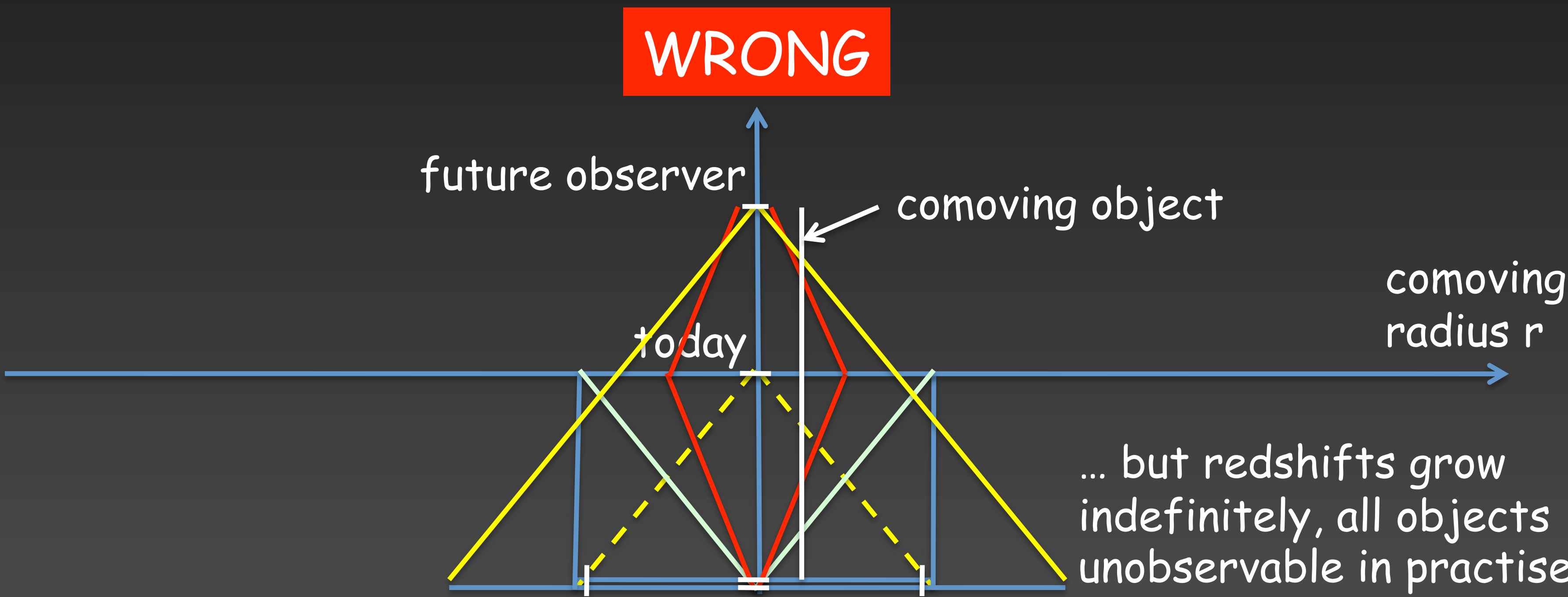
• Answer to Cosmo Quizz !!!

… but what is more relevant is that it is SMALLER IN UNITS OF COMOVING HUBBLE RADIUS …

(actually, r_{obs} must be smaller than r_H at t_i)



- Answer to Cosmo Quizz !!!
  3. In case of everlasting Λ domination, future observers in the MW will ultimately see no other comoving objects (galaxies, …) because they will all be out of causal contact



… but redshifts grow indefinitely, all objects unobservable in practise

Cosmological Perturbations

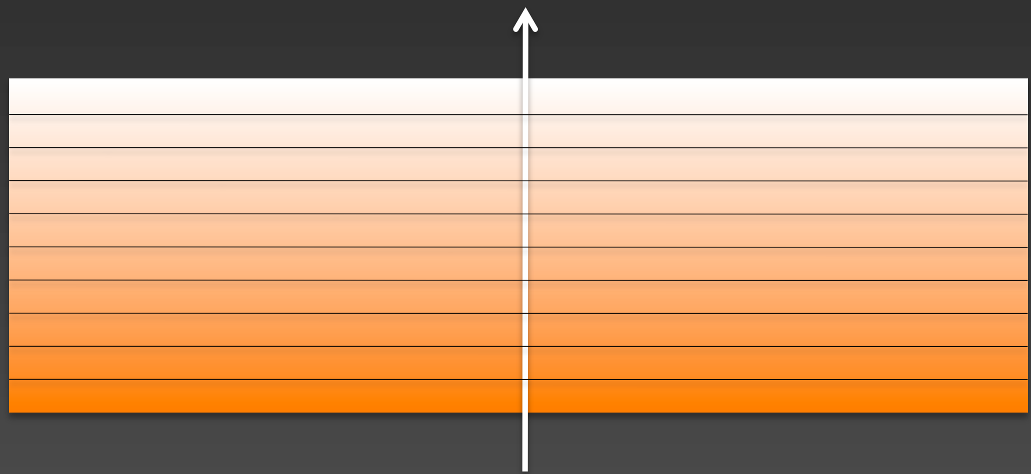
Matter Power Spectrum

Perturbations in Fourier space

- k = comoving wavenumber, $\lambda(t)=2\pi\, a(t)/k$
- Power spectra: $<|\text{perturbation}(k)|^2>$
  - theorists : <…> = average over many realization of theory (many universes)
  - observers : <…> = average over many independent modes/directions
  - in practise, for calculations : forget <…> and normalize initially all perturbations to +- 1 r.m.s for each k

Perturbations in Fourier space

- *gauge ambiguity :*
  - GR allows many possible time slicings of a spacetime manifold
  - FLRW model is perfectly homogeneous : unique time slicing compatible with homogeneity

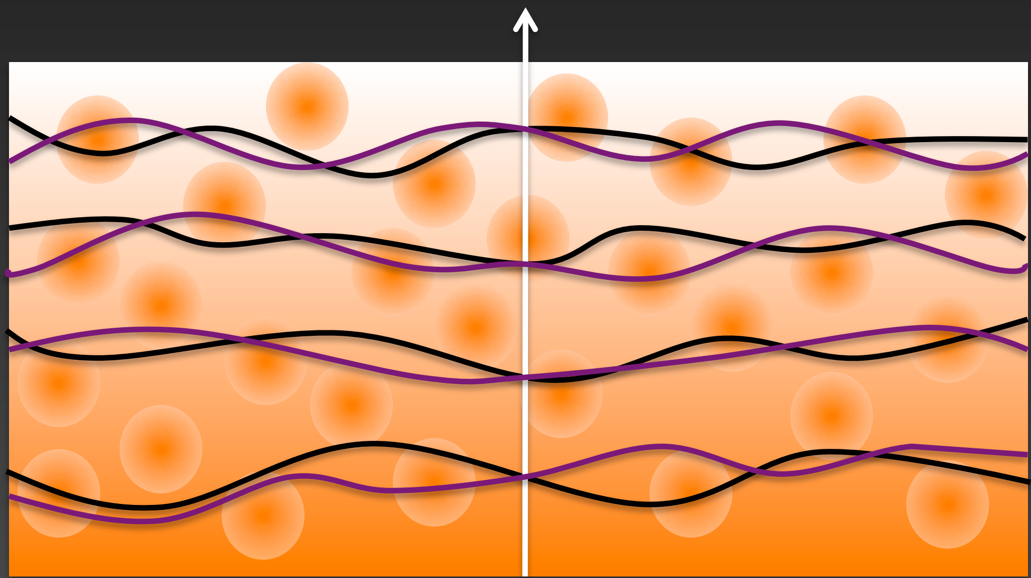


Perturbations in Fourier space

- gauge ambiguity :
  - GR allows many possible time slicings of a spacetime manifold
  - FLRW model is perfectly homogeneous : unique time slicing compatible with homogeneity
  - actual universe not quite homogeneous: time slicing not quite unique either…

Perturbations in Fourier space

- gauge ambiguity :
  - one choice of time-slicing = one way to define spatial averages = one way to define perturbations



Perturbations in Fourier space

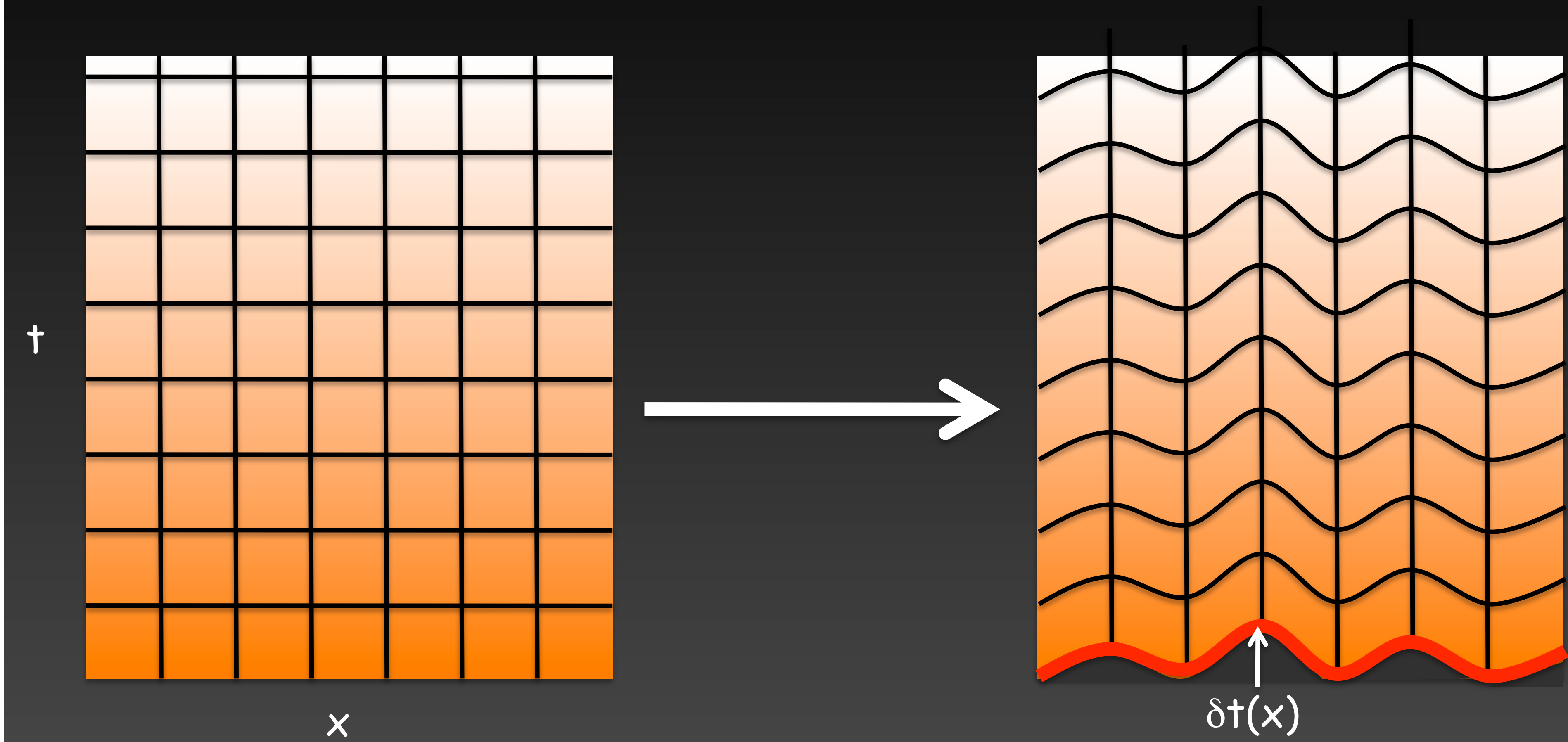
- gauge ambiguity :
  - one choice of time-slicing = one way to define spatial averages = one way to define perturbations
  - existence of gauge-invariant quantities
  - all observables quantities are automatically gauge-invariant

Perturbations in Fourier space

- what are adiabatic initial conditions ?
  - assume we are only interested in large (super-Hubble scales)
  - N fluids; N gauge-invariant density contrasts $\delta_i(t,\mathbf{k})$

    (their sum induces metric perturbations: curvature perturbation R, gravitational potential ϕ)
  - for each $\mathbf{k}$ with $|\mathbf{k}| > aH$, initial conditions = 2N numbers $\{ \delta_i(t,\mathbf{k}), \delta_i'(t,\mathbf{k}) \}$
  - one special case : homogenous universe + initial time shift

Perturbations in Fourier space

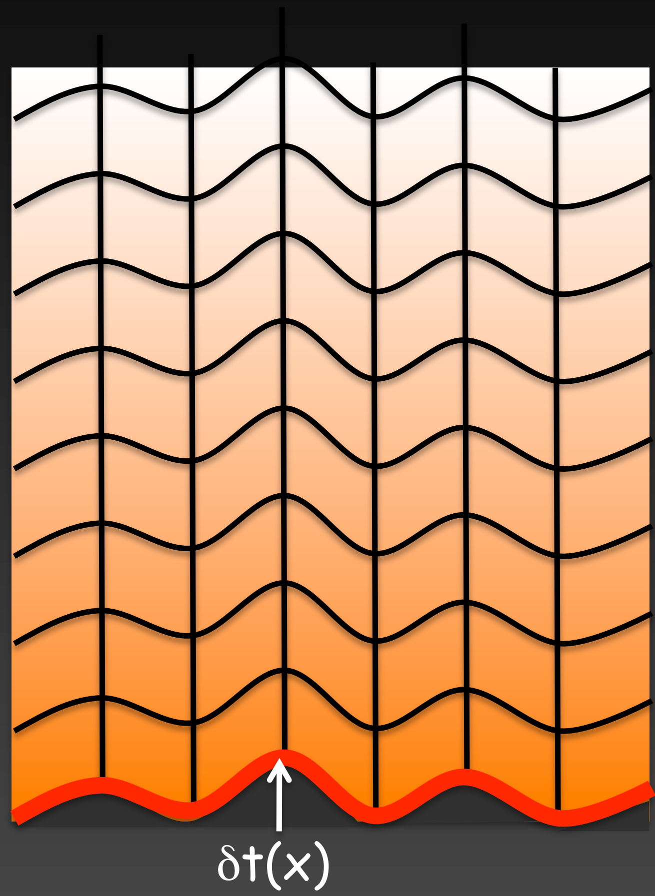


$\bar{\Delta}_i(t)$

$\Delta_i(x,t)=\bar{\Delta}_i(t+\delta t(x))$

Perturbations in Fourier space

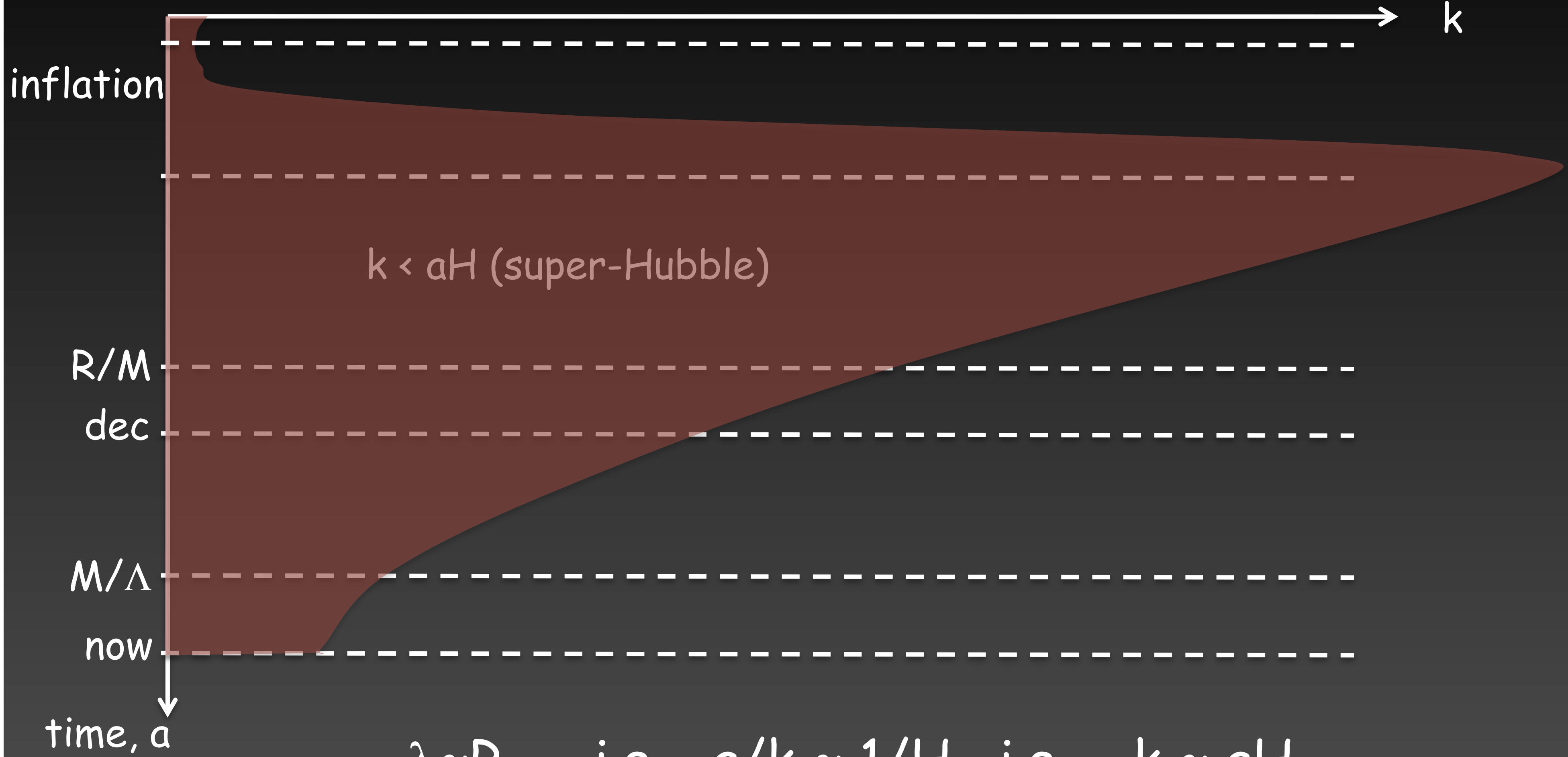
- In this case, all gauge-invariant pertrubations are proportional to single function $\delta t(x)$
- Generic when perturbations originate from single d.o.f. (e.g. growing mode of inflaton fluctuations)
- Called adiabatic : $\delta p_{tot} = c_s^2 \, \delta \rho_{tot}$
- Out of 2N i.c., 2 match this condition (one growing, one decaying)
- Other cases require mechanism with several d.o.f; still, thermal equilibrium usually enforces adiabatic conditions



$\Delta_i\,(x,t)=\Delta_i\,(t+\delta t(x))$

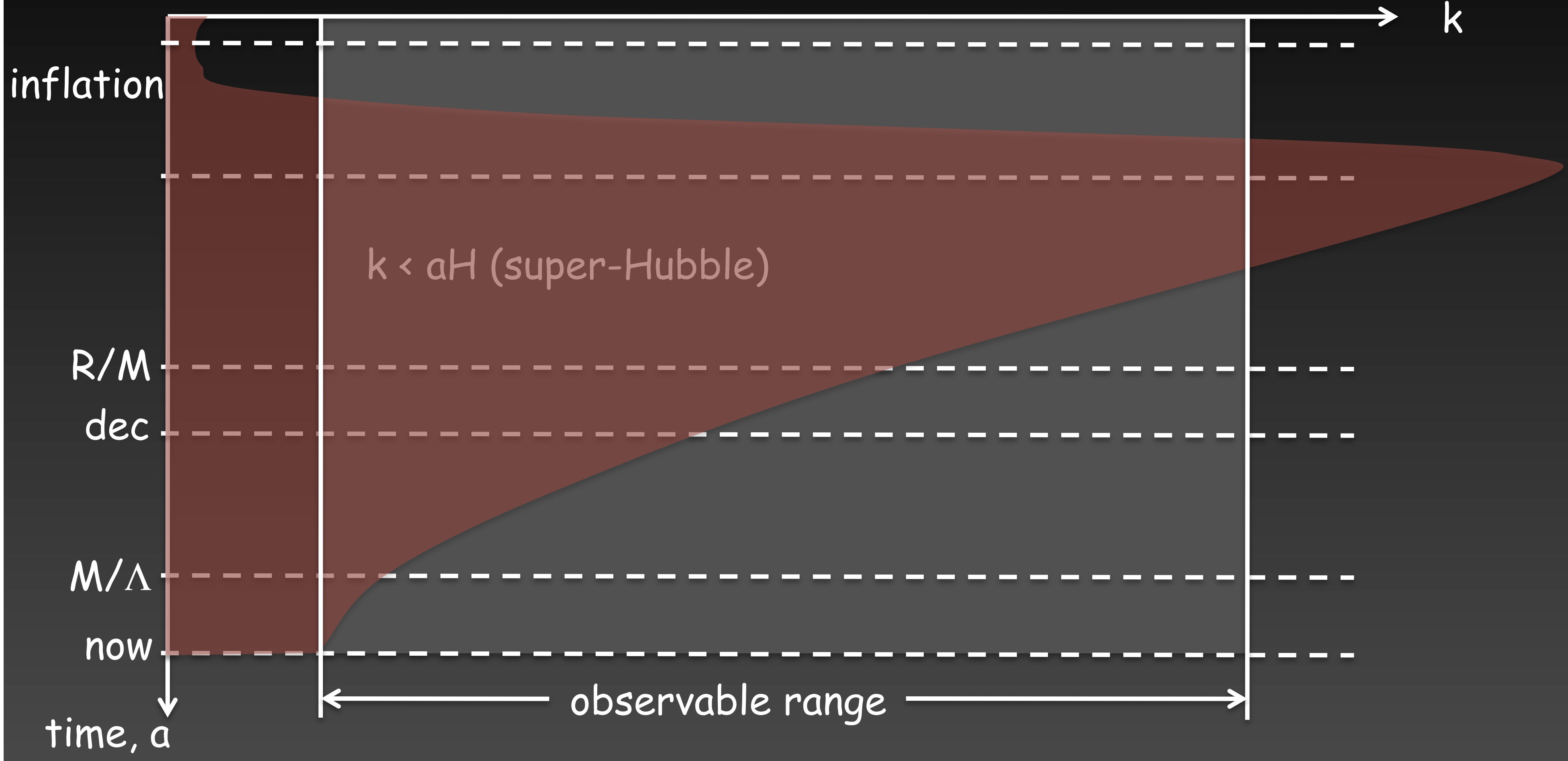
Perturbations in Fourier space

Perturbations in Fourier space

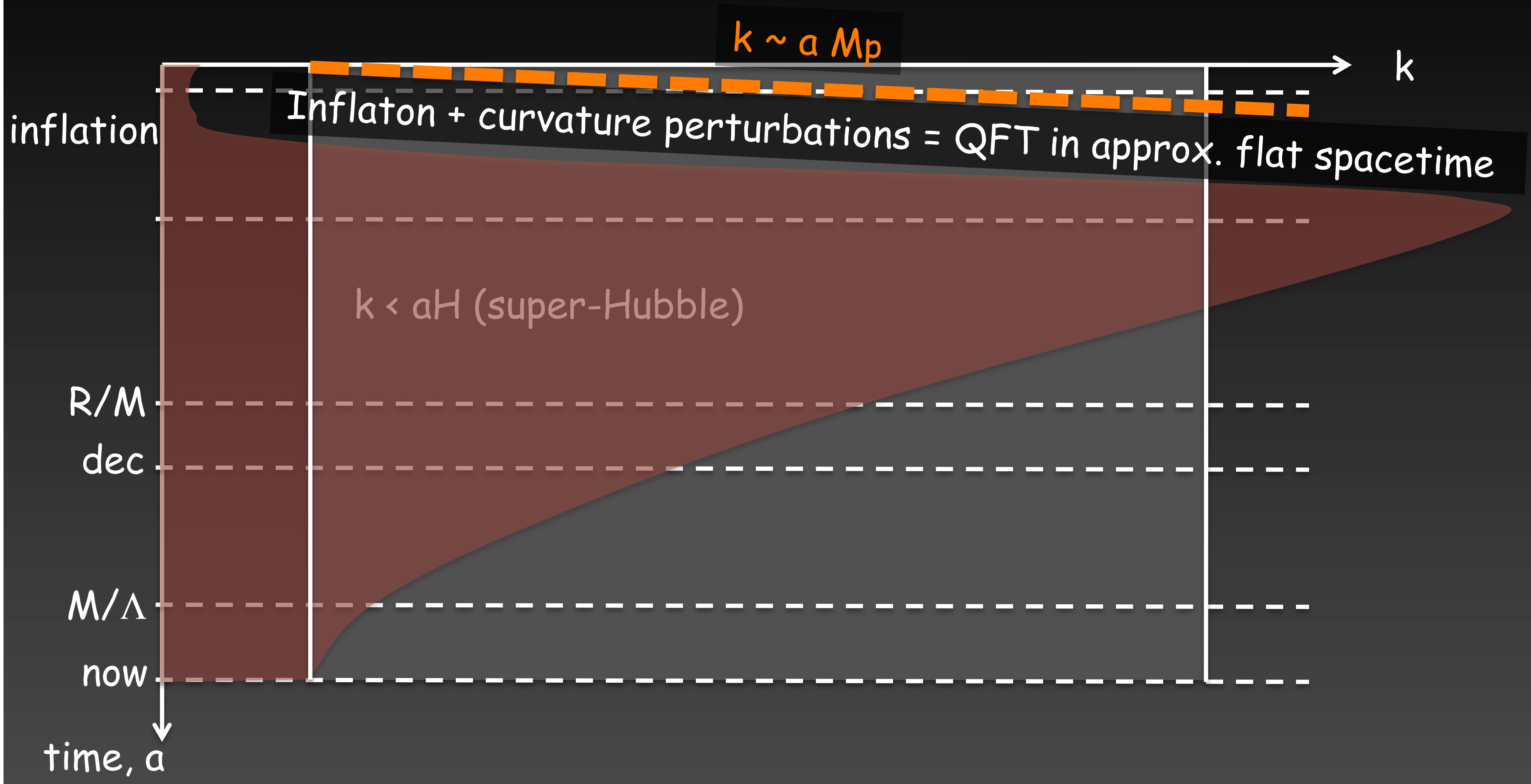


$\lambda \sim R_H$ i.e. $a/k \sim 1/H$ i.e. $k \sim aH$

Perturbations in Fourier space



Perturbations in Fourier space



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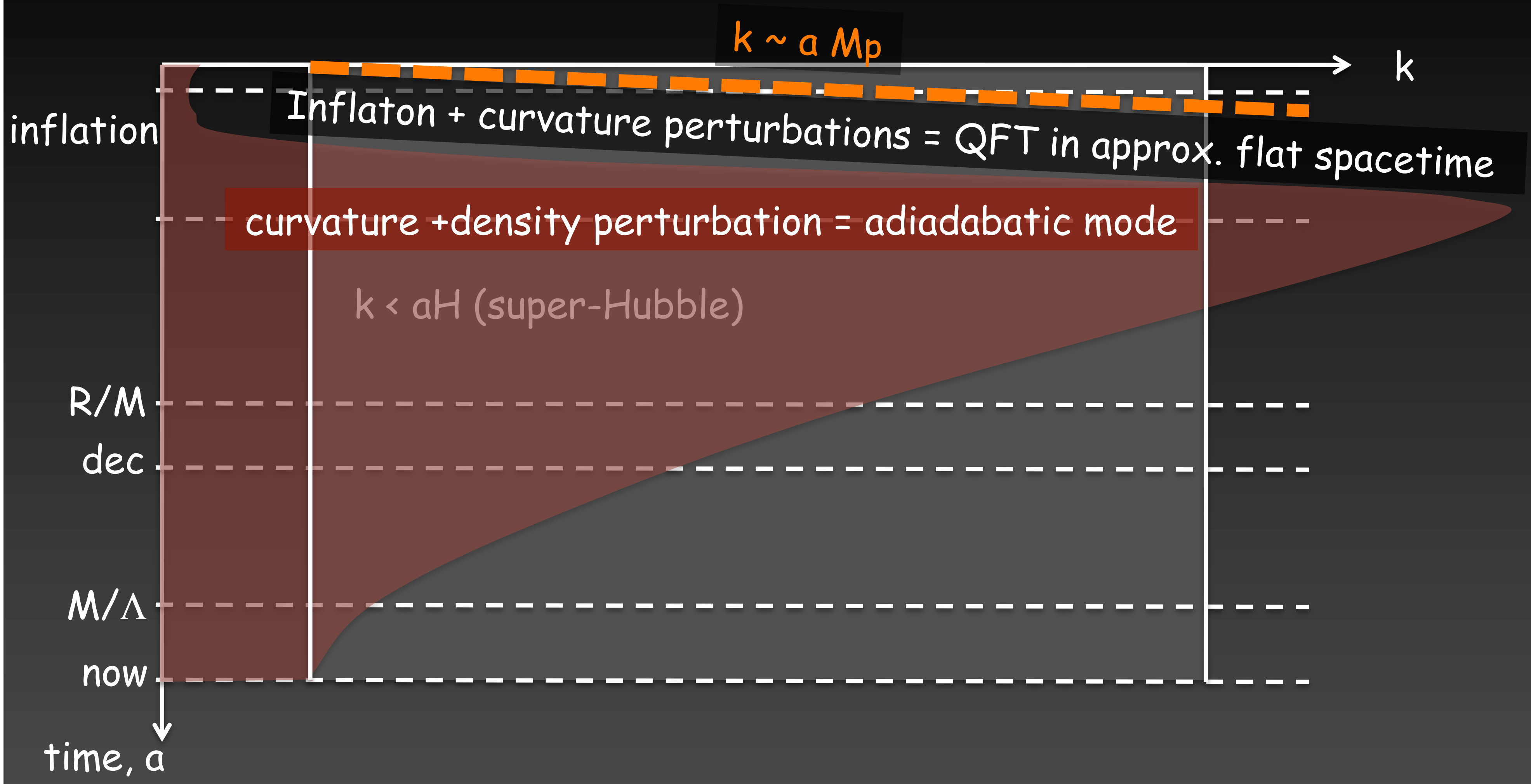

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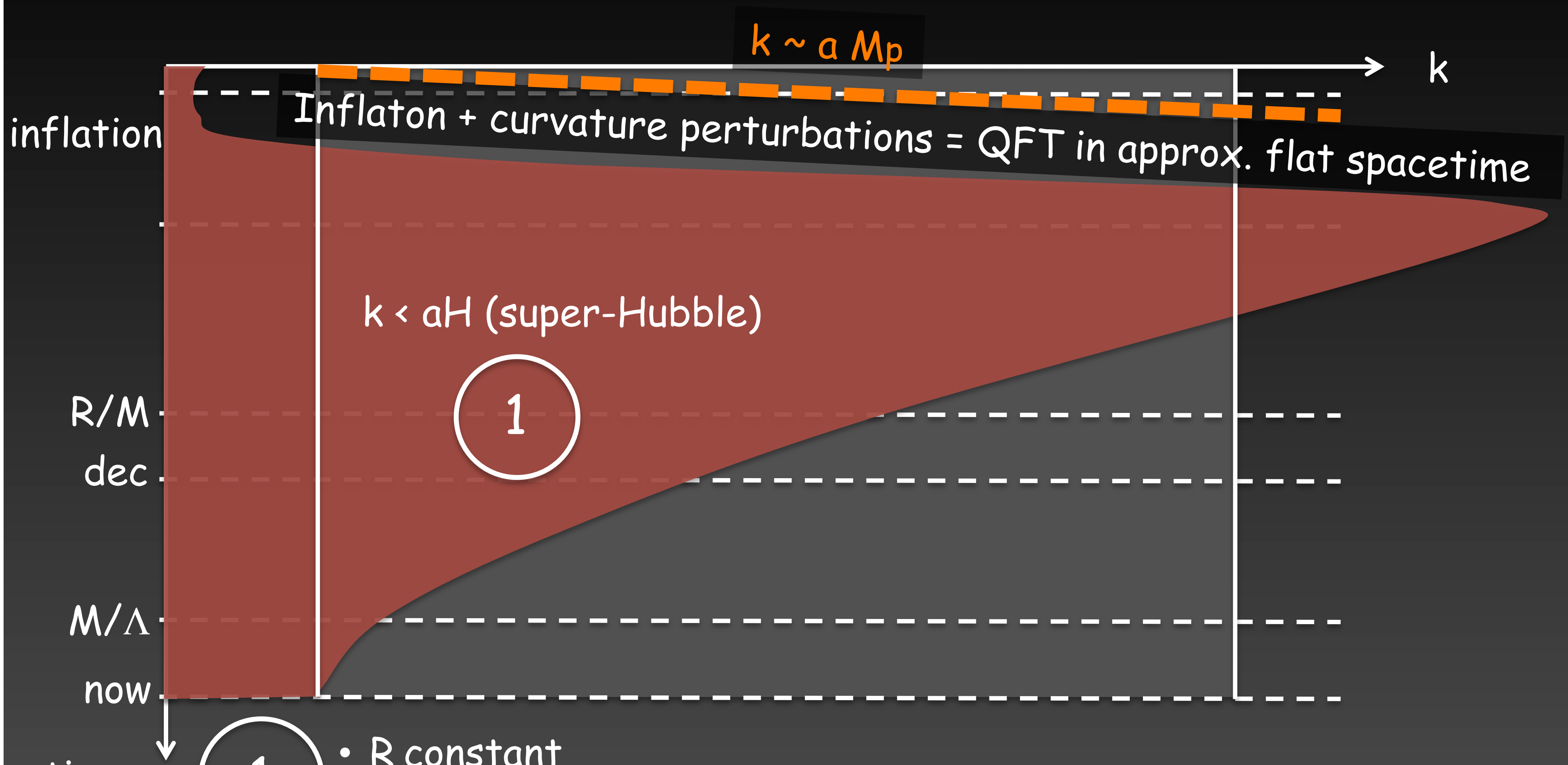
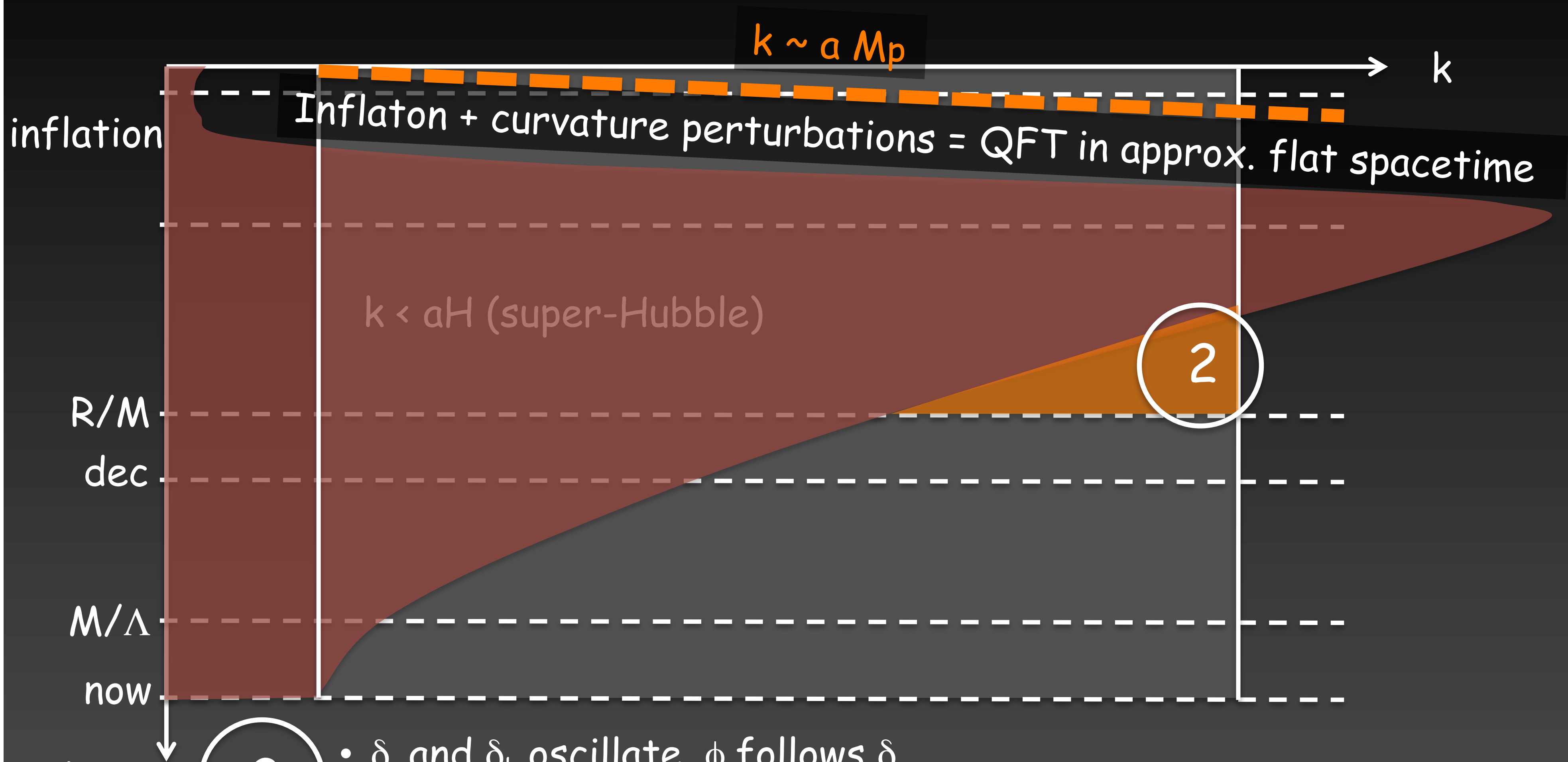


1

- R constant
- $\phi, \delta_m, \delta_\gamma$ essentially constant

Perturbations in Fourier space

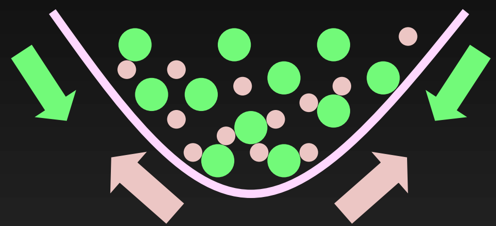


2

- δ_γ and δ_b oscillate, ϕ follows δ_γ
- cdm = test fluid, δ_{cdm} grows slowly

- Accoustic oscillations:

Tightly-coupled baryons, electrons, photon fluid

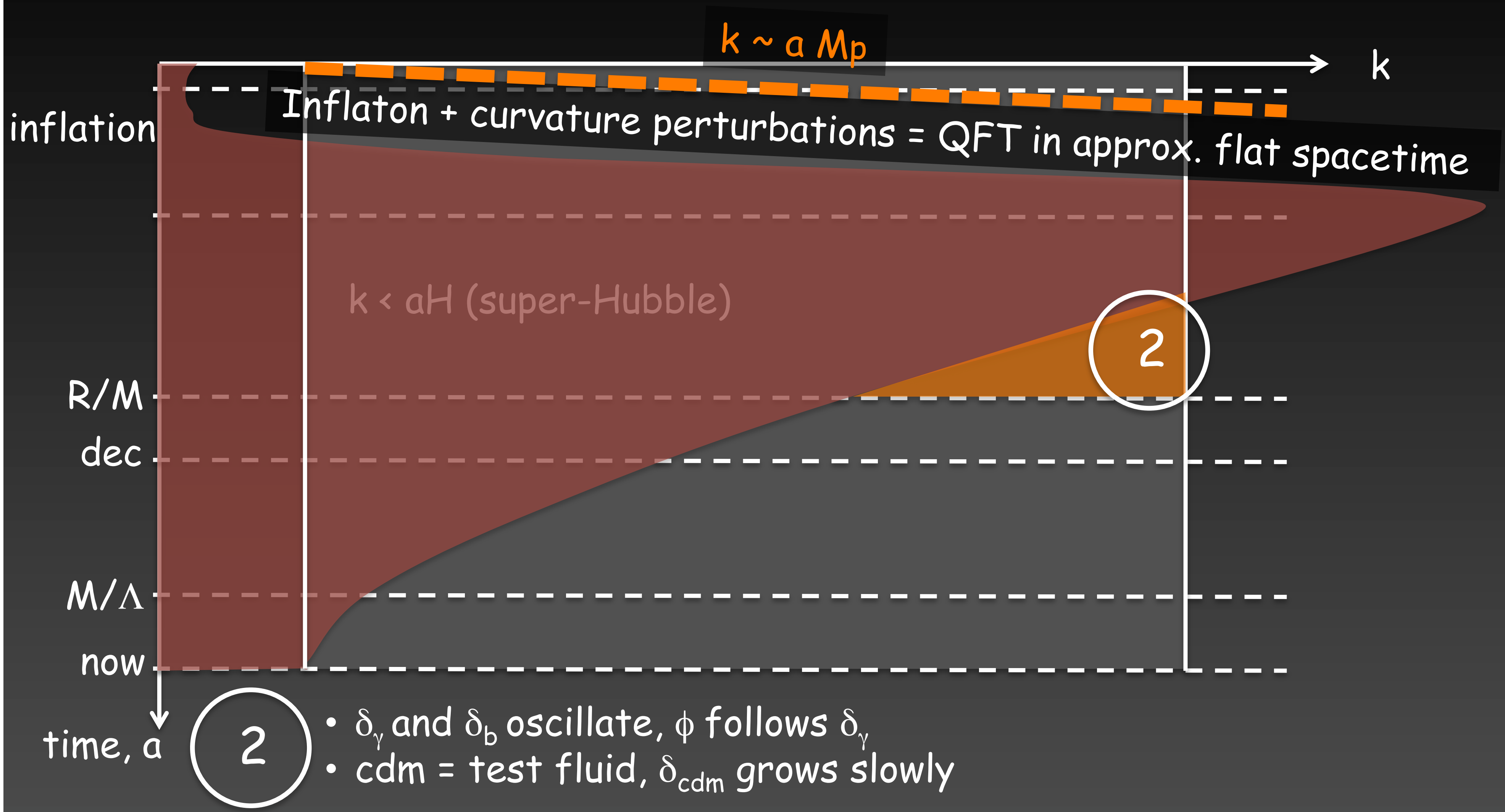


- sound speed $c_s^2 = \delta p / \delta\rho = 1/3$ during RD
- sound horizon:

$$d_s = a(t) \int^t \frac{c_s(t')\,dt'}{a(t')} \simeq \frac{d_H(t)}{\sqrt{3}}$$

- if computed starting from RD, close to $R_H(t)$

Perturbations in Fourier space



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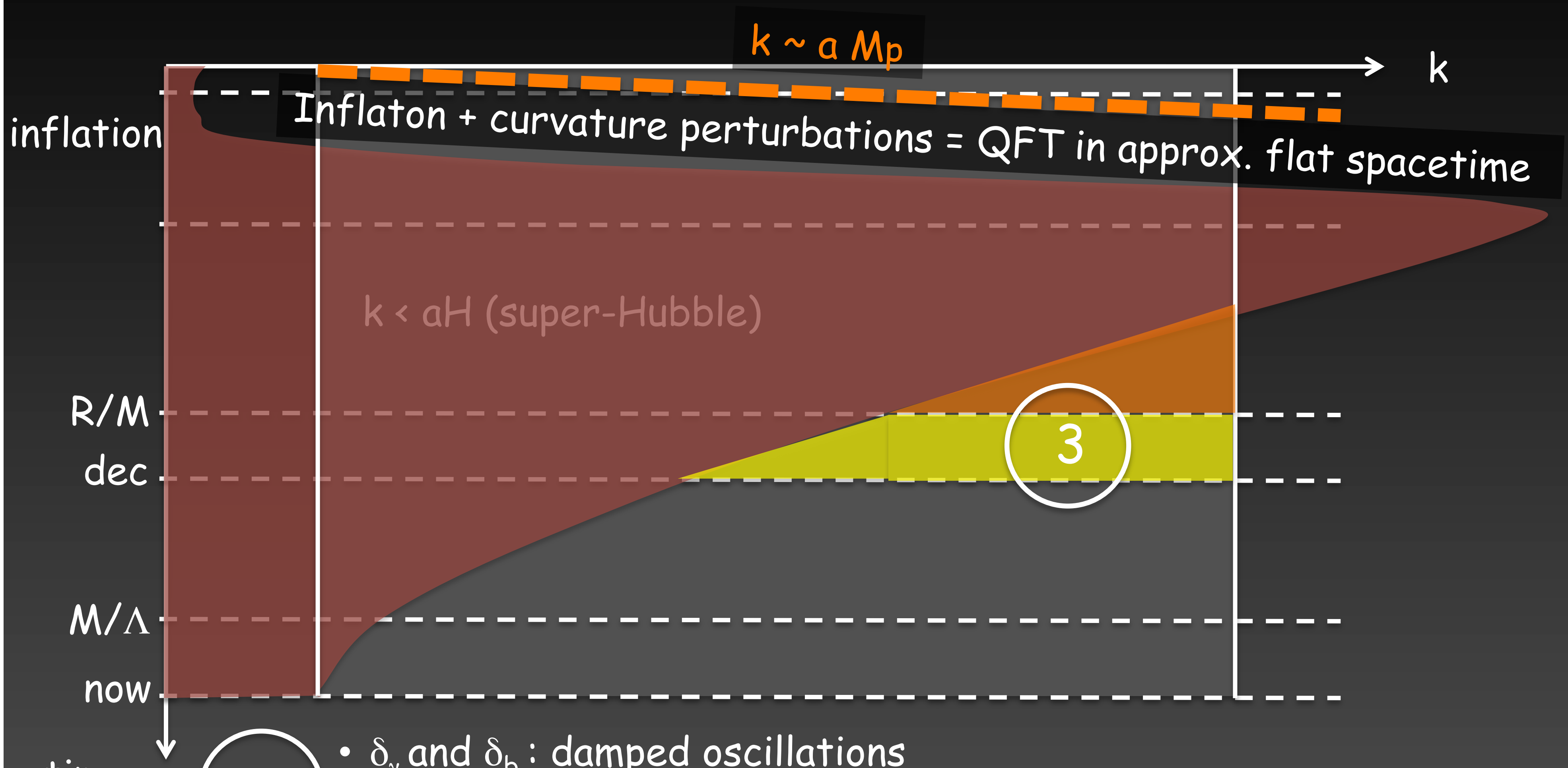
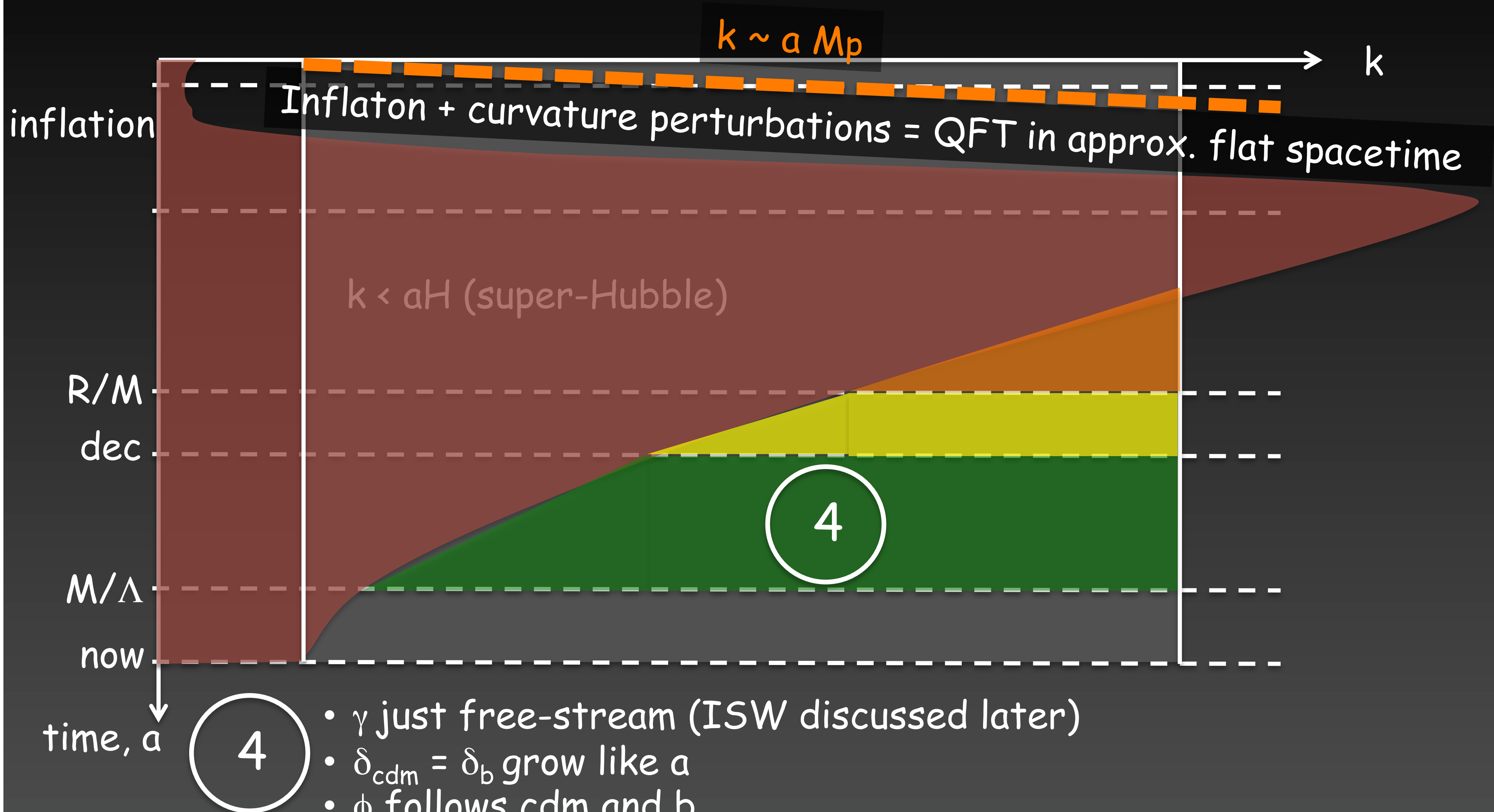
Perturbations in Fourier space



3

- δ_γ and δ_b : damped oscillations
- δ_{cdm} grows less slowly
- ϕ follows weigthed mixture of the two (depends on Ω_b/Ω_{cdm})

Perturbations in Fourier space



- γ just free-stream (ISW discussed later)
- $\delta_{cdm} = \delta_b$ grow like a
- ϕ follows cdm and b

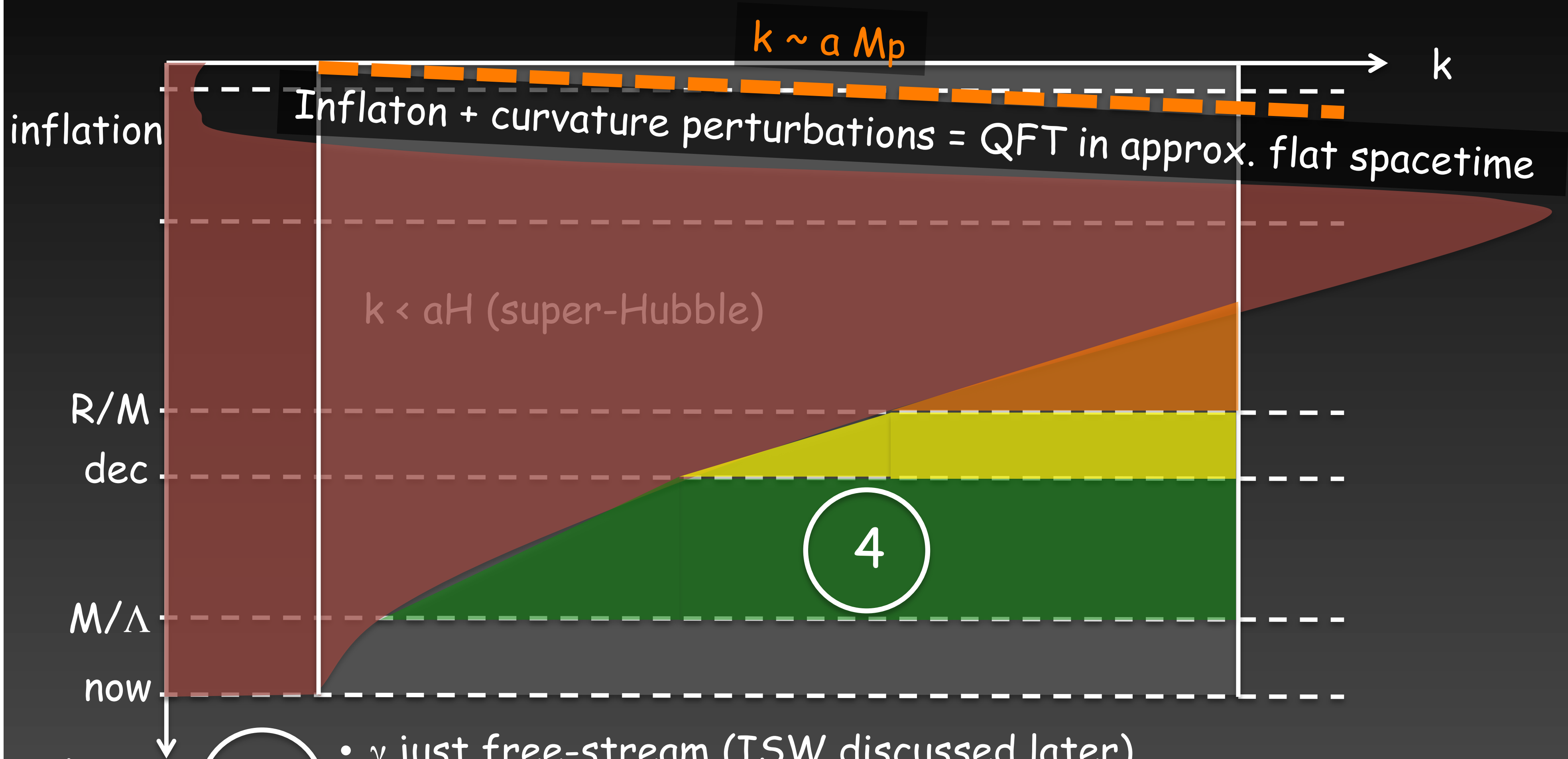
- Poisson equation:
  - Einstein equation in small-scale limit:

$$\frac{\Delta\phi}{a^2} = 4\pi G \ \delta\rho$$

  - comoving Fourier space:

$$-k^2\phi = 4\pi G \ a^2 \ \delta\rho$$

Perturbations in Fourier space



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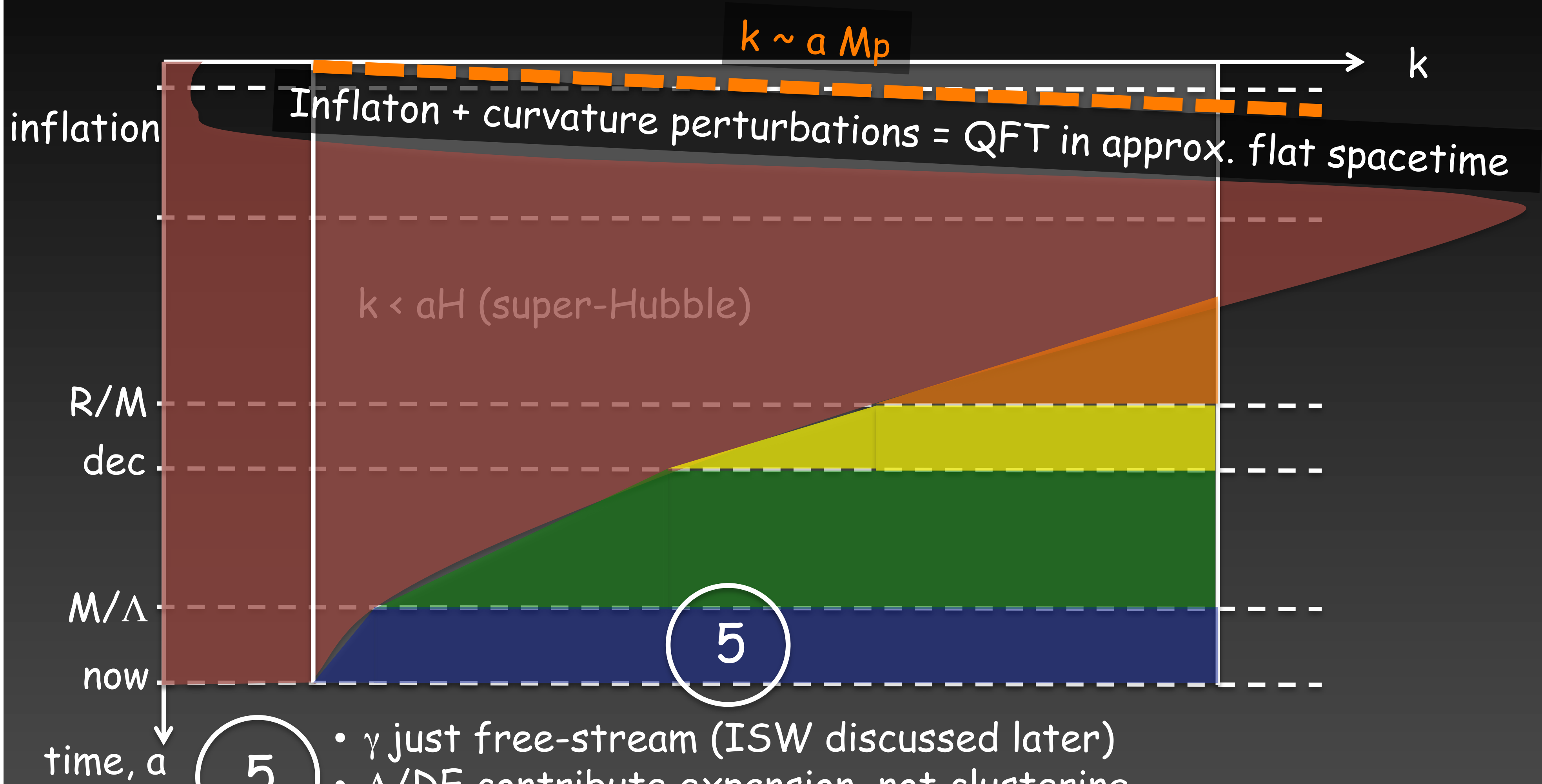
- γ just free-stream (ISW discussed later)
- $\delta_{cdm} = \delta_b$ grow like a
- ϕ follows cdm and b (Poisson : $\phi \sim \delta/a$ = cst ! clustering/expansion)

Perturbations in Fourier space

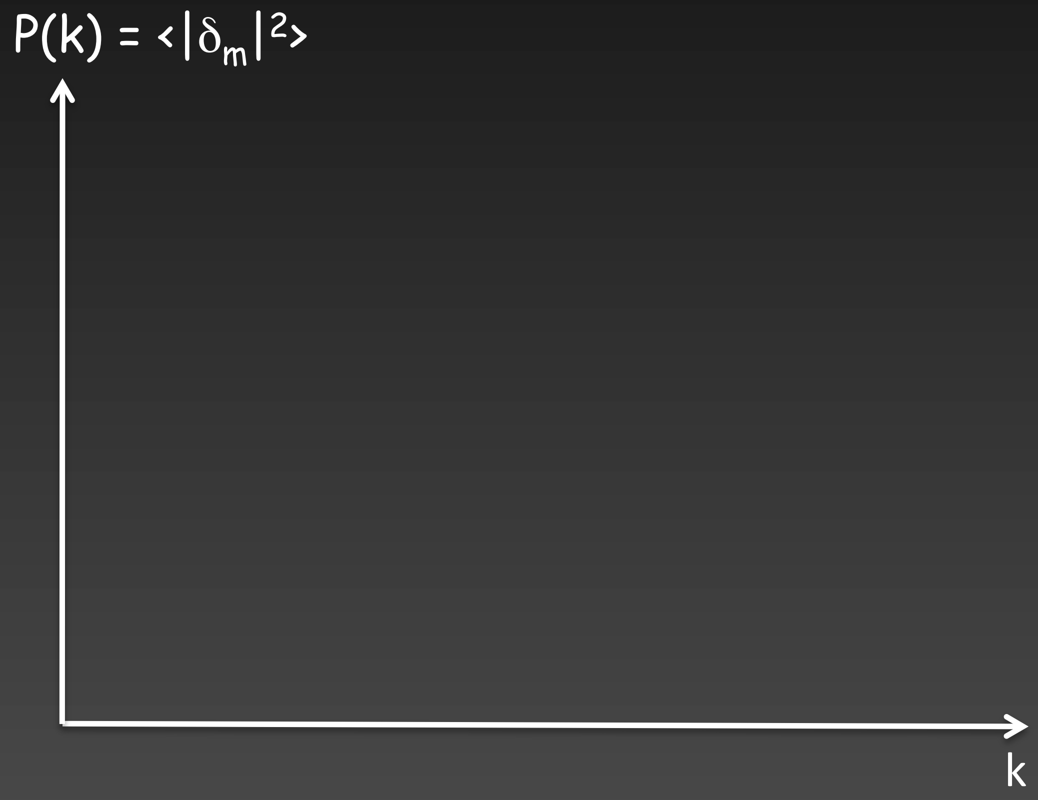


5

- γ just free-stream (ISW discussed later)
- Λ/DE contribute expansion, not clustering
- ϕ decays, $\delta_{cdm} = \delta_b$ grow slowlier than a

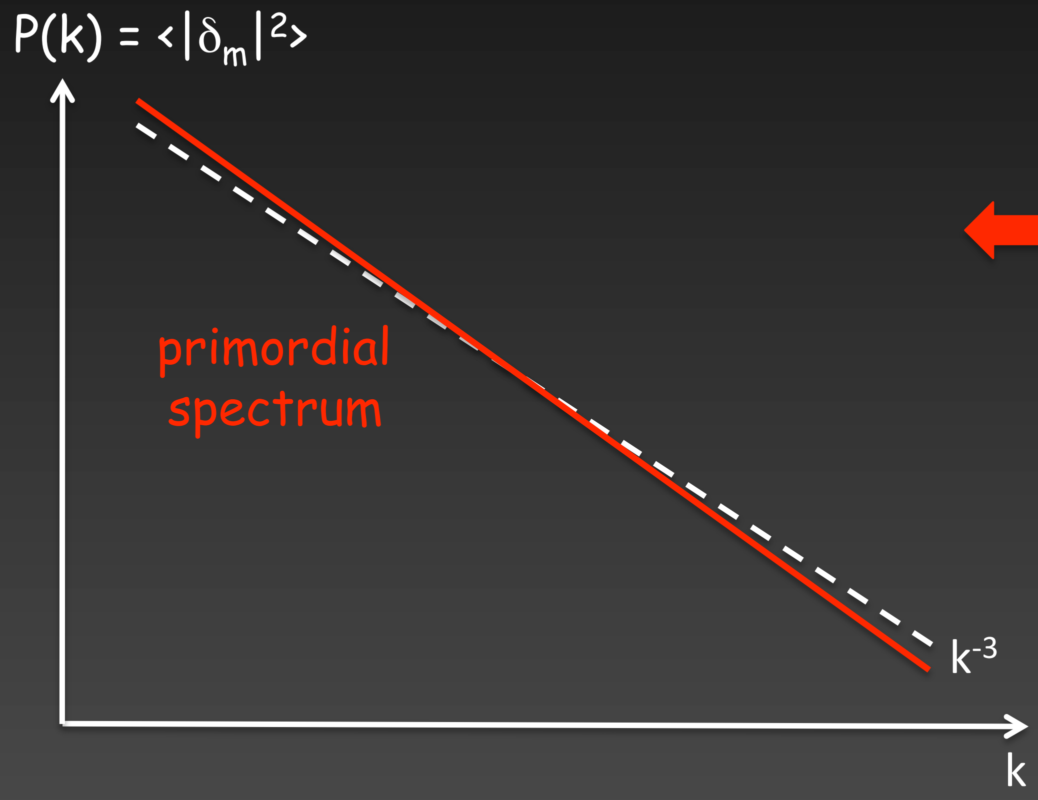
Matter power spectrum

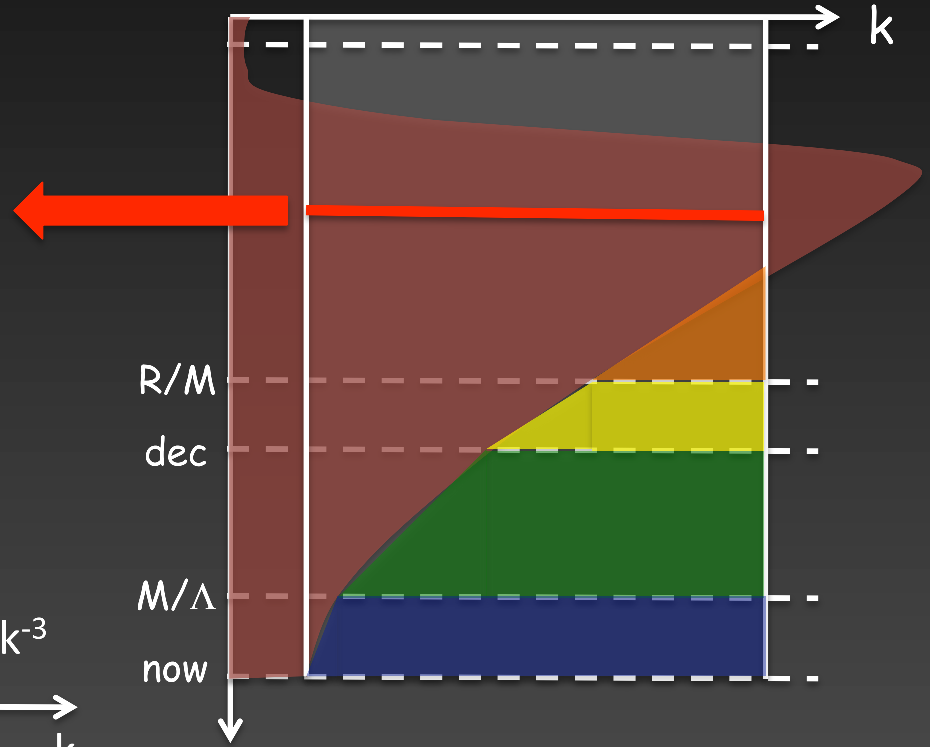
- Case $\Omega_{cdm} \gg \Omega_b$



Matter power spectrum

- Case Ωcdm >> Ωb





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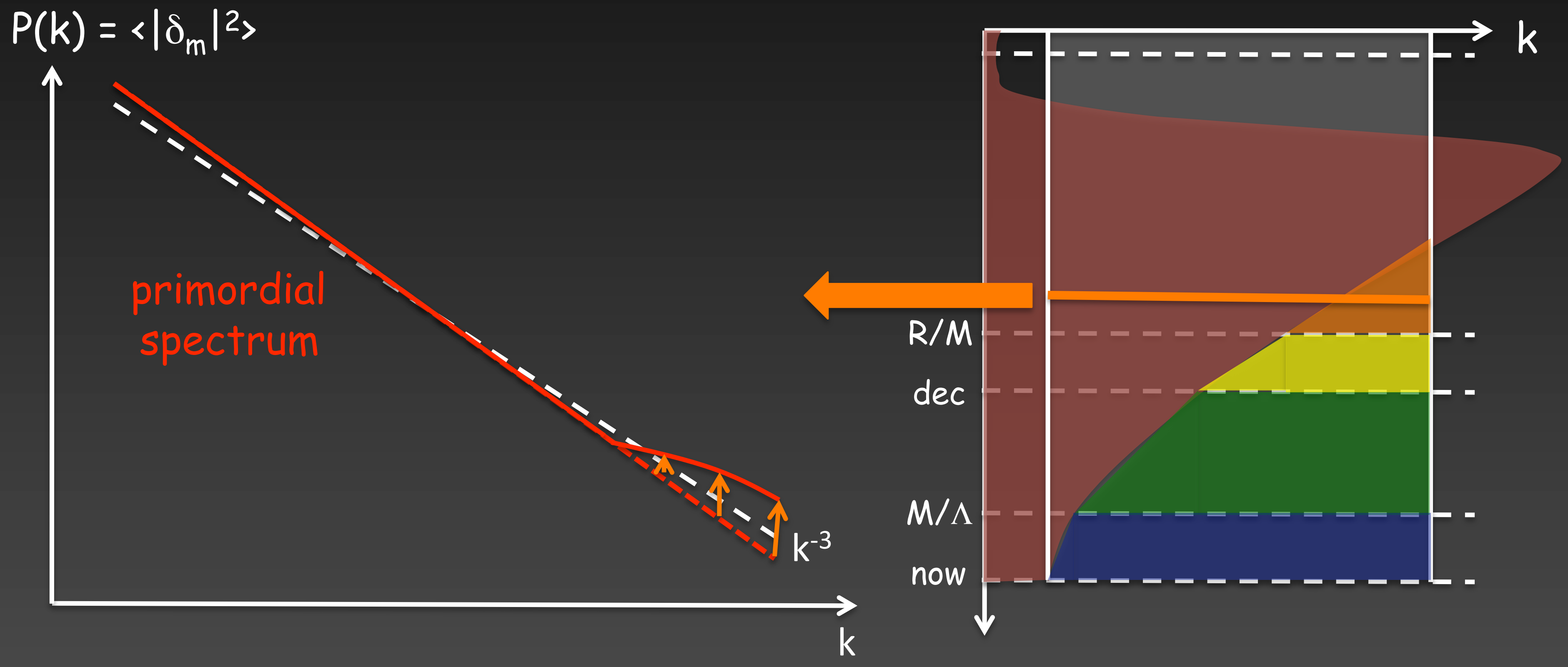

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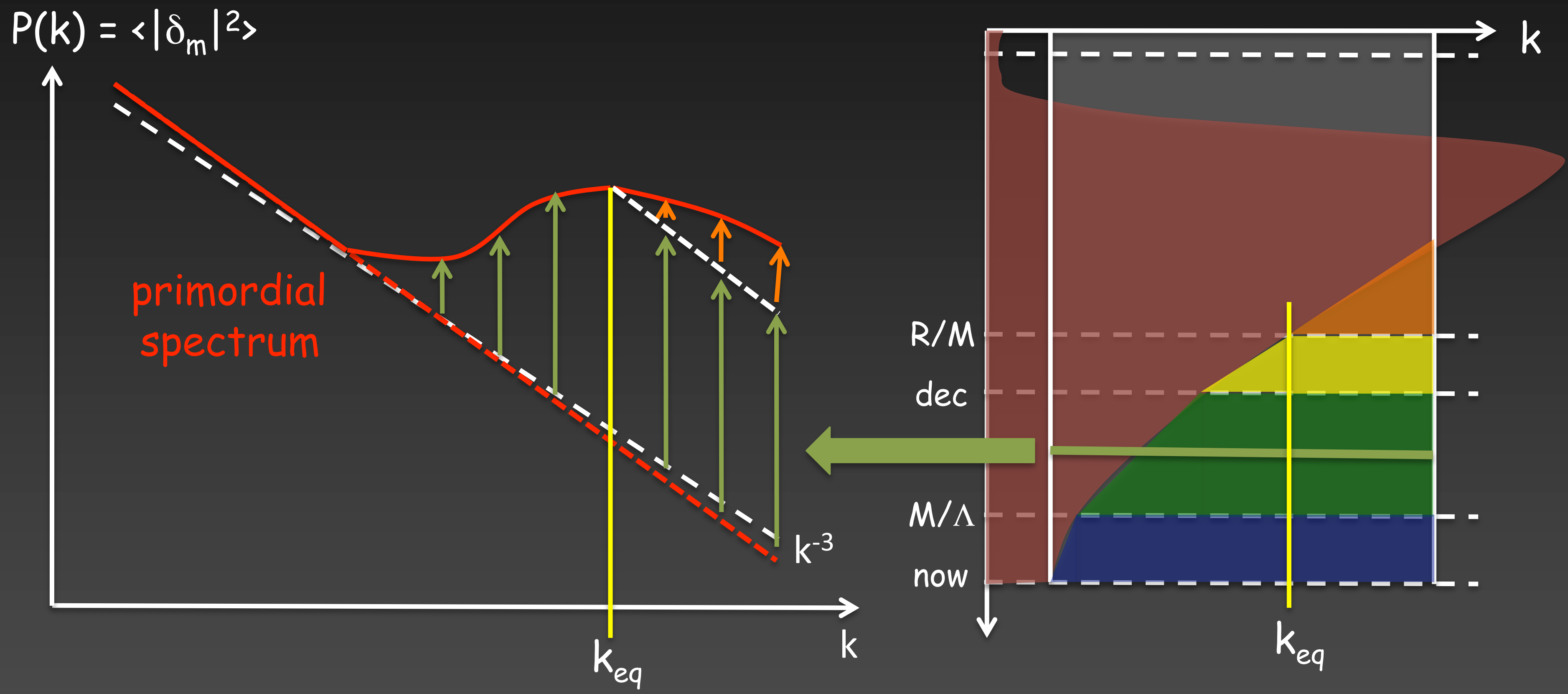

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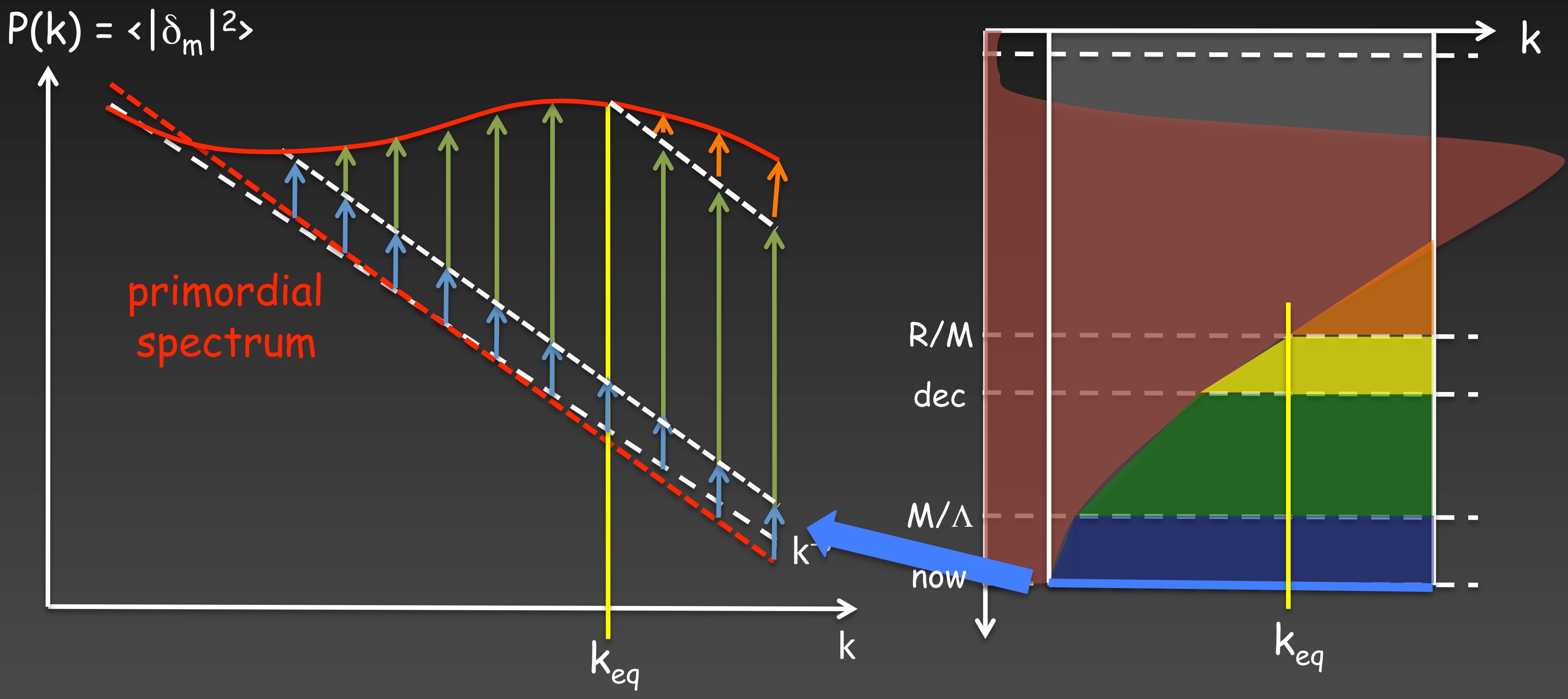

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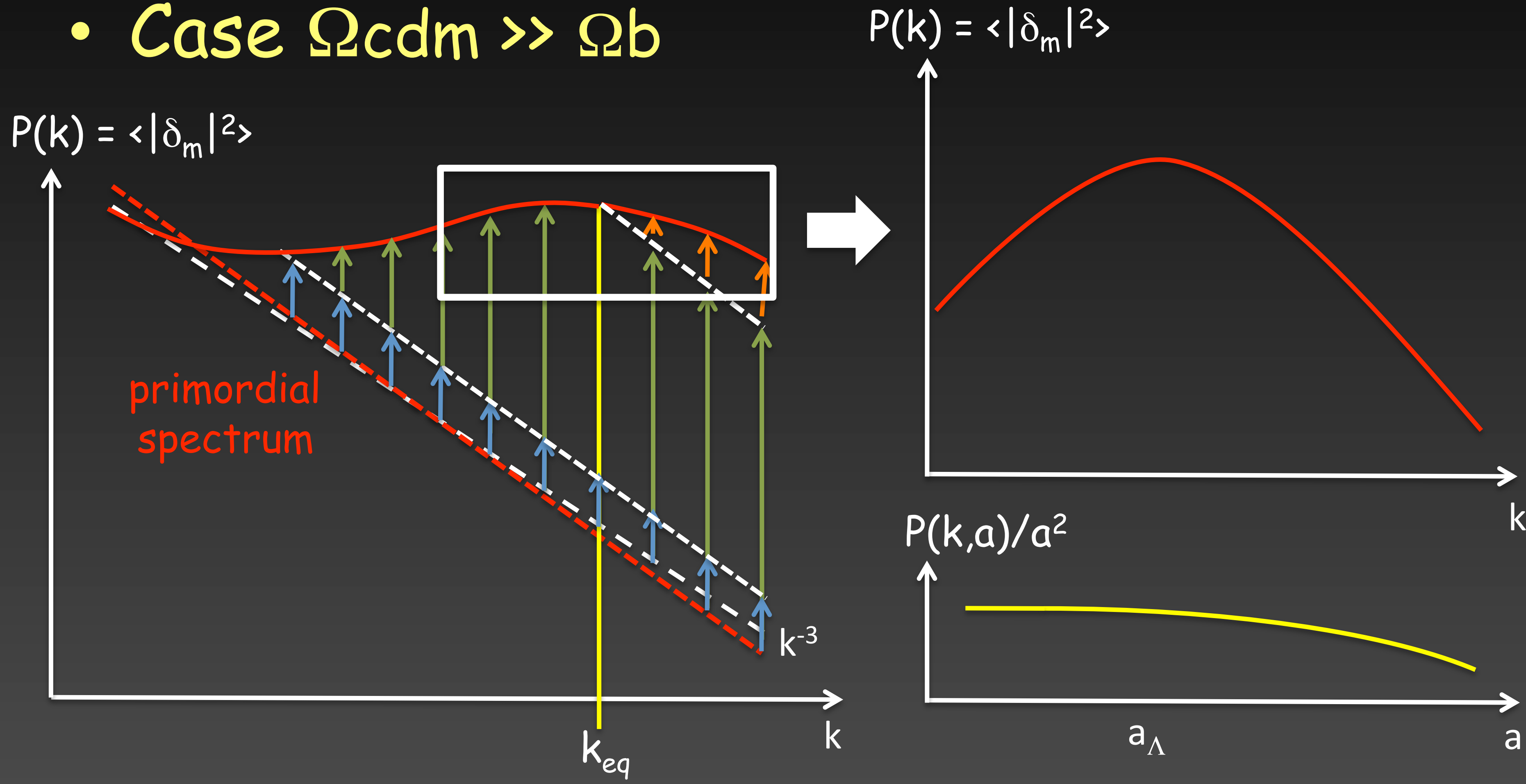
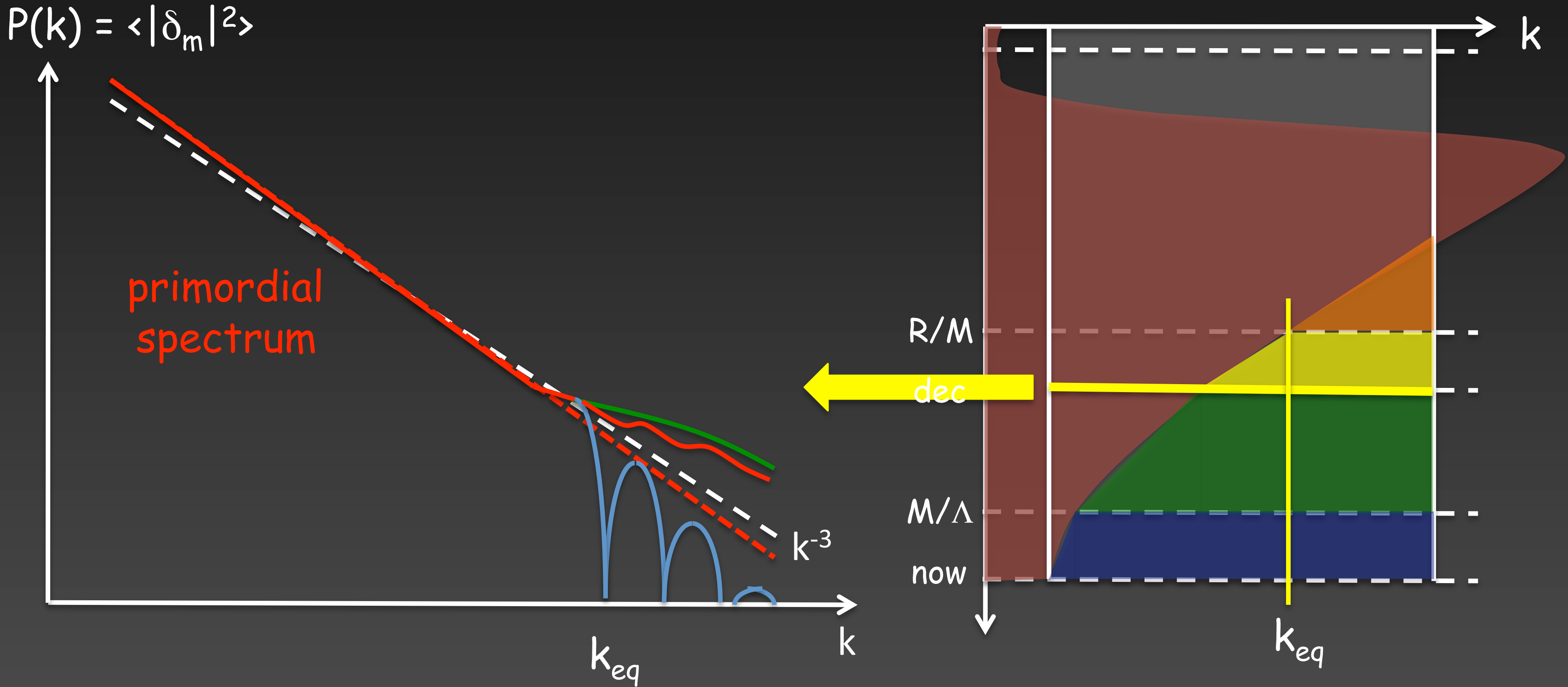


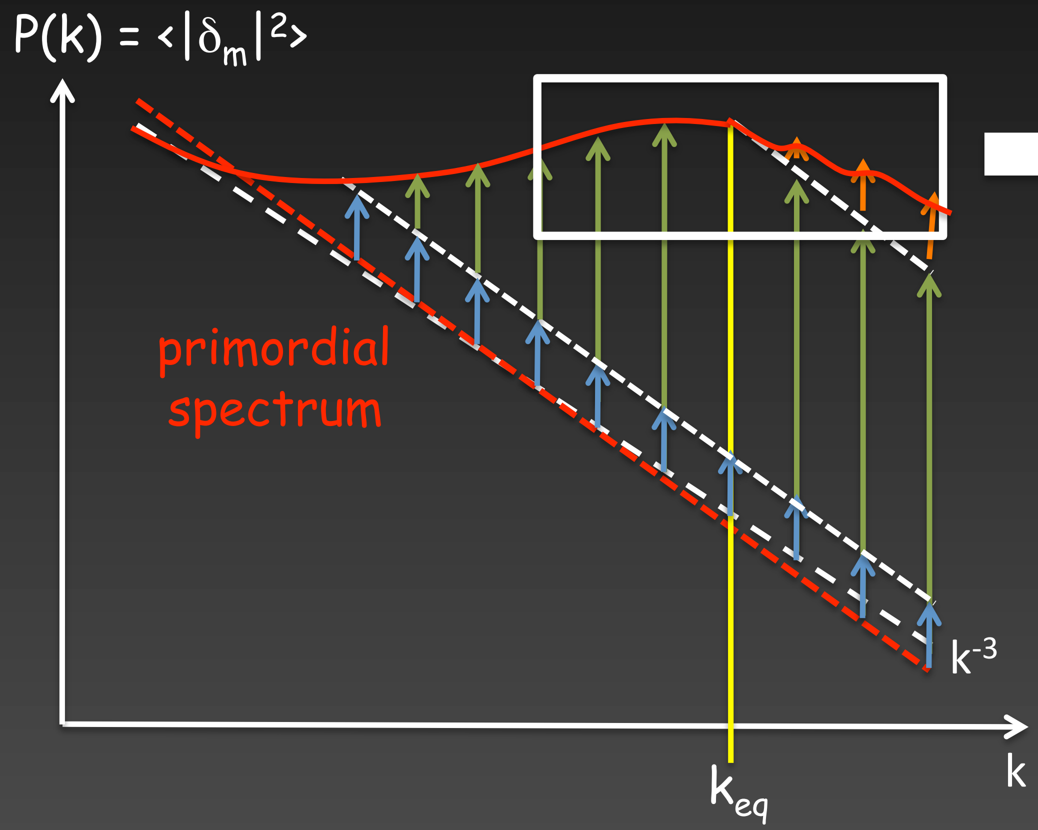
Matter power spectrum

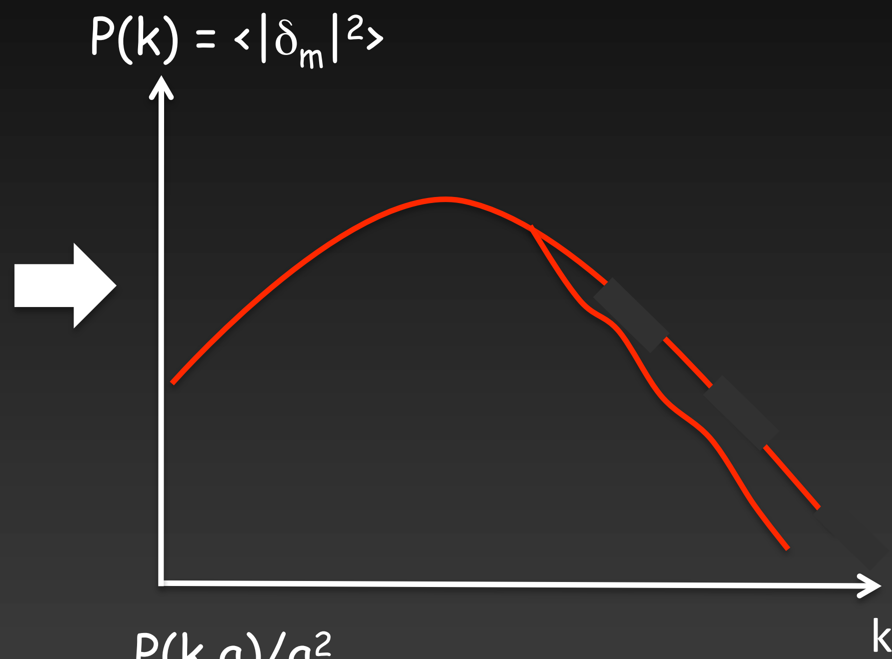
- Including baryons

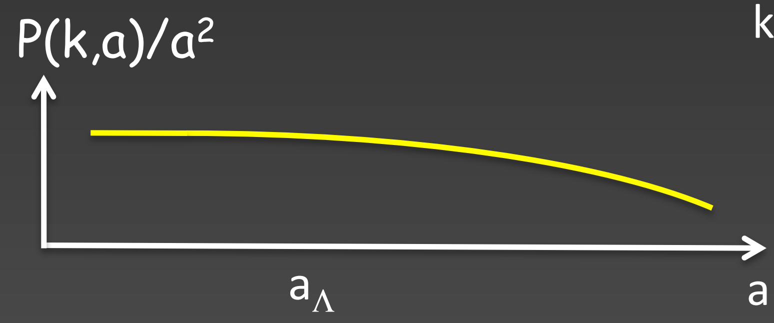


Matter power spectrum

- Including baryons

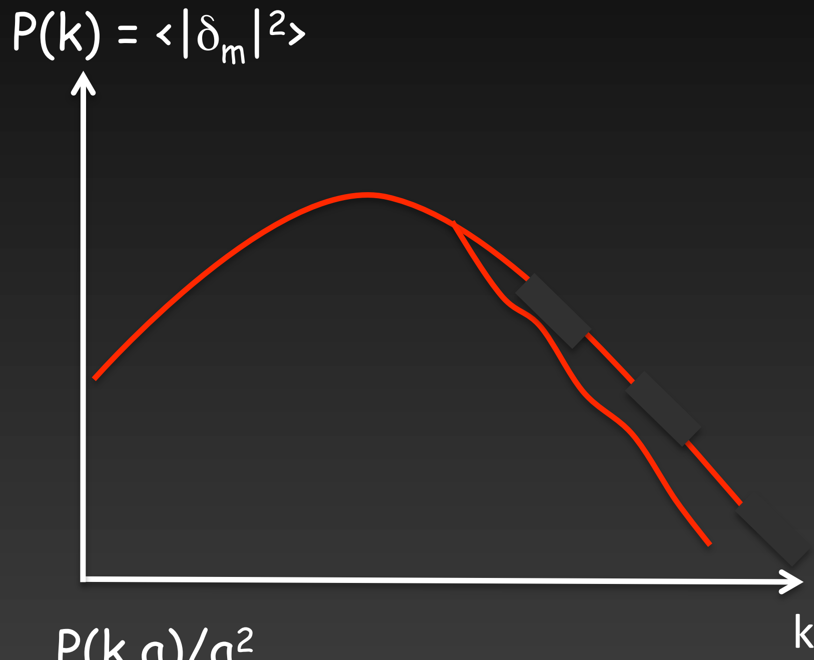


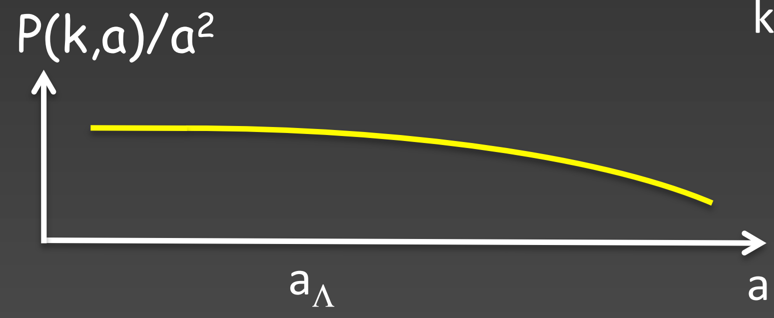




Matter power spectrum

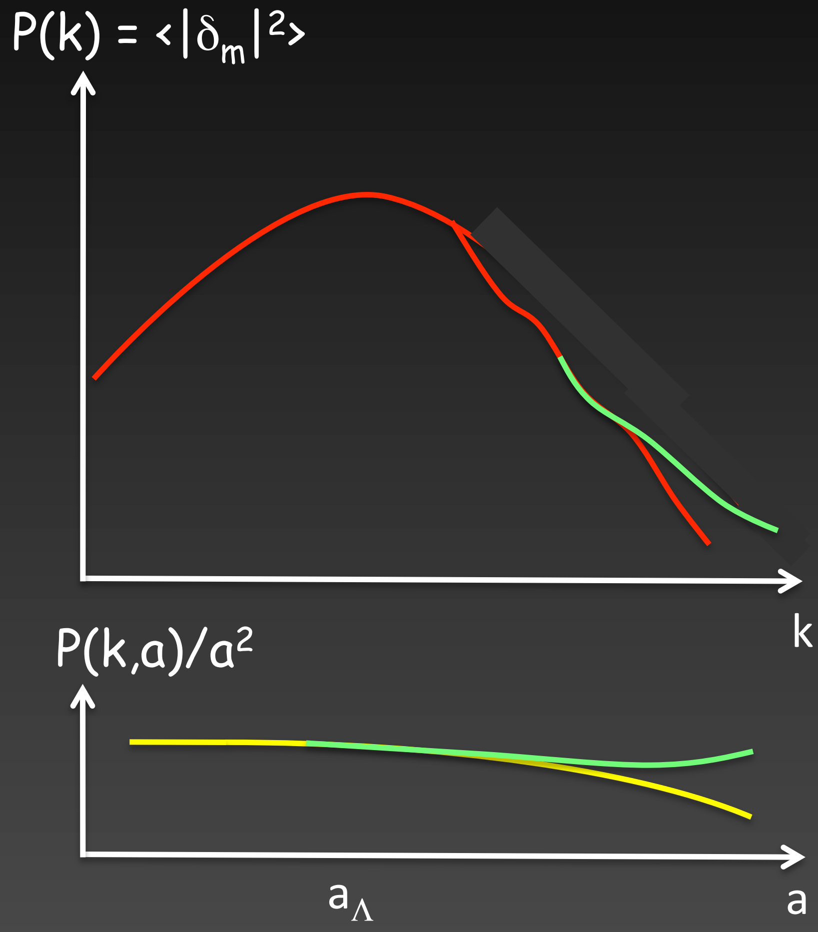
- Parameter effects :
  1. primordial spectrum :
     trivial multiplicative effect
  2. $\Omega_\Lambda/\Omega_{DE}$:
     global amplitude today
     growth factor
  3. $\omega_m=\Omega_m h^2$ (time of R/M eq.) :
     $k_{eq}=a_{eq}H_{eq}$, scale of maximum
  4. ω_b/ω_{cdm} :
     slope for $k>k_{eq}$ and oscillations





Matter power spectrum

- non-linear corrections :
  - N-body
  - Perturbation theory, renormalization…



Matter power spectrum

- observations :
  - galaxy/cluster redhsift surveys
  - weak lensing surveys (cosmic shear)
  - Lyman-α forests in quasar spectra
  - CMB lensing
  - 21cm in the future?