

Large-scale model-to-data comparison of ultrarelativistic heavy-ion collisions using TRENTo-3D initial conditions

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Initial Stages 2023
Copenhagen

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T_RENTo Collision Model

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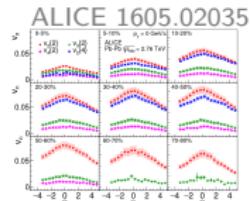
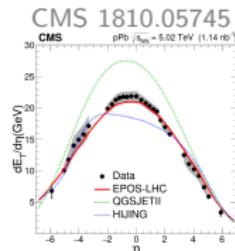
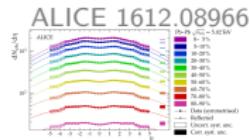
Model-to-Data Comparison

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► Introduction

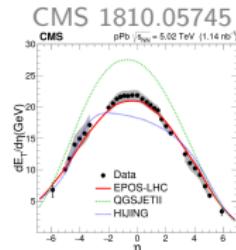
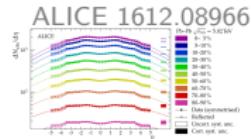
Introduction

- Boost-invariant (2D) studies of QGP have been remarkably successful—but they are not the whole story
- 3D: an additional dimension of observables
 - Rapidity-differential measurements
 - Rapidity (de)correlations
 - Rapidity cuts, rapidity gaps, ...
- 3D: an additional dimension of challenges
 - (3+1)D hydrodynamics is expensive
 - Model degrees of freedom are not constrained by symmetry (no boost invariance)
 - 3D initial conditions are an open question
- Goal: propose T_RENTo-3D as a fast, flexible, 3D initial-conditions model
- This talk: describe and demonstrate the model

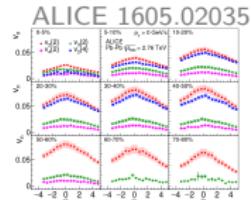


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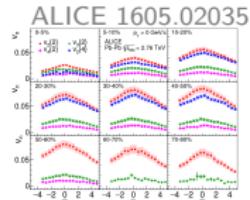
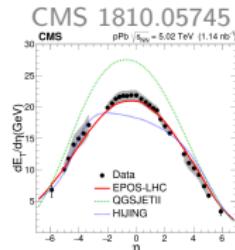
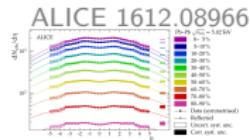


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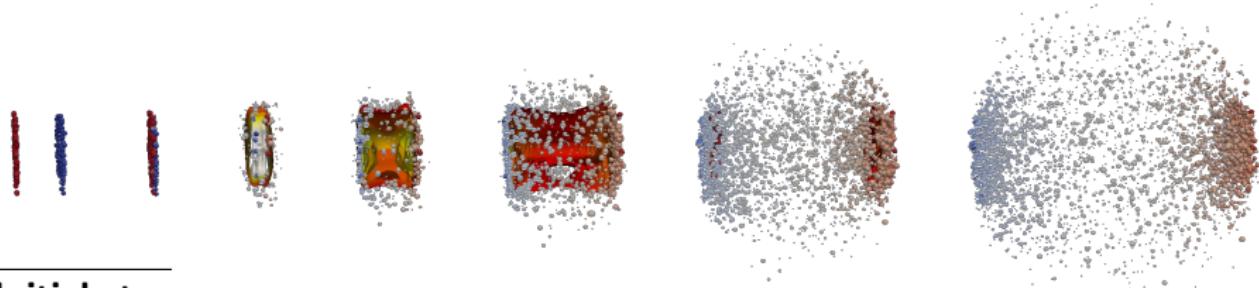
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► Background

Background



✓ Initial stage

✗ Pre-equilibrium

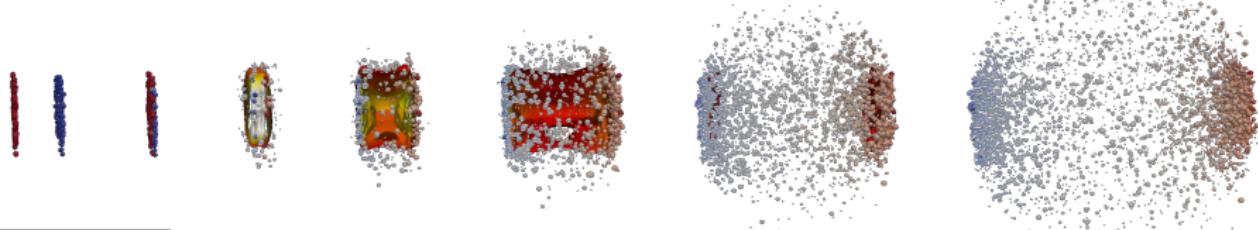
✓ Hydrodynamics

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✗ Hadronic transport

Image: J. Bernhard (1804.06469) / H. Elfner / MADAI

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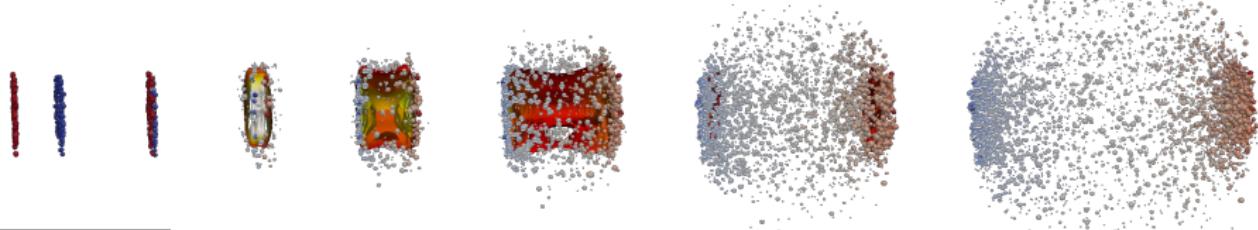
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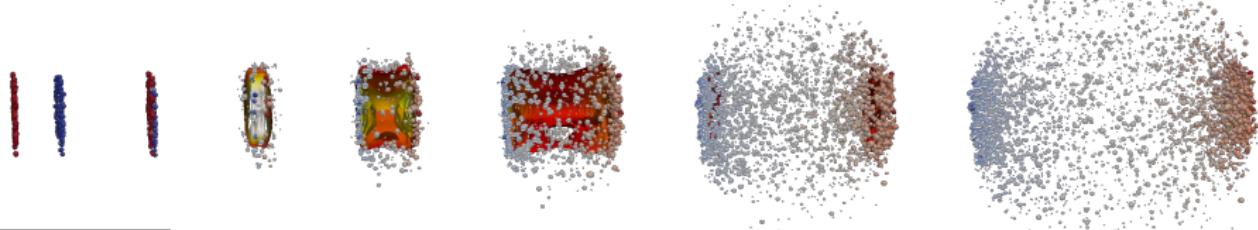
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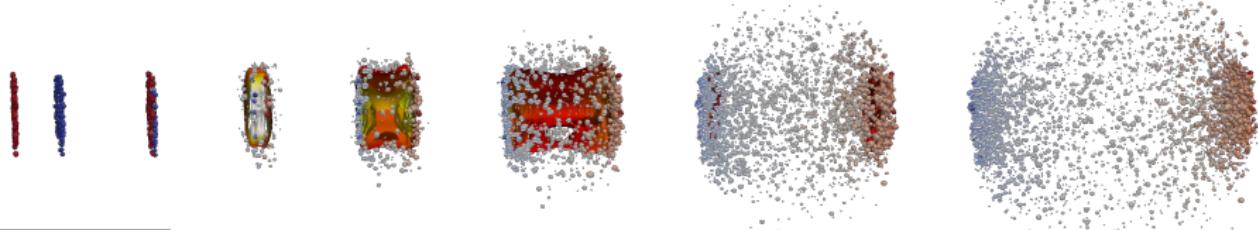
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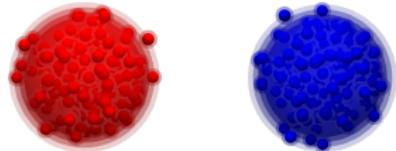
Background: TRENTo Collision Model

1412.4708

① Construct nuclei

Woods–Saxon distribution:

$$p(r) \propto \frac{r^2}{1+e^{(r-R)/a}}$$



② Sample impact parameter

$$p(b) \propto b \Theta(b_{\max} - b)$$

③ Interact nucleons

Nucleon–nucleon interaction probability
at separation b_{NN} (no substructure):

$$p(b_{NN}) \propto \exp \left\{ -\frac{\sigma_{NN}^{\text{inel}}}{4\pi w^2} e^{-\frac{b_{NN}^2}{4w^2}} \right\}$$

④ Compute thicknesses

$$T_X(\vec{x}_\perp) = \sum_{p \in \text{particip.}\{X\}} \frac{\gamma_p}{2\pi w^2} e^{-\frac{|\vec{x}_\perp - \vec{x}_\perp^{(p)}|^2}{2w^2}}$$

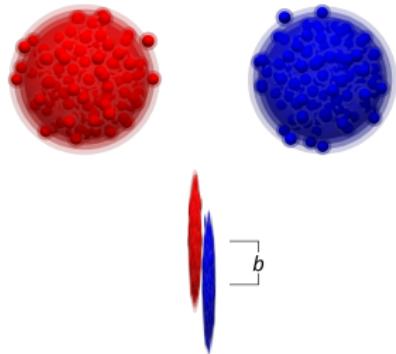
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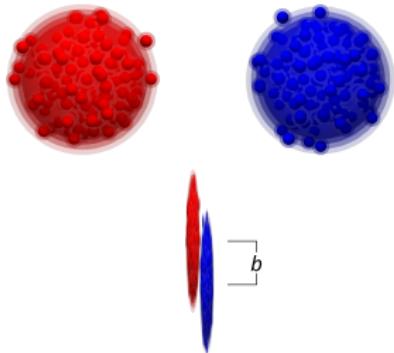
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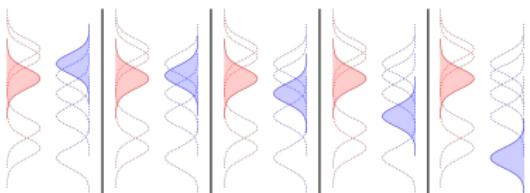
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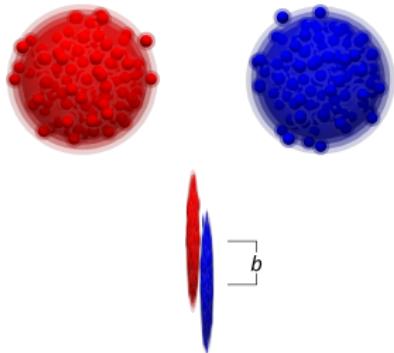
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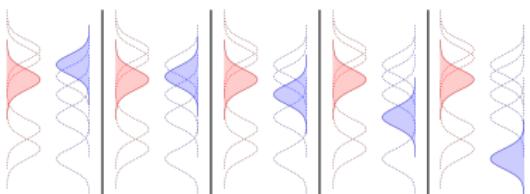
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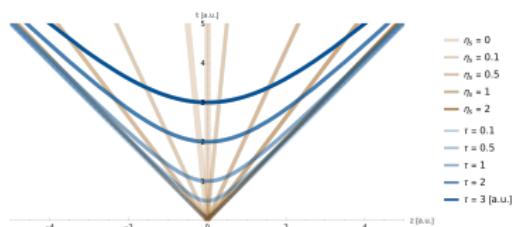
► 3D Ansatz

3D Ansatz

- Principles
 - Ultrarelativistic approximation
 $z = 0$ at $t = 0$
 - Reproduce 2D T_RENTo at midrapidity
 - Energy-momentum conservation
- “Fireball” and “fragments”
- Coordinates: (τ, x, y, η_s)
 - $\tau = \sqrt{t^2 - z^2}$
 - $\eta_s = \text{arctanh}(z/t)$
- Quantity: energy density \times initial time
 - E_T : energy in Bjorken frame
 - $\frac{dE_T}{dx dy dz} \rightarrow \frac{dE_T}{dx dy \tau d\eta_s}$ (GeV/fm³)
 - T_RENTo-3D: $\frac{dE_T}{dx dy d\eta_s}$ (GeV/fm²)

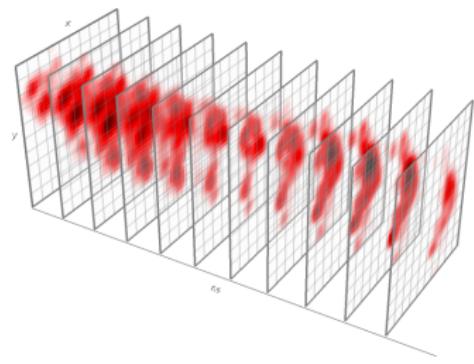
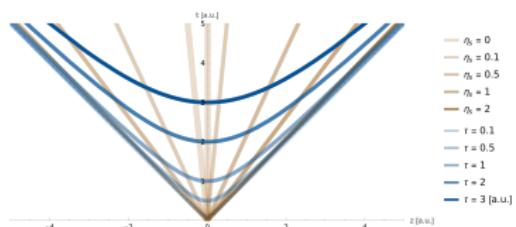
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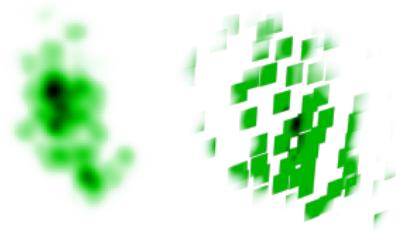
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3D Ansatz: Central Fireball

- Portions t_X of participant thicknesses combine and explode, creating fireball around $A-B$ c.o.m.

$$P_{\text{cm}}^\mu = t_A P_A^\mu + t_B P_B^\mu$$
$$\eta_{s,\text{cm}} = \operatorname{arctanh} \left(\sqrt{1 - \frac{4mp^2}{s_{\text{NN}}} \frac{t_A - t_B}{t_A + t_B}} \right)$$



- Fireball profile: rapidity distribution of explosion

$$f_{\text{fb}}(\Delta\eta_s) = \exp \left\{ - \left(\frac{\Delta\eta_s^2}{2\lambda} \right)^f \right\} \left(1 - \left(\frac{\Delta\eta_s}{\eta_{s,\text{max}}} \right)^4 \right)^4$$

- $f = 1$: Gaussian
- $f \rightarrow \infty$: plateau (rectangular function)

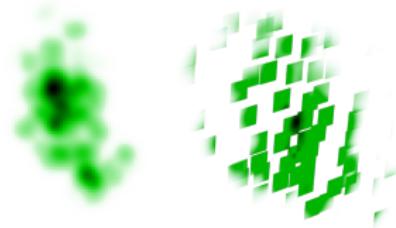
- Conservation of energy

$$\langle \gamma \rangle \sqrt{s_{\text{NN}}} = N_{\text{fb}} \int \cosh(\eta_s) d\eta_s f_{\text{fb}}(\eta_s)$$

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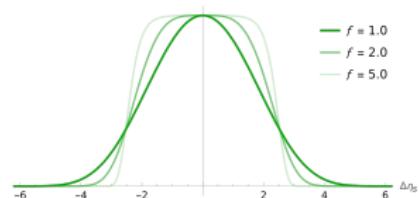
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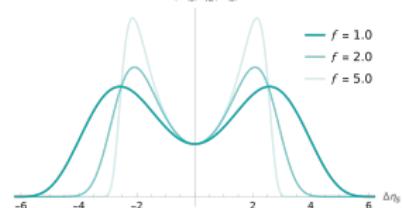
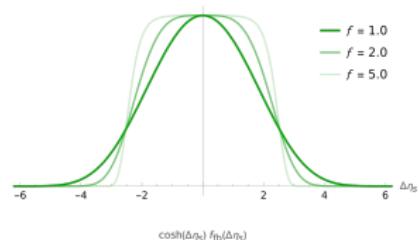
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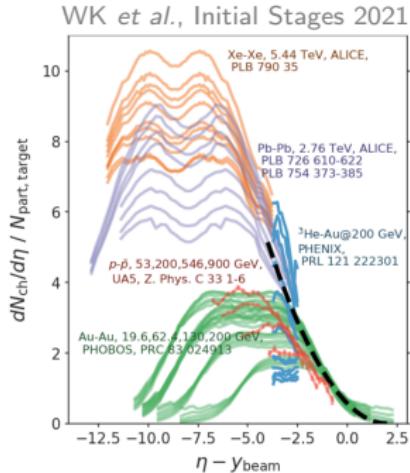
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3D Ansatz: Fragments

- Nuclear matter is damaged but not stopped by collision, and breaks apart
- Limiting fragmentation (Benecke *et al.* (1969))
 - Shape of particle spectrum near y_{beam} becomes independent of $\sqrt{s_{NN}}$
 - Well supported by experiment →
- Model for trailing spectrum motivated by PDFs (PRL 91, 052302 (2003))
- Fragment profile:

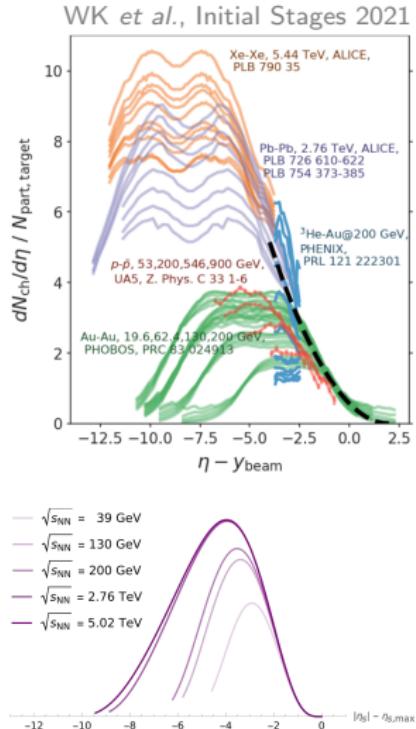
$$f_{\text{frag}}(x) = (-\ln(x))^{\alpha} x^{\beta+1} \exp\left\{-\frac{2k_{T,\min}}{x\sqrt{s_{NN}}}\right\}$$
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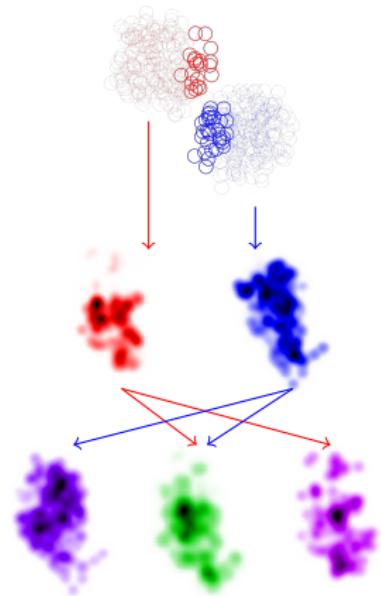
3D Ansatz: Fluctuation

- Participant thickness is allocated between fireball and fragments

$$T_X(\vec{x}_\perp) = \sum_{p \in \text{particip.}\{X\}} \frac{\gamma_p}{2\pi w^2} e^{-\frac{|\vec{x}_\perp - \vec{x}_\perp^{(p)}|^2}{2w^2}}$$

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- $\gamma \sim \text{Beta}\left\{\langle\gamma\rangle \frac{1-k}{k}, (1 - \langle\gamma\rangle) \frac{1-k}{k}\right\}$
 - $\langle\gamma\rangle$ is the mean of the distribution, determines average energy content of fireball
 - k shapes the distribution:



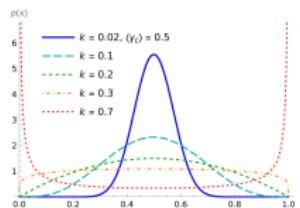
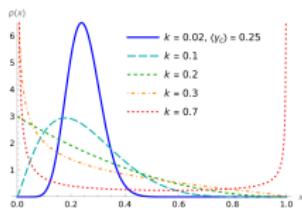
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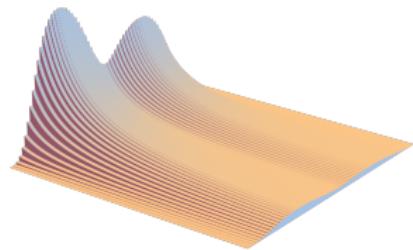
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► Simplified Evolution Model

Simplified Evolution Model

- Longitudinal smoothing and broadening means initial and final distributions will differ in shape
→ Some evolution is needed to compare with data



① (1+1)D linearized hydrodynamics

- $T^{\mu\nu} \approx \begin{pmatrix} 1 & (1+c_s^2) \frac{v_z}{\tau} \\ (1+c_s^2) \frac{v_z}{\tau} & \frac{c_s^2}{\tau^2} \end{pmatrix} \varepsilon$
- $\nabla_\mu T^{\mu\nu} = \delta(\tau - \tau_0) \varepsilon_0(\eta_s) \delta_{\nu,0}$

② Transverse Gaussianization

- Make 1D energy density 3D by assuming transversely Gaussian distribution

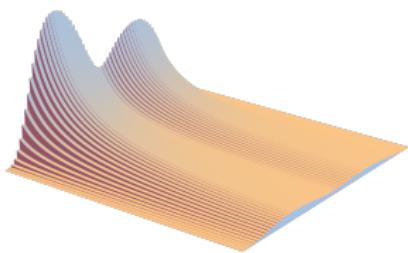
$$r_{\text{FO}}(\tau, \eta_s) = \sqrt{2\sigma_G^2 \ln \left\{ \frac{\varepsilon(\tau, \eta_s)}{2\pi\sigma_G^2 \varepsilon_{\text{FO}}} \right\}}$$

③ Cooper–Frye: (PRD 10 186 (1974))

$$\begin{aligned} \frac{dN_i}{d\eta} &= \frac{1}{(2\pi\hbar)^3} \int_0^\infty dp_T \int_0^{2\pi} p_T d\phi_p \sqrt{1 + \frac{m_i}{p_T^2 \cosh^2(\eta)}} \\ &\quad \times \int_{\Sigma} d^3\Sigma_\mu p^\mu \frac{1}{\exp\{u_\mu p^\mu / T_{\text{FO}}\} \pm 1} \end{aligned}$$

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$$r_{\text{FO}}(\tau, \eta_s) = \sqrt{2\sigma_G^2 \ln \left\{ \frac{\varepsilon(\tau, \eta_s)}{2\pi\sigma_G^2 \varepsilon_{\text{FO}}} \right\}}$$

③ Cooper–Frye: (PRD 10 186 (1974))

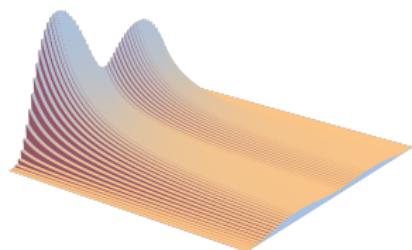
$$\begin{aligned} \frac{dN_i}{d\eta} &= \frac{1}{(2\pi\hbar)^3} \int_0^\infty dp_T \int_0^{2\pi} p_T d\phi_p \sqrt{1 + \frac{m_i}{p_T^2 \cosh^2(\eta)}} \\ &\quad \times \int_{\Sigma} d^3\Sigma_\mu p^\mu \frac{1}{\exp\{u_\mu p^\mu / T_{\text{FO}}\} \pm 1} \end{aligned}$$

Simplified Evolution Model

- Longitudinal smoothing and broadening means initial and final distributions will differ in shape
→ Some evolution is needed to compare with data

① (1+1)D linearized hydrodynamics

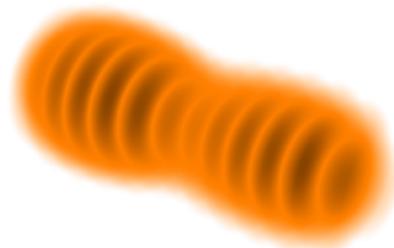
- $T^{\mu\nu} \approx \begin{pmatrix} 1 & (1 + c_s^2) \frac{v_z}{\tau} \\ (1 + c_s^2) \frac{v_z}{\tau} & \frac{c_s^2}{\tau^2} \end{pmatrix} \varepsilon$
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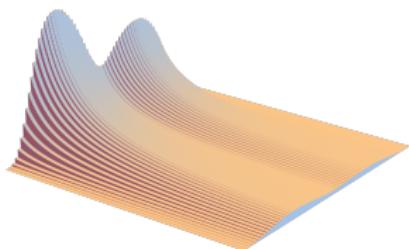
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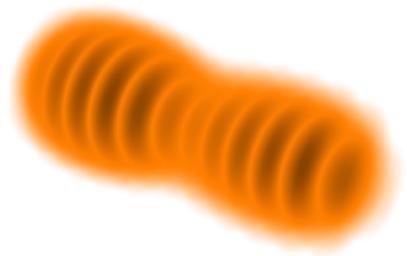
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► Calibration

Calibration

- Want: parameter values that best reproduce data
“Maximum a posteriori” (MAP) point in parameter space
- Have: 16 parameters and a fast—but not free—model

① Latin hypercube design (LHD)

- No two points share any coordinate
- Rule of thumb is $N_{\text{points}} \propto N_{\text{params}}$

② Gaussian process emulator (GP)

- Reproduces model at training points
- Predicts model away from training points
- Qualifies prediction with emulator uncertainty

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- Efficiently estimates posterior in high dimensions
- Common to sample millions of points

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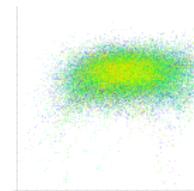
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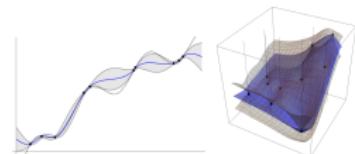
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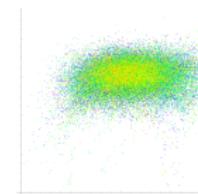
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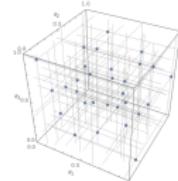


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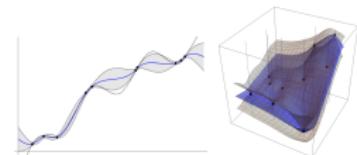
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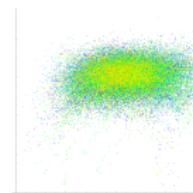
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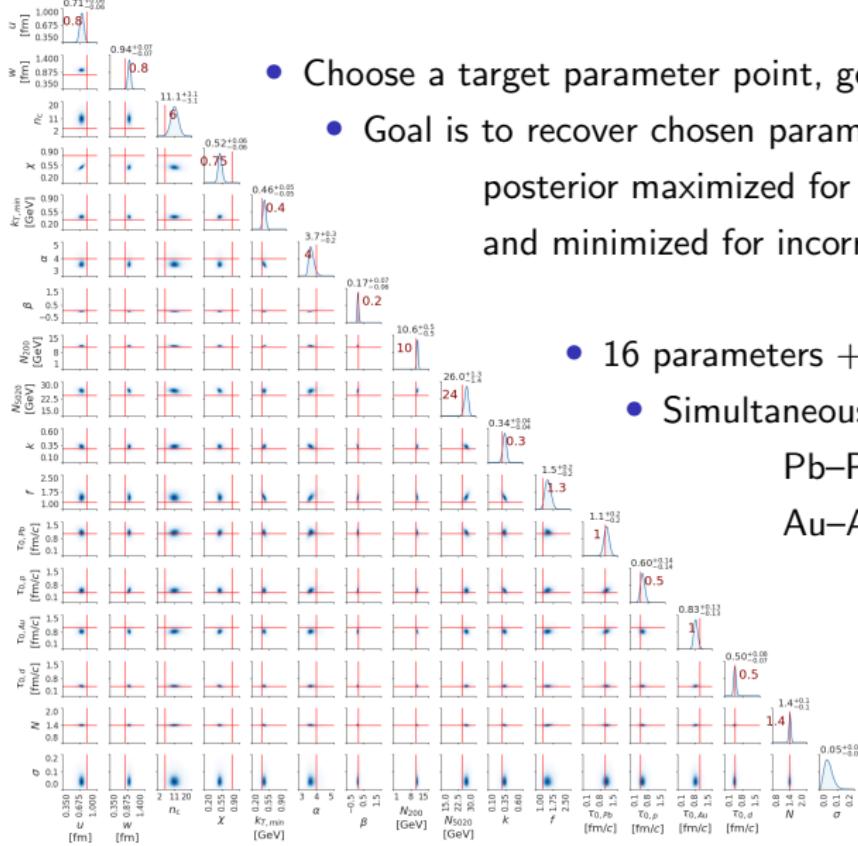


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Calibration: Closure Test



- Choose a target parameter point, generate mock data

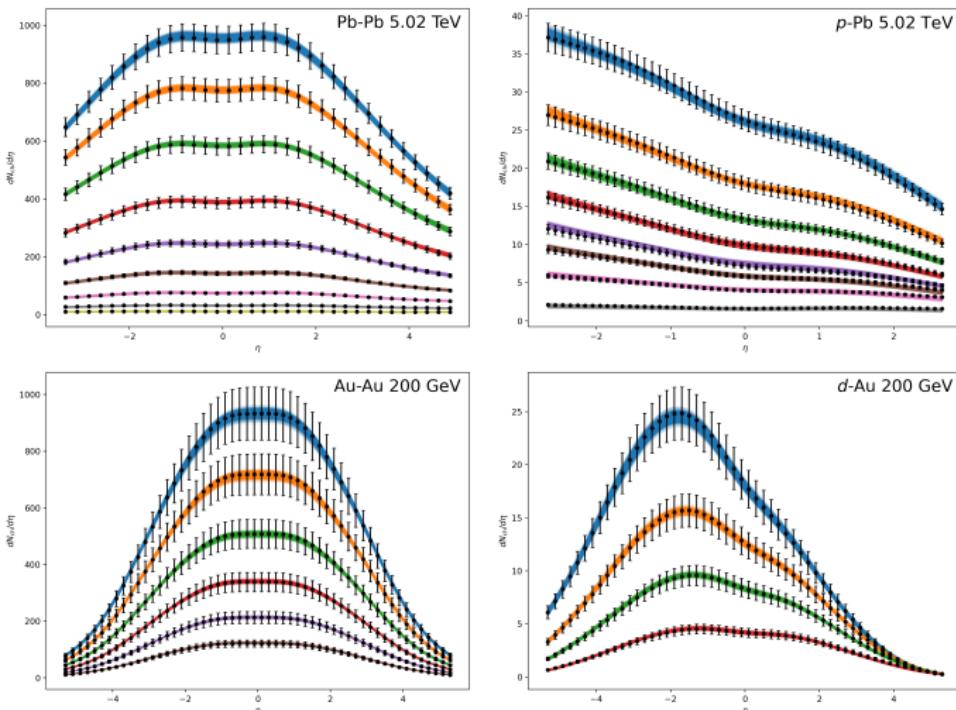
- Goal is to recover chosen parameter values:
posterior maximized for correct values,
and minimized for incorrect values

- 16 parameters + model uncertainty

- Simultaneous fit to four systems:

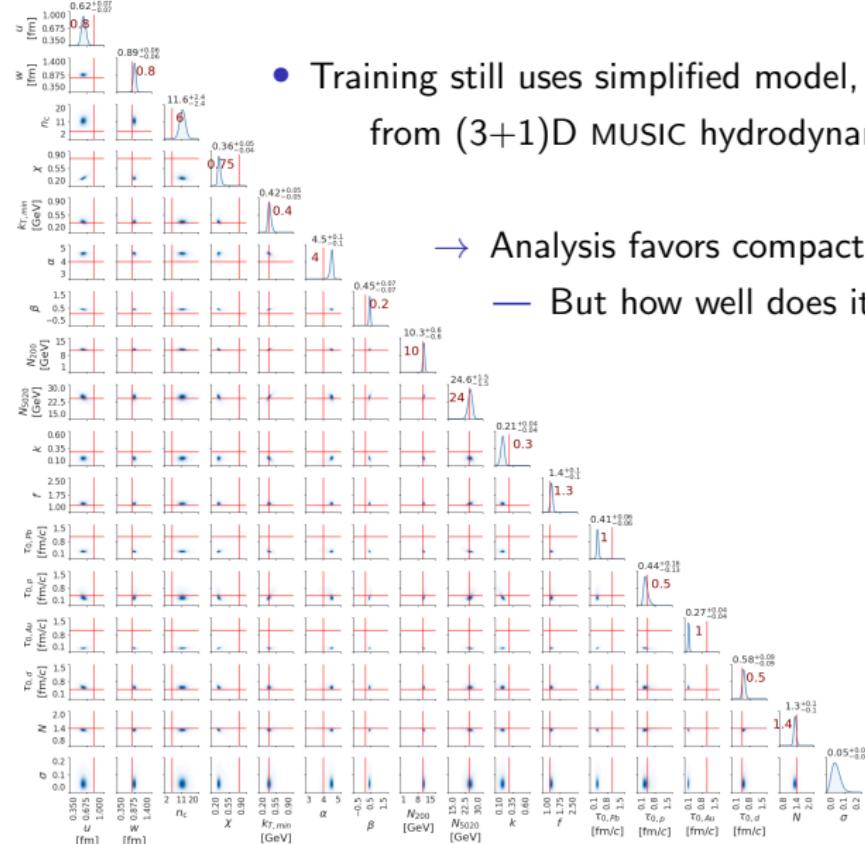
Pb–Pb and p –Pb at 5.02 TeV,
Au–Au and d –Au at 200 GeV

Calibration: Closure Test



- 1,000 emulated $dN_{ch}/d\eta$ per system \otimes centrality \otimes pseudorapidity
→ Excellent match, validates analysis

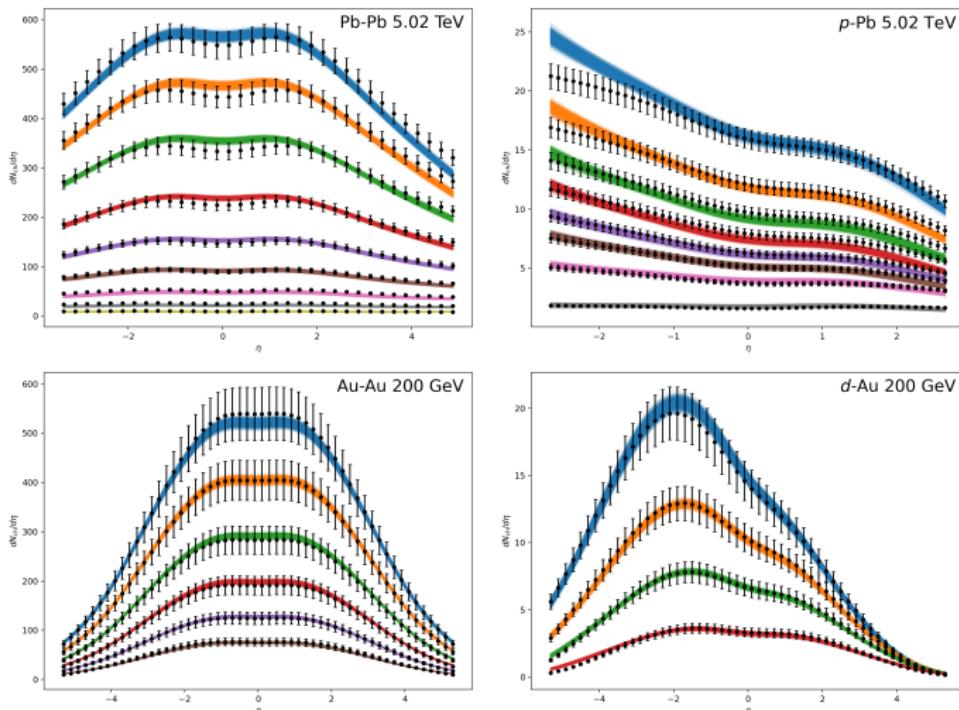
Calibration: Cross-Model Validation



- Training still uses simplified model, while mock data is now from (3+1)D MUSIC hydrodynamics and particlization

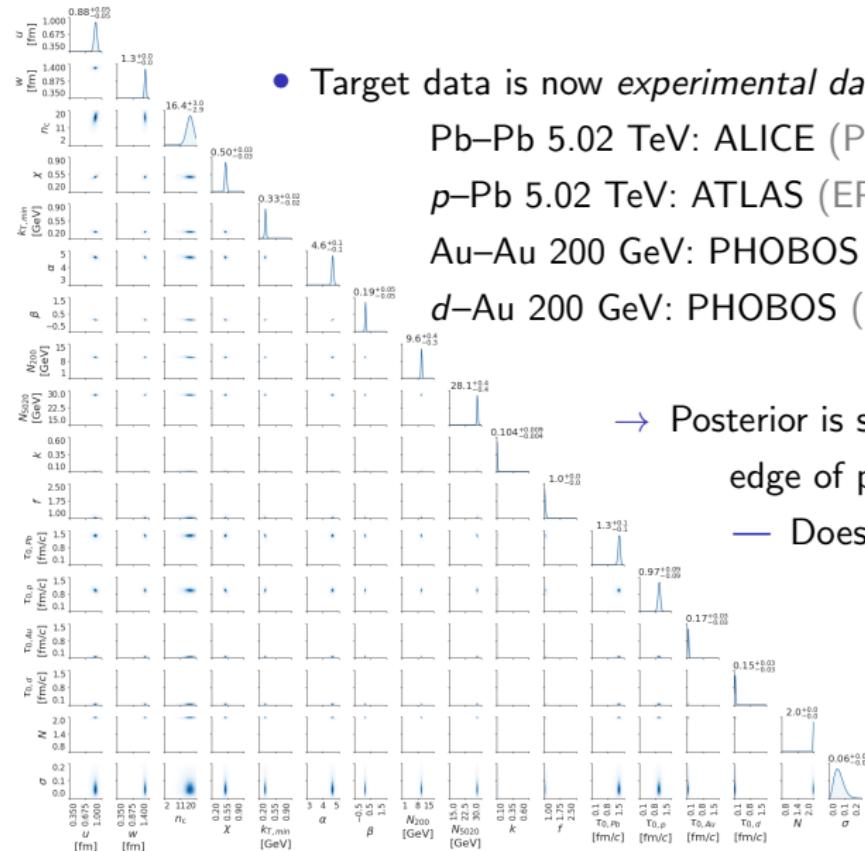
→ Analysis favors compact region in parameter space
— But how well does it reproduce mock data?

Calibration: Cross-Model Validation



→ Good match (fair for p -Pb), demonstrates expressiveness of TRENTo-3D

Calibration: Model-to-Data Comparison



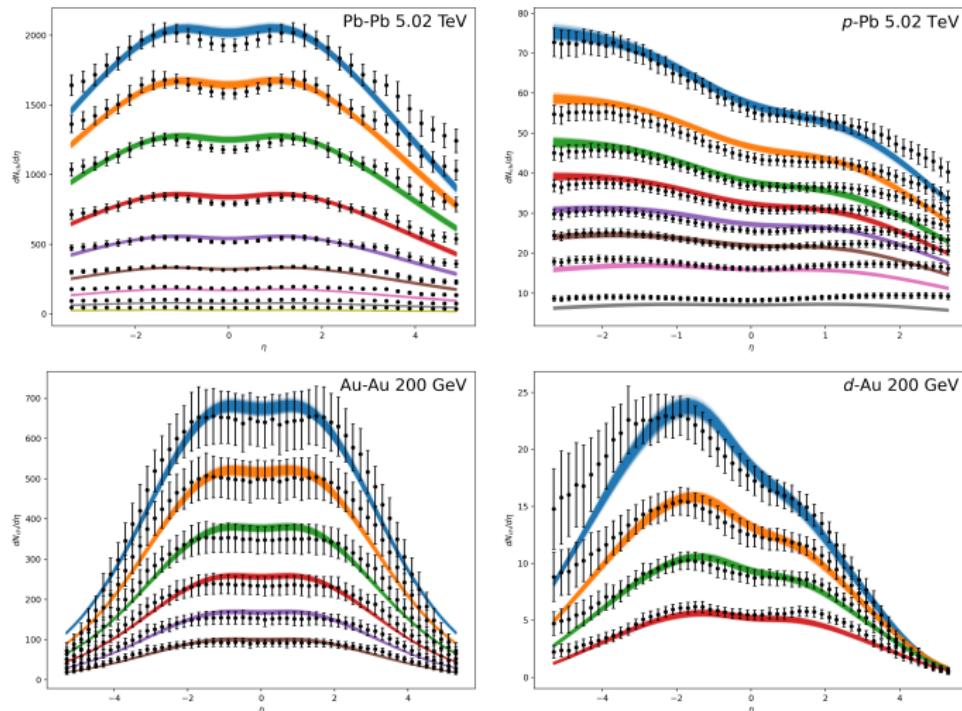
- Target data is now *experimental data*:

Pb–Pb 5.02 TeV: ALICE (PLB 772, 567 (2017))
p–Pb 5.02 TeV: ATLAS (EPJC 76, 199 (2016))
Au–Au 200 GeV: PHOBOS (PRL 91, 052303 (2003))
d–Au 200 GeV: PHOBOS (PRC 72, 031901 (2005))

→ Posterior is sharply peaked—but near edge of prior for some parameters
— Does it reproduce data?

Calibration: Model-to-Data Comparison

ALICE (Pb–Pb), ATLAS (p -Pb), PHOBOS (Au–Au), PHOBOS (d -Au)



- Moderate agreement, expect improvement with higher-fidelity evolution
- Shows T_RENTo-3D initial conditions *can result in realistic final states*

► Conclusion

Conclusion

- T_RENTo-3D is a parametric model of initial 3D energy distribution
- Tests validate analysis and demonstrate expressiveness of model
- Next: calibrate using (3+1)D viscous relativistic hydrodynamics
 - (3+1)D: can compute observables unavailable to (1+1)D (e.g., $v_n(\eta)$)
 - Higher-fidelity physical model: more meaningful comparison to data
- Code coming soon—watch <https://github.com/Duke-QCD> or contact me (derek.soeder@duke.edu)