

Closed string disk amplitudes in the pure spinor formalism

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Overview

- Motivation
- Review of pure spinor formalism
- Adaption to purely closed string disk amplitudes
- Application to 2- and 1-point functions

Why disk amplitudes?

- First quantum correction to sphere level

- Higher derivative corrections to DBI action?

E.g. $\sim e^{-\Phi} \epsilon_{10} \epsilon_{10} R^4$

leads to correction
to 4D EH-term

[Antoniadis, Ferrara,
Minasian, Narain (1997)]

- Heterotic / Type I duality seems to predict such a term

[Green, Rudra (2016)]

- Origin: Disk and/or projective plane
- Direct check requires 5-point graviton amplitude!
- Alternatively: Check for EH-term correction from disk / projective plane in type I on Calabi-Yau Y_3
$$\sim e^{-\Phi} \chi_{Y_3} R$$
- How can disk / projective plane amplitude of gravitons depend on Euler number χ_{Y_3} of Calabi-Yau Y_3 ?

Pure spinor formalism

[Berkovits (2000)]

- Manifestly super-Poincare covariant
- Loop amplitudes calculable via non-minimal formulation
- Equivalence to RNS formalism shown in many examples (mainly amplitudes of massless states; for examples with massive states cf.


[Chakrabarti, Kashyap, Verma (2018))

- Can lead to simplified calculations of amplitudes
 - ★ Complete quartic effective action of type II at tree level [Policastro, Tsimpis (2006)]
 - ★ Closed string 4-point *3-loop* amplitude in type II at low energy [Gomez, Mafra (2013)]
 - ★ Arbitrary n -point amplitude of massless open strings on the disk [Mafra, Schlotterer, Stieberger (2011)]

CFT

- Type IIB-action for 10D flat space-time:

$$S = \frac{1}{2\pi} \int d^2z \left(\frac{1}{2} \partial X^m \bar{\partial} X_m + p_\alpha \bar{\partial} \theta^\alpha + \bar{p}_\alpha \partial \bar{\theta}^\alpha + w_\alpha \bar{\partial} \lambda^\alpha + \bar{w}_\alpha \partial \bar{\lambda}^\alpha \right)$$


$$(\lambda \gamma^m \lambda) = 0$$

i.e. λ pure spinor

- λ^α, w_α commuting
- $h(\theta^\alpha) = h(\lambda^\alpha) = 0, h(p_\alpha) = h(w_\alpha) = 1$

- Supersymmetric fields (relevant for vertex operators):

$$\Pi^m = \partial X^m + \frac{1}{2}(\theta\gamma^m\partial\theta) ,$$

$$d_\alpha = p_\alpha - \frac{1}{2}(\partial X^m + \frac{1}{4}(\theta\gamma^m\partial\theta))(\gamma_m\theta)_\alpha$$

(antiholomorphic analogs)

- OPEs (needed to perform contractions):


$$X^m(z, \bar{z})X^n(w, \bar{w}) = -\eta^{mn} \ln |z - w|^2$$

$$p_\alpha(z)\theta^\beta(w) = \frac{\delta_\alpha^\beta}{z - w} \quad , \quad w_\alpha(z)\lambda^\beta(w) = -\frac{\delta_\alpha^\beta}{z - w}$$

- Nilpotent BRST operator:

$$Q = \oint \frac{dz}{2\pi i} \lambda^\alpha(z) d_\alpha(z)$$

$$(\lambda \gamma^m \lambda) = 0 \quad \Longrightarrow \quad Q^2 = 0$$

$$d_\alpha(z) d_\beta(w) = - \frac{\gamma_{\alpha\beta}^m \Pi_m(w)}{z - w}$$


- Cohomology of Q coincides with superstring spectrum

[Berkovits (2000)]

Massless vertex operators


- Open string

- ★ Unintegrated

$$V^{(0)}(z) = [\lambda^\alpha A_\alpha(X, \theta)](z)$$

- ★ Integrated

$$V^{(1)}(z) = [\partial\theta^\alpha A_\alpha(X, \theta) + \Pi^m A_m(X, \theta) + d_\alpha W^\alpha(X, \theta) + \frac{1}{2} N^{mn} \mathcal{F}_{mn}(X, \theta)](z)$$

$\frac{1}{2}(w\gamma^{mn}\lambda)$


- ★ $QV^{(0)} = 0$, $QV^{(1)} = \partial V^{(0)}$ (cf. RNS)

★ E.g.: Gauge field with polarisation ξ_m :

$$A_\alpha(X, \theta) = e^{ik \cdot X} \left\{ \frac{\xi_m}{2} (\gamma^m \theta)_\alpha - \frac{1}{16} (\gamma_p \theta)_\alpha (\theta \gamma^{mnp} \theta) ik_{[m} \xi_{n]} + \mathcal{O}(\theta^5) \right\}$$

$$A_m(X, \theta) = e^{ik \cdot X} \left\{ \xi_m - \frac{1}{4} ik_p (\theta \gamma_m^{pq} \theta) \xi_q + \mathcal{O}(\theta^4) \right\}$$

$$W^\alpha(X, \theta) = e^{ik \cdot X} \left\{ -\frac{1}{2} ik_{[m} \xi_{n]} (\gamma^{mn} \theta)^\alpha + \mathcal{O}(\theta^3) \right\}$$

$$\mathcal{F}_{mn}(X, \theta) = e^{ik \cdot X} \left\{ 2ik_{[m} \xi_{n]} - \frac{1}{2} ik_{[p} \xi_{q]} ik_{[m} (\theta \gamma_n^{pq} \theta) + \mathcal{O}(\theta^4) \right\}$$

- Closed string (G_{mn}, B_{mn}, Φ , i.e. NS-NS-fields)

- ★ $\epsilon_{mn} = \xi_m \otimes \bar{\xi}_n$

- ★ $V^{(a,b)}(z, \bar{z}) = V^{(a)}(z) \otimes \bar{V}^{(b)}(\bar{z})$, $a, b \in \{0, 1\}$

- ★ $V^{(a,b)}$ contains factor

$$e^{ik \cdot X(z)} e^{ik \cdot \bar{X}(\bar{z})} = e^{ik \cdot [X(z) + \bar{X}(\bar{z})]} = e^{ik \cdot X(z, \bar{z})}$$

Tree level correlators

- After fixing conformal Killing group:

$$\mathcal{A}_{S^2}^{\text{closed}}(1, 2, \dots, n) = \left\langle V_1^{(0,0)}(z_1, \bar{z}_1) \prod_{i=2}^{n-2} \int d^2 z_i V_i^{(1,1)}(z_i, \bar{z}_i) V_{n-1}^{(0,0)}(z_{n-1}, \bar{z}_{n-1}) V_n^{(0,0)}(z_n, \bar{z}_n) \right\rangle$$

$$\mathcal{A}_{D_2}^{\text{open}}(1, 2, \dots, n) = \left\langle V_1^{(0)}(z_1) \prod_{i=2}^{n-2} \int_{z_{i-1}}^{z_{i+1}} dz_i V_i^{(1)}(z_i) V_{n-1}^{(0)}(z_{n-1}) V_n^{(0)}(z_n) \right\rangle$$

take this as example

- Integrate out non-zero modes via Wick's theorem, using

$$\langle X^m(z) X^n(w) \rangle = -\eta^{mn} \ln(z - w)$$

$$\langle p_\alpha(z) \theta^\beta(w) \rangle = \frac{\delta_\alpha^\beta}{z - w} \quad , \quad \langle w_\alpha(z) \lambda^\beta(w) \rangle = -\frac{\delta_\alpha^\beta}{z - w}$$

- Tree level: only $h = 0$ fields $X^m, \theta^\alpha, \lambda^\alpha$ have zero modes
 - \implies All $h = 1$ fields $\partial\theta^\alpha, \Pi^m, d_\alpha, N^{mn}$ have to be integrated out via Wick's theorem
 - \implies After Wick contractions and integrating out X^m zero modes, one ends up with:

$$\left\langle V_1^{(0)}(z_1) \prod_{i=2}^{n-2} V_i^{(1)}(z_i) V_{n-1}^{(0)}(z_{n-1}) V_n^{(0)}(z_n) \right\rangle = \delta(\sum_{i=1}^n k_i) \langle \lambda^\alpha \lambda^\beta \lambda^\gamma f_{\alpha\beta\gamma}(\theta; z_i) \rangle_0$$

zero mode
integration
of X^m

zero mode
integration
of $\theta^\alpha, \lambda^\alpha$

- Zero mode prescription:

$$\underbrace{\langle (\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta) \rangle_0}_{= 1}$$

unique element of Q -cohomology
with 3 factors of λ

- Projects out coefficients of $\lambda^3\theta^5$ -terms of

$$\lambda^\alpha\lambda^\beta\lambda^\gamma f_{\alpha\beta\gamma}(\theta; z_i)$$

- Need to integrate over z_i , $i = 2, \dots, n - 2$

Closed string disk amplitudes

[Bischof, M.H.]

- Earlier results on 3-pt fct of 1 closed and 2 open states in type I
[Alencar (2009); Alencar, Tahim, Landim, Costa Filho (2011)]

- Type IIB with Dp-brane along X^1, \dots, X^p

- $V^{(a,b)}(z, \bar{z}) = V^{(a)}(z) \bar{V}^{(b)}(\bar{z})$, $z \in \mathbb{H}_+$

- Employ doubling trick: $\bar{X}^m(\bar{z}) = D^m_n X^n(\bar{z})$

$$D^{mn} = \begin{cases} \eta^{mn} & m, n \in \{0, 1, \dots, p\} \\ -\eta^{mn} & m, n \in \{p+1, \dots, 9\} \\ 0 & \text{otherwise} \end{cases}$$

- Doubling trick for spinors

[Garousi, Myers; Hashimoto, Klebanov;
Gubser, Hashimoto, Klebanov, Maldacena
(1996)]

$$\bar{\Psi}^\alpha(\bar{z}) = M^\alpha_\beta \Psi^\beta(\bar{z}) \quad , \quad \bar{\Psi}_\alpha(\bar{z}) = N_\alpha^\beta \Psi_\beta(\bar{z})$$

- Relations between D, M, N (e.g. $N = (M^T)^{-1}$) allow rewriting:

$$\bar{V}^{(0)}(\bar{z}) = \left(\bar{\lambda}^\alpha \bar{A}_\alpha[\bar{\xi}, k](\bar{X}, \bar{\theta}) \right)(\bar{z}) = \left(\lambda^\alpha A_\alpha[D \cdot \bar{\xi}, D \cdot k](X, \theta) \right)(\bar{z})$$

$$\begin{aligned} \bar{V}^{(1)}(\bar{z}) = & \left(\bar{\partial} \theta^\alpha A_\alpha[D \cdot \bar{\xi}, D \cdot k](X, \theta) + \Pi^m A_m[D \cdot \bar{\xi}, D \cdot k](X, \theta) \right. \\ & \left. + d_\alpha W^\alpha[D \cdot \bar{\xi}, D \cdot k](X, \theta) + \frac{1}{2} N^{mn} \mathcal{F}_{mn}[D \cdot \bar{\xi}, D \cdot k](X, \theta) \right)(\bar{z}) \end{aligned}$$

\implies Can use same contractions as above, but allow both z and \bar{z}

- Conformal Killing group $PSL(2, \mathbb{R})$ only allows to fix one and a half closed string vertex operators

$$\implies \mathcal{A}_{D_2}^{\text{closed}}(1, \dots, n) =$$

following [Hoogeveen, Skenderis (2007)]

$$= 2ig_c^n \tau_p \int_0^1 dy \left\langle V_1^{(0)}(iy) \bar{V}_1^{(1)}(-iy) \prod_{j=2}^{n-1} \int_{\mathbb{H}_+} d^2 z_j V_j^{(1,1)}(z_j, \bar{z}_j) V_n^{(0)}(i) \bar{V}_n^{(0)}(-i) \right\rangle$$

D-brane
tension

cf. also [Grassi, Tamassia (2004); Alencar, Tahim, Landim, Costa Filho (2011)]

2-pt fct

$$\begin{aligned}
 \mathcal{A}_{D_2}^{\text{closed}}(1, 2) &= 2ig_c^2 \tau_p \int_0^1 dy \left\langle V_1^{(0)}(iy) \bar{V}_1^{(1)}(-iy) V_2^{(0)}(i) \bar{V}_2^{(0)}(-i) \right\rangle \\
 &= 2ig_c^2 \tau_p \int_0^1 dy \left\langle (\lambda A_1[\xi_1, k_1])(iy) \left(\bar{\partial} \theta^\alpha A_{1\alpha}[D \cdot \bar{\xi}_1, D \cdot k_1] + \Pi^m A_{1m}[D \cdot \bar{\xi}_1, D \cdot k_1] \right. \right. \\
 &\quad \left. \left. + d_\alpha W_1^\alpha[D \cdot \bar{\xi}_1, D \cdot k_1] + \frac{1}{2} N^{mn} \mathcal{F}_{1mn}[D \cdot \bar{\xi}_1, D \cdot k_1] \right) (-iy) (\lambda A_2[\xi_2, k_2])(i) (\lambda A_2[D \cdot \bar{\xi}_2, D \cdot k_2])(-i) \right\rangle
 \end{aligned}$$

$$\sim g_c^2 \tau_p \int_0^1 dy \underbrace{\left(\frac{4y}{(1+y)^2} \right)^{k_1 \cdot D \cdot k_1} \left(\frac{(1-y)^2}{(1+y)^2} \right)^{k_1 \cdot k_2}}_{\text{Koba-Nielsen factor}} \left(\frac{d_1}{2y} + \frac{d_2}{1+y} + \frac{d_3}{1-y} \right)$$

Koba-Nielsen factor

kinematic factors

- E.g.

$$d_1 = \langle i(\lambda A_1[\xi_1, k_1])k_1 \cdot A_1[D \cdot \bar{\xi}_1, D \cdot k_1](\lambda A_2[\xi_2, k_2])(\lambda A_2[D \cdot \bar{\xi}_2, D \cdot k_2]) \\ + A_{1m}[\xi_1, k_1](\lambda \gamma^m W_1[D \cdot \bar{\xi}_1, D \cdot k_1])(\lambda A_2[\xi_2, k_2])(\lambda A_2[D \cdot \bar{\xi}_2, D \cdot k_2]) \rangle_0$$

we applied zero mode prescription using *Cadabra* [Peeters]

- $$A_{D_2}^{\text{closed}}(1, 2) \sim g_c^2 \tau_p \frac{\Gamma(-t/2)\Gamma(2q^2)}{\Gamma(1 - t/2 + 2q^2)} \left(2q^2 a_1 + \frac{t}{2} a_2 \right)$$

$\frac{1}{2}k_1 \cdot D \cdot k_1$ (pointing to $2q^2$) and $-2k_1 \cdot k_2$ (pointing to t)

$$a_1 = \text{Tr}(\epsilon_1 \cdot D)k_1 \cdot \epsilon_2 \cdot k_1 - k_1 \cdot \epsilon_2 \cdot D\epsilon_1 \cdot k_2 - k_1 \cdot \epsilon_2 \cdot \epsilon_1^T \cdot D \cdot k_1 \\ - k_1 \cdot \epsilon_2^T \cdot \epsilon_1 \cdot D \cdot k_1 - k_1 \cdot \epsilon_2 \cdot \epsilon_1^T \cdot k_2 + q^2 \text{Tr}(\epsilon_1 \cdot \epsilon_2^T) + \{1 \leftrightarrow 2\}$$


Same as
RNS

[Hashimoto, Klebanov;
Garousi, Myers (1996)]

$$a_2 = \text{Tr}(\epsilon_1 \cdot D)(k_2 \cdot D \cdot \epsilon_2 \cdot D \cdot k_2 + k_1 \cdot \epsilon_2 \cdot D \cdot k_2 + k_2 \cdot D \cdot \epsilon_2 \cdot k_1) \\ + k_1 \cdot D \cdot \epsilon_1 \cdot D \cdot \epsilon_2 \cdot D \cdot k_2 - k_2 \cdot D \cdot \epsilon_2 \cdot \epsilon_1^T \cdot D \cdot k_1 + q^2 \text{Tr}(\epsilon_1 \cdot D \cdot \epsilon_2 \cdot D) \\ - q^2 \text{Tr}(\epsilon_1 \cdot \epsilon_2^T) - (q^2 - \frac{t}{4}) \text{Tr}(\epsilon_1 \cdot D) \text{Tr}(\epsilon_2 \cdot D) + \{1 \leftrightarrow 2\}$$

1-pt fct

- Earlier work in bosonic theory and RNS [Douglas, Grinstein (1987); Liu, Polchinski (1988); Ohta (1987)]
- Pure spinor: At most 2 factors of λ^α (in $V^{(0,0)}$) !?
- Alternative zero mode prescription (equivalent for higher-point tree amplitudes): [Berkovits (2016)]

$$\langle 1 \rangle_0 = 1$$


corresponds to only alternative scalar element of Q -cohomology

- $$\mathcal{A}_{D_2}^{\text{closed}}(1) = g_c \tau_p \int_{\mathbb{H}_+} \frac{d^2 z}{V_{\text{CKG}}} \underbrace{\left\langle V^{(1)}(z, \bar{z}) \right\rangle}_{V^{(1)}(z) \bar{V}^{(1)}(\bar{z})}$$

infinite

\implies Fix position of $V^{(1)}$ to $z = i$ and divide by volume of $K \subset PSL(2, \mathbb{R})$ leaving $z = i$ invariant

$$K = \left\{ \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \mid \theta \in [0, 2\pi] \right\}, \quad \frac{\cos \theta i + \sin \theta}{-\sin \theta i + \cos \theta} = i$$

volume: 2π

$\implies \mathcal{A}_{D_2}^{\text{closed}}(1) \sim g_c \tau_p \text{Tr}(\epsilon \cdot D)$

For graviton this corresponds to linearisation of $\tau_p \int d^p x \sqrt{-G}$

- Alternative approach [Kashyap (2020)]

Outlook

- RR-fields & fermions
- Projective plane cf. [Garousi (2006)] for RNS
- Higher n -points
- Higher derivative corrections to DBI

Thank you