

Four Applications of SFT

1. RR flux
2. D-instantons
3. Time-dependent background
4. open / closed duality

Standard paradigm for perturbative string

- worldsheet CFT w/ BRST sym,

$$T = Q_B \cdot b$$

↑
may not exist?

Consider: type II string in RR flux

RR deformation represented by
vertex operator $\mathcal{V}^{-\frac{1}{2}, -\frac{1}{2}}$ ↔ picture #

$$e^{-(S + \Delta S)}$$

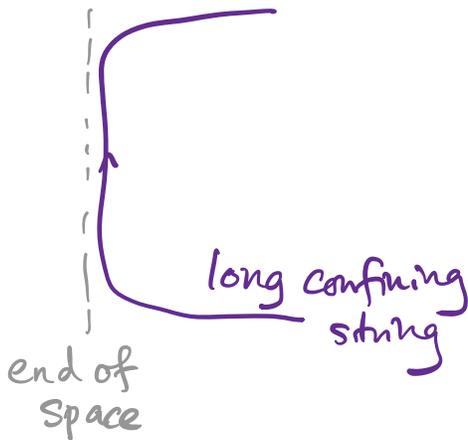
picture-raised
↓

$$\rightsquigarrow e^{-S} \sum_{n \geq 0} \frac{1}{(2n)!} \left(\int \nu^{-\frac{1}{2}, -\frac{1}{2}} \right)^n \left(\int \nu^{\frac{1}{2}, \frac{1}{2}} \right)^n$$

[Berenstein, Leigh '99]

"local up to BRST transf." ?

Note: RR flux needed in known constructions of holographic confining string bkgnd...



Is there a local worldsheet CFT for the flux string in 4D Yang-Mills @ large N?

A first-principle approach to RR flux based on NSR closed super-SFT

E.O.M.

$$Q_B \Psi + \sum_{n \geq 2} \frac{1}{n!} G[\Psi^{\otimes n}] = 0$$

[as reviewed in de Lacroix et al.
1703.06410]

RR flux bkgnd

= solution Ψ_0 of the form

$$\Psi_0 = \mu V^{-\frac{1}{2}, -\frac{1}{2}} + \mathcal{O}(\mu^2)$$

↑ RR deformation parameter

Expand $\Psi = \Psi_0 + \hat{\Psi}$

↑ fluctuation S.F.
linearize in $\hat{\Psi}$
to find string spectrum

$AdS_3 \times S^3 \times M_4$ order μ^2 ✓

[Collier, Cho, xY '18]

μ^4 & higher

$AdS_5 \times S^5$,

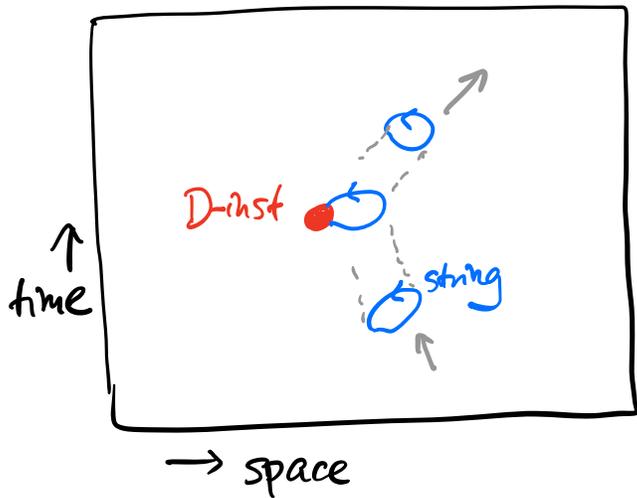
expansion in $\frac{1}{R}$

} work to do !

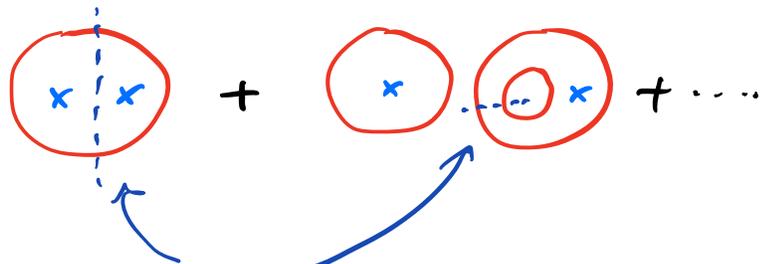
D-instantons

- worldsheet formalism can capture $\mathcal{O}(e^{-1/g_s})$ effects

[Polchinski '94,
Green, Gutperle '97]



worldsheet with boundaries

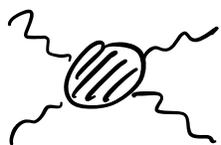


open string divergences
require (open+closed) SFT
to regularize

[Balthazar, Rodriguez, XY '19]

[Sen '19, '20]

Example: $2 \rightarrow 2$ graviton scattering in IIB superstring



$$A = \mathcal{A}_{\text{string tree}} \times M(s, t | \tau, \bar{\tau})$$

low E expansion

$$\begin{aligned}
 & stu \left(\frac{1}{stu} + f_0(\tau, \bar{\tau}) + H_2(s, t) \right) \\
 & + f_4(\tau, \bar{\tau}) (s^2 + t^2 + u^2) + f_6(\tau, \bar{\tau}) (s^3 + t^3 + u^3) \\
 & + f_8(\tau, \bar{\tau}) (s^4 + t^4 + u^4) + f_0(\tau, \bar{\tau}) H_8(s, t) + \dots
 \end{aligned}$$

$R^4 \checkmark$



$D^6 R^4 \checkmark$

$D^4 R^4 \checkmark$

$D^8 R^4 ?$



g_s -expansion

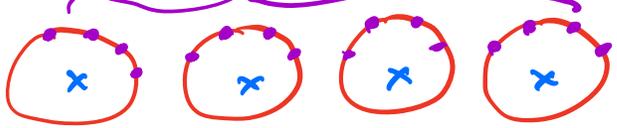
[Note: $\frac{\alpha'}{l_p^2} \sim \tau_2^{\frac{1}{2}}$]

$$\sum_{h \geq 0} M_h(\alpha's, \alpha't) \tau_2^{-2h} \quad \leftarrow \text{closed string loops}$$

$$+ e^{2\pi i \tau} \sum_{L \geq 0} M_L^{1\text{-inst}}(\alpha's, \alpha't) \tau_2^{-L} \quad \leftarrow \text{open string loops}$$

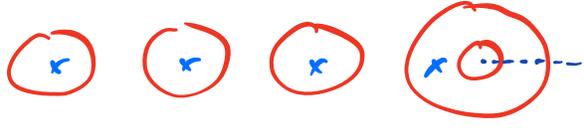
$$+ e^{-2\pi i \bar{\tau}} (\dots)$$

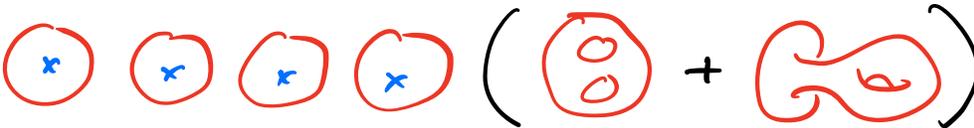
$$+ \dots$$

$$M_{L=0}^{1\text{-inst}} \sim \int d^{10}x \quad \overbrace{\text{S}^1 \text{ (bdry spin fields)}}^{\text{S}^1 \text{ (bdry spin fields)}}$$


$$\propto stu \quad (\text{only contributes to } \mathbb{R}^4)$$

$$M_{L=1}^{1\text{-inst}} \sim$$


$$+$$


$$+$$


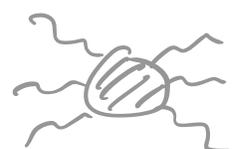
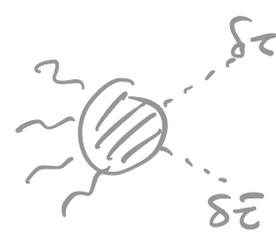
$$\propto stu \left(\alpha's \psi \left(1 - \frac{\alpha's}{4} \right) + (s \rightarrow t) + (s \rightarrow u) + A \right)$$

[Agmon, Bakthazar, Cho,
Rodriguez, XY, W.I.P.]

ambiguous in on-shell calc.
need SFT to fix.
($e^{2\pi i \tau} \tau_2^{-1} R^4$ coeff.)

* A shortcut: we can fix A using susy
[Green, Sethi '98, Wang, XY '14]

$$(\tau_2^2 \partial_\tau \partial_{\bar{\tau}} - \frac{3}{16}) f_0(\tau, \bar{\tau}) = 0$$

follows from susy ward id on 
and soft limit  $\Rightarrow \partial_\tau \partial_{\bar{\tau}}$ 

* Naive on-shell worldsheet calc. gets
wrong answer for A , incompatible w/ susy

* In SFT approach, susy ensured by

BV formalism

$$\Psi \rightarrow (\Phi, \Phi^\dagger)$$

↑
field

↑
anti-field

$$\int D\Phi \Big|_{\Phi^\dagger=0} e^{-S[\Phi, \Phi^\dagger]}$$

↑ BV gauge fixing

Susy: $\delta\Phi = \{\Psi, \delta_\lambda S\}_{BV} \Rightarrow S \text{ inv.}$

$$\Lambda = c\tilde{c} \left(\Lambda^\alpha e^{-\frac{\phi}{2}} S^\alpha e^{-2\hat{\phi}} \bar{\partial} \tilde{\xi} + (L \leftrightarrow R) \right)$$

- BV gauge condition $\bar{\Phi}^\dagger = 0$ (~~Siegel~~ Sen gauge)
not manifestly susy inv., but phys. obs. guaranteed to be susy-inv due to gauge-independence.

What about uncharged D-instantons?

e.g. D- \bar{D} instantons



open string "tachyon"

\leadsto singularity in moduli space

Is $\int_{\mathcal{M}_{D-\bar{D}}}$ defined?

[Wick rotⁿ of open SF contour?]

- $c=1$ string, contour prescription for $\int_{\mathcal{M}_{D-\text{inst}}}$ ✓

[Balthazar, Rodriguez, XY '19]

- IIB superstring, D- \bar{D} contribution

unclear beyond $D^6 R^4$ in momentum expansion

(\Rightarrow ...)

(\leftrightarrow how to sum up closed string pert. series?)

Time - dependent background

- worldsheet CFT $[\mu=0, 1, \dots, D-1]$

$$S = \int G_{\mu\nu}(x) \partial X^\mu \bar{\partial} X^\nu + \dots$$

usually defined by "wick rotation"

$$X^0 \rightarrow iX^D$$

\rightsquigarrow unitary "matter" CFT

for **static** target spacetime

($G_{\mu\nu}(x)$ independent of X^0)

- construct $\mathcal{H}_{\text{CFT}} \ni e^{ik \cdot X}$
- compute correlators,
- analytically continue k^μ to Lorentzian sig.

* Doesn't work for generic X^0 -dep. bkgnd.

- for good reasons?

- need choice of vacuum state

- a priori unclear how to define asymp. states / S-matrix.

(what are the phys. obs.?)

In SFT,

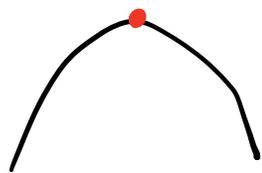
$$\Psi = \Psi_0 + \hat{\Psi}$$

↑
time-dependent
"bkgrd" solution

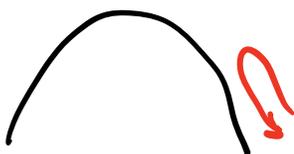
- Can do perturbation theory in $\hat{\Psi}$

* Consider (closed string) tachyon

"potential"



classical $T=0$ solution unstable,
does not approx. a quantum state
pert. theory ill-defined.



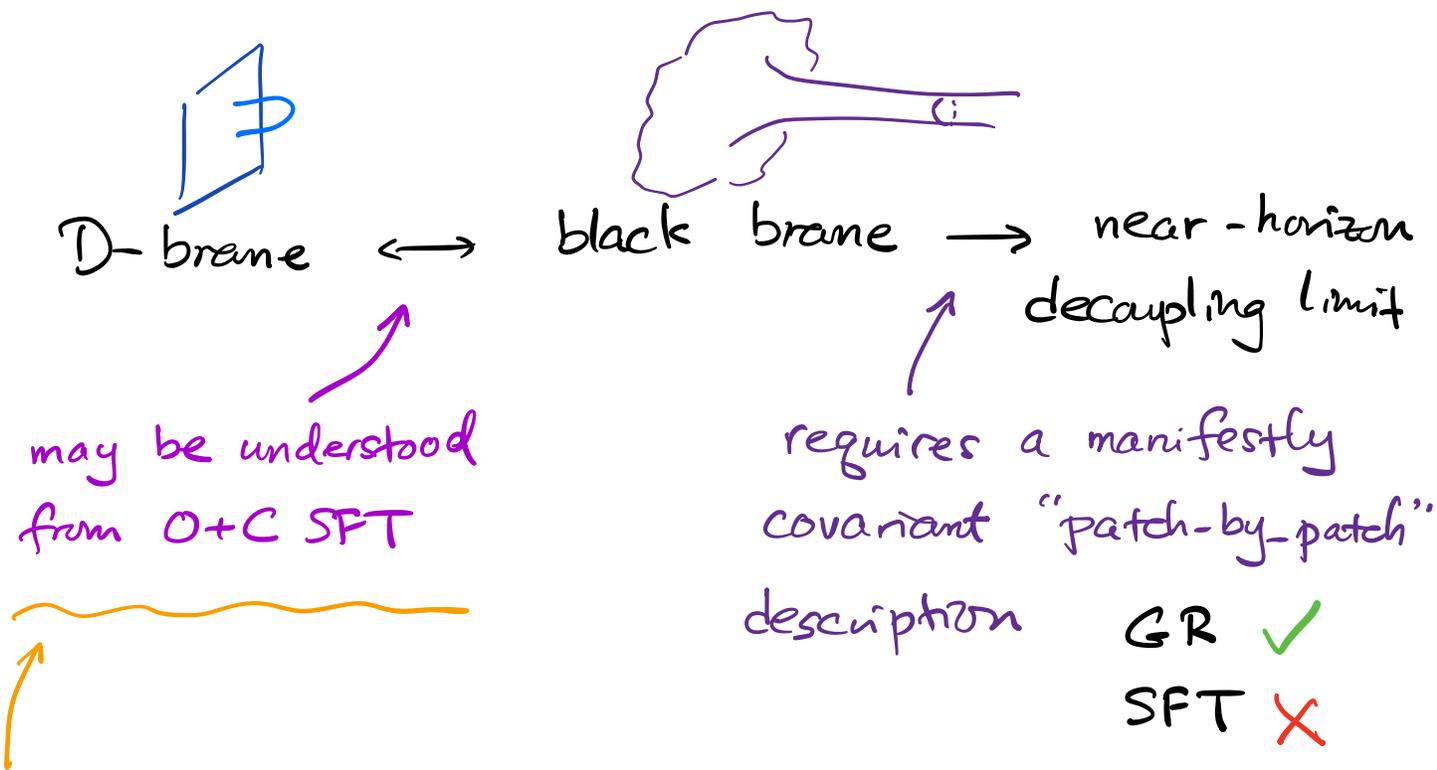
rolling tachyon, does approx.
a quantum state,
time-dependent pert. theory?

[Tachyon condensation]

[In contrast, tachyon-condensed vacuum may or may not exist...]

open/closed duality.

. The logic of AdS/CFT from decoupling

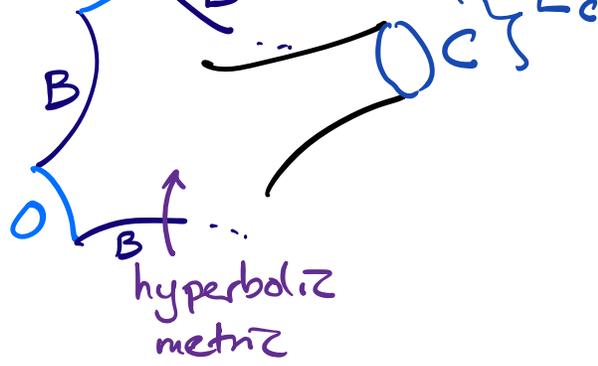


Cho '19: (following Costello & Zwiebach)

Hyperbolic vertices for O+C SFT

labeled by (L_o, L_c)

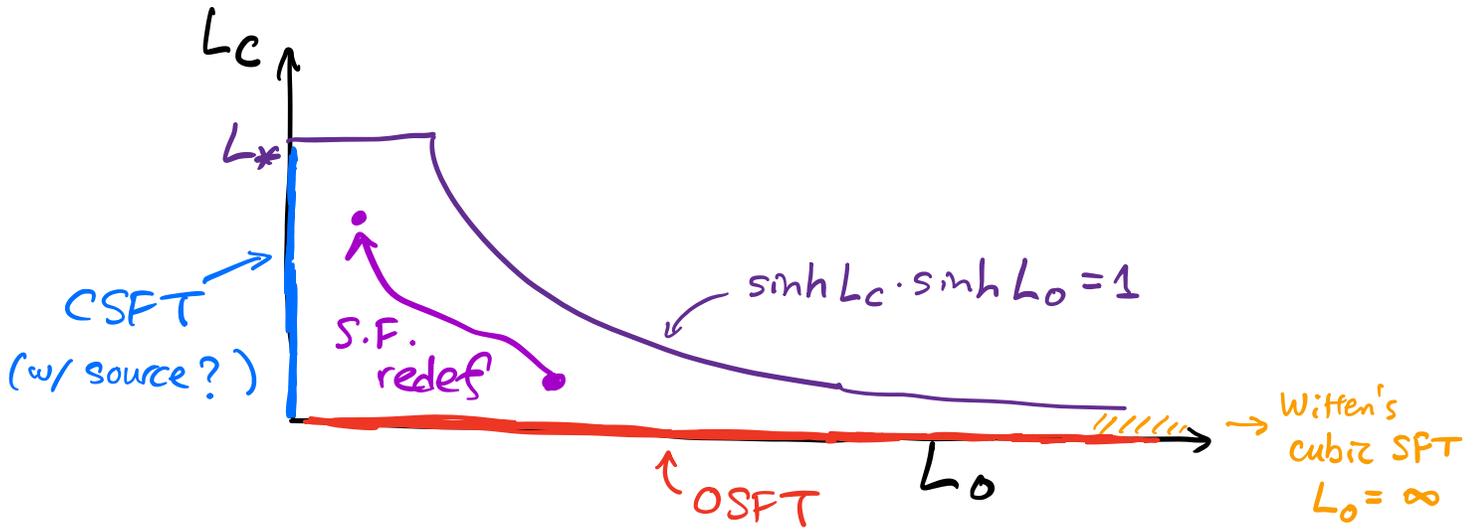
L_o  . $L_o \rightarrow 0$, "cut open string"



→ "mostly closed" SFT

• $L_c \rightarrow 0$, "out closed string"

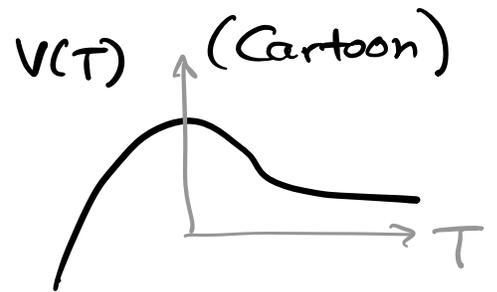
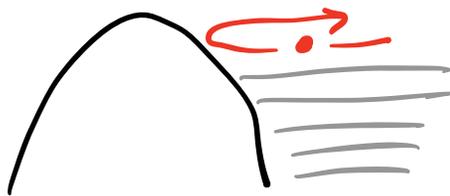
→ "mostly open" SFT



A clean setup: rolling open string tachyon on a ZZ-brane in $c=1$ string theory

[Sen, '02-'04]

MQM



• classical OSFT on ZZ-brane

\cong Witten's cubic OSFT

truncated to vac. module

of $c=25$ matter CFT

- quantum $O+C$ SFT ?

- Need to expand around rolling G.S. tachyon sol'n
- time-dep. perturbation theory

rolling OS tachyon $\xleftrightarrow{\text{S.F. redef?}}$ closed string radiation

Conclusion:

The formalism of SFT can overcome a number of deficiencies of the on-shell approach.

- the technology is available
- much work to do !