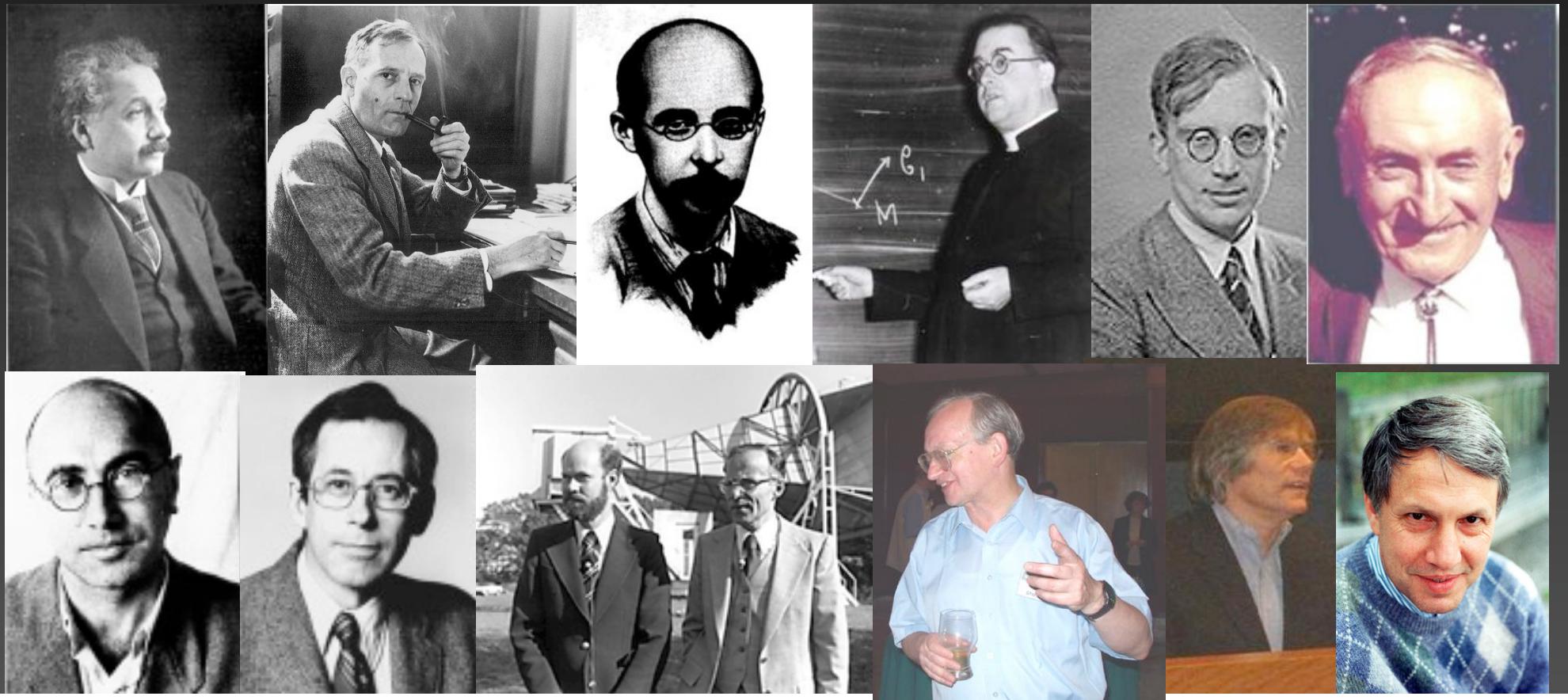


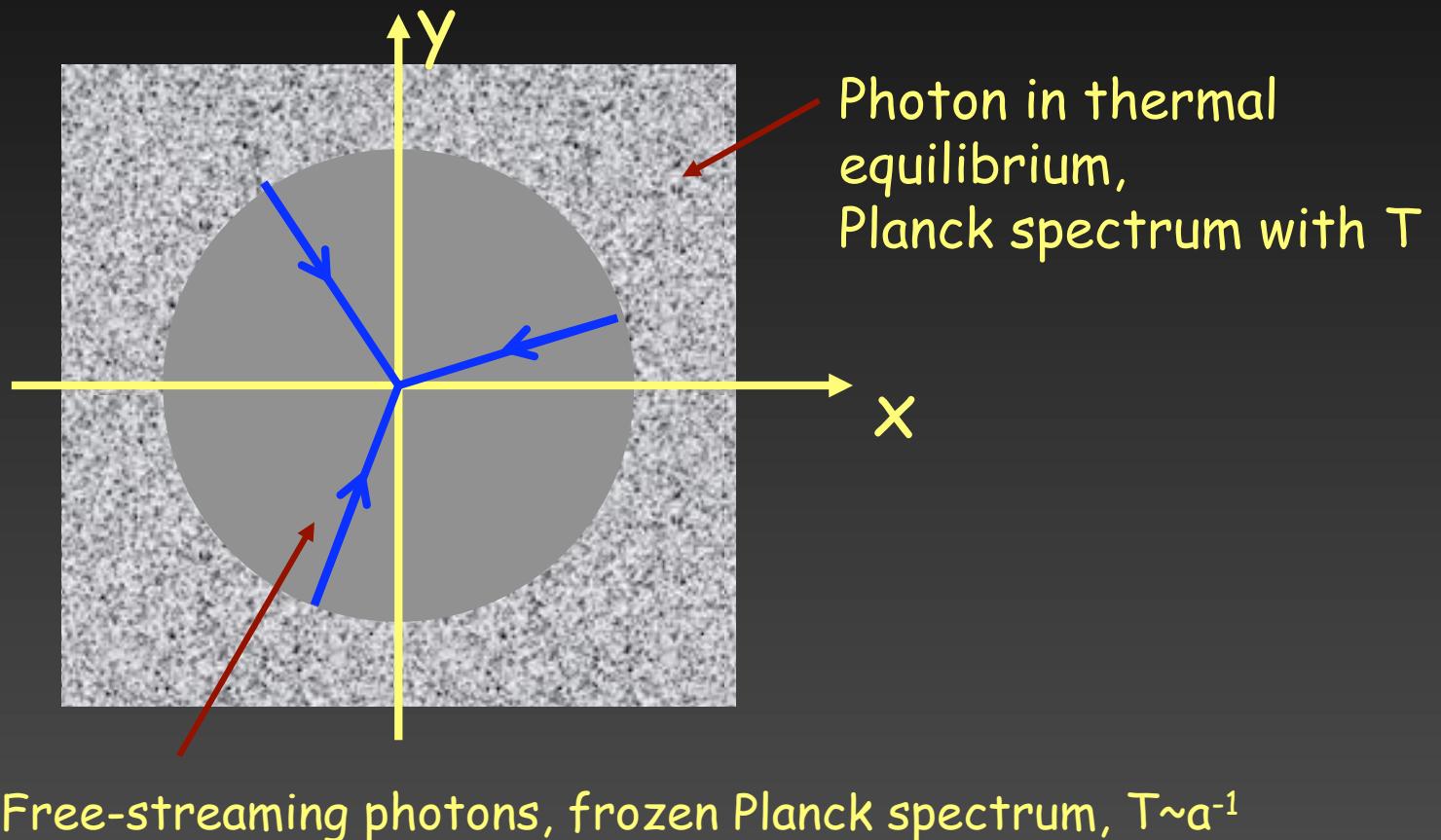
Part III: Cosmic Microwave Background

CHIPP PhD Winter School 2011, Leukerbad
Julien Lesgourgues (CERN & EPFL)



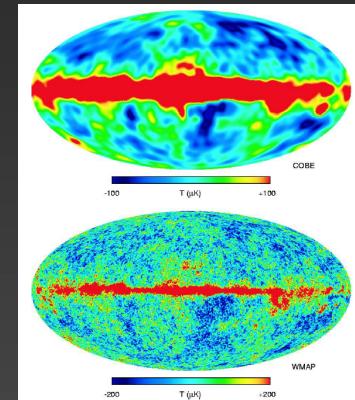
Photon decoupling

- around $t_{dec} \sim 380,000$ yr, $z_{dec} \sim 1100$, beginning of MD ($z_{eq} \sim 10^4$)
 - last scattering surface:



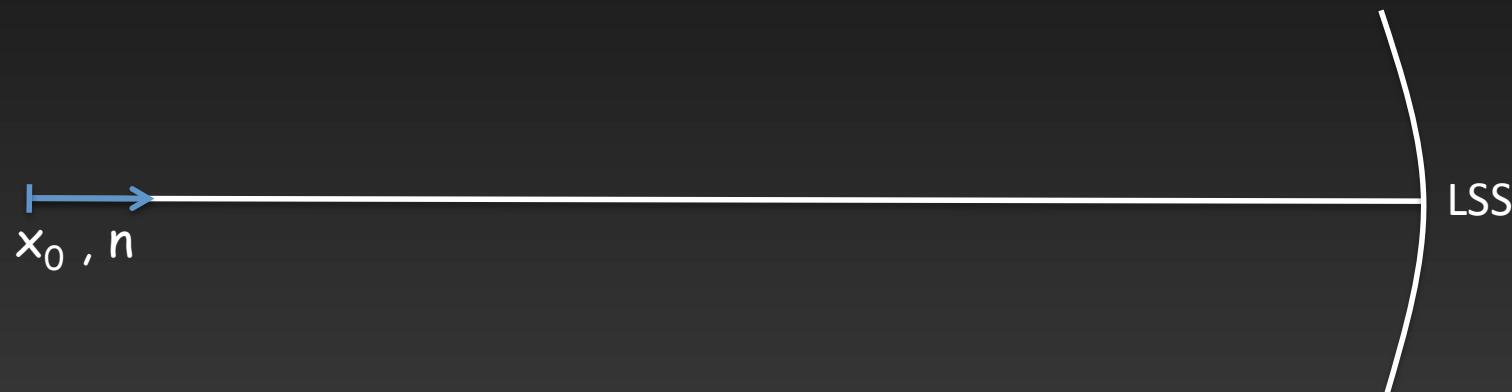
CMB anisotropies

- Penzias & Wilson and followers :
monopole $T \sim 2.726\text{K}$
- later : dipole induced by earth peculiar motion (10^{-3} , like v/c)
- primordial anisotropies predicted to be $\sim 10^{-5}, 10^{-6}$
- 90's : COBE and followers see
primordial temperature anisotropies (10^{-5})
- also polarization anisotropies, not discussed here



CMB power spectrum

- Instantaneous decoupling: Sachs-Wolfe
 - Integrate Boltzmann equation along geodesics:



$$\frac{\delta T}{T_{obs}}(\vec{x}_0, \hat{n}) = \frac{1}{4} \delta_\gamma LSS + \phi_{LSS} - \hat{n} \cdot \vec{v}_{bLSS} - \phi(\vec{x}_0) + \int_{LSS}^0 d\tau (\dot{\phi} + \dot{\psi})$$

CMB power spectrum

- Instantaneous decoupling: Sachs-Wolfe
 - Integrate Boltzmann equation along geodesics:

large scales : $\frac{1}{4}\delta_\gamma \sim -\frac{2}{3}\phi$ so $\left[\frac{1}{4}\delta_\gamma + \phi\right] \sim \frac{1}{3}\phi$ hot spot = underdensity!

$$\frac{\delta T}{T}_{obs}(\vec{x}_0, \hat{n}) = \underbrace{\frac{1}{4}\delta_\gamma}_{LSS} + \phi_{LSS} - \hat{n} \cdot \vec{v}_{bLSS} - \phi(\vec{x}_0) + \left\{ \int_{LSS}^0 d\tau (\dot{\phi} + \dot{\psi}) \right\}$$

CMB power spectrum

- Instantaneous decoupling: Sachs-Wolfe
 - Integrate Boltzmann equation along geodesics:

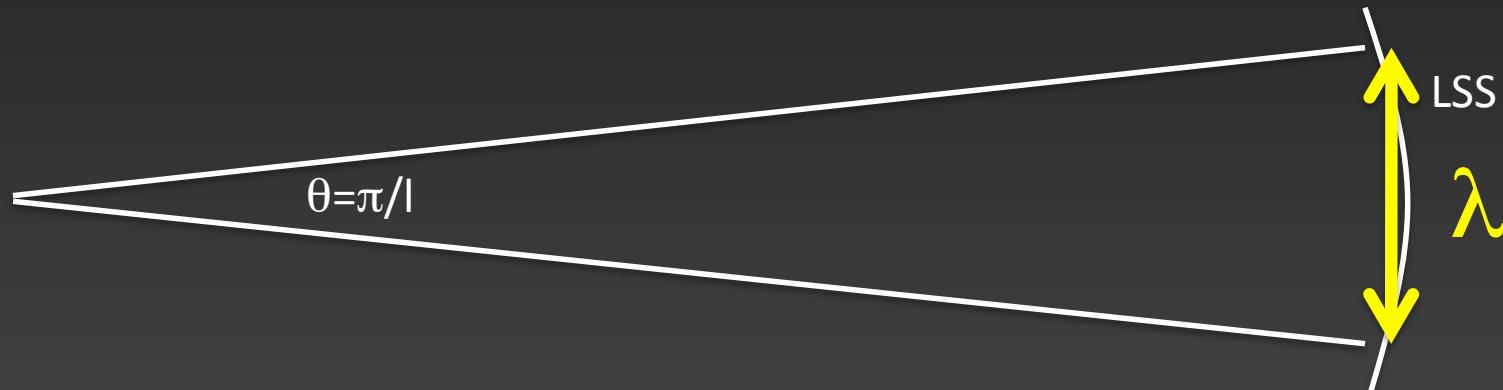
$$\text{large scales : } \frac{1}{4}\delta_\gamma \sim -\frac{2}{3}\phi \quad \text{so} \quad \left[\frac{1}{4}\delta_\gamma + \phi \right] \sim \frac{1}{3}\phi \quad \text{hot spot = underdensity!}$$

$$\frac{\delta T}{T}_{obs}(\vec{x}_0, \hat{n}) = \underbrace{\frac{1}{4}\delta_\gamma}_{LSS} + \underbrace{\phi_{LSS}}_{- \hat{n} \cdot \vec{v}_{bLSS}} - \phi(\vec{x}_0) + \underbrace{\int_{LSS}^0 d\tau (\dot{\phi} + \dot{\psi})}_{\text{}}$$

small scales : need to go from real space to Fourier space

CMB power spectrum

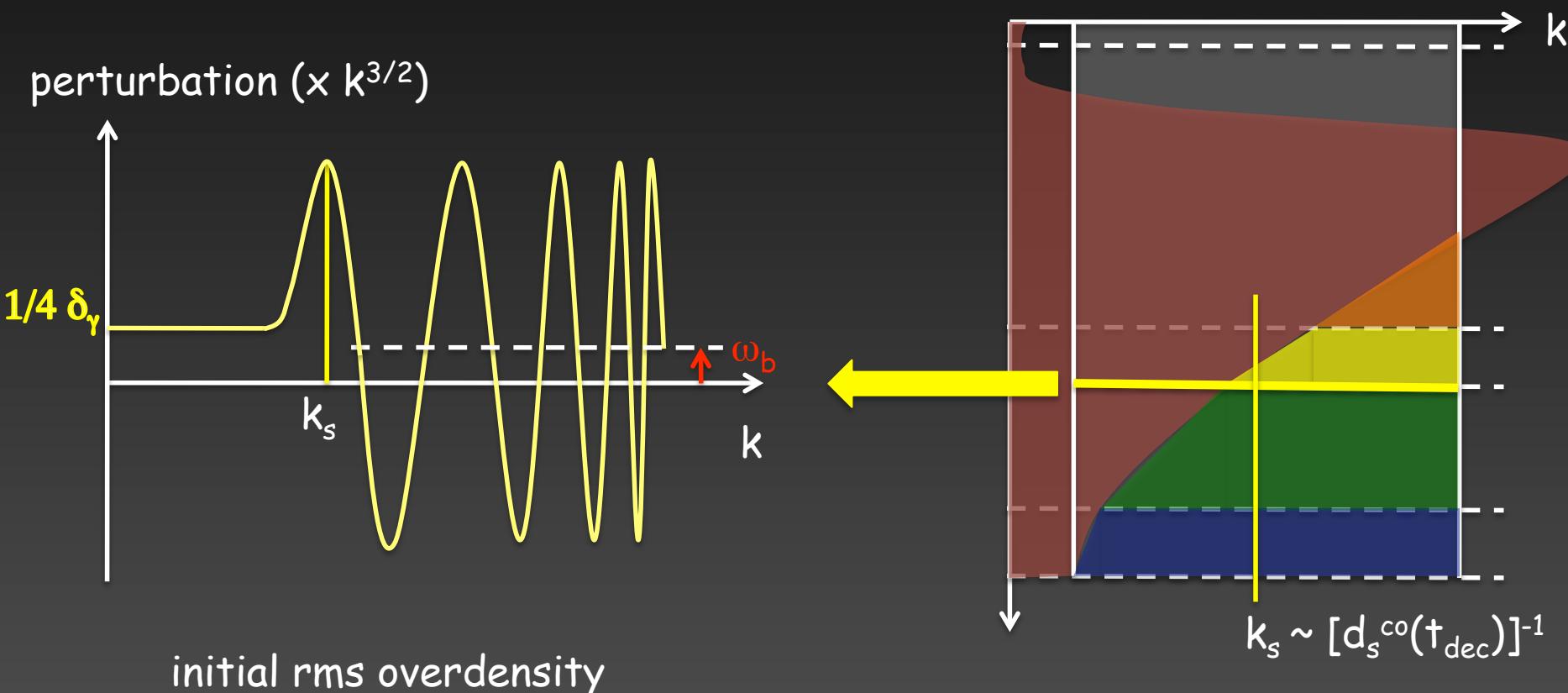
- approximate relation btw C_l and Fourier spectra on LSS:
 - $C_l = \delta T/T$ power spectrum for Legendre multipole l
 - l subtends an angle π/θ and a wavelength $\lambda = d_A(\text{LSS}) \theta$



- C_l related to Fourier spectra on LSS with $k \sim l a(\text{LSS})/d_A(\text{LSS})$
- contributions from $\langle |1/4 \delta_\gamma + \phi|^2 \rangle_k$, $\langle |v_b|^2 \rangle_k$

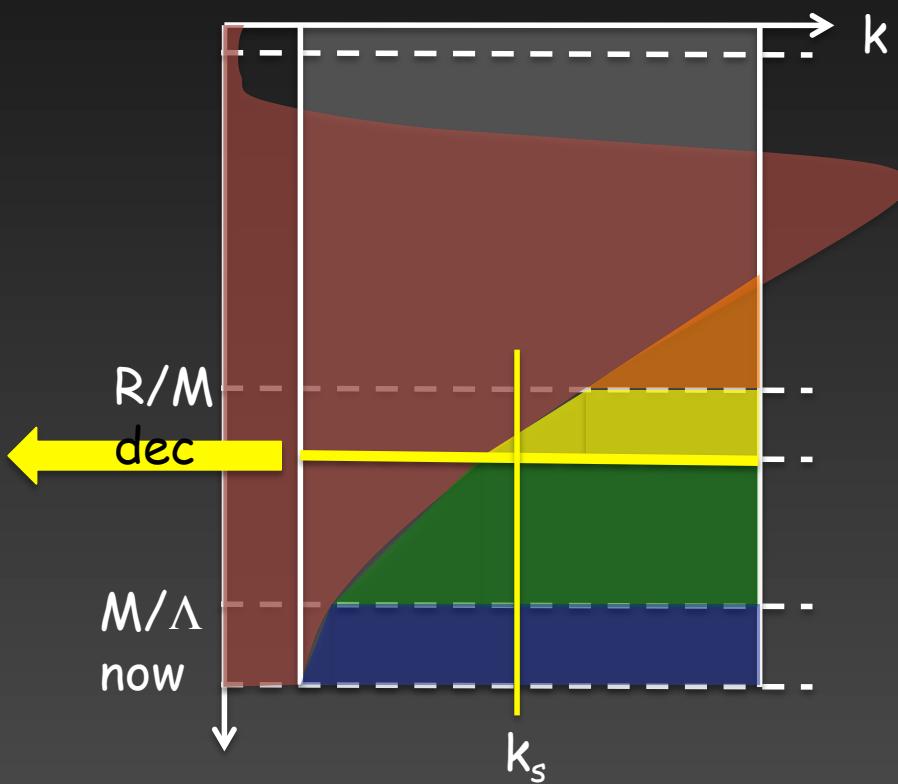
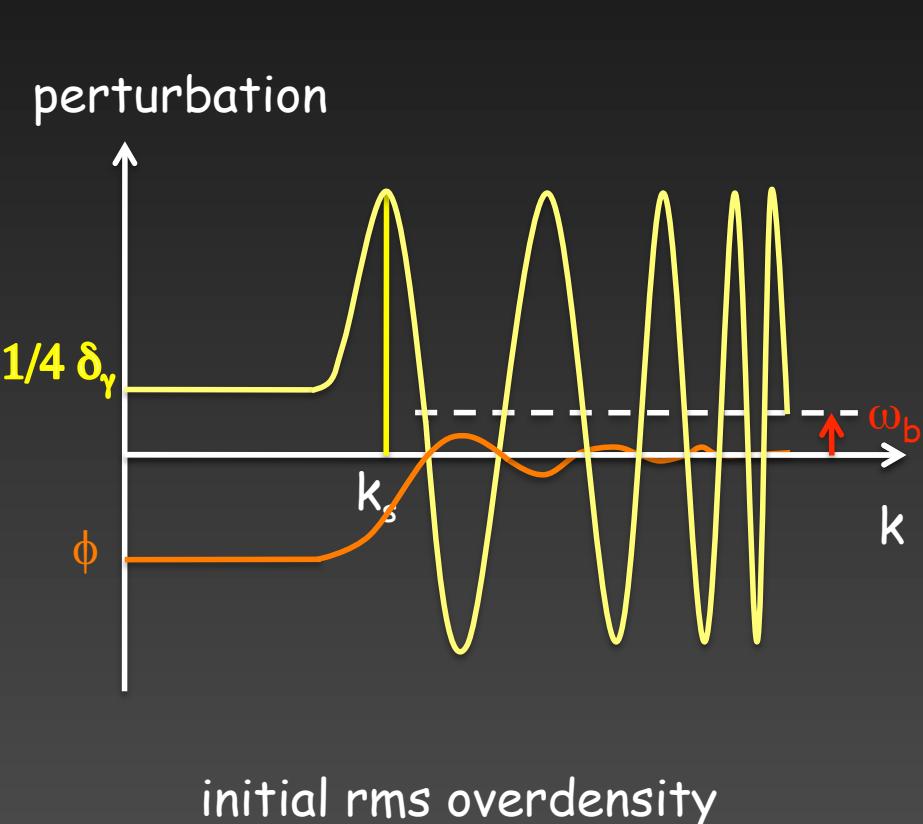
CMB power spectrum

- Fourier spectra at decoupling:



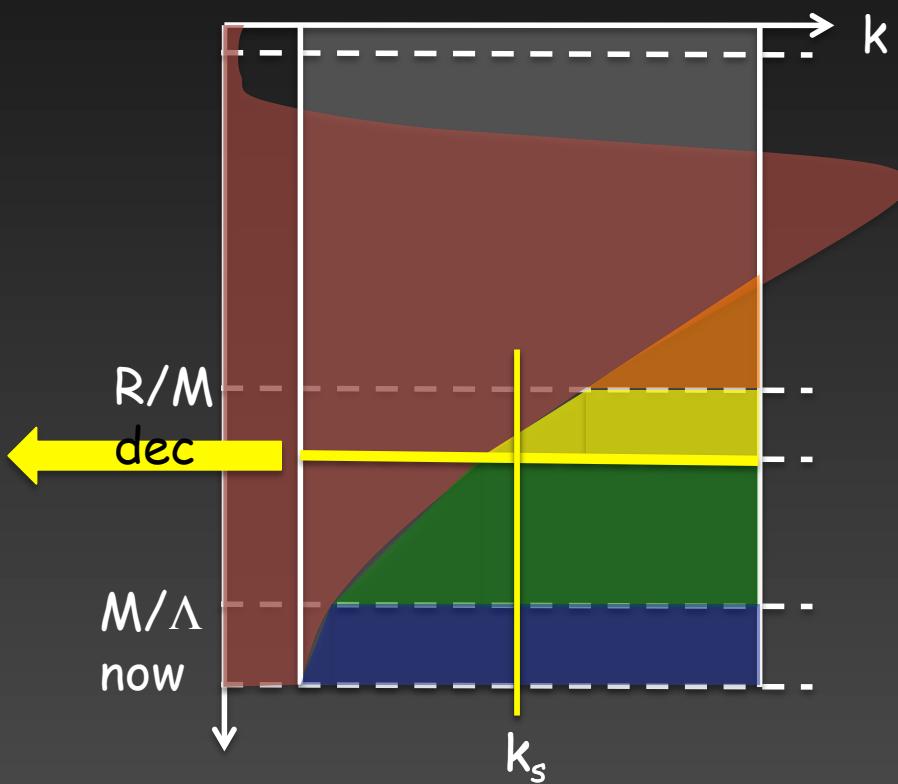
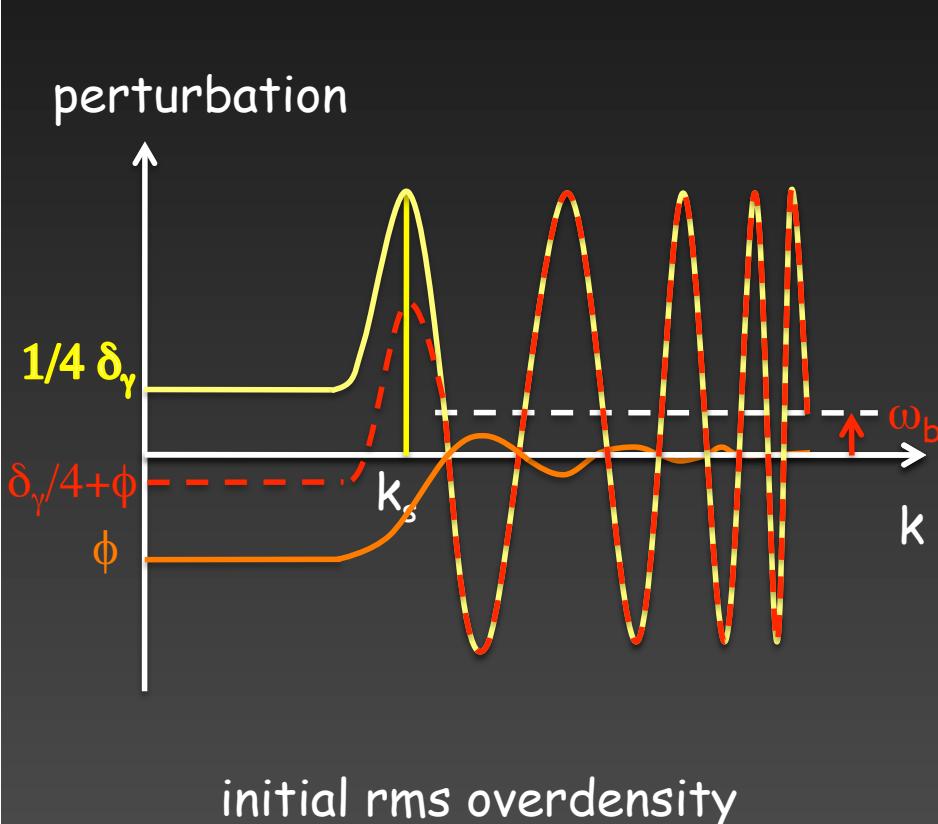
CMB power spectrum

- Fourier spectra at decoupling:



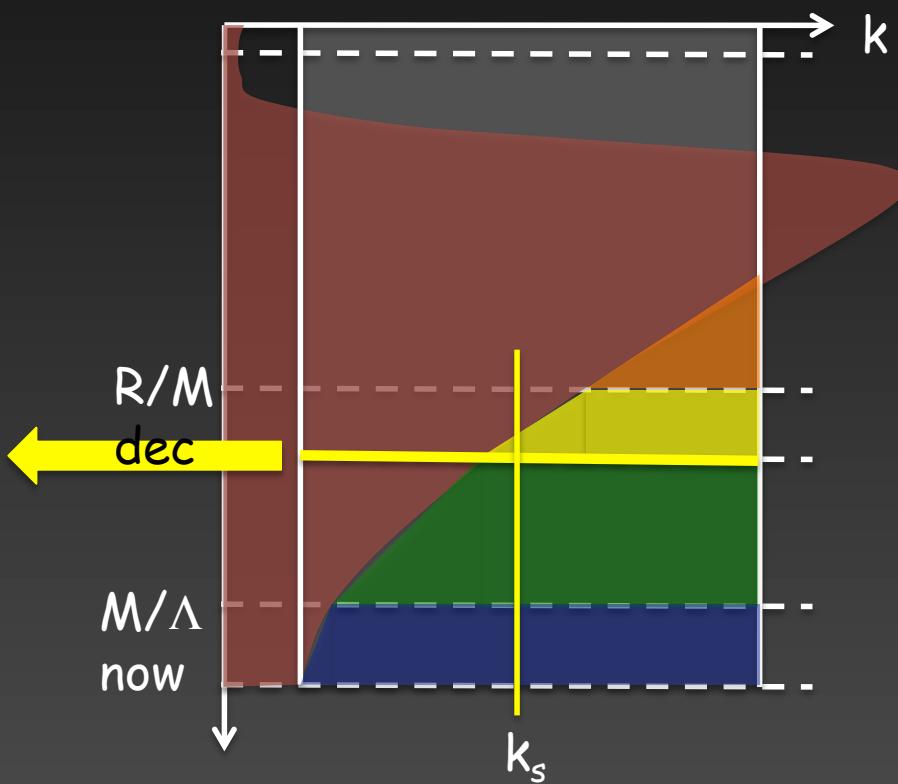
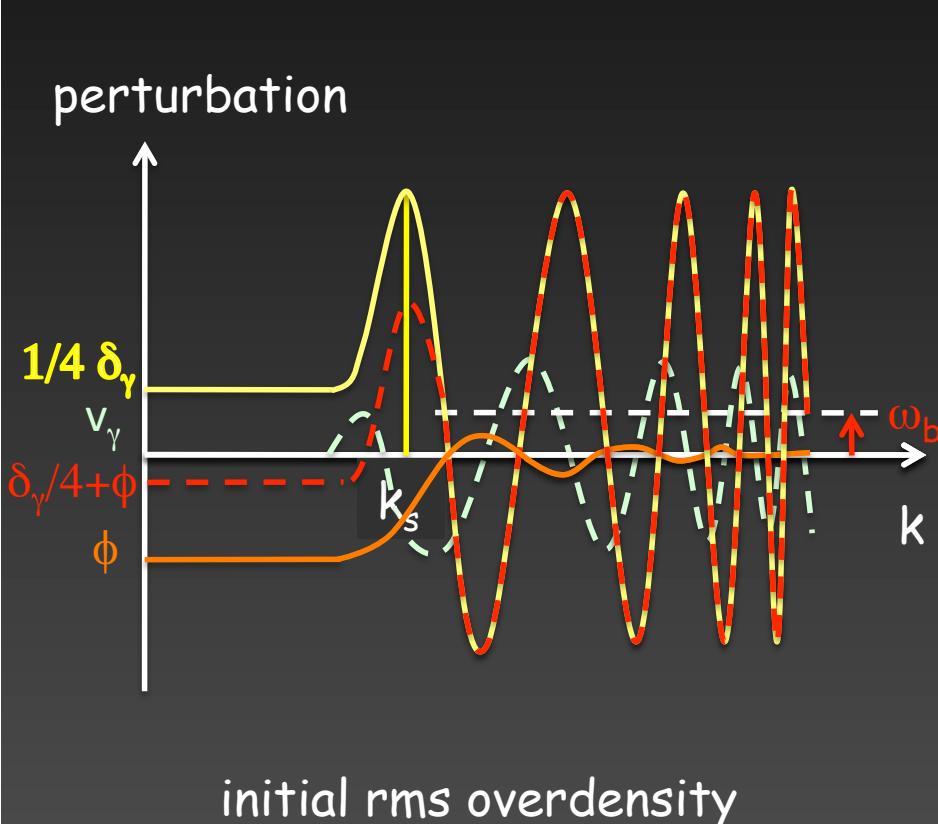
CMB power spectrum

- Fourier spectra at decoupling:



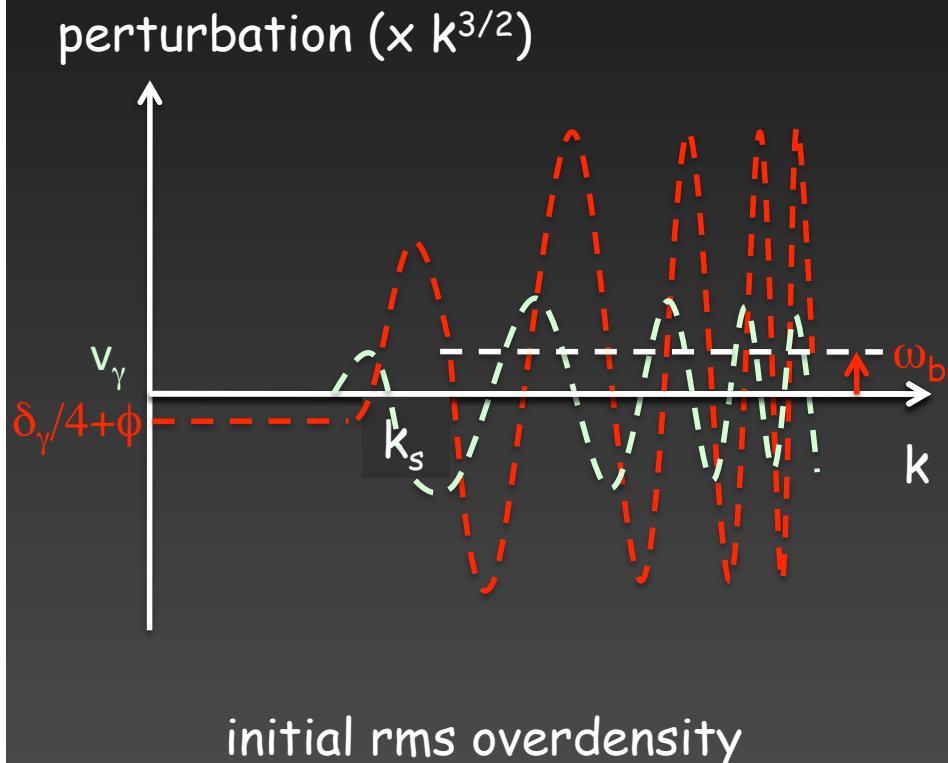
CMB power spectrum

- Fourier spectra at decoupling:



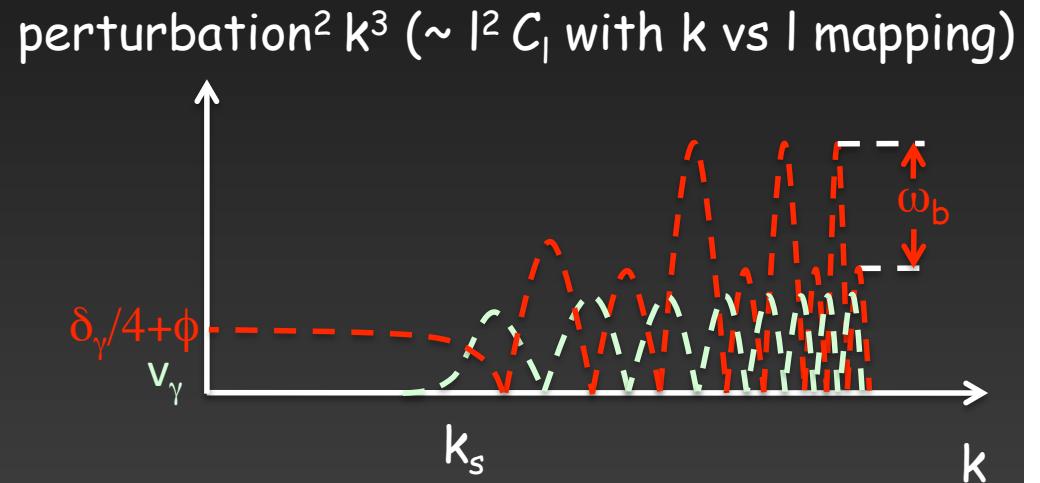
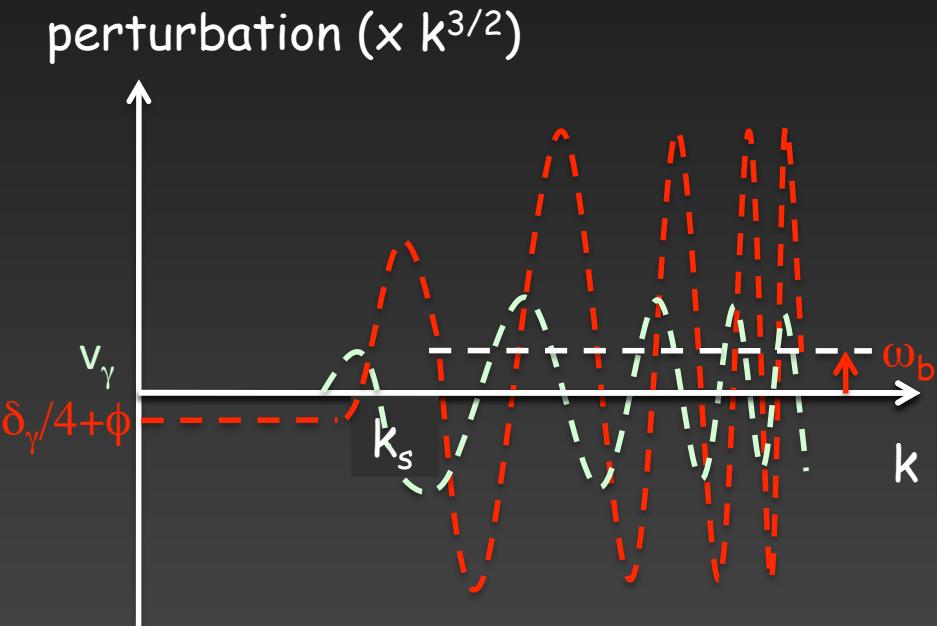
CMB power spectrum

- Fourier spectra at decoupling:



CMB power spectrum

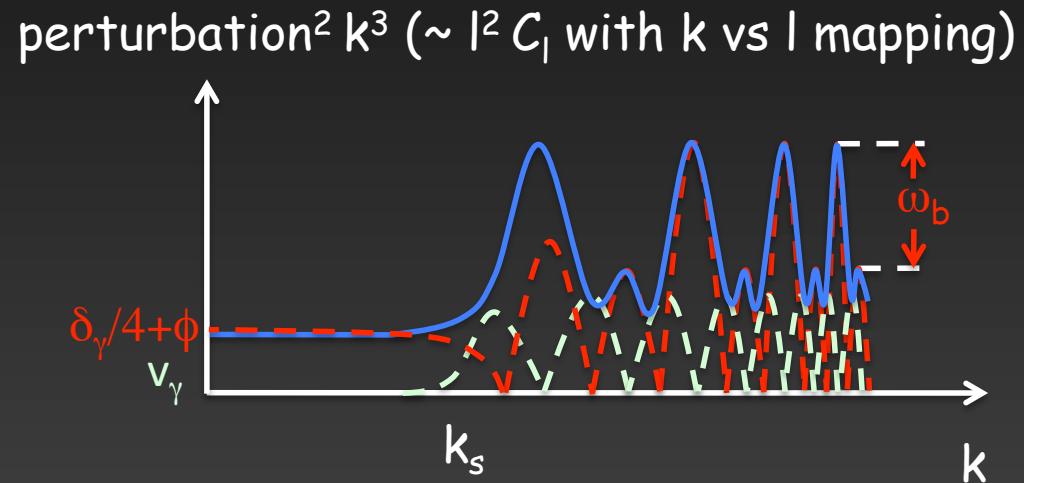
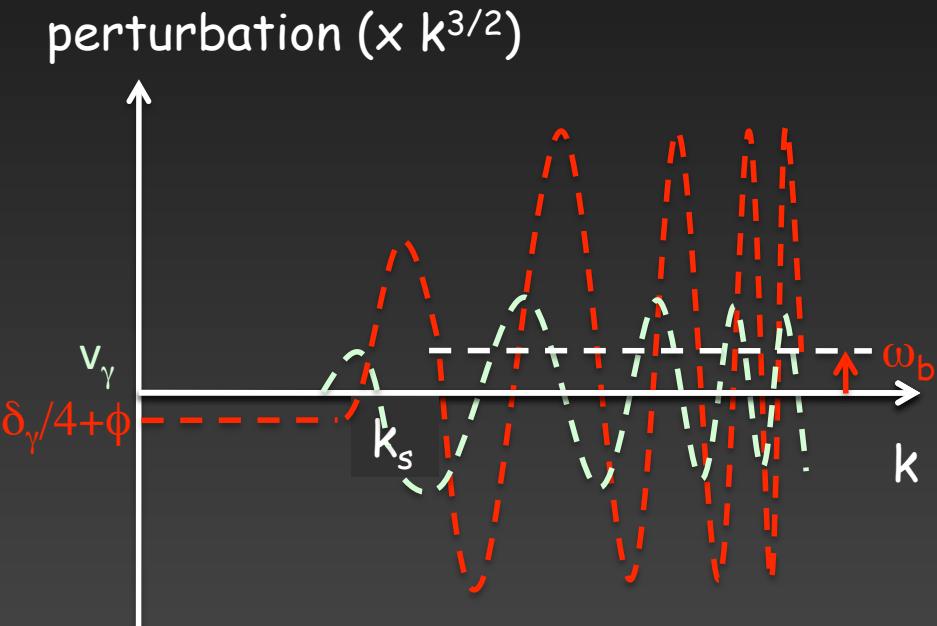
- Fourier spectra at decoupling:



initial rms overdensity

CMB power spectrum

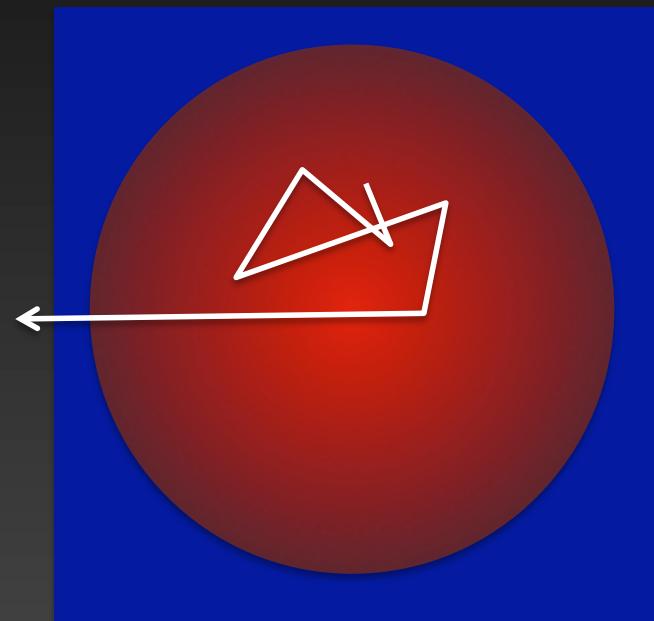
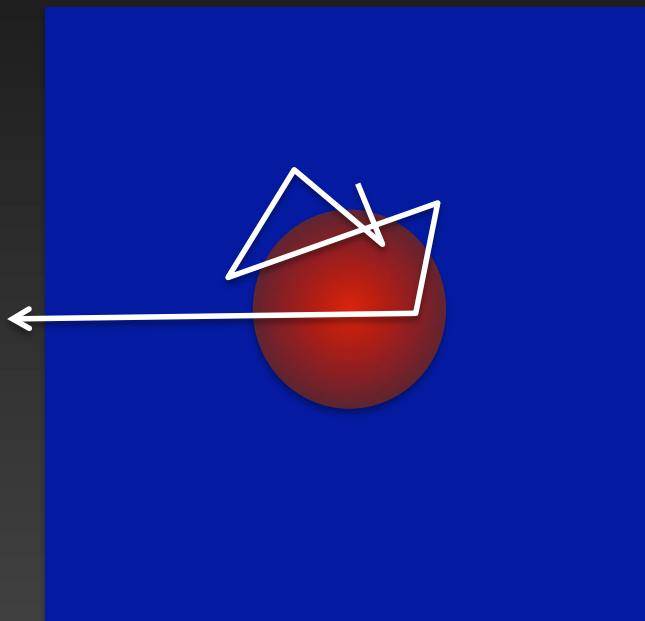
- Fourier spectra at decoupling:



initial rms overdensity

CMB power spectrum

- Non-instantaneous decoupling:

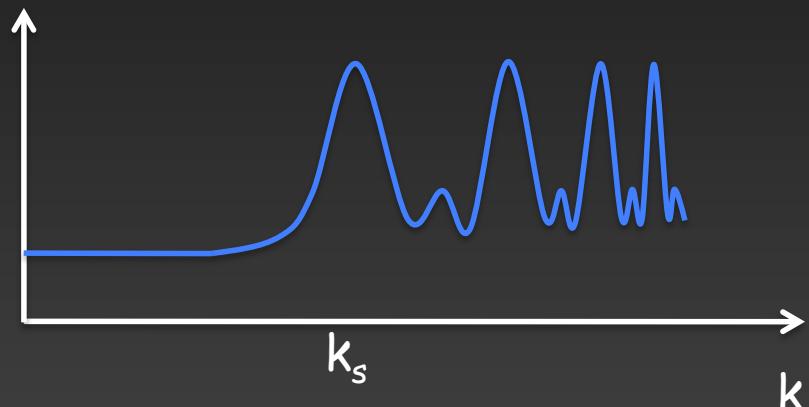


CMB power spectrum

- Non-instantaneous decoupling:

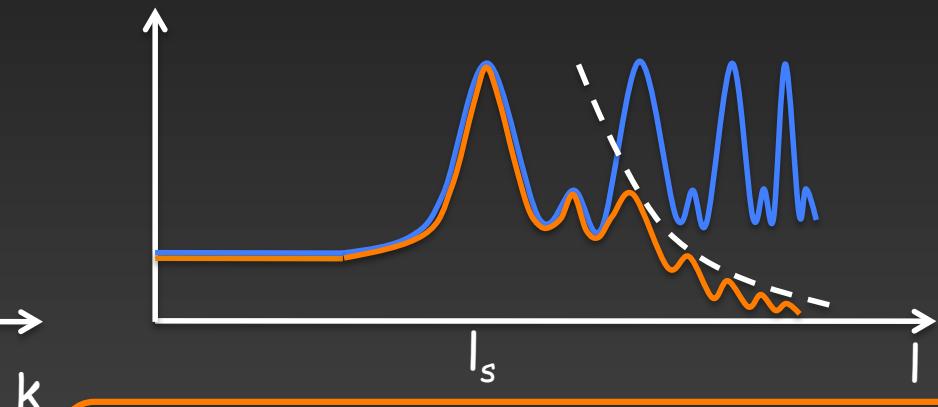
Fourier, instantaneous dec.

perturbation $^2 k^3$



Legendre, non-instantaneous dec.

$l(l+1)C_l$



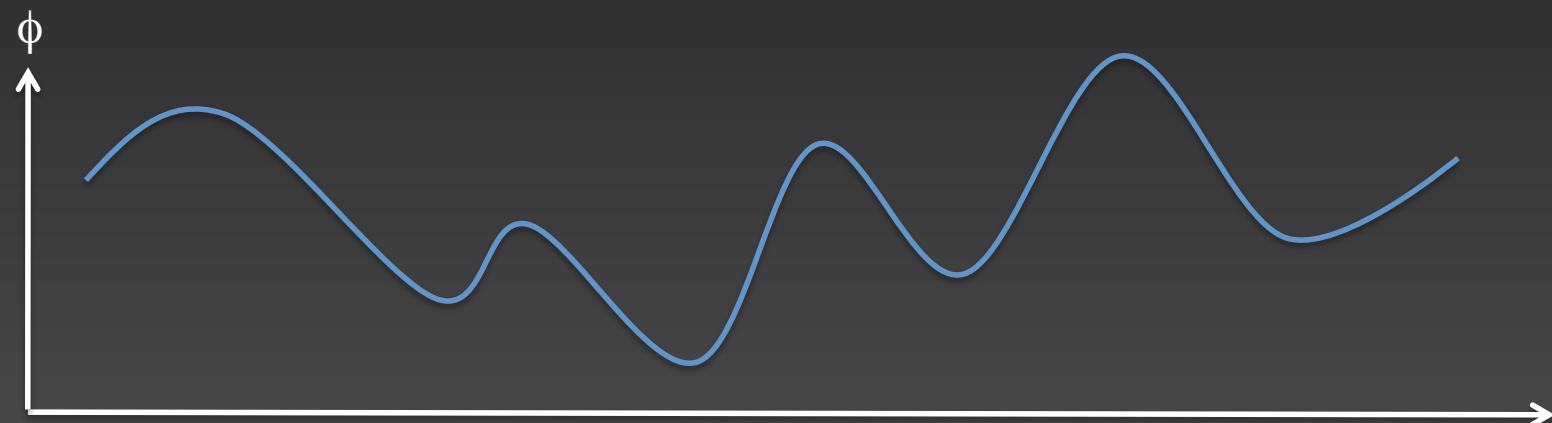
k to l mapping : $k \sim l a_{\text{LSS}}/d_A(\text{LSS})$
+ Silk damping

$$l_s \sim d_A(\text{LSS})/d_s(\text{LSS}) \sim 1^\circ$$

CMB power spectrum

- Integrated Sachs-Wolfe effect

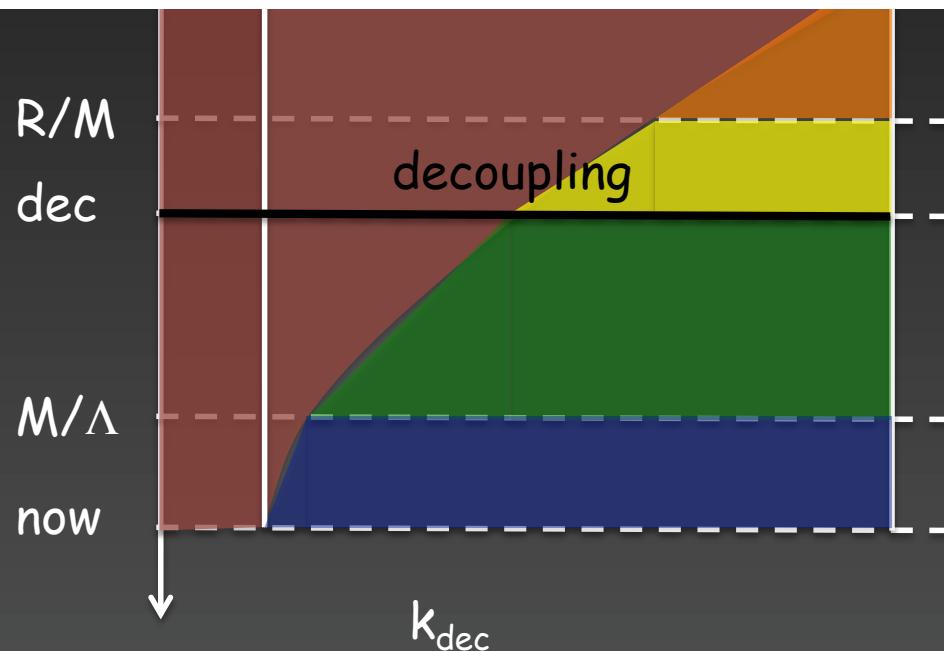
$$\frac{\delta T}{T}_{obs}(\vec{x}_0, \hat{n}) = \frac{\delta T}{T}_{LSS} + \phi_{LSS} - \hat{n} \cdot \vec{v}_{bLSS} - \phi(\vec{x}_0) + \int_{LSS}^0 d\tau (\dot{\phi} + \dot{\psi})$$



CMB power spectrum

- Integrated Sachs-Wolfe effect

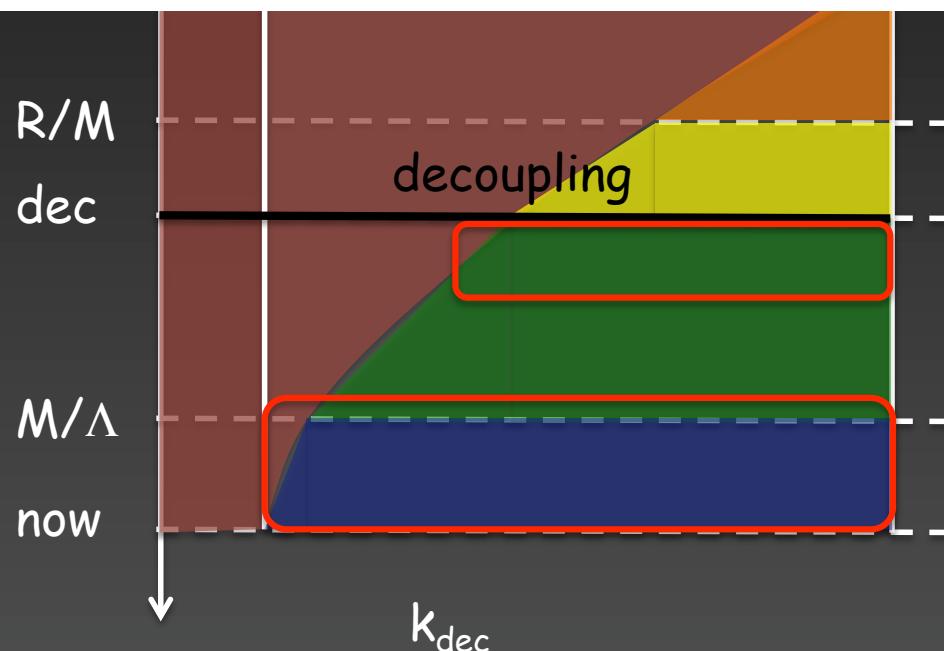
$$\frac{\delta T}{T}_{obs}(\vec{x}_0, \hat{n}) = \frac{\delta T}{T}_{LSS} + \phi_{LSS} - \hat{n} \cdot \vec{v}_{bLSS} - \phi(\vec{x}_0) + \int_{LSS}^0 d\tau (\dot{\phi} + \dot{\psi})$$



CMB power spectrum

- Integrated Sachs-Wolfe effect

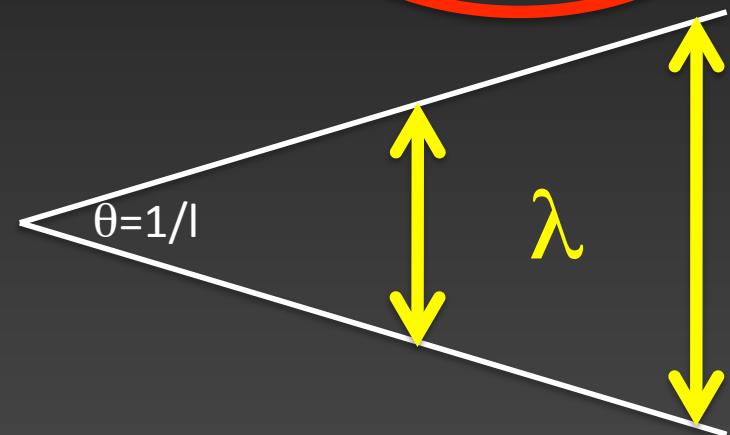
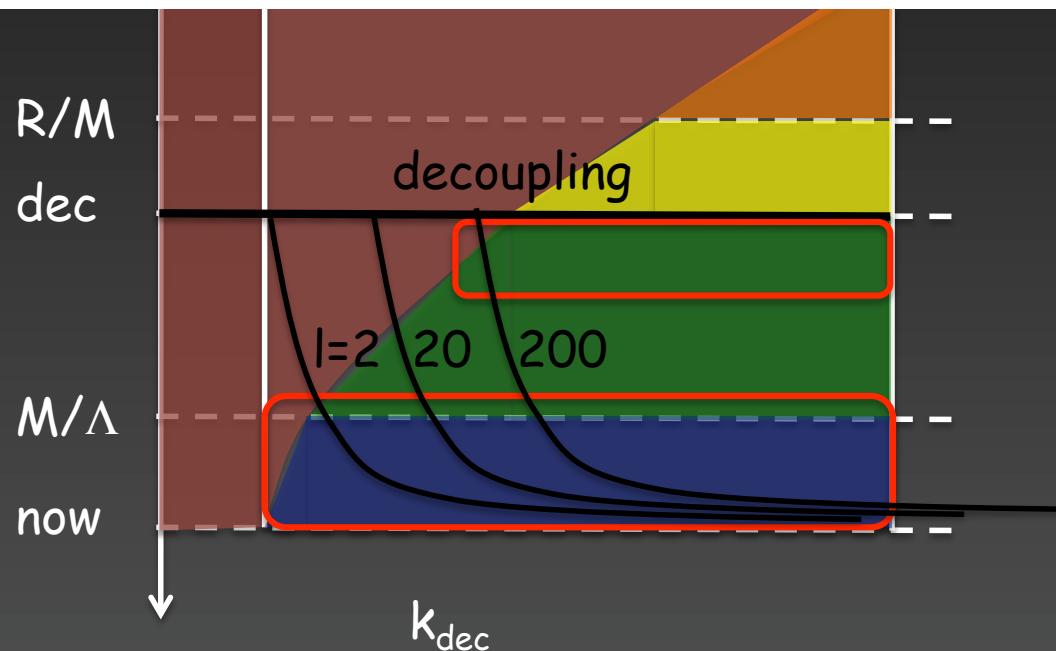
$$\frac{\delta T}{T}_{obs}(\vec{x}_0, \hat{n}) = \frac{\delta T}{T}_{LSS} + \phi_{LSS} - \hat{n} \cdot \vec{v}_{bLSS} - \phi(\vec{x}_0) + \int_{LSS}^0 d\tau (\dot{\phi} + \dot{\psi})$$



CMB power spectrum

- Integrated Sachs-Wolfe effect

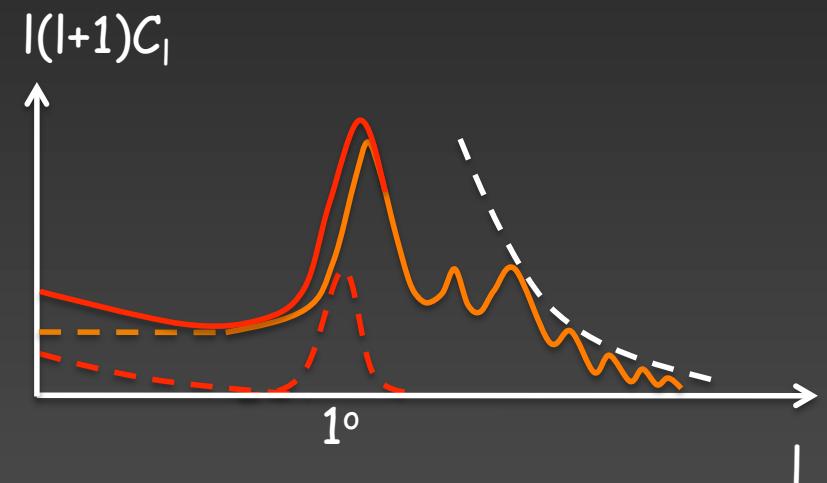
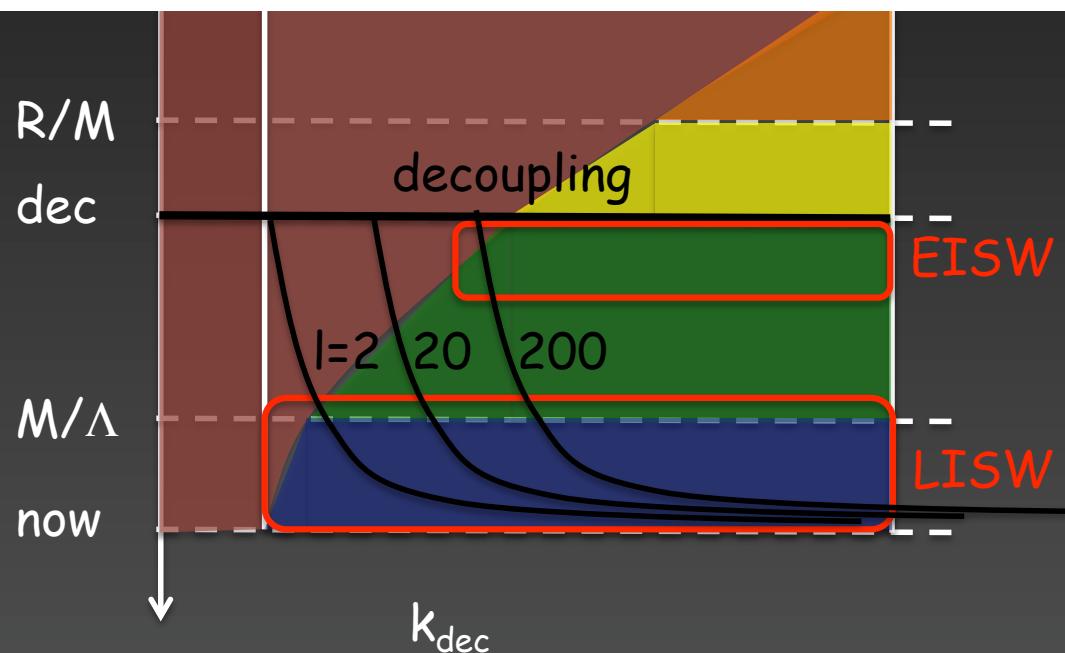
$$\frac{\delta T}{T}_{obs}(\vec{x}_0, \hat{n}) = \frac{\delta T}{T}_{LSS} + \phi_{LSS} - \hat{n} \cdot \vec{v}_{bLSS} - \phi(\vec{x}_0) + \int_{LSS}^0 d\tau (\dot{\phi} + \dot{\psi})$$



CMB power spectrum

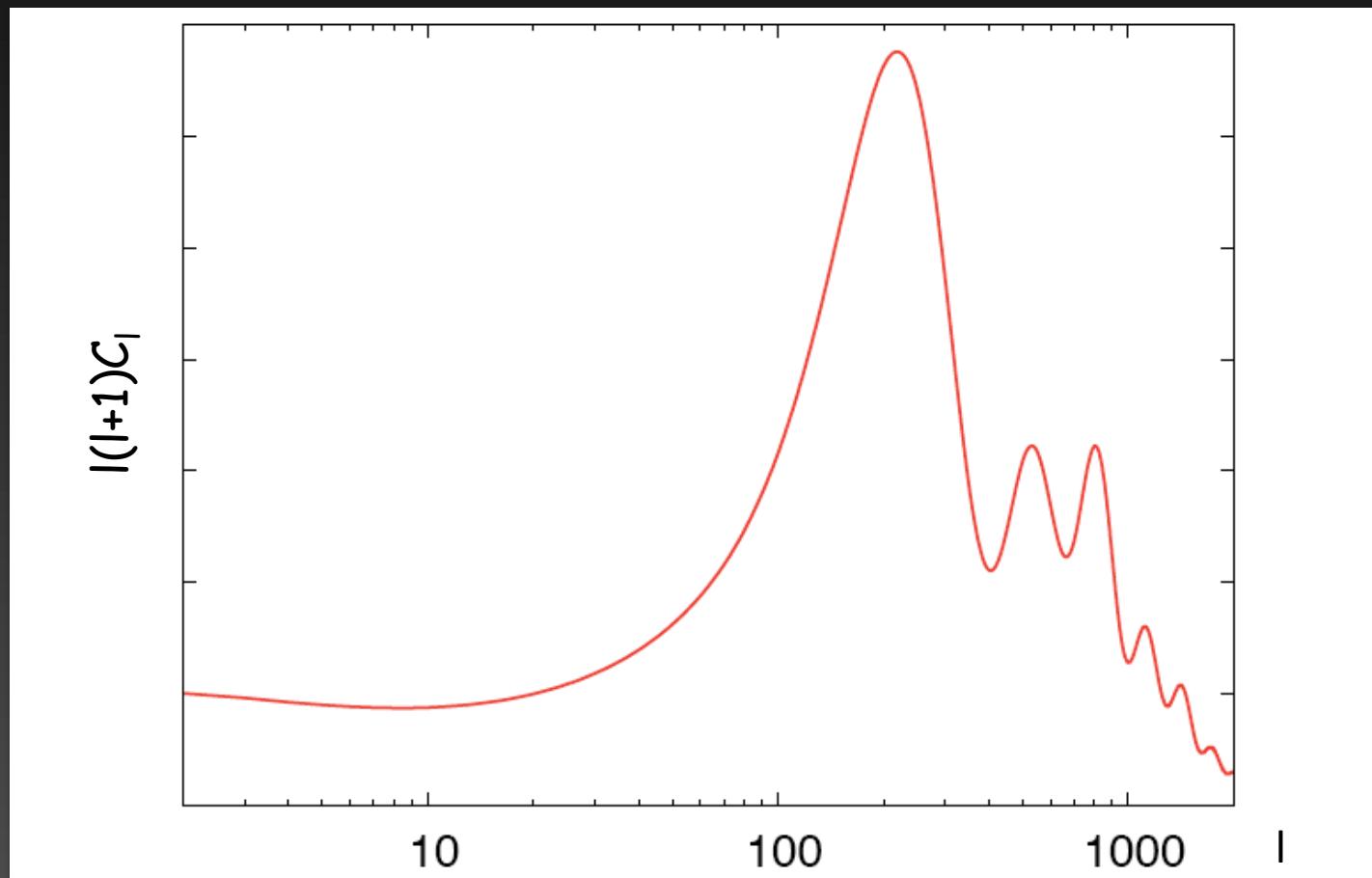
- Integrated Sachs-Wolfe effect

$$\frac{\delta T}{T}_{obs}(\vec{x}_0, \hat{n}) = \frac{\delta T}{T}_{LSS} + \phi_{LSS} - \hat{n} \cdot \vec{v}_{bLSS} - \phi(\vec{x}_0) + \int_{LSS}^0 d\tau (\dot{\phi} + \dot{\psi})$$



CMB power spectrum

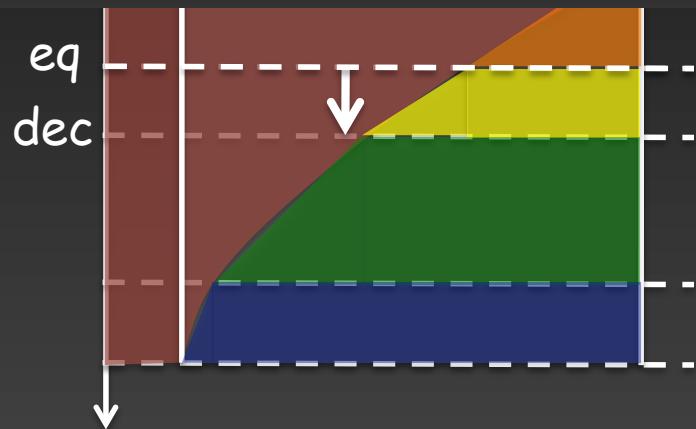
- Numerical result from Boltzman code



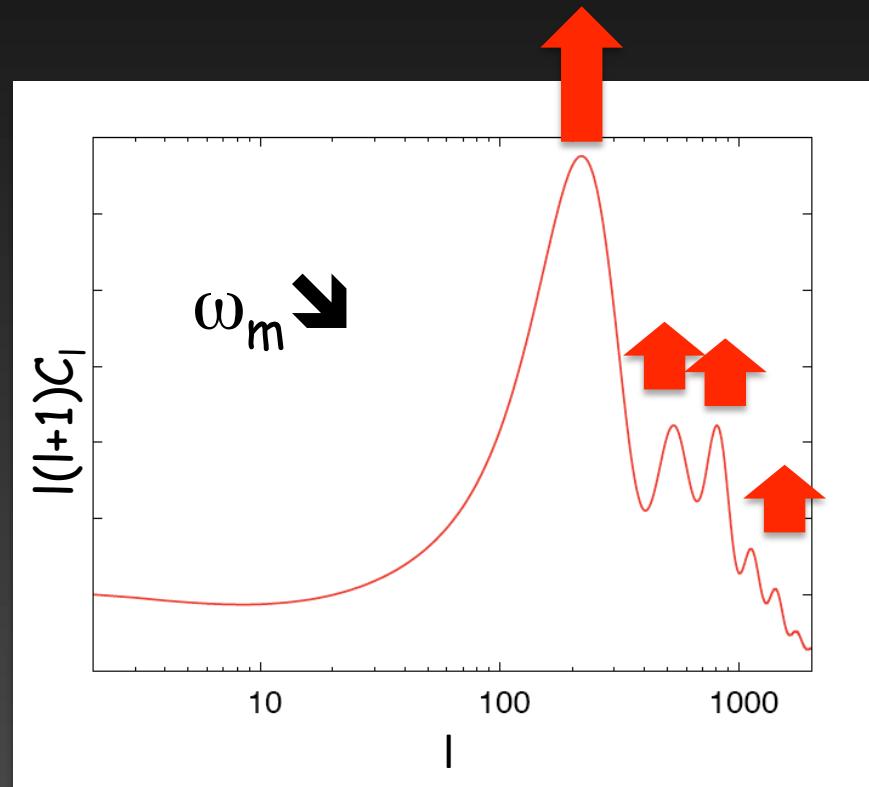
CMB power spectrum

- Quantities affecting C_l 's

1. Time of R/M equality
delay equality:



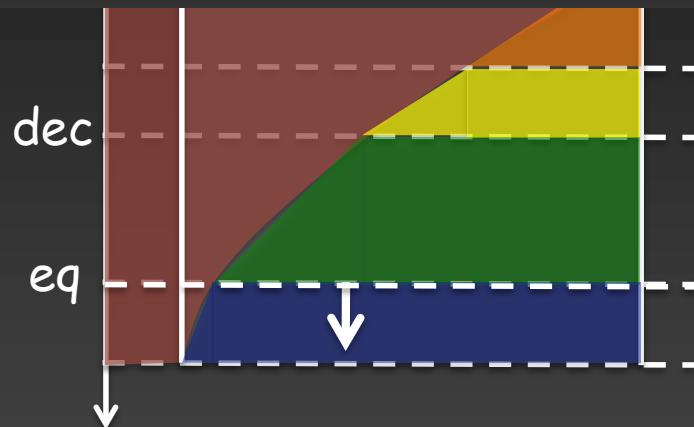
- more EISW
- less oscillation damping



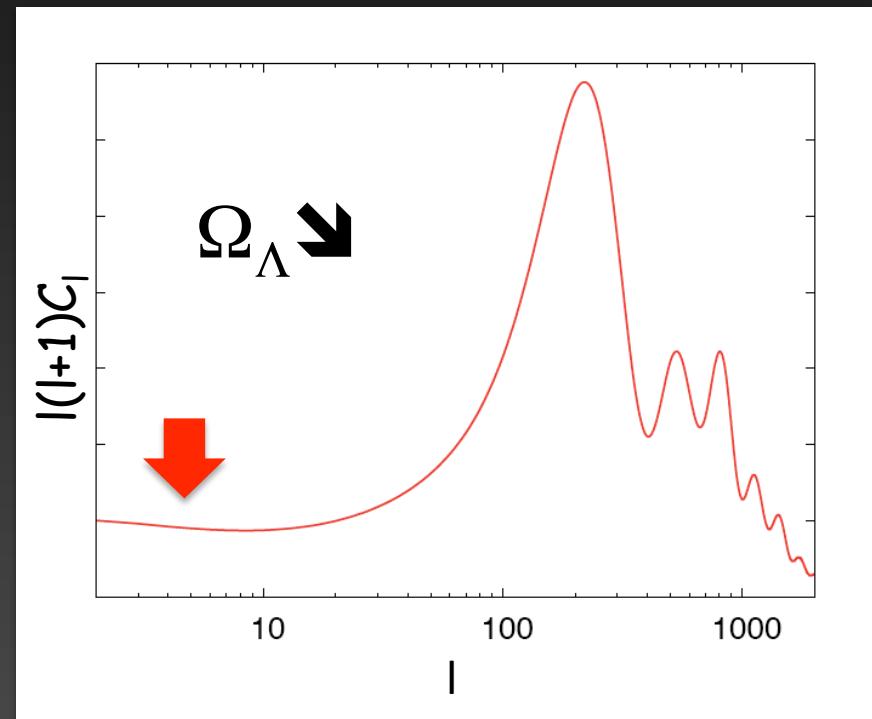
CMB power spectrum

- Quantities affecting C_l 's

2. Time of M/Λ equality
delay equality:



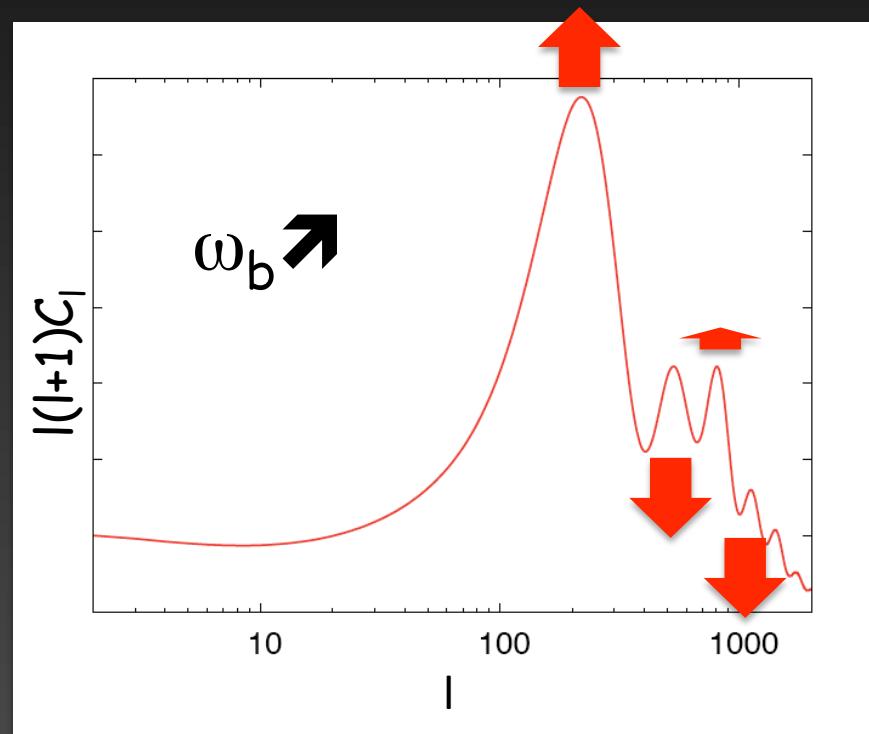
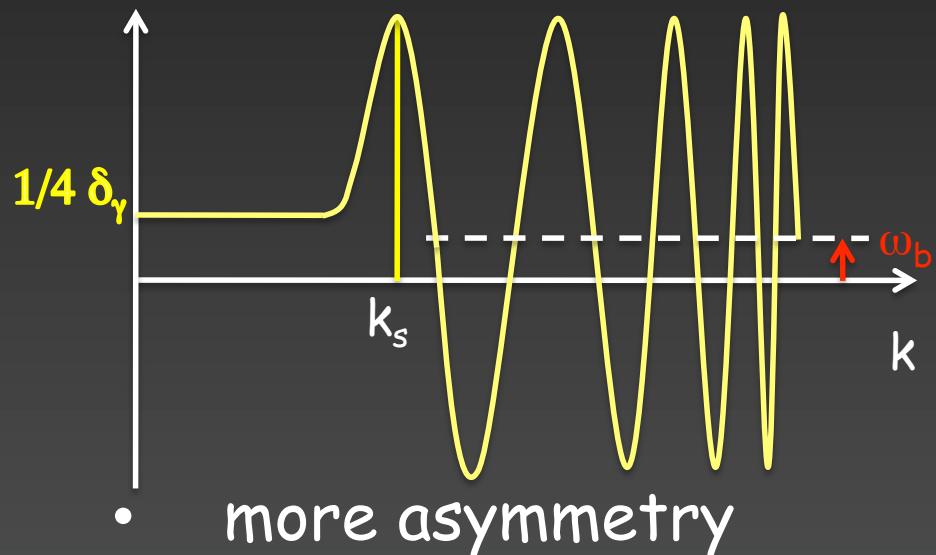
- Less LISW



CMB power spectrum

- Quantities affecting C_l 's

3. Balance gravity/pressure in photon-baryon fluid
increase baryon density:



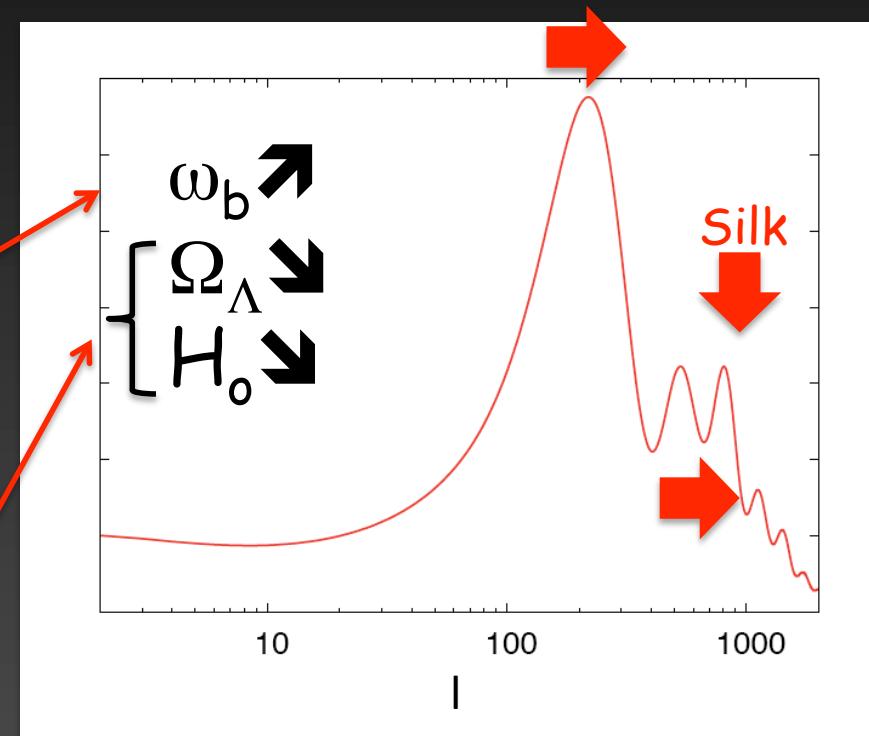
CMB power spectrum

- Quantities affecting C_l 's

4. angular scale of sound horizon at recombination

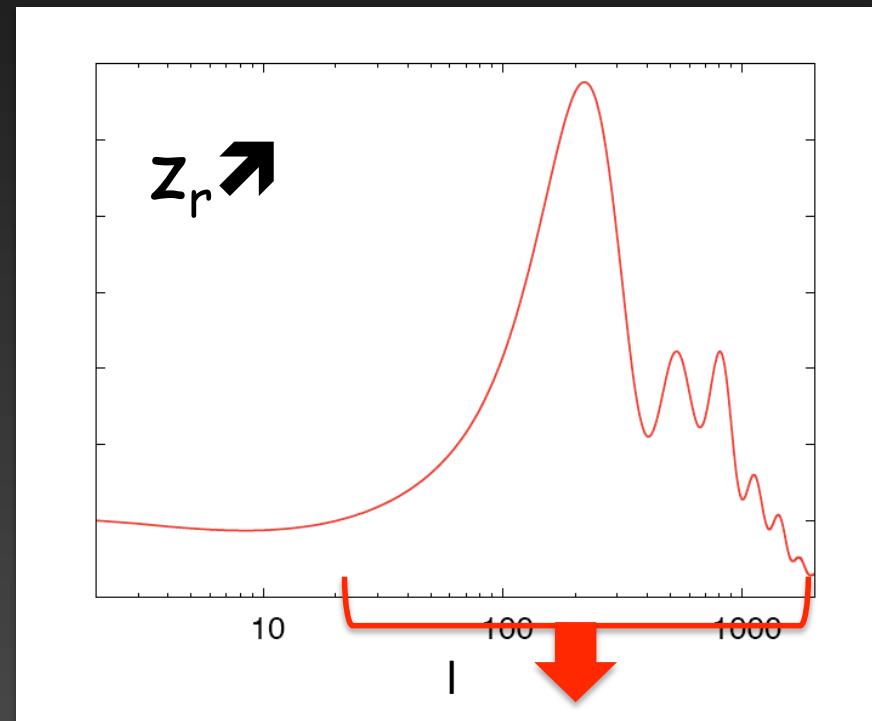
$$l_{\text{peak}} \sim d_A(\text{LSS})/d_S(\text{LSS})$$

$$d_S^{co}(t_{dec}) = \int_0^{t_{dec}} \frac{c_s dt}{a}$$
$$d_A^{co}(t_{dec}) = \frac{1}{1+z_{dec}} \int_0^{z_{dec}} \frac{dz}{H}$$



CMB power spectrum

- Quantities affecting C_l 's
- 5. redshift of reionization
- Damp anisotropies below Hubble scale at that time

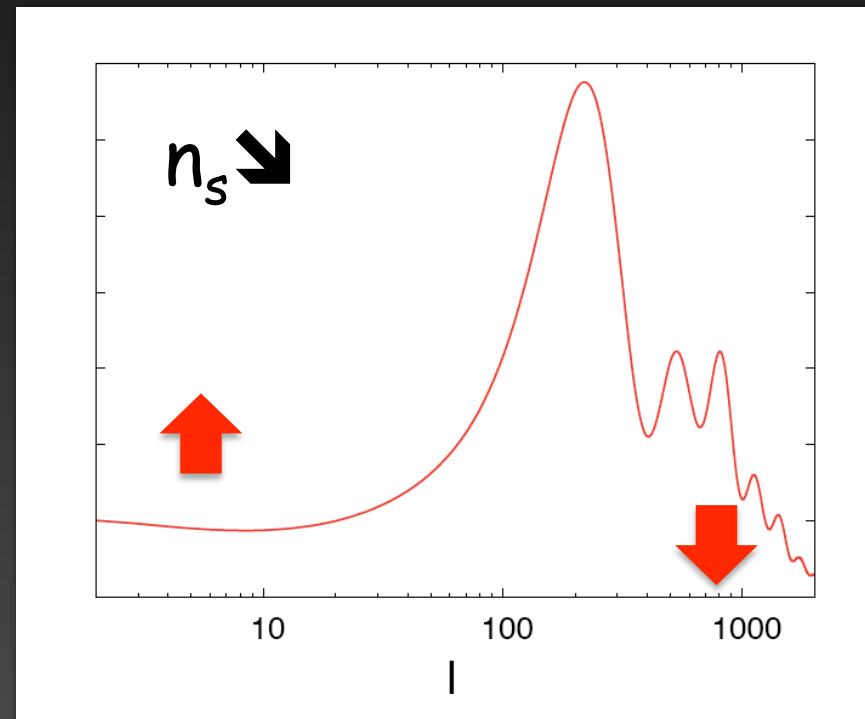


CMB power spectrum

- Quantities affecting C_l 's

6. Primordial spectrum

- amplitude
- tilt



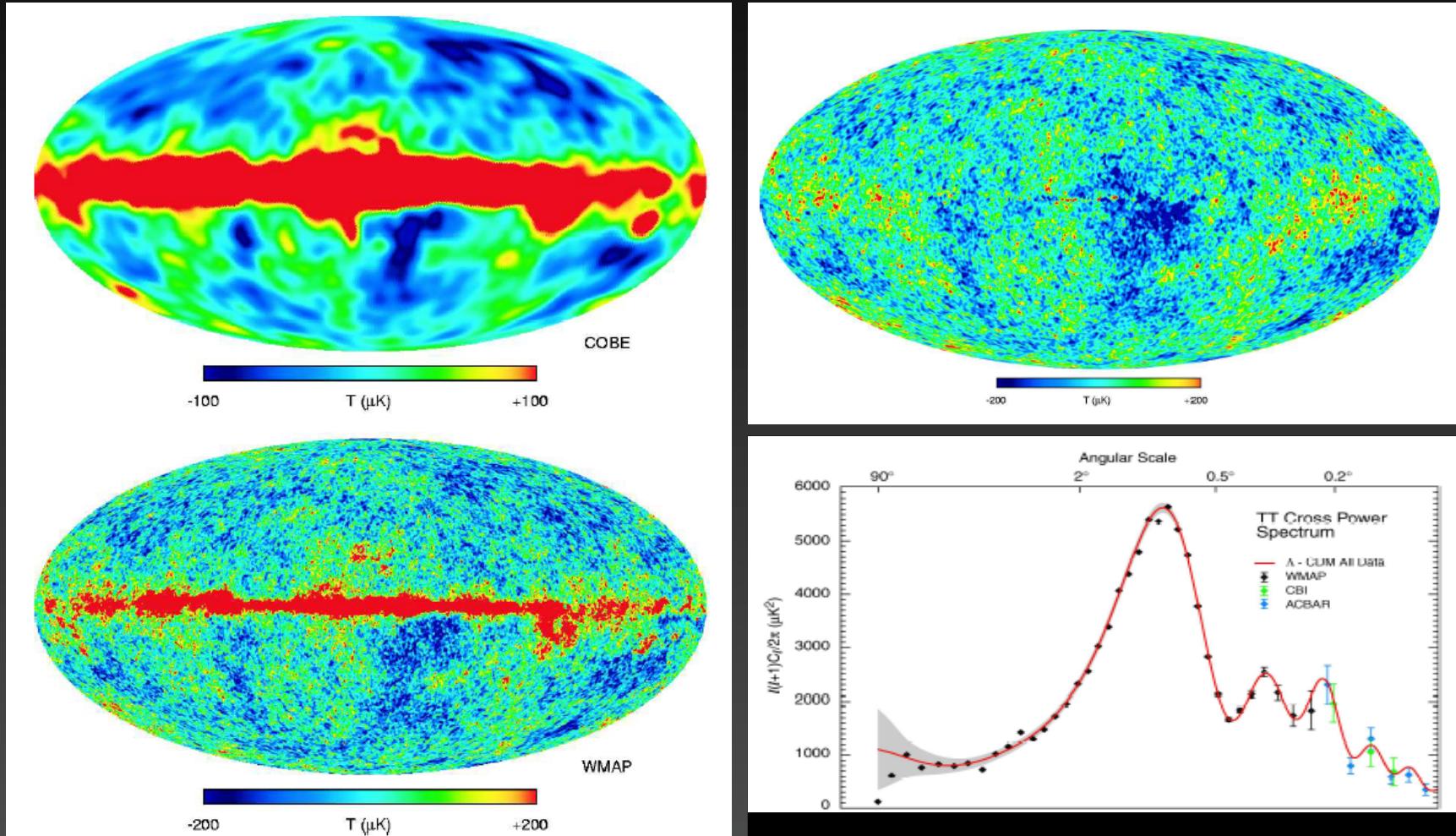
CMB power spectrum

- Summary for minimal Λ CDM :
 - 6 parameters $\{\omega_b, \omega_m, \Omega_\Lambda, A, n_s, z_r\}$
 - 6 effects:
 - 1) peak enhancement, especially 1st (EISW)
 - 2) peak asymmetry
 - 3) small l enhancement (LISW) HARDLY MESURABLE
 - 4) peak position
 - 5) global amplitude
 - 6) global tilt

CMB power spectrum

- Summary for minimal Λ CDM :
 - CMB temperature alone can measure all 6 parameters !!!
 - correlation with other data sets welcome to reduce degeneracies allowed by instrument noise
- Other datasets unavoidable for extended models
 - e.g. open/closed, DE with w or $w(z)$, extra d.o.f (more params, same effects)
 - to a lesser extent : massive neutrinos, primordial spectrum (more params, more effects)

- CMB OBSERVATIONS:
WMAP (2003)

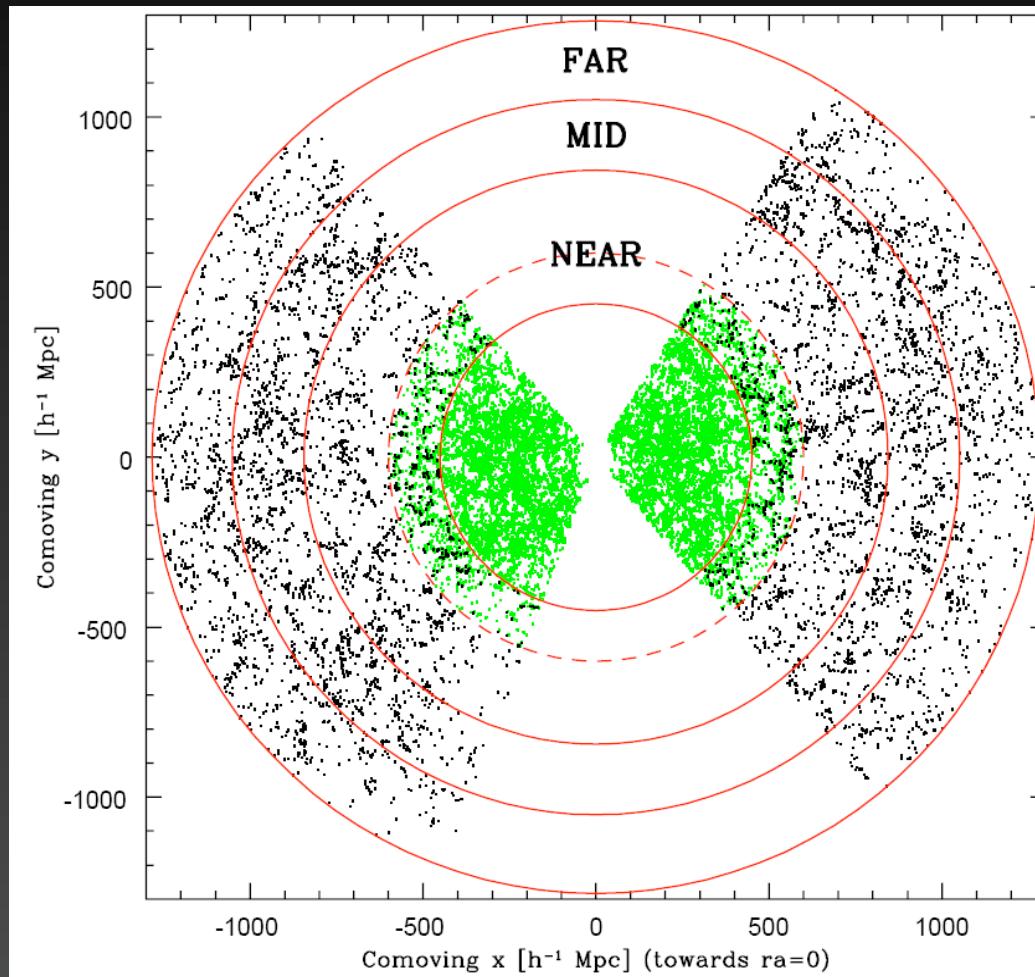


- **CMB observations:**
 - Parameters from WMAP 2009:

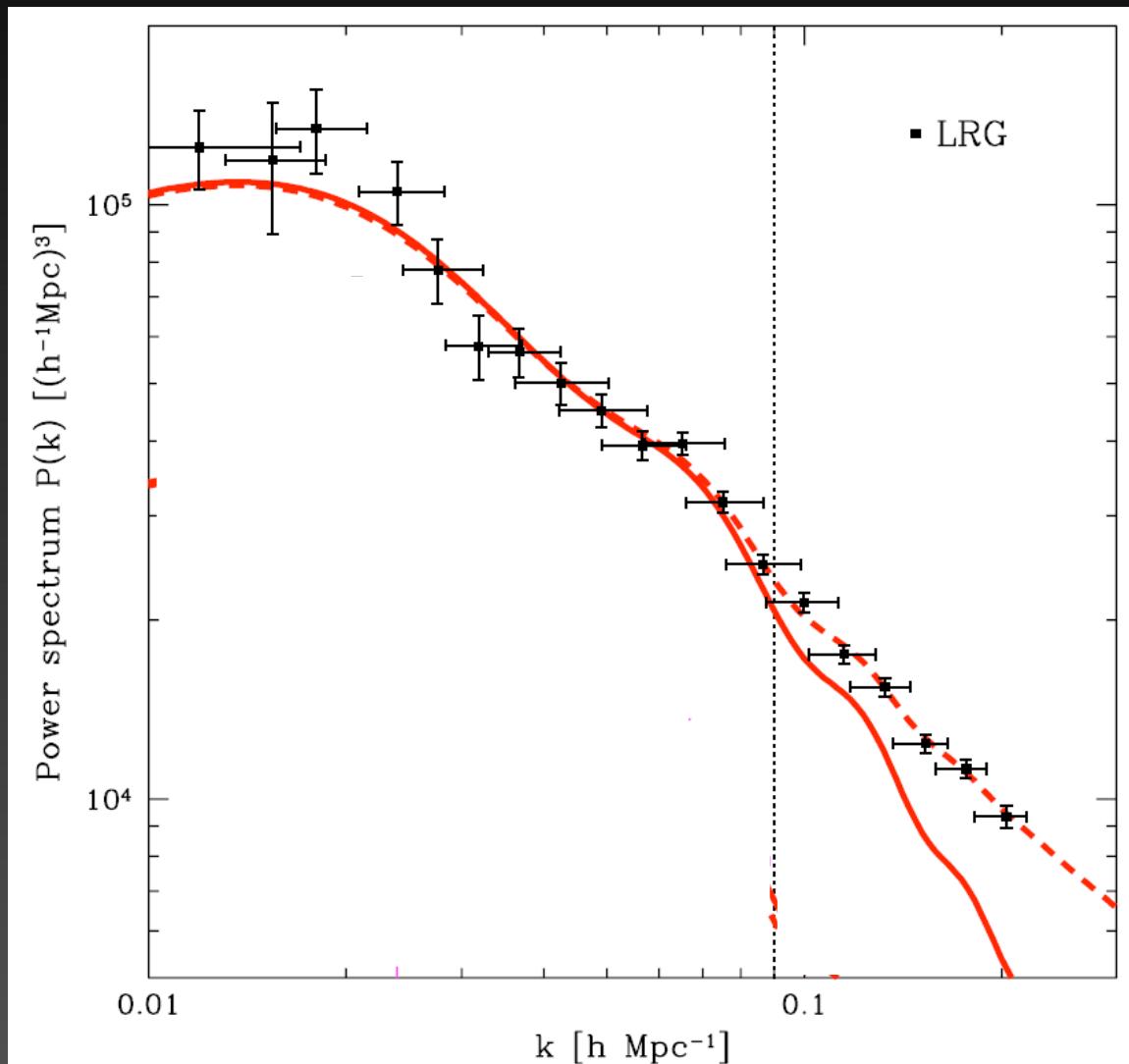
Class	Parameter	WMAP 5-year ML
Primary	$100\Omega_b h^2$	2.268
	$\Omega_c h^2$	0.1081
	Ω_Λ	0.751
	n_s	0.961
	τ	0.089
	$\Delta_{\mathcal{R}}^2(k_0 e)$	2.41×10^{-9}
Derived	σ_8	0.787
	H_0	72.4 km/s/Mpc
	Ω_b	0.0432
	Ω_c	0.206

- Non-flat models: from WMAP+distance measurements
 $\Omega_0 = \Omega_M + \Omega_\Lambda = 1.05 \pm 0.013$, and if $\Omega_0 = 1$:

- MATTER POWER SPECTRUM:
 - *SDSS-LRG (2007), SDSS halos (2009), ...*



- MATTER POWER SPECTRUM:
 - *SDSS-LRG (2007), SDSS halos (2009), ...*



Part IV Inflation

- Accelerated expansion

- requires :

Friedman law

conservation equation

$$\ddot{a}(t) > 0 \Leftrightarrow \rho + 3p < 0$$

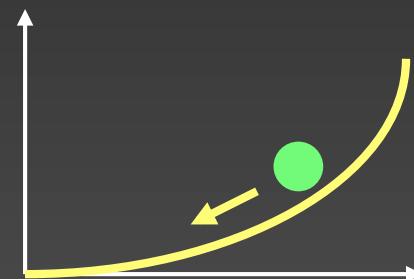
- candidates :

- Λ : $\rho + 3p = -2p < 0$ but inflation forever ...

- slow-rolling scalar field : $V(\phi)$

$$\begin{cases} \rho = \dot{\phi}^2/2 + V(\phi) \\ p = \dot{\phi}^2/2 - V(\phi) \end{cases}$$

- need $|V'/V|$ small, $|V''/V|$ small



- Slow-roll dynamics:

- Slow-roll conditions :

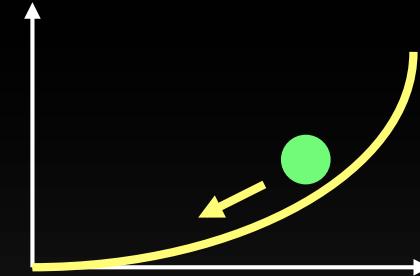
$$|V'/V| \ll M_p^{-1}, |V''/V| \ll M_p^{-2}$$

- Equation of motion reduces to

$$\dot{\phi} = -V'/3H \quad \text{nearly constant}$$

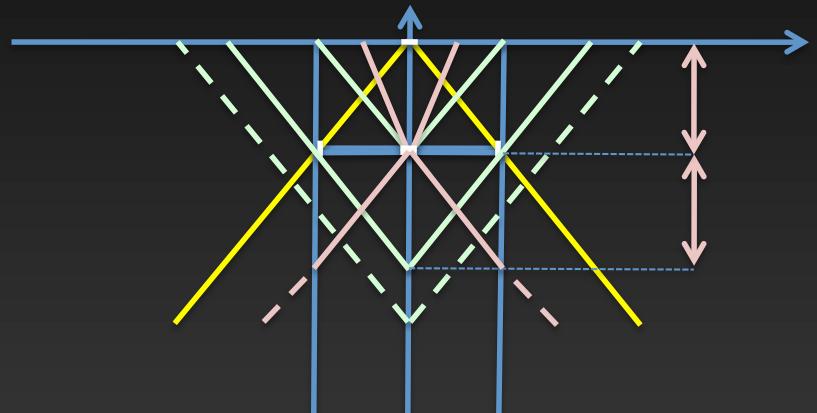
- Friedmann reduces to

$$3 H^2 M_p^2 = V \quad \text{nearly constant}$$



- duration of inflation
 - definition of e-fold number: $N = \ln a$
 - horizon problem requires :

$$\Delta N_{\text{inflation}} \geq \Delta N_{\text{post-inflation}}$$



- between BBN and now: 30 e-folds
- if RD starts at GUT scale: 60 e-folds
- Flatness problem ($|\Omega_k| \sim [aH]^{-2}$) requires the same

- INFLATION with slow-rolling scalar field :

2 BONUS !!!

- mechanism for generation of cosmological perturbations

quantum fluctuations
of scalar field

wavelength
amplification

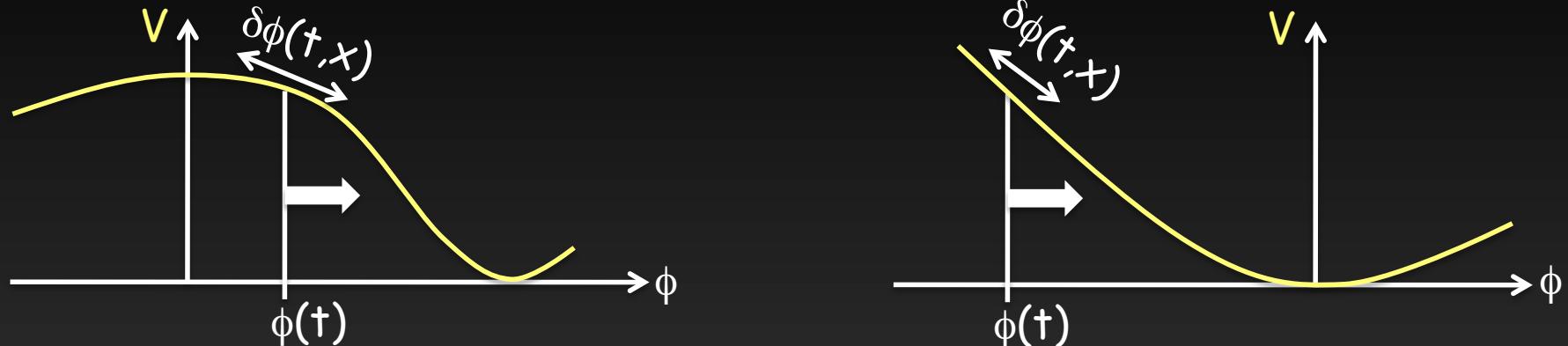
stochastic background of
curvature perturbations

predictions : coherent, gaussian,
adiabatic, scale-invariant

VALIDATED BY
OBSERVATIONS

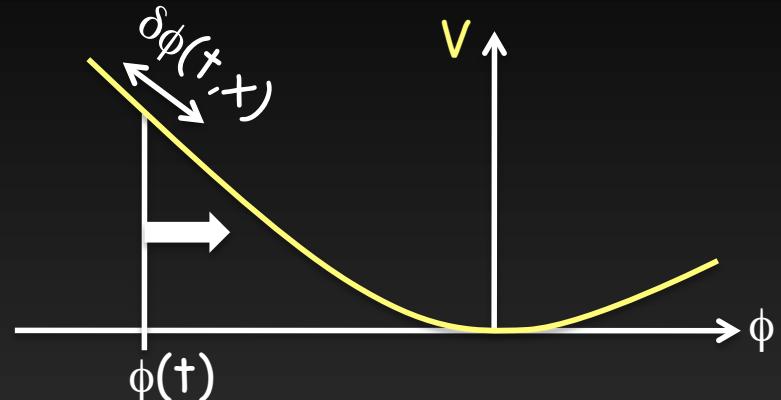
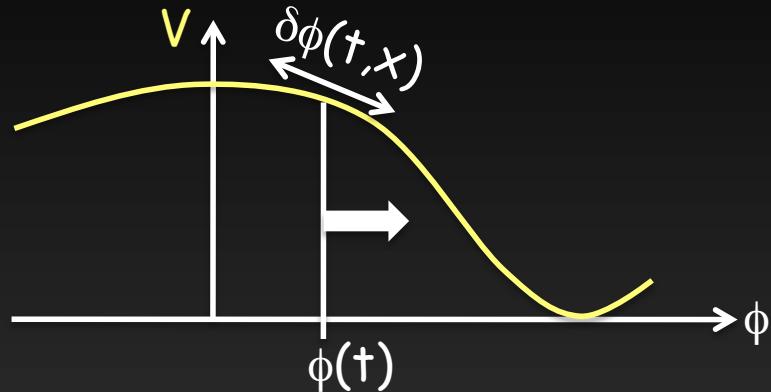
- mechanism for generation of first particles : PREHEATING
- end of inflation \longrightarrow oscillations of scalar field \longrightarrow particle production

Constraints on inflation



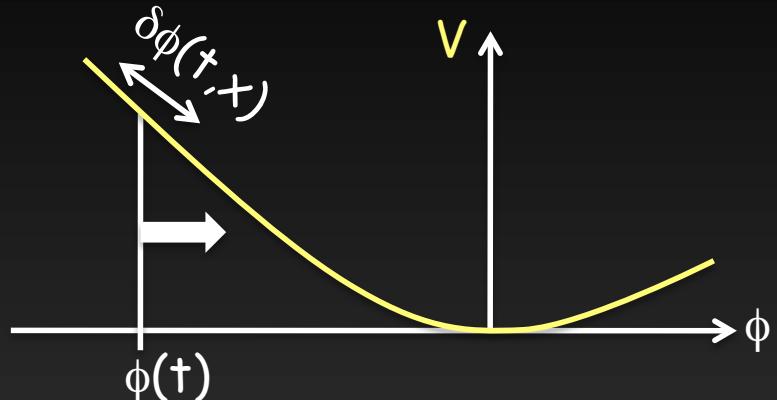
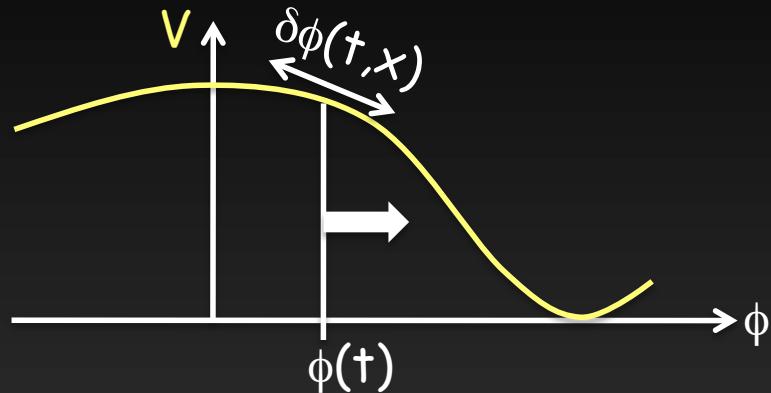
- Much before Hubble crossing:
 - use quantum field theory in flat space-time, define vacuum fluctuations normalized by $[x,p]=i\hbar$
 - non-trivial evolution near Hubble crossing
- Near Hubble crossing
 - when $k \sim aH$, power spectrum $\langle |\delta\phi_k|^2 \rangle \sim [H_k^2 / k^3]$

Constraints on inflation



- After Hubble crossing:
 - R_k frozen, depends on δN_k when $|k| \sim aH$ (i.e. at freeze-out)
 - $\delta N = [d \ln a / d\phi] \delta\phi = [H/\phi'] \delta\phi$
 - $\langle |R_k|^2 \rangle \sim \langle |\delta N_k|^2 \rangle = [H/\phi']_k^2 \langle |\delta\phi_k|^2 \rangle \sim [H/\phi']_k^2 [H_k^2 / k^3]$
 - so $k^3 \langle |R_k|^2 \rangle \sim [V/\phi']_k^2$

Constraints on inflation



- primordial spectrum : $k^3 \langle |R_k|^2 \rangle \sim [V/\phi']_k^2 \sim [V^3/V'^2]_k$
- V and ϕ' nearly constant, so $k^3 \langle |R_k|^2 \rangle$ nearly scale-invariant
- next correction: tilt n , $k^3 \langle |R_k|^2 \rangle \sim k^{n-1}$ depends on V', V''

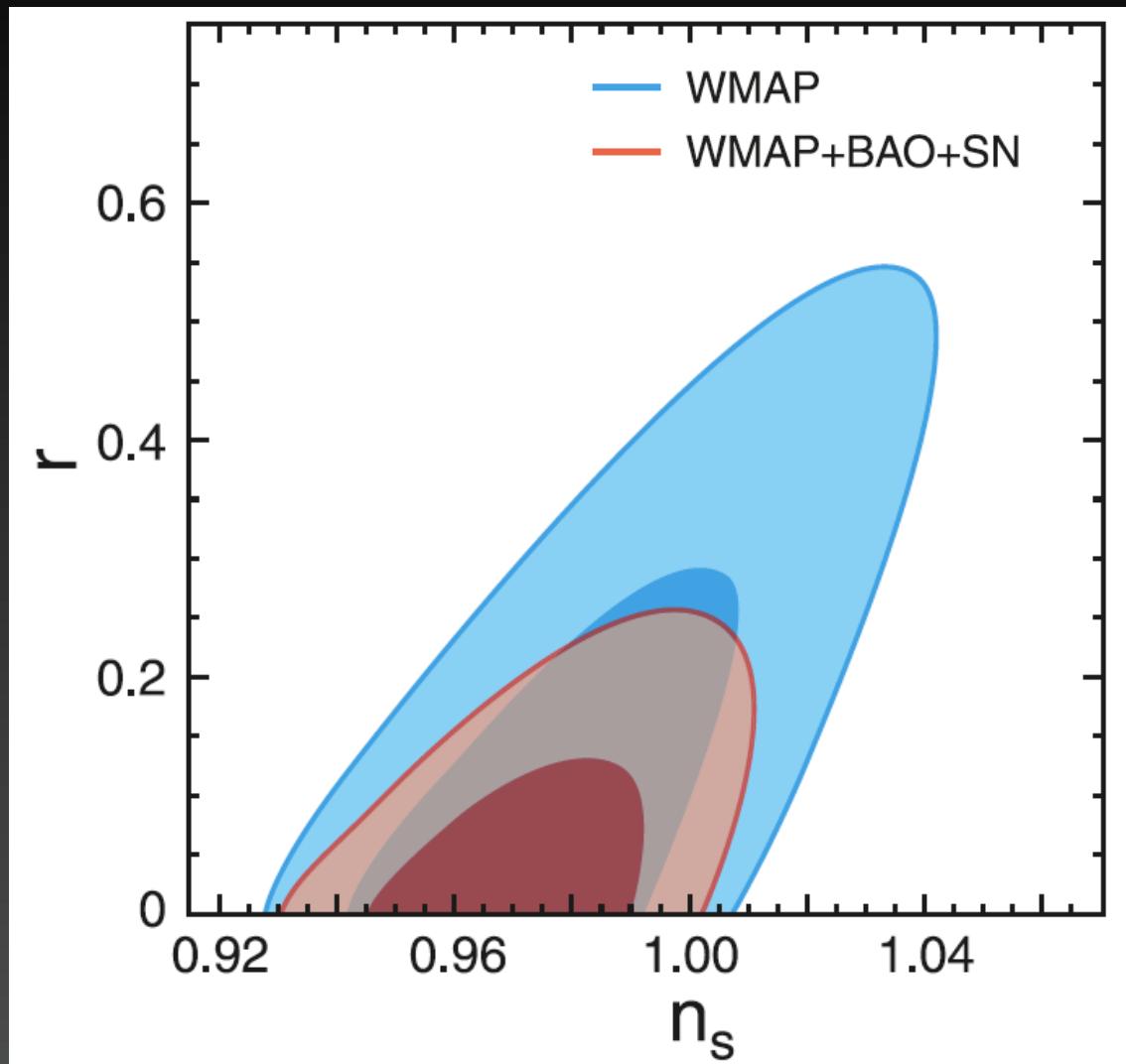
Constraints on inflation

- sign of $(n-1)$ given by evolution of $[V/\phi']_k^2$ versus k .
As time passes, smaller wavelenghts (larger k 's) cross Hubble radius, and:
 - V can only decrease (excepted fine-tuned situations)
 - ϕ' usually increases in viable models, to end inflation
 - So $[V/\phi']_k^2$ decreases when k increases: red spectrum ($n < 1$)
 - ϕ' can decrease faster than V if inflation ends differently (e.g. hybrid inflation : phase transition for another field coupled to inflaton)

Constraints on inflation

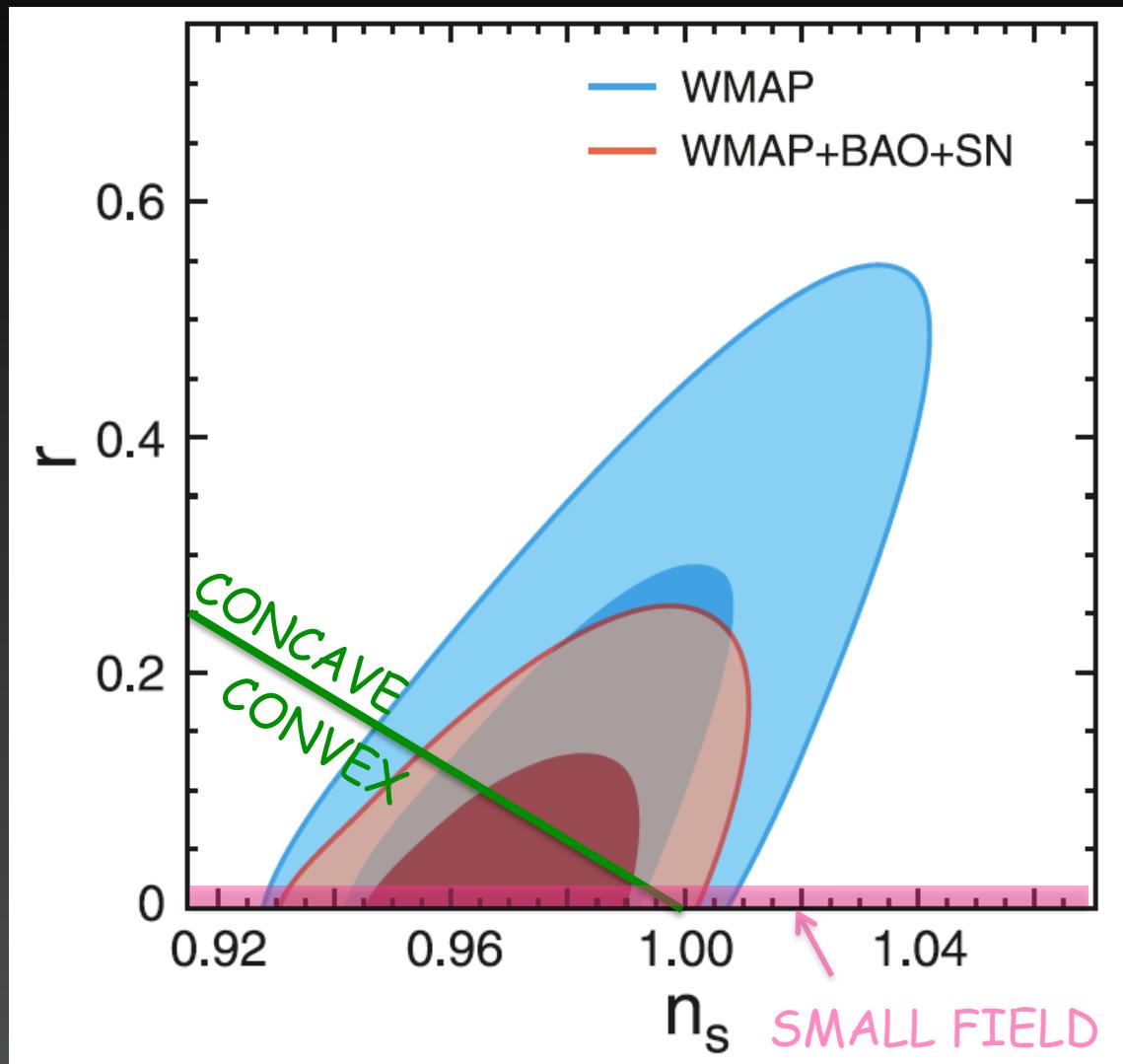
- Primordial gravitational waves:
 - quantization of metric tensor perturbations
 - primordial spectrum of gravitational waves, given by H^2_k i.e. V_k
 - GW contribute to CMB temperature (small l's) and polarization
 - Observational bounds on $r \sim T/S$; S being known, $r \sim V_k$

Constraints on slow-roll models



WMAP5, Komatsu et al.

Constraints on slow-roll models



WMAP5, Komatsu et al.

Constraints on slow-roll models

Monomial inflation:

$$V=m_p^4(\phi/\mu)^\alpha$$

$$\alpha=4$$

$$\alpha=2$$

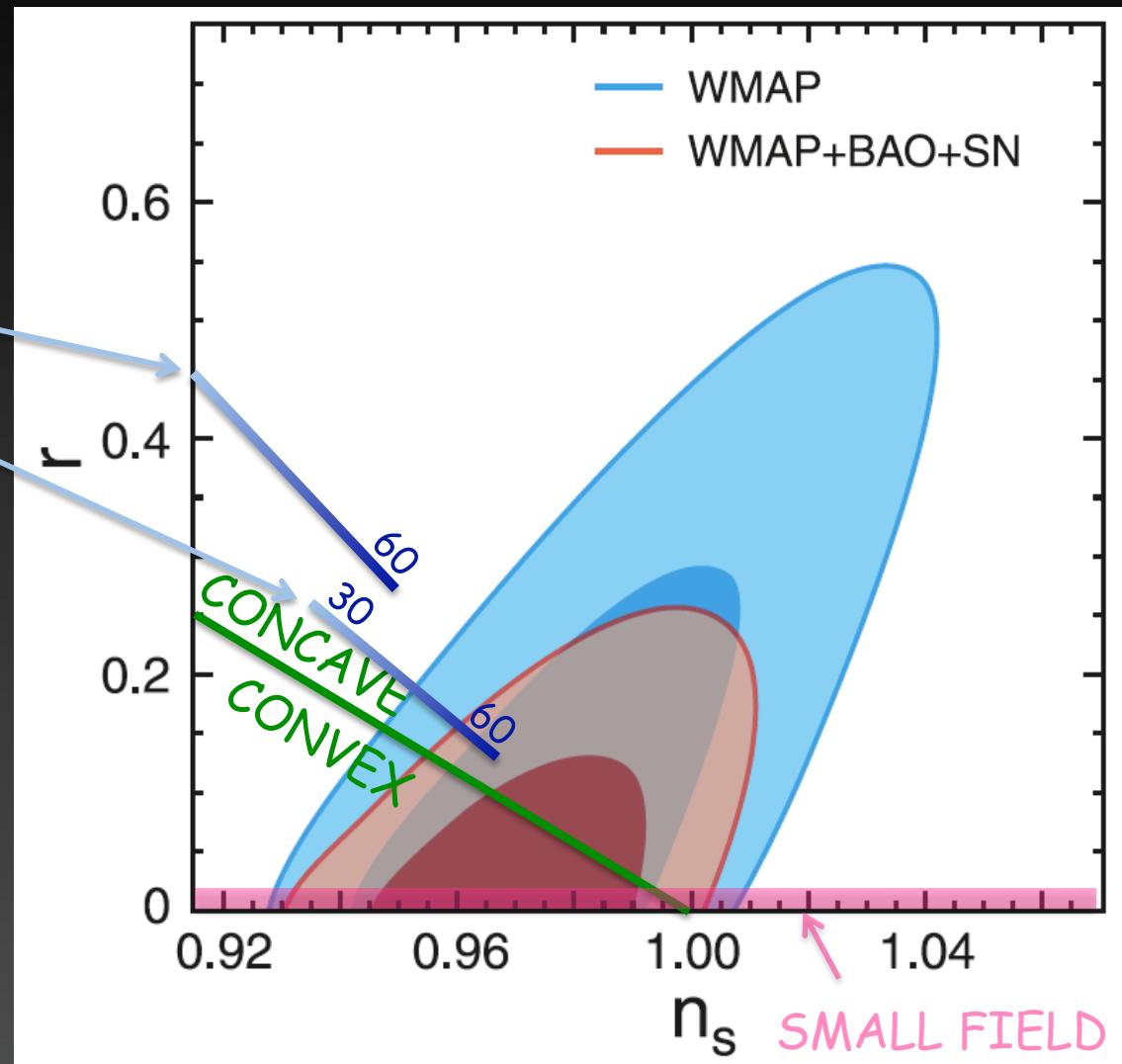
Hybrid inflation with
Higgs field

New inflation:

$$V=V_0[1-(\phi/\mu)^\alpha]$$

$$\alpha=2$$

$$\alpha=4$$



WMAP5, Komatsu et al.

Constraints on slow-roll models

Monomial inflation:

$$V=m_p^4(\phi/\mu)^\alpha$$

$\alpha=4$

$\alpha=2$

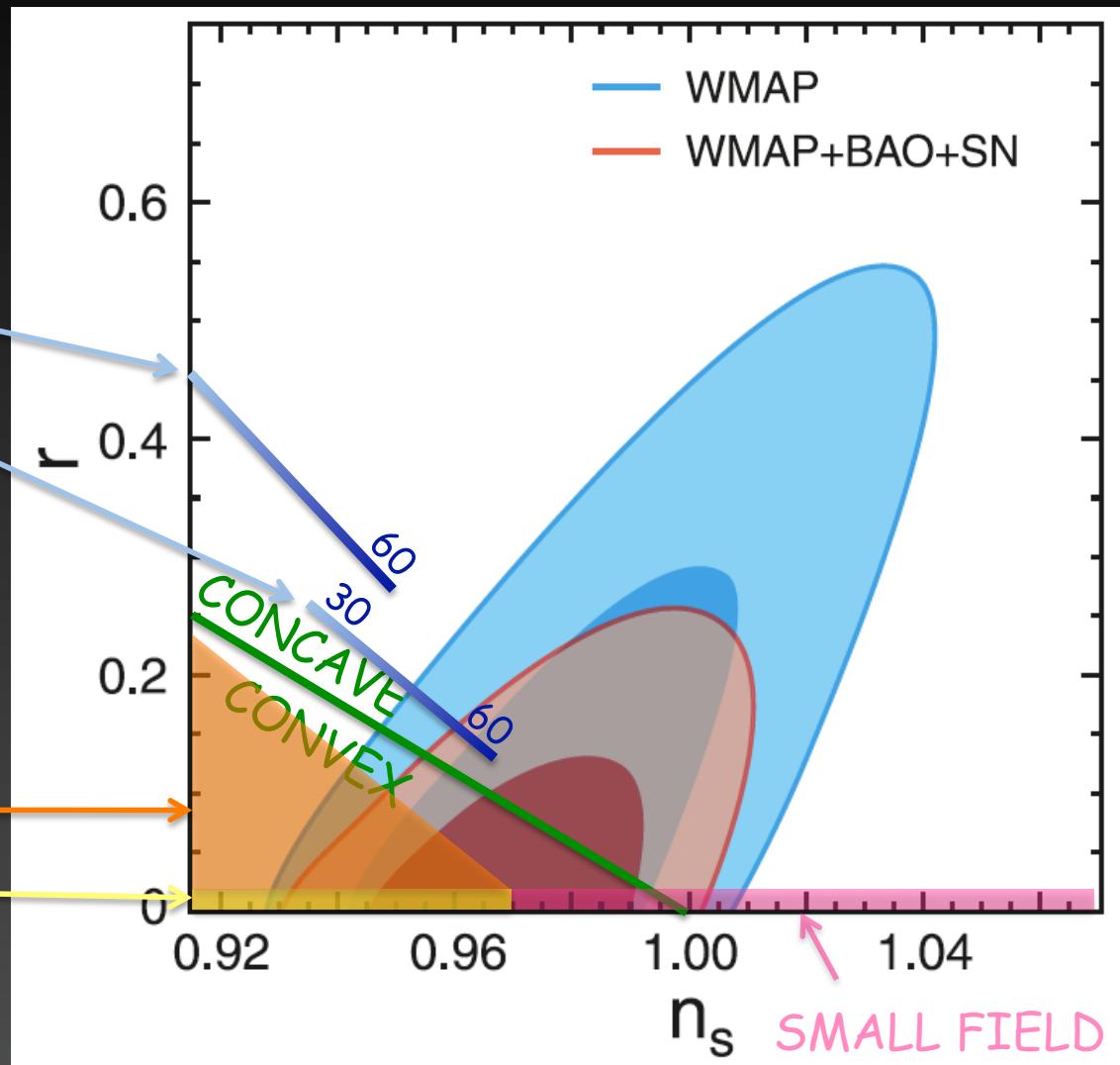
Hybrid inflation with
Higgs field

New inflation:

$$V=V_0[1-(\phi/\mu)^\alpha]$$

$\alpha=2$

$\alpha=4$



WMAP5, Komatsu et al.

Constraints on slow-roll models

Monomial inflation:

$$V=m_p^4(\phi/\mu)^\alpha$$

$$\alpha=4$$

$$\alpha=2$$

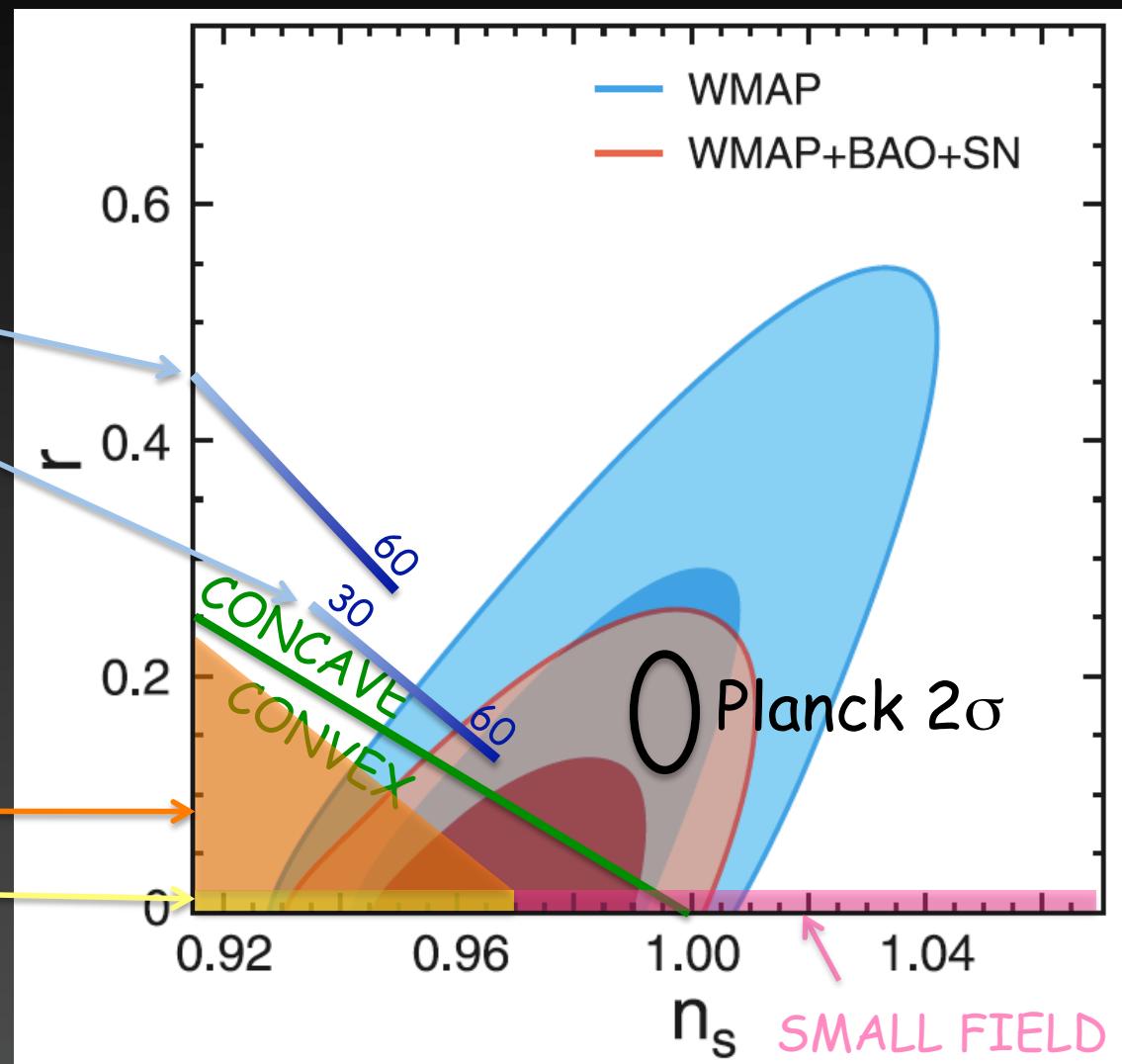
Hybrid inflation with
Higgs field

New inflation:

$$V=V_0[1-(\phi/\mu)^\alpha]$$

$$\alpha=2$$

$$\alpha=4$$



WMAP5, Komatsu et al.

Part V

Cosmological signature of

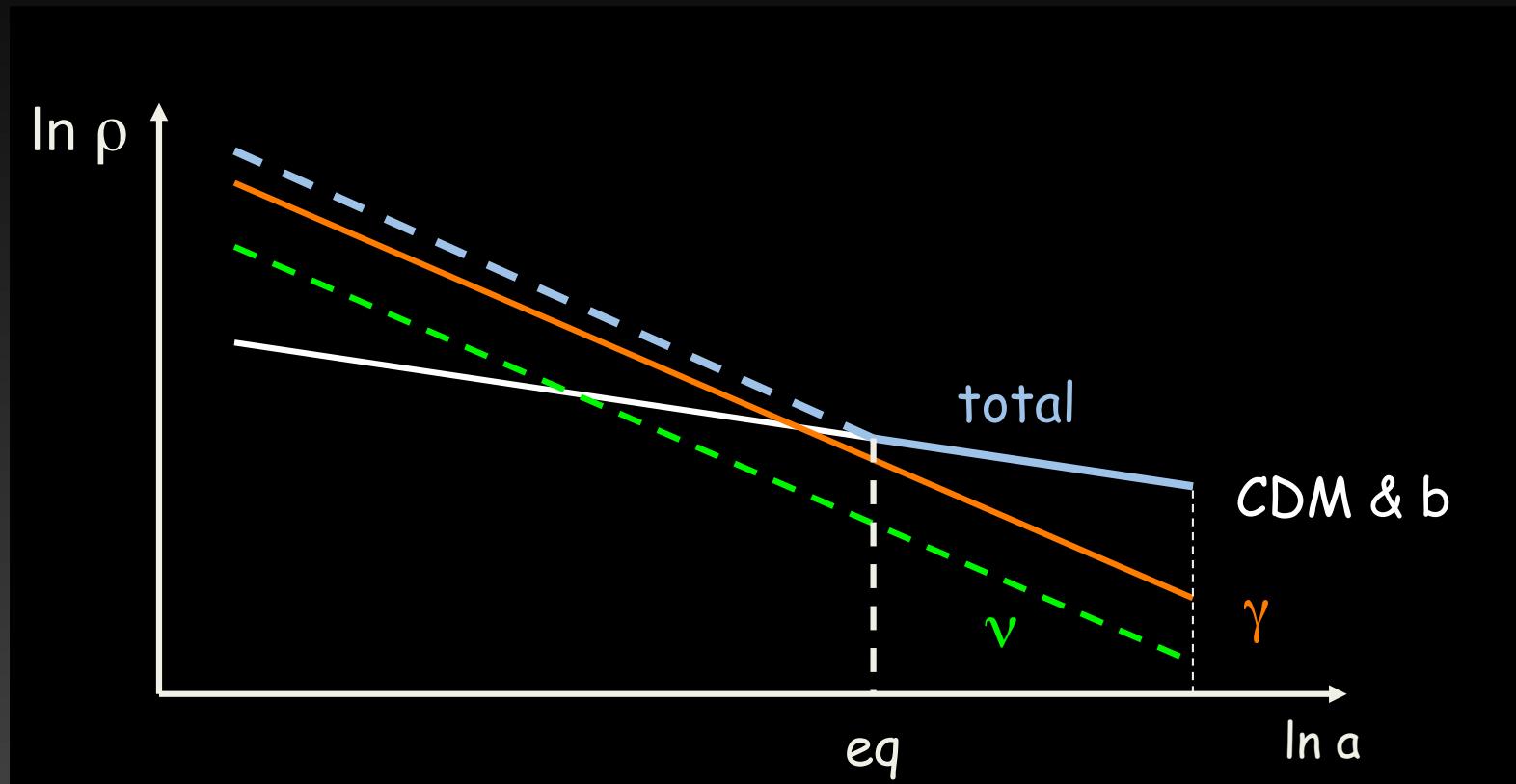
neutrinos

and

exotic dark matter

candidates

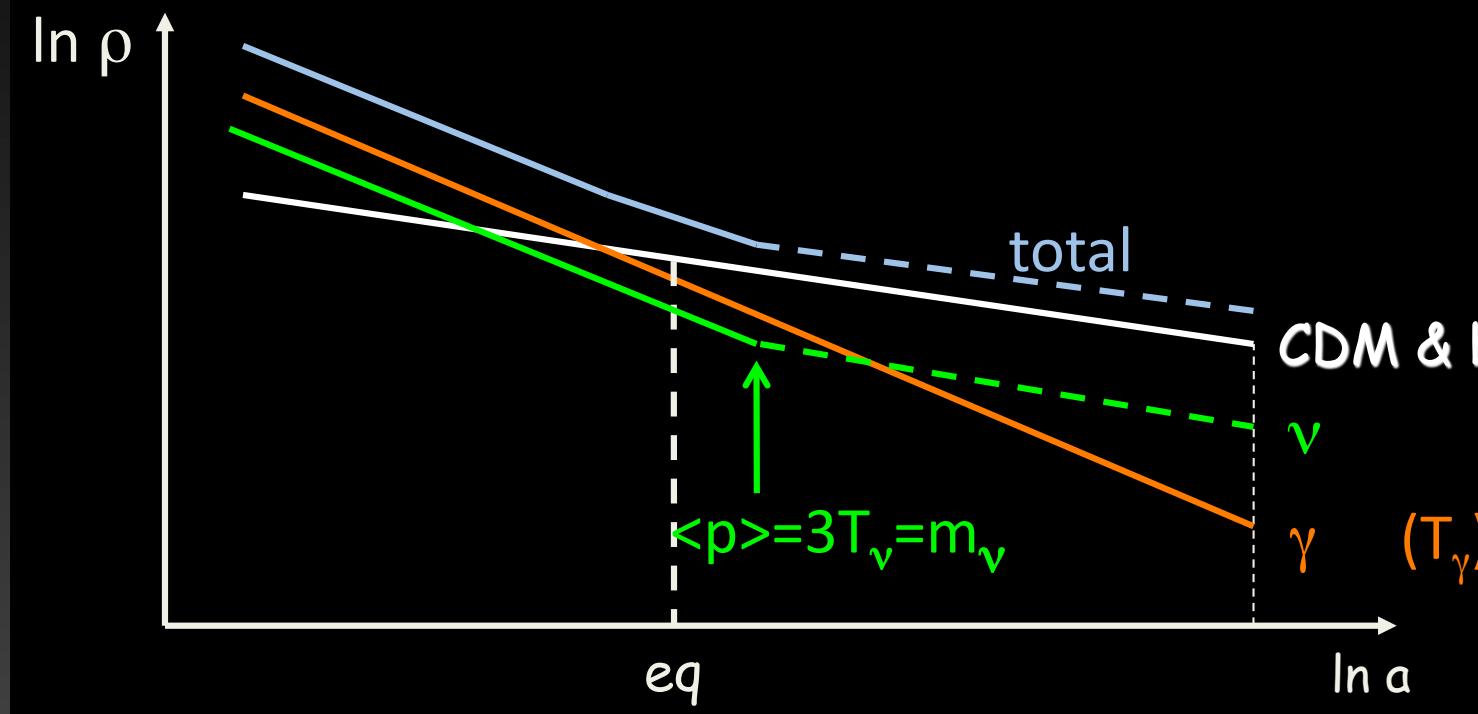
Density evolution for massless neutrinos



$$\rho_\nu \sim T_\nu^4 \quad \text{with} \quad T_\nu = (4/11)^{1/3} T_\gamma$$

Density evolution for massive neutrinos

oscillations: { $m_1 > 0.05\text{eV}$, $m_2 > 0.01\text{eV}$, $m_3?$ } while $T_\nu^0 \sim 10^{-3}\text{eV}$



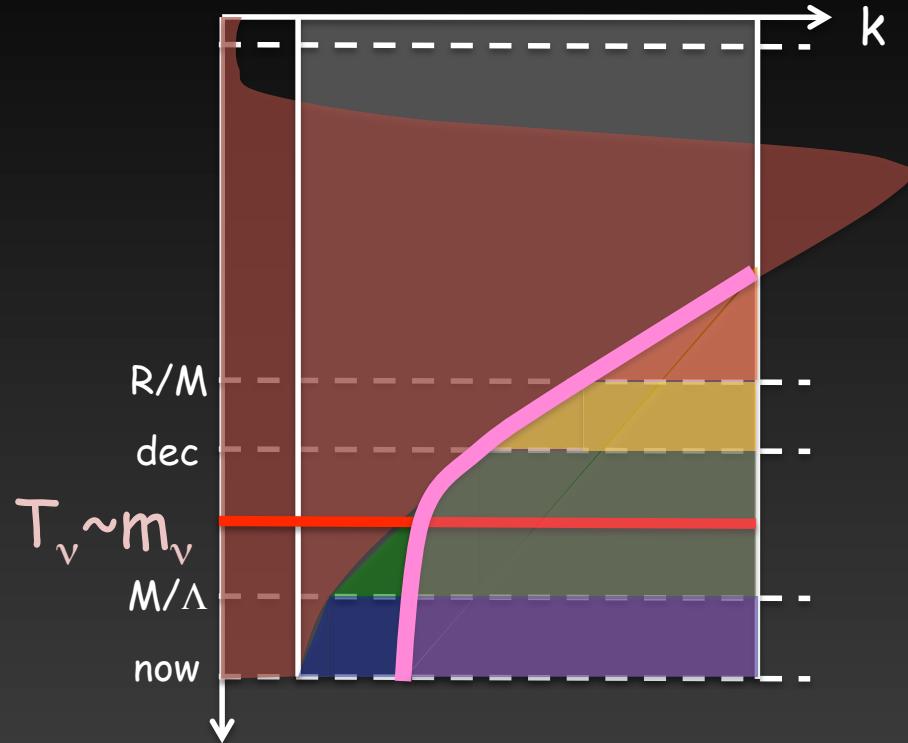
$\rho_\nu = m_\nu n_\nu$: current density given by $(\sum_i m_{\nu i})$

Neutrinos free-streaming horizon

$$d_{\nu} = a(t) \int^t \frac{\langle v \rangle dt'}{a(t')}$$

- Relativistic regime : $\langle v \rangle = 1$
- Non-relativistic regime : $\langle v \rangle = \langle p \rangle / m \sim 3 T_\nu / m \sim 1/a$
- neutrinos do not cluster below this scale

Neutrinos free-streaming horizon



Below free-streaming horizon, neutrinos :

- do not cluster
- participate to background expansion

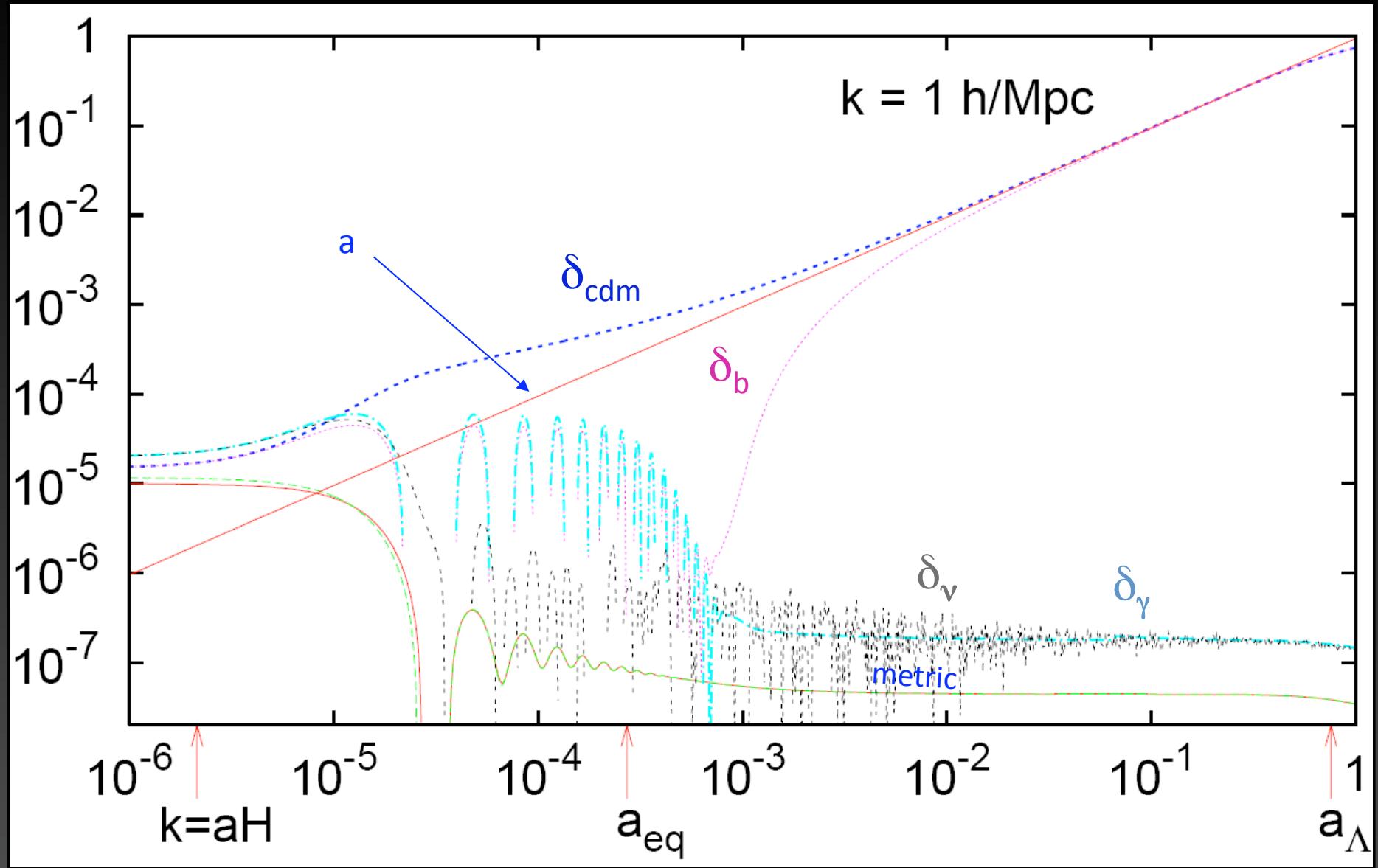


Growth rate of δ_m modified
(like for Λ /DE domination: break balance gravity / expansion)

$$\delta_m \sim a^{1-3/5f_\nu} \text{ with } f_\nu = \Omega_\nu/\Omega_m$$

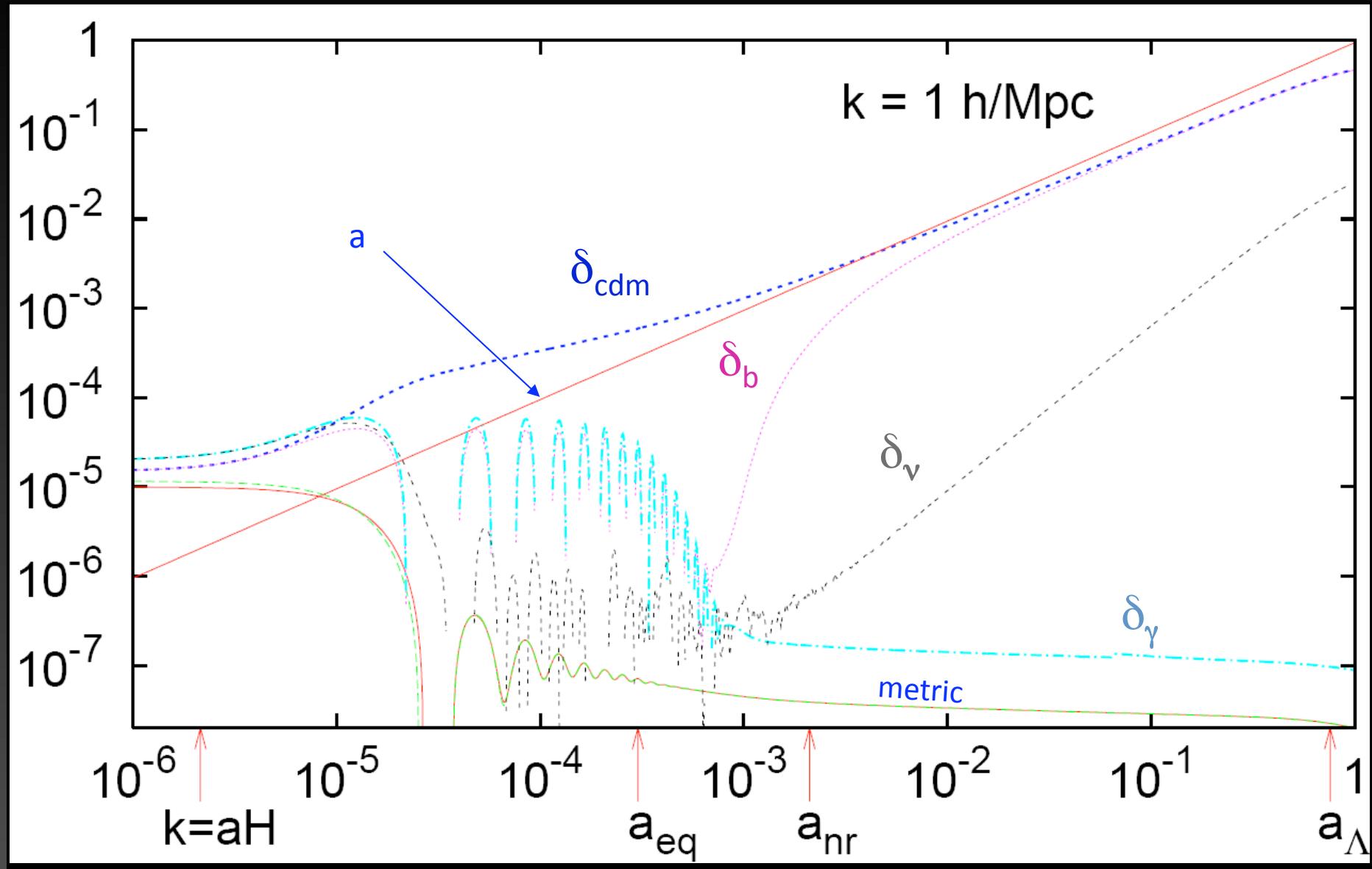
Free-streaming and structure formation

J.L. & S. Pastor, Physics Reports [astro-ph/0603494]



Free-streaming and structure formation

J.L. & S. Pastor, Physics Reports [astro-ph/0603494]

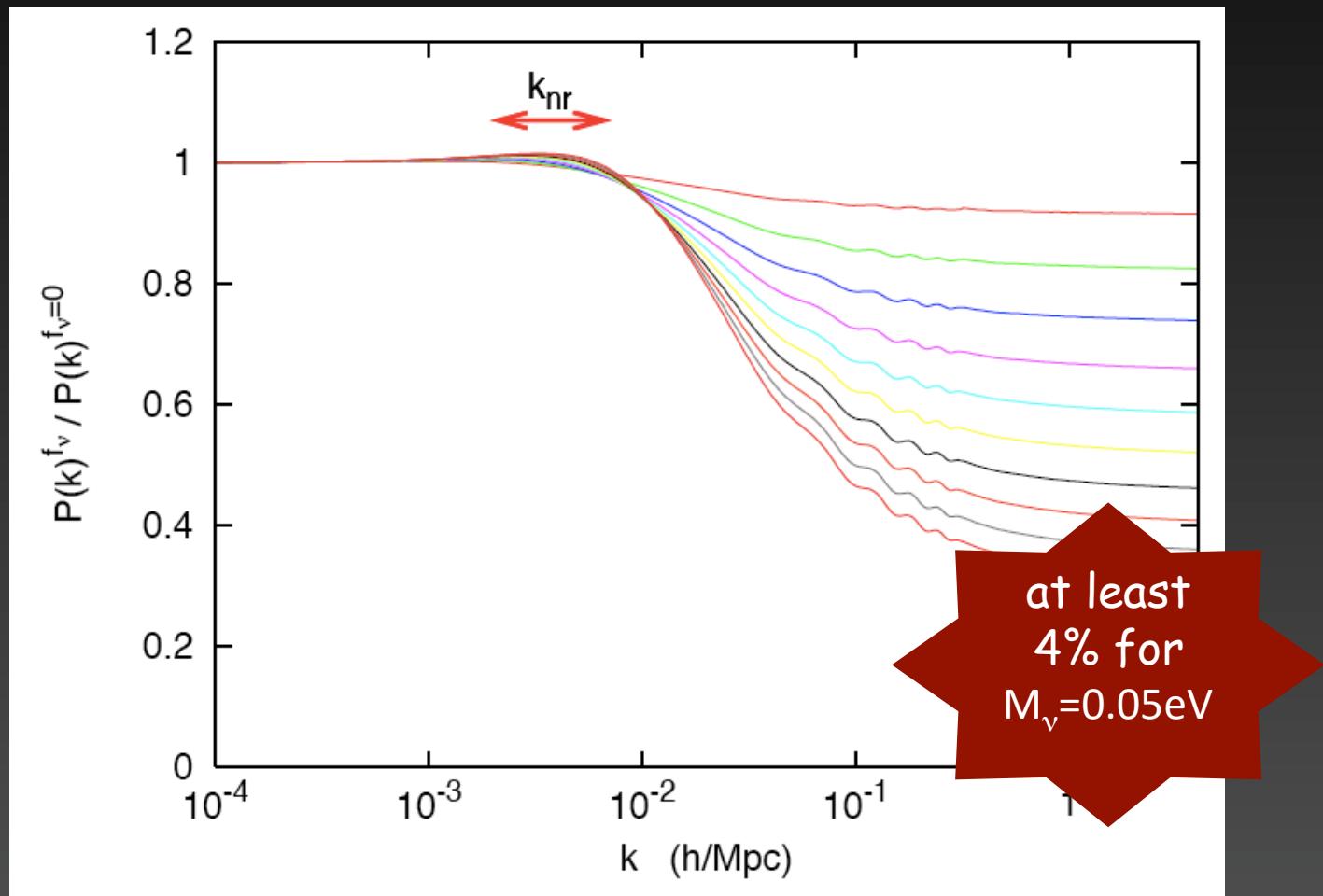


Effect of neutrino masses on (linear) structure formation

ν mass effect
at fixed z ,
with M_ν varying

OR

at fixed M_ν ,
with z varying



Limits on neutrino mass

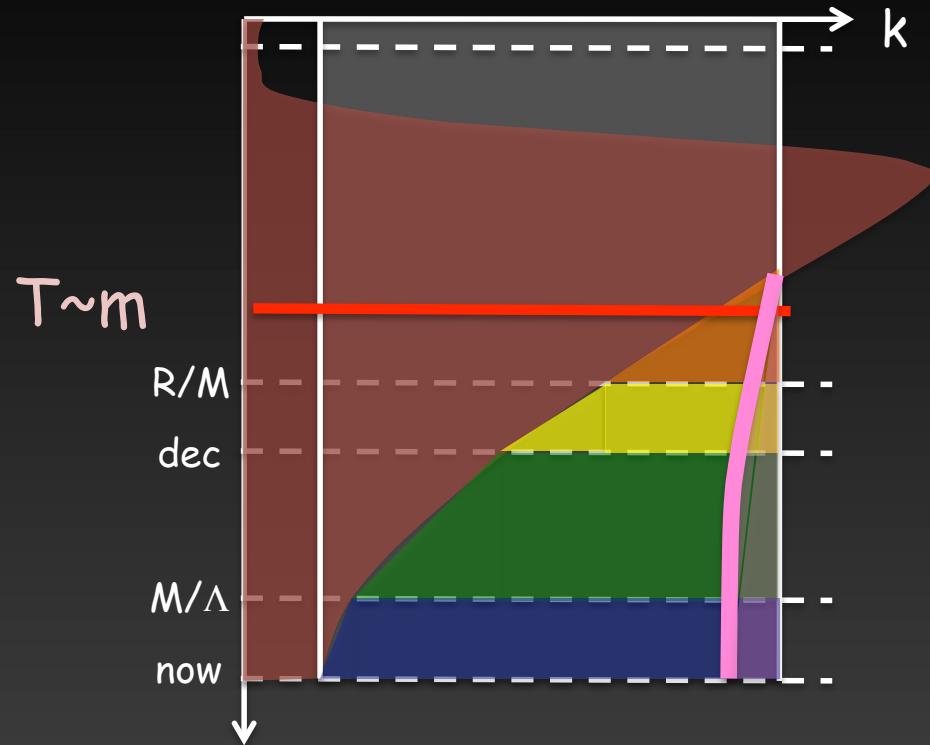
- WMAP+galaxy redshift surveys : $M_\nu < 0.3 \text{ eV}$ at 95%
(Lahav et al., Reid et al.)
- < 5 years: Planck + SDSSIII, DES : $\sigma(M_\nu) \sim 0.05 \text{ eV}$:
can exclude inverted hierarchy
- 5-15 years: Planck + LSST, Euclid, ... : $\sigma(M_\nu) \sim 0.01-0.02 \text{ eV}$:
ensures detection
- 15 -20 years: 21 cm surveys: individual masses

Free-streaming horizon for main DM component

$$d_v = a(t) \int^t \frac{\langle v \rangle dt'}{a(t')}$$

- CDM : tiny $\langle v \rangle$, free-streaming length \ll galaxy size
- Warm Dark Matter candidates (e.g. axions, sterile neutrinos with keV mass) : length might coincide with galaxy scales

Neutrinos free-streaming horizon



Below free-streaming horizon, break in matter power spectrum :

Attempts to detect such feature give negative results so far

