Panel discussion on “String field theory beyond string theory”

Yuji Okawa
The University of Tokyo, Komaba

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I am afraid that my comments might be "within string theory," but I would like to mention directions which have not been explored much in the past conventional research of sting field theory.

Some of the slides at Strings 2021

Achievements, Progress and Open Questions in String Field Theory

Strings 2021

ICTP-SAIFR, São Paulo

June 22, 2021

Yuji Okawa and Barton Zwiebach

Recent directions

- Homological perturbation theory of homotopy algebras
  - Traditionally, homotopy algebras (such as A_∞ algebra or L_∞ algebra) were used in construction of string field theory. arXiv:hep-th/9206084 by Zwiebach
  - Recently, homotopy algebras are used in different contexts.
    * Low-energy effective theory
      arXiv:1609.00459 by Sen
    * Scattering amplitudes
      arXiv:math/0306332 by Kajiura

The A_∞ algebra describes relations among multi-string products of open string fields.

Consider an action of the form:

$$S = \, -\frac{1}{2}\,\langle\,\Psi, Q\Psi\,\rangle - \frac{g}{3}\,\langle\,\Psi, V_2(\Psi,\Psi)\,\rangle - \frac{g^2}{4}\,\langle\,\Psi, V_3(\Psi,\Psi,\Psi)\,\rangle + O(g^3)\,.$$

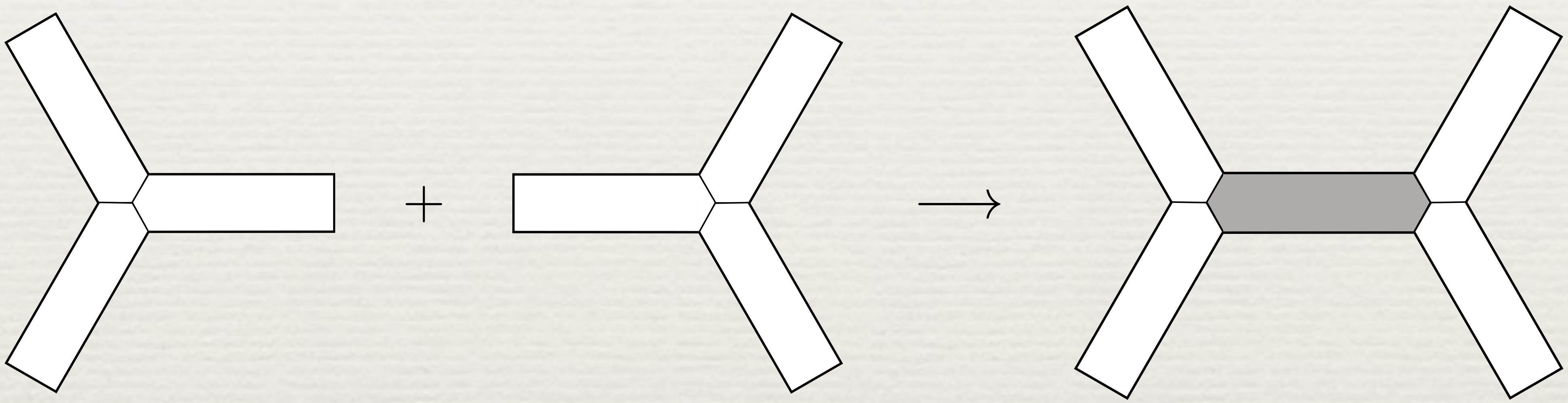
This action is invariant under the gauge transformation given by

$$\begin{aligned}\delta_\Lambda \Psi = Q\Lambda &+ g\,(\,V_2(\Psi,\Lambda) - V_2(\Lambda,\Psi)\,) \\ &+ g^2\,(\,V_3(\Psi,\Psi,\Lambda) - V_3(\Psi,\Lambda,\Psi) + V_3(\Lambda,\Psi,\Psi)\,) + O(g^3)\end{aligned}$$

if multi-string products satisfy a set of relations called A_∞ relations:

$$\begin{aligned}&Q^2 A_1 = 0\,, \\ &Q\,(V_2(A_1,A_2)) - V_2(QA_1,A_2) - (-1)^{A_1} V_2(A_1,QA_2) = 0\,, \\ &Q\,(V_3(A_1,A_2,A_3)) + V_3(QA_1,A_2,A_3) + (-1)^{A_1} V_3(A_1,QA_2,A_3) \\ &\;+ (-1)^{A_1+A_2} V_3(A_1,A_2,QA_3) - V_2(V_2(A_1,A_2),A_3) + V_2(A_1,V_2(A_2,A_3)) = 0\,, \\ &\qquad\qquad\vdots\end{aligned}$$

The path integral over massive fields to obtain the effective action for massless fields generates new multi-string products, but the A_∞ structure is preserved.



A_∞ structure in the original theory

$\downarrow$ Homological perturbation theory

A_∞ structure in the low-energy effective theory
or A_∞ structure in the scattering amplitudes

Multi-string products satisfying the A_∞ relations can be efficiently described by linear operators acting on the vector space $T\mathcal{H}$ defined by

$$T\mathcal{H} = \mathcal{H}^{\otimes 0} \oplus \mathcal{H} \oplus \mathcal{H}^{\otimes 2} \oplus \mathcal{H}^{\otimes 3} \oplus \ldots\,,$$

where we denoted the tensor product of n copies of the Hilbert space $\mathcal{H}$ by $\mathcal{H}^{\otimes n}$.

The A_∞ relations can be compactly expressed in terms of a linear operator $\mathbf{M}$ on $T\mathcal{H}$ which squares to zero:

$$\mathbf{M}^2 = 0\,.$$

For the action of open bosonic string field theory, the A_∞ structure can be described in terms of $\mathbf{M}$ given by

$$\mathbf{M} = \mathbf{Q} + \boldsymbol{m}_2\,,$$

where $\mathbf{Q}$ is associated with the BRST operator and $\boldsymbol{m}_2$ is associated with the two-string product (the star product).

The homological perturbation theory provides $\mathbf{M}$ for the A_∞ structure in the effective action for massless fields as

$$\mathbf{M} = \mathbf{P}\,\mathbf{Q}\,\mathbf{P} + \mathbf{P}\,\boldsymbol{m}_2\,\frac{1}{\mathbf{I} + \boldsymbol{h}\,\boldsymbol{m}_2}\,\mathbf{P}\,,$$

where $\mathbf{P}$ is associated with the projection onto the massless sector, $\boldsymbol{h}$ is associated with the propagator, and $\mathbf{I}$ is the identity operator.

Examples from recent string field theory papers

- Classical algebraic structures in string theory effective actions
  arXiv:2006.16270 by Erbin, Maccaferri, Schnabl and Vošmera

- Closed string deformations in open string field theory
  arXiv:2103.04919, 2103.04920, 2103.04921
  by Maccaferri and Vošmera

- Mapping between Witten and Lightcone String Field Theories
  arXiv:2012.09521 by Erler and Matsunaga

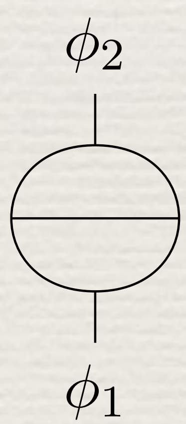
- Gauge-invariant operators of open bosonic string field theory in the low-energy limit
  arXiv:2006.16710 by Koyama, Okawa and Suzuki

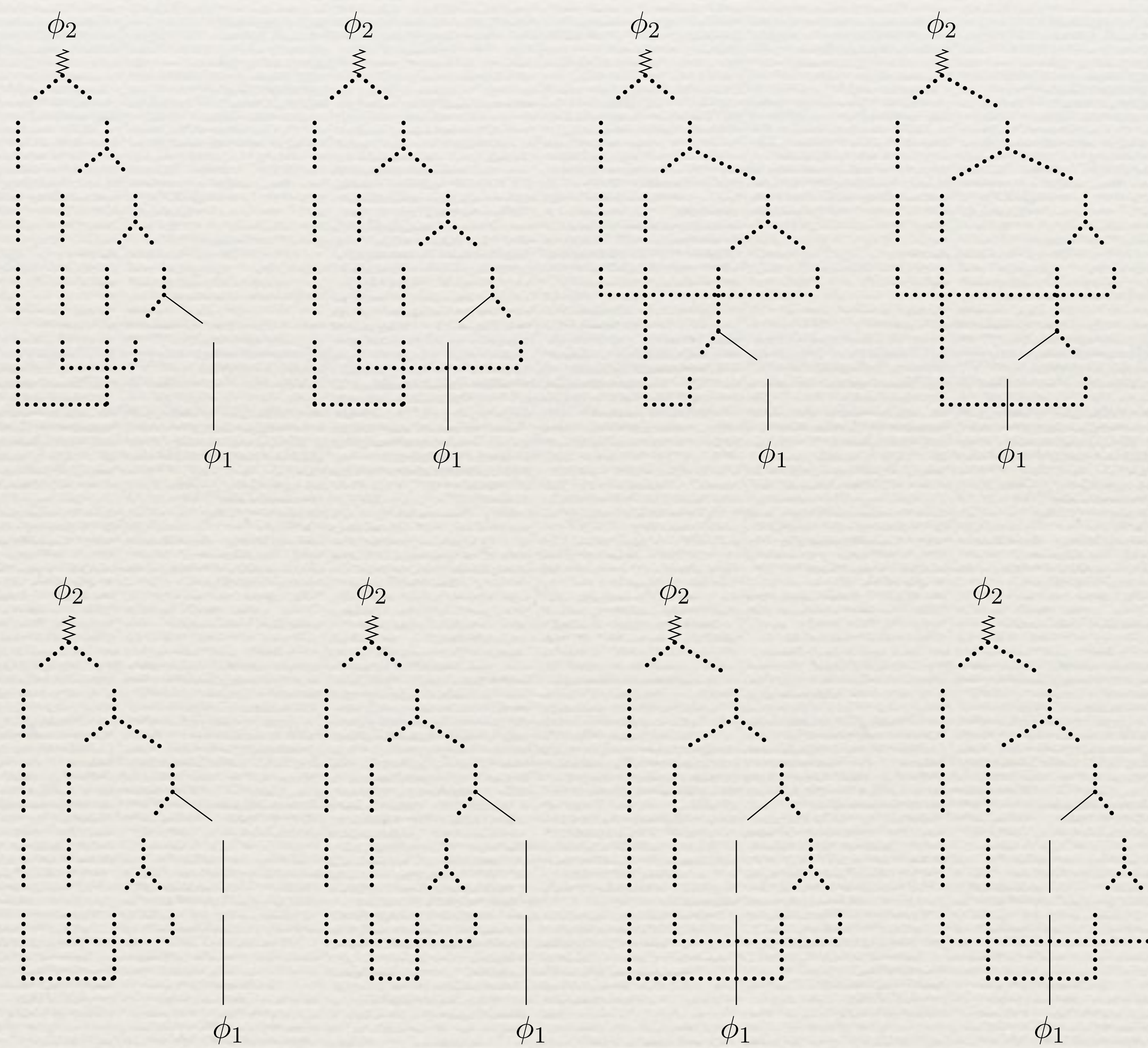
These papers are based on totally different motivations, but essentially the same tool from homological perturbation theory is used.

Homotopy algebras can be also useful in ordinary quantum field theory.

- We can handle Feynman diagrams (tree and loop) algebraically.
- We expect fruitful interactions among various research areas.

Figures from arXiv:2009.12616 by Saemann and Sfinarolakis





While we can construct a homotopy algebra for any decent theory, we feel (or I feel?) that the "stringy realization" of the homotopy algebra (based on the decomposition of the moduli space) uniquely determines the theory up to field redefinition.

Can we characterize the stringy homotopy algebra?

If the low-energy effective theory inherits the stringy homotopy algebra via homotopy transfer, we may be able to investigate the low-energy effective theory using the stringy homotopy algebra.

$\rightarrow$ homotopy algebraic approach to the Swampland program

One more slide from Strings 2021

- Proof of the AdS/CFT correspondence!
  - The AdS/CFT correspondence is typically realized in the low-energy limit of the theory on D-branes.
  - The theory before taking the low-energy limit is considered to be open-closed string field theory (which is difficult to be defined nonperturbatively), but I claim that open string field theory can do the job.
  - Instead of on-shell scattering amplitudes of open strings we are interested in correlation functions of gauge-invariant operators in this context.
  - The $1/N$ expansion of such correlation functions should be a perturbation theory containing gravity.
  - Is open superstring field theory a consistent quantum theory? If yes, use it to prove the AdS/CFT correspondence!

Whether you like my approach or not, gauge invariance of superstring field theory at the quantum level should be an important issue for everyone. (To me gauge invariance of open superstring field theory with source terms for gauge-invariant operators at the quantum level is most important.)

While Sen and Witten told us what to do if we encounter spurious singularities in the superconformal ghost sector, it is difficult to tell *when* we encounter them (as we discussed many times during the discussion sessions of this workshop).

It would be difficult to generalize the beautiful proof based on the minimal-area metric for the bosonic string to the superstring because we cannot use length or area.

An alternative approach based on the supersymmetric generalization of hyperbolic vertices?

FROM FIELDS TO STRINGS AND BACK

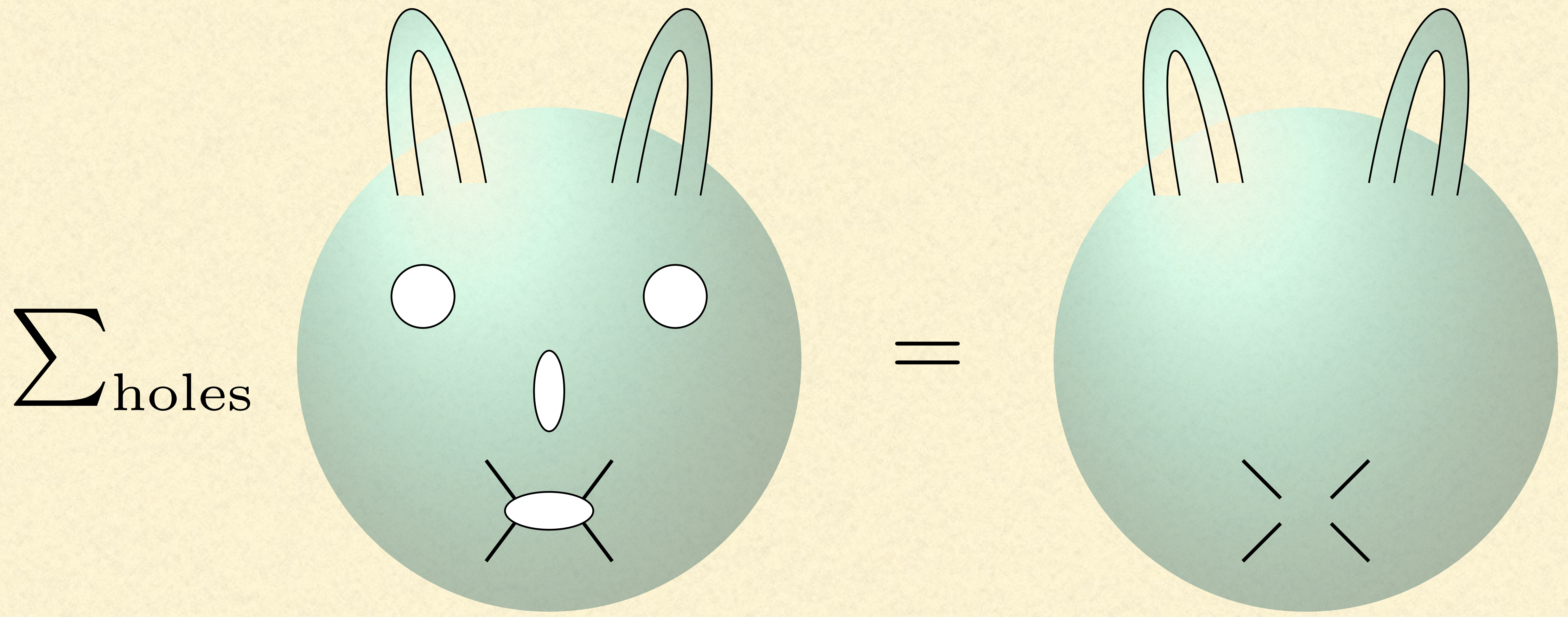
Rajesh Gopakumar,
ICTS-TIFR, Bengaluru

Collaboration with: Matthias Gaberdiel
Also P. Maity, B. Knighton

SFT21@Cloud,
23rd Sep. 2021

DERIVING AdS/CFT

- How exactly do large N QFTs reorganise themselves into theories of strings? ['t Hooft -'74]
- D-brane physics indicates open-closed string duality as the underlying reason [Maldacena-'97].
- Holes close up and backreaction alters the background.
- But difficult to see this explicitly happen at large $g_s N = \lambda$.
- Therefore, cannot delineate scope of gauge-string duality beyond examples.



To sum up infinitely many holes, would it be useful to organize Feynman diagrams using the homotopy algebra?