BV formalism and perturbative algebraic quantum field theory

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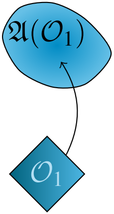

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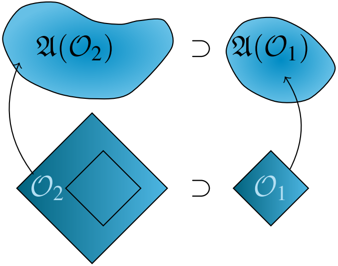
- The physical notion of subsystems is realized by the condition of isotony, i.e.: $\mathcal{O}_1 \subset \mathcal{O}_2 \Rightarrow \mathfrak{A}(\mathcal{O}_1) \subset \mathfrak{A}(\mathcal{O}_2)$. We obtain a net of algebras.



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- Dynamics: we use a modification of the Lagrangian formalism (fully covariant).

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- Classical observables are smooth functionals on $\mathcal{E}(M)$, i.e. elements of $\mathcal{C}^\infty(\mathcal{E}(M),\mathbb{C})$. For simplicity of notation, I will drop M, if no confusion arises.

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- F is local, $F\in\mathcal{F}_{\mathrm{loc}}$ if it is of the form: $F(\varphi)=\int\limits_M f(j^k_x(\varphi))\,d\mu_g(x)$, where f is a function on the jet bundle over M and $j^k_x(\varphi)$ is the k-th jet of φ at the point x. $\mathcal{F}$ denotes the space of multilocal functionals (products of local).

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- A functional is regular, $F\in\mathcal{F}_{\mathrm{reg}}$ if $F^{(n)}(\varphi)$ is as smooth section (in general it would be distributional).

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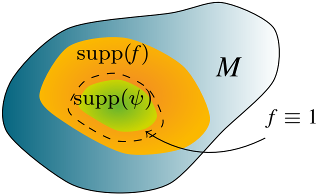

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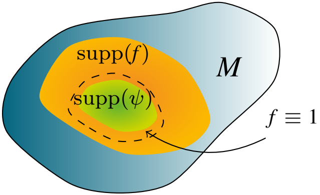
- The field equation is: $dS(\varphi) = 0$, so geometrically, the solution space is the zero locus of the 1-form dS.



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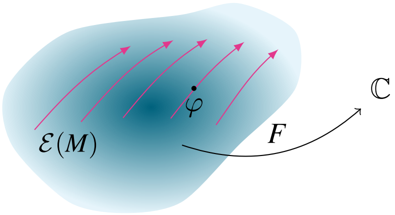

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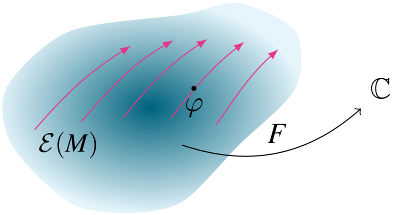
- We denote vector fields that are multilocal and compactly supported by $\mathcal{V}$. They act on $\mathcal{F}$ as derivations:

$$\partial_X F(\varphi) := \langle F^{(1)}(\varphi), X(\varphi)\rangle$$



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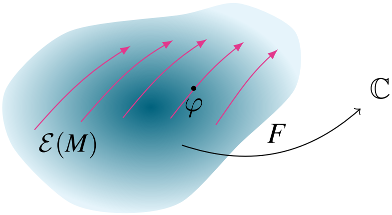

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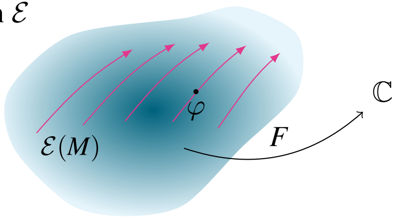
- A symmetry of S is a direction in $\mathcal{E}$ in which the action is constant, i.e. it is a vector field $X \in \mathcal{V}$ such that: $\forall \varphi \in \mathcal{E}$: $\delta_S(X) \equiv 0$.



Homological interpretation

- Space of solutions: $\mathcal{E}_S \subset \mathcal{E}$. Denote functionals that vanish on $\mathcal{E}_S$ by $\mathcal{F}_0$. In all physically relevant models, they are of the form: $\delta_S(X)$ for some $X \in \mathcal{V}$.

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- The space of sunctions on $\overline{\mathcal{E}}$ is the underlying algebra of the Chevalley-Eilenberg complex: $\mathcal{CE} \doteq \left(\bigwedge \mathfrak{s}^* \widehat{\otimes} \mathcal{F}, \gamma\right)$.

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- Now we combine gauge invariant and on-shell, to be able to characterize the space $\mathcal{F}_S^{\mathrm{inv}}$ iusing the BV complex, $\mathcal{BV}(M)$. Its underlying algebra is the algebra of multivector fileds on $\overline{\mathcal{E}}$, i.e. functions on shifted cotangent bundle $T^*[-1]\overline{\mathcal{E}}$.

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- $\mathcal{BV}$ is equipped with the BV differential, which in simple cases is just $s = \delta + \gamma$ (in general, more work needed).
- We have $H^0(s) = H^0(H_0(\delta), \gamma) = \mathcal{F}_S^{\mathrm{inv}}$, which is the reason to work with $\mathcal{BV}$ as it contains the same information as $\mathcal{F}_S^{\mathrm{inv}}$, but has a simpler algebraic structure (quotients and spaces of orbits are resolved).

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- The BV differential s has to be nilpotent, i.e.: $s^2 = 0$, which leads to the classical master equation (CME):

$$\{S^{\text{ext}}(f), S^{\text{ext}}(f)\} = 0\,,$$

modulo terms that vanish in the limit of constant f.

Propagators and Green functions

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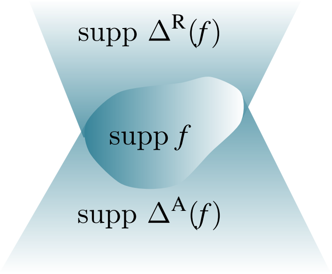

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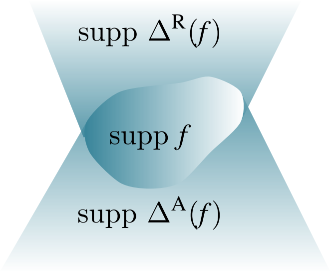

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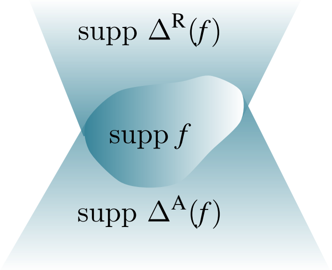

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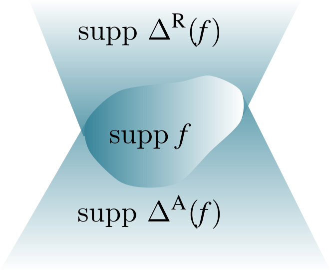

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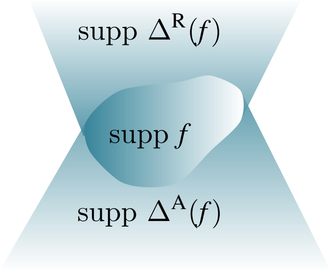
- Assume that S^{ext} has been constructed in such a way that P is a normally hyperbolic operator.

Propagators and Green functions



- Linearize S^{ext} around a fixed configuration φ_0, and write $S^{\mathrm{ext}} = S_0 + V$, where S_0 might contain both fields and antifields.
- Let $S_0 = S_{00} + \theta_0$ where S_{00} is the term with no antifields.
- Denote the operator induced by the second derivative of S_{00} by P.
- Assume that S^{ext} has been constructed in such a way that P is a normally hyperbolic operator.
- For M globally hyperbolic, P admits retarded and advanced Green's functions Δ^{R}, Δ^{A}.

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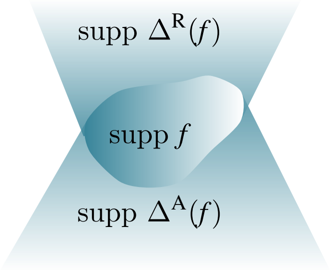

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- Their difference is the Pauli-Jordan (commutator) function
$$\Delta \doteq \Delta^R - \Delta^A .$$

Poisson structure and the $\star$-product

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$$(F \star G)(\varphi) \doteq \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \left\langle F^{(n)}(\varphi), W^{\otimes n} G^{(n)}(\varphi) \right\rangle ,$$

where W is the 2-point function of a Hadamard state (on Minkowski spacetime this is just the Wightman 2-point function) and it differs from $\frac{i}{2}\Delta$ by a symmetric bidistribution:

$$W = \frac{i}{2}\Delta + H .$$

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- The free QFT is defined as an appropriate completion of $\mathcal{F}(M)[[\hbar]]$, equipped with $\star$ and the conjugation $*$, where $F^*(\varphi) \doteq \overline{F(\varphi)}$.

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- Formally it corresponds to the operator of convolution with the oscillating Gaussian measure "with covariance $i\hbar\Delta^{\mathrm{F}}$",

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- Define the time-ordered product $\cdot_{\mathcal{T}}$ on $\mathcal{F}_{\mathrm{reg}}(M)[[\hbar]]$ by:

$$F \cdot_{\mathcal{T}} G \doteq \mathcal{T}(\mathcal{T}^{-1}F \cdot \mathcal{T}^{-1}G)$$

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- Renormalization problem: extend $\cdot_{\mathcal{T}}$, and all the above structures, to V local and non-linear.

QME on regular functionals

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- This expression can be rewritten as:

$$\{e_{\mathcal{T}}^{iV/\hbar}, S_0\} = e_{\mathcal{T}}^{iV/\hbar} \cdot_{\mathcal{T}} \left(\frac{1}{2}\{S_0 + V, S_0 + V\} - i\hbar \bigtriangleup (S_0 + V)\right)\,,$$

where $\bigtriangleup$ is the BV Laplacian, which on regular functionals is

$$\bigtriangleup X = (-1)^{(1+|X|)} \int dx \frac{\delta^2 X}{\delta\varphi^{\ddagger}(x)\delta\varphi(x)}\,.$$

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- We obtain the standard form of the QME (as a condition on V):

$$\frac{1}{2}\{S_0 + V, S_0 + V\} = i\hbar \bigtriangleup (S_0 + V).$$

Quantum BV operator I

- The free quantum BV operator $\hat{s}_0$ is defined on regular functionals by:

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- The 0th cohomology of $\hat{s}$ characterizes quantum gauge invariant observables.

Quantum BV operator II

- Assuming QME, we have:

$$\hat{s}X = e_{\mathcal{T}}^{-iV/\hbar} \cdot_{\mathcal{T}} \left(\{ e_{\mathcal{T}}^{iV/\hbar} \cdot_{\mathcal{T}} X, S_0 \} \right) .$$

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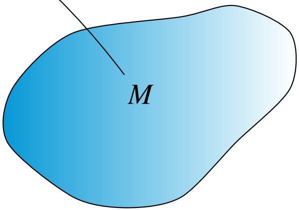
- In our framework this is a mathematically rigorous result, no path integral needed (in contrast to other approaches).

Theories with boundary: BV-BFV

- $\boxed{(\mathcal{X}, \Omega, S, Q)}$



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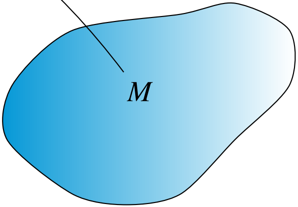

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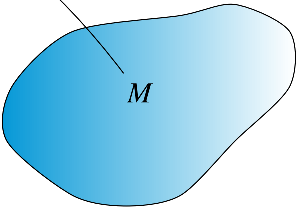

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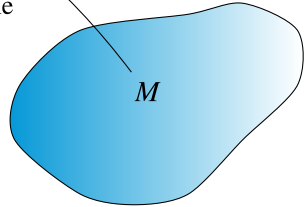

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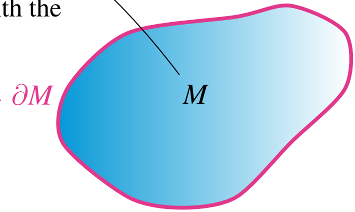

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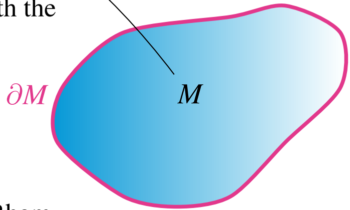

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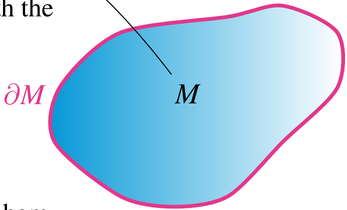

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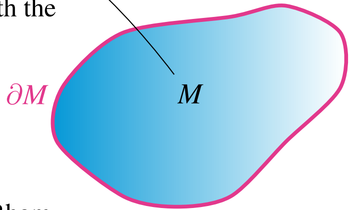
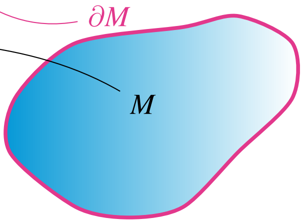


BV-BFV data

$\boxed{(\mathcal{X}, \Omega, S, Q)}$ and $\boxed{(\mathcal{X}^\partial, \Omega^\partial, S^\partial, Q^\partial)}$ connected by

$$\pi : \mathcal{X} \to \mathcal{X}^\partial$$

such that:



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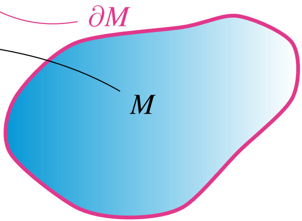

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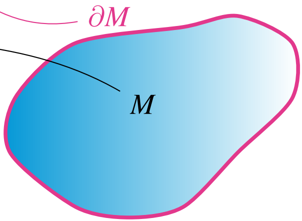

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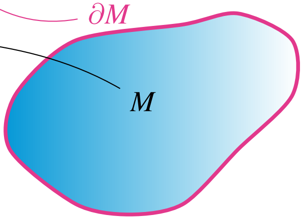

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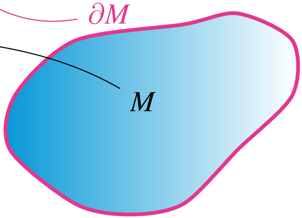

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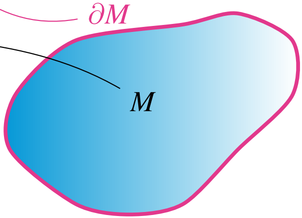

We can generalize this and assign data to corners, etc.

Generalization to asymptotic boundary

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- Quantization: work in progress with Schiavina.

Thank you very much for your attention!