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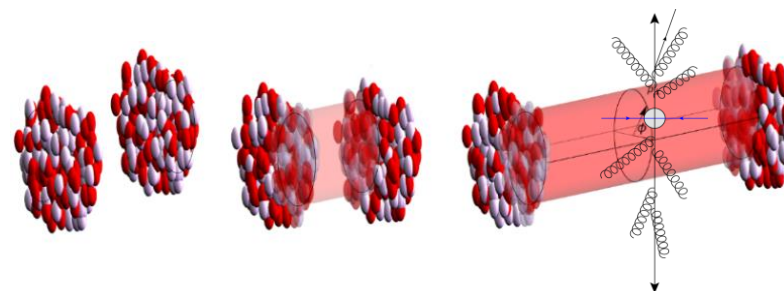
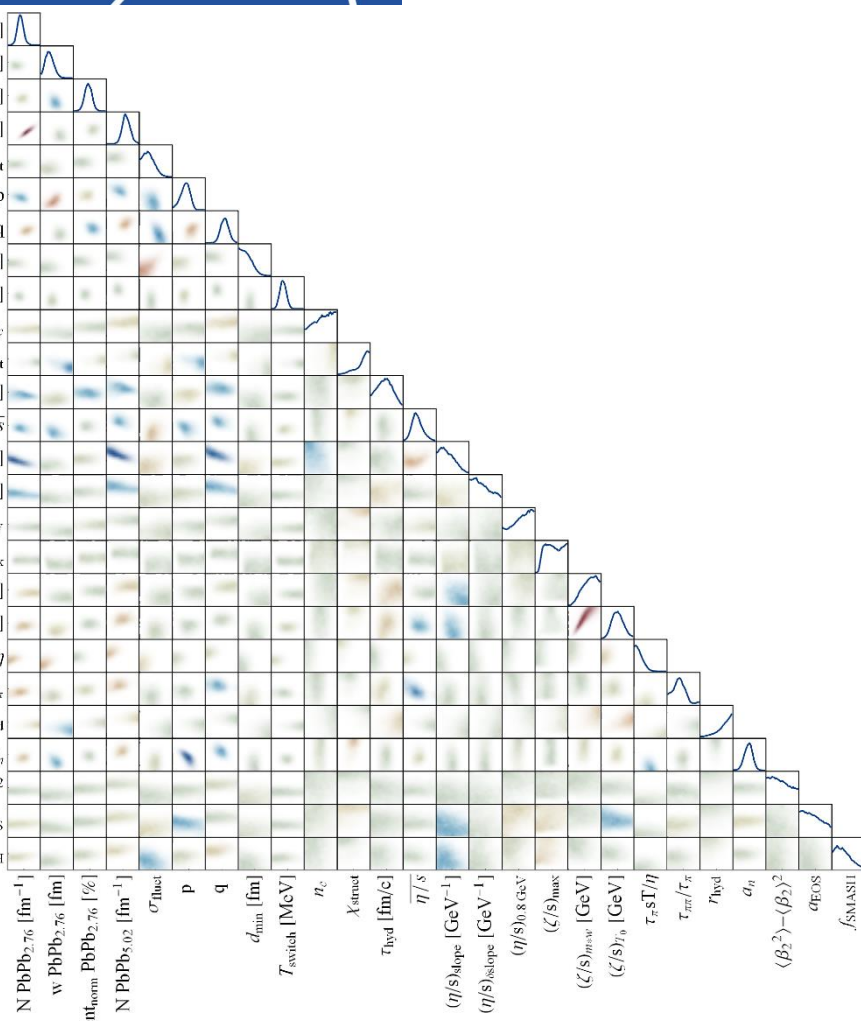


The VII-th International Conference on the  
**Initial Stages** of High-Energy Nuclear  
Collisions (IS2023), Copenhagen.

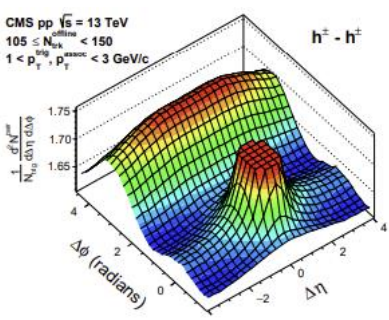
# Anisotropic flow in small systems

Hydrodynamics at its limits

Together with Govert Nijs



**Wilke van der Schee**  
Initial Stages, Copenhagen  
21 June 2023



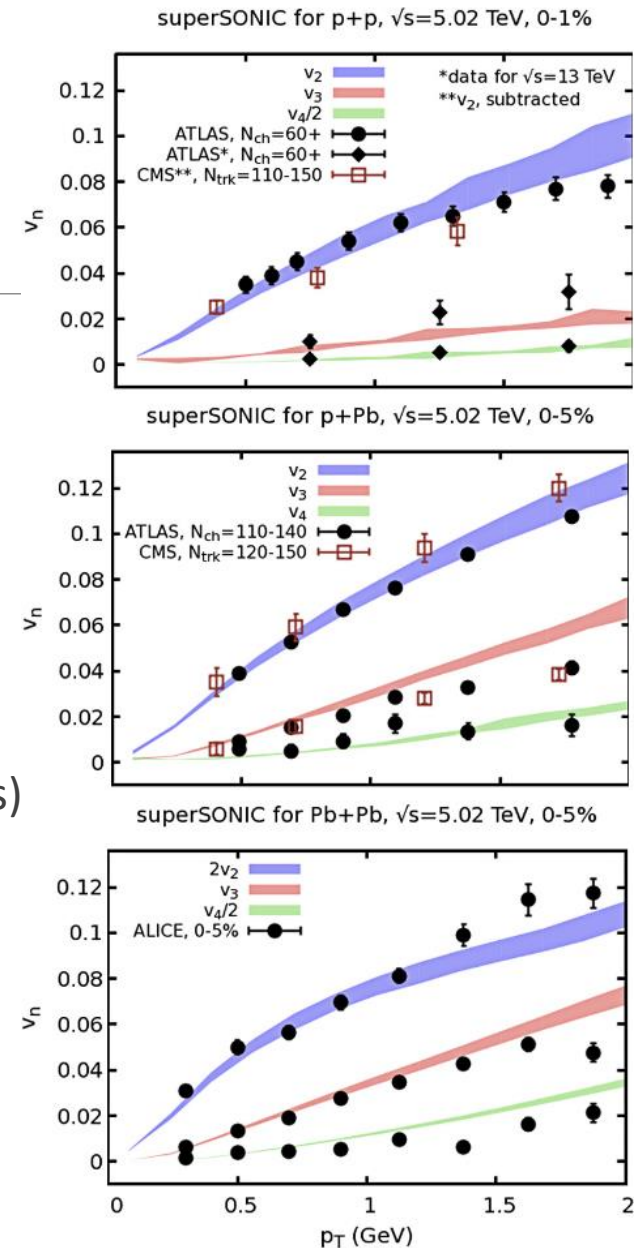
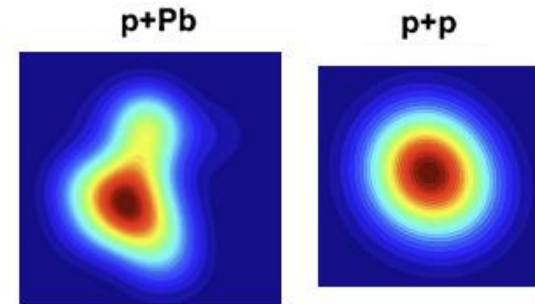
# One fluid to rule them all --- Panta Rei

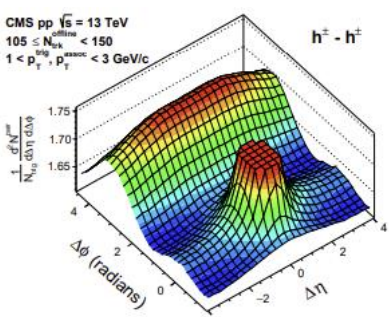
## Long range rapidity correlations seen in near-side ridge

- Most natural explanation in terms of a geometric effect --- hydro?
- Low multiplicity: correlations dominated by 'non-flow' effects:
  - Momentum conservation (away side ridge)
  - Fragmentation (also: *resonance decays*)

## Hydrodynamic models describe the data well (?)

- In the case of superSONIC a relatively simple initial Glauber model (3 constituents)





See also talk by Parker Gardner (Wed 16:10)

See also talk by Huichao Song (Wed 15:20)

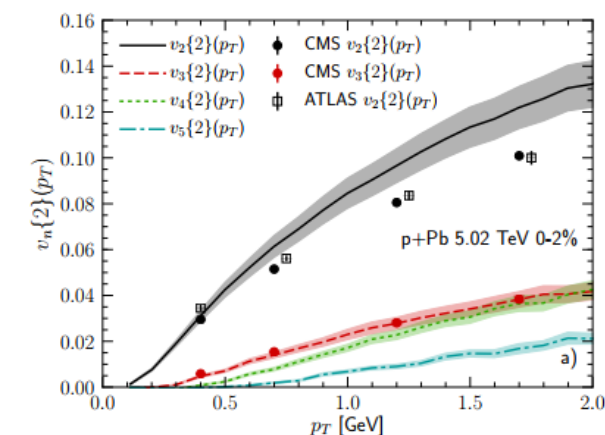
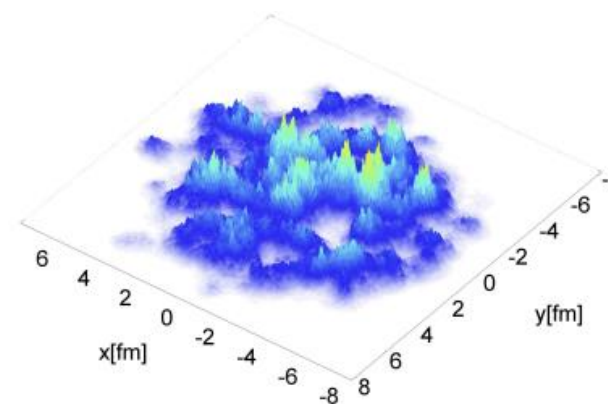
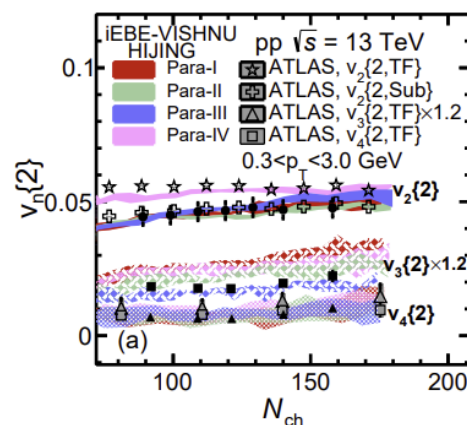
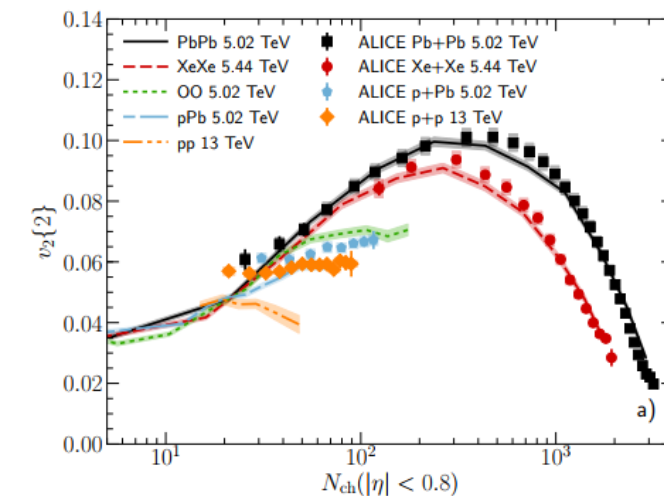
# One fluid to rule them all --- Panta Rei

## Similar results from IP-Glasma + MUSIC + UrQMD

- Initial state in this case very 'bumpy'
- PbPb/XeXe elliptic flow larger than small systems at same small multiplicity

## Similar results from HIJING/Trento + iEBE-VISHNU for pp collisions

- Initial state is varied from HIJING to Trento
- Describes data well





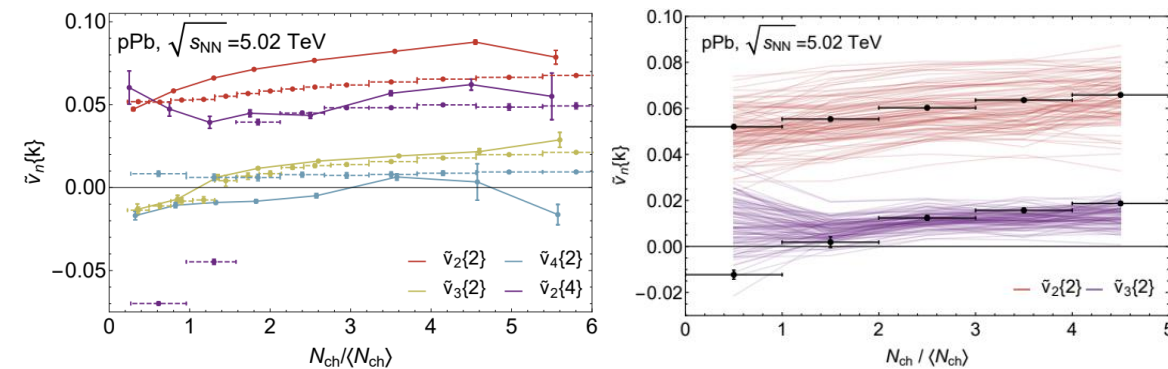
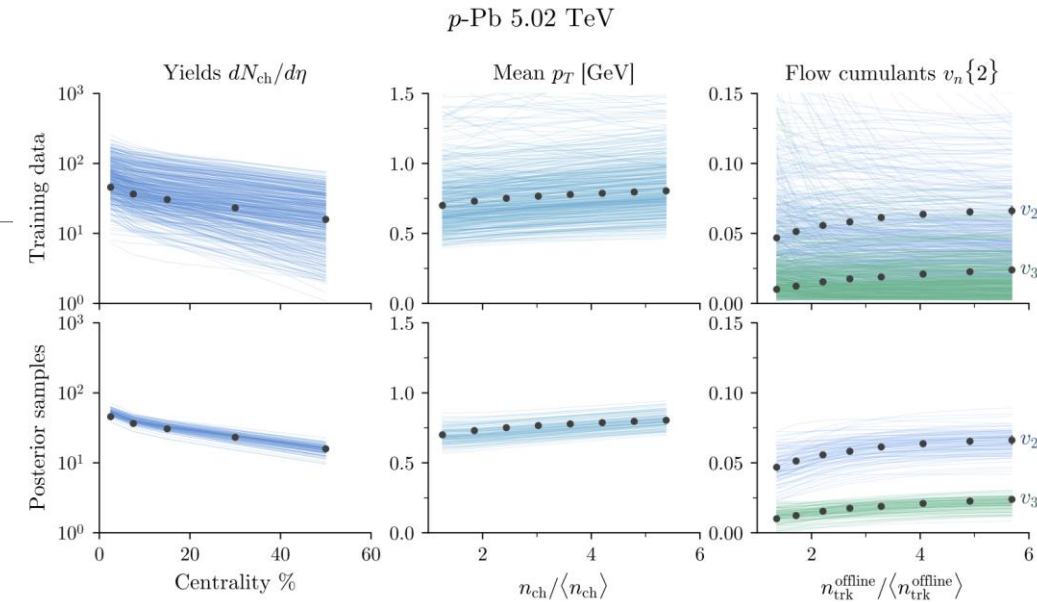
# Bayesian analysis of p-Pb

## Bayesian analysis including p-Pb collisions

- Requires nucleon 'substructure':  $n$  constituents of width  $\nu$
- More challenging to emulate/compute than PbPb

## MAP comparison of Trajectum in 2020

- Reasonable agreement
  - Curious imaginary values both in theory and data of  $v_3\{2\}$



# Anisotropic flow in small systems

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## The success of hydro or anything goes?

- Requires a systematic study: vary parameters consistent with PbPb data
  - Including changes in initial state and changes in viscosities
- Do we do an apples-to-apples comparison?
  - Small multiplicity: need to understand ‘non-flow subtraction’
  - Resonance decays lead to non-trivial correlations

**This talk:** use *Trajectum* to study this in detail

**Future aim:** what are the limits of hydro?

The limits of hydrodynamics is a great motivation in theory and experiment: upcoming OO and pO runs in 2024 😊. Possibility of Neon run in 2025?

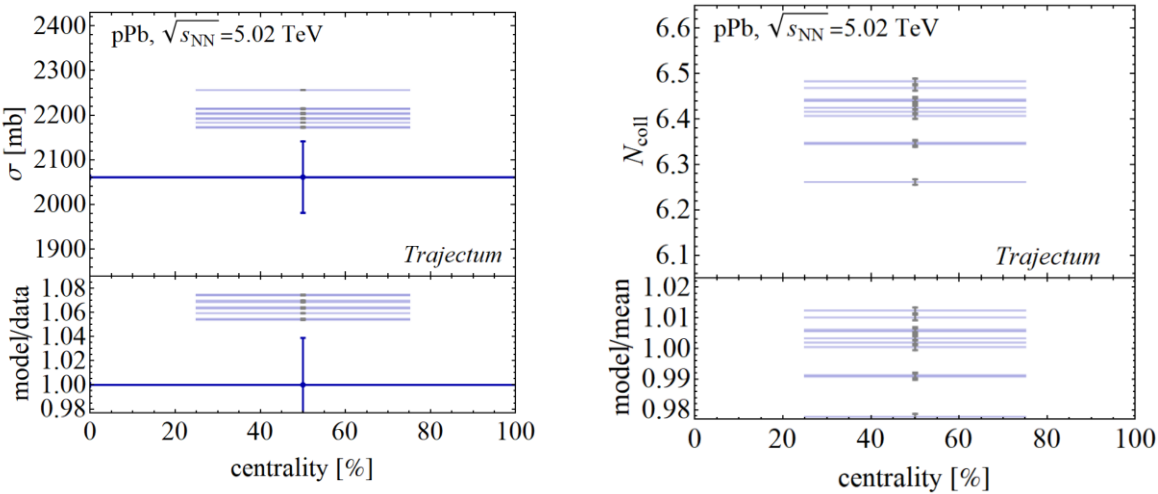
# Back to basics: the total hadronic cross section

## Total cross section on the high side

- Based on 10 ‘likely’ parameter settings
- Even with a nucleon width of ~0.6 fm

$$\sigma_{AB} = A B \sigma_{NN} / \langle N_{\text{col}} \rangle$$

- Average  $N_{\text{coll}}$  hence a bit low



## Implies reduce nuclear thickness of about 7% higher than CMS value

- Could have serious implications for pPb  $R_{AA}$

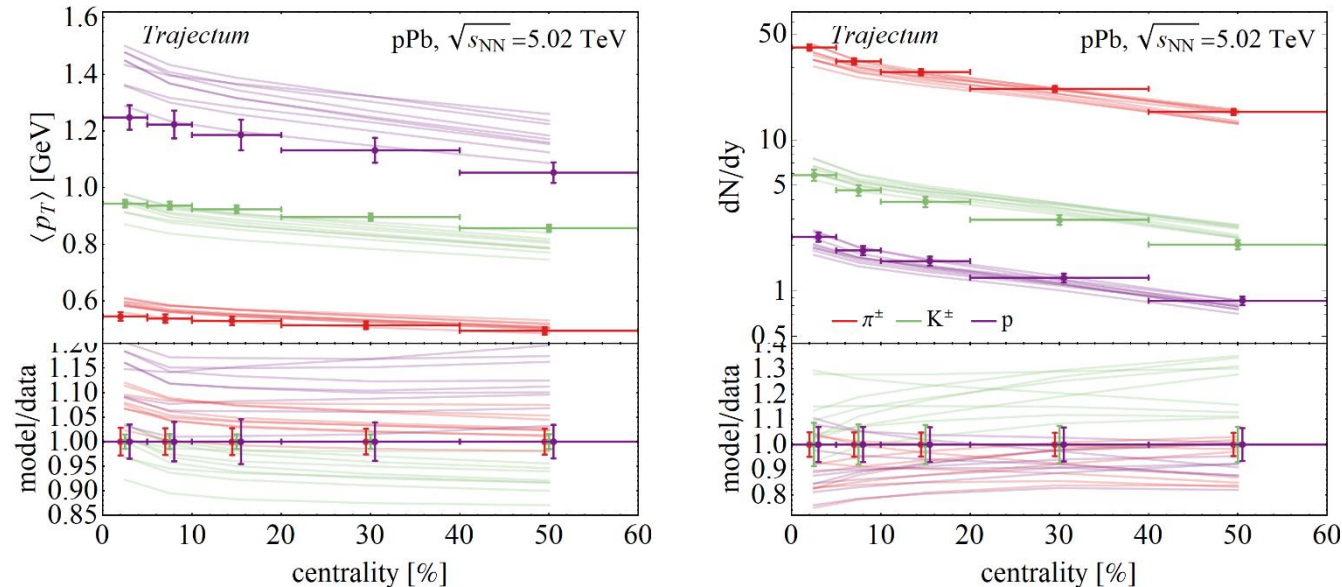
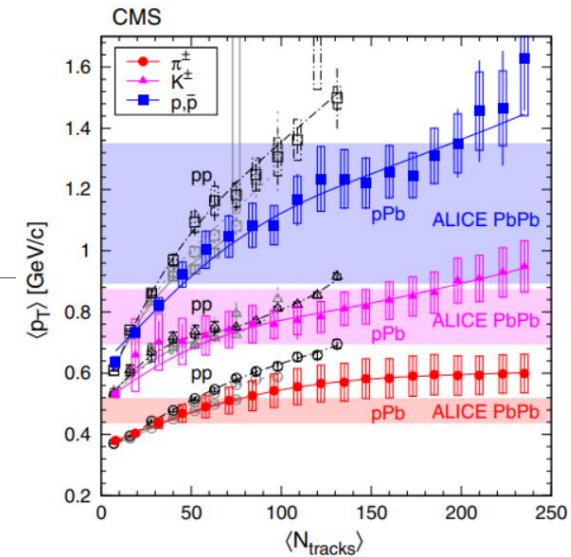
	$T_{AA} \text{ [mb}^{-1}\text{]}$
Trajectum	$0.0947 \pm 0.0010$
CMS	$0.0983 \pm 0.0044$

$$\langle T_{AB} \rangle = \langle N_{\text{col}} \rangle / \sigma_{NN}$$

# Yields and mean transverse momentum pPb

## Same 10 likely settings that compare well with pPb

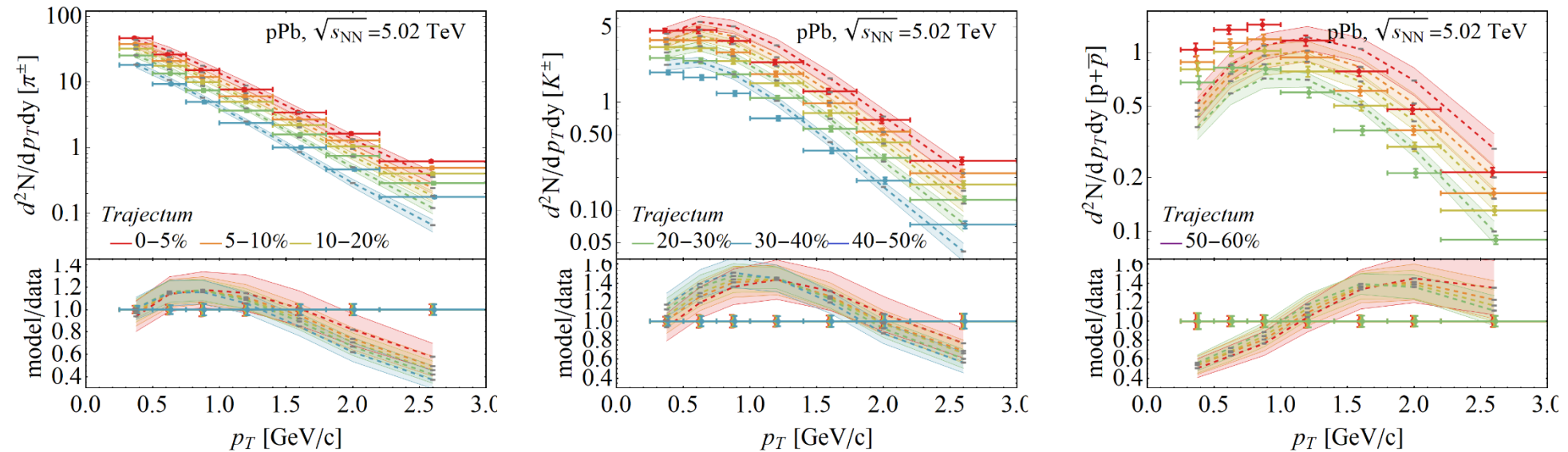
- No new fitting parameters (uses norm of PbPb)
- Proton mean  $p_T$  overestimated compared to ALICE (see however CMS)
- Significant systematic uncertainty: constrain by Bayesian global analysis?



# Spectra

## More differential: identified spectra

- Significant distortions in shape
- Hadronisation is difficult, also uncertainty due to afterburner

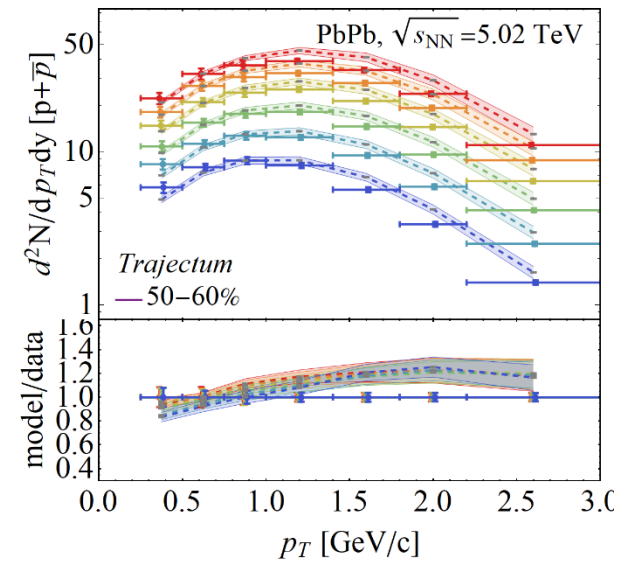
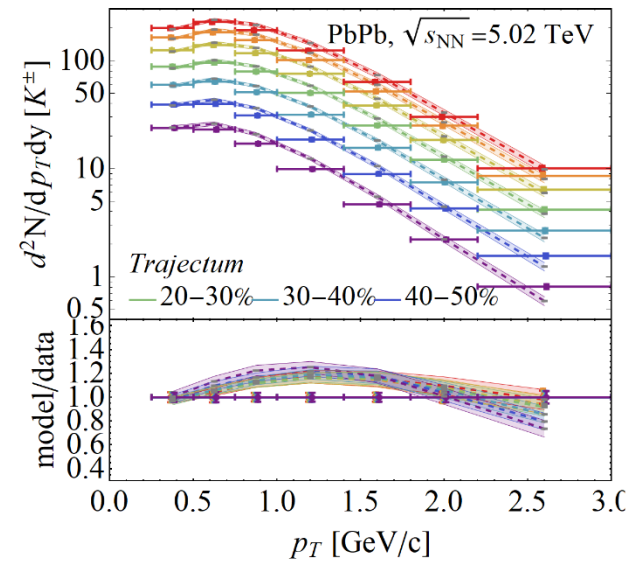
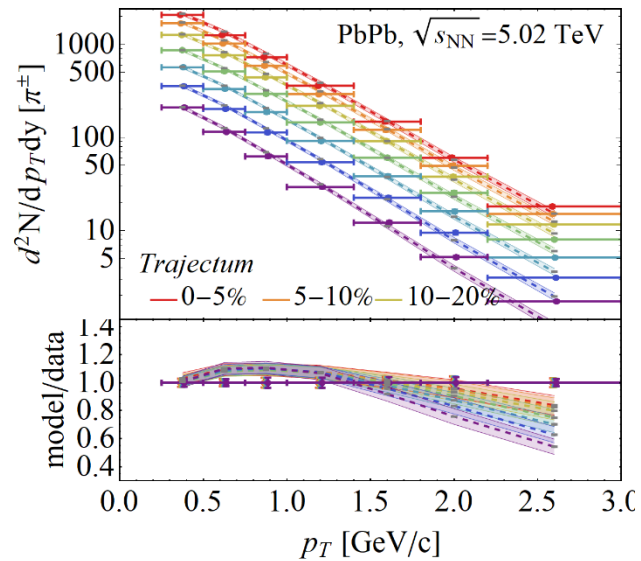




# Spectra

## More differential: identified spectra

- Significant distortions in shape
- Hadronisation is difficult, also uncertainty due to afterburner
- Smaller but similar tensions with data for PbPb



# The elephant in the room: `non-flow' subtraction

**Theory:** usually ignored in hydro (hydro does not have `non-flow'?)

- Exceptions: Zhao, Ko, Liu, Qin and Song (pp, 2001.06742) and Zhao, Ryu, Shen and Schenke (dAu,  $\gamma$ -Pb 2211.16376, 2203.06094)

**Experiment:** almost always subtracted as much as possible (?) by imposing largest  $\Delta\eta$  gap

- $\Delta\eta$  gap depends on experiment and is rarely varied

Method 1: cumulants

$$v_n\{2\}^2 = \langle\langle e^{in(\phi_i - \phi_j)} \rangle\rangle$$

**First average:** within acceptance all particle pairs  $i, j$  in a single event

**Second average\*:** average over ensemble of events

Method 2: subevents

$$v_n\{2, |\Delta\eta| > \gamma\}^2 = \langle\langle e^{in(\phi_i - \phi_j)} \rangle\rangle$$

**First average:** within acceptance all particle pairs  $i$  (with  $\eta < \gamma/2$ ) and  $j$  (with  $\eta > \gamma/2$ )

**Second average\*:** average over ensemble of events

Note: not equivalent even for  $\gamma = 0$  (!)

\* In practice one first averages over small centrality classes and then averages over those results to obtain a larger bin. For some observables this is extremely important (SC,  $\rho(v_2\{2\}, p_T)$ ), but for  $v_n\{2\}$  it makes only a small difference. Often it is not explicitly mentioned how the third averaging is done.

# Cumulants

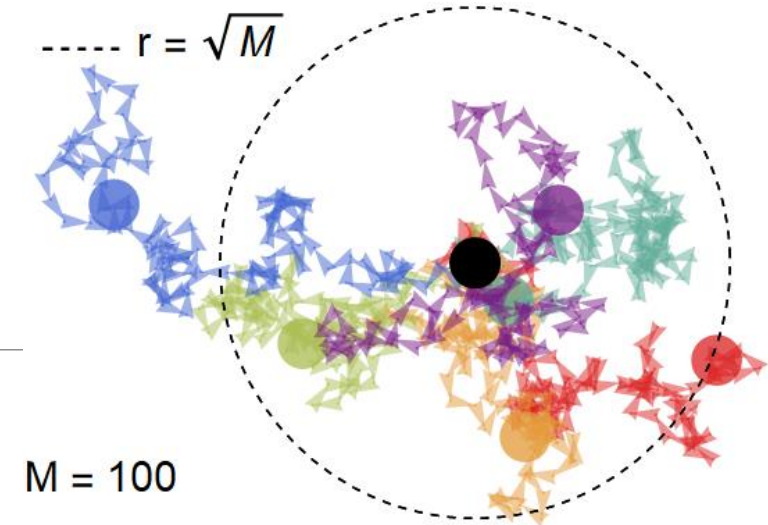
## Method 1: cumulants

$$v_n\{2\}^2 = \langle \langle e^{in(\phi_i - \phi_j)} \rangle \rangle$$

### Efficient computation:

$$Q_n = \sum_{a=i}^M e^{in\phi_i}$$
$$v_n\{2\}^2 = \left\langle \frac{|Q_n|^2 - M}{M(M-1)} \right\rangle$$

Requires  $M$  computations instead of  $M^2$ .  
The  $-M$  subtracts  $i = j$  in double sum



### Important theorem:

If  $\phi_i$  are randomly drawn from  $f(\phi)$  then  $v_n\{2\}$  will converge to the true Fourier coefficients of  $f(\phi)$

Easy to understand for  $f(\phi) = \text{constant}$ :

$Q_n$  is a random walk, so for many events:

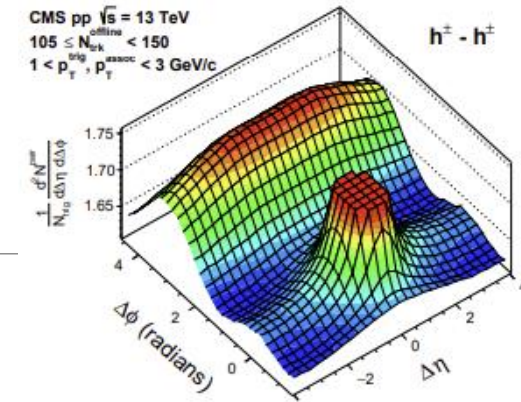
$$|Q_n| = \sqrt{M} \rightarrow v_n\{2\} = 0 \text{ (for } n > 0)$$

### Important caveat:

$\phi_i$  are not drawn randomly

In this talk we focus on correlations due to **resonance decays**

# Subevent



## Method 2: subevents

$$v_n\{2, |\Delta\eta| > \gamma\}^2 = \langle\langle e^{in(\phi_i - \phi_j)} \rangle\rangle$$

### Efficient computation:

$$Q_{n,a} = \sum_{i \in \{\eta < \gamma/2\}}^{M_a} e^{in\phi_i}$$

$$Q_{n,b} = \sum_{i \in \{\eta > \gamma/2\}}^{M_b} e^{in\phi_i}$$

$$v_n\{2\}^2 = \left\langle \frac{\Re(Q_{n,a} Q_{n,b}^*)}{M_a M_b} \right\rangle$$

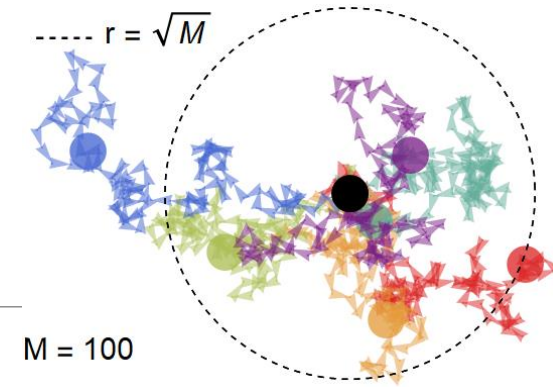
(for infinite # events the imaginary part vanishes anyway)

## Three comments

- Particles from different regions: less effect from resonances
- Smaller phase space: fewer particles, harder statistically
- For large  $\gamma$  event-plane decorrelation is important



# Intuition: random walk with resonances



Method 1: cumulants

$$v_n\{2\}^2 = \langle \langle e^{in(\phi_i - \phi_j)} \rangle \rangle$$

**Efficient computation:**

$$Q_n = \sum_{a=i}^M e^{in\phi_i}$$

$$v_n\{2\}^2 = \left\langle \frac{|Q_n|^2 - M}{M(M-1)} \right\rangle$$

Requires  $M$  computations instead of  $M^2$ .  
The  $-M$  subtracts  $i = j$  in double sum

***Oversimplified ansatz:***

Zero intrinsic flow and every particle decays into two particles with the same transverse direction.

**New 'flow' due to resonances:**

$$M \rightarrow 2M$$

$$Q_n \rightarrow 2Q_n$$

$$v_n\{2\}^2 : 0 \rightarrow \left\langle \frac{M}{M(M-1)} \right\rangle \approx 1/M$$

**When is can this be ignored?**

$$v_n \gtrsim 1/\sqrt{M}$$

$$\text{pPb: } v_n \sim 0.05, 1/\sqrt{M} \sim 0.1$$

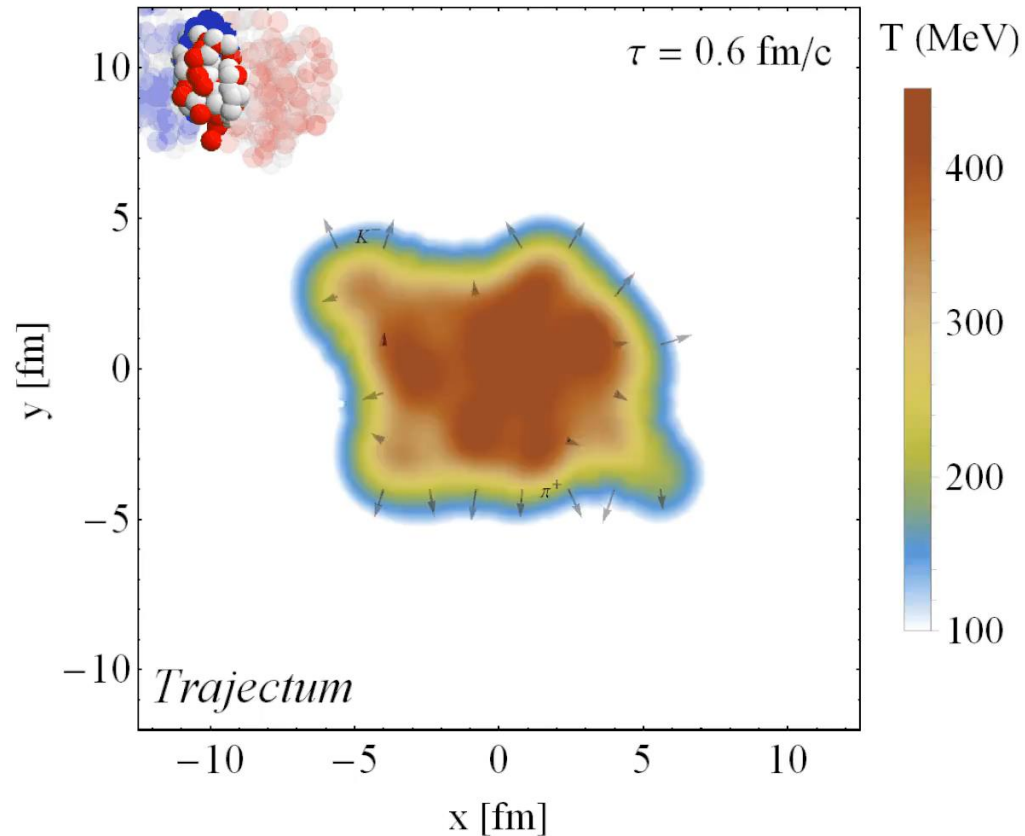
$$\text{PbPb: } v_n \sim 0.1, 1/\sqrt{M} \sim 0.03$$

**For pPb and cumulants this can be the dominant effect**

**Also present in all hydro codes that include resonance decays (!)**

# Now more serious: using *Trajectum*

1. Straightforward to use
2. Fast and publicly available
  - ~1 event/second on a laptop
3. Fully parallelized
  - Can run unlimited number of events
4. Resonance decays/interactions handled by SMASH



```
general{
  output=out
  format=smash
  f0500=false
  numevents=1
  seed=7398984.747399307
  debugoutput=true
  numthreads=2
}

entropyacceptanceprobability{
  0:0.0
  24:0.0
  24.5:0.05
  25.5:0.05
  26:0.0
  100:0.0
}

trentosubstructurePbPb{
  dmin=0.63933
  w=0.701919
  sigmann=70.0
  sigmafluct=0.73579
  p=0.14388
  q=1.0
  Eref=0.2
  norm=23.507
  freestreamingreferencetime=1.1708
  freestreamingvelocity=0.62672
  weaktostrong=0.0
  nref=20
  alpha=0
  nc=3.2747
  voverw=0.4892041602706295
}

secondorderhydro{
  numlatticesites=166.0
  latticesize=33.2
}

musclsolverkmtminmodfastmidpoint{
  cflconstant=0.08
}

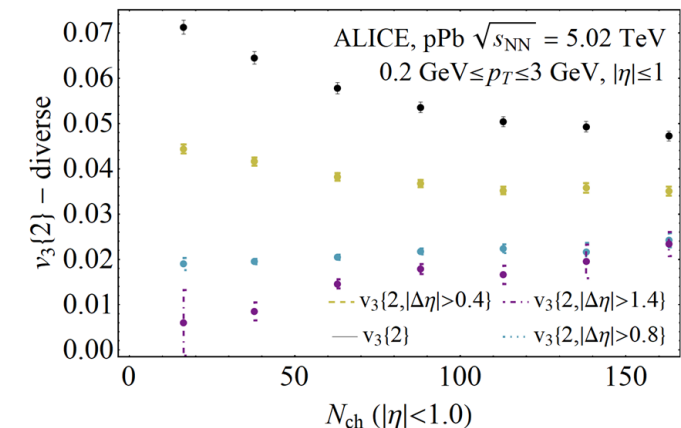
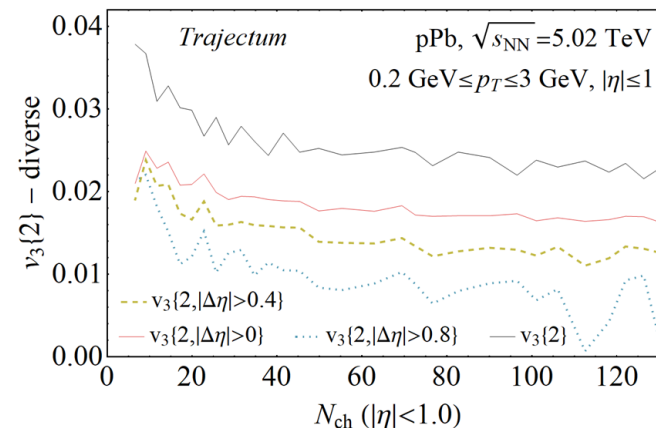
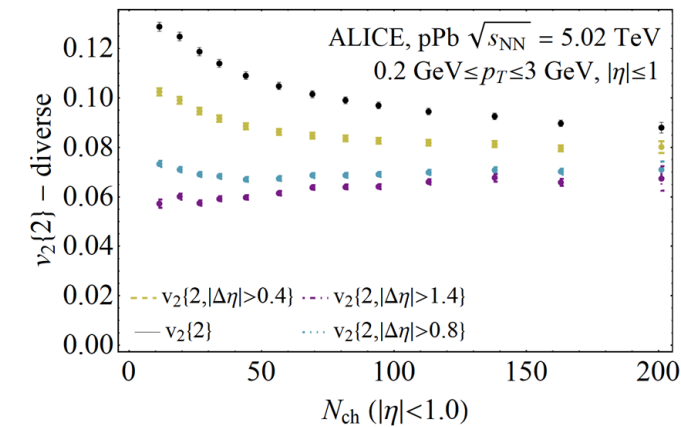
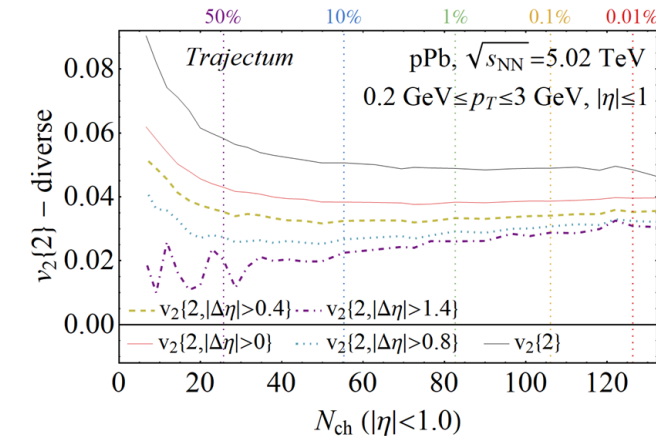
LatticeE0StempdepDuke{
  shearhg=0.0895066
  shearmin=0.0895066
  shearslope=0.43252
  shearcrv=0.231195
  shearrelaxationtime=6.318855
  bulkmax=0.0030138
  bulkT0=0.21471
  bulkwidth=0.10906
  bulkrelaxationtime=0.0687
  deltapiiovertaui=1.3333333333333
  phi7overpressure=0.128571
  taupiovertaui=1.61033
  lambdaPiPiovertaui=1.2
  deltaPiPiovertaui=0.6666666666666
  lambdaPiPiovertaui=1.6
  phi1overpressure=0
  phi3overpressure=0
  phi6overpressure=0
}

cooperfryehadronizer{
  freezeouttemp=153.456
  rapidityrange=0.1
}
```

# $\Delta\eta$ in pPb collisions – MAP setting with varying cuts

## Large and significant effects

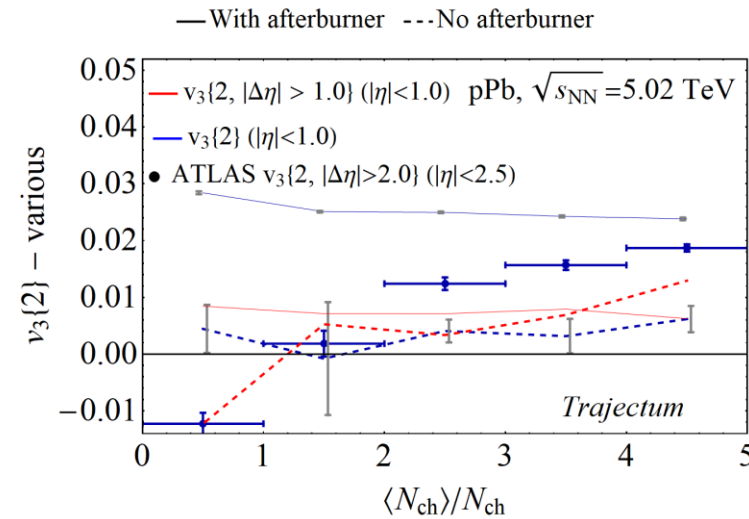
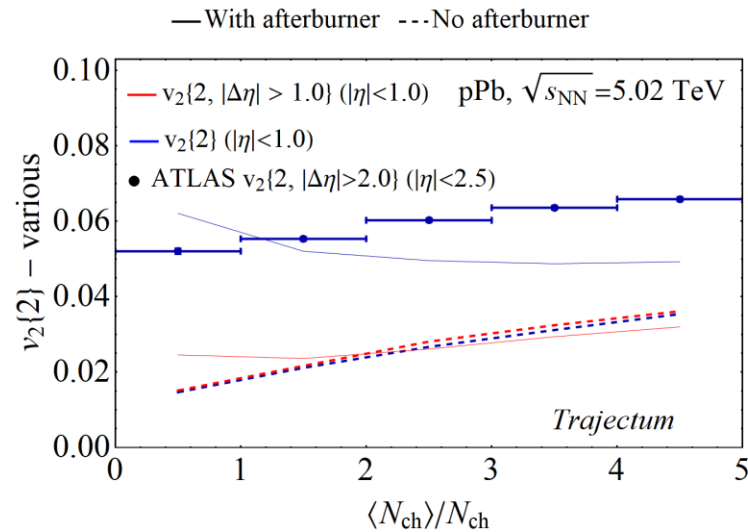
- Even  $\Delta\eta=0$  decreases flow by  $\sim 20\%$
- For low multiplicity  $\Delta\eta$  is dominant effect
  - Even in ‘hydro’
- Relatively unique measurement from ALICE
- Qualitative agreement
- Does not agree quantitatively
  - Not fitted in Bayesian analysis



# $\Delta\eta$ in pPb collisions – Resonances and afterburner (SMASH)

## Verification that effects are due to hadronic phase

- Cumulants (blue) and Subevent (red)
- Without afterburner the two methods agree (dashed)

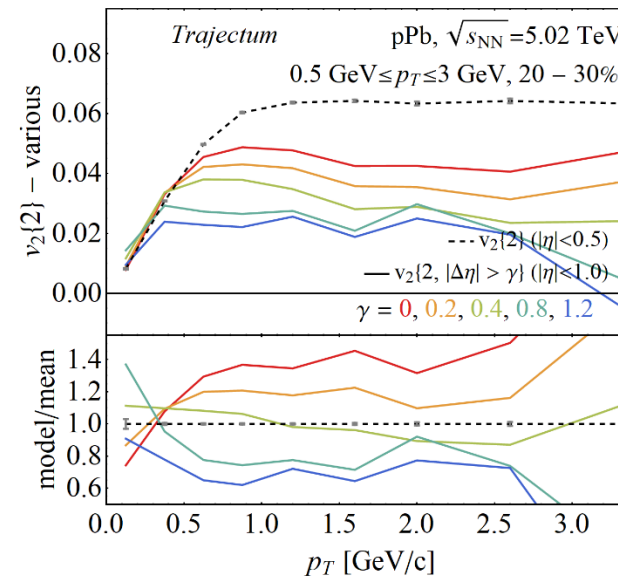
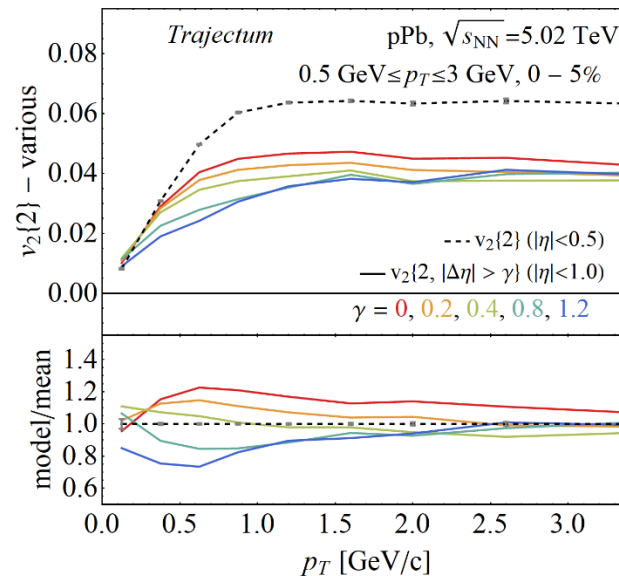




# $\Delta\eta$ in pPb collisions – $p_T$ - differential

## Similar effect when looking at $p_T$ differential flow

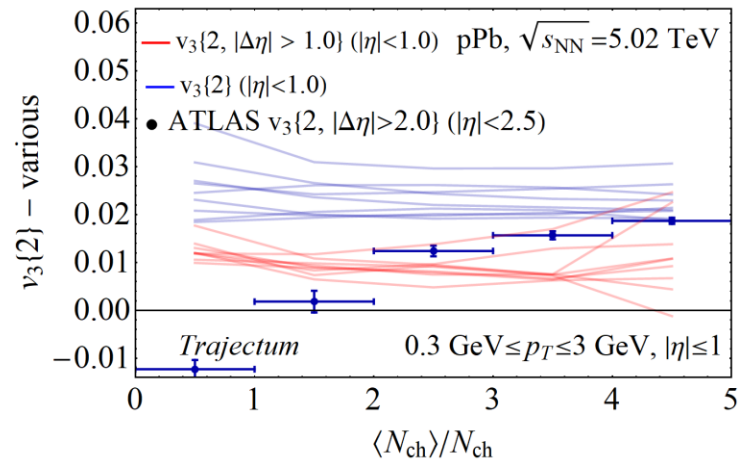
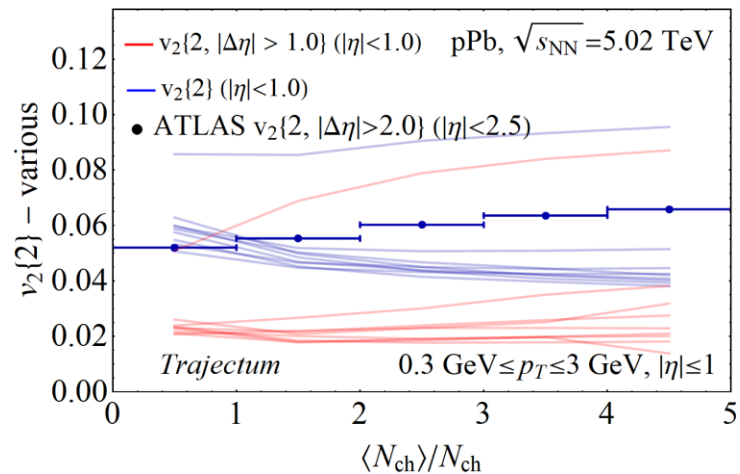
- Stronger effect at lower multiplicity
- Converges at low  $p_T$  (attractor ;))



# Elliptic flow in pPb collisions – a systematic analysis

## Ten likely settings from posterior distribution

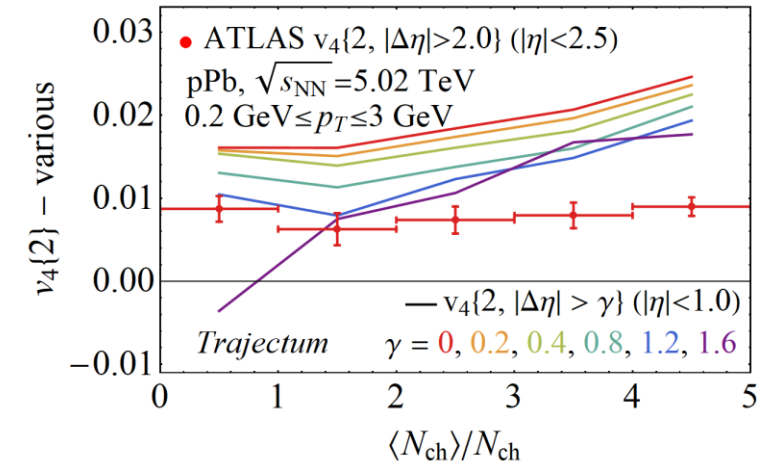
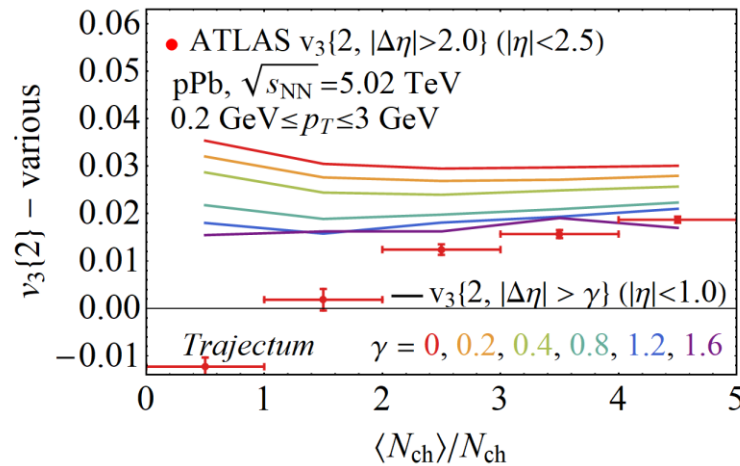
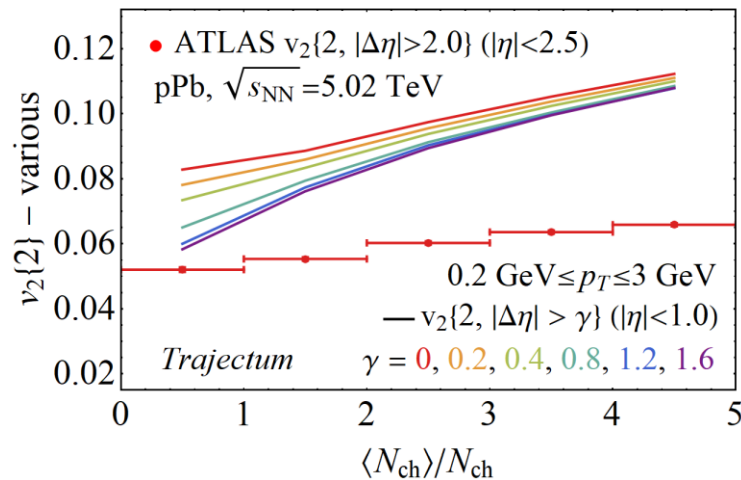
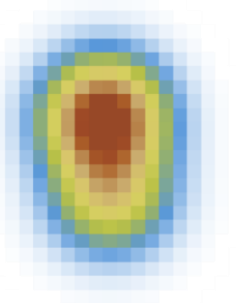
- Subevent method much smaller flow than cumulant method
- Realistically typically a factor two too low
  - Points to caveat to all previous pPb flow studies that include an afterburner/resonances
- One exception (next slide)



# Elliptic flow in pPb collisions – two constituent model (?)

**Guess based on previous parameters: two constituents with subwidth 0.3 fm (rest is MAP)**

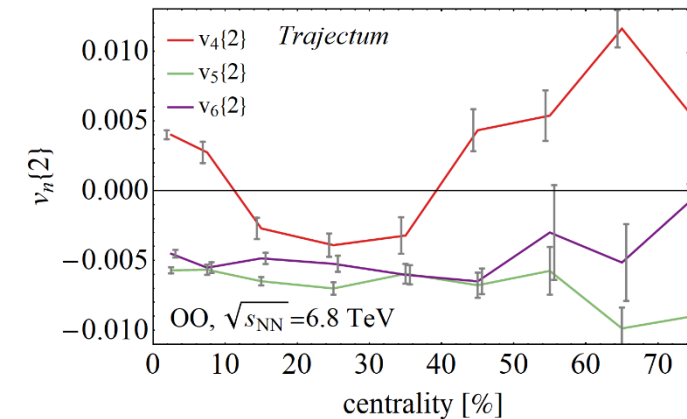
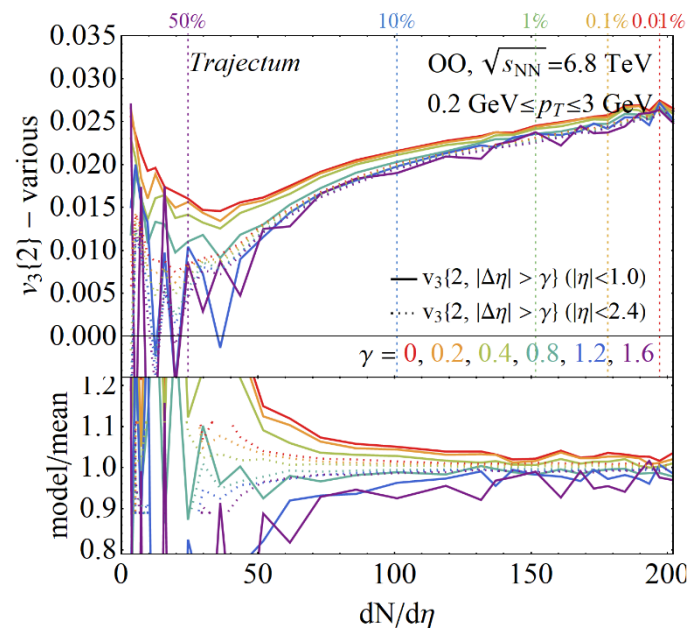
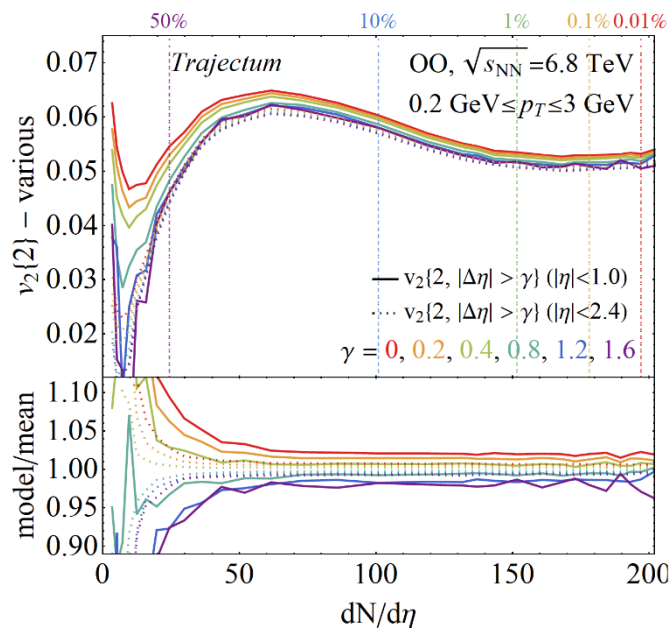
- Clearly much more elliptic flow (even too much)
- Effects from  $\Delta\eta$  gap smaller at large multiplicity (also because of higher flow:  $v_n \gtrsim 1/\sqrt{M}$ )



# Exciting upcoming Oxygen run

## Perfect system at moderate multiplicity: hydro at its limits

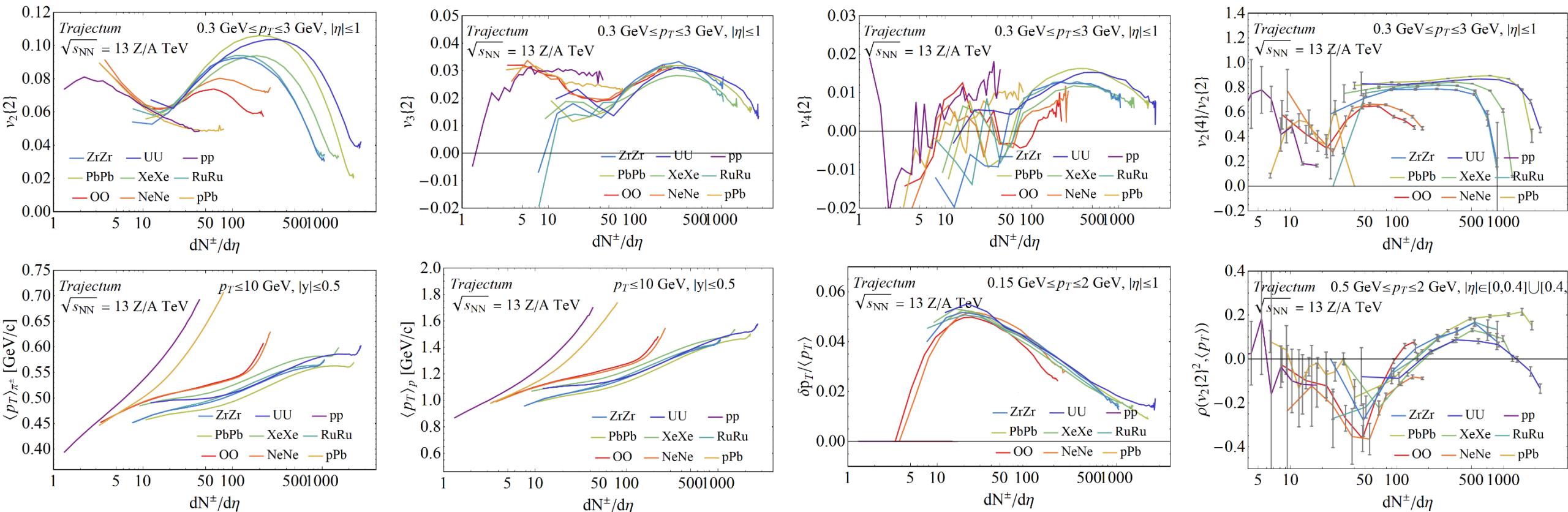
- $\Delta\eta=0$  important for ALICE coverage, but not dominant
- Important caveat: nuclear structure is not that well understood (fix this with Neon collisions?)
- Curious: resonance decays lead to negative  $v_5\{2\}$  and  $v_6\{2\}$





# Bonus slide

## Several systems with MAP settings (systematic analysis with ratios for some to appear)



# Important questions for small systems

Hydrodynamic signature is anisotropic flow

- Flow is small, signal  $v_n \sim 1/\sqrt{M}$  and hence need to be careful
- Important: no discussion of 'real non-flow', e.g. jets and hard QCD

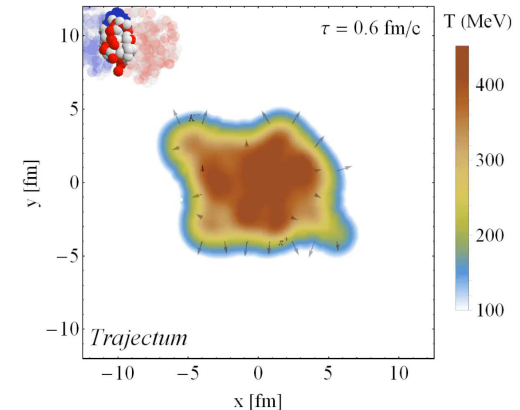
Collectivity is a broader concept than hydrodynamics

- pPb results depend sensitively on model parameters
  - Hard to conclude that 'panta rei'; **systematics is important**
- At the moment unclear if any parameters fit consistently (also spectra?)

Useful to have a tool like *Trajectum*

- **Fast:** each curve for pPb takes only about 5k CPU hours, Runs ~400M Trento IC, ~1M hydro runs and ~30M SMASH runs
- Message both to theory + experiment: **details and specifically cuts matter**

**Exciting time ahead with Oxygen and other small systems (Ne?);**



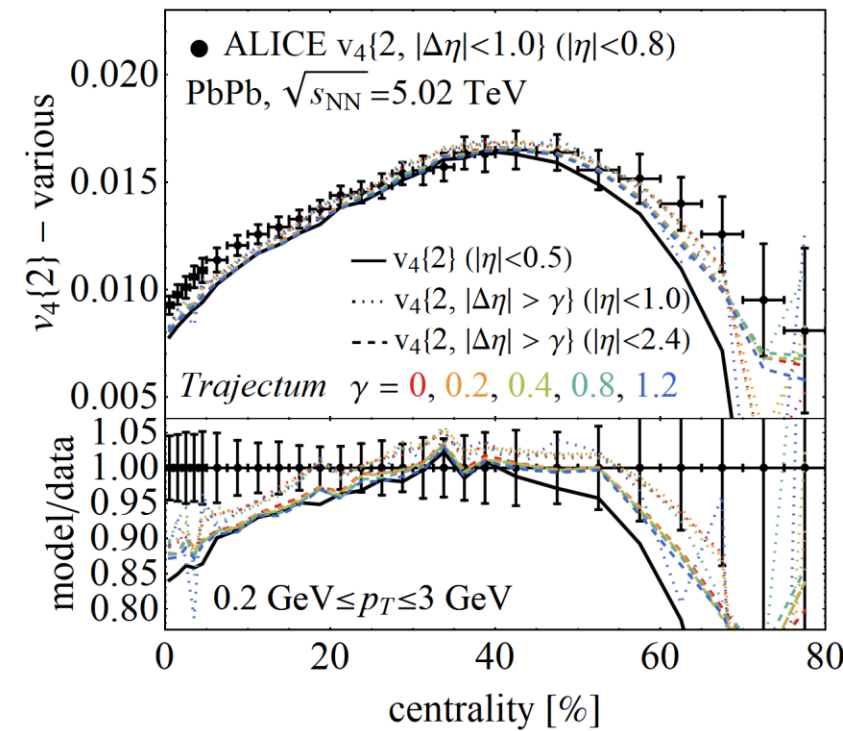
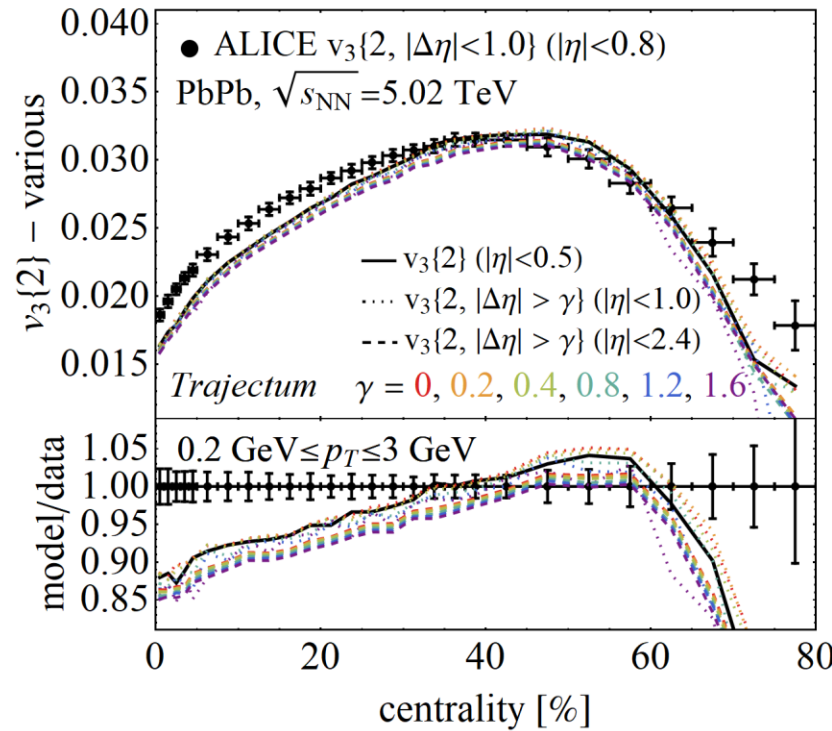
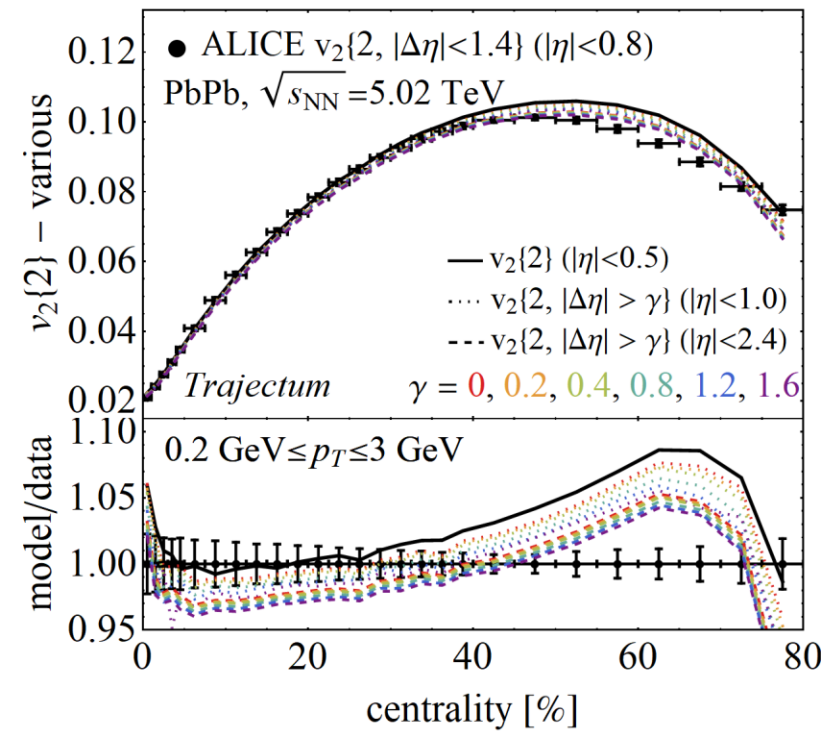
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BACK-UP

# $\Delta\eta$ in PbPb collisions – MAP setting with varying cuts

## Small but significant effects

- Etagap always reduces  $v_2$ , always increases  $v_4$  (even for  $\Delta\eta = 0$ )
- Settings fit to method 1 (black), but etacut potentially improves agreement (important within ALICE uncertainties)
- Larger effect for smaller detector acceptance (e.g. more important for ALICE than for ATLAS)
- In ratio with data effects are constant versus centrality (however small absolute effect: percent level \* percent level)





# MAP settings

## For reference the settings used for most runs

- Except 10 settings drawn from posterior

	MAP
$N_{\text{PbPb}2.76} \text{ [fm}^{-1}\text{]}$	17.9188
$\sigma_{\text{NN PbPb}2.76} \text{ TeV [mb]}$	61.8
$w_{\text{PbPb}2.76} \text{ [fm]}$	0.577622
$\text{cent}_{\text{norm PbPb}2.76} \text{ [\%]}$	97.9491
$N_{\text{PbPb}5.02} \text{ [fm}^{-1}\text{]}$	22.3675
$\sigma_{\text{NN PbPb}5.02} \text{ TeV [mb]}$	67.6
$\sigma_{\text{fluct}}$	0.510595
$p$	-0.0790886
$q$	1.2493
$d_{\text{min}} \text{ [fm]}$	0.99714
$T_{\text{switch}} \text{ [MeV]}$	149.742
$n_c$	2.30004
$\chi_{\text{struct}}$	0.792271
$\tau_{\text{hyd}} \text{ [fm/c]}$	0.544193
$\overline{\eta/s}$	0.18022
$(\eta/s)_{\text{slope}} \text{ [GeV}^{-1}\text{]}$	0.132154
$(\eta/s)_{\text{slope}} \text{ [GeV}^{-1}\text{]}$	0.371218
$(\eta/s)_{0.8 \text{ GeV}}$	0.324098
$(\zeta/s)_{\text{max}}$	0.0372411
$(\zeta/s)_{\text{m+w}} \text{ [GeV]}$	0.00517364
$(\zeta/s)_{T_0} \text{ [GeV]}$	0.35605
$\tau_{\pi} s T / \eta$	1.65841
$\tau_{\pi\pi} / \tau_{\pi}$	3.48342
$r_{\text{hyd}}$	0.829136
$a_n$	0.655107
$\langle \beta_2^2 \rangle - \langle \beta_2 \rangle^2$	0.0689689
$a_{\text{EOS}}$	-9.41155
$f_{\text{SMASH}}$	0.903164