AdS/CFT to all loop orders

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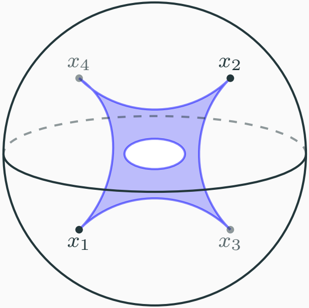
A cartoon picture of AdS/CFT



CFT correlators on the boundary

$\Longleftrightarrow$

Worldsheet correlators in the bulk

AdS:

Observables in string theory are organized as a sum over the genus of the worldsheet:

$$\sum_{\text{genus}} g_s^{2g-2} \int_{\mathcal{M}_{g,n}} \mathcal{O}_{\text{string},g}$$

CFT:

Observables in CFTs also admit a genus expansion in terms of the genus of Feynman graphs:

$$\sum_{\text{genus}} N^{2-2g} \mathcal{O}_{\text{CFT},g}$$

The AdS/CFT correspondence should hold at all orders in this genus expansion with $g_s \sim 1/N$

A free-field limit

Possibly tractable regimes:

- $\ell_{\text{string}} \ll L_{\text{AdS}}$: SUGRA approximation, dual CFT strongly coupled
- $\ell_{\text{string}} \gg L_{\text{AdS}}$: spacetime is no longer geometric, dual CFT weakly coupled

Most AdS/CFT calculations are done in the SUGRA (large tension) regime, where string theory reduces to (semiclassical) QFT

However, the opposite (tensionless) regime has its advantages:

$$\text{free dual CFT} \implies \text{free worldsheet theory?}$$

The tensionless string

One specific example realises this idea fully:

'tensionless' IIB strings on $\mathsf{AdS}_3 \times \mathsf{S}^3 \times \mathsf{T}^4$

$$\Longleftrightarrow$$

the symmetric product CFT $\mathsf{Sym}^N(\mathsf{T}^4)$

- Lower-dimensional analogue of the free limit of $\mathsf{AdS}_5/\mathsf{CFT}_4$
- Spectra and correlation functions of both sides can be shown to agree [Eberhardt, Gaberdiel, Gopakumar, '18], [Eberhardt, Gaberdiel, Gopakumar, '19]

This talk: argue a simple geometric picture relating the two sides of this duality

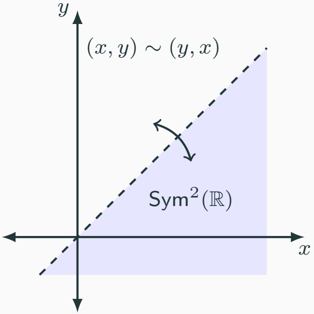
The symmetric product CFT

Given a conformal field theory X, we can consider the theory

$$\text{Sym}^N(X) = (X \otimes \cdots \otimes X)/S_N$$

Coordinate fields are labelled by Φ_i, and we have the identification $\{\Phi_1, \ldots, \Phi_N\} \sim \{\Phi_{\pi(1)}, \ldots, \Phi_{\pi(N)}\}$

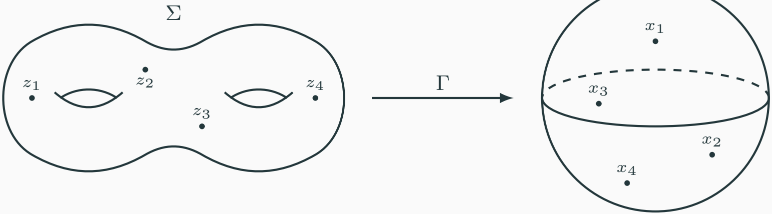
Concretely if X is a sigma-model on $\mathcal{M}$, $\text{Sym}^N(X)$ is a sigma-model on $\text{Sym}^N(\mathcal{M})$



States in $\text{Sym}^N(X)$ are of the form $\mathcal{O}_{[\pi]}$ where $[\pi]$ is a conjugacy class in S_N

We will focus on conjugacy classes of *single-cycle* permutations $(1 \ldots w)$, which form the w-twisted sector

The states $\mathcal{O}_w$ are analogous to single-trace operators in gauge theory, and correspond to *single-string* states in AdS



Correlation functions in $\mathrm{Sym}^N(X)$ can be expressed as correlation functions of X on *covering spaces* [Lunin, Mathur, '00]

$$\langle \mathcal{O}_1^{w_1}(x_1) \cdots \mathcal{O}_n^{w_n}(x_n) \rangle_{\mathrm{S}^2} = \sum_{\Gamma:\Sigma\to\mathrm{S}^2} C_\Gamma \, \langle \mathcal{O}_1(z_1) \cdots \mathcal{O}_n(z_n) \rangle_\Sigma$$

The *covering map* Γ has to satisfy

$$\Gamma(z) \sim x_i + \mathcal{O}((z-z_i)^{w_i}) \ , \quad z \to z_i$$

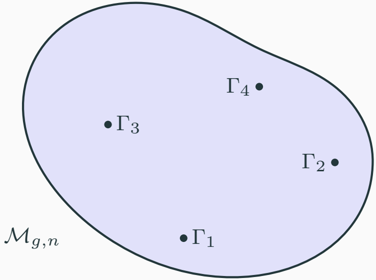
The sum runs over all coverings of S^2 with appropriate branching $\implies$ a genus expansion for the symmetric product CFT!

A localisation principle

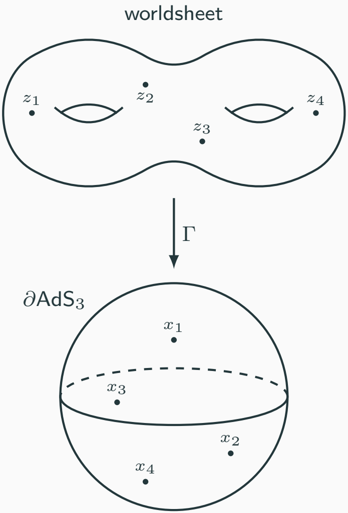
$$\underbrace{\sum_{\Gamma:\Sigma\to S^2} C_\Gamma \left\langle \mathcal{O}_1(z_1)\cdots\mathcal{O}_n(z_n)\right\rangle_\Sigma}_{\text{symmetric product CFT}} \overset{!}{=} \underbrace{\sum_g g_s^{2g-2}\int_{\mathcal{M}_{g,n}} \left\langle V^{w_1}(x_1,z_1)\cdots V^{w_n}(x_n,z_n)\right\rangle}_{\text{AdS}_3\text{ string theory}}$$

CFT correlator is a *sum* over surfaces, string theory correlator is an *integral* over surfaces

String theory correlators should localise!
$\Longrightarrow$ We think of the string as *covering* the boundary of AdS_3 [Pakman, Rastelli, Razamat, 2009]



A localisation principle



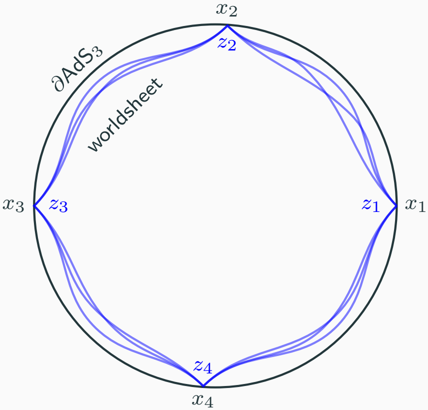


Figure adapted from [Eberhardt, Gaberdiel, Gopakumar, 2019]

The tensionless string

The 'tensionless' string on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{T}^4$ is based on the supergroup WZW model $\mathfrak{psu}(1,1|2)_k \oplus \mathbb{T}^4$ at level $k = 1$

This model admits a description in terms of free-fields

$$\mathfrak{psu}(1,1|2)_1 = \left(\begin{array}{c|c} \mathfrak{sl}(2,\mathbb{R})_1 & \text{supercharges} \\ \hline \text{supercharges} & \mathfrak{su}(2)_1 \end{array} \right) = \left(\begin{array}{c|c} \xi\eta & \xi\chi \\ \hline \eta\psi & \psi\chi \end{array} \right)$$

$$\underbrace{\xi^{\pm}, \quad \eta^{\pm}}_{\text{spin-}\frac{1}{2}\text{ bosons}}, \quad \underbrace{\psi^{\pm}, \quad \chi^{\pm}}_{\text{spin-}\frac{1}{2}\text{ fermions}}$$

States in the theory are descendents of 'spectrally flowed' states

$$V_h^w(x,z) \otimes \mathcal{O}_{\mathsf{T}^4}(z)$$

We are, in the end, interested in the correlators

$$\left\langle \prod_{i=1}^{n} V_{h_i}^{w_i}(x_i, z_i) \right\rangle$$

Proving localisation

Define the spin-$1/2$ functions on Σ:

$$\omega^{\pm}(z) = \left\langle \xi^{\pm}(z) \prod_{i=1}^{n} V_{h_i}^{w_i}(x_i, z_i) \right\rangle_{\text{phys.}}$$

By the OPEs between $\xi^{\pm}$ and V_h^w, we know

- $\omega^{\pm}$ both have poles of order $\frac{w_i+1}{2}$ at $z = z_i$
- The combinations $\omega^{-} + x_i\,\omega^{+}$ have zeroes of order $\frac{w_i-1}{2}$ at $z = z_i$
- (*non-trivial*) ω^{+} and ω^{-} have $n + 2g - 2$ shared zeroes

This turns out to be enough to completely constrain the forms of $\omega^{\pm}$, and the function

$$\Gamma(z) = -\frac{\omega^{-}(z)}{\omega^{+}(z)}$$

satisfies *exactly* the properties of the covering map $\Gamma : \Sigma \to \mathsf{S}^2$ used in the symmetric orbifold

If such a covering map does not exist, **correlation functions vanish** $\implies$ *localisation!*

With a little more effort, one can show the h_i dependence also matches the symmetric orbifold answer

Takeaway

Summary:

- Symmetric orbifold correlators can be 'geometrised' in terms of covering spaces $\Gamma : \Sigma \to \mathsf{S}^2$
- Tensionless string correlators define functions $\omega^{\pm}$ such that $\Gamma = -\omega^-/\omega^+$ is the covering map
- This implies that string theory amplitudes localise on $\mathcal{M}_{g,n}$

Future directions:

- Tensionless AdS_3 admits a natural generalisation to AdS_5 [Gaberdiel, Gopakumar, 2021]
    - Use similar geometric picture to compute correlation functions in $\mathrm{AdS}_5/\mathrm{CFT}_4$ [Gaberdiel, Gopakumar, BK, Maity, *work in progress*]
- One can study the effects of D-branes on this duality [Gaberdiel, BK, Vošmera, *work in progress*]
    - Similar localisation: correlators in the presence of D-branes $\implies$ covering maps between surfaces with boundaries

Thank you for your attention