Massless RR Sector in Superstring Field Theory

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The idea of p-brane democracy has a long history   Townsend

– all p-branes should play equal role in string theory

In perturbation theory there is a limited version of this statement

– all D-branes should play equal role

Since a Dp-brane couples to RR (p+1)-form potential, this means that all RR (p+1)-form potentials should play equal role

– contradicts conventional wisdom since the field strengths of (p+1)-form and (7-p)-form potentials are dual to each other

– normally only one of the two potentials is included in the field theory action.

In our analysis we shall use the version of superstring field theory (SFT) that requires doubling of the number of fields.

One set of fields is free and decouple from the system.

The other set describes an interacting theory, reproducing the usual string amplitudes.

We also know that this theory can be extended to include open strings living on any set of D-branes Moosavian, A.S., Verma

– must include (p+1)-form potential for all allowed p.

How?

Reference:

Alexandrov, A.S., Stefanski, 2108.04265, to appear (in appendix)

A brief review of classical SFT:

$H_{m,n}$: vector space of closed string states of picture number m,n, and ghost number 2, subject to the condition

$$b_0^- H_{m,n} = 0, \qquad L_0^- H_{m,n} = 0$$

We introduce two sets of string fields:

$$|\psi\rangle \in H \equiv H_{-1,-1} \oplus H_{-1,-1/2} \oplus H_{-1/2,-1} \oplus H_{-1/2,-1/2}$$

$$|\phi\rangle \in \widetilde{H} \equiv H_{-1,-1} \oplus H_{-1,-3/2} \oplus H_{-3/2,-1} \oplus H_{-3/2,-3/2}$$

Action

$$S = \langle\phi|c_0^- Q_B|\psi\rangle - \frac{1}{2}\langle\phi|c_0^- Q_B\, G|\phi\rangle + \sum_n \frac{1}{n!}\{\psi^n\}$$

BRST operator Q_B: preserves picture numbers

Zero modes of PCO G: maps $\widetilde{H} \mapsto H$ (identity on $H_{-1,-1}$)

$\{\psi^n\}$: Some interaction term whose details will not be important for our analysis.

$$\mathbf{S} = \langle \phi | \mathbf{c}_0^- \mathbf{Q}_\mathbf{B} | \psi \rangle - \frac{1}{2} \langle \phi | \mathbf{c}_0^- \mathbf{Q}_\mathbf{B}\, \mathbf{G} | \phi \rangle + \sum_\mathbf{n} \frac{1}{\mathbf{n}!} \{ \psi^\mathbf{n} \}$$

Derive equations of motion from S:

$$\mathbf{Q}_\mathbf{B} (|\psi\rangle - \mathbf{G} |\phi\rangle) = \mathbf{0}$$

$$\mathbf{Q}_\mathbf{B} |\phi\rangle + \sum_\mathbf{N} \frac{1}{\mathbf{N}!} [\psi^\mathbf{N}] = \mathbf{0}$$

$[\psi^\mathbf{n}]$ is constructed from variation of $\{\psi^{\mathbf{n}+1}\}$ with respect to ψ but the details will not be important for us.

Combine the two equations to get the interacting field equation:

$$\mathbf{Q}_\mathbf{B} |\psi\rangle + \sum_\mathbf{N} \frac{1}{\mathbf{N}!} \mathbf{G}\, [\psi^\mathbf{N}] = \mathbf{0}$$

$$\mathbf{Q_B}|\psi\rangle + \sum_{\mathbf{N}} \frac{\mathbf{1}}{\mathbf{N}!}\,\mathbf{G}\,[\psi^{\mathbf{N}}] = \mathbf{0}$$

$$\mathbf{Q_B}|\phi\rangle + \sum_{\mathbf{N}} \frac{\mathbf{1}}{\mathbf{N}!}[\psi^{\mathbf{N}}] = \mathbf{0}$$

Once we solve the first equation for $|\psi\rangle$, the solution for $|\phi\rangle$ to the second equation is fixed up to addition of solutions to free field equations:

$$\mathbf{Q_B}|\phi\rangle = \mathbf{0}$$

Furthermore which solution we choose does not affect the equations of motion for $|\psi\rangle$.

$$Q_B(|\psi\rangle - G|\phi\rangle) = 0, \qquad Q_B|\phi\rangle + \sum_N \frac{1}{N!}[\psi^N] = 0$$

$$|\phi\rangle \in H_{-3/2,-3/2}, \qquad |\psi\rangle \in H_{-1/2,-1/2}$$

At generic momentum, a practical way to eliminate the free field degrees of freedom is to set $|\psi\rangle - G|\phi\rangle = 0$ and write the equations as:

Berkovits, Zwiebach

$$|\psi\rangle = G|\phi\rangle, \qquad Q_B|\phi\rangle + \sum_N \frac{1}{N!}[(G\phi)^N] = 0$$

We shall work with the massless fields in the RR sector and ignore the interaction terms

Later we shall restore the action, interactions terms and lost dof

The second equation reduces to $Q_B|\phi\rangle = 0$.

We shall now expand the fields in components and examine how the various potentials and field strengths enter in the expansion.

We shall describe the analysis for IIB but the analysis for IIA is almost identical.

$$|\psi\rangle = \int \frac{d^{10}p}{(2\pi)^{10}}\, F^{\alpha\beta}(p)\, c\,\bar c\, e^{-\phi/2} S_\alpha\, e^{-\bar\phi/2} \bar S_\beta\, e^{ip.X}(0)|0\rangle$$

$$\begin{aligned}|\phi\rangle = \int \frac{d^{10}p}{(2\pi)^{10}} \Big[& A_{\alpha\beta}(p)\, c\,\bar c\, e^{-3\phi/2}\, S^\alpha\, e^{-3\bar\phi/2}\, \bar S^\beta(0) \\ & + B_\alpha^{\ \beta}(p)\, (\partial c + \bar\partial \bar c)\, c\,\bar c\, e^{-3\phi/2}\, S^\alpha\, \bar\partial\bar\xi\, e^{-5\bar\phi/2} \bar S_\beta(0) \\ & + D^\alpha_{\ \beta}(p)\, (\partial c + \bar\partial \bar c)\, c\,\bar c\, \partial\xi\, e^{-5\phi/2} S_\alpha\, e^{-3\bar\phi/2}\, \bar S^\beta(0)\Big] e^{ip.X}(0)|0\rangle\end{aligned}$$

$\xi, \eta, \phi, \bar\xi, \bar\eta, \bar\phi$: bosonized superghosts

$S_\alpha, \bar S_\alpha, S^\alpha, \bar S^\alpha$: spin fields in matter sector

$$|\psi\rangle = G|\phi\rangle, \qquad Q_B|\phi\rangle = 0$$

Gauge invariance:

$A \to A + U + V, \qquad B \to B + U\not p\,, \qquad D \to D - \not p\, V\,, \qquad U, V$: arbitrary

$|\psi\rangle = \mathbf{G}|\phi\rangle$ **gives**

$$\mathbf{F}^{\alpha\beta} = \frac{1}{4}\,(\not{p}\,\mathbf{A}\,\not{p} - \not{p}\,\mathbf{B} + \mathbf{D}\,\not{p})^{\alpha\beta}\ .$$

$\mathbf{Q_B}|\phi\rangle = \mathbf{0}$ **gives**

$$\mathbf{p}^2\mathbf{A} - \mathbf{B}\,\not{p} + \not{p}\,\mathbf{D} = \mathbf{0}\,,$$

Using the gauge transformation

$$\mathbf{A} \to \mathbf{A} + \mathbf{U} + \mathbf{V}, \qquad \mathbf{B} \to \mathbf{B} + \mathbf{U}\,\not{p}\,, \qquad \mathbf{D} \to \mathbf{D} - \not{p}\,\mathbf{V}\,, \qquad \mathbf{U}, \mathbf{V} : \textbf{arbitrary}$$

we can set A=0.

$$\mathbf{F} = \frac{1}{4}\,(-\not{p}\,\mathbf{B} + \mathbf{D}\,\not{p})^{\alpha\beta}\,, \qquad -\mathbf{B}\,\not{p} + \not{p}\,\mathbf{D} = \mathbf{0}\,.$$

We now expand F, B, D in terms of tensor fields:

$$F^{\alpha\beta} = \frac{i}{2} \sum_k \frac{1}{(2k+1)!} F^{(2k+1)}_{M_1 \cdots M_{2k+1}} (\gamma^{M_1 \cdots M_{2k+1}})^{\alpha\beta}$$

$$B_\alpha{}^\beta = \frac{1}{2} \sum_k \frac{1}{(2k)!} B^{(2k)}_{M_1 \cdots M_{2k}} (\gamma^{M_1 \cdots M_{2k}})_\alpha{}^\beta \,,$$

$$D^\alpha{}_\beta = \frac{1}{2} \sum_k \frac{1}{(2k)!} D^{(2k)}_{M_1 \cdots M_{2k}} (\gamma^{M_1 \cdots M_{2k}})^\alpha{}_\beta$$

$$*B^{(2k)} = (-1)^{k+1} B^{(10-2k)}, \quad *D^{(2k)} = (-1)^k D^{(10-2k)}, \quad *F^{(2k+1)} = (-1)^k F^{(9-2k)}$$

– follow from the chirality constraint on the spinors

$$(\gamma^{01 \cdots 9})_\alpha{}^\beta = \delta_\alpha{}^\beta$$

$$F = \frac{1}{4}\left(-\not{p}\, B + D\, \not{p}\right)^{\alpha\beta}, \qquad -B\,\not{p} + \not{p}\, D = 0\,.$$

In terms of tensor components, these become

$$F^{(2k+1)}_{M_1 M_2 M_{2k+1}} = \frac{i}{4} p^M \left(D^{(2k+2)} + B^{(2k+2)}\right)_{MM_1\cdots M_{2k+1}}$$

$$-\frac{i}{4}\left[p_{M_1}\left(D^{(2k)} - B^{(2k)}\right)_{M_2\cdots M_{2k+1}} + \text{cyclic perm. of } M_1,\cdots,M_{2k+1} \text{ with sign}\right]$$

$$p^M \left(D^{(2k+2)} + B^{(2k+2)}\right)_{MM_1\cdots M_{2k+1}}$$

$$= -\left[p_{M_1}\left(D^{(2k)} - B^{(2k)}\right)_{M_2\cdots M_{2k+1}} + \text{cyclic perm. of } M_1,\cdots,M_{2k+1} \text{ with sign}\right].$$

In position space these become

$$F^{(2k+1)} = \frac{1}{4} * d * \left(B^{(2k+2)} + D^{(2k+2)}\right) - \frac{1}{4} d\left(D^{(2k)} - B^{(2k)}\right)$$

$$*d * \left(B^{(2k+2)} + D^{(2k+2)}\right) = -d\left(D^{(2k)} - B^{(2k)}\right)$$

$$*B^{(2k)} = (-1)^{k+1} B^{(10-2k)}, \quad *D^{(2k)} = (-1)^k D^{(10-2k)}, \quad *F^{(2k+1)} = (-1)^k F^{(9-2k)}$$

$$F^{(2k+1)} = \frac{1}{4} * d * \left(B^{(2k+2)} + D^{(2k+2)}\right) - \frac{1}{4} d \left(D^{(2k)} - B^{(2k)}\right) \ : \ |\psi\rangle = Q_B|\phi\rangle$$

$$*d * \left(B^{(2k+2)} + D^{(2k+2)}\right) = -d \left(D^{(2k)} - B^{(2k)}\right) \ : \ Q_B|\phi\rangle = 0$$

Using the first equations, the last two equations take the form:

$$F^{(2k+1)} = \frac{1}{4}(-1)^{k+1} * d \left(D^{(8-2k)} - B^{(8-2k)}\right) - \frac{1}{4} d \left(D^{(2k)} - B^{(2k)}\right)$$

$$(-1)^{k+1} * d \left(D^{(8-2k)} - B^{(8-2k)}\right) = -d \left(D^{(2k)} - B^{(2k)}\right)$$

Define $C^{(2k)} = \left(B^{(2k)} - D^{(2k)}\right)/2$ and write these as:

$$F^{(2k+1)} = \frac{1}{2}(-1)^k * dC^{(8-2k)} + \frac{1}{2} dC^{(2k)}, \quad (-1)^k * dC^{(8-2k)} = dC^{(2k)}$$

Now $C^{(2k)}$'s are all independent and $*F^{(2k+1)} = (-1)^k F^{(9-2k)}$ is automatic.

$$F^{(2k+1)} = \frac{1}{2}(-1)^k * dC^{(8-2k)} + \frac{1}{2} dC^{(2k)}$$

$$(-1)^k * dC^{(8-2k)} = dC^{(2k)}$$

We can interpret $C^{(2k)}$'s for different k as RR potential and $F^{(2k+1)}$ as RR field strength

– we have all the potentials!

The second equation sets $dC^{(2k)}$ and $dC^{(8-2k)}$ to be dual to each other and therefore we do not have independent field strengths for all the potentials

With the help of the second equation the first equation can be written as

$$F^{(2k+1)} = (-1)^k * dC^{(8-2k)} \qquad \text{or} \qquad dC^{(2k)}$$

We shall now examine how SFT action leads to these equations.

$$F^{(2k+1)} = \frac{1}{2}(-1)^k * dC^{(8-2k)} + \frac{1}{2}dC^{(2k)}$$

$$(-1)^k * dC^{(8-2k)} = dC^{(2k)}$$

String field theory action (in $A_{\alpha\beta} = 0$ gauge) is proportional to

$$\sum_{k=0}^{4} \int \left[F^{(2k+1)} - \frac{1}{4}dC^{(2k)} - (-1)^k \frac{1}{4} * dC^{(8-2k)}\right] \wedge * \left[dC^{(2k)} - (-1)^k * dC^{(8-2k)}\right]$$

Equations of motion of $F^{(2k+1)}$ leads to the second equation.

Equations of motion of $C^{(8-2k)}$ leads to

$$d\left[F^{(2k+1)} - \frac{1}{2}(-1)^k * dC^{(8-2k)} - \frac{1}{2}dC^{(2k)}\right] = 0$$

$\Rightarrow$ the first equation up to addition of plane wave solutions to $F^{(2k+1)}$

– reflects that $Q_B(|\psi\rangle - G|\phi\rangle) = 0$ has solution $|\psi\rangle = G|\phi\rangle$ up to addition of BRST invariant states.

In the interacting theory, SFT action takes the form:

$$\sum_{k=0}^{4}\int\left[F^{(2k+1)}-\frac{1}{4}dC^{(2k)}-(-1)^k\frac{1}{4}*dC^{(8-2k)}\right]\wedge*\left[dC^{(2k)}-(-1)^k*dC^{(8-2k)}\right]$$
$$+S_I[F,\cdots]$$

$\cdots$ represents other fields but not C.

The equations of motion of C remain unchanged but those of F get corrections.

$$d\left[F^{(2k+1)}-\frac{1}{2}(-1)^k*dC^{(8-2k)}-\frac{1}{2}dC^{(2k)}\right]=0$$

$$(-1)^k*dC^{(8-2k)}-dC^{(2k)}=S_I'[F,\cdots]$$

$$\mathbf{d}\left[\mathbf{F}^{(2k+1)} - \frac{1}{2}(-1)^k * \mathbf{dC}^{(8-2k)} - \frac{1}{2}\mathbf{dC}^{(2k)}\right] = 0$$

$$(-1)^k * \mathbf{dC}^{(8-2k)} - \mathbf{dC}^{(2k)} = \mathbf{S}'_\mathbf{I}[\mathbf{F}, \cdots]$$

Using the second equation, the first equation can be written as,

$$\mathbf{d}\left[\mathbf{F}^{(2k+1)} - \frac{1}{2}\mathbf{S}'_\mathbf{I}[\mathbf{F}, \cdots]\right] = 0$$

– correct interacting theory of RR fields in IIB.

Once we have a solution to this, we can solve

$$(-1)^k * \mathbf{dC}^{(8-2k)} - \mathbf{dC}^{(2k)} = \mathbf{S}'_\mathbf{I}[\mathbf{F}, \cdots]$$

Only freedom is in adding solutions to

$$(-1)^k * \mathbf{dC}^{(8-2k)} - \mathbf{dC}^{(2k)} = 0$$

$$\Rightarrow \qquad \mathbf{d} * \mathbf{dC}^{(8-2k)} = 0 \qquad \Rightarrow \qquad \textbf{plane waves}$$

– does not affect interacting F equations

Conclusion

Massless RR sector in type IIB superstring field theory contains

– a set of field strengths $F^{(2k+1)}$ satisfying $*F^{(2k+1)} = (-1)^k F^{(9-2k)}$

– a set of RR potentials $C^{(2k)}$ without any self-duality constraint.

$F^{(2k+1)}$ is determined by a set of interacting field equations that depend on other fields but not C

Once $F^{(2k+1)}$ is given, $C^{(2\ell)}$ is fixed upto addition of solutions to the equations

$$(-1)^\ell * dC^{(8-2\ell)} - dC^{(2\ell)} = 0$$

– the solutions are plane waves and addition of these do not affect the equations of motion of any other field

– a set of free fields that completely decouple from the theory and are invisible.