

Transverse momentum broadening from NLL BFKL to all orders in pQCD

Yacine Mehtar-Tani (BNL & RBRC)

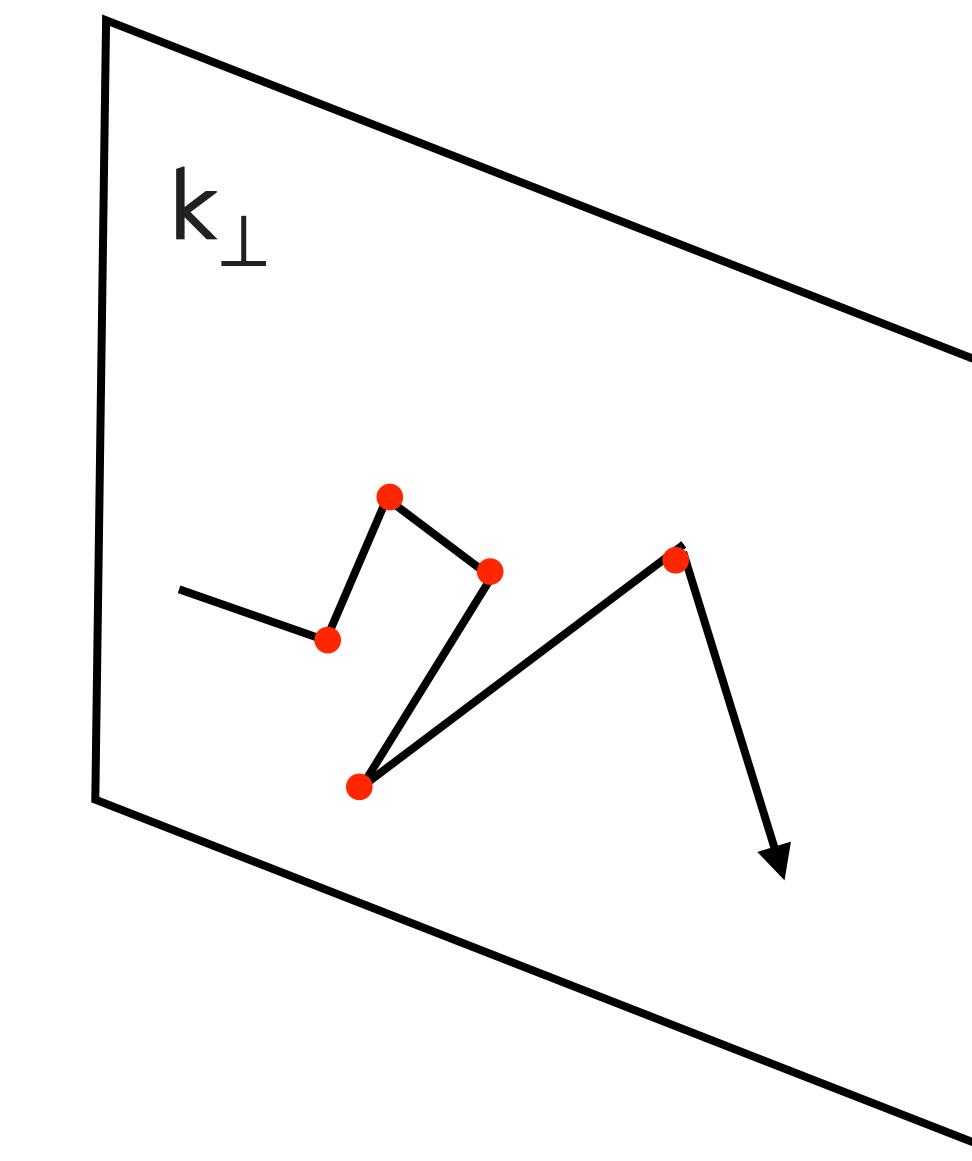
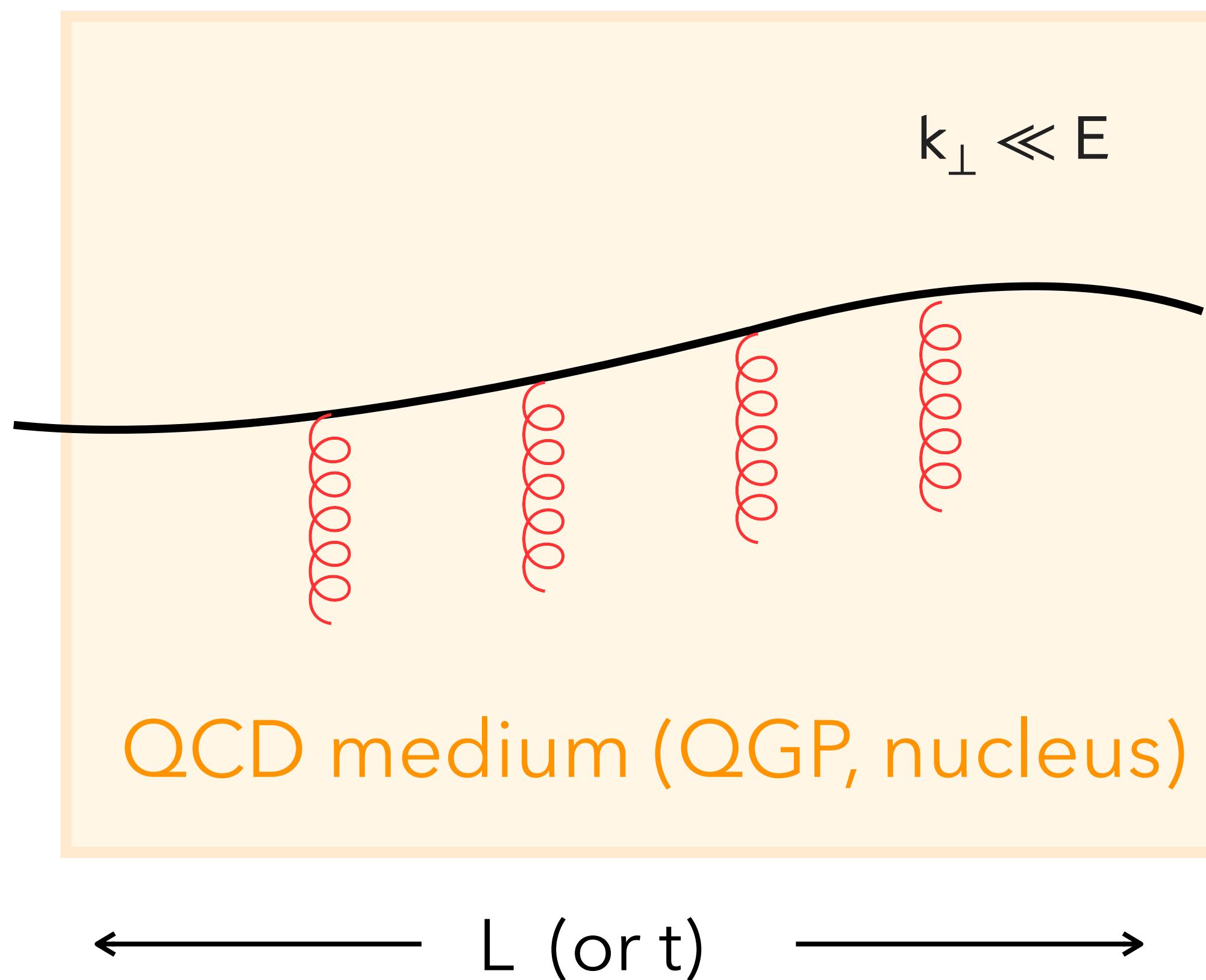
Initial Stages, June 20, 2023 @ Copenhagen

Based on: 2109.12041 [hep-ph] 2203.09407 [hep-ph] 2209.08900 [hep-ph]
In collaboration with Paul Caucal



Diffusion of partons in transverse momentum space

High energy partons experience random kicks in hot or cold nuclear matter that cause their transverse momentum to increase over time



Diffusion in transverse plane

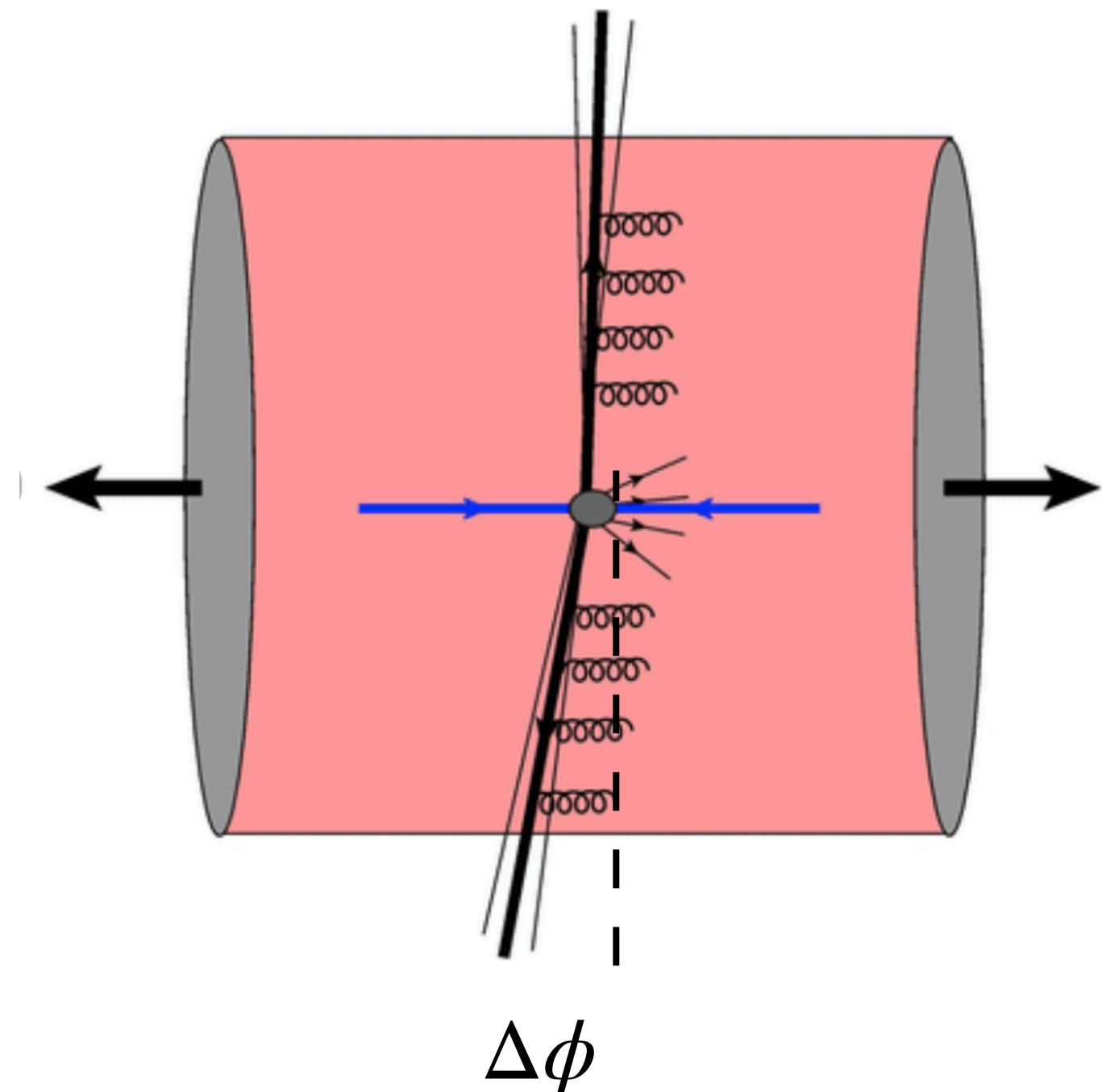
Observables

Heavy Ion Collisions:

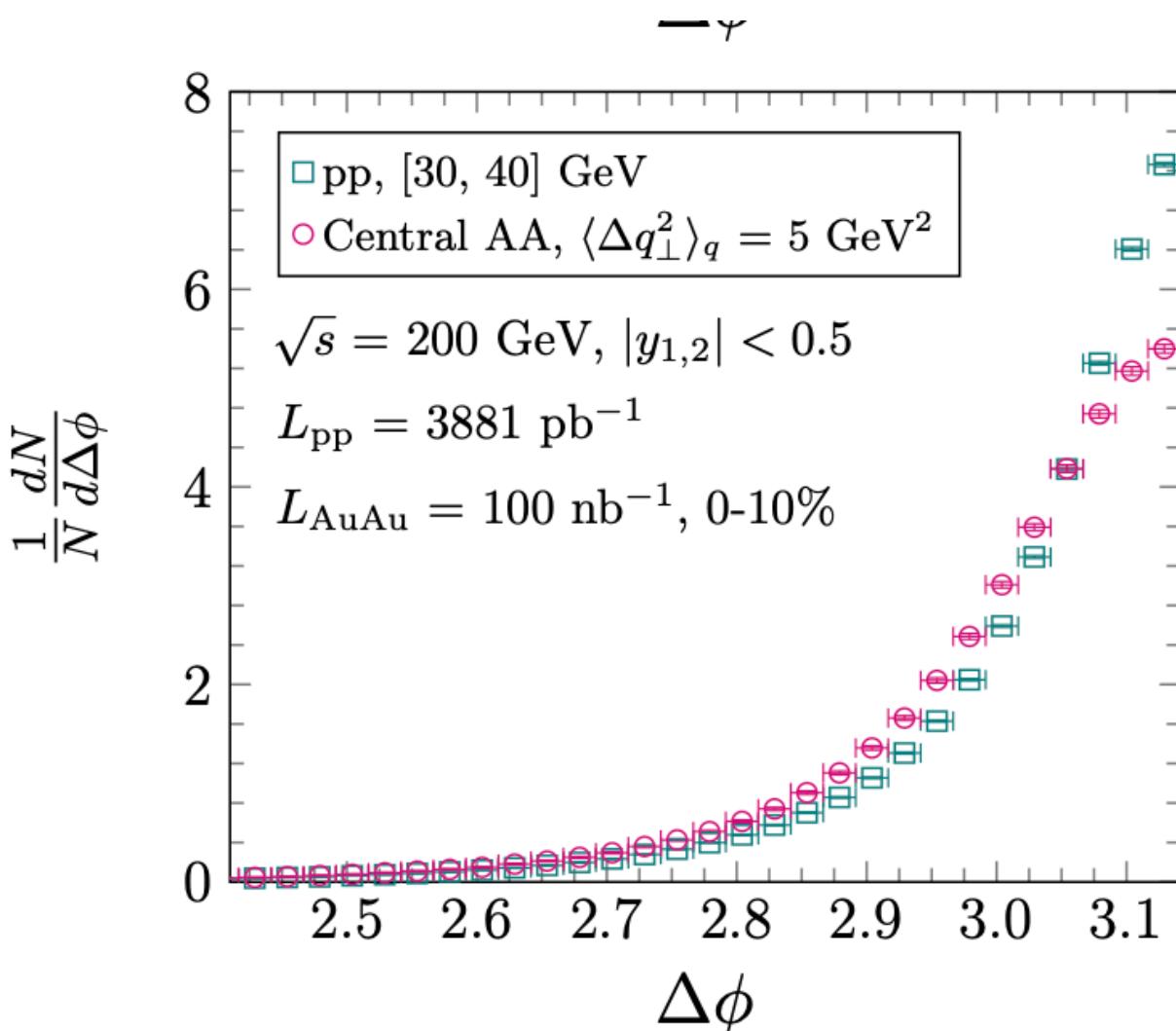
jet quenching

dijet azimuthal de-correlation

Mueller, Wu, Xiao, Yuan (2016)

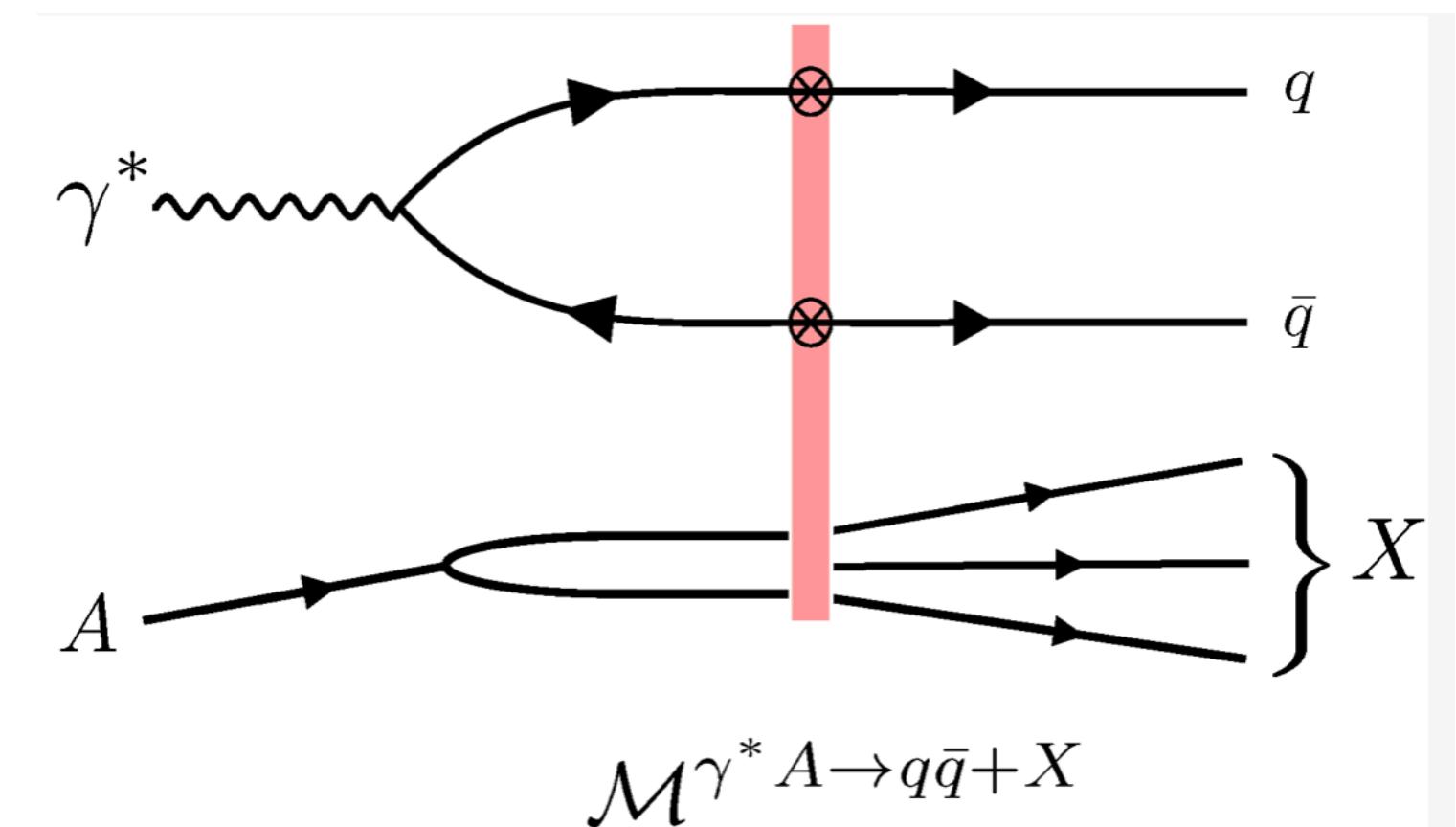


Jia, Xiao, Yuan (2019)

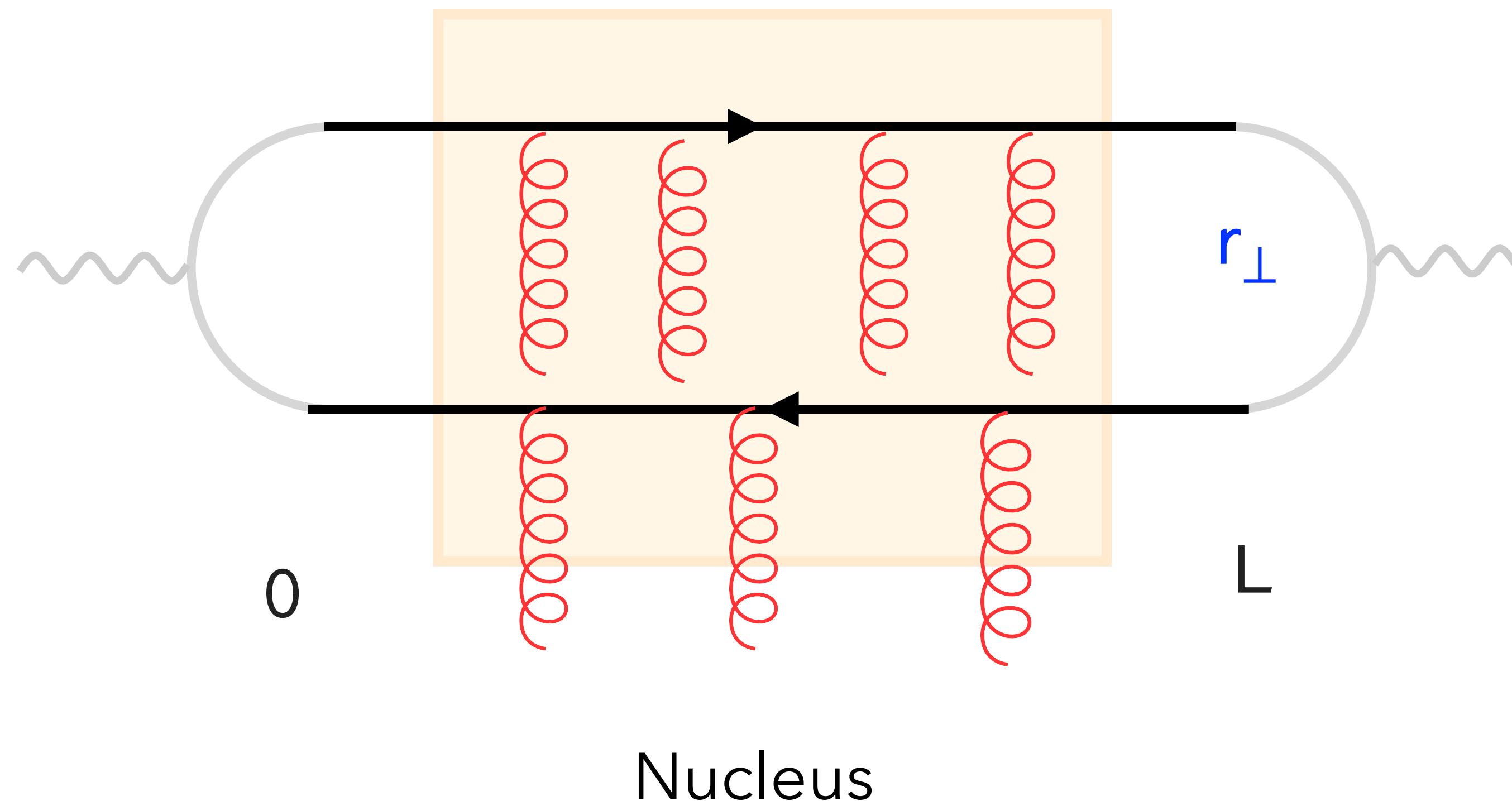


Cold nuclear matter:

SIDIS, forward dijet
(dihadron) production
in eA/pA , etc



Cold matter: small- x initial condition



Dipole S-matrix

$$S(x_\perp) \simeq \exp \left[-\frac{1}{4} Q_s^2 r_\perp^2 \right]$$

Saturation scale

$$Q_s^2 \equiv \hat{q}L \sim A^{1/3}$$

Mueller, McLerran-Venugopalan, Golec-Biernat-Wüsthof...

Hot matter: transverse momentum broadening

- Probability: Gaussian for $k_\perp \sim Q_s \sim \hat{q} L$ and power law tail k_\perp^{-4} for $k_\perp > Q_s$

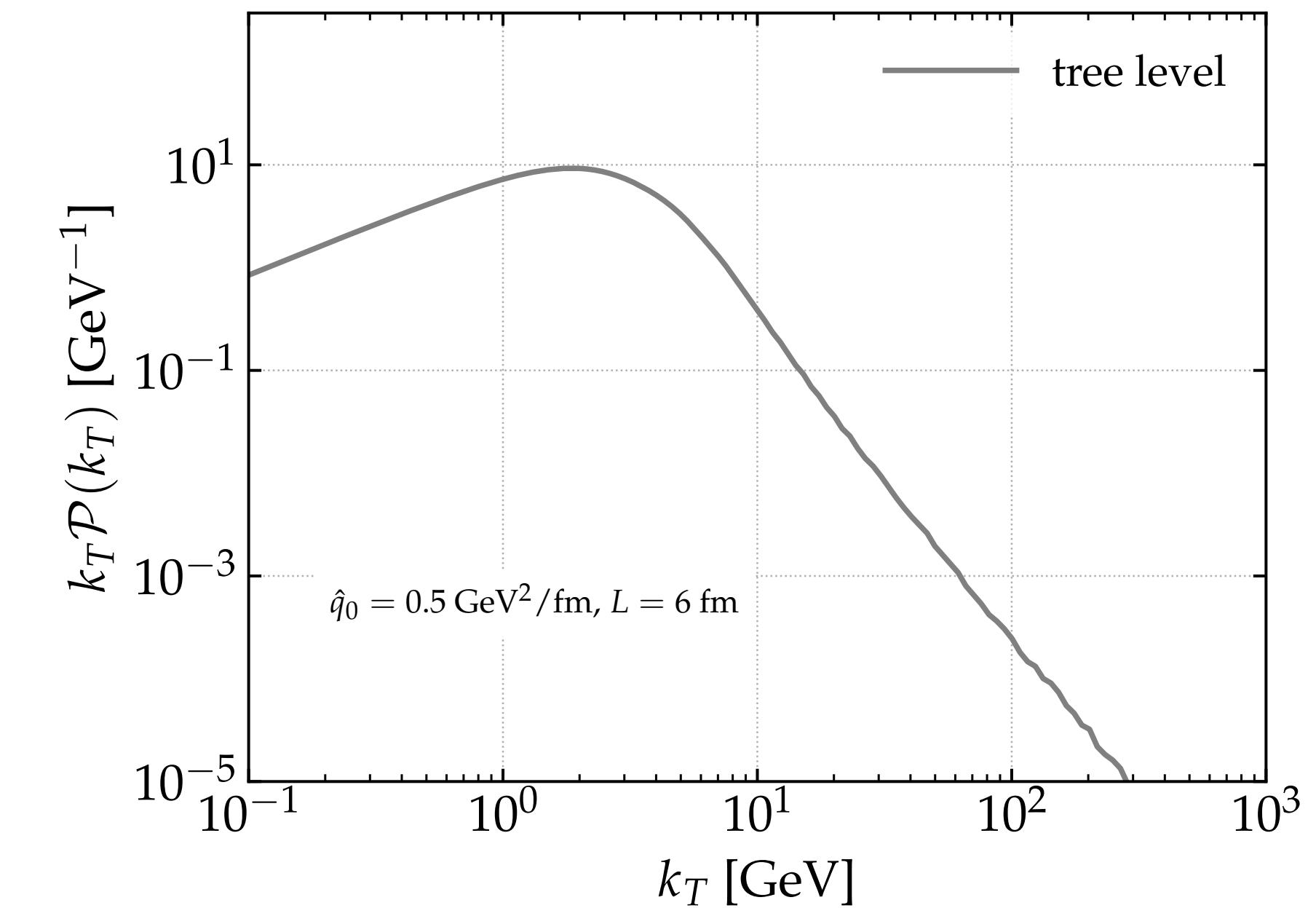
$$P(k_\perp) \equiv \int d^2x_\perp S(x_\perp) e^{-ix_\perp \cdot k_\perp}$$

$$\approx \frac{4\pi}{Q_s^2} \exp\left(-k_\perp^2/Q_s^2\right)$$

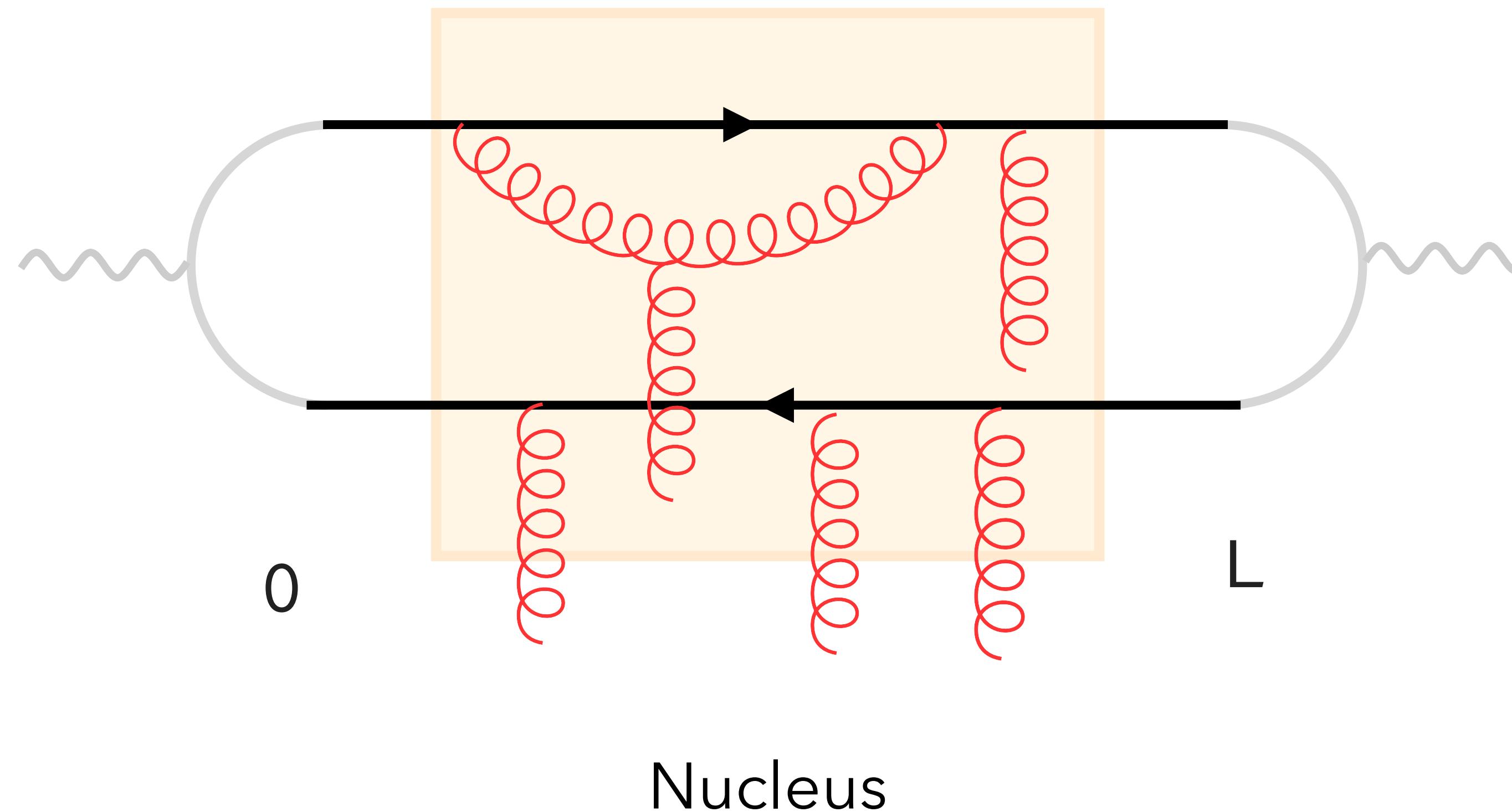
- Normal diffusion in transverse momentum

$$\langle k_\perp^2 \rangle_{\text{typ}} \propto \hat{q} t$$

$$\hat{q} = C_R n \int_{q_\perp} q_\perp^2 \frac{d^2\sigma_{\text{el}}}{d^2q_\perp} \sim \bar{\alpha}_s^2 n \log \frac{q_{\max}^2}{\mu^2}$$



Effects of quantum corrections on transverse momentum broadening?



- Short-lived quantum fluctuations inside the nucleus enhanced by system size $\ln L \sim \ln A^{1/3}$

[See talk by P. Arnold on Monday]

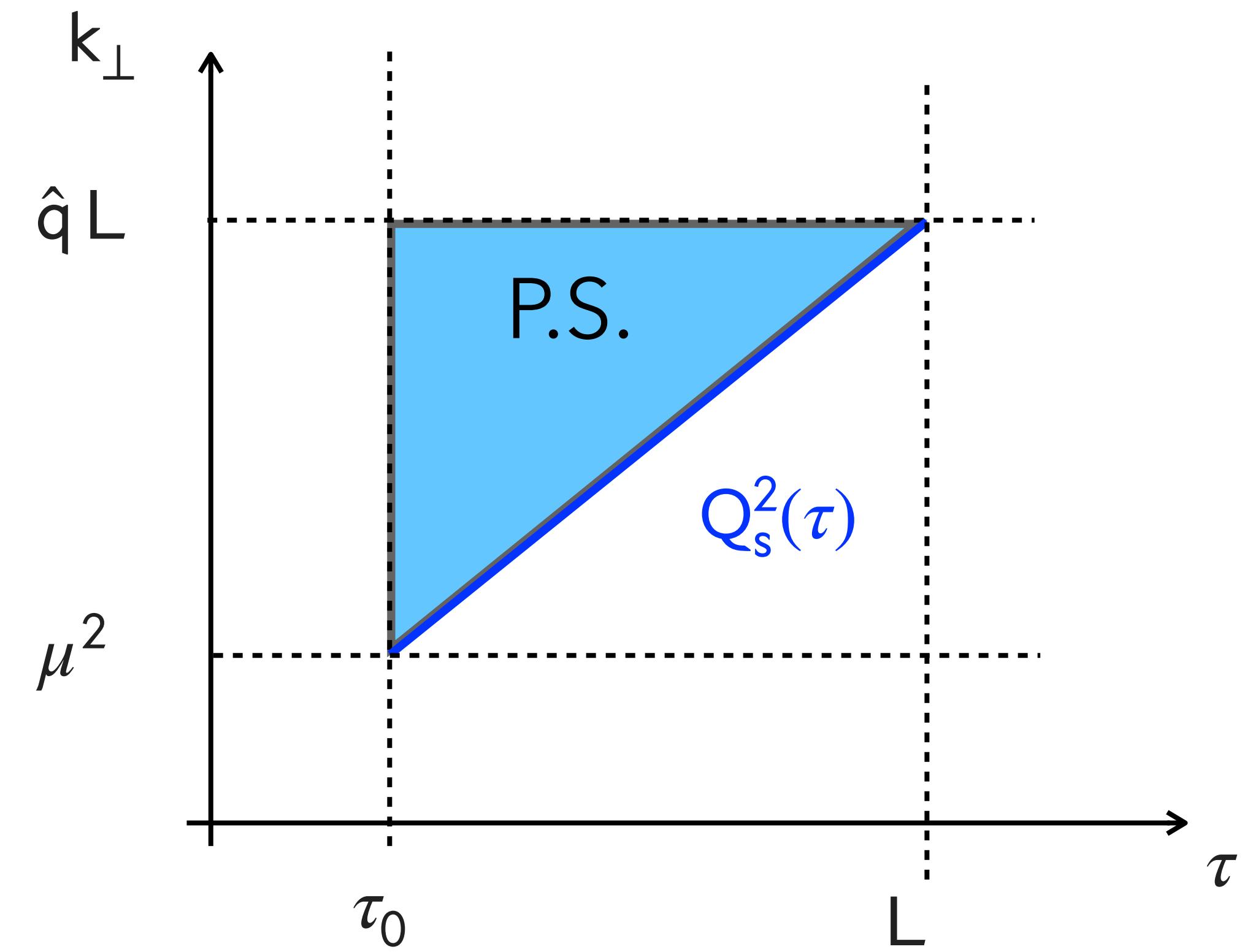
- Double log result for TMB:

$$\langle k_\perp^2 \rangle = \hat{q}_0 L \left(1 + \frac{\bar{\alpha}}{2} \log^2 \frac{L}{\tau_0} \right)$$

[Liou, Mueller, Wu (2013)
Blaizot, Dominguez, Iancu, MT (2014)]

- Single log (connection to HTL):

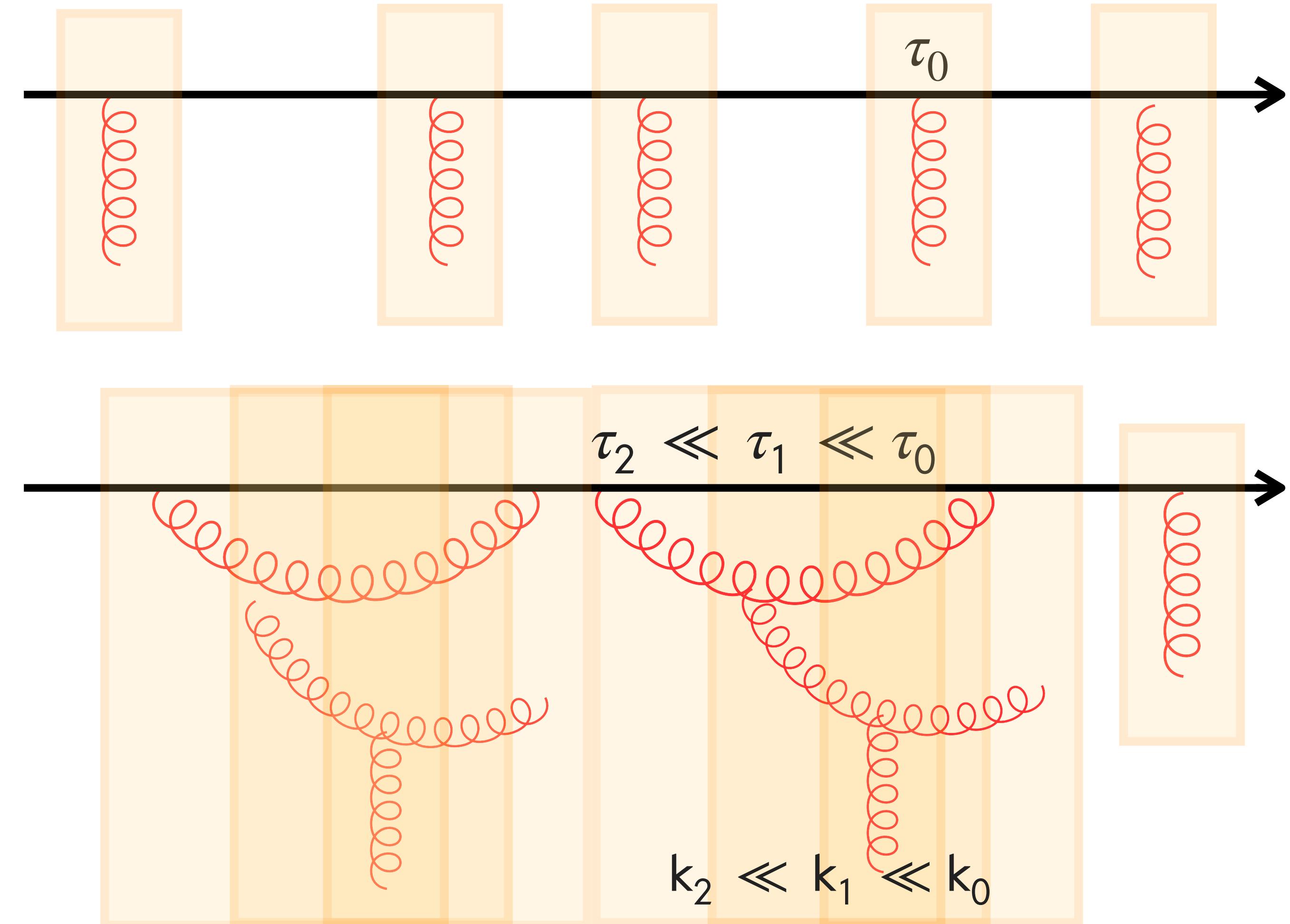
[Arnold (2021) Ghiglieri, Weitz (2022)]



- Saturation line $Q_s^2(\tau) \equiv \hat{q}(Q_s, \tau) \tau$: Multiple-scattering screen mass singularity
- Not the standard DGLAP double log: the factor 1/2 reflects the presence of multiple scattering constraint $\hat{q}\tau \ll k_\perp \ll \hat{q}L$

Physical picture (all orders)

- LO: local/instantaneous interactions
- DLA + saturation: quasi-local interactions
- Exponentiation of the double logs with adequate phase space constraints



Emissions strongly ordered in k_\perp and τ

Anomalous diffusion

- Scaling solution for large L : $x = r_\perp^2 Q_s^2(L)$ (akin to geometric scaling at small x)
- From Gaussian to Stretched exponential:

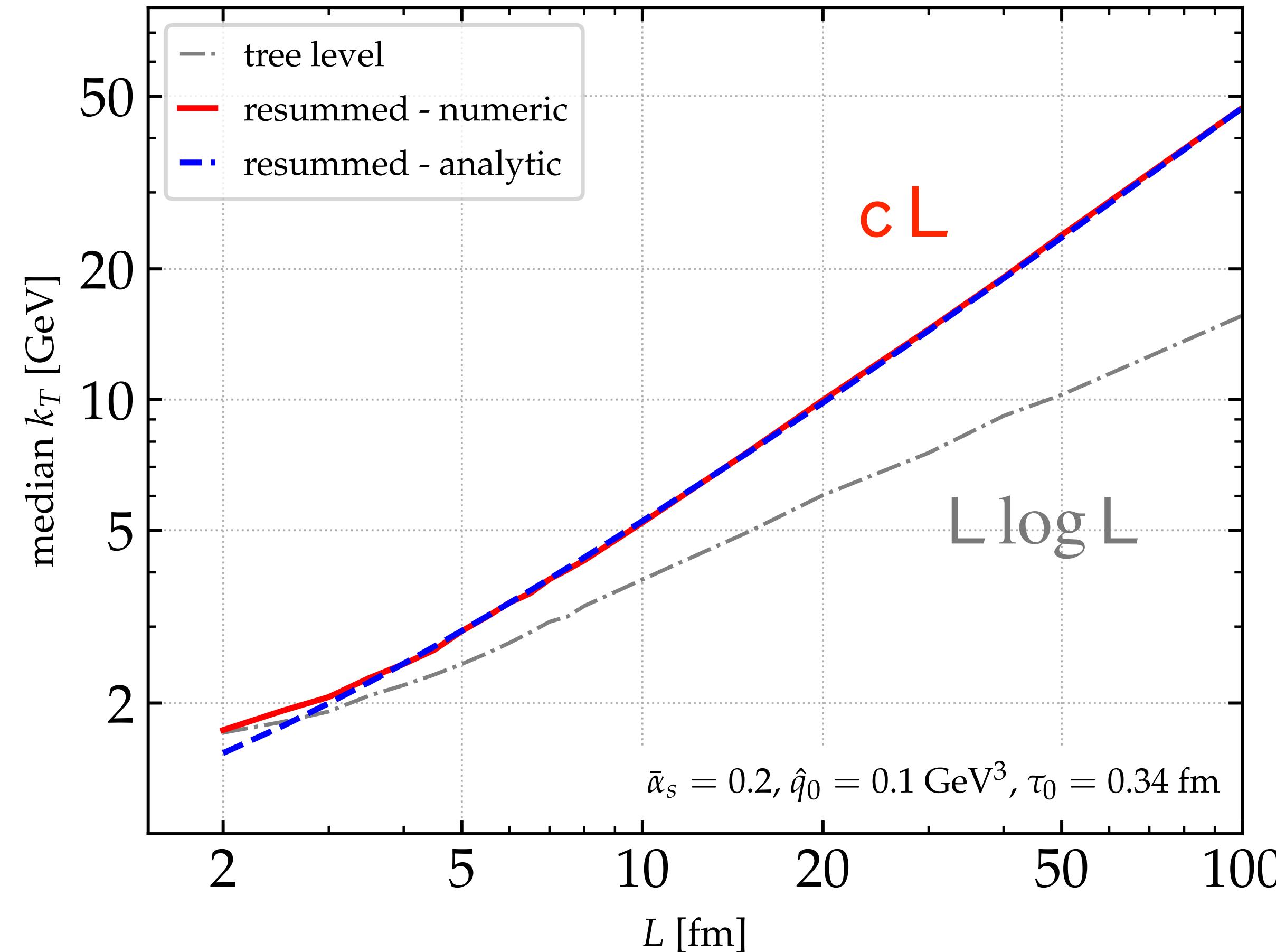
$$S(r_\perp, L) \simeq \exp \left[-\frac{1}{4} (\hat{q}^{(0)} + \bar{\alpha} \hat{q}^{(1)} + O(\bar{\alpha}^2)) L r_\perp^2 \right] \rightarrow \exp [-(r_\perp^2 Q_s^2(L))^\gamma]$$

- Saturation scale:
- Anomalous dimension:

$$\langle k_\perp^2 \rangle_{\text{typ}} \sim Q_s^2(L) \sim L^c$$

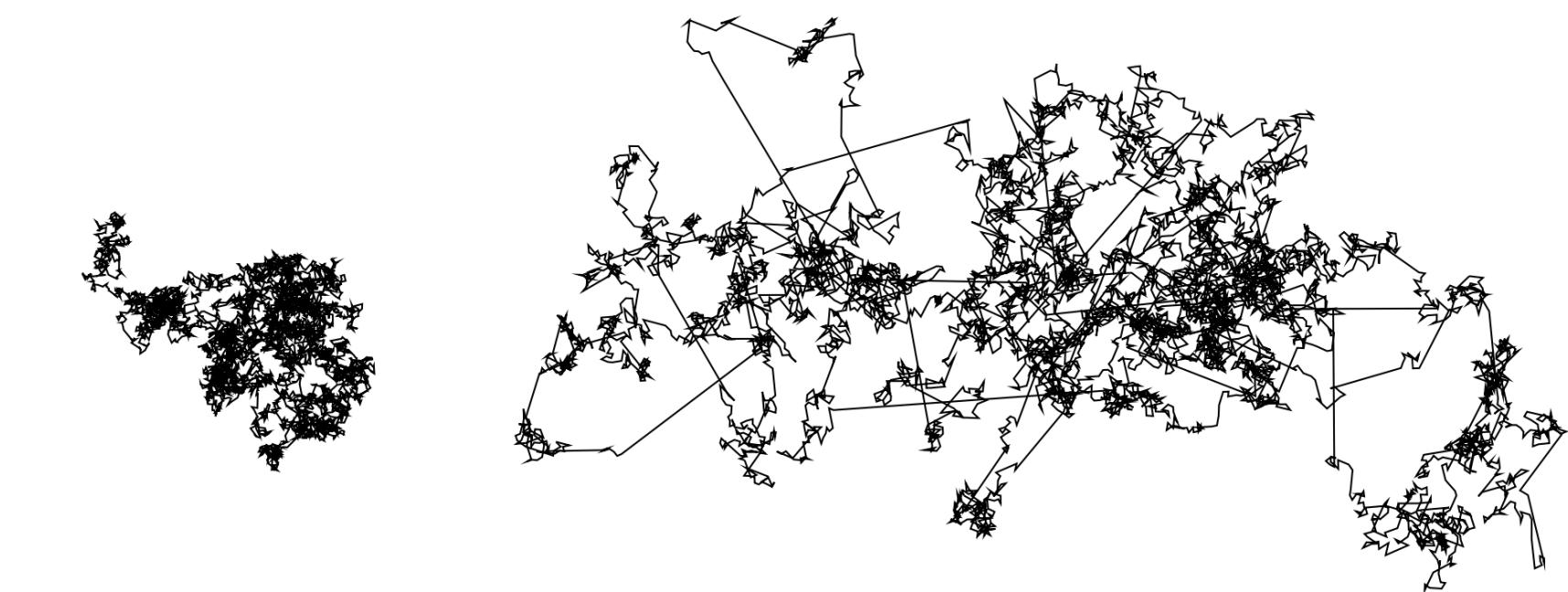
$$\gamma = 1 - 2\sqrt{\bar{\alpha}} \quad c = 1 + 2\sqrt{\bar{\alpha}}$$

L dependence of transverse momentum



Super diffusion

Normal diffusion



Brownian motion

Lévy flight

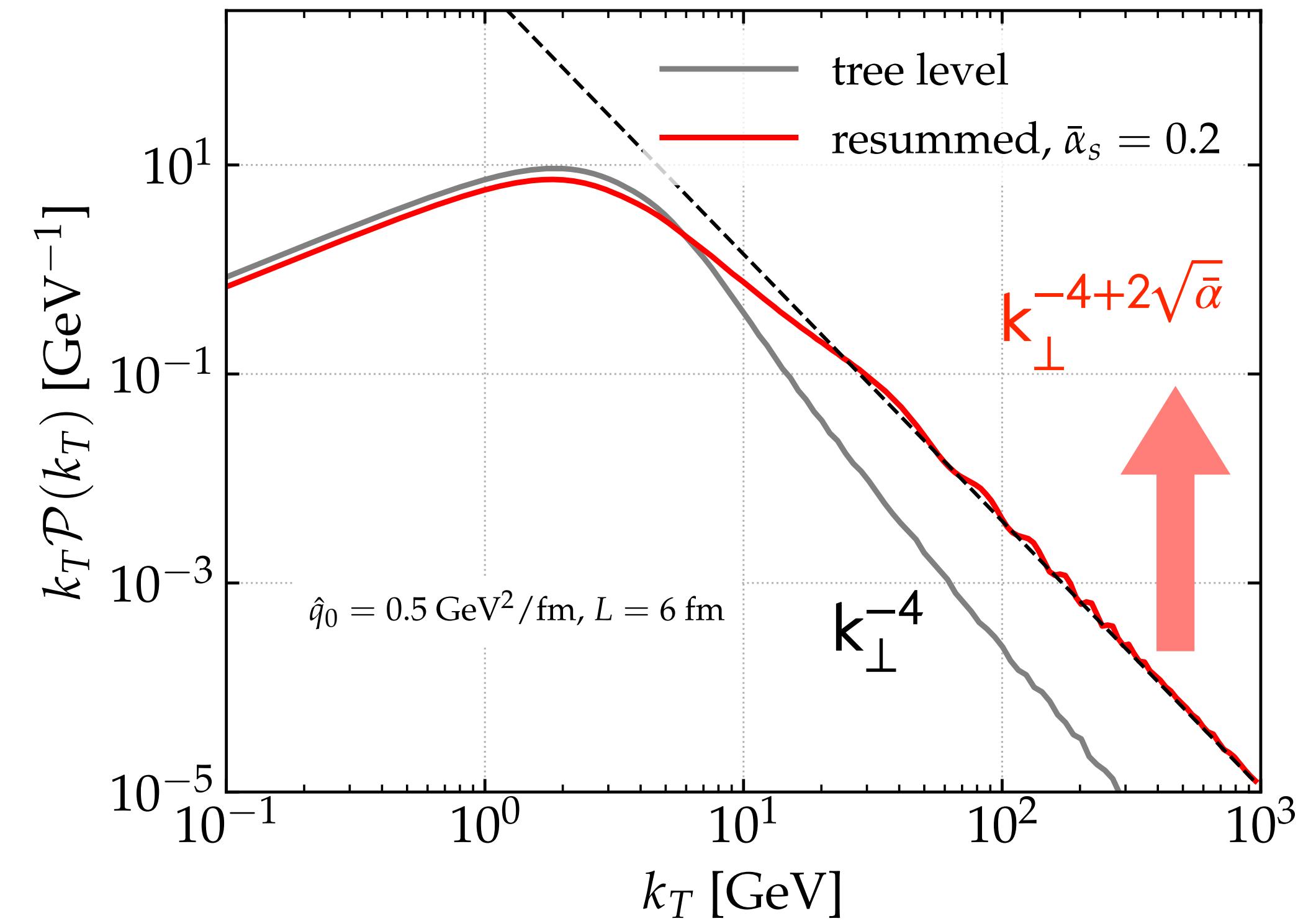
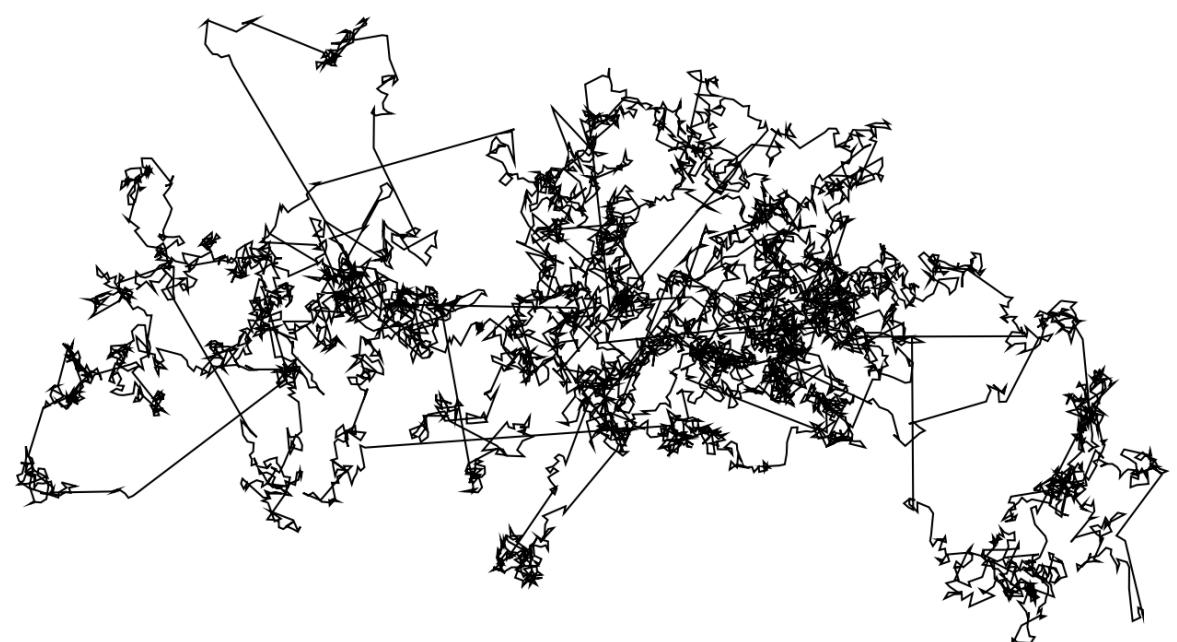
$$c = 1$$

$$c > 1$$

Heavy tail - Lévy random walk

- Lévy flight: deviations from Rutherford scattering (LO)

$$P(k_{\perp}) \rightarrow f(k_{\perp}/Q_s(L)) \simeq \left(\frac{Q_s(L)}{k_{\perp}} \right)^{4-2\sqrt{\bar{\alpha}}}$$



→ Quantum evolution yields heavy power law tail

Universal pre-asymptotic solution at fixed coupling

YMT, P. Caucal 2109.12041 [hep-ph]

- Saturation scale : $\ln Q_s^2(Y) = cY + b \log Y + \text{const.}$ $Y \equiv \ln \frac{L}{\tau_0}$ $x \equiv \ln \frac{1}{Q_s^2 r_\perp^2}$

- Shape of the wave front $r_\perp < 1/Q_s$:

$$\ln S(x_\perp, L) = \frac{1}{4} \exp \left((1 + \beta)x - \frac{\beta x^2}{4cY} \right) \left[1 + \beta x + \frac{bx}{c^2 Y} \left(1 + \frac{\beta(c+4)x}{6} \right) + \mathcal{O}(Y^{-2}) \right]$$

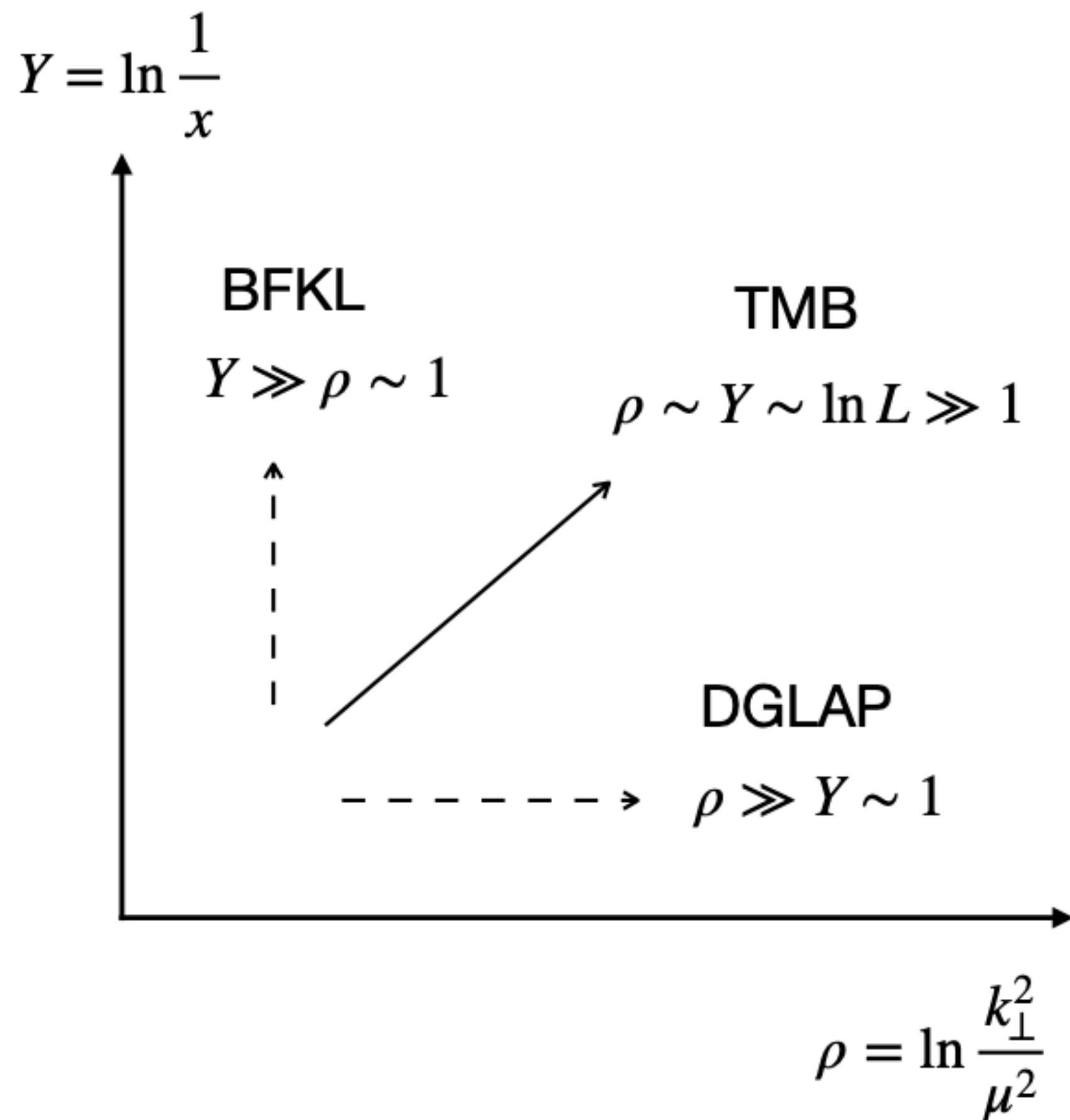
- Velocity of the wave front:

$$c \simeq 1 + 2\sqrt{\bar{\alpha}}$$

$$\beta = \frac{c-1}{2c} \quad b = -\frac{2}{3(1-\beta)}$$

Next-to-double logarithm (NDL) evolution

- Resummation of NDL from BFKL or DGLAP + saturation boundary



- Running coupling scaling variable

$$x \equiv \ln \frac{k_\perp^2}{Q_s^2} \rightarrow x \equiv \frac{\ln \frac{k_\perp^2}{Q_s^2}}{\sqrt{Y}} \sim \sqrt{\bar{\alpha}(Y)} \ln \frac{k_\perp^2}{Q_s^2}$$

NLL BFKL + saturation boundary

Caucal, MT, 2209.08900 [hep-ph]

- Solve linear NLL-BFKL imposing $\rho > \rho_s(Y) = \ln Q_s^2(Y)/\mu^2$ (non-linear effects)

$$\frac{\partial \hat{q}}{\partial Y} = \chi_{\text{LL}}(\partial_\rho) [\bar{\alpha}_s(\rho) \hat{q}(\rho, Y)] + \bar{\alpha}_s^2(\rho) \tilde{\chi}_{\text{NLL}}(\partial_\rho) \hat{q}(\rho, Y)$$

$$Y \equiv \ln \frac{L}{\tau_0}$$
$$\rho \equiv \ln \frac{1}{\mu^2 r_\perp^2}$$

[Mueller, Triantafyllopoulos (2002) (Iancu, Itakura, McLerran (2002) Munier, Peschanski (2003)]

- Expansion of BFKL kernel around DLA: $\gamma = 0$

$$\chi_{\text{LL}}(\gamma) = \frac{1}{\gamma} + 2\zeta(3)\gamma^2 + \mathcal{O}(\gamma^4), \quad \tilde{\chi}_{\text{NLL}} \sim B_g/\gamma^2 + a_{1,-1}/\gamma$$

Saturation scale $\rho_s(Y) = \ln Q_s^2(Y)/\mu^2$

$$Y \equiv \ln \frac{L}{\tau_0}$$

Caucal, MT, 2109.12041 [hep-ph] 2203.09407 [hep-ph]

$$\begin{aligned} \rho_s(Y) = & Y + 2\sqrt{4b_0Y} + 3\xi_1(4b_0Y)^{1/6} + \left(\frac{1}{4} - 2b_0\right) \ln(Y) + \kappa \\ & + \frac{7\xi_1^2}{180} \frac{1}{(4b_0Y)^{1/6}} + \xi_1 \left(\frac{5}{108} + 18b_0\right) \frac{1}{(4b_0Y)^{1/3}} + b_0 (1 - 8b_0) \frac{\ln(Y)}{\sqrt{4b_0Y}} + \mathcal{O}(Y^{-1/2}) \end{aligned}$$

First four terms conjectured by Iancu and Triantafyllopoulos (2015)

- Non universal terms start at order $Y^{-1/2}$ as can be seen by the substitution $Y \rightarrow Y + Y_0$
- NNLO BFKL and beyond do not contribute to the universal terms

Summary

- TMB in QCD is a **super-diffusive process** (non-gaussian) due to logarithmically enhanced quantum corrections → **anomalous system size dependence**
- Systematic approach for computing **universal asymptotic** and pre-asymptotic solutions for transverse momentum broadening
- Using DGLAP or **NLL BFKL** (due to the DL nature of the problem) we have **computed all of the universal terms** in the asymptotic expansion of the **saturation scale**
- Outlook: Jet quenching phenomenology, **non-Gaussian initial condition** for BK evolution at small x

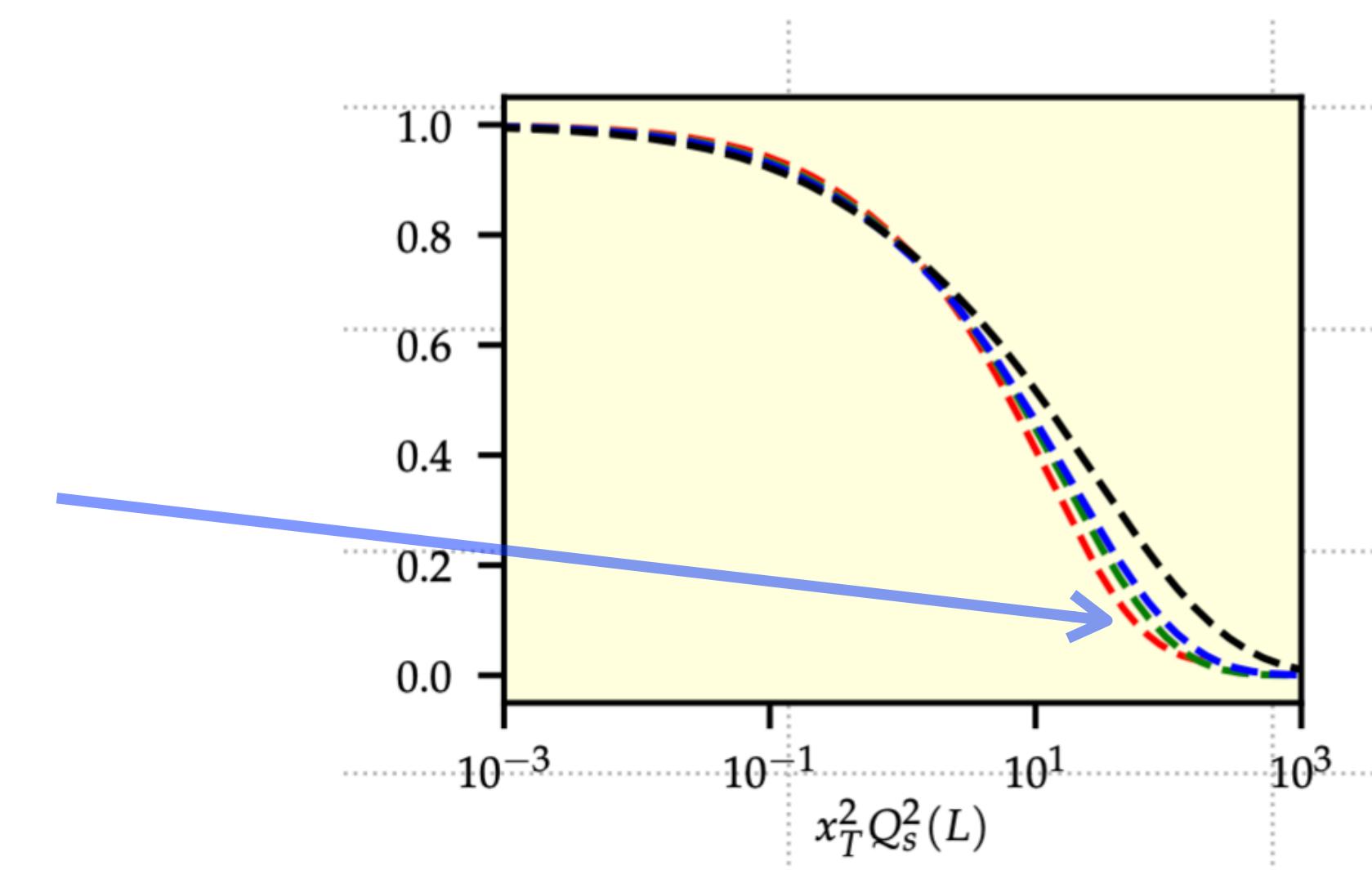
Backup

- We look for a solution of the form (around the scaling solution)

$$\hat{q}(\rho, Y) = e^{\rho_s(Y)-Y} e^{\beta x} f(x, Y), \quad x = \frac{\rho - \rho_s(Y)}{\sqrt{Y}}$$

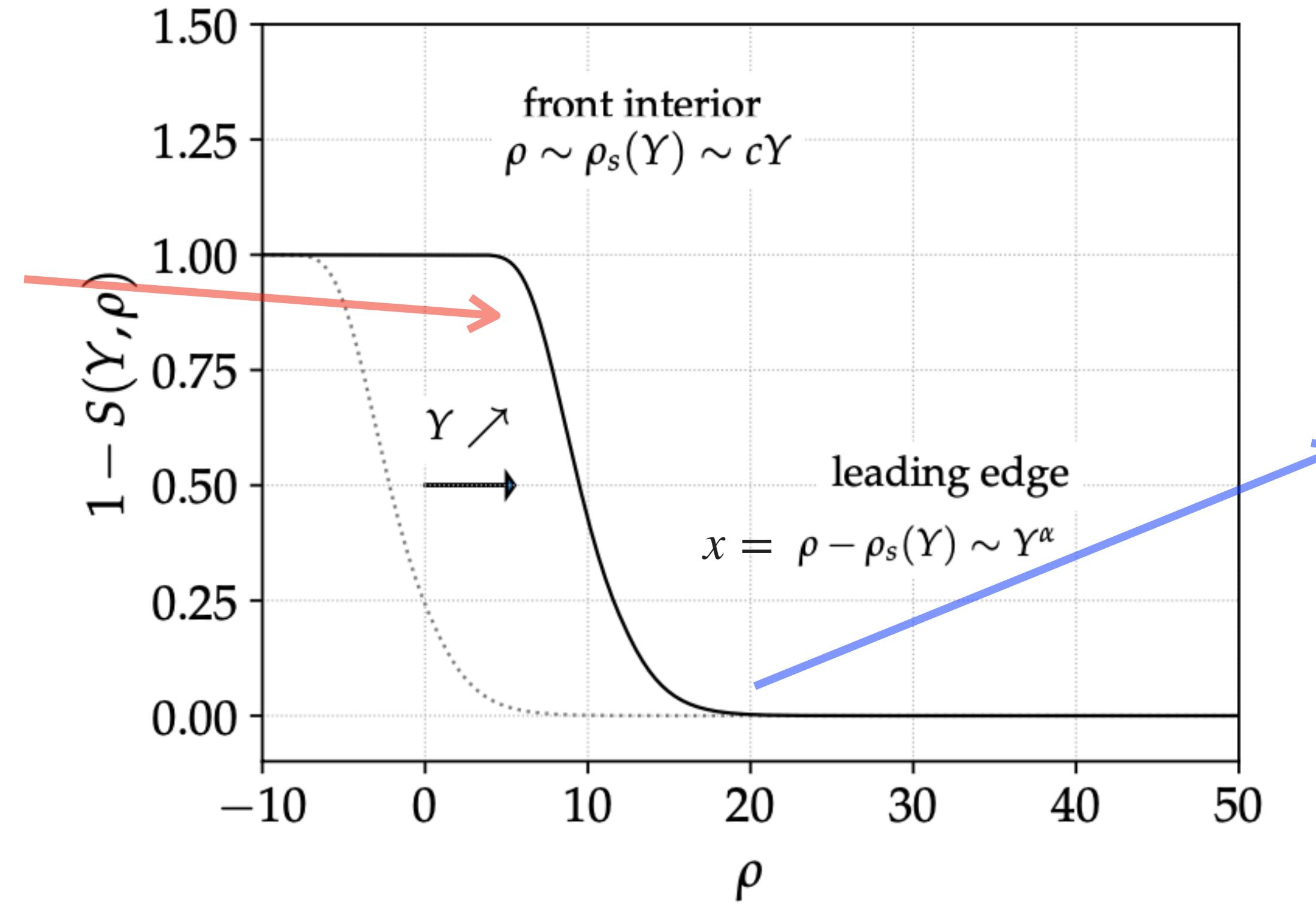
- Pre-asymptotic corrections obtained using the leading-edge expansion

$$f(x, Y) = \sum_{n=-1}^{\infty} Y^{-n/6} G_n \left(\frac{x}{Y^{1/6}} \right)$$



Front interior and leading edge expansions

Front interior
expansion
(Saturation line)



Leading edge: growth of perturbations around the unstable state $S = 0$
(diffusion of the wave front)

U. Ebert and W. van Saarloos (2000)

$$\frac{\hat{q}(Y, k_{\perp}) L}{Q_s^2} = e^{\beta x} \left[f_0(x) + \frac{1}{Y^{1/2}} f_1(x) + \frac{1}{Y} f_2(x) + \dots \right]$$

$$\frac{\hat{q}(Y, k_{\perp}) L}{Q_s^2} = e^{\beta x} \left[Y^{1/2} G_1 \left(\frac{x}{Y^{1/2}} \right) + G_0 \left(\frac{x}{Y^{1/2}} \right) + \frac{1}{Y^{1/2}} G_{-1} \left(\frac{x}{Y^{1/2}} \right) + \dots \right]$$

Boundary conditions at $\rho = \ln k_{\perp}^2 / \mu^2 \rightarrow \infty$ constrain the saturation line $Q_s(Y)$

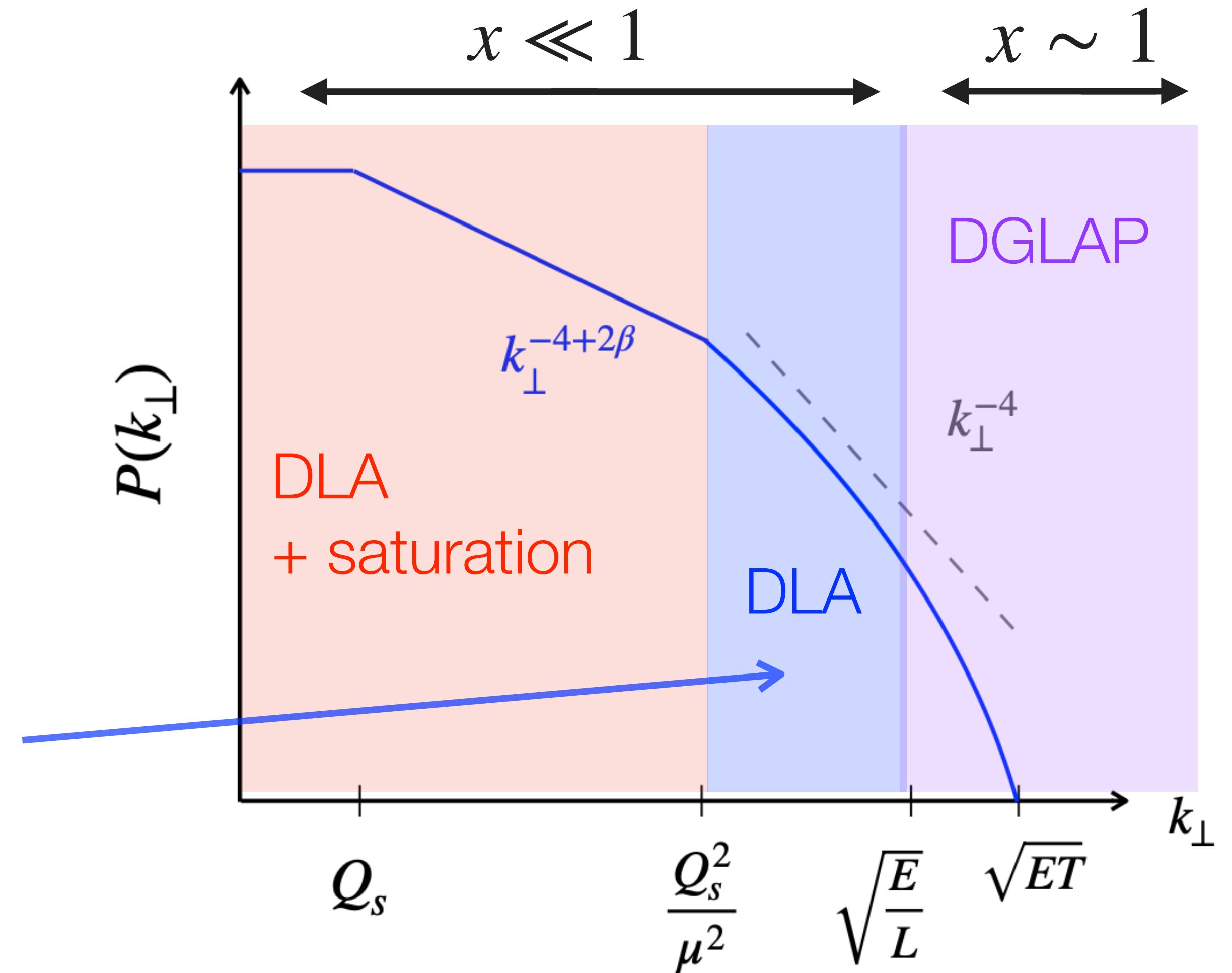
Larger momentum transfer

- Coulomb power law k_\perp^{-4} for $k_\perp \gg Q_s^2/\mu$

$$P(k_\perp) \simeq \frac{L}{k_\perp^4} \frac{\partial}{\partial \log k_\perp^2} \hat{q}(k_\perp^2, L)$$

- Hard scattering regime: gluon PDF of the plasma at DLA:

$$\hat{q}(k_\perp^2) \propto x g(x, k_\perp^2) \simeq \exp \left(\sqrt{\log \frac{k_\perp^2}{\mu^2} \log \frac{L}{\tau_0}} \right)$$



Caucal, MT

2203.09407 [hep-ph]

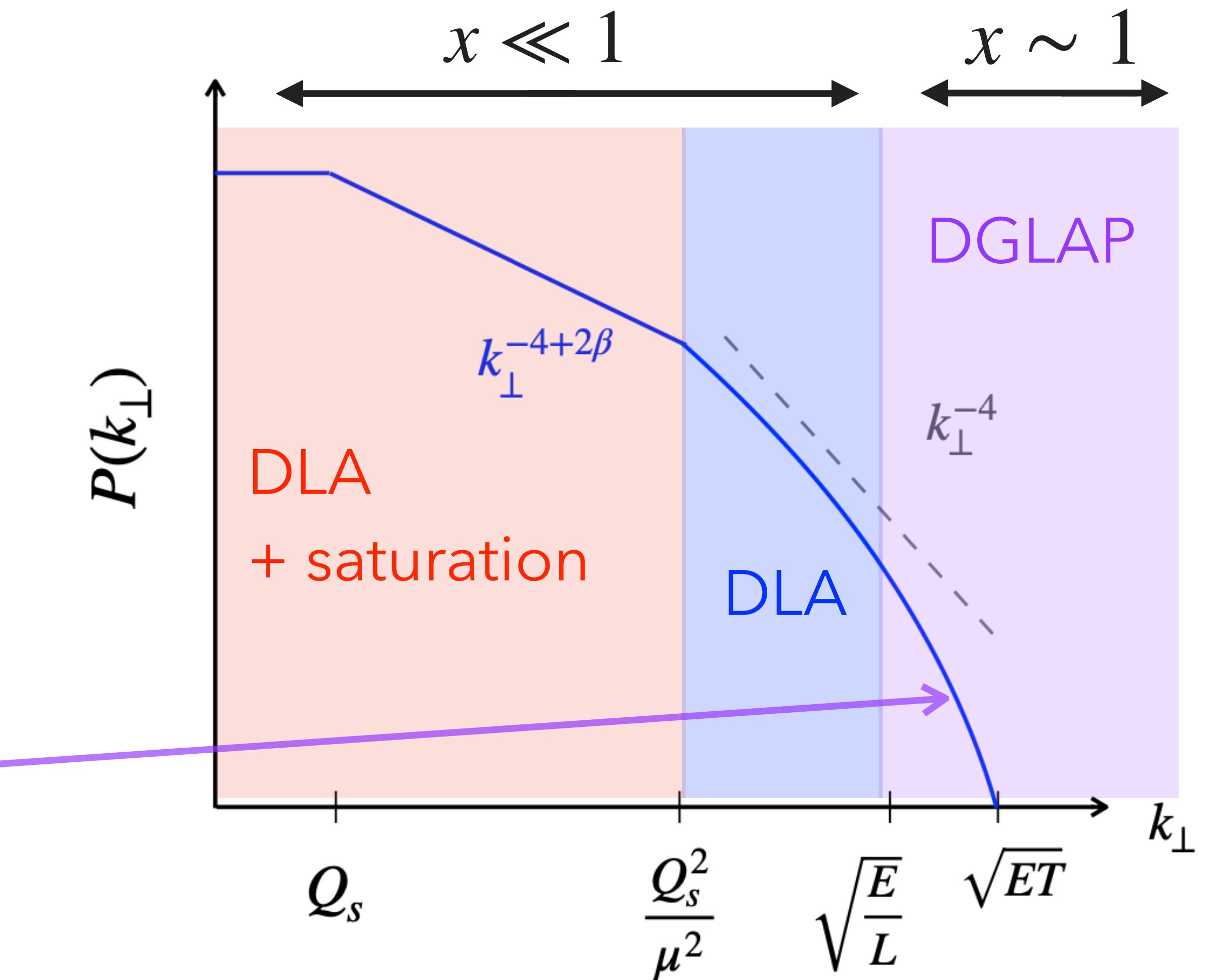
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- Hard scattering regime: gluon PDF of the plasma at DLA:

$$\hat{q}(k_\perp^2) \propto x g(x, k_\perp^2) \simeq \exp \left(\sqrt{\log \frac{k_\perp^2}{\mu^2} \log \frac{ET}{k_\perp^2}} \right)$$



- $k_\perp > \sqrt{E/L}$, expect energy dependence through $x \sim k_\perp^2/ET < 1/LT$

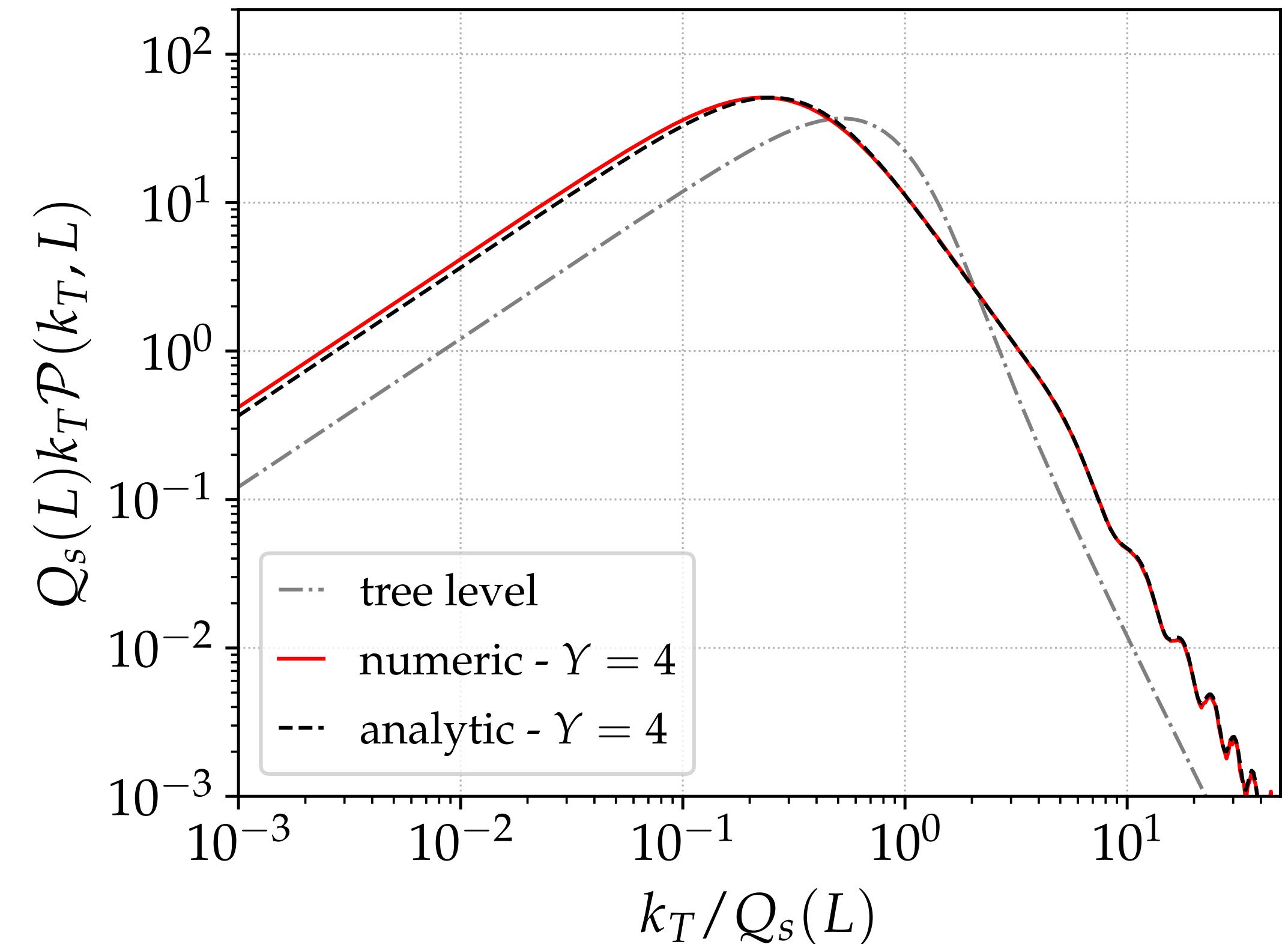
Wang, Casalderrey-Solana (2007) Kumar, Majumder, Chen (2019)

Running coupling case (vs numerics)

- At large Y (leading contribution):

$$G(\zeta) = \frac{2^{1/3} b_0^{1/6}}{\text{Ai}'(\xi_1)} \text{Ai} \left[\xi_1 + 2^{-1/3} b_0^{1/3} \zeta \right]$$

- Small Y (leading edge expansion diverges). Instead expand around the saturation line



$$\hat{q}_<(Y, x) = \hat{q}_0 e^{\rho_s(Y)-Y} \exp \left(\frac{\dot{\rho}_s - 1}{\dot{\rho}_s} x + \frac{1}{2} \frac{\ddot{\rho}_s}{\dot{\rho}_s^3} x^2 + \mathcal{O}(x^3) \right)$$

Operator definition of \hat{q}

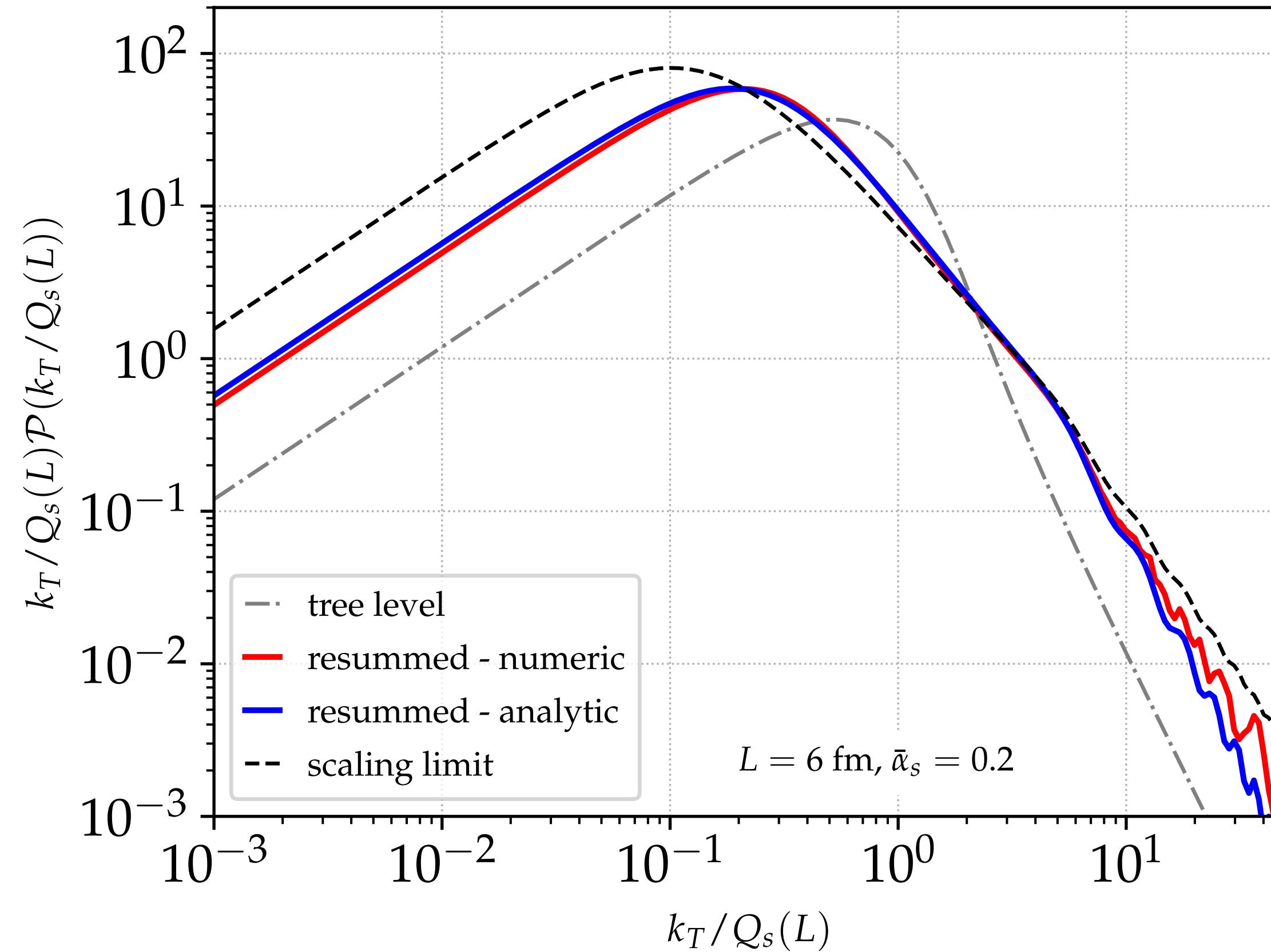
- \hat{q} is in the double log regime and can be accessed with OPE or kt-factorization

$$\hat{q} \equiv \frac{4\pi^2 \alpha_s n}{P^-} \int_0^\infty \frac{dx^+}{2\pi} e^{ixP^-x^+} \langle P | F^{i-}(x^+) [x^+, 0^+] F^{i-}(0^+) | P \rangle \Theta(x^+ < L)$$

- No x (coherence length) dependence when

$$(xP^-)^{-1} \gg L$$

Analytic vs numerics



→ wider distribution
due to heavy Lévy tail

→ Universal pre-asymptotic solution provides a good description of numerical simulations for $L = 6 \text{ fm}$ and $\bar{\alpha} = 0.2$

Renormalization of \hat{q}

- DL resummed to all orders in jet quenching parameter

$$\frac{\partial}{\partial \ln \tau} \hat{q}(k_\perp, \tau) = \bar{\alpha} \int_{Q_s^2(\tau)}^{k_\perp^2} \frac{dk'_\perp^2}{k'_\perp^2} \hat{q}(k'_\perp, \tau)$$

[Blaizot, MT (2014), Iancu (2014)]

Liou, Mueller, Wu (2013)

Iancu, Triantafyllopoulos (2015)

- Linear solution ($Q_s^2(\tau) \sim \hat{q}_0 \tau$)

$$Q_s^2(L) \equiv \hat{q}(Q_s(L), L) L = \hat{q}_0 \frac{I_0(2\sqrt{\bar{\alpha}}Y)}{\sqrt{\bar{\alpha}Y}} \simeq L^{2\sqrt{\bar{\alpha}}} \quad Y \equiv \log \frac{L}{\tau_0}$$