Attractors in the Adiabatic Hydrodynamization framework

How and Where to Find Them

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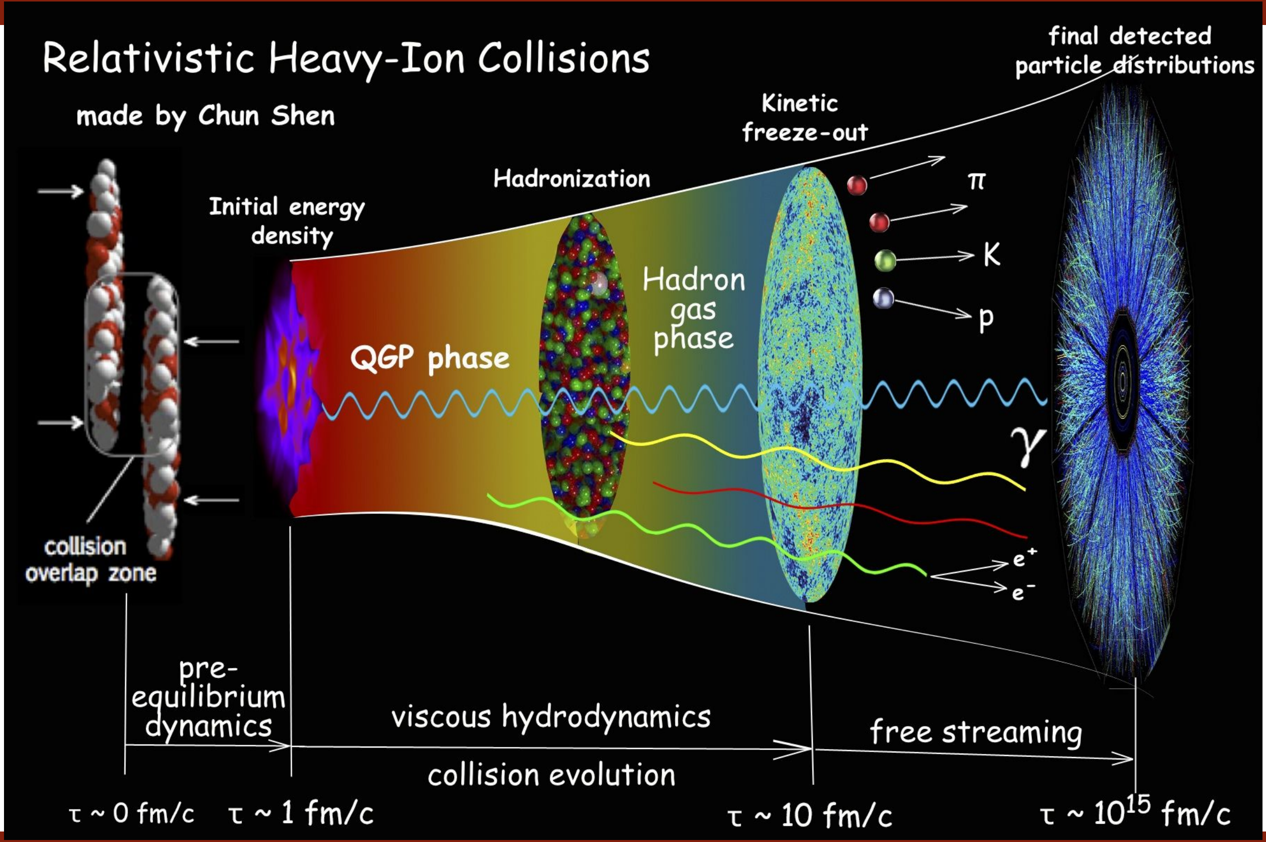
with Bruno Scheihing-Hitschfeld and Krishna Rajagopal

Hydrodynamization in Heavy Ion Collisions



Hydrodynamization in Heavy Ion Collisions

- How can we describe early out-of-equilibrium pre-hydro stage?
  - QCD Kinetic Theory
  - Holography
  - Glasma
- Many descriptions have been shown to have “attractor” solutions
  - See e.g. Kurkela, van der Schee, Wiedemann, Wu, arXiv:1907.08101
- Adiabatic Hydrodynamization framework: understand attractors in kinetic theory as the time-dependent ground state of an evolving effective Hamiltonian, long before hydrodynamization
  - Brewer, Yan, Yin, arXiv:1910.00021
  - Brewer, Scheihing-Hitschfeld, Yin, arXiv:2203.02427

Kinetic Theory and Rescaling

$$\frac{\partial f}{\partial \tau} + \frac{p_\perp}{p} \nabla_{x_\perp} f + \frac{p_\eta}{\tau p} \frac{\partial f}{\partial \eta} + \frac{p_\eta}{\tau} \frac{\partial f}{\partial p_\eta} = \frac{g_s^4 N_c^2}{4\pi} l_{Cb}[f] \left(q \nabla_p^2 f + \lambda \nabla_p \cdot (\hat{p} f(1+f)) \right)$$

collision kernel assuming small-angle scattering

where $q = \int_p f(1+f)$ and $\lambda = \int_p \frac{2f}{p}$

- Goal: dynamically rescale f and $\boldsymbol{p}$ to write theory as $H_{eff}\, w = -\partial_\tau w$ in such a way that H_{eff} is gapped and w decays to a ground state "attractor"

- System well described by adiabatic approximation if

$$\delta_A \equiv \left| \frac{<w_n|\partial_\tau|w_0>}{\epsilon_n - \epsilon_0} \right| \ll 1$$

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neglect transverse expansion

assume boost invariance

collision kernel assuming small-angle scattering

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- Previous work: longitudinally expanding, highly occupied approximation (early times) [BSY arXiv:2203.02427]
  - Found analytic expression eigenstates; scaling such that $\partial_\tau |w_0 >= 0$
- One step forward, one step back: keep full small-angle collision kernel, but neglect longitudinal term and take $f \ll 1$.
- Suppose: $f(p,\tau) = A(\tau)w(p/D(\tau),\tau) = A(\tau)w(\chi,\tau)$. Then

$$H_{eff}w = -\partial_\tau w = \frac{\dot{A}}{A}w - \frac{\dot{D}}{D}\chi\partial_\chi w - \frac{q}{\chi^2 D^2}(2\chi\partial_\chi w + \chi^2\partial_\chi^2 w) - \frac{\lambda}{\chi^2 D}(2\chi w + \chi^2 \partial_\chi w)$$

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previous work: simplified collision kernel

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Kinetic Theory and Rescaling

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for now: neglect
longitudinal expansion

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Kinetic Theory and Rescaling

$$H_{eff} = \frac{\dot{A}}{A} - \frac{\dot{D}}{D}\chi\partial_\chi - \frac{q}{\chi^2 D^2}(2\chi\partial_\chi + \chi^2\partial_\chi^2) - \frac{\lambda}{\chi^2 D}(2\chi + \chi^2\partial_\chi)$$

- Express H_{eff} in terms of convenient basis:

$$\psi_L^{(n)} = p_n(\chi), \quad \psi_R^{(n)} = p_n(\chi)e^{-\chi}$$

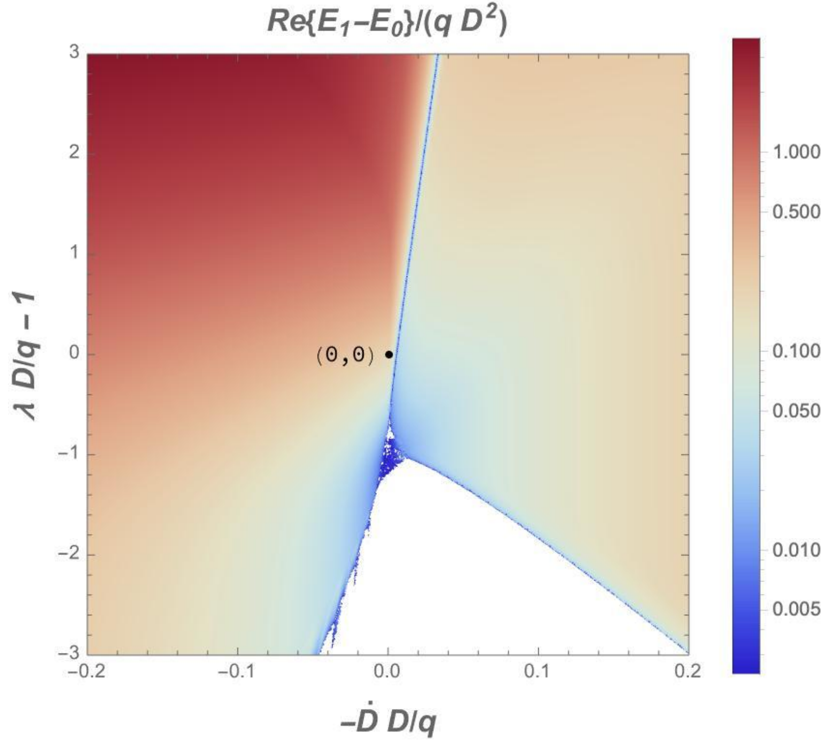
Note: p_n are generalized Laguerre polynomials with $\alpha=2$

$$H_{nm} = \int d^3\chi\psi_L^{(n)} H_{eff}\psi_R^{(m)}$$

- Given an initial condition $f(p, t=0) = \sum_n b_n(t=0)\psi_R^{(n)}$ and a momentum rescaling *D(t)*, we can:
  - solve for *w(t)* using effective Hamiltonian evolution equation
  - solve for instantaneous eigenstates to see how well it satisfies adiabaticity

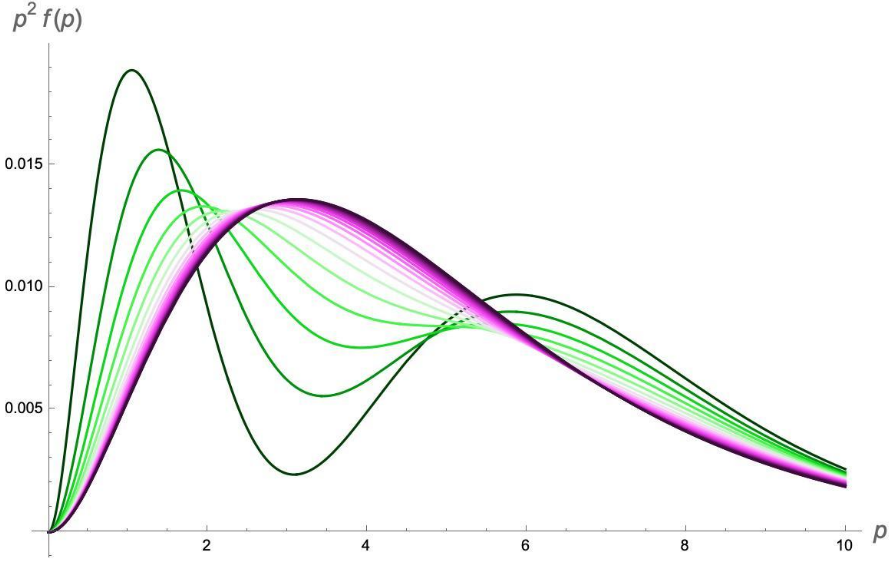
The Energy Gap as a Function of *D*

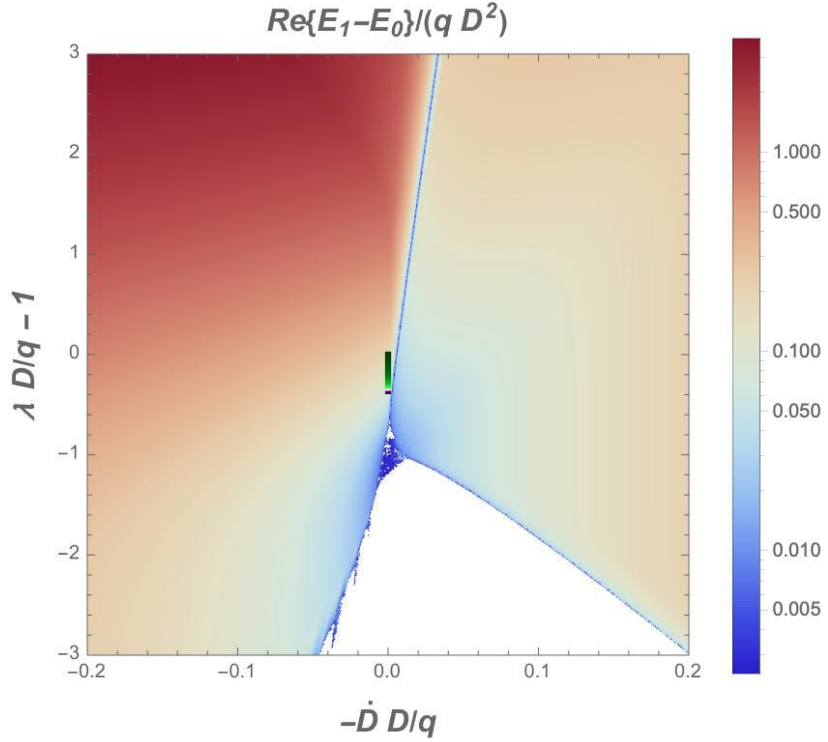
- The energy gap above the instantaneous ground state depends on the choice of the scaling variable *D*.
- The evolution of the system will traverse a 1D path on this parameter space
    - How adiabatic is the evolution on these paths?



Example path #1

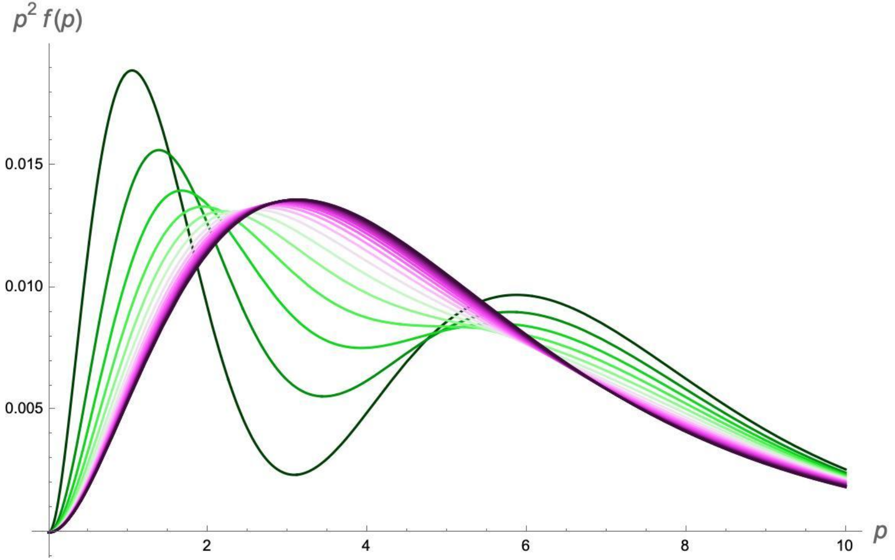
Given some choice of initial distribution function, we can test difference possible choices for *D(t)*. **One choice: *D* constant**

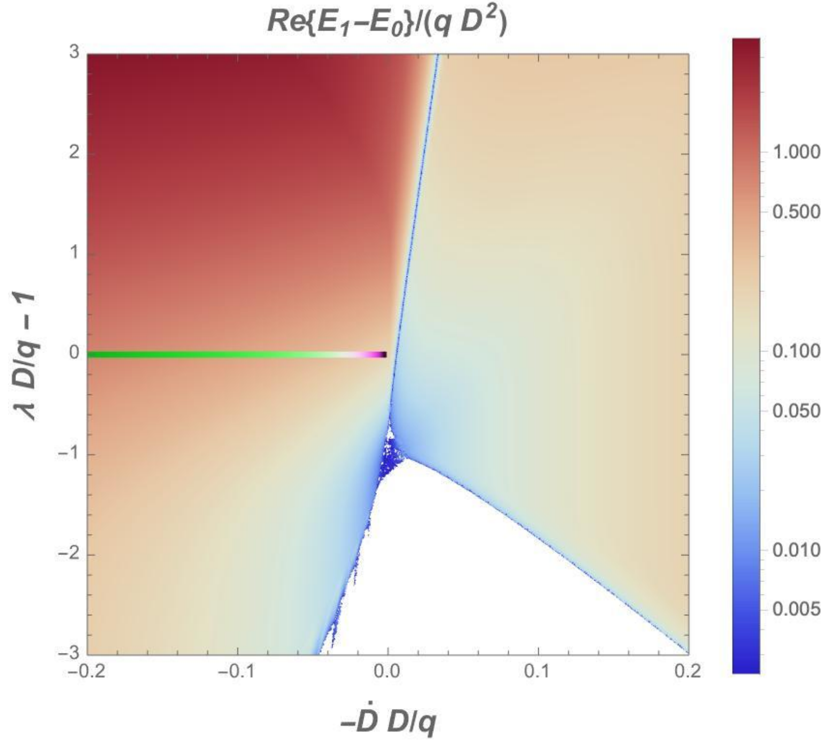




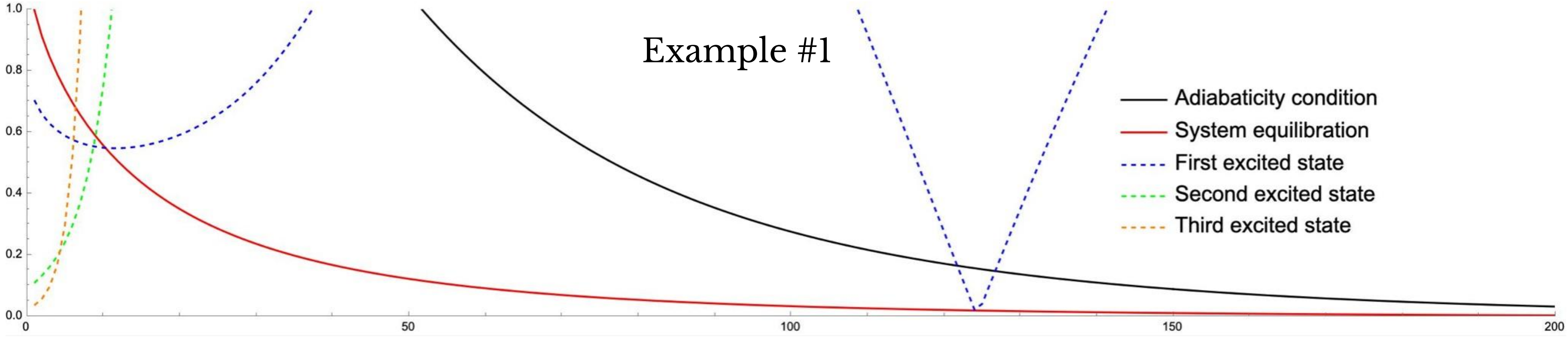
Example path #2

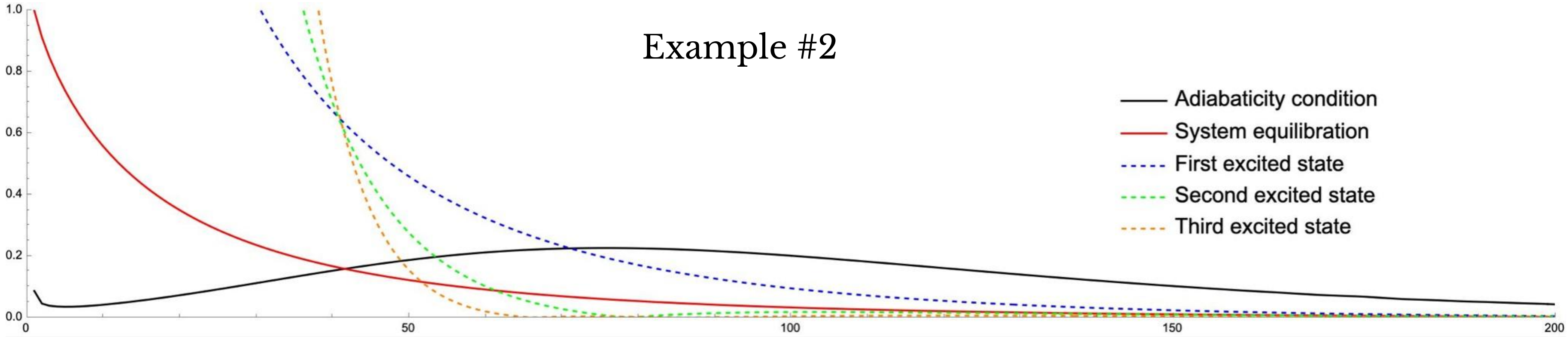
Another possible choice: $D(t) = q/\lambda(t)$





Adiabaticity for example paths



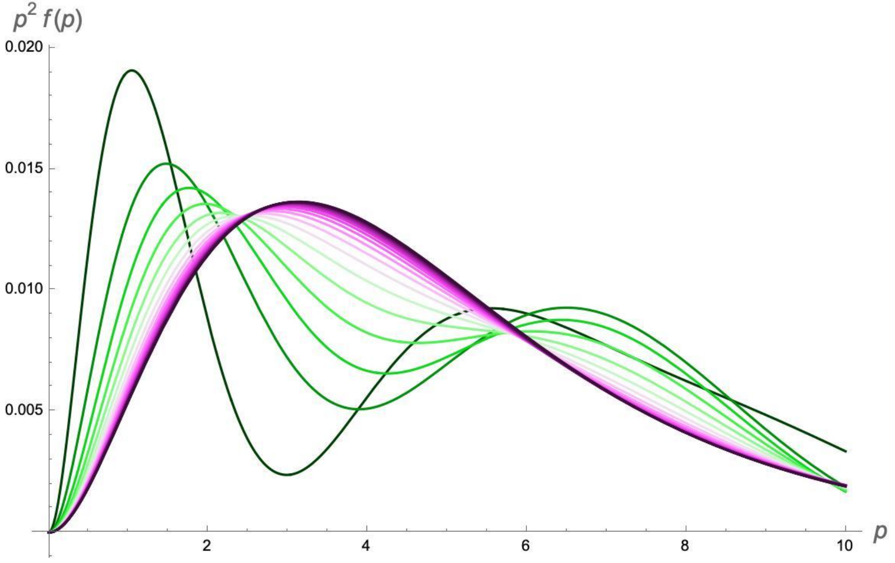


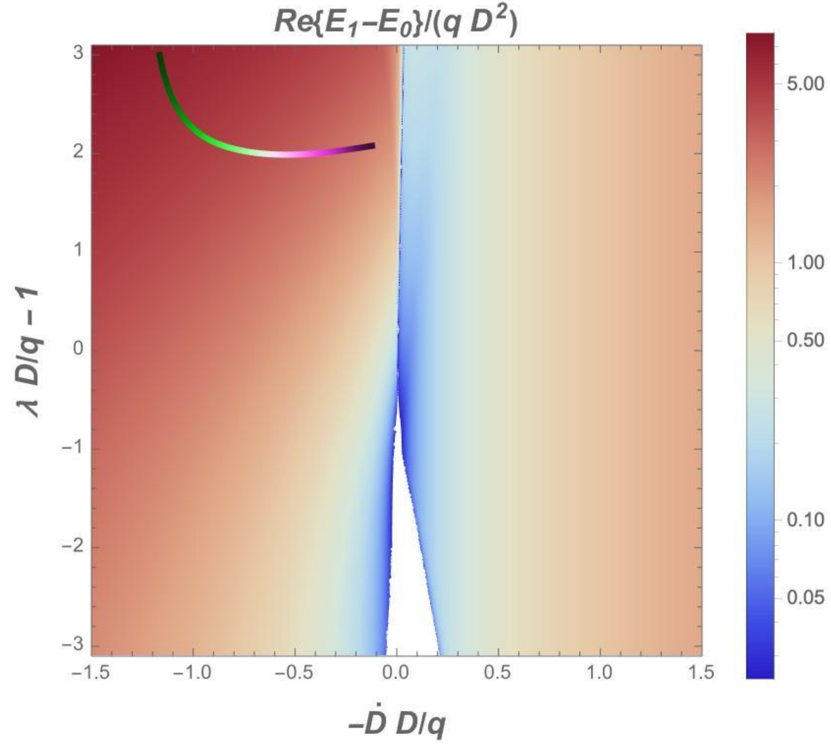
Optimizing Adiabaticity

- To find an adiabatic rescaling, write down δ_A and at each time step minimize over $\ddot{D}$
  - Gives an evolution equation for the rescaling coupled to the system evolution
- Given $f(p, t=0) = \sum_n b_n(t=0)\psi_R^{(n)}$, simultaneously solve for $D(t)$ and $b_n(t)$

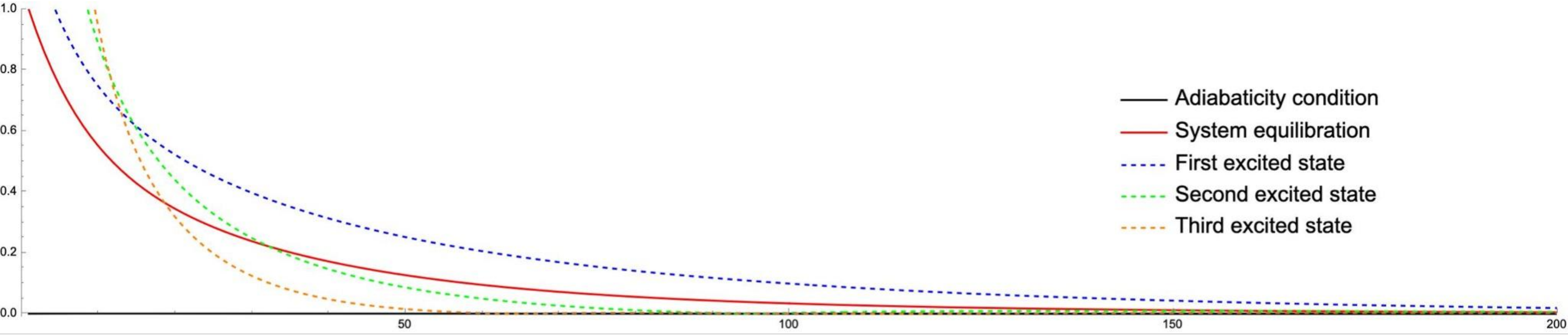
Optimizing Adiabaticity

Choosing D to maximize adiabaticity





Optimizing Adiabaticity



As expected, we have a picture in which slow modes dominate before the system has fully thermalized. The excited states decay sequentially, and the evolution is extremely close to being exactly adiabatic.

Conclusion and Next Steps

- We are able to find an adiabatic frame for a kinetic equation whose effective eigenstates are not known analytically
- Here, as in previous work, adiabatic approach describes attractor behavior long before hydro
- Still very much a work in progress!
- Extending this analysis to more general kinetic equations
    - Restore longitudinal expansion, add transverse expansion
- Using attractors as an ingredient in Bayesian analyses of heavy ion collision data (e.g. Trajectum)