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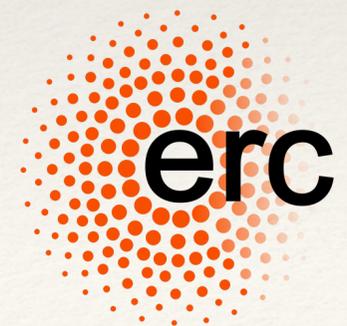
Precision probe of the structure of atomic nuclei with heavy-ion collisions

The VIIth International Conference on the Initial Stages of High-Energy Nuclear Collisions

June 19-23, 2023

THE VELUX FOUNDATIONS

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DANMARKS FRIE FORSKNINGSFOND

Nuclear structure

Nuclear structure is a key challenge in low-energy nuclear physics

❖ Many different approaches:

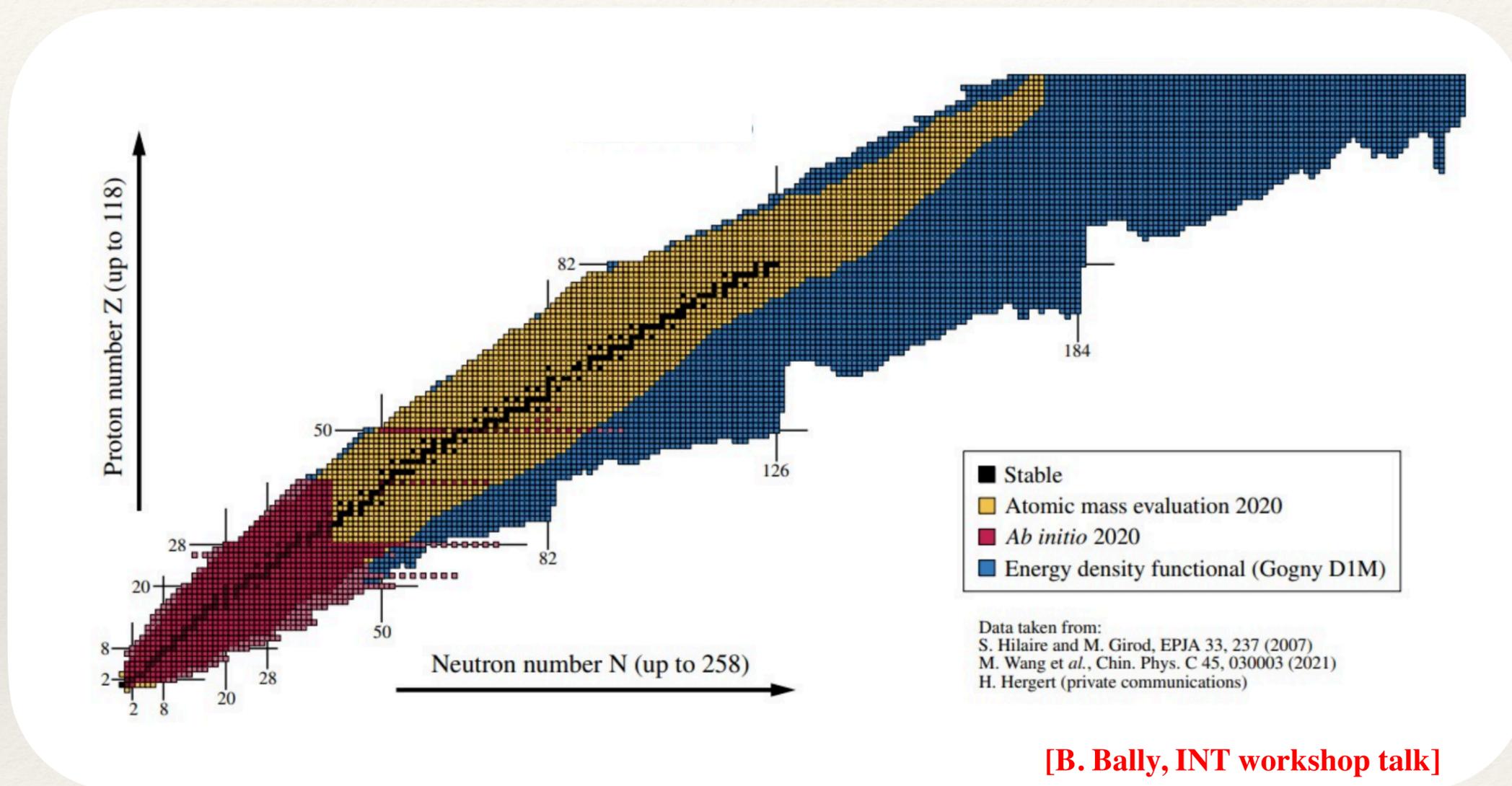
- Ab initio: Solve Schrödinger eq.
- EDF: Mean field theory
- Shell models

Consistent nuclear structure?

Low energy  High energy

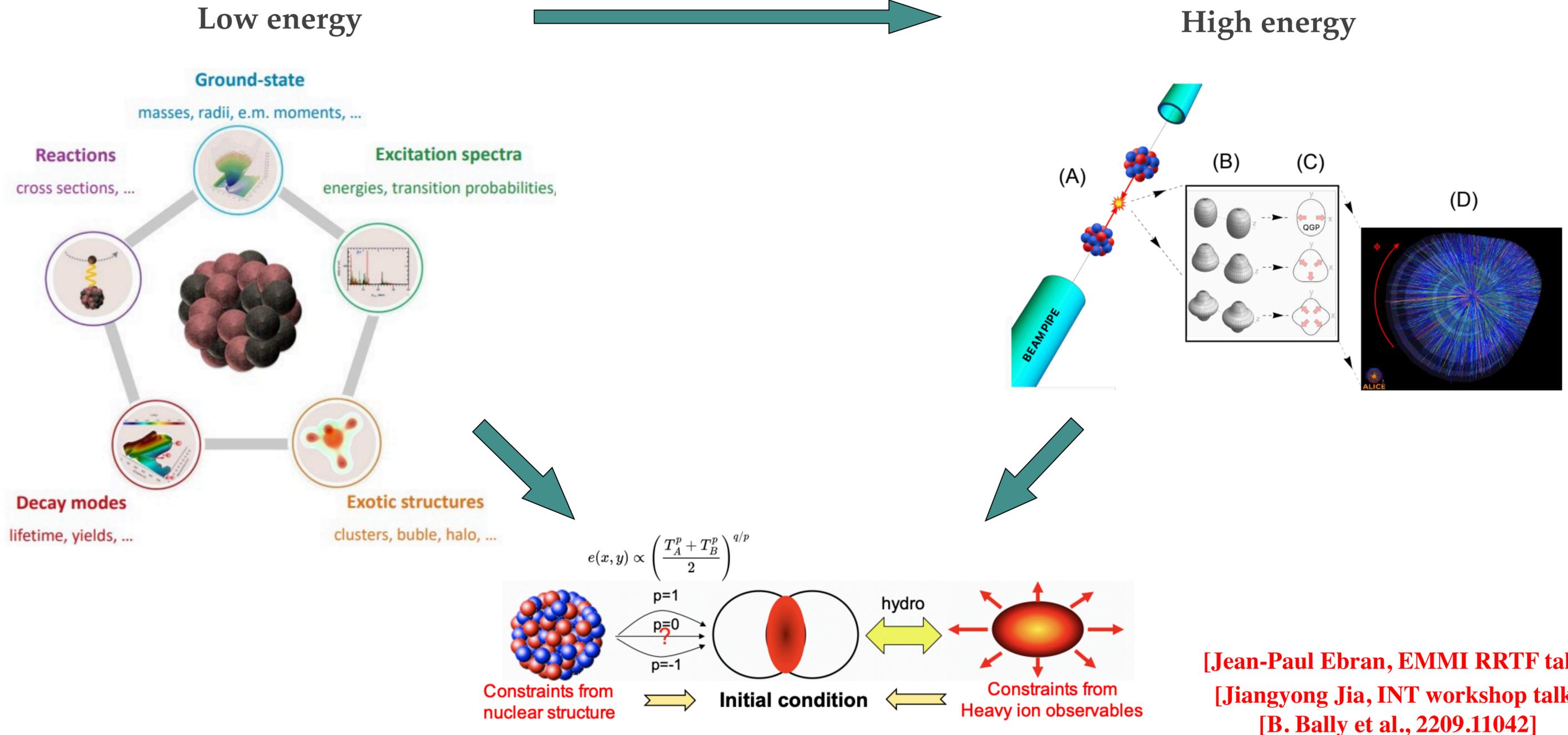
❖ Complement the low-energy efforts

- Extract initial conditions of heavy-ion collisions



Heavy-ion collisions: snapshot of geometry at time of collisions

Collaboration across the fields



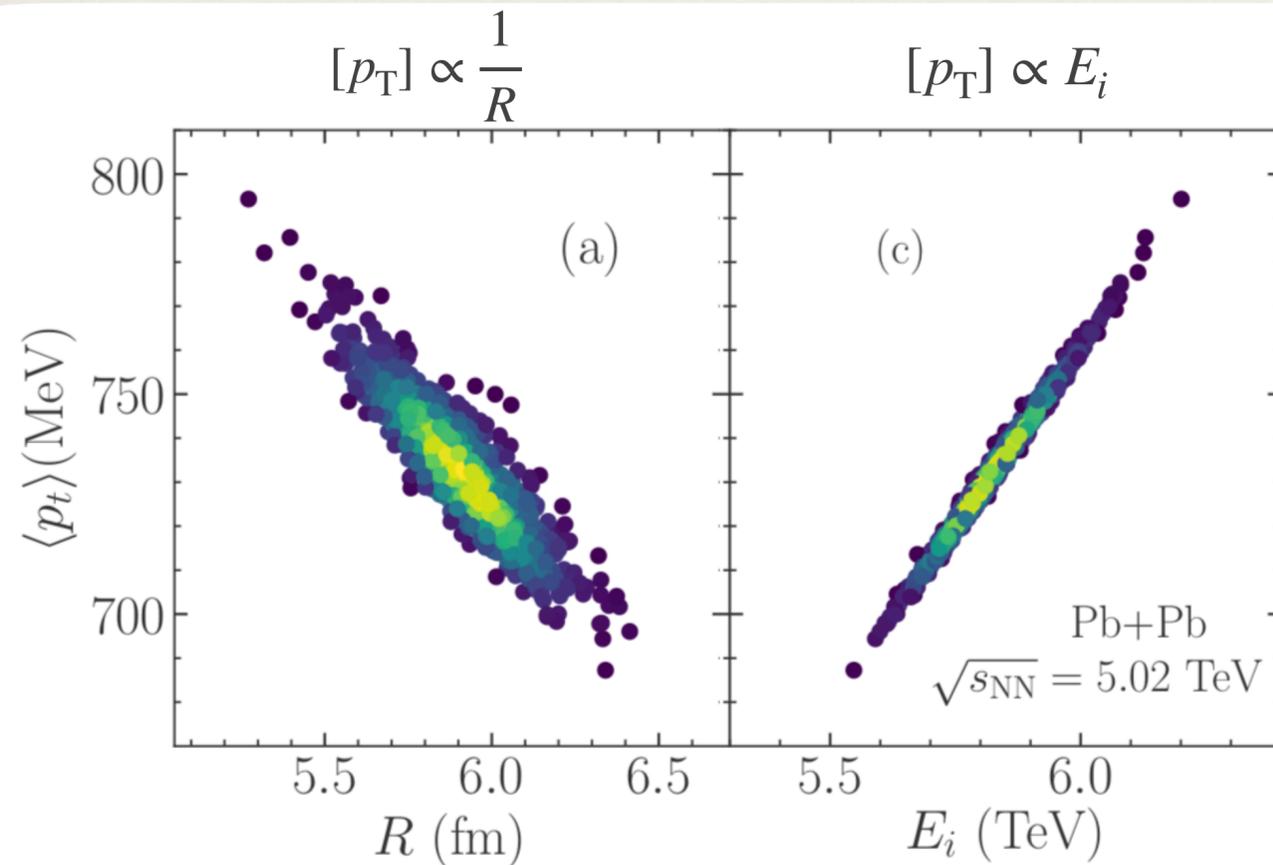
[Jean-Paul Ebran, EMMI RRTF talk]
 [Jiangyong Jia, INT workshop talk]
 [B. Bally et al., 2209.11042]

Connecting final and initial state

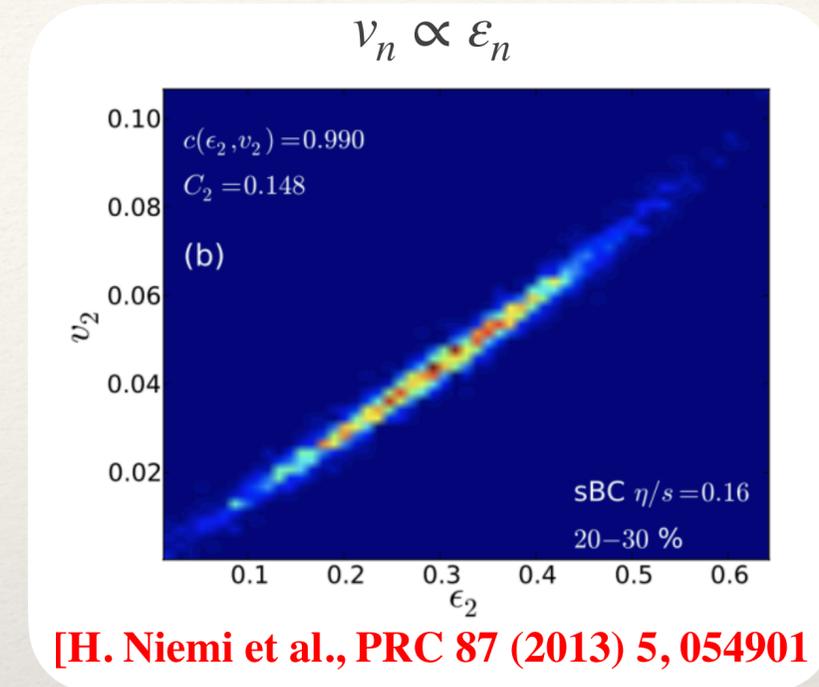


Key observables:

- ❖ Shape of the fireball: v_n
- ❖ Size of the fireball: $[p_T]$



[G. Giacalone et al., PRC103 (2021) 2, 024909]



- ❖ Pearson Correlation Coefficient

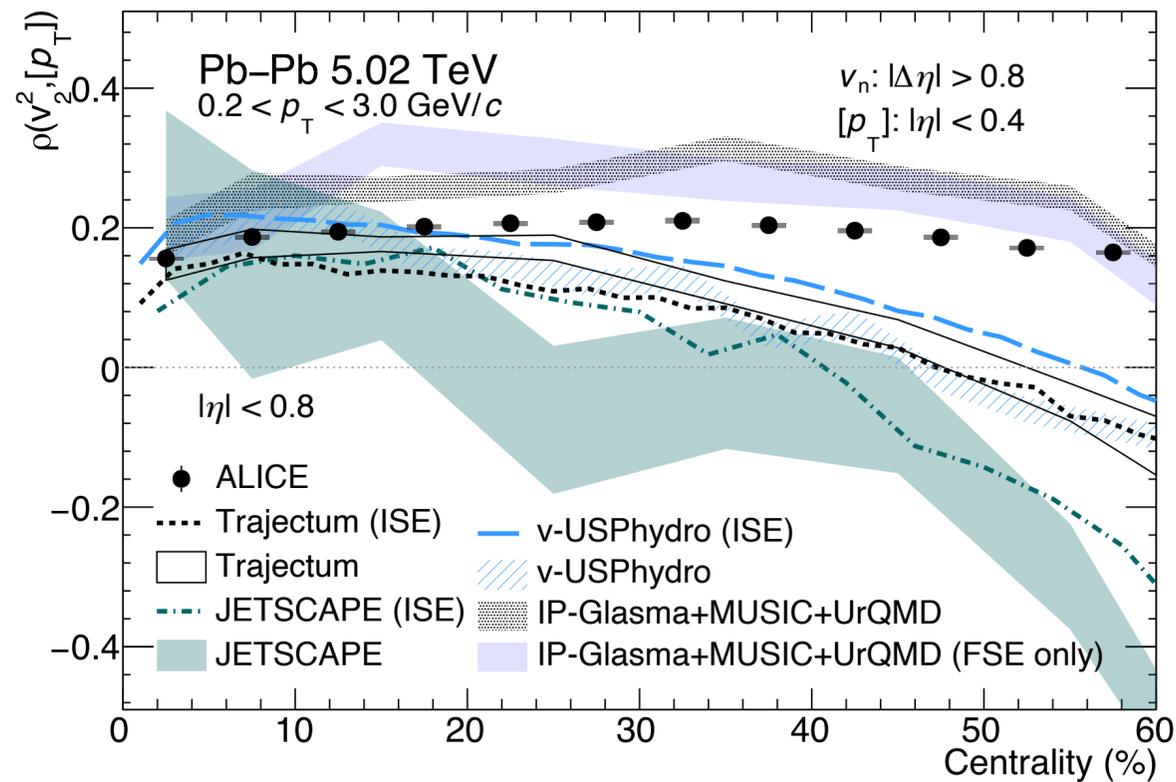
$$\rho(v_n^2, [p_T]) = \frac{\text{Cov}(v_n^2, [p_T])}{\sqrt{\text{var}(v_n^2)} \sqrt{c_k}}$$

Correlation of v_n and $[p_T] \rightarrow$ Initial geometry and fluctuations of size and shape

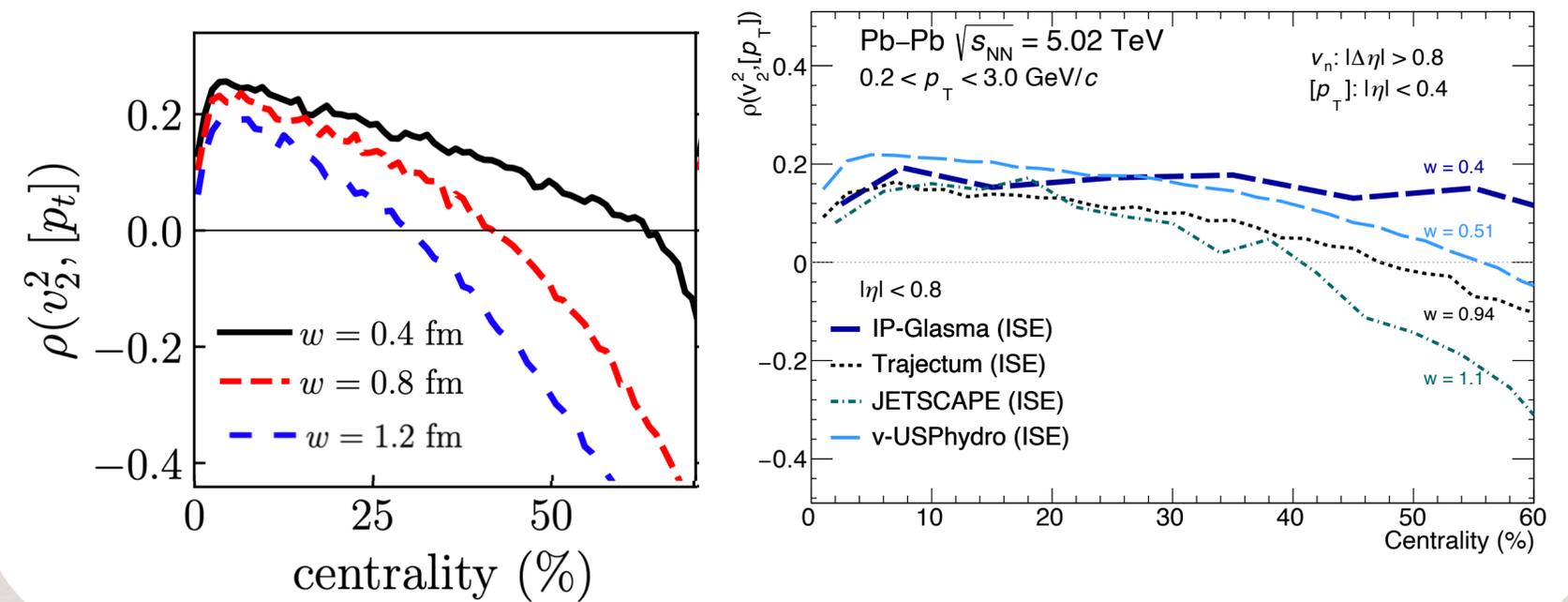
Probing the initial state



ALICE Collaboration, PLB 834, 137393 (2022)



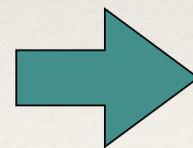
G. Giacalone et al., PRL 128, 042301 (2022)



Initial state dependence

- ❖ Large variation in description from different models

Resolved



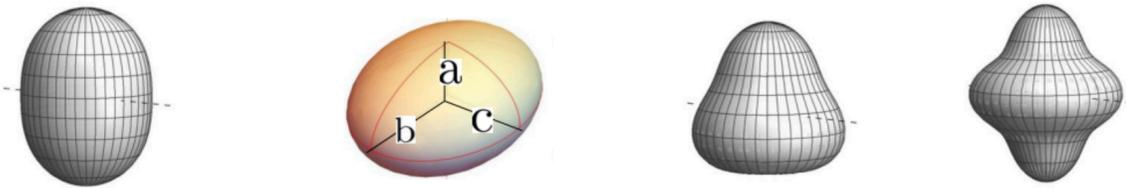
Differing nucleon width explains discrepancy

Nuclear structure in heavy-ion collisions

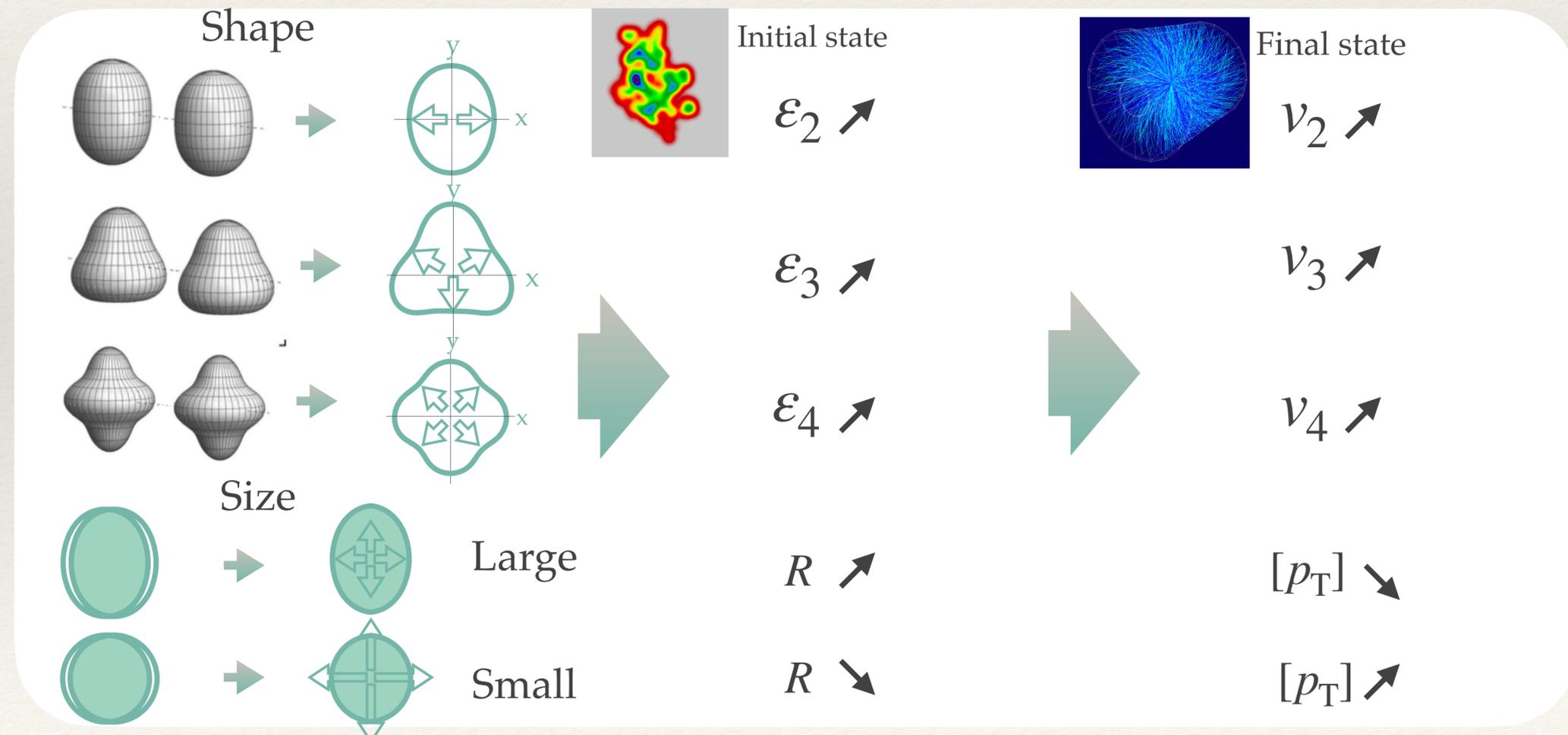


$$\rho(r, \Theta, \Phi) \propto \frac{1}{1 + \exp([r - R(\Theta, \Phi)]/a)}, \quad R(\Theta, \Phi) = R_0 \left[1 + \beta_2 \left(\cos \gamma Y_{20}(\Theta) + \sin \gamma Y_{22}(\Theta, \Phi) \right) + \beta_3 Y_{30}(\Theta) + \beta_4 Y_{40}(\Theta) \right]$$

Generalised Woods-Saxon profile



- ❖ Collision of deformed nuclei reveal the structure in **central** collisions
- ❖ Random orientation (body-body, tip-tip)



Collision selects one orientation

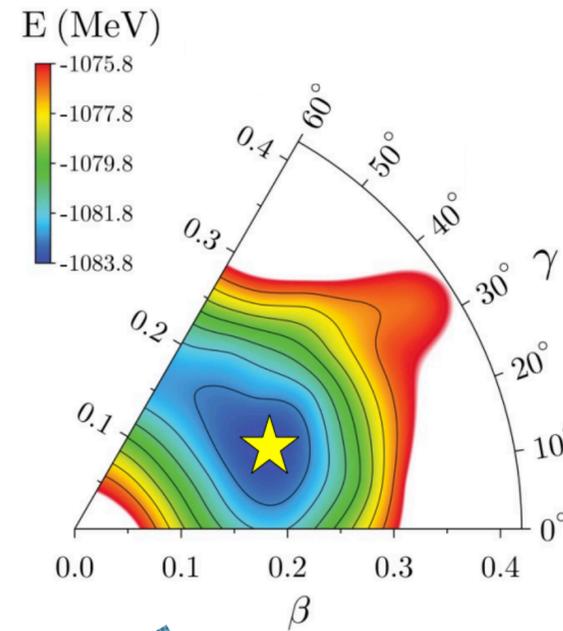
Structure of ^{129}Xe



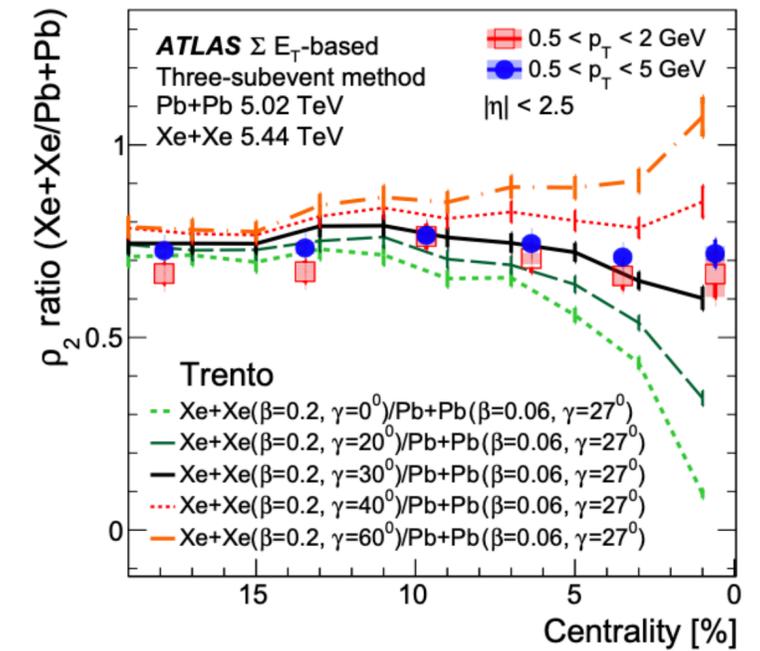
❖ ^{129}Xe predicted to have triaxial structure

Good agreement with measurements from heavy-ion collisions

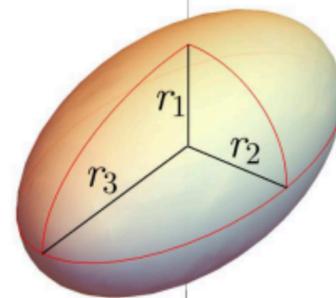
[B. Bally et al., PRL 128 (2022) 8, 082301]



[ATLAS Collaboration, PRC 107 (2023) 5, 054910]

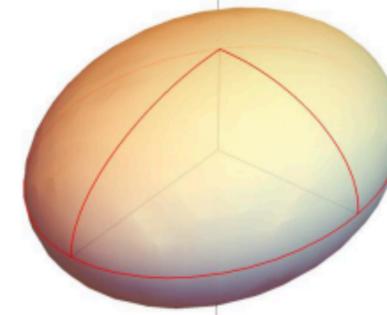


Prolate



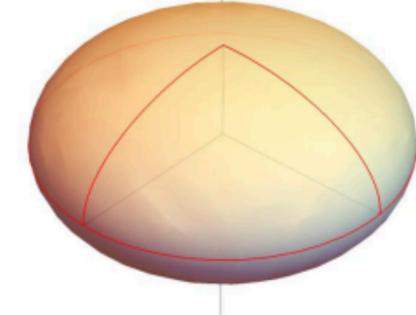
$$r_1 = r_2 < r_3$$

Triaxial



$$r_1 \neq r_2 \neq r_3$$

Oblate

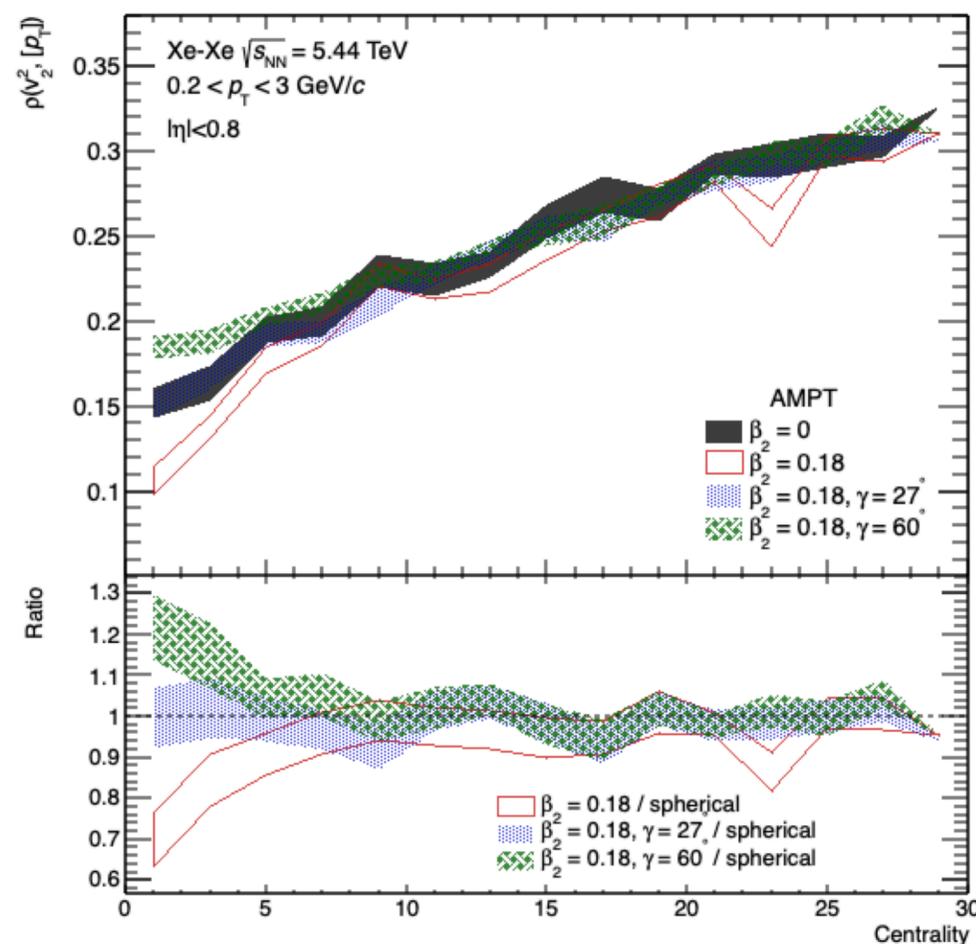


$$r_1 < r_2 = r_3$$

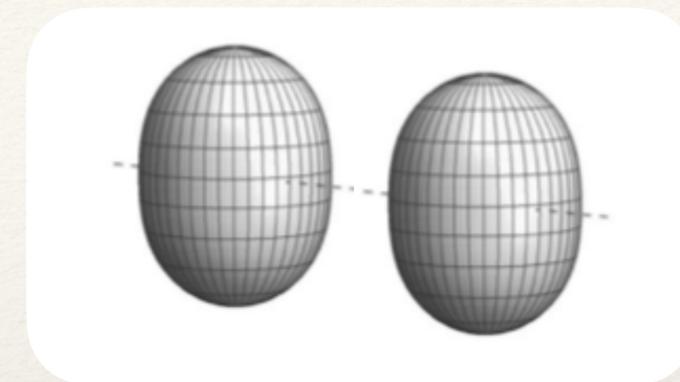
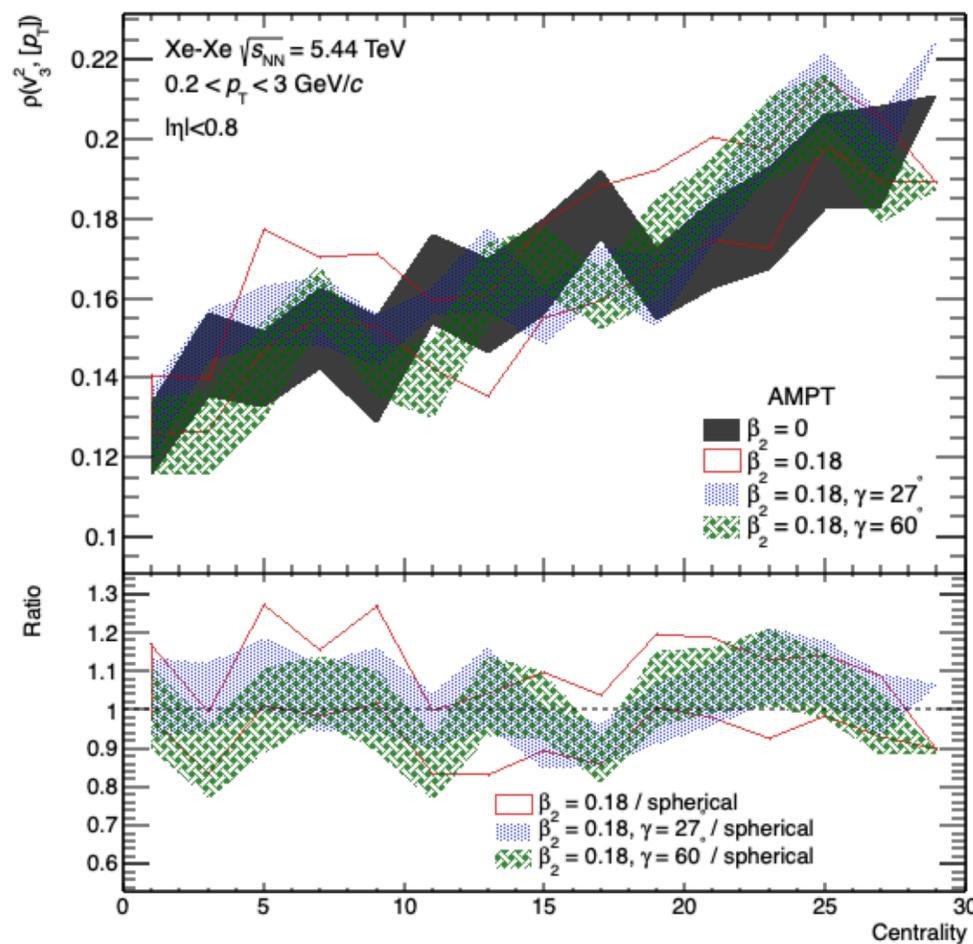
$\rho(v_n^2, [p_T])$ in AMPT

AMPT

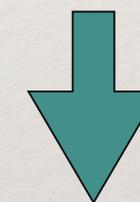
$v_2^2 - [p_T]$



$v_3^2 - [p_T]$



Large quadrupole deformation



- Larger v_2 in central collisions
- v_3 unaffected
- Clear distinction of triaxial state

Ratio to spherical baseline

$\rho(v_n^2, [p_T])$ in T_RENTo

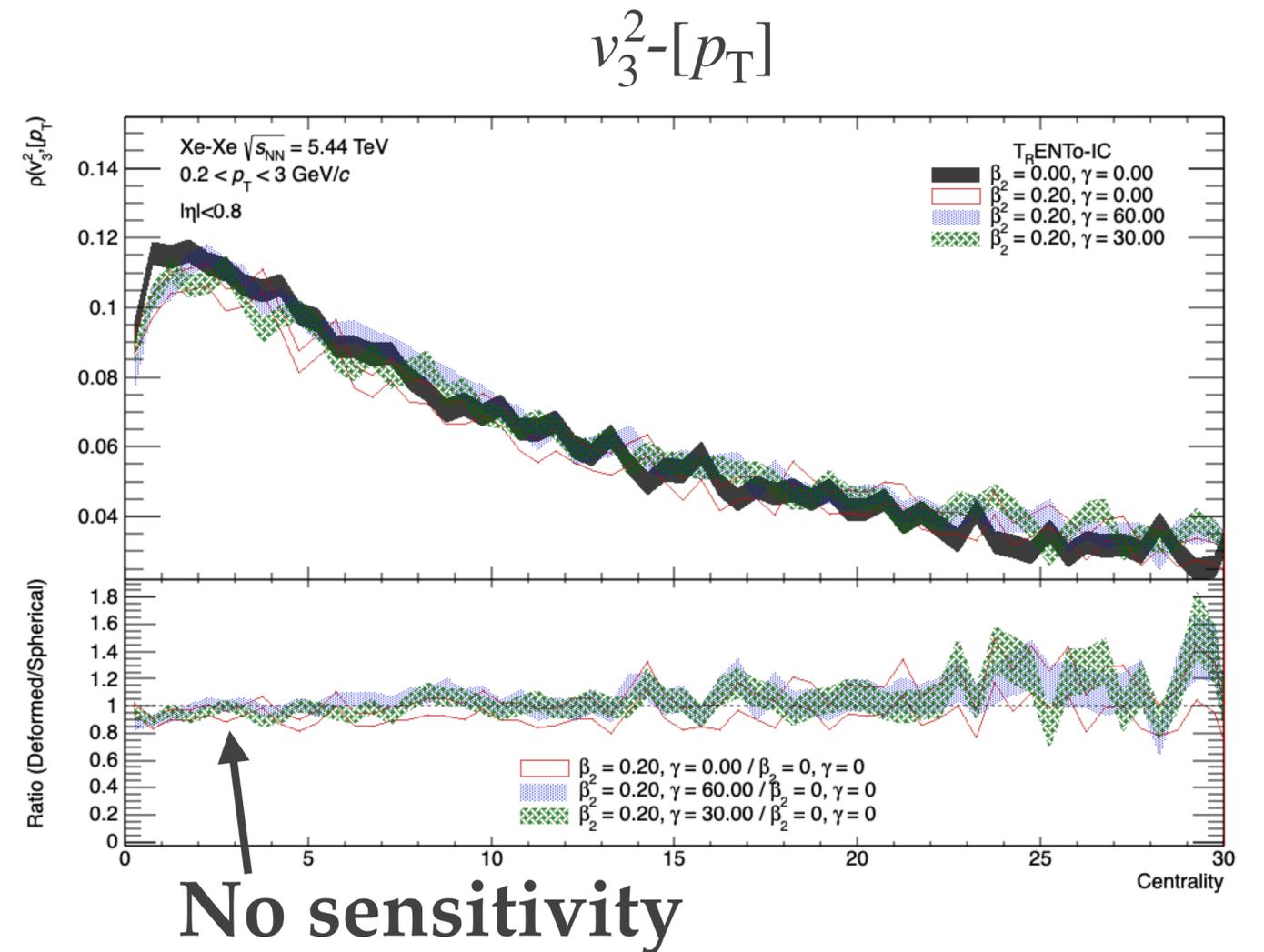
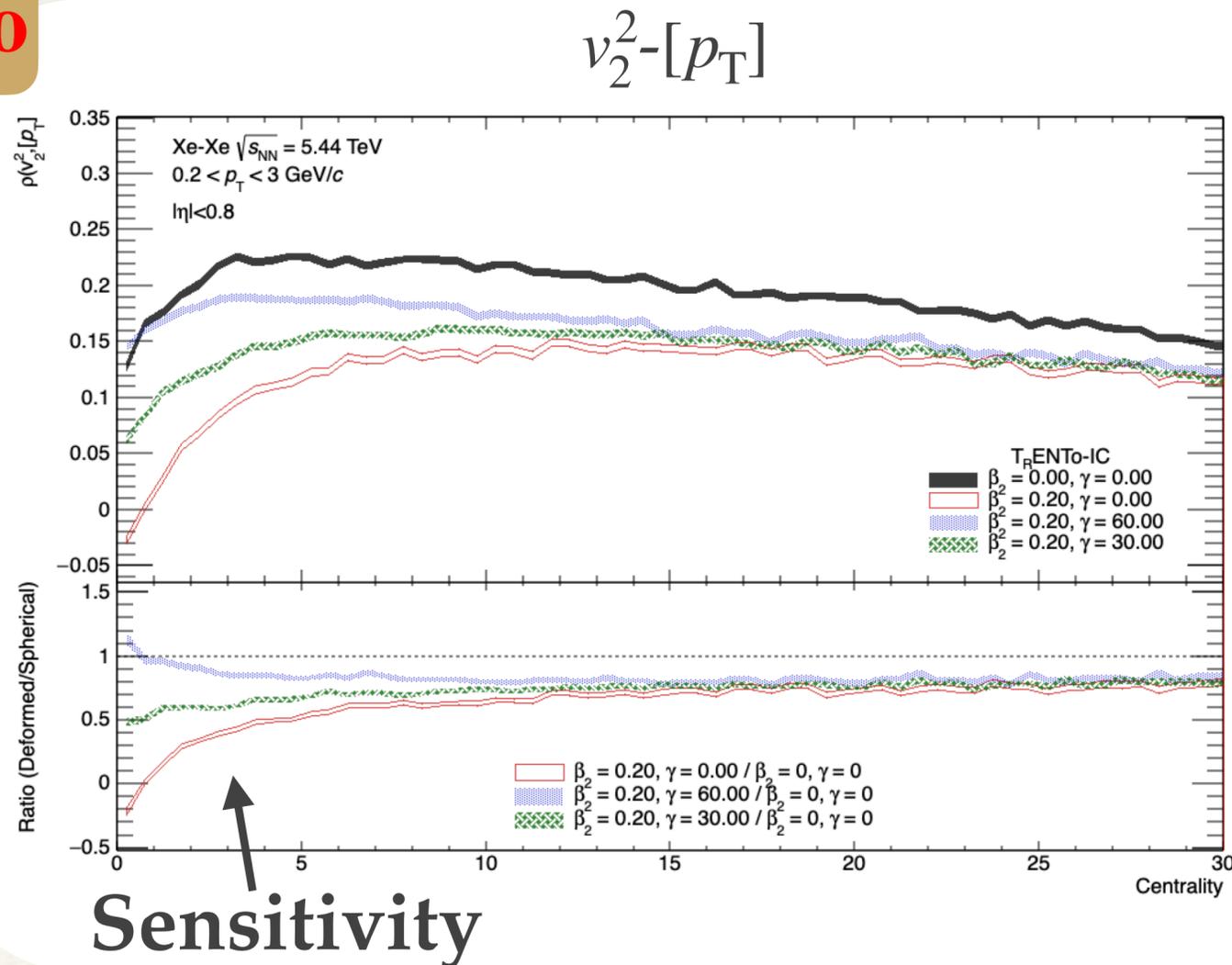
❖ Initial state estimator (T_RENTo)

$$v_2 \leftrightarrow \varepsilon_2 \quad [p_T] \leftrightarrow E_i$$

Parametrised initial state

$$T = \left(\frac{T_A^p + T_B^p}{2} \right)^{1/p}, \quad T_A = \sum_i^{N_{\text{part},A}} \frac{\lambda_i}{2\pi\omega^2} e^{-\frac{(x-x_i)^2}{2\omega^2}}$$

T_RENTo





Higher-order correlations

Can we increase our sensitivity to the nuclear structure?

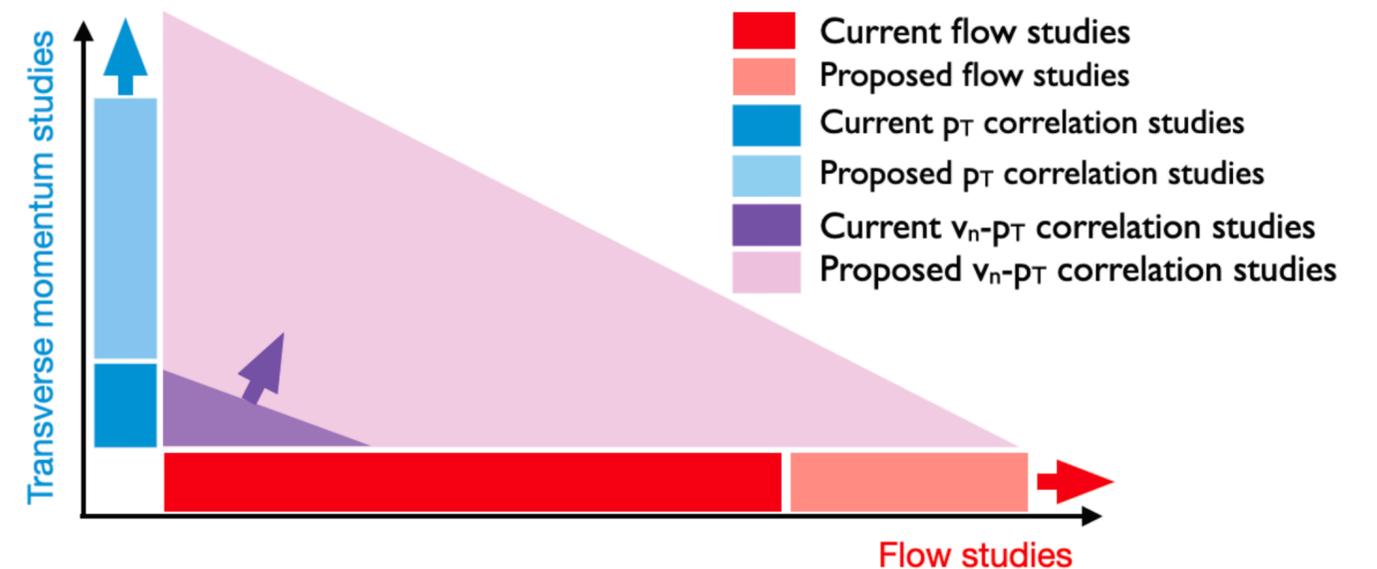
So far only lowest order v_n - $[p_T]$ has been explored

$[p_T^{(k)}]$: k-particle p_T correlation
 $\langle p_{T,i_1} \cdots p_{T,i_k} \rangle$

❖ Derive the higher-order cumulants of the v_n - p_T correlations

$$\rho(v_n^m, [p_T^{(k)}]) = \frac{C(v_n^m, [p_T^{(k)}])}{\sqrt{\text{var}(v_n^m)} \sqrt{\text{var}([p_T^{(k)}])}}$$

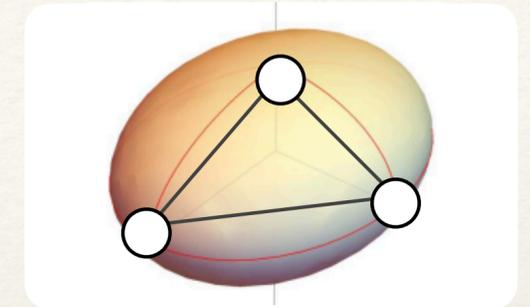
All lower orders subtracted
→ Genuine correlation



Higher-order correlations in AMPT

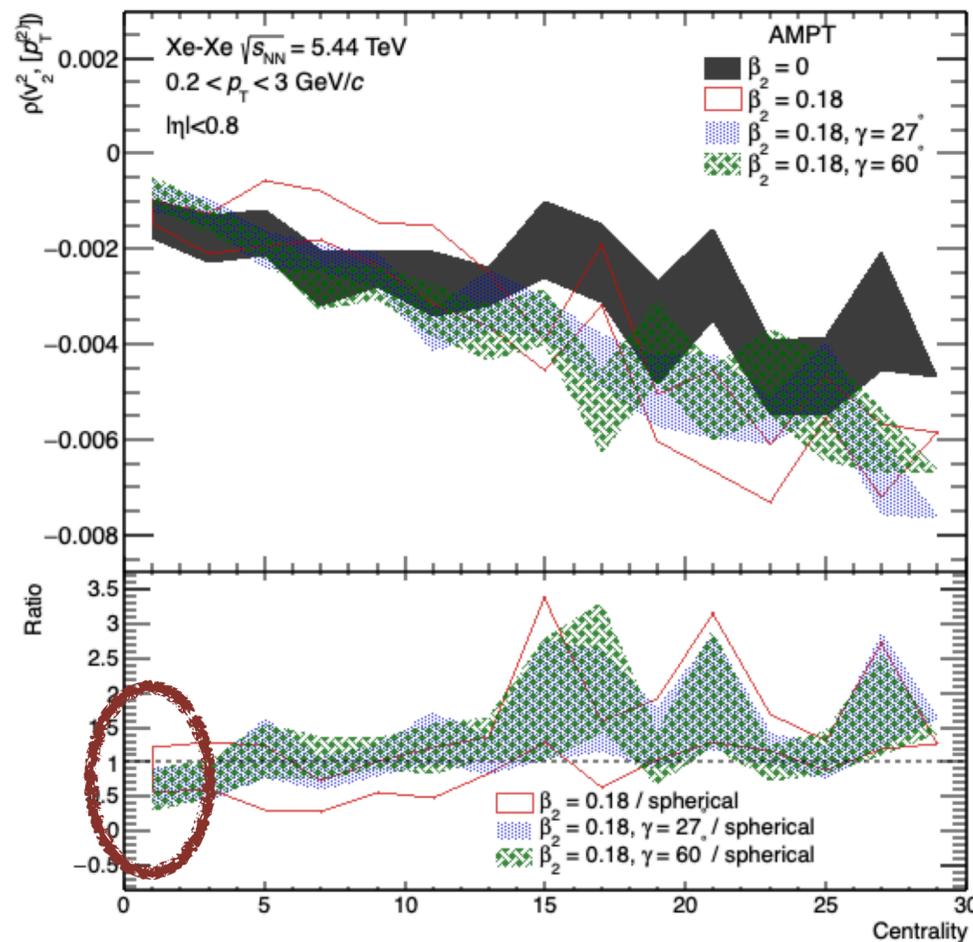


- ❖ No sensitivity in four-particle cumulant $\rho(v_n^2, [p_T^{(2)}])$

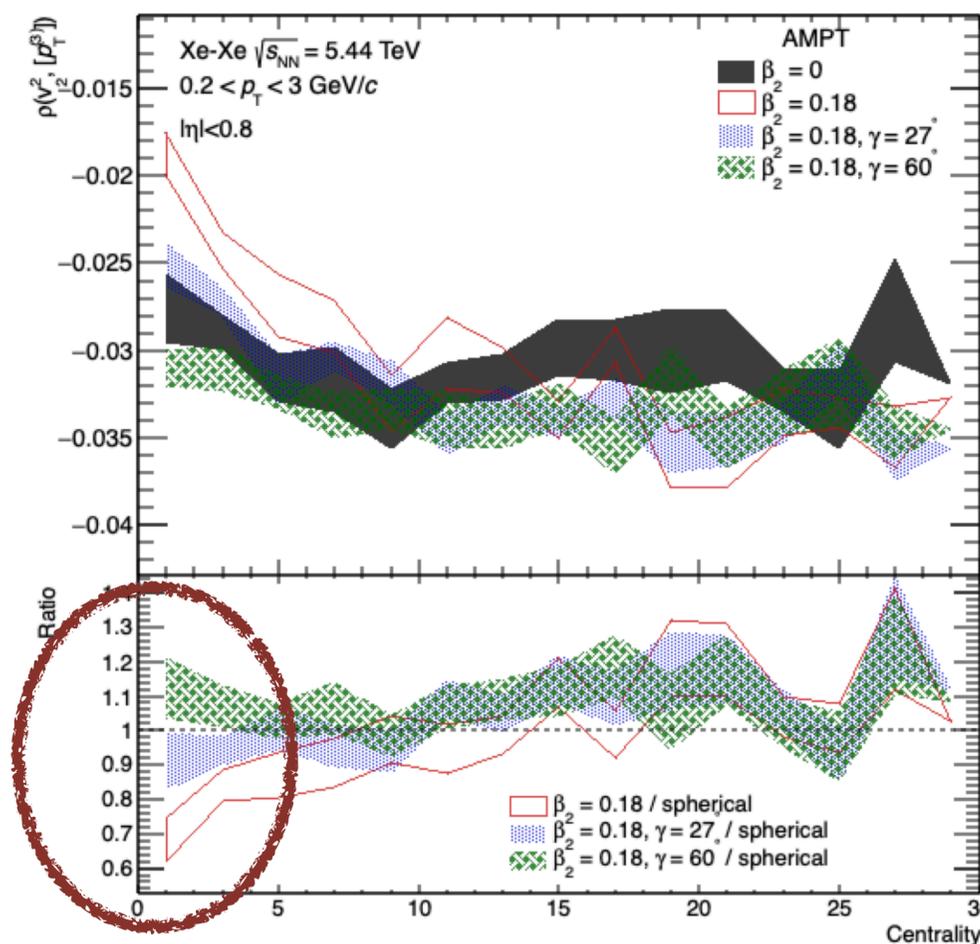


AMPT

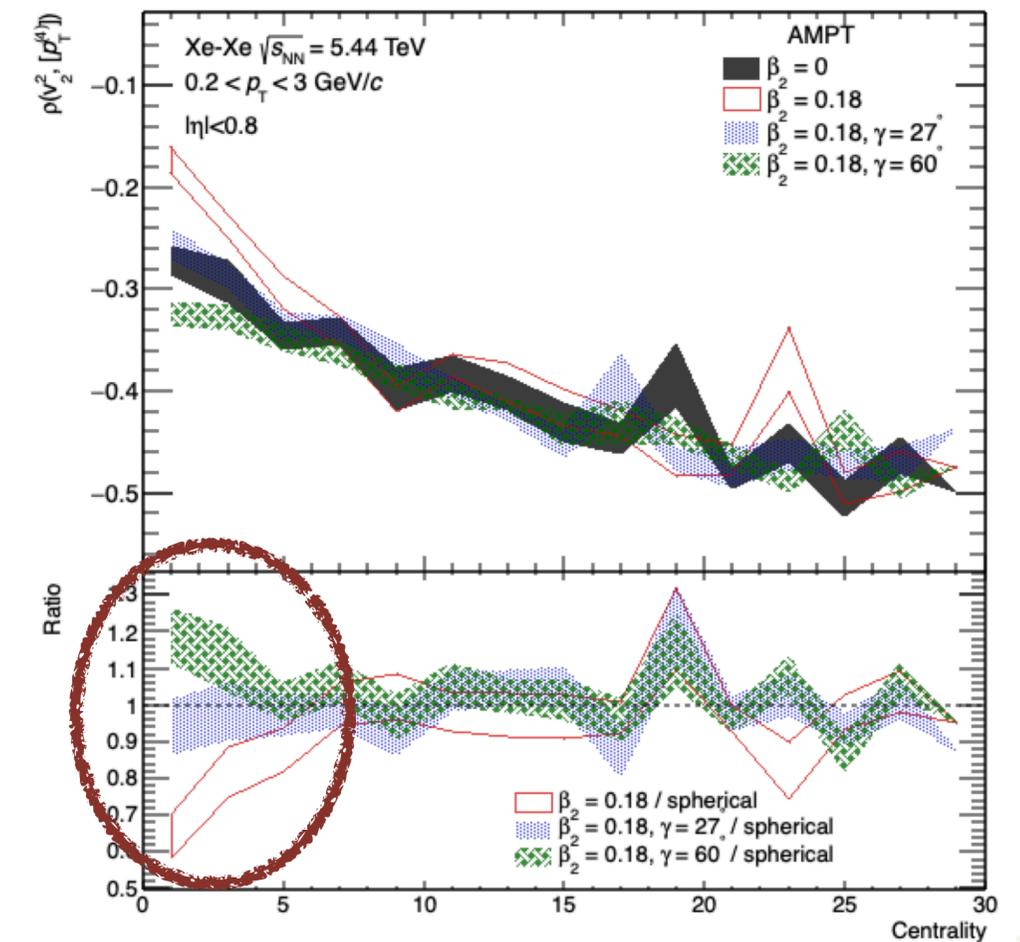
$v_2^2 - [p_T^{(2)}]$



$v_2^2 - [p_T^{(3)}]$



$v_2^2 - [p_T^{(4)}]$



Higher-order correlations in T_RENTo

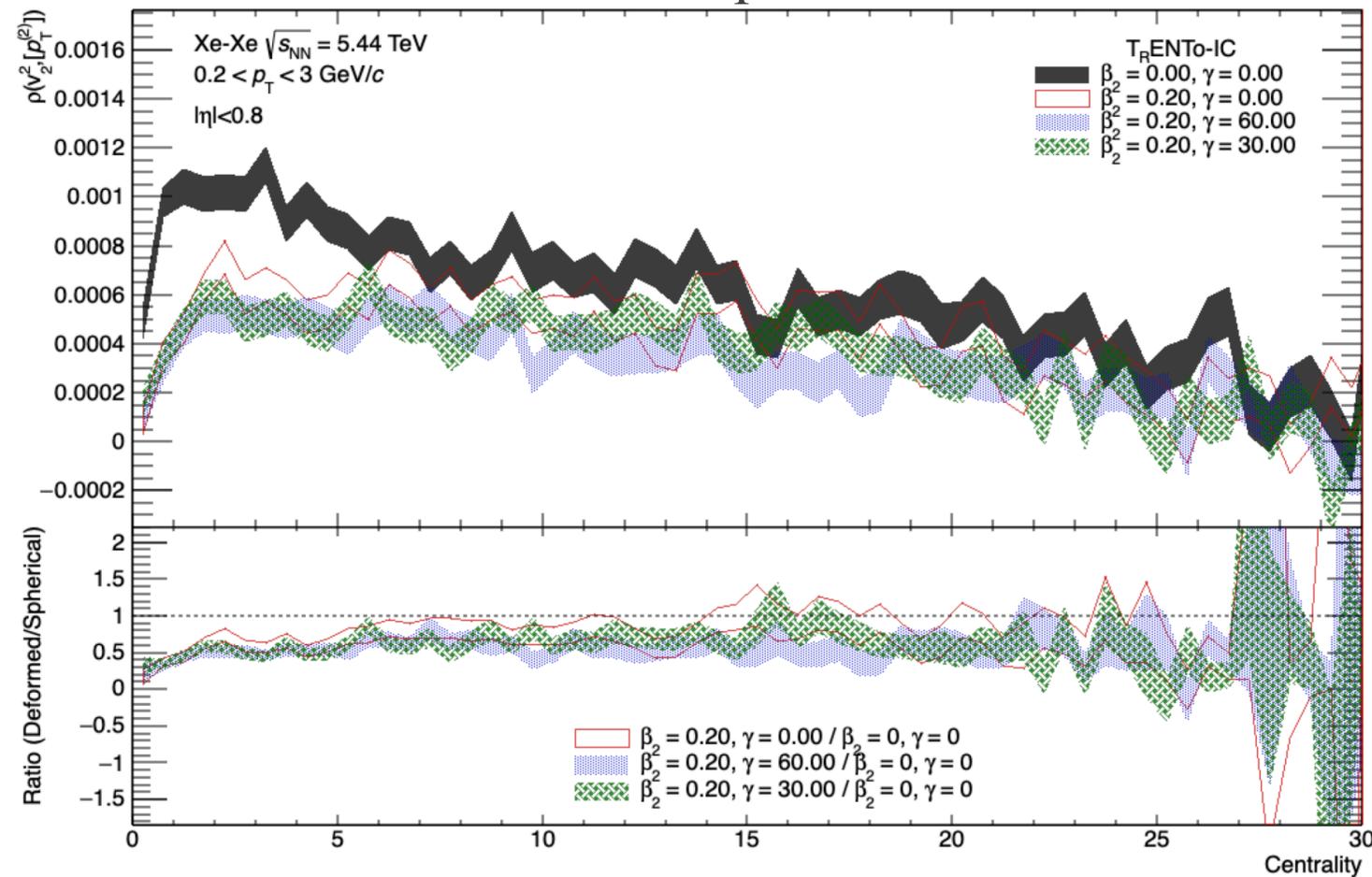


❖ Even clearer with T_RENTo:

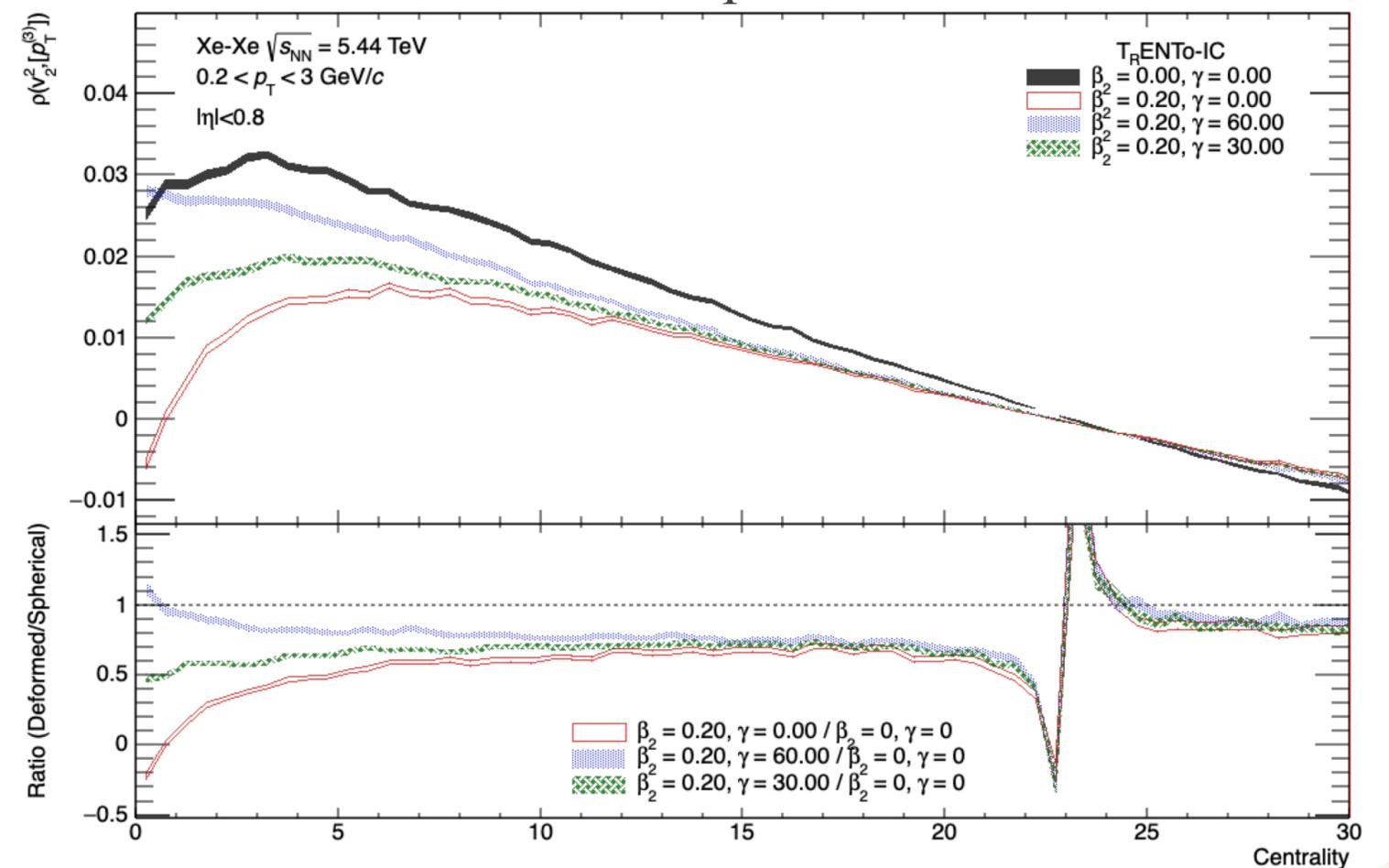
- Triaxial sensitivity in $v_2^2 - [p_T^{(3)}]$
- β_2 sensitivity in $v_2^2 - [p_T^{(2)}]$

T_RENTo

$v_2^2 - [p_T^{(2)}]$



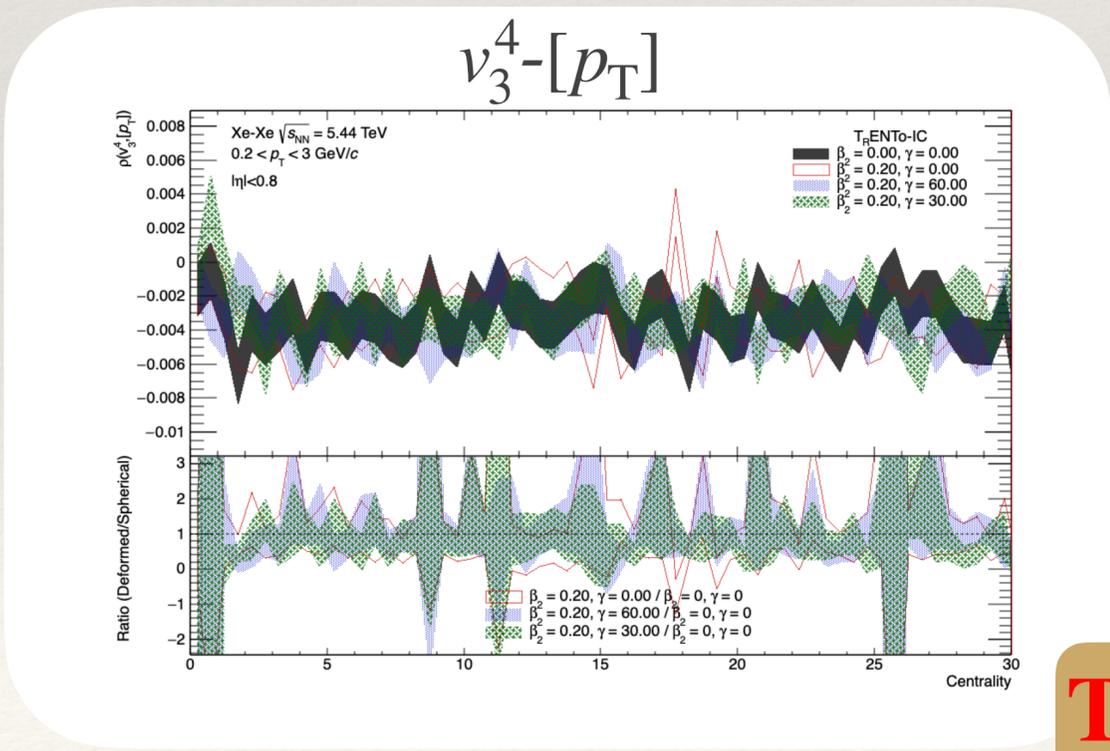
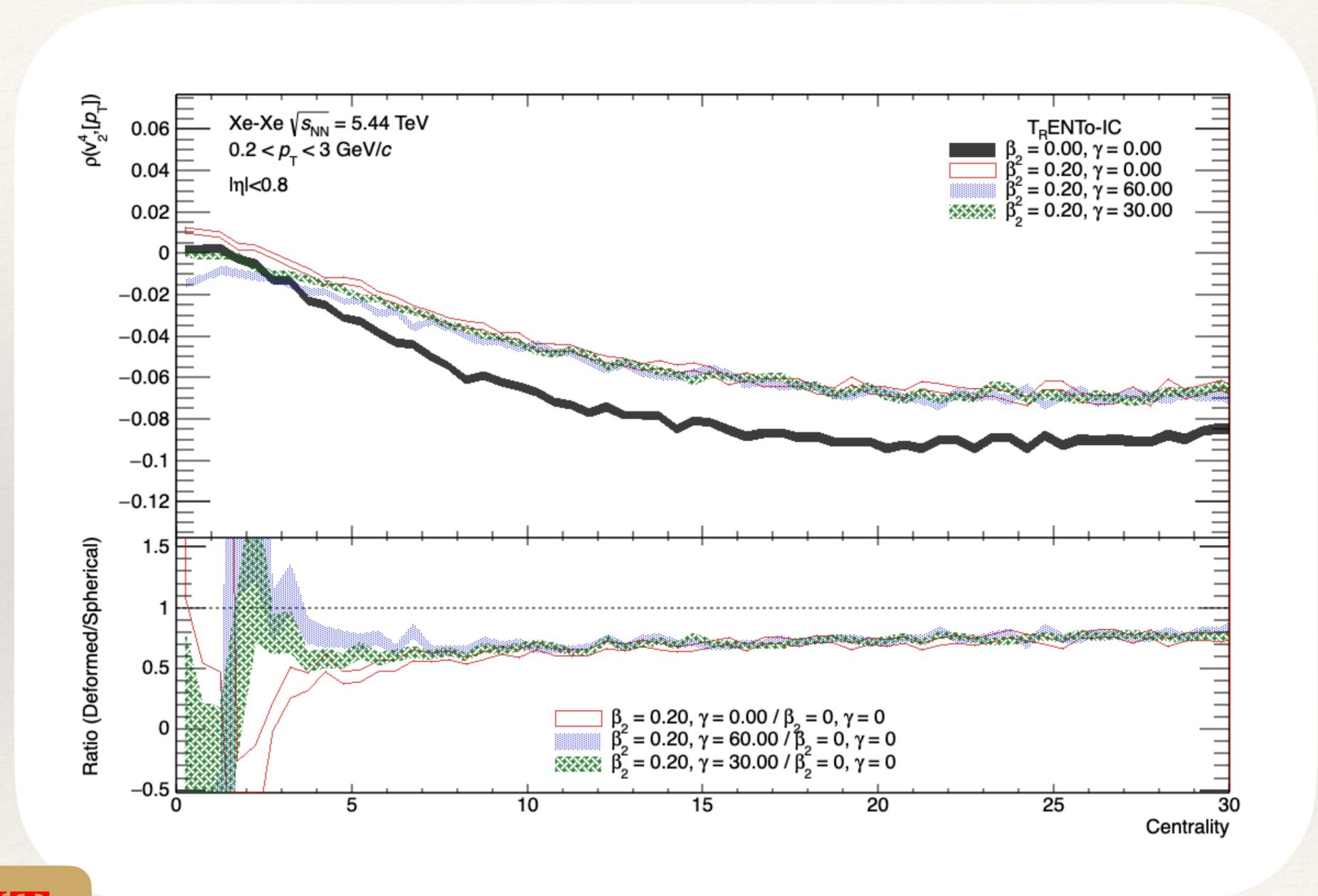
$v_2^2 - [p_T^{(3)}]$



Higher-order correlation: v_2^4 -[p_T]



- Higher moment of eccentricity: $v_2^4 \leftrightarrow \varepsilon_2^4$
- Weighs shape over size

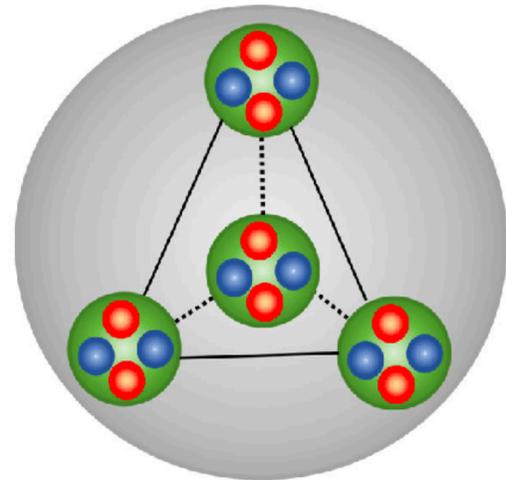


T_RENTo

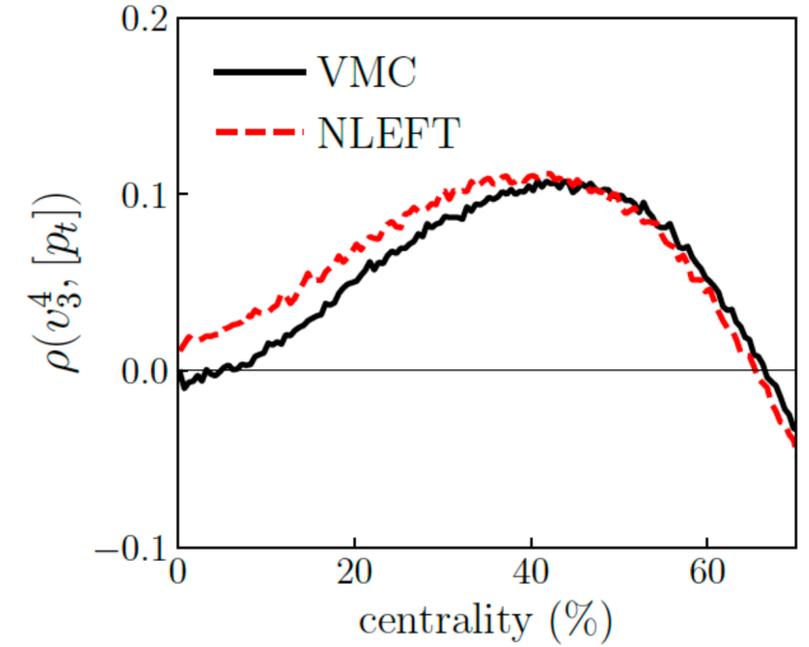
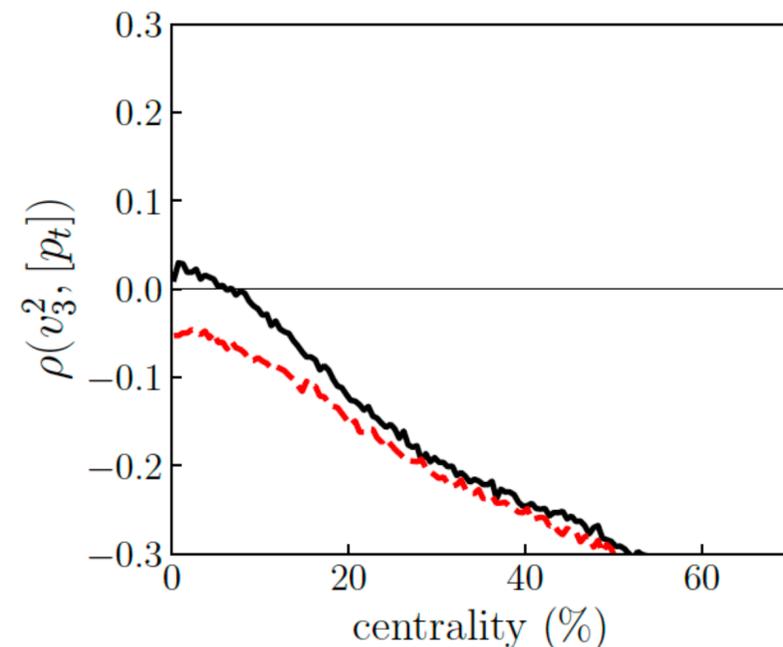
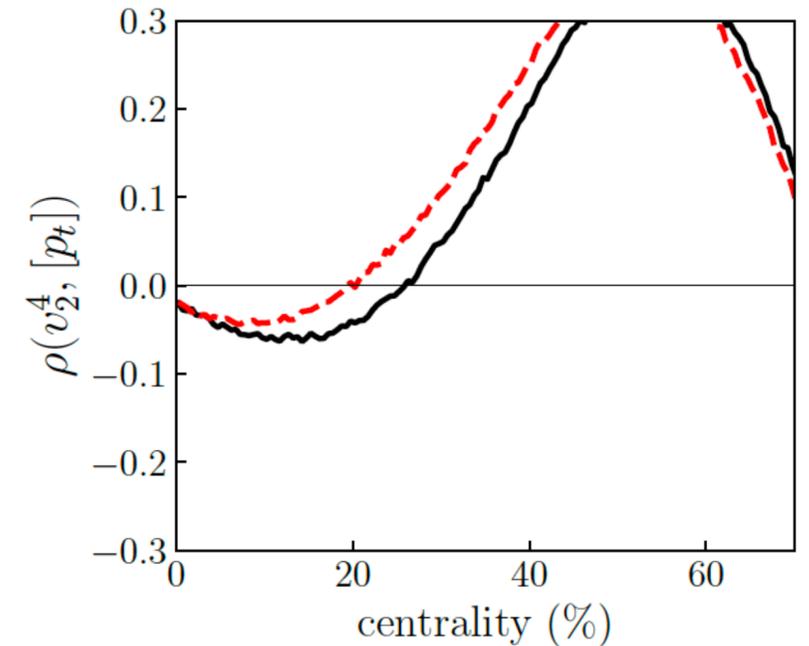
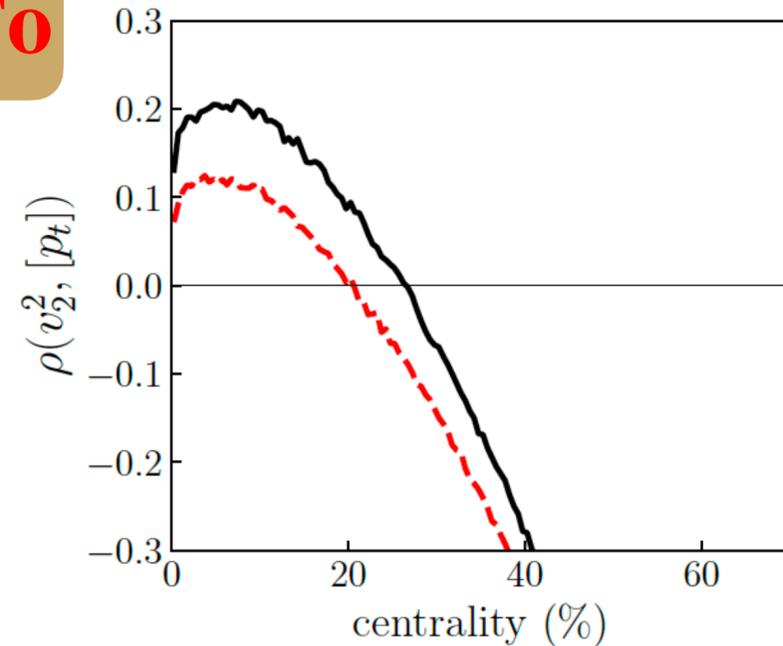
α -cluster structure of ^{16}O



D. Behera et al., EPJA (2022) 58:175



T_RENTo



❖ O-O structure:

- Nuclear Lattice Effective Field Theory
[N. Summerfield et al., PRC 104 (2021) 4, L041901]
- Variational Monte Carlo
[D. Lonardi et al., PRC 96 (2017) 2, 024326]

Nuclear physics \Leftrightarrow Heavy-ion collisions

Summary



- ❖ **Heavy-ion collisions offer a unique, precision imaging tool of the nuclear structure with cross field applications**
Complements low-energy nuclear physics, which rely on modelling to obtain the structure
- ❖ **Anisotropic flow and mean transverse momentum are the two key observables in probing the nuclear structure**
Higher-order correlations can reveal additional sensitivities to more complex shapes (octupole, hexadecapole, etc.)
- ❖ **Future runs at the LHC (O-O, Ne-Ne) allow us to test and predict cutting edge *ab-initio* nuclear structure methods**
Reducing our uncertainty on QGP properties while providing valuable information for hydrodynamic and transport model simulations and nuclear modification of pdfs

Formulas



$$C(v_2^2, [p_T^{(2)}]) = \langle v_n^2 \delta p_T^2 \rangle - \langle v_n^2 \rangle \langle \delta p_T^2 \rangle = \langle v_n^2 [p_T^{(2)}] \rangle - 2 \langle v_n^2 [p_T] \rangle \langle [p_T] \rangle - \langle v_n^2 \rangle \langle [p_T^{(2)}] \rangle + 2 \langle v_n^2 \rangle \langle [p_T] \rangle^2$$

$$\delta p_T = [p_T] - \langle [p_T] \rangle$$

$$\begin{aligned} C(v_2^2, [p_T^{(3)}]) &= \langle v_n^2 \delta p_T^3 \rangle - \langle v_n^2 \rangle \langle \delta p_T^3 \rangle - 3 \langle \delta p_T^2 \rangle \langle v_n^2 \delta p_T \rangle \\ &= \langle v_n^2 [p_T^{(3)}] \rangle - 3 \langle v_n^2 [p_T^{(2)}] \rangle \langle [p_T] \rangle - \langle v_n^2 \rangle \langle [p_T^{(3)}] \rangle - 3 \langle v_n^2 [p_T] \rangle \langle [p_T^{(2)}] \rangle + 6 \langle [p_T] \rangle \langle v_n^2 \rangle \langle [p_T^{(2)}] \rangle + 6 \langle [p_T] \rangle^2 \langle v_n^2 [p_T] \rangle - 6 \langle [p_T] \rangle^3 \langle v_n^2 \rangle \end{aligned}$$

$$\begin{aligned} C(v_2^2, [p_T^{(4)}]) &= \langle v_n^2 \delta p_T^4 \rangle - \langle v_n^2 \rangle \langle \delta p_T^4 \rangle - 6 \langle \delta p_T^2 \rangle \langle v_n^2 \delta p_T^2 \rangle - 4 \langle v_n^2 \delta p_T \rangle \langle \delta p_T^3 \rangle + 6 \langle \delta p_T^2 \rangle^2 \langle v_n^2 \rangle \\ &= \langle v_n^2 [p_T^{(4)}] \rangle - 4 \langle v_n^2 [p_T^{(3)}] \rangle \langle [p_T] \rangle - 6 \langle [p_T^{(2)}] \rangle \langle v_n^2 [p_T^{(2)}] \rangle - \langle v_n^2 \rangle \langle [p_T^{(4)}] \rangle - 4 \langle [p_T^{(3)}] \rangle \langle v_n^2 [p_T] \rangle \\ &\quad + 6 \langle [p_T^{(2)}] \rangle^2 \langle v_n^2 \rangle + 12 \langle v_n^2 [p_T^{(2)}] \rangle \langle [p_T] \rangle^2 + 24 \langle [p_T] \rangle \langle [p_T^{(2)}] \rangle \langle v_n^2 [p_T] \rangle + 8 \langle [p_T] \rangle \langle [p_T^{(3)}] \rangle \langle v_n^2 \rangle - 36 \langle [p_T] \rangle^2 \langle [p_T^{(2)}] \rangle \langle v_n^2 \rangle \\ &\quad - 24 \langle [p_T] \rangle^3 \langle v_n^2 [p_T] \rangle + 24 \langle [p_T] \rangle^4 \langle v_n^2 \rangle \end{aligned}$$

$$C(v_2^4, [p_T]) = \langle v_n^4 \delta p_T \rangle - 4 \langle v_n^2 \delta p_T \rangle \langle v_n^2 \rangle = \langle v_n^4 [p_T] \rangle - \langle v_n^4 \rangle \langle [p_T] \rangle - 4 \langle v_n^2 \rangle \langle v_n^2 [p_T] \rangle + 4 \langle v_n^2 \rangle^2 \langle [p_T] \rangle$$



Formulas

m-particle p_T correlation with auto-correlations subtracted

$$\langle [p_T^{(k)}] \rangle = \frac{N\langle k \rangle_{p_T}}{D\langle k \rangle_{p_T}}$$

$$N\langle m \rangle_{p_T} = \sum_{k=1}^m (-1)^{k-1} N\langle m-k \rangle_{p_T} \frac{(m-1)!}{(m-k)!} \sum_i^M w_i^k p_{T,i}^k$$

$$D\langle m \rangle_{p_T} = \sum_{k=1}^m (-1)^{k-1} D\langle m-k \rangle_{p_T} \frac{(m-1)!}{(m-k)!} \sum_i^M w_i^k$$