Collider physics with no PDFs

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Speaker: Mirja Tevio
In collaboration with: Tuomas Lappi Heikki Mäntysaari Hannu Paukkunen

University of Jyväskylä

Helsinki Institute of Physics

Center of Excellence in Quark Matter

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JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

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- Problems with PDFs
  - Parametrize non-observable quantities
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- The novelty of our work:
  - Momentum space
  - Full three-flavor basis in NLO

Straightforward example with only two observables in LO

Singlet and gluon approximation

$$(x, Q^2) = \sum_{q}^{n_f} \left[q(x, Q) + \overline{q}(x, Q^2) \right], \quad \text{where } n_f = 3$$

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Express F_2 and F_L in terms of and gluon PDF:

$$F_2(x, Q^2) = x h e^2 i \quad (x, Q^2), \quad \text{where } h e^2 i \quad \frac{1}{n_f} \sum_{q}^{n_f} e_q^2$$

$$\frac{F_L(x, Q^2)}{\frac{\alpha_s(Q^2)}{2\pi}} = x h e^2 i \left[C_{F_L}^{(1)} \qquad + n_f C_{F_L g}^{(1)} \quad g \right]$$

First non-zero order in α_s ⟶ $F_2, \frac{F_L}{\alpha_s} / 1$

Straightforward example with only two observables

We need to invert this linear mapping

$$\begin{bmatrix} F_2 \\ F_L / \frac{\alpha_s}{2\pi} \end{bmatrix} = \begin{bmatrix} x\langle e^2 \rangle & 0 \\ x\langle e^2 \rangle C^{(1)}_{F_L} & x\langle e^2 \rangle n_f C^{(1)}_{F_L g} \end{bmatrix} \begin{bmatrix} \\ g \end{bmatrix}$$

Straightforward example with only two observables

We need to invert this linear mapping

$$\begin{bmatrix} F_2 \\ F_\mathrm{L} / \frac{\alpha_\mathrm{s}}{2\pi} \end{bmatrix} = \begin{bmatrix} x he^2 i & 0 \\ x he^2 i C^{(1)}_{F_\mathrm{L}} & x he^2 i n_\mathrm{f} C^{(1)}_{F_\mathrm{L} g} \end{bmatrix} \begin{bmatrix} \\ g \end{bmatrix}$$

Singlet and gluon PDF in physical basis

$$(x, Q^2) = \frac{1}{he^2 i} 1 \quad \widetilde{F}_2$$

$$g(x, Q^2) = \int_x^1 \frac{\mathrm{d}z}{z} \delta(1 \quad z) \Bigg\{ \frac{C_\mathrm{F}}{4 T_\mathrm{R} n_\mathrm{f} he^2 i} \left[\frac{x}{z} \frac{\mathrm{d}}{\mathrm{d}\frac{x}{z}} \quad 2 \right] \frac{F_2\left(\frac{x}{z}, Q^2\right)}{\frac{x}{z}}$$

$$+ \frac{2\pi}{\alpha_\mathrm{s}(Q^2)} \frac{1}{8 T_\mathrm{R} n_\mathrm{f} he^2 i} \left[\frac{x^2}{z^2} \frac{\mathrm{d}^2}{\mathrm{d}\left(\frac{x}{z}\right)^2} \quad 2 \frac{x}{z} \frac{\mathrm{d}}{\mathrm{d}\frac{x}{z}} + 2 \right] \frac{F_\mathrm{L}\left(\frac{x}{z}, Q^2\right)}{\frac{x}{z}} \Bigg\}$$

$$\frac{1}{n_\mathrm{f} he^2 i} \left\{ C_{g\widetilde{F}^{0}_2} \quad \widetilde{F}^{0}_2 + C_{g\widetilde{F}_2} \quad \widetilde{F}_2 + C_{g\widetilde{F}^{00}_\mathrm{L}} \quad \widetilde{F}^{00}_\mathrm{L} + C_{g\widetilde{F}^{0}_\mathrm{L}} \quad \widetilde{F}^{0}_\mathrm{L} + C_{g\widetilde{F}_\mathrm{L}} \quad \widetilde{F}_\mathrm{L} \right\}$$

Notation:

$$\widetilde{F}_2(x, Q^2) \quad \frac{F_2(x, Q^2)}{x}, \quad \widetilde{F}_\mathrm{L}(x, Q^2) \quad \frac{2\pi}{\alpha_\mathrm{s}(Q^2)} \frac{F_\mathrm{L}(x, Q^2)}{x},$$

$$\widetilde{F}^{0}_{2,L}(x, Q^2) \quad x \frac{\mathrm{d}}{\mathrm{d}x} \widetilde{F}_{2,L}(x, Q^2), \quad \widetilde{F}^{00}_\mathrm{L}(x, Q^2) \quad x^2 \frac{\mathrm{d}^2}{\mathrm{d}x^2} \widetilde{F}_\mathrm{L}(x, Q^2)$$

Results for two observable basis

DGLAP equations of F_2 and F_L/α_s in physical basis:

$$\frac{dF_2(x,Q^2)}{d\log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} x \Big\{ 2P_{qg} \quad \Big[C_{g\widetilde{F^{00}}_L} \quad \widetilde{F^{00}}_L + C_{g\widetilde{F^{0}}_L} \quad \widetilde{F^{0}}_L + C_{g\widetilde{F}_L} \quad \widetilde{F}_L \Big]$$

$$+ P_{qq} \quad 1 \quad \widetilde{F}_2 + 2P_{qg} \quad \Big[C_{g\widetilde{F^{0}}_2} \quad \widetilde{F^{0}}_2 + C_{g\widetilde{F}_2} \quad \widetilde{F}_2 \Big] \Big\}$$

$$\frac{d}{d\log(Q^2)} \left(\frac{F_L(x,Q^2)}{\frac{\alpha_s(Q^2)}{2\pi}} \right) =$$

$$\frac{\alpha_s(Q^2)}{2\pi} x \Big\{ \Big[2C^{(1)}_{F_L} \quad P_{qg} + C^{(1)}_{F_L g} \quad P_{gg} \Big] \quad \Big[C_{g\widetilde{F^{00}}_L} \quad \widetilde{F^{00}}_L + C_{g\widetilde{F^{0}}_L} \quad \widetilde{F^{0}}_L + C_{g\widetilde{F}_L} \quad \widetilde{F}_L \Big]$$

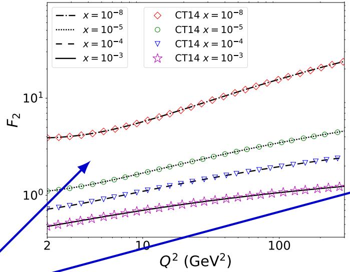
$$+ \Big[C^{(1)}_{F_L} \quad \Big(P_{qq} \quad 1 + 2P_{qg} \quad C_{g\widetilde{F}_2} \Big) + C^{(1)}_{F_L g} \quad \Big(n_f P_{gq} \quad 1 + P_{gg} \quad C_{g\widetilde{F}_2} \Big) \Big] \quad \widetilde{F}_2$$

$$+ \Big[2C^{(1)}_{F_L} \quad P_{qg} + C^{(1)}_{F_L g} \quad P_{gg} \Big] \quad C_{g\widetilde{F^{0}}_2} \quad \widetilde{F^{0}}_2 \Big\}$$

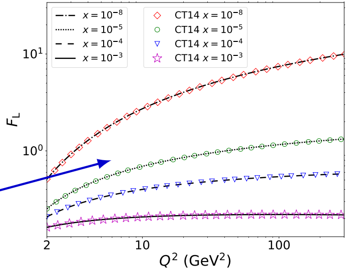
Scheme dependence of PDFs only at next order in α_s

! Should agree with evolution in terms of PDFs

Comparison with conventional DGLAP evolution

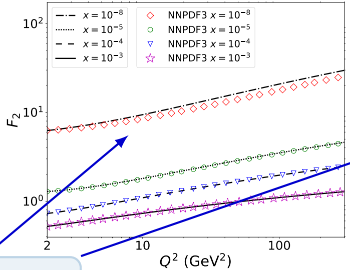


F_2 CT14 LO

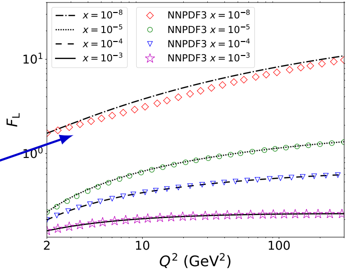


F_L CT14 LO

Nice match



F_2 NNPDF3.0 LO



F_L NNPDF3.0 LO

Small discrepancy after x 10^{6}

Six observable basis

- Full three-flavor basis: $u, u, d, d, s = s$, and g
  - ! Need six linearly independent DIS structure functions

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- Full three-flavor basis: $u, u, d, d, s = s$, and g
  - ! Need six linearly independent DIS structure functions
- We choose structure functions: (again in first non-zero order in α_s)

Neutral current

$$F_2 = x \sum_q^{n_f} e_q^2 (q + \bar{q})$$

$$F_3 = 2 \sum_q^{n_f} (L_q^2 \quad R_q^2)(q \quad \bar{q}),$$

$$\text{where } L_q = T_q^3 \quad 2e_q \sin^2\theta_W$$

$$\text{and } R_q = \quad 2e_q \sin^2\theta_W$$

$$\frac{F_L(x, Q^2)}{\frac{\alpha_s(Q^2)}{2\pi}} = x\left[C^{(1)}_{F_L \tilde{F}_2} \quad \sum_q^{n_f} e_q^2 (q + \bar{q}) + \sum_q^{n_f} e_q^2 C^{(1)}_{F_L g} \quad g\right]$$

Charged current

$$F_2^W = 2x(u + \bar{d} + \bar{s})$$

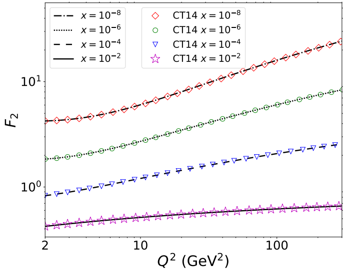
$$F_3^W = 2(u \quad \bar{d} \quad \bar{s})$$

$$F_{2c}^W = 2x\bar{s}$$

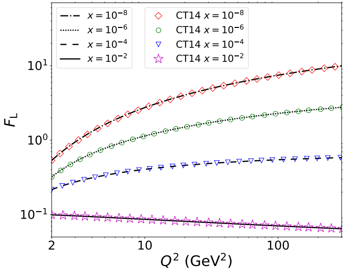
Derive DGLAP equations for these

Again, should agree with conventional DGLAP evolution

Comparison with conventional DGLAP evolution

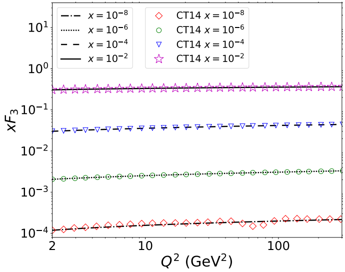


F_2 CT14 LO

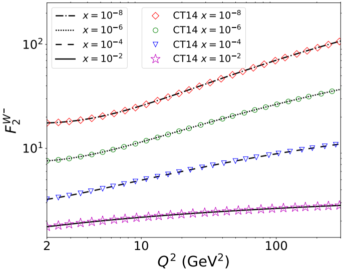


F_L CT14 LO

Match in full grid



xF_3 CT14 LO



F_2^W CT14 LO

Inverting g at NLO without going to Mellin space

(Work in progress)

Simple case without quarks

Invert $g(x)$ from $\widetilde{F}_L = C^{(1)}_{F_L g} \quad g + \frac{\alpha_s(Q^2)}{2\pi} C^{(2)}_{F_L g} \quad g \qquad \widetilde{F}_L(x;Q^2) \quad \frac{2\pi}{\alpha_s(Q^2)} \frac{F_L(x,Q^2)}{x}$

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Define differential operator $\hat{P}(x) \qquad \frac{1}{8 T_{\mathrm{R}} n_{\mathrm{f}} h e^2 i} \left[x^2 \frac{\mathrm{d}^2}{\mathrm{d}x^2} \quad 2x \frac{\mathrm{d}}{\mathrm{d}x} + 2 \right]$

Notice $g(x) = \hat{P}(x) \left[C^{(1)}_{F_{\mathrm{L}}g} \quad g \right]$

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Get $C^{(1)}_{F_{\mathrm{L}}g} \quad g$ from $\widetilde{F}_{\mathrm{L}}$: $\qquad C^{(1)}_{F_{\mathrm{L}}g} \quad g = \widetilde{F}_{\mathrm{L}} \quad \frac{\alpha_{\mathrm{s}}(Q^2)}{2\pi} C^{(2)}_{F_{\mathrm{L}}g} \quad g$

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$$g(x) = \hat{P}(x) \left[\widetilde{F}_L(x) \qquad \frac{\alpha_s(Q^2)}{2\pi} C^{(2)}_{F_L g} \qquad g \right]$$

Plug in $g(x) = \hat{P}(x) \widetilde{F}_L(x) + O\left(\quad _s(Q^2) \right)$ to the right hand side

$$g(x) = \hat{P}(x) \widetilde{F}_L(x) \qquad \frac{\alpha_s(Q^2)}{2\pi} \hat{P}(x) \left[C^{(2)}_{F_L g} \qquad \hat{P} \widetilde{F}_L \right] + O\left(\alpha_s^2(Q^2) \right)$$

Scheme dependence at NLO (work in progress)

DGLAP evolution in physical basis

$$\frac{\mathrm{d}F_i(x, Q^2)}{\mathrm{d}\log Q^2} = \sum_j P_{ij} \quad F_j(Q^2)$$

Kernels P_{ij} are independent of the factorization scheme

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P_{ij}'s determined by:

- Splitting functions
- Coefficient functions
  - ! The scheme dependence exactly cancels between these two

Summary

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- Scheme dependence of PDFs starts to play part at NLO in α_s
  ! By using a physical basis in second non-zero order in α_s, we will be able to avoid scheme dependence
- What next:
  - Establish physical basis at NLO (ongoing work)
  - Study how LHC cross sections, e.g. Drell-Yan, are expressed in physical basis
  - Obtain physical basis including also heavy quarks

Backup: Inverting the gluon PDF

Gluon PDF in mellin space

$$g(n) = \frac{1}{n_{\mathrm{f}} C^{(1)}_{F_{\mathrm{L}} g}(n)} \left[\frac{1}{he^2 i} \widetilde{F}_{\mathrm{L}}(n) \quad C^{(1)}_{F_{\mathrm{L}}}\ (n) \quad (n) \right]$$

$$\frac{1}{C^{(1)}_{F_{\mathrm{L}} g}(n)} = \frac{1}{8 T_{\mathrm{R}} z_0^n} \int_0^1 \mathrm{d}z z^{n+2} \delta^{00}(z \quad z_0),$$

where z_0 $2]0,1[$ is an arbitrary constant that cancels in final result.

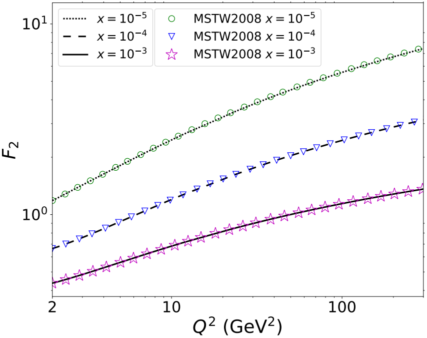
$$g(x,Q^2) = \int_x^1 \frac{\mathrm{d}z}{z} \delta(1 \quad z) \Bigg\{ \frac{C_{\mathrm{F}}}{4 T_{\mathrm{R}} n_{\mathrm{f}} he^2 i} \left[\frac{x}{z} \frac{\mathrm{d}}{\mathrm{d}\frac{x}{z}} \quad 2 \right] \frac{F_2\left(\frac{x}{z}, Q^2\right)}{\frac{x}{z}}$$

$$+ \frac{2\pi}{\alpha_{\mathrm{s}}(Q^2)} \frac{1}{8 T_{\mathrm{R}} n_{\mathrm{f}} he^2 i} \left[\frac{x^2}{z^2} \frac{\mathrm{d}^2}{\mathrm{d}\left(\frac{x}{z}\right)^2} \quad 2 \frac{x}{z} \frac{\mathrm{d}}{\mathrm{d}\frac{x}{z}} + 2 \right] \frac{F_{\mathrm{L}}\left(\frac{x}{z}, Q^2\right)}{\frac{x}{z}} \Bigg\}$$

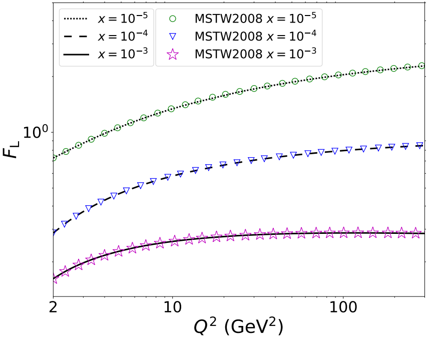
$$\frac{1}{n_{\mathrm{f}} he^2 i} \left\{ C_{g\widetilde{F^0}_2} \quad \widetilde{F^0}_2 + C_{g\widetilde{F}_2} \quad \widetilde{F}_2 + C_{g\widetilde{F^{00}}_{\mathrm{L}}} \quad \widetilde{F^{00}}_{\mathrm{L}} + C_{g\widetilde{F^0}_{\mathrm{L}}} \quad \widetilde{F^0}_{\mathrm{L}} + C_{g\widetilde{F}_{\mathrm{L}}} \quad \widetilde{F}_{\mathrm{L}} \right\}$$

Backup: Comparison with MSTW2008 PDF set

DGLAP evolution for F_2 and F_L in two observable physical basis



F_2 MSTW LO



F_L MSTW LO

Backup: PDFs in six observable physical basis

$$\begin{bmatrix} F_2 \\ F_3 \\ F_2^{W} \\ F_3^{W} \\ F_{2c}^{W} \end{bmatrix} = \begin{bmatrix} xe_d^2 & xe_d^2 & xe_u^2 & xe_u^2 & 2xe_s^2 \\ 2A_d & 2A_d & 2A_u & 2A_u & 0 \\ 0 & 2x & 2x & 0 & 2x \\ 0 & 2 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2x \end{bmatrix} \begin{bmatrix} d \\ \overline{d} \\ u \\ \overline{u} \\ \overline{s} \end{bmatrix},$$

where we have defined $A_q \quad L_q^2 \quad R_q^2$ in order to simplify equations.

$$xd(x,Q^2) = \frac{1}{A_u e_d^2 + A_d e_u^2}\Bigg[A_u F_2(x,Q^2) + \frac{e_u^2}{2} xF_3(x,Q^2) \quad \frac{A_u(2e_u^2 + e_d^2) \quad A_d e_u^2}{4} F_2^{W} \quad (x,Q^2)$$

$$\frac{A_u(2e_u^2 \quad e_d^2) + A_d e_u^2}{4} xF_3^{W} \quad (x,Q^2) + \frac{A_u(e_d^2 \quad 2e_s^2) \quad A_d e_u^2}{2} F_{2c}^{W} \quad (x,Q^2)\Bigg]$$

$$x\overline{d}(x,Q^2) = \frac{1}{4} F_2^{W} \quad (x,Q^2) \quad \frac{1}{4} xF_3^{W} \quad (x,Q^2) \quad \frac{1}{2} F_{2c}^{W}$$

$$xu(x,Q^2) = \frac{1}{4} F_2^{W} \quad (x,Q^2) + \frac{1}{4} xF_3^{W} \quad (x,Q^2)$$

$$x\overline{u}(x,Q^2) = \frac{1}{A_u e_d^2 + A_d e_u^2}\Bigg[A_d F_2(x,Q^2) \quad \frac{e_d^2}{2} xF_3(x,Q^2) \quad \frac{A_d(2e_d^2 + e_u^2) \quad A_u e_d^2}{4} F_2^{W} \quad (x,Q^2)$$

$$+ \frac{A_d(2e_d^2 \quad e_u^2) + A_u e_d^2}{4} xF_3^{W} \quad (x,Q^2) + A_d(e_d^2 \quad e_s^2) F_{2c}^{W} \quad (x,Q^2)\Bigg]$$

$$x\overline{s}(x,Q^2) = xs(x,Q^2) = \frac{1}{2} F_{2c}^{W} \quad (x,Q^2)$$

Backup: Results in six observable basis

$$\frac{dF_2(x,Q^2)}{d\log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi}\left[P_{qq} \quad \widetilde{F}_2 + 2\sum_q^{n_f} e_q^2 P_{qg} \quad g\right], \quad \text{where } \widetilde{F}_2(x,Q^2) \quad \frac{F_2(x,Q^2)}{x}$$

$$\frac{dxF_3(x,Q^2)}{d\log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} xP_{qq} \quad F_3$$

$$\frac{dF_2^W \quad (x,Q^2)}{d\log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} x\left[P_{qq} \quad \widetilde{F}_2^W \quad + 6P_{qg} \quad g\right], \quad \text{where } \widetilde{F}_2^W \quad (x,Q^2) \quad \frac{F_2^W \quad (x,Q^2)}{x}$$

$$\frac{dxF_3^W \quad (x,Q^2)}{d\log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} x\left[P_{qq} \quad F_3^W \quad 2P_{qg} \quad g\right]$$

$$\frac{dF_{2c}^W \quad (x,Q^2)}{d\log(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} x\left[P_{qq} \quad \widetilde{F}_{2c}^W \quad + 2P_{qg} \quad g\right], \quad \text{where } \widetilde{F}_{2c}^W \quad (x,Q^2) \quad \frac{F_{2c}^W \quad (x,Q^2)}{x}$$

$$P_{qg} \quad g = \frac{C_F}{4\sum_q^{n_f} e_q^2}\left[\quad \widetilde{F}_2 + 2(z \quad 1) \quad \widetilde{F}_2\right] + \frac{1}{4\sum_q^{n_f} e_q^2}\left[\left(\frac{1}{x} \quad \frac{1}{2}\frac{d}{dx}\right) F_L(x,Q^2) + 1 \quad \widetilde{F}_L\right]$$

Backup: Results in six observable basis

$$\frac{d}{d\log(Q^2)}\left(\frac{F_L(x,Q^2)}{\frac{\alpha_s(Q^2)}{2\pi}}\right) =$$

$$\frac{\alpha_s(Q^2)}{2\pi}x\Big\{\left[2C^{(1)}_{F_L\widetilde{F}_2}\quad P_{qg} + C^{(1)}_{F_L g}\quad P_{gg}\right]\quad\left[C_{g\widetilde{F^{00}}_L}\quad \widetilde{F^{00}}_L + C_{g\widetilde{F^{0}}_L}\quad \widetilde{F^{0}}_L + C_{g\widetilde{F}_L}\quad \widetilde{F}_L\right]$$

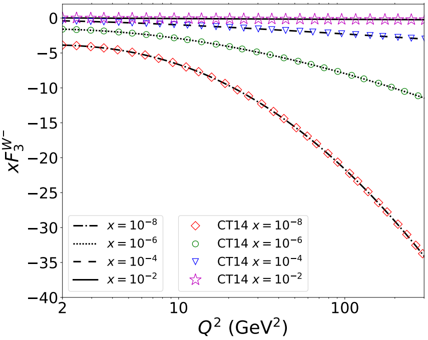
$$+\left[C^{(1)}_{F_L\widetilde{F}_2}\quad\left(P_{qq} + 2P_{qg}\quad C_{g\widetilde{F}_2}\right) + C^{(1)}_{F_L g}\quad P_{gg}\quad C_{g\widetilde{F}_2}\right]\quad \widetilde{F}_2$$

$$+\left[2C^{(1)}_{F_L\widetilde{F}_2}\quad P_{qg} + C^{(1)}_{F_L g}\quad P_{gg}\right]\quad C_{g\widetilde{F^{0}}_2}\quad \widetilde{F^{0}}_2 + \sum_q^{n_f} e_q^2 C^{(1)}_{F_L g}\quad P_{gq}\quad \sum_q (q+\overline{q})\Big\}$$

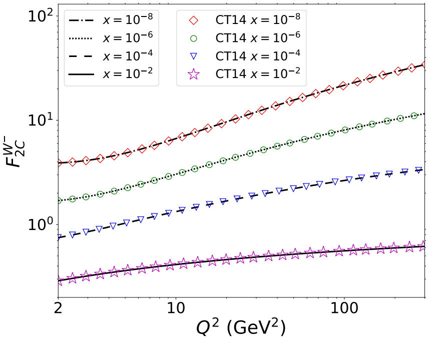
where $\sum_q (q+\overline{q})$ can be expressed in physical basis (see slide 13/10)

Backup: Comparison with conventional DGLAP evolution

Comparison for structure functions F_3^{W} and F_{2c}^{W} in six observable physical basis



F_3^{W} CT14 LO



F_{2c}^{W} CT14LO