Conformal defects from Open String Field Theory

Kasia Budzik
w/ Miroslav Rapcak and Jairo M. Rojas

Perimeter Institute for Theoretical Physics

SFT@Cloud
22 September 2021

Motivation

- **Conformal defects** describe the low energy behaviour of impurities.
- Topological defects
  - implement symmetries and dualities
  - constrain bulk and boundary RG flows
  - realize mathematical concepts like fusion categories and modular tensor categories
- Conformal defects are only classified for the Ising model $c = 1/2$ and the Lee-Yang model $c = -22/5$
- Find new conformal defects by RG flows, extended symmetry ...
- OSFT can be used to search for new conformal boundary conditions

Plan

- Boundaries and defects in Minimal Models
- Defects RG flows:
  topological defect $D_{(1,2)}$ in minimal models [Kormos, Runkel, Watts]
- OSFT and classical solutions
- Apply level truncation to conformal defects
- Future directions

Minimal models

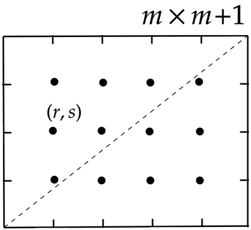
The Hilbert space of a **unitary diagonal minimal model** $\mathcal{M}_{m,m+1}$, $m \geq 3$ is

$$\bigoplus_{(r,s)} R_{(r,s)} \otimes \bar{R}_{(r,s)}, \qquad (r,s) \in \text{ Kac table}$$

where $R_{(r,s)}$ is an irreducible Verma module (conformal family).

$$c(m) = 1 - \frac{6}{m(m+1)}$$

$$h_{r,s}(m) = \frac{((m+1)r - ms)^2 - 1}{4m(m+1)}$$



Boundary CFT

- Boundary conditions represented by boundary states $||B\rangle\rangle \in \mathcal{H}_{\text{bulk}}$:

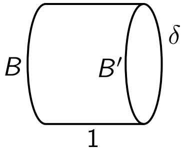
$$\langle\langle B||\phi\rangle = \langle\phi(0,0)\rangle_B^{\text{disk}}\,, \qquad \phi(z,\bar{z}) \text{ any bulk primary}$$

- Conformal boundary condition:

$$(L_n - \bar{L}_{-n})\,||B\rangle\rangle = 0$$

- Cardy condition:

$$\sum_h n_h^{BB'} \chi_h(q) = Z_{BB'}(\delta) = \langle\langle B||\, e^{-H/\delta}\, ||B'\rangle\rangle$$



Boundary CFT

- For diagonal CFTs there is a bijection between Cardy states and bulk primaries

$$||\phi_{(r,s)}\rangle\rangle\,, \quad (r,s) \in \text{ Kac table}$$

- Boundary spectrum of $||\phi_{(r,s)}\rangle\rangle$ is given by the fusion rules

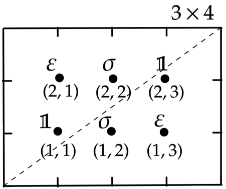
$$\bigoplus_{(k,l)} R_{(k,l)}, \qquad \phi_{(r,s)} \times \phi_{(r,s)} \supset \phi_{(k,l)}$$

Ising model $\mathcal{M}_{3,4}$

- Describes continuum limit of the critical point of the 2D Ising model
- Fusion rules:

$$\varepsilon \times \sigma \sim \sigma$$
$$\sigma \times \sigma \sim \mathbb{1} + \varepsilon$$
$$\varepsilon \times \varepsilon \sim \mathbb{1}$$

- Kac table:

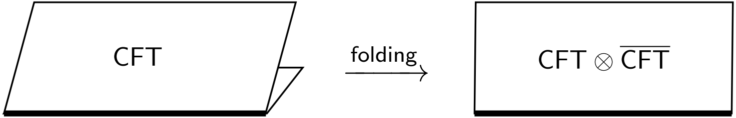


- Cardy boundary conditions and their spectrum:

$\|\|\mathbb{1}\rangle\rangle$	spins up	$R_{\mathbb{1}}$
$\|\|\varepsilon\rangle\rangle$	spins down	$R_{\mathbb{1}}$
$\|\|\sigma\rangle\rangle$	free	$R_{\mathbb{1}} \oplus R_{\varepsilon}$

Folding trick

Folding trick [Affleck, Wong] relates a defect CFT to a boundary CFT



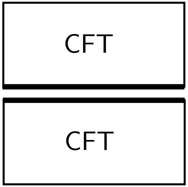
conformal defect	$\longleftrightarrow$	conformal boundary
defect spectrum	$\longleftrightarrow$	boundary spectrum
defect g-function	$\longleftrightarrow$	boundary g-function

$$g(B) = \langle\langle B || 0 \rangle = \langle \mathbb{1} \rangle_B$$

Conformal defects

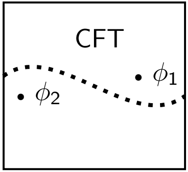
Factorizing defects

- (sums of) products of boundaries
- correlation functions factorize



Topological defects

- can be continuously deformed
- in Minimal Models labeled by Kac labels
- defect fields labelled by two Kac labels $\phi_{(r,s)(k,l)}$



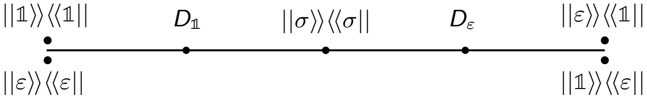
E.g. for the Ising model:

9 factorizing:	$\|\|\varepsilon\rangle\rangle\langle\langle\sigma\|\|\,,\ \|\|\mathbb{1}\rangle\rangle\langle\langle\varepsilon\|\|\,,\ \|\|\sigma\rangle\rangle\langle\langle\sigma\|\|\,,\ \ldots$
3 topological:	$D_{\mathbb{1}},\ D_{\varepsilon},\ D_{\sigma}$

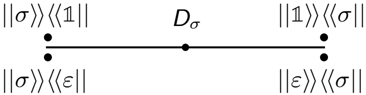
Conformal defects

- the Ising model $\mathcal{M}_{3,4} \xrightarrow{\text{folding}}$ free boson on $S^1/\mathbb{Z}_2$ [Oshikawa, Affleck]

• Dirichlet boundary conditions $||D(\varphi)\rangle\rangle$, $\varphi \in (0, \pi)$



• Neumann boundary conditions $||N(\tilde{\varphi})\rangle\rangle$, $\tilde{\varphi} \in (0, \pi/2)$



• Exceptional boundary conditions

- the Lee Yang model $\mathcal{M}_{2,5} \xrightarrow{\text{folding}} D_6$-invariant of $\mathcal{M}_{3,10}$ [Quella, Runkel, Watts]
- interfaces of Lee-Yang $\times$ Ising, Lee-Yang $\times$ $\mathcal{M}_{2,7}$ [Quella, Runkel, Watts]

Defect flows

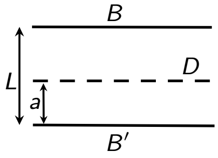
Perturbative analysis
Relevant defect fields ϕ_i have

$$\Delta_i = h_i + \bar{h}_i < 1.$$

New conformal defects determined from the zeros of the beta function:

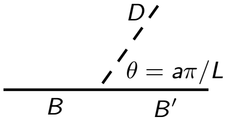
$$\beta(\lambda_i) = (1 - \Delta_i)\lambda_i + \sum_{jk} C_{ijk}\lambda_j\lambda_k + \mathcal{O}(\lambda^3),$$

where C_{ijk} are **defect** structure constants.



Truncated conformal space approach (TCSA)
Restrict the perturbed hamiltonian to a finite dim. space:

$$H = \frac{\pi}{L}\left[\left(L_0 - \frac{c}{24}\right) + \sum_i \lambda_i e^{i(h_i - \bar{h}_i)\theta}\phi_i\right]$$



Find approximated spectrum of the perturbed system.

Defect flows

Topological defect $D_{(1,2)}$ placed in $\mathcal{M}_{m,m+1}$ [Kormos, Runkel, Watts]

Spectrum of $D_{(1,2)}$:

$$\phi_{(1,1)(1,1)},\ \phi_{(1,3)(1,1)},\ \phi_{(1,1)(1,3)},\ (\phi_{(1,3)(1,3)})^{\oplus 2},\ (\phi_{(1,2)(1,2)})^{\oplus 2},\ \ldots$$

$\phi_{(1,3)(1,1)}$, $\phi_{(1,1)(1,3)}$ have conformal weights

$$(h,0) \quad \text{and} \quad (0,h) \quad \text{with} \quad h = \frac{m-1}{m+1}$$

We can perturb defect $D_{(1,2)}$ by

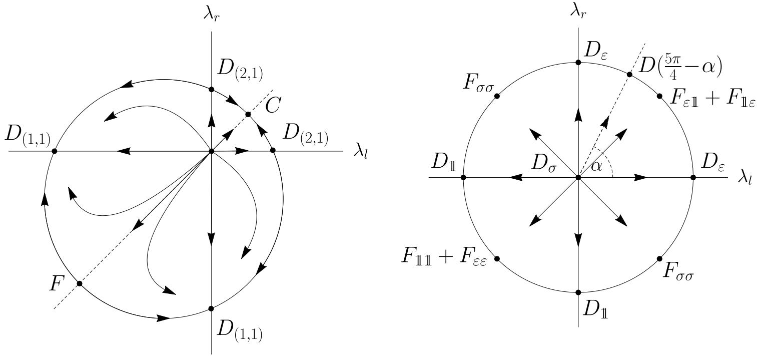
$$\lambda_l\, \phi_{(1,3)(1,1)} + \lambda_r\, \phi_{(1,1)(1,3)}$$

Defect flows

Topological defect $D_{(1,2)}$ perturbed by

$$\lambda_l\, \phi_{(1,3)(1,1)} + \lambda_r\, \phi_{(1,1)(1,3)}$$

[Kormos, Runkel, Watts]



(a) 6 fixed points in the $m > 3$ minimal model case.

(b) Continuum of fixed points in the Ising model case.

Open String Field Theory

OSFT is a second quantized version of open bosonic string theory [Witten]

- OSFT background is given by

$$\underbrace{\mathrm{BCFT}^{\mathrm{matter}}_{c=26}}_{\mathrm{BCFT}_c \otimes \mathrm{BCFT}^{\mathrm{aux}}_{26-c}} \otimes \mathrm{BCFT}^{\mathrm{ghost}}_{c=-26}$$

- Classical string field $\Psi \iff$ linear combination of boundary operators of ghost number $+1$
- The action is

$$S[\Psi] = -\frac{1}{g_o^2}\left(\frac{1}{2}\langle\Psi|Q_B\Psi\rangle + \frac{1}{3}\langle\Psi|\Psi * \Psi\rangle\right)$$

- n-vertex $\langle\Psi_1|\Psi_2 * \cdots * \Psi_n\rangle$ is given in terms of a BCFT correlator of n boundary operators

Classical solutions in OSFT

- Classical solutions Ψ of the OSFT action represent classical vacua (a tachyon vacuum or other D-branes associated to $||B_\Psi\rangle\rangle$) [Sen]
- g-function given by the action

$$g(B_\Psi) = 2\pi^2 S[\Psi] + g(B_0)$$

- Gauge invariant observables: Ellwood invariants

$$\langle E[\Phi]|\Psi\rangle \equiv \langle \Phi(i,-i)\ f_I \circ \Psi(0)\rangle_{B_0}^{\text{UHP}}$$

- Boundary state $||B_\Psi\rangle\rangle$ can be recovered from Ellwood invariants of Ψ [Kiermaier, Okawa, Zwiebach], [Kudrna, Maccaferri, Schnabl]

$$\langle\phi||B_\Psi\rangle\rangle = 2\pi i \left\langle E[c\bar{c}\,\phi\,\omega^{(aux)}]\middle|\Psi - \Psi_{\text{TV}}\right\rangle,$$

where Ψ_{TV} is the tachyon vacuum solution and $\omega^{(aux)}$ has $(h,\bar{h}) = (1-h_\phi, 1-\bar{h}_\phi)$.

Classical solutions in OSFT

Some of the analytic solutions:

- Tachyon vacuum solution [Schnabl 2006]
- Marginal deformations [many]
- Analytic solution relating any two BCFTs (sharing closed string background) [Erler, Maccaferri 2014 & 2020]

Level truncation

Generic string field has an infinite number of components:

$$|\Psi\rangle = \sum t_{iIJKM}\, L_{-I}\bar{L}_{-J}\,|\phi_i\rangle \otimes L^{\text{aux}}_{-K}\,|0\rangle \otimes \hat{L}^{\text{gh}}_{-M}c_1\,|0\rangle \qquad (*)$$

In level truncation we

- restrict to fields with eigenvalue under $L_0 + 1 \leq$ fixed level L
- find saddles of the truncated action
- check stability of the solution under the increase of the level L (e.g. Newton's method)
- level truncation produces a lot of solutions e.g. honycomb $c = 2$ BCFT [Kudrna, Schnabl, Vosmera], universal sector solutions [Kudrna, Schnabl]

$(*)$ in the Siegel gauge i.e. $b_0\,|\Psi\rangle = 0$ (other gauges possible)

Conformal defects from OSFT

Minimal model CFT with
a topological defect $D_{(1,2)}$

$\Downarrow$

Boundary CFT

$\Downarrow$

OSFT background

$$\mathrm{B}\big[\,\mathrm{CFT} \otimes \overline{\mathrm{CFT}}\,\big] \otimes \underbrace{\mathrm{BCFT}^{\mathrm{aux}}}_{26-2c} \otimes \mathrm{BCFT}^{\mathrm{gh}}$$

$\Downarrow$

String field

subset of the boundary spectrum
closed under OPE

Truncation at level 1

String field truncated to level $L = 1$:

$$|\Psi_{L=1}\rangle = \left(t_1 |0\rangle + t_2 \left|\phi_{(1,3)(1,1)}\right\rangle + t_3 \left|\phi_{(1,1)(1,3)}\right\rangle\right) \otimes |0\rangle^{\text{aux}} \otimes c_1 |0\rangle^{\text{gh}}$$

OSFT action truncated to level $L = 3$:

$$\begin{aligned} S[\Psi_{L=1}] = &- \frac{g_0}{2}\left(-t_1^2 - (t_2^2 + t_3^2)(h-1)d_{\phi\phi}\right) \\ &- \frac{g_0}{3}\left(t_1^3 K^3 + (t_2^3 + t_3^3)K^{3-3h} d_{\phi\phi} C_{\phi\phi}^{\phi} + 3(t_1 t_2^2 + t_1 t_3^2)K^{3-2h} d_{\phi\phi}\right), \end{aligned}$$

where

$K = 3\sqrt{3}/4$

$h = \dfrac{m-1}{m+1}$ is the conformal weight of $\phi_{(1,3)(1,1)}$ and $\phi_{(1,1)(1,3)}$

$d_{\phi\phi}$ and $C_{\phi\phi}^{\phi}$ are **defect** structure constants [Makabe, Watts]

g_0 is the g-function of the initial defect $D_{(1,2)}$

Solutions for $\mathcal{M}_{m,m+1}$, $m > 3$

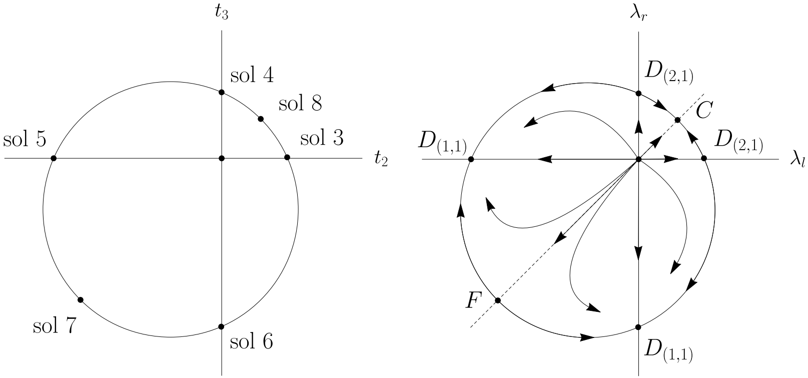


Figure: Structure of OSFT solutions at $L = 1$ matches the structure of RG fixed points found by KRW.

Defect C

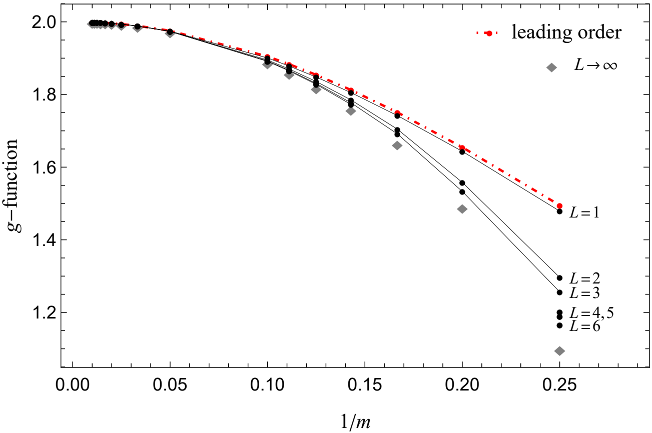


Figure: The g-function of the OSFT solution at different levels of truncation compared to the leading order g-function of defect C.

Defect $D_{(2,1)}$

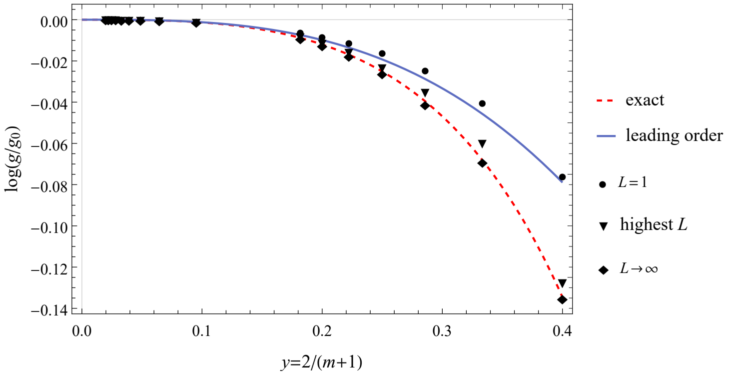


Figure: Comparison of the exact g-function, the leading order g-function and g-functions of OSFT solutions at the levels $L = 1, 4, \infty$.

Solutions for Ising model

OSFT action truncated to level $L = 1.5$:

$$S[\Psi_{L=0.5}] = -\frac{1}{\sqrt{2}}\left(-t_1^2 - \frac{1}{2}(t_2^2 + t_3^2)d_{\phi\phi}\right) - \frac{\sqrt{2}}{3}\left(t_1^3 K^3 + 3t_1(t_2^2 + t_3^2)K^2 d_{\phi\phi}\right),$$

Continuum of solutions with $g \xrightarrow{L\to\infty} 1.007$

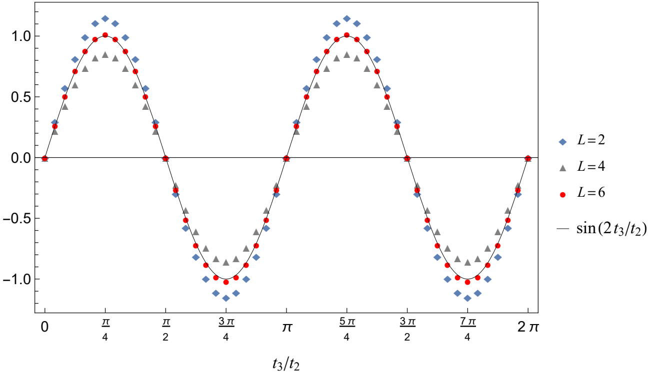


Figure: The value of the Ellwood invariant associated to $(\varepsilon \otimes \mathbb{1} + \mathbb{1} \otimes \varepsilon)/2$ at levels 2, 4 and 6.

Future directions

OSFT & level truncation can be used to study conformal defects.

Limitations:

- finite number of defect/boundary fields below any level
- need to know defect/boundary structure constants
- sensible restriction of string field needed.

Generalizations and future directions:

- interfaces between different theories eg. domain walls
- defects/boundaries with higher g-function [Kudrna]
- find a BCFT analog of the method.

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Thank you!

Defect $D_{(2,1)}$

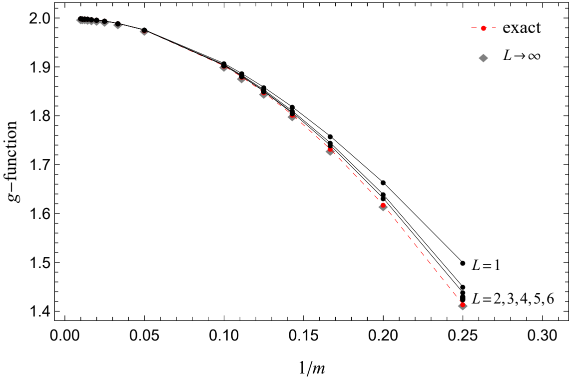


Figure: The g-function of the OSFT solution at different levels of truncation compared to the exact g-function of defect $D_{(2,1)}$.

Defect $D_{(1,1)}$

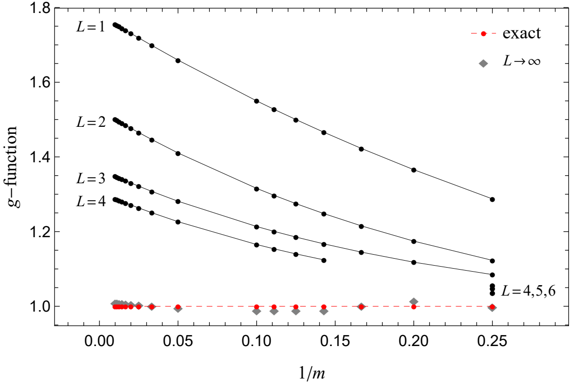
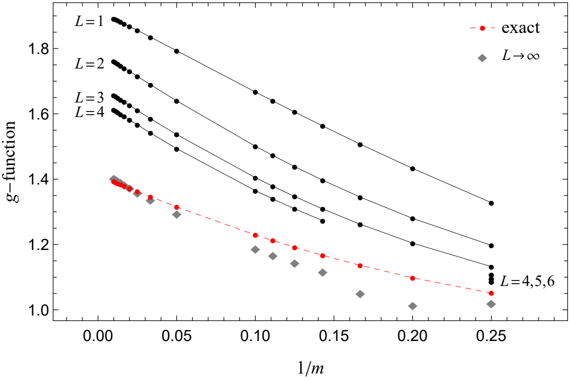


Figure: The g-function of the OSFT solution at different levels of truncation compared to the exact g-function of defect $D_{(1,1)}$.

Defect F

[Kormos, Runkel, Watts]:

$$F = \sum_{r=1}^{m-1} ||\phi_{(r,1)}\rangle\rangle \, \langle\langle\phi_{(r,1)}||$$



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