Curing negativity of quarkonium NLO cross-sections with High-Energy Factorization

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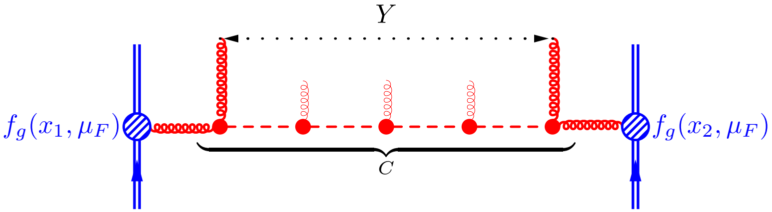
Outline

1. High-Energy Factorization $\Rightarrow$ resummation of $\ln 1/z$
2. Reproducing NLO results at $z \to 0$ from HEF
3. Matching HEF and NLO CPM calculations

High-Energy factorization in a nutshell

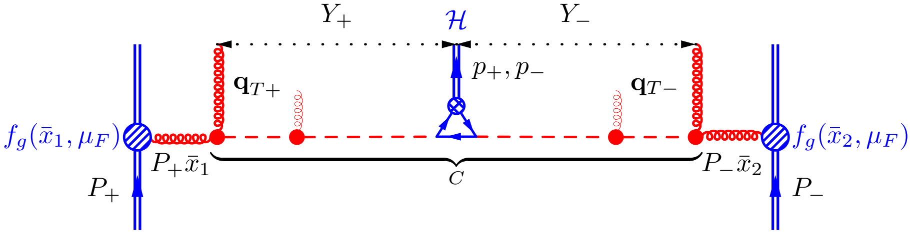
Reminder: Müller-Navelet dijet production (p_T of both jets is fixed):



Hard-scattering coefficient C contains higher-order corrections $\propto (\alpha_s Y)^n$ (LLA) or $\alpha_s(\alpha_s Y)^n$ (NLLA), which can be resummed at leading power w.r.t. e^{-Y} using Balitsky-Fadin-Kuraev-Lipatov (BFKL)-formalism.

High-Energy factorization in a nutshell

High-Energy Factorization [Collins, Ellis, 91'; Catani, Ciafaloni, Hautmann, 91',94']:



Using the same formalism one can resum corrections to C enhanced by

$$Y_\pm = \ln\left(\frac{\mu_Y}{|\mathbf{q}_{T\pm}|}\frac{1-z_\pm}{z_\pm}\right) \simeq \cancel{\ln\frac{\mu_Y}{|\mathbf{q}_{T\pm}|}} + \ln\frac{1}{z_\pm}, \text{ in LP w.r.t. } \cancel{\frac{|\mathbf{q}_{T\pm}|}{\mu_Y}}\,\frac{z_\pm}{1-z_\pm}$$

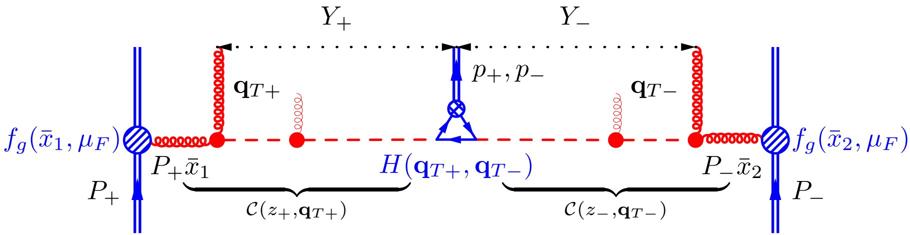
in inclusive observables (e.g. inclusive quarkonium production). Here

$$z_+ = \frac{p_+}{P_+\bar{x}_1},\ z_- = \frac{p_-}{P_-\bar{x}_2} \text{ and } \mu_Y = p_+e^{-y_{\mathcal{H}}} = p_-e^{y_{\mathcal{H}}},$$

e.g. $\mu_Y^2 = m_{\mathcal{H}}^2 + \mathbf{p}_T^2$.

High-Energy factorization in a nutshell

High-Energy Factorization [Collins, Ellis, 91'; Catani, Ciafaloni, Hautmann, 91',94']:



Hard-scattering coefficient is re-factorized, *unintegrated PDF* is introduced:

$$\Phi_g(x, \mathbf{q}_T, \mu_F, \mu_R) = f_g\left(\frac{x}{z}, \mu_F\right) \otimes \mathcal{C}(z, \mathbf{q}_T, \mu_F, \mu_R).$$

- *Collinear divergences* from additional emissions are subtracted inside UPDF.
- New coefficient function H depends on $x_{1,2}$ as well as $\mathbf{q}_{T\pm}$ (k_T-*factorization*).
- Factorization with single type of factors $\mathcal{C}$ and H is proven at LL and NLL approximation [Fadin *et.al.*, early 2000s], and known to be violated at N^{2}LL. Factorization with several types of $\mathcal{C}$ and H should be introduced then.

LLA evolution w.r.t. $\ln 1/z$

In the LL($\ln 1/z$)-approximation, the $Y = \ln 1/z$-evolution equation for *collinearly un-subtracted* $\tilde{\mathcal{C}}$-factor has the form:

$$\tilde{\mathcal{C}}(x, \mathbf{q}_T) = \delta(1-z)\delta(\mathbf{q}_T^2) + \hat{\alpha}_s \int\limits_x^1 \frac{dz}{z} \int d^{2-2\epsilon}\mathbf{k}_T K(\mathbf{k}_T^2, \mathbf{q}_T^2) \tilde{\mathcal{C}}\left(\frac{x}{z}, \mathbf{q}_T - \mathbf{k}_T\right)$$

with $\hat{\alpha}_s = \alpha_s C_A/\pi$ and

$$K(\mathbf{k}_T^2, \mathbf{p}_T^2) = \delta^{(2-2\epsilon)}(\mathbf{k}_T) \frac{(\mathbf{p}_T^2)^{-\epsilon}}{\epsilon} \frac{(4\pi)^\epsilon \Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} + \frac{1}{\pi(2\pi)^{-2\epsilon}\mathbf{k}_T^2}.$$

It is convenient to go from $(z, \mathbf{q}_T)$-space to $(N, \mathbf{x}_T)$-space:

$$\tilde{\mathcal{C}}(N, \mathbf{x}_T) = \int d^{2-2\epsilon}\mathbf{q}_T \; e^{i\mathbf{x}_T\mathbf{q}_T} \int\limits_0^1 dx \; x^{N-1} \; \tilde{\mathcal{C}}(x, \mathbf{q}_T),$$

because:

- Mellin convolutions over z turn into products: $\int \frac{dz}{z} \to \frac{1}{N}$
- Large logs map to poles at $N = 0$: $\boxed{\alpha_s^{k+1} \ln^k \frac{1}{z} \to \frac{\alpha_s^{k+1}}{N^{k+1}}}$
- All *collinear divergences* are contained inside $\mathcal{C}$ in $\mathbf{x}_T$-space.

Collinear divergences

Exact (up to terms $O(\epsilon)$) solution for $\tilde{\mathcal{C}}$ can be obtained [Catani, Hautmann, 94'] . It contains *collinear divergences*, which can be removed (absorbed into PDFs) in the $\overline{MS}$-scheme to all orders in α_s:

$$Z^{-1}_{\text{coll.}} = \exp\left[-\frac{1}{\epsilon}\int\limits_0^{\hat{\alpha}_s S_\epsilon \mu_F^{-2\epsilon}} \frac{d\alpha}{\alpha}\gamma_{gg}(\alpha, N)\right], \quad S_\epsilon = \exp\left[\epsilon\left(\ln 4\pi - \gamma_E\right)\right],$$

$$\tilde{\mathcal{C}}(N, \mathbf{x}_T) = Z^{-1}_{\text{coll.}}\mathcal{C}(N, \mathbf{x}_T, \mu_F)$$

Exact LL solution

In $(N, \mathbf{q}_T)$-space, subtracted $\mathcal{C}$, which resums all terms $\propto (\hat{\alpha}_s/N)^n$ has the form:

$$\mathcal{C}(N, \mathbf{q}_T, \mu_F) = R(\gamma_{gg}(N,\alpha_s))\frac{\gamma_{gg}(N,\alpha_s)}{\mathbf{q}_T^2}\left(\frac{\mathbf{q}_T^2}{\mu_F^2}\right)^{\gamma_{gg}(N,\alpha_s)},$$

where $\gamma_{gg}(N,\alpha_s)$ is the solution of [Jaroszewicz, 82']:

$$\frac{\hat{\alpha}_s}{N}\chi(\gamma_{gg}(N,\alpha_s)) = 1, \text{ with } \chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma),$$

where $\psi(\gamma) = d\ln\Gamma(\gamma)/d\gamma$ – Euler's ψ-function. The first few terms:

$$\gamma_{gg}(N,\alpha_s) = \frac{\hat{\alpha}_s}{N} + 2\zeta(3)\frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5)\frac{\hat{\alpha}_s^6}{N^6} + \dots$$

The function $R(\gamma)$ is

$$R(\gamma_{gg}(N,\alpha_s)) = 1 + O(\alpha_s^3).$$

Doubly-logarithmic (Collins-Ellis-)Blümlein UPDF

Taking the LO result for $\gamma_{gg}(N,\alpha_s) \to \boxed{\gamma_N = \frac{\hat{\alpha}_s}{N}}$ we obtain:

$$\boxed{\mathcal{C}_{\rm DL}(N,\mathbf{q}_T,\mu_F) = \frac{\gamma_N}{\mathbf{q}_T^2}\left(\frac{\mathbf{q}_T^2}{\mu_F^2}\right)^{\gamma_N},}$$

which resums $\left(\frac{\hat{\alpha}_s}{N}\ln\frac{\mathbf{q}_T^2}{\mu_F^2}\right)^n \leftrightarrow \hat{\alpha}_s^n \ln^n\left(\frac{\mathbf{q}_T^2}{\mu_F^2}\right)\ln^{n-1}\left(\frac{1}{z}\right)$.

In $(z,\mathbf{q}_T)$-space it is:

$$\mathcal{C}_{\rm DL}(z,\mathbf{q}_T,\mu_F) = \frac{\hat{\alpha}_s}{\mathbf{q}_T^2}\begin{cases} J_0\left(2\sqrt{\hat{\alpha}_s \ln\frac{\mu_F^2}{\mathbf{q}_T^2}\ln\frac{1}{z}}\right), & |\mathbf{q}_T| < \mu_F, \\ I_0\left(2\sqrt{\hat{\alpha}_s \ln\frac{\mathbf{q}_T^2}{\mu_F^2}\ln\frac{1}{z}}\right), & |\mathbf{q}_T| > \mu_F, \end{cases}$$

where J_0/I_0 is the Bessel function of the first/second kind.

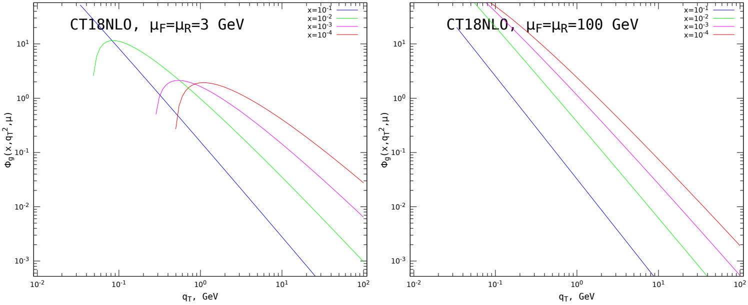
Doubly-logarithmic (Collins-Ellis-)Blümlein UPDF

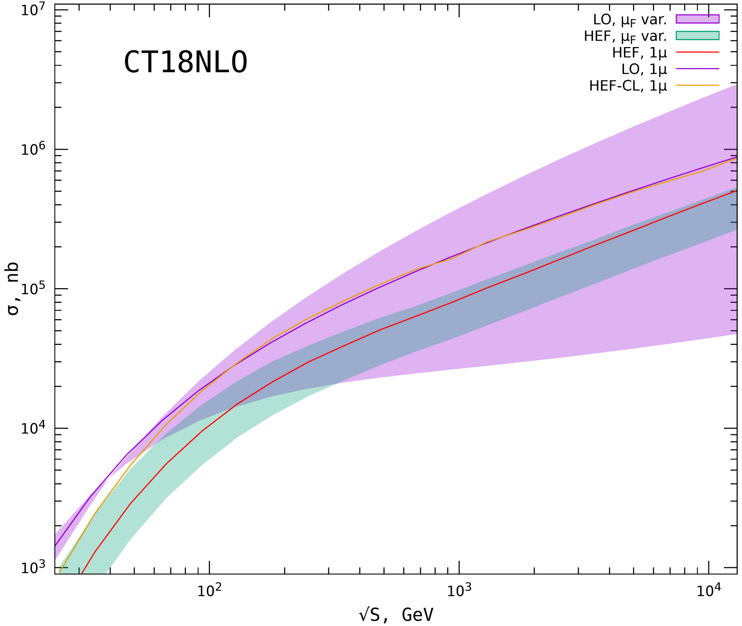
Relation wit PDF:

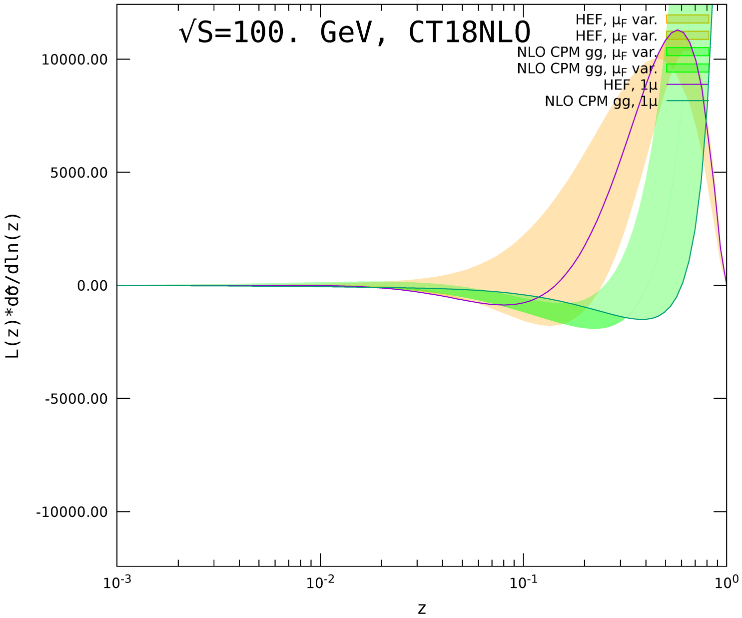
$$\Phi_g^{(\mathrm{DL})}(x, \mathbf{q}_T^2, \mu_F, \mu_R) = \int\limits_x^1 \frac{dz}{z} \mathcal{C}_{\mathrm{DL}}(z, \mathbf{q}_T^2, \mu_F, \mu_R) \frac{x}{z} f_g\left(\frac{x}{z}, \mu_F\right),$$

integration property (*hence the name – UPDF*):

$$\int\limits_0^{\mu_F^2} d\mathbf{q}_T^2 \; \Phi_g^{(\mathrm{DL})}(x, \mathbf{q}_T^2, \mu_F, \mu_R) = x f_g(x, \mu_F^2).$$




CT18NLO
LO, μF var.
HEF, μF var.
HEF, 1μ
LO, 1μ
HEF-CL, 1μ
σ, nb
√S, GeV
10^7
10^6
10^5
10^4
10^3
10^2
10^3
10^4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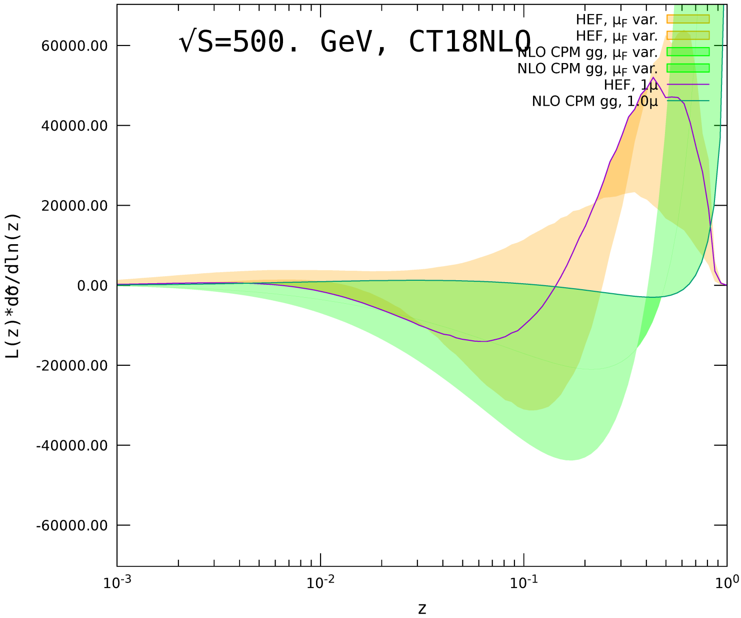
HEF, μF var.
HEF, μF var.
NLO CPM gg, μF var.
NLO CPM gg, μF var.
HEF, 1μ
NLO CPM gg, 1μ
10000.00
5000.00
0.00
-5000.00
-10000.00
L(z)*dσ/dln(z)
10-3
10-2
10-1
100
z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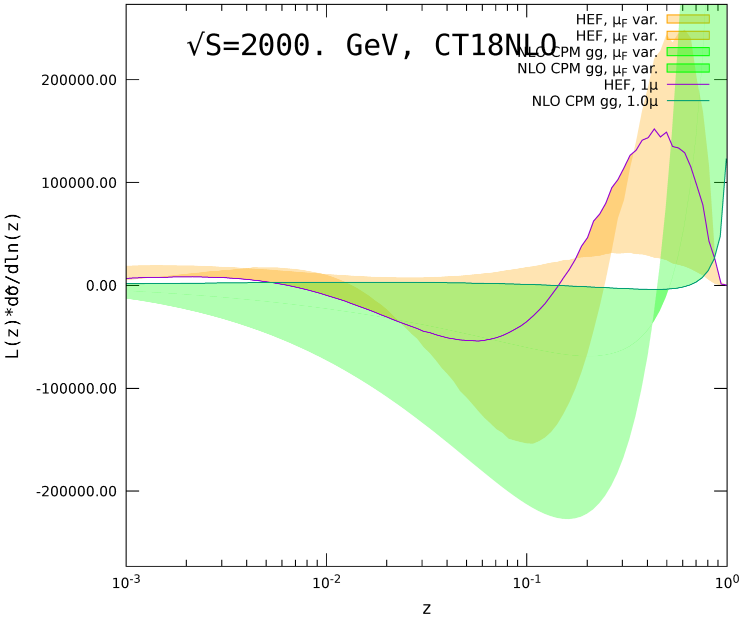
HEF, μ_F var.
HEF, μ_F var.
NLO CPM gg, μ_F var.
NLO CPM gg, μ_F var.
HEF, 1μ
NLO CPM gg, 1.0μ
60000.00
40000.00
20000.00
0.00
-20000.00
-40000.00
-60000.00
L(z)*dσ/dln(z)
10^{-3}
10^{-2}
10^{-1}
10^{0}
z


√S=2000. GeV, CT18NLO
HEF, μF var.
HEF, μF var.
NLO CPM gg, μF var.
NLO CPM gg, μF var.
HEF, 1μ
NLO CPM gg, 1.0μ
L(z)*dσ/dln(z)
z
200000.00
100000.00
0.00
-100000.00
-200000.00
10-3
10-2
10-1
100