Curing negativity of quarkonium NLO cross-sections with High-Energy Factorization

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Outline

1. High-Energy Factorization ⇒ resummation of $\ln 1/z$
2. Reproducing NLO results at $z \to 0$ from HEF
3. Matching HEF and NLO CPM calculations
Reminder: Müller-Navelet dijet production ($p_T$ of both jets is fixed):

$$f_g(x_1, \mu_F) \rightarrow Y \rightarrow \ldots \ldots \ldots \ldots \ldots \rightarrow f_g(x_2, \mu_F)$$

Hard-scattering coefficient $C$ contains higher-order corrections $\propto (\alpha_s Y)^n$ (LLA) or $\alpha_s (\alpha_s Y)^n$ (NLLA), which can be resummed at leading power w.r.t. $e^{-Y}$ using Balitsky-Fadin-Kuraev-Lipatov (BFKL)-formalism.
High-Energy factorization in a nutshell

High-Energy Factorization [Collins, Ellis, 91'; Catani, Ciafaloni, Hautmann, 91',94']:

\[
\begin{align*}
  P^+ + P^- & \to P^+ \bar{x}_1 + P^- \bar{x}_2 + q_T^+ + q_T^- \\
  f_g(\bar{x}_1, \mu_F) & \to P_+ x_1 \\
  f_g(\bar{x}_2, \mu_F) & \to P_- x_2
\end{align*}
\]

Using the same formalism one can resum corrections to \( C \) enhanced by

\[
Y_\pm = \ln \left( \frac{\mu_Y}{|q_T^\pm|} \frac{1 - z_\pm}{z_\pm} \right) \simeq \ln \frac{\mu_Y}{|q_T^\pm|} + \ln \frac{1}{z_\pm}, \text{ in LP w.r.t. } \frac{q_T^\pm}{\mu_Y} \frac{z_\pm}{1 - z_\pm}
\]

in inclusive observables (e.g. inclusive quarkonium production). Here

\[
z_+ = \frac{p_+}{P_+ \bar{x}_1}, \quad z_- = \frac{p_-}{P_- \bar{x}_2} \quad \text{and} \quad \mu_Y = p_+ e^{-y_H} = p_- e^{y_H},
\]

e.g. \( \mu_Y^2 = m_H^2 + p_T^2 \).
High-Energy factorization in a nutshell

High-Energy Factorization [Collins, Ellis, 91'; Catani, Ciafaloni, Hautmann, 91',94']:

\[
P_+ + P_− + \bar{p}_1 + \bar{p}_2 + q_T^+ + q_T^- - H(q_T^+, q_T^-) p_+ , p_− \]

\[
f_g(\bar{x}_1, \mu_F) \quad \text{\(\otimes\)} \quad f_g(\bar{x}_2, \mu_F)
\]

Hard-scattering coefficient is re-factorized, *unintegrated* PDF is introduced:

\[
\Phi_g(x, q_T, \mu_F, \mu_R) = f_g \left( \frac{x}{z}, \mu_F \right) \otimes C(z, q_T, \mu_F, \mu_R).
\]

- **Collinear divergences** from additional emissions are subtracted inside UPDF.
- New coefficient function \(H\) depends on \(x_{1,2}\) as well as \(q_T^\pm\) (\(k_T\)-factorization).
- Factorization with single type of factors \(C\) and \(H\) is proven at LL and NLL approximation [Fadin et.al., early 2000s], and known to be violated at \(N^2\)LL. Factorization with several types of \(C\) and \(H\) should be introduced then.
LLA evolution w.r.t. $\ln 1/z$

In the LL($\ln 1/z$)-approximation, the $Y = \ln 1/z$-evolution equation for *collinearly un-subtracted $\tilde{C}$-factor* has the form:

\[
\tilde{C}(x, q_T) = \delta(1-z)\delta(q_T^2) + \hat{\alpha}_s \int_x^1 \frac{dz}{z} \int d^{2-2\epsilon}k_T K(k_T^2, q_T^2) \tilde{C}
\left(\frac{x}{z}, q_T - k_T\right)
\]

with $\hat{\alpha}_s = \alpha_s C_A/\pi$ and

\[
K(k_T^2, p_T^2) = \delta^{(2-2\epsilon)}(k_T) \left(\frac{p_T^2}{\epsilon}\right)^{-\epsilon} \left(\frac{4\pi}{\epsilon}\right)^\epsilon \Gamma(1+\epsilon) \Gamma^2(1-\epsilon) + \frac{1}{\pi(2\pi)^{-2\epsilon} k_T^2}.
\]

It is convenient to go from $(z, q_T)$-space to $(N, x_T)$-space:

\[
\tilde{C}(N, x_T) = \int d^{2-2\epsilon}q_T \ e^{ix_T q_T} \int_0^1 dx \ x^{N-1} \tilde{C}(x, q_T),
\]

because:

- **Mellin convolutions over $z$ turn into products:** $\int \frac{dz}{z} \to \frac{1}{N}$

- **Large logs map to poles at $N = 0$:**

  \[
  \alpha_s^{k+1} \ln^k z \to \frac{\alpha_s^{k+1}}{N^{k+1}}
  \]

- **All *collinear divergences* are contained inside $C$ in $x_T$-space.
Collinear divergences

Exact (up to terms $O(\epsilon)$) solution for $\tilde{C}$ can be obtained \cite{Catani, Hautmann, 94}. It contains *collinear divergences*, which can be removed (absorbed into PDFs) in the $\overline{MS}$-scheme to all orders in $\alpha_s$:

$$Z_{\text{coll.}}^{-1} = \exp \left[ -\frac{1}{\epsilon} \int_0^{\hat{\alpha}_s S_\epsilon \mu_F^{-2\epsilon}} \frac{d\alpha}{\alpha} \gamma_{gg}(\alpha, N) \right], \quad S_\epsilon = \exp \left[ \epsilon (\ln 4\pi - \gamma_E) \right],$$

$$\tilde{C}(N, x_T) = Z_{\text{coll.}}^{-1} \mathcal{C}(N, x_T, \mu_F)$$
Exact LL solution

In \((N, q_T)\)-space, subtracted \(C\), which resums all terms \(\propto (\hat{\alpha}_s/N)^n\) has the form:

\[
C(N, q_T, \mu_F) = R(\gamma_{gg}(N, \alpha_s)) \frac{\gamma_{gg}(N, \alpha_s)}{q_T^2} \left( \frac{q_T^2}{\mu_F^2} \right) \gamma_{gg}(N, \alpha_s),
\]

where \(\gamma_{gg}(N, \alpha_s)\) is the solution of [Jaroszewicz, 82']:

\[
\frac{\hat{\alpha}_s}{N} \chi(\gamma_{gg}(N, \alpha_s)) = 1, \text{ with } \chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma),
\]

where \(\psi(\gamma) = d\ln \Gamma(\gamma)/d\gamma\) – Euler’s \(\psi\)-function. The first few terms:

\[
\gamma_{gg}(N, \alpha_s) = \frac{\hat{\alpha}_s}{N} + 2\zeta(3) \frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5) \frac{\hat{\alpha}_s^6}{N^6} + \ldots
\]

The function \(R(\gamma)\) is

\[
R(\gamma_{gg}(N, \alpha_s)) = 1 + O(\alpha_s^3).
\]
Doubly-logarithmic (Collins-Ellis-) Blümlein UPDF

Taking the LO result for $\gamma_{gg}(N, \alpha_s) \to \gamma_N = \frac{\hat{\alpha}_s}{N}$ we obtain:

$$C_{DL}(N, q_T, \mu_F) = \gamma_N \left( \frac{\frac{q_T^2}{\mu_F^2}}{q_T^2} \right)^{\gamma_N},$$

which resums $\left( \frac{\hat{\alpha}_s}{N} \ln \frac{q_T^2}{\mu_F^2} \right)^{\hat{\alpha}_s} \ln \ln n = \frac{\hat{\alpha}_s}{N} \ln \left( \frac{q_T^2}{\mu_F^2} \right) \ln^{-1} \left( \frac{1}{z} \right)$.

In $(z, q_T)$-space it is:

$$C_{DL}(z, q_T, \mu_F) = \frac{\hat{\alpha}_s}{q_T^2} \left\{ \begin{array}{ll} \frac{J_0}{I_0} \left( 2 \sqrt{\hat{\alpha}_s \ln \frac{\mu_F^2}{q_T^2} \ln \frac{1}{z}} \right), & |q_T| < \mu_F, \\ J_0 \left( 2 \sqrt{\hat{\alpha}_s \ln \frac{q_T^2}{\mu_F^2} \ln \frac{1}{z}} \right), & |q_T| > \mu_F, \end{array} \right.$$
Doubly-logarithmic (Collins-Ellis-)Blümlein UPDF

Relation with PDF:

\[
\Phi_{g}^{(DL)}(x, q_T^2, \mu_F, \mu_R) = \int_x^{1} \frac{dz}{z} C_{DL}(z, q_T^2, \mu_F, \mu_R) \frac{x}{z} f_g \left( \frac{x}{z}, \mu_F \right),
\]

integration property (hence the name – UPDF):

\[
\mu_F^2 \int_0^{\mu_F^2} dq_T^2 \Phi_{g}^{(DL)}(x, q_T^2, \mu_F, \mu_R) = x f_g(x, \mu_F^2).
\]