

# Collider physics with no PDFs

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- The novelty of our work:
  - ▶ Momentum space
  - ▶ Full three-flavor basis in NLO

# Straightforward example with only two observables in LO

## Singlet and gluon approximation

$$\Sigma(x, Q^2) = \sum_q^{n_f} [q(x, Q) + \bar{q}(x, Q^2)], \quad \text{where } n_f = 3$$

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## Singlet and gluon approximation

$$\Sigma(x, Q^2) = \sum_q^{n_f} [q(x, Q) + \bar{q}(x, Q^2)], \quad \text{where } n_f = 3$$

Express  $F_2$  and  $F_L$  in terms of  $\Sigma$  and gluon PDF:

$$F_2(x, Q^2) = x \langle e^2 \rangle \Sigma(x, Q^2), \quad \text{where } \langle e^2 \rangle \equiv \frac{1}{n_f} \sum_q^{n_f} e_q^2$$

$$\frac{F_L(x, Q^2)}{\frac{\alpha_s(Q^2)}{2\pi}} = x \langle e^2 \rangle \left[ C_{F_L \Sigma}^{(1)} \otimes \Sigma + n_f C_{F_L g}^{(1)} \otimes g \right]$$

First non-zero order in  $\alpha_s$    $F_2, \frac{F_L}{\alpha_s} \propto 1$

## Straightforward example with only two observables

We need to invert  
this linear mapping

$$\xrightarrow{\hspace{1cm}} \begin{bmatrix} F_2 \\ F_L/\frac{\alpha_s}{2\pi} \end{bmatrix} = \begin{bmatrix} x\langle e^2 \rangle & 0 \\ x\langle e^2 \rangle C_{F_L\Sigma}^{(1)} & x\langle e^2 \rangle n_f C_{F_Lg}^{(1)} \end{bmatrix} \otimes \begin{bmatrix} \Sigma \\ g \end{bmatrix}$$

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Singlet and gluon PDF in physical basis

$$\Sigma(x, Q^2) = \frac{1}{\langle e^2 \rangle} 1 \otimes \tilde{F}_2$$

$$\begin{aligned} g(x, Q^2) &= \int_x^1 \frac{dz}{z} \delta(1-z) \left\{ \frac{C_F}{4 T_R n_f \langle e^2 \rangle} \left[ \frac{x}{z} \frac{d}{d \frac{x}{z}} - 2 \right] \frac{F_2(\frac{x}{z}, Q^2)}{\frac{x}{z}} \right. \\ &\quad + \frac{2\pi}{\alpha_s(Q^2)} \frac{1}{8 T_R n_f \langle e^2 \rangle} \left[ \frac{x^2}{z^2} \frac{d^2}{d (\frac{x}{z})^2} - 2 \frac{x}{z} \frac{d}{d \frac{x}{z}} + 2 \right] \frac{F_L(\frac{x}{z}, Q^2)}{\frac{x}{z}} \Big\} \\ &\equiv \frac{1}{n_f \langle e^2 \rangle} \left\{ C_{g\tilde{F}'_2} \otimes \tilde{F}'_2 + C_{g\tilde{F}_2} \otimes \tilde{F}_2 + C_{g\tilde{F}'_L} \otimes \tilde{F}''_L + C_{g\tilde{F}'_L} \otimes \tilde{F}'_L + C_{g\tilde{F}_L} \otimes \tilde{F}_L \right\} \end{aligned}$$

Notation:

$$\tilde{F}_2(x, Q^2) \equiv \frac{F_2(x, Q^2)}{x}, \quad \tilde{F}_L(x, Q^2) \equiv \frac{2\pi}{\alpha_s(Q^2)} \frac{F_L(x, Q^2)}{x},$$

$$\tilde{F}'_{2,L}(x, Q^2) \equiv x \frac{d}{dx} \tilde{F}_{2,L}(x, Q^2), \quad \tilde{F}''_{L}(x, Q^2) \equiv x^2 \frac{d^2}{dx^2} \tilde{F}_L(x, Q^2)$$

# Results for two observable basis

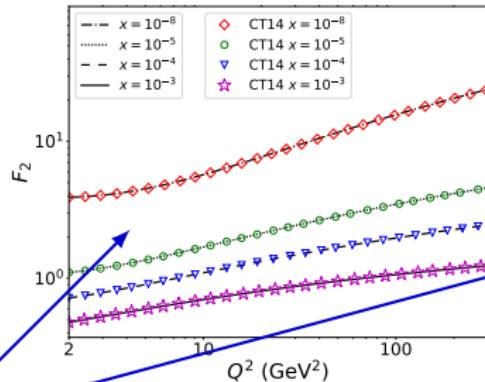
DGLAP equations of  $F_2$  and  $F_L/\alpha_s$  in physical basis:

$$\frac{dF_2(x, Q^2)}{d \log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} x \left\{ 2P_{qg} \otimes \left[ C_{g\widetilde{F''}_L} \otimes \widetilde{F''}_L + C_{g\widetilde{F'}_L} \otimes \widetilde{F'}_L + C_{g\widetilde{F}_L} \otimes \widetilde{F}_L \right] \right.$$
$$\left. + P_{qq} \otimes 1 \otimes \widetilde{F}_2 + 2P_{qg} \otimes \left[ C_{g\widetilde{F'}_2} \otimes \widetilde{F'}_2 + C_{g\widetilde{F}_2} \otimes \widetilde{F}_2 \right] \right\}$$

$$\frac{d}{d \log(Q^2)} \left( \frac{F_L(x, Q^2)}{\frac{\alpha_s(Q^2)}{2\pi}} \right) =$$
$$\frac{\alpha_s(Q^2)}{2\pi} x \left\{ \left[ 2C_{F_L\Sigma}^{(1)} \otimes P_{qg} + C_{F_Lg}^{(1)} \otimes P_{gg} \right] \otimes \left[ C_{g\widetilde{F''}_L} \otimes \widetilde{F''}_L + C_{g\widetilde{F'}_L} \otimes \widetilde{F'}_L + C_{g\widetilde{F}_L} \otimes \widetilde{F}_L \right] \right.$$
$$+ \left[ C_{F_L\Sigma}^{(1)} \otimes \left( P_{qq} \otimes 1 + 2P_{qg} \otimes C_{g\widetilde{F}_2} \right) + C_{F_Lg}^{(1)} \otimes \left( n_f P_{gq} \otimes 1 + P_{gg} \otimes C_{g\widetilde{F}_2} \right) \right] \otimes \widetilde{F}_2$$
$$\left. + \left[ 2C_{F_L\Sigma}^{(1)} \otimes P_{qg} + C_{F_Lg}^{(1)} \otimes P_{gg} \right] \otimes C_{g\widetilde{F'}_2} \otimes \widetilde{F'}_2 \right\}$$

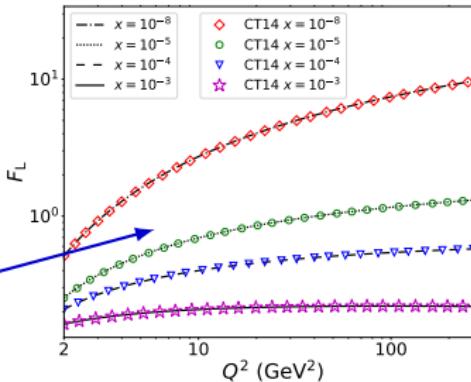
Scheme dependence of PDFs only at next order in  $\alpha_s$   
→ Should agree with evolution in terms of PDFs

# Comparison with conventional DGLAP evolution

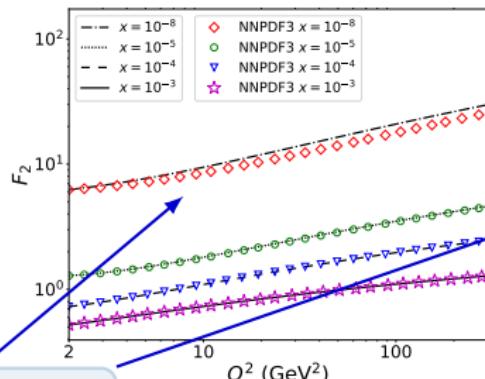


Nice match

$F_2$  CT14 LO

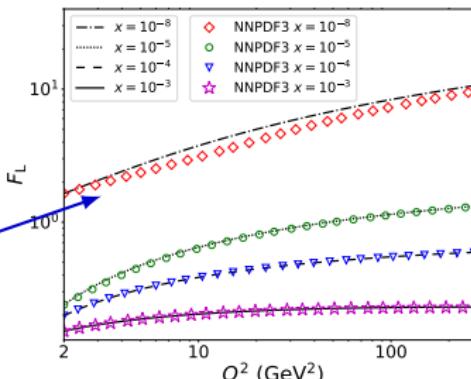


$F_L$  CT14 LO



Small discrepancy  
after  $x \sim 10^{-6}$

$F_2$  NNPDF3.0 LO



$F_L$  NNPDF3.0 LO

## Six observable basis

- Full three-flavor basis:  $u, \bar{u}, d, \bar{d}, s = \bar{s}$ , and  $g$   
→ Need six linearly independent DIS structure functions

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→ Need six linearly independent DIS structure functions
- We choose structure functions: (again in first non-zero order in  $\alpha_s$ )

## Neutral current

$$F_2 = x \sum_q^{n_f} e_q^2 (q + \bar{q})$$

$$F_3 = 2 \sum_q^{n_f} (L_q^2 - R_q^2) (q - \bar{q}),$$

$$\text{where } L_q = T_q^3 - 2e_q \sin^2 \theta_W$$

$$\text{and } R_q = -2e_q \sin^2 \theta_W$$

$$\frac{F_L(x, Q^2)}{\frac{\alpha_s(Q^2)}{2\pi}} = x \left[ C_{F_L \tilde{F}_2}^{(1)} \otimes \sum_q^{n_f} e_q^2 (q + \bar{q}) \right. \\ \left. + \sum_q^{n_f} e_q^2 C_{F_L g}^{(1)} \otimes g \right]$$

## Charged current

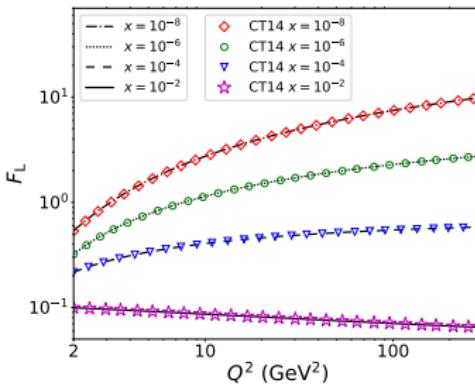
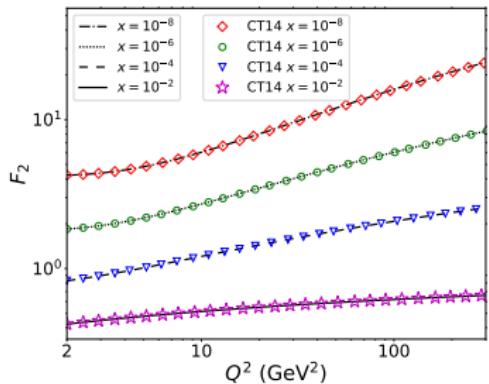
$$F_2^{W^-} = 2x(u + \bar{d} + \bar{s})$$

$$F_3^{W^-} = 2(u - \bar{d} - \bar{s})$$

$$F_{2c}^{W^-} = 2x\bar{s}$$

Derive DGLAP equations for these  
Again, should agree with conventional  
DGLAP evolution

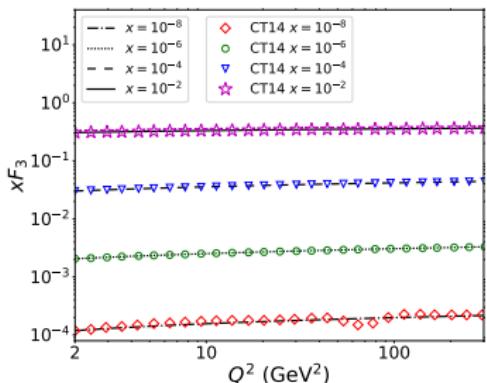
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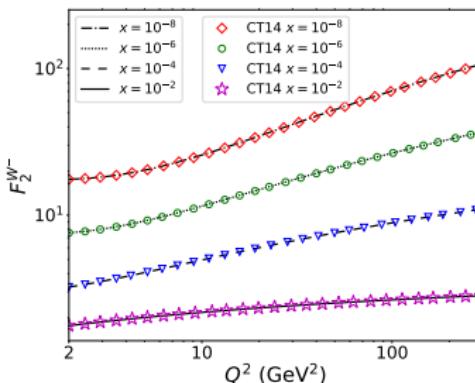
$F_2$  CT14 LO

Match in full grid

$F_L$  CT14 LO



$xF_3$  CT14 LO



$F_2^{W^-}$  CT14 LO

# Inverting $g$ at NLO without going to Mellin space

(Work in progress)

## Simple case without quarks

$$\text{Invert } g(x) \text{ from } \tilde{F}_L = C_{F_L g}^{(1)} \otimes g + \frac{\alpha_s(Q^2)}{2\pi} C_{F_L g}^{(2)} \otimes g \quad \tilde{F}_L(x, Q^2) \equiv \frac{2\pi}{\alpha_s(Q^2)} \frac{F_L(x, Q^2)}{x}$$

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$$\text{Define differential operator } \hat{P}(x) \equiv \frac{1}{8T_R n_f \langle e^2 \rangle} \left[ x^2 \frac{d^2}{dx^2} - 2x \frac{d}{dx} + 2 \right]$$

$$\text{Notice } g(x) = \hat{P}(x) \left[ C_{F_L g}^{(1)} \otimes g \right]$$

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Plug in  $g(x) = \hat{P}(x)\tilde{F}_L(x) + \mathcal{O}(\alpha_s(Q^2))$  to the right hand side

$$g(x) = \hat{P}(x)\tilde{F}_L(x) - \frac{\alpha_s(Q^2)}{2\pi} \hat{P}(x) \left[ C_{F_L g}^{(2)} \otimes \hat{P}\tilde{F}_L \right] + \mathcal{O}(\alpha_s^2(Q^2))$$

## Scheme dependence at NLO (work in progress)

DGLAP evolution in physical basis

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Kernels  $P_{ij}$  are independent of the factorization scheme

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$P_{ij}$ 's determined by:

- Splitting functions
  - Coefficient functions
- The scheme dependence exactly cancels between these two

# Summary

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- Scheme dependence of PDFs starts to play part at NLO in  $\alpha_s$ 
  - By using a physical basis in second non-zero order in  $\alpha_s$ , we will be able to avoid scheme dependence
- What next:
  - ▶ Establish physical basis at NLO (ongoing work)
  - ▶ Study how LHC cross sections, e.g. Drell-Yan, are expressed in physical basis
  - ▶ Obtain physical basis including also heavy quarks

# Backup: Inverting the gluon PDF

Gluon PDF in mellin space

$$g(n) = \frac{1}{n_f C_{F_L g}^{(1)}(n)} \left[ \frac{1}{\langle e^2 \rangle} \tilde{F}_L(n) - C_{F_L \Sigma}^{(1)}(n) \Sigma(n) \right]$$

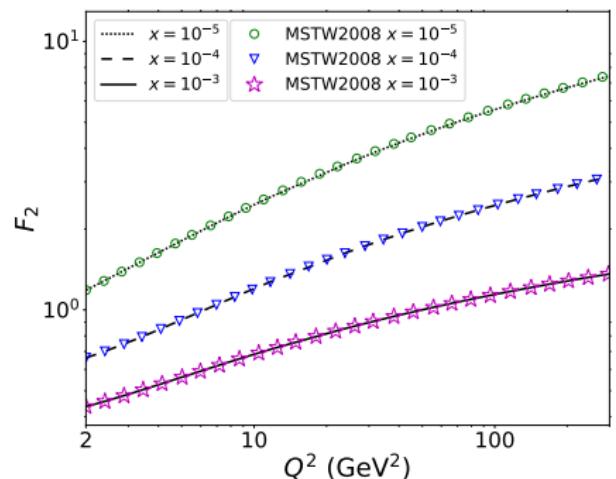
$$\frac{1}{C_{F_L g}^{(1)}(n)} = \frac{1}{8 T_R z_0^n} \int_0^1 dz z^{n+2} \delta''(z - z_0),$$

where  $z_0 \in ]0, 1[$  is an arbitrary constant that cancels in final result.

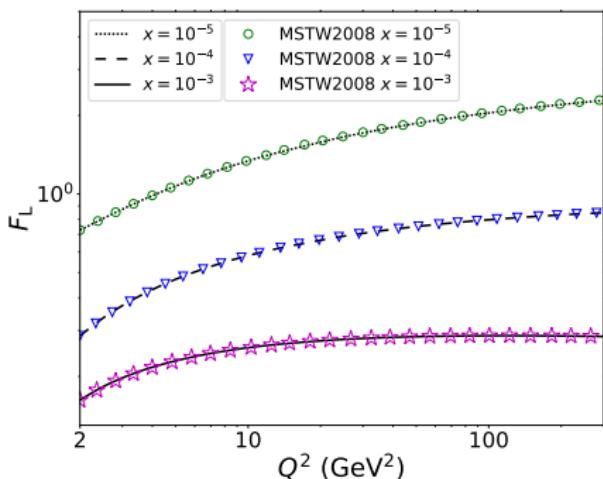
$$\begin{aligned} g(x, Q^2) &= \int_x^1 \frac{dz}{z} \delta(1-z) \left\{ \frac{C_F}{4 T_R n_f \langle e^2 \rangle} \left[ \frac{x}{z} \frac{d}{d \frac{x}{z}} - 2 \right] \frac{F_2 \left( \frac{x}{z}, Q^2 \right)}{\frac{x}{z}} \right. \\ &\quad + \frac{2\pi}{\alpha_s(Q^2)} \frac{1}{8 T_R n_f \langle e^2 \rangle} \left[ \frac{x^2}{z^2} \frac{d^2}{d \left( \frac{x}{z} \right)^2} - 2 \frac{x}{z} \frac{d}{d \frac{x}{z}} + 2 \right] \frac{F_L \left( \frac{x}{z}, Q^2 \right)}{\frac{x}{z}} \Big\} \\ &\equiv \frac{1}{n_f \langle e^2 \rangle} \left\{ C_{g \widetilde{F}'_2} \otimes \widetilde{F}'_2 + C_{g \widetilde{F}_2} \otimes \widetilde{F}_2 + C_{g \widetilde{F}''_L} \otimes \widetilde{F}''_L + C_{g \widetilde{F}'_L} \otimes \widetilde{F}'_L + C_{g \widetilde{F}_L} \otimes \widetilde{F}_L \right\} \end{aligned}$$

# Backup: Comparison with MSTW2008 PDF set

DGLAP evolution for  $F_2$  and  $F_L$  in two observable physical basis



$F_2$  MSTW LO



$F_L$  MSTW LO

# Backup: PDFs in six observable physical basis

$$\begin{bmatrix} F_2 \\ F_3 \\ F_2^{\text{W}^-} \\ F_3^{\text{W}^-} \\ F_{2c}^{\text{W}^-} \end{bmatrix} = \begin{bmatrix} xe_d^2 & xe_d^2 & xe_u^2 & xe_u^2 & 2xe_s^2 \\ 2A_d & -2A_d & 2A_u & -2A_u & 0 \\ 0 & 2x & 2x & 0 & 2x \\ 0 & -2 & 2 & 0 & -2 \\ 0 & 0 & 0 & 0 & 2x \end{bmatrix} \times \begin{bmatrix} d \\ \bar{d} \\ u \\ \bar{u} \\ \bar{s} \end{bmatrix},$$

where we have defined  $A_q \equiv L_q^2 - R_q^2$  in order to simplify equations.

$$xd(x, Q^2) = \frac{1}{A_u e_d^2 + A_d e_u^2} \left[ A_u F_2(x, Q^2) + \frac{e_u^2}{2} x F_3(x, Q^2) - \frac{A_u(2e_u^2 + e_d^2) - A_d e_u^2}{4} F_2^{\text{W}^-}(x, Q^2) \right]$$

$$- \frac{A_u(2e_u^2 - e_d^2) + A_d e_u^2}{4} x F_3^{\text{W}^-}(x, Q^2) + \frac{A_u(e_d^2 - 2e_s^2) - A_d e_u^2}{2} F_{2c}^{\text{W}^-}(x, Q^2)$$

$$x\bar{d}(x, Q^2) = \frac{1}{4} F_2^{\text{W}^-}(x, Q^2) - \frac{1}{4} x F_3^{\text{W}^-}(x, Q^2) - \frac{1}{2} F_{2c}^{\text{W}^-}$$

$$xu(x, Q^2) = \frac{1}{4} F_2^{\text{W}^-}(x, Q^2) + \frac{1}{4} x F_3^{\text{W}^-}(x, Q^2)$$

$$x\bar{u}(x, Q^2) = \frac{1}{A_u e_d^2 + A_d e_u^2} \left[ A_d F_2(x, Q^2) - \frac{e_d^2}{2} x F_3(x, Q^2) - \frac{A_d(2e_d^2 + e_u^2) - A_u e_d^2}{4} F_2^{\text{W}^-}(x, Q^2) \right. \\ \left. + \frac{A_d(2e_d^2 - e_u^2) + A_u e_d^2}{4} x F_3^{\text{W}^-}(x, Q^2) + A_d(e_d^2 - e_s^2) F_{2c}^{\text{W}^-}(x, Q^2) \right]$$

$$x\bar{s}(x, Q^2) = xs(x, Q^2) = \frac{1}{2} F_{2c}^{\text{W}^-}(x, Q^2)$$

## Backup: Results in six observable basis

$$\frac{dF_2(x, Q^2)}{d \log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} \left[ P_{qq} \otimes \tilde{F}_2 + 2 \sum_q^{n_f} e_q^2 P_{qg} \otimes g \right], \quad \text{where } \tilde{F}_2(x, Q^2) \equiv \frac{F_2(x, Q^2)}{x}$$

$$\frac{dx F_3(x, Q^2)}{d \log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} x P_{qq} \otimes F_3$$

$$\frac{dF_2^{W^-}(x, Q^2)}{d \log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} x \left[ P_{qq} \otimes \tilde{F}_2^{W^-} + 6 P_{qg} \otimes g \right], \quad \text{where } \tilde{F}_2^{W^-}(x, Q^2) \equiv \frac{F_2^{W^-}(x, Q^2)}{x}$$

$$\frac{dx F_3^{W^-}(x, Q^2)}{d \log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} x \left[ P_{qq} \otimes F_3^{W^-} - 2 P_{qg} \otimes g \right]$$

$$\frac{dF_{2c}^{W^-}(x, Q^2)}{d \log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} x \left[ P_{qq} \otimes \tilde{F}_{2c}^{W^-} + 2 P_{qg} \otimes g \right], \quad \text{where } \tilde{F}_{2c}^{W^-}(x, Q^2) \equiv \frac{F_{2c}^{W^-}(x, Q^2)}{x}$$

$$P_{qg} \otimes g = \frac{C_F}{4 \sum_q^{n_f} e_q^2} \left[ -\tilde{F}_2 + 2(z-1) \otimes \tilde{F}_2 \right] + \frac{1}{4 \sum_q^{n_f} e_q^2} \left[ \left( \frac{1}{x} - \frac{1}{2} \frac{d}{dx} \right) F_L(x, Q^2) + 1 \otimes \tilde{F}_L \right]$$

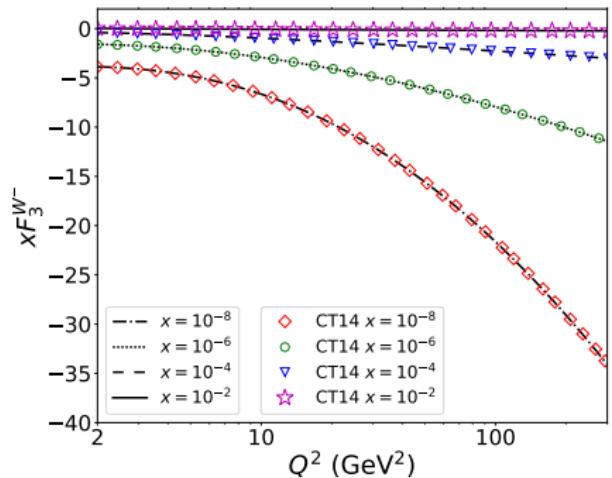
## Backup: Results in six observable basis

$$\frac{d}{d \log(Q^2)} \left( \frac{F_L(x, Q^2)}{\frac{\alpha_s(Q^2)}{2\pi}} \right) =$$
$$\frac{\alpha_s(Q^2)}{2\pi} \times \left\{ \left[ 2C_{F_L \tilde{F}_2}^{(1)} \otimes P_{qg} + C_{F_L g}^{(1)} \otimes P_{gg} \right] \otimes \left[ C_{g \tilde{F}'_L} \otimes \tilde{F}''_L + C_{g \tilde{F}'_L} \otimes \tilde{F}'_L + C_{g \tilde{F}_L} \otimes \tilde{F}_L \right] \right.$$
$$+ \left[ C_{F_L \tilde{F}_2}^{(1)} \otimes (P_{qq} + 2P_{qg} \otimes C_{g \tilde{F}_2}) + C_{F_L g}^{(1)} \otimes P_{gg} \otimes C_{g \tilde{F}_2} \right] \otimes \tilde{F}_2$$
$$+ \left. \left[ 2C_{F_L \tilde{F}_2}^{(1)} \otimes P_{qg} + C_{F_L g}^{(1)} \otimes P_{gg} \right] \otimes C_{g \tilde{F}'_2} \otimes \tilde{F}'_2 + \sum_q^{n_f} e_q^2 C_{F_L g}^{(1)} \otimes P_{gq} \otimes \sum_q (q + \bar{q}) \right\}$$

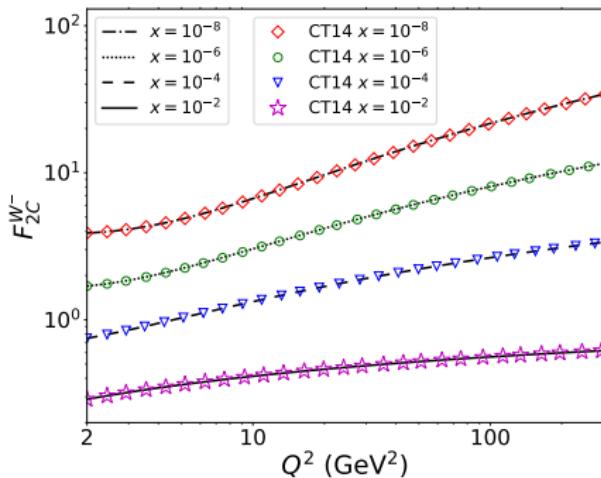
where  $\sum_q (q + \bar{q})$  can be expressed in physical basis (see slide 13/10)

# Backup: Comparison with conventional DGLAP evolution

Comparison for structure functions  $F_3^{W^-}$  and  $F_{2c}^{W^-}$  in six observable physical basis



$F_3^{W^-}$  CT14 LO



$F_{2c}^{W^-}$  CT14 LO