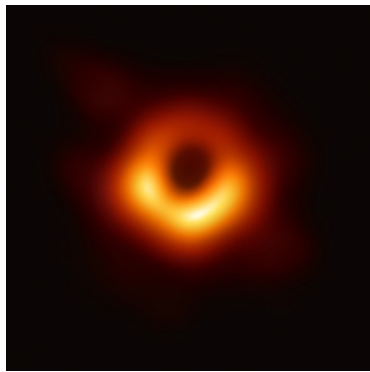
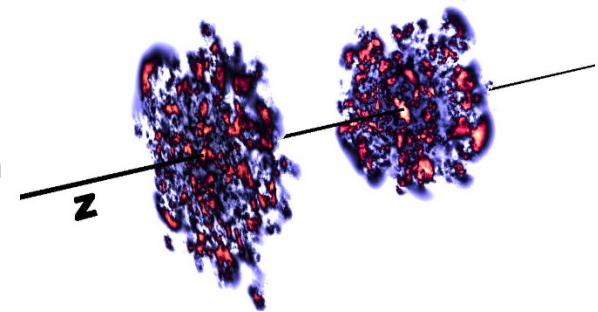


The CGC-Black Hole Double Copy and gravitational radiation from primordial Black Hole mergers



$M_{\text{BH}} = (6.5 \pm 0.2_{\text{stat}} \pm 0.7_{\text{sys}}) \times 10^9 M_{\odot}$ at center of Messier 87
Event Horizon Telescope image of photon ring

$10^9 \text{ km} \longleftrightarrow 10^{-19} \text{ km}$



Collisions of Color Glass Condensate
gluon states in nuclei, arXiv:1206.6805

Raju Venugopalan
Brookhaven National Laboratory

Initial Stages 2023, June 19-23, 2023

Black Holes “demystified”

A great achievement of physics in my lifespan: Black Holes from “exotic” solutions of General Relativity to observable phenomena



2020



Roger Penrose



Reinhard Genzel



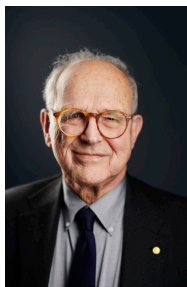
Andrea Ghez

For the discovery that black hole formation is a robust prediction of the general theory of relativity

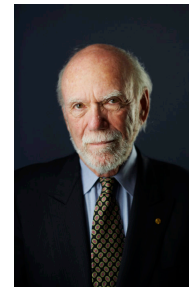
For the discovery of a supermassive compact object at the centre of our galaxy



2017



Reiner Weiss



Barry Barish



Kip Thorne

For decisive contributions to the LIGO detector and the observation of gravitational waves...from Black Hole mergers

Black Holes “demystified”: The Black Hole N Portrait (BHNP)

Dvali, Gomez, arXiv:1203.6575

Dvali, Gomez, arXiv:1112.3359

Dvali, Guidice, Gomez, Kehagis, arXiv:1010.1415

Classical description: Macroscopic objects in GR with geometric, thermodynamic properties

QFT understanding in the BHNP: Black Holes are highly occupied

“leaky” bound states of **soft gravitons** ($N = M_{\text{BH}}^2 / M_{\text{P}}^2 \gg 1$, $\sim 10^{66}$ for solar mass BHs)

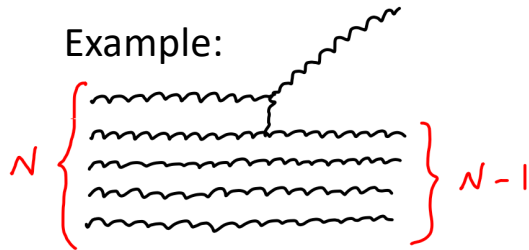
$$L_P^2 = \hbar G$$

$$M_P^2 = \frac{\hbar}{G}$$

Semi-classical limit: $N \rightarrow \infty$, $L_P \rightarrow 0$, Schwarzschild radius $R_S = 2 G M_{\text{BH}} = L_P \sqrt{N}$ = finite, \hbar = finite

Event horizon, BH thermodynamics, no-hair theorem, understood simply in this limit

Example:



Black Hole evaporation rate

$$\Gamma = 6 n_h (1+N) \sim \frac{\hbar^2}{G^2 R_S^2} \frac{N^2}{R_S^3}$$

$$\equiv \frac{1}{R_S}$$

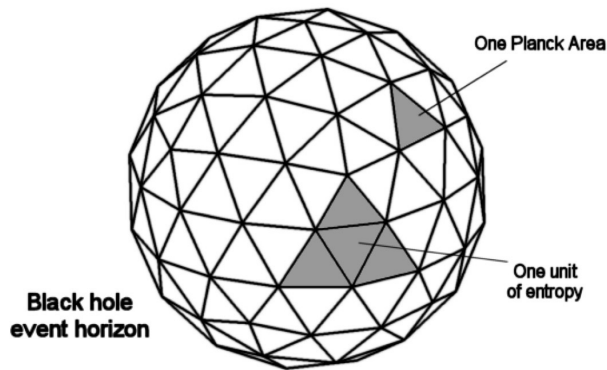
Hawking Temperature

$$T_H = \frac{\hbar}{R_S} = \frac{\hbar}{L_P \sqrt{N}}$$

Rate can be equivalently be written as $\frac{dM}{dt} = - \frac{T_H^2}{\hbar^2}$ with Black Hole half-life (“Page time”) $t_{\text{BH}} = \frac{\hbar^2}{T_H^3 G} = N^{3/2} L_P$

Black Hole N Portrait and entropy bound

Bekenstein-Hawking bound



(for a nice discussion,
see Bousso, arxiv:1810.01880)

The Bekenstein entropy bound is given by $S \leq 2\pi ER/\hbar$

Define $E = N Q_S$ as energy in a critically packed volume $= R_S^3$ of quanta ("qubits") saturating **unitarity** (maximal information) and $Q_S = 1/R_S$

Then in this BHNP picture, $S \leq 2\pi N Q_S R_S$
saturated when $N = \frac{1}{\alpha_{gr}} \rightarrow S_{Bek} = \frac{1}{\alpha_{gr}}$

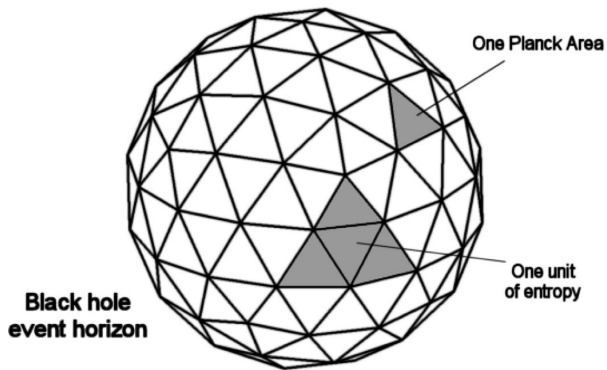
"classical" high occupancy lump of matter

In turn,
$$S_{Bek} = \frac{1}{\alpha_{gr}} = \frac{R_S^2}{L_P^2} = \frac{Area}{4G} = S_{BH}$$

Famous area law for BHs

Black Hole N Portrait and entropy bound

Bekenstein-Hawking bound

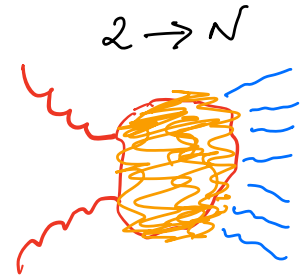


(for a nice discussion, see Bousso, arxiv:1810.01880)

The Bekenstein entropy bound is given by $S \leq 2\pi ER/\hbar$

Define $E = N Q_S$ as energy in a critically packed volume $= R_S^3$ of quanta ("qubits") saturating **unitarity** (maximal information) and $Q_S = 1/R_S$

What does this lump have to do with unitarity?



$$P_{2 \rightarrow N} \sim e^S \alpha_S^N N! \quad \Rightarrow \quad \text{If } N \sim \frac{1}{\alpha_S} \quad P_{2 \rightarrow N} \sim e^S \alpha_S^N \left(\frac{1}{\alpha_S}\right)^N e^{-1/\alpha_S}$$

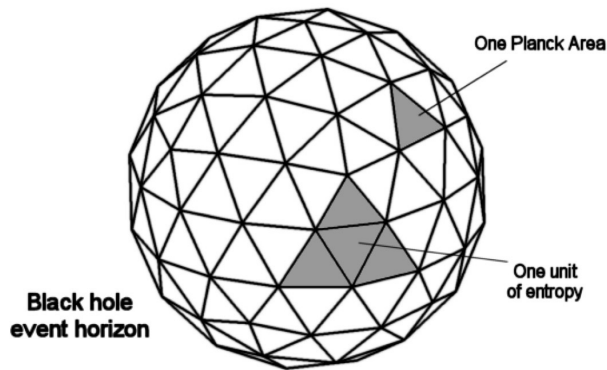
Exponential suppression of high occupancy states (classical lumps) unless

$$S \sim 1/\alpha_S \sim N$$

$$\Rightarrow P_{2 \rightarrow N} \sim O(1)$$

Black Hole N Portrait and entropy bound

Bekenstein-Hawking bound



(for a nice discussion,
see Bousso, arXiv:1810.01880)

The Bekenstein entropy bound is given by $S \leq 2\pi ER/\hbar$

Define $E = N Q_S$ as energy in a critically packed volume $= R_S^3$ of quanta ("qubits") saturating unitarity (maximal information) and $Q_S = 1/R_S$

Then in this BHNP picture, $S \leq 2\pi N Q_S R_S$

saturated when $N = \frac{1}{\alpha_{gr}} \rightarrow S_{Bek} = \frac{1}{\alpha_{gr}}$

The entropy can be expressed in terms of a Goldstone decay constant f_G
- spontaneous breaking of Poincare invariance by graviton condensate:

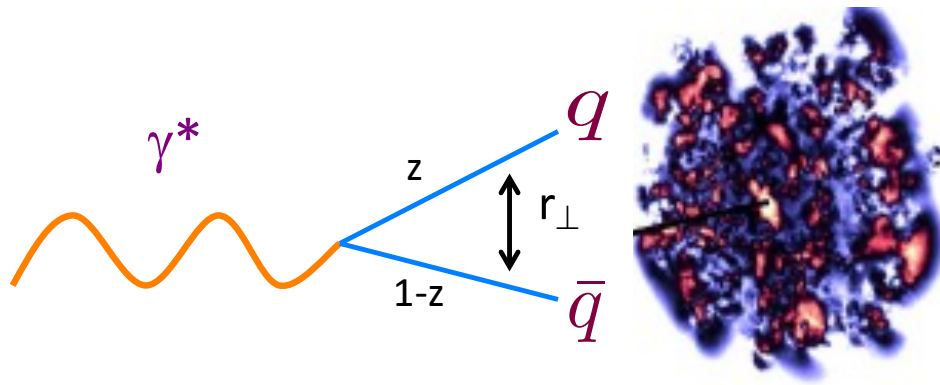
Decay constant of Goldstone field ϕ is $f = R_S \partial_x \phi = \frac{\sqrt{N}}{R_S}$

In gravity, f_G is nothing but M_{Planck}

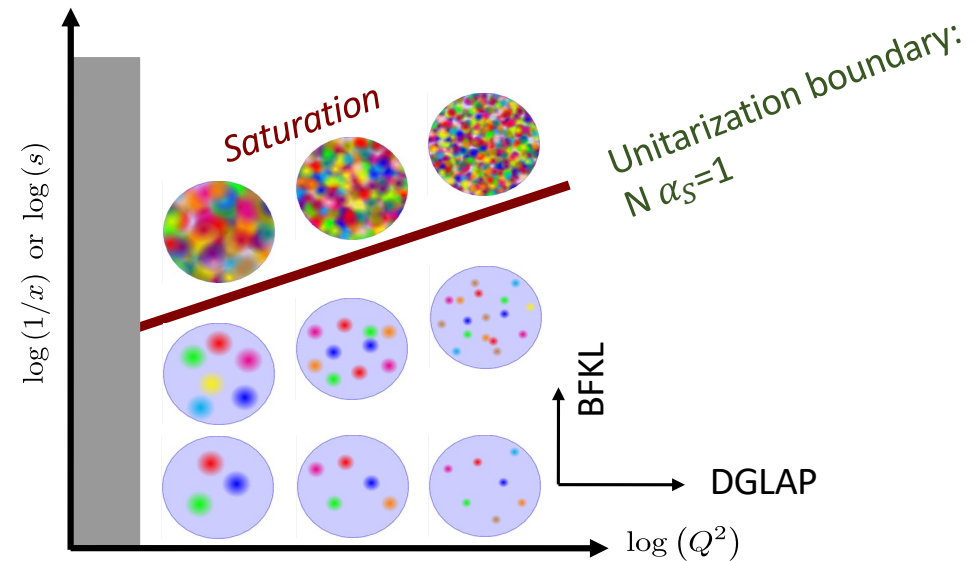
Hence one can equivalently express the entropy as
 $S_{\text{BH}} = \text{Area} \times f_G^2$

Dvali, arXiv:1907.07332

Gluon saturation: classicalization and unitarization in QCD



Perturbative unitarization at fixed impact parameter
when size of saturated blob $\sim 1/Q_s \ll 1/\Lambda_{QCD}$



Conjecture: Both CGC / BH are classical lumps (max occupancy/information) of gluons / gravitons (respectively) that unitarize the cross-section at emergent scale Q_s

– color screening or saturation scale in QCD and inverse of the Schwarzschild radius characterizing the event horizon of a Black Hole

The physics at this scale is universal and independent of the microscopic details of the two theories

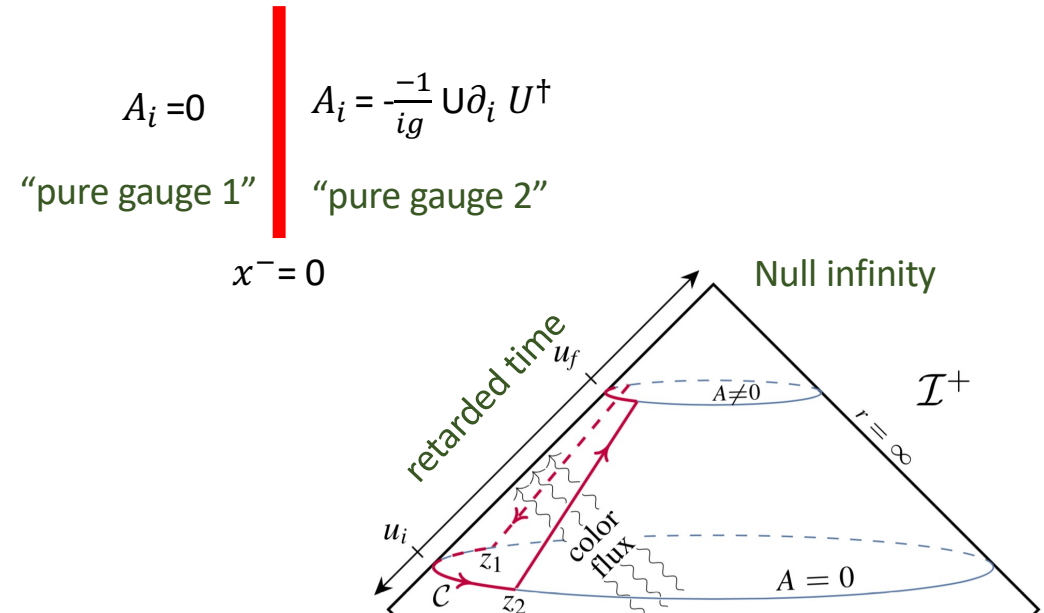
Gluon saturation: classicalization and unitarization in QCD

In the CGC, the classical field (color glass condensate) breaks Poincare invariance and a global sub-group of large gauge transformations

The breaking of this symmetry is responsible for the exact colored analog of **gravitational memory**: dipole acquires a net kick from light-like Wilson loop

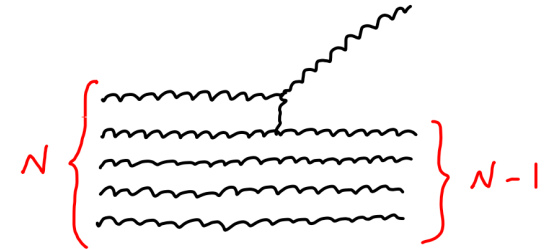
Pate, Raclariu, Strominger, PRL (2017)

Ball, Pate, Raclariu, Strominger, RV, Annals of Physics 407 (2019) 15



Gluon saturation: classicalization and unitarization in QCD

The quantum excitations of the condensate are Goldstone modes; the scattering of these modes (with energy levels separated by $1/N$) leads to decay of condensate



$$S_{\text{CGC}} = 1/\alpha_S = N = f_G^2 * \text{Area} \quad \text{where } f_G^2 = N Q_S^2 \text{ is the Goldstone decay scale}$$

See also Kharzeev, Levin, arXiv:1701.03489
Duan, Kovner, Skokov, arXiv:2111.06475
for same result from different perspectives

This decay time scale ($1/f_G$) of the “quantum” CGC (which differs qualitative and quantitatively from Eikonal power counting) is consistent with the bottom-up thermalization scenario

Baier, Mueller, Schiff, Son, hep-ph/0009237
Dvali, RV, arXiv:2106.11989

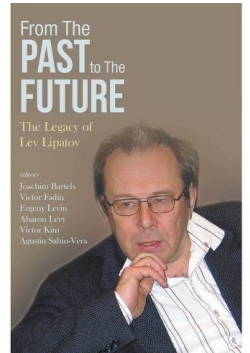
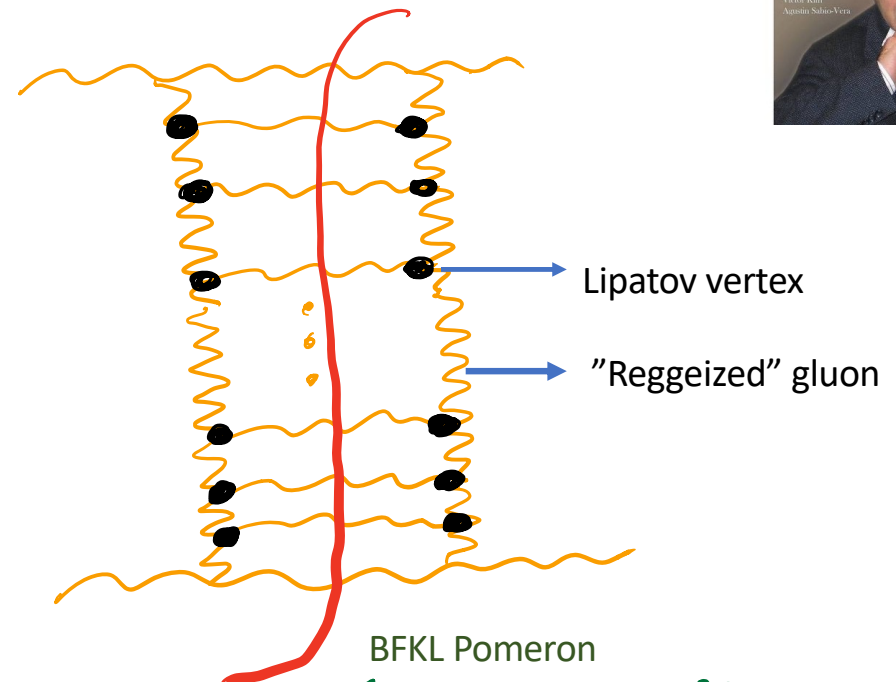
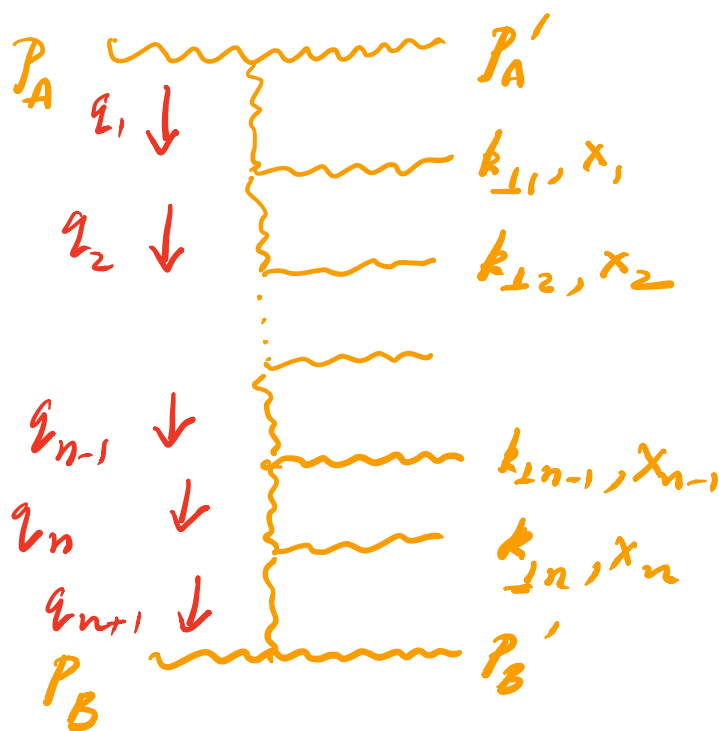
In many CGC calculations, one assumes that the classical saddle point is robust

- but this is only so if occupancy $N \rightarrow \infty$ For finite N , the condensate is “leaky” and will decay

Lipatov's EFT for wee partons in QCD and gravity

Powerful double copy between QCD and gravity was discovered by Lipatov 40 years ago

Computing $2 \rightarrow N + 2$ amplitude in the Regge limit of QCD: the BFKL equation



Rapid growth of cross-section with rapidity

2 → N + 2 amplitudes in the Regge limit of gravity

Double copy between QCD and Gravity amplitudes in Regge asymptotics

Lipatov, PLB 116B (1982) 411, Spinor helicity derivation, Liu, arXiv:1811.11710

$$\mathcal{M}_n \simeq -s^2 \mathcal{C}(2; 3) \frac{-1}{|q_4^\perp|^2} \mathcal{V}(q_4; 4; q_5) \cdots \frac{-1}{|q_{n-1}^\perp|^2} \mathcal{V}(q_{n-1}; n-1; q_n) \frac{-1}{|q_n^\perp|^2} \mathcal{C}(1; n)$$

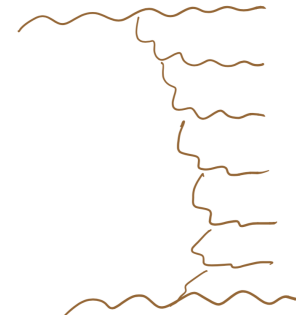
Gravitational effective vertex: $\mathcal{V}(q_i, i, q_{i+1}) = \Gamma_{i, \mu\nu}(q_i, q_{i+1}) \epsilon_i^{\mu\nu}(k_i)$

is the double copy $\Gamma_i^{\mu\nu}(q_i, q_{i+1}) \equiv 2(C_i^\mu C_i^\nu - N_i^\mu N_i^\nu)$

QCD Lipatov vertex

QED Bremsstrahlung vertex

High energy scattering of Gravitons



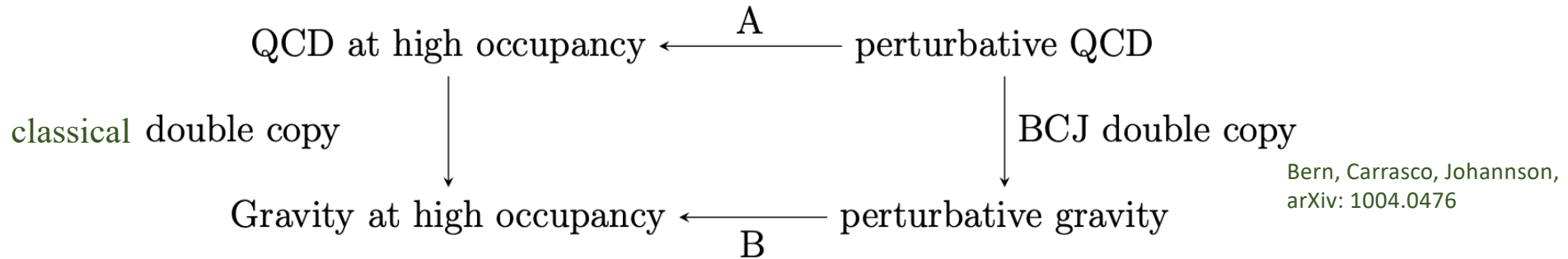
Same multi-Regge kinematics as previously

$\mathcal{C}(2;3)$ and $\mathcal{C}(1;n)$: gravitational “impact factors”
double copies of Lipatov’s gluon-gluon-reggeized gluon vertex

Johansson, Sabio Vera, Campillo, Vasquez-Mozo, arXiv:1310.1680

Large body of work by Amati, Ciafaloni, Veneziano, et al.;
also interesting work in AdS/CFT by Strassler and Polchinski

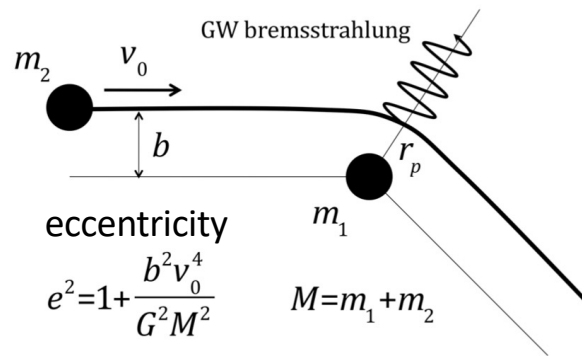
Double Copy: enabling computation of gravitational radiation



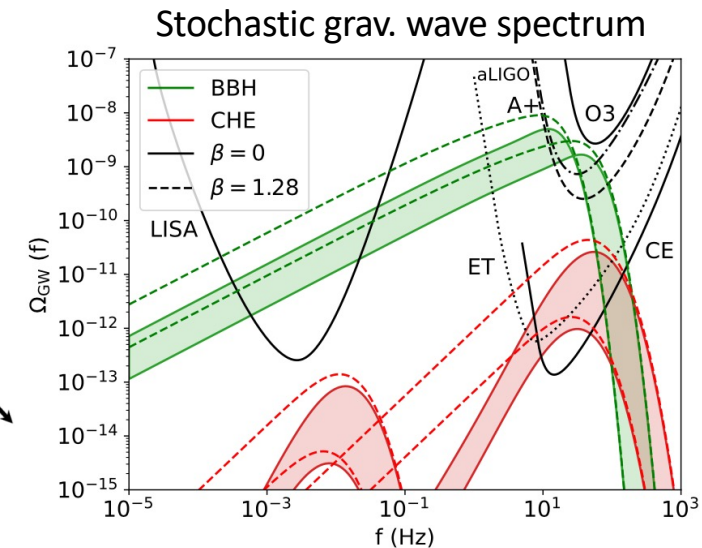
Monteiro, O'Connell, White, arXiv:1410.0239
Goldberger, Ridgeway, arXiv:1611.03493

Can we use semi-classical strong field methods developed for the CGC to compute gravitational wave radiation from primordial Black Holes ?

H. Raj, RV, in preparation



Garcia-Bellido, Nesseris, arXiv:1706.02111
Garcia-Bellido, Jaraba, Kuroyanagi, arXiv:2109.11376



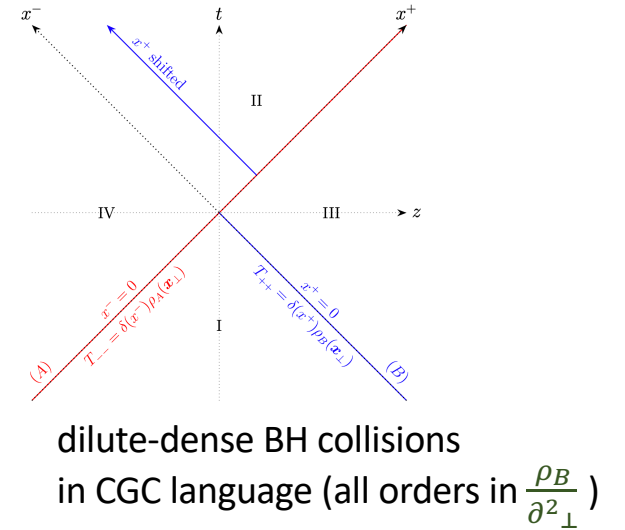
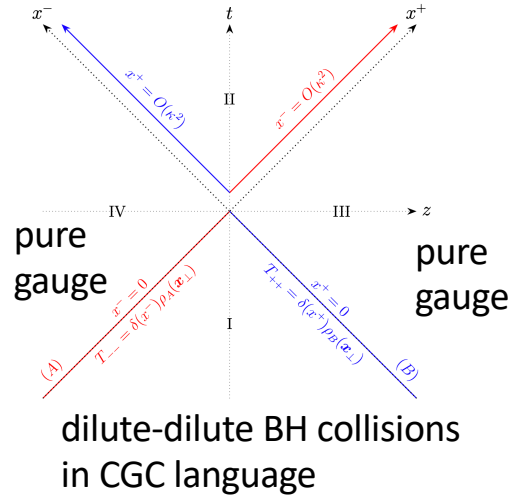
Double Copy: enabling computation of gravitational radiation

H. Raj, RV, in preparation

Shockwave collisions:

Solve Einstein's equations
for linearized perturbations
 $h_{\mu\nu}$ around strong field metric
in light cone gauge $h_{+i} = 0$

For dilute-dilute, expand eqns.
to $O(\frac{\rho_A}{\partial_{\perp}^2} \frac{\rho_B}{\partial_{\perp}^2})$



$$2\partial_+\partial_-\tilde{h}_{ij} + \square_{\perp}\tilde{h}_{ij} = (2\partial_i\partial_j - \square_{\perp}\delta_{ij}) \frac{1}{\partial_+^2} T_{++} - 2\delta_{ij}\delta(x^-)\delta(x^+) \frac{\rho_A}{\square_{\perp}} \rho_B + 2\delta_{ij} \frac{\partial_-}{\partial_+} T_{++} \\ + 2\delta_{ij} T_{+-} + 2T_{ij} - \delta_{ij} T - \frac{2}{\partial_+} (\partial_i T_{+j} + \partial_j T_{+i}) - \delta(x^-)\delta(x^+) \frac{\rho_A}{\square_{\perp}} \left(\frac{2\partial_i\partial_j}{\square_{\perp}} - \delta_{ij} \right) \rho_B$$

The components of the energy-momentum tensor $T_{\mu\nu}$ are constrained by conservation laws
-and by matching on the light cone – just as for the case of current conservation for gluon shockwave collisions

Double copy: enabling computation of gravitational radiation

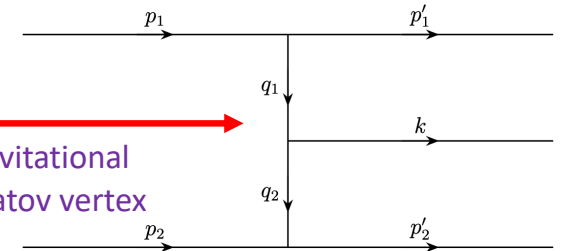
H. Raj, RV, in preparation

Graviton field produced

In close hyperbolic encounters

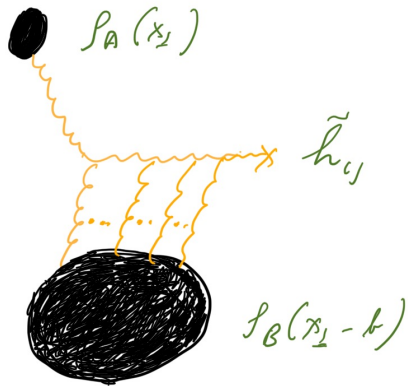
$$-k^2 \tilde{h}_{ij} = \int \frac{d\mathbf{q}_2}{(2\pi)^2} \frac{\rho_A(\mathbf{q}_1)}{\mathbf{q}_1^2} \frac{\rho_B(\mathbf{q}_2)}{\mathbf{q}_2^2} \Gamma_{ij}$$

$$\Gamma_{ij} = 2 \left[\left(q_{2i} - k_i \frac{\mathbf{q}_{2\perp}^2}{\mathbf{k}_\perp^2} \right) \left(q_{2j} - k_j \frac{\mathbf{q}_{2\perp}^2}{\mathbf{k}_\perp^2} \right) - k_i k_j \frac{\mathbf{q}_{1\perp}^2 \mathbf{q}_{2\perp}^2}{\mathbf{k}_\perp^4} \right]$$



Thus a semi-classical computation (completely analogous to QCD example)
can reproduce Lipatov's Feynman $2 \rightarrow N$ Feynman diagram computation

In QCD, Blaizot, Gelis, RV, hep-ph/0402256
Gelis, Mehtar-Tani, hep-ph/0512079



In the dilute-dense case, the usual QCD Wilson line (representing coherent multiple scattering) is replaced by its double copy counterpart...a la BCJ

$$V(x^-, \mathbf{x}_\perp) \equiv \exp \left(\frac{1}{2} \int_{x_0^-}^{x^-} dz^- \bar{g}_{--}(z^-, \mathbf{x}_\perp) \partial_+ \right)$$

$\nearrow A^{+,a}$
 $\nearrow T^a$

Yet, many subtle features to the correspondence remain to be understood

Prolegomenon to a future program...

Key current issues are in establishing the uniqueness of the gravitational wave solutions

A subsequent computation is of the shockwave (graviton-graviton-reggeon) propagators

This will be necessary to understand the impact parameter dependence of $2 \rightarrow N$ scattering (including such quantum effects) via an RG equation.

An outstanding goal would be to understand if next generation GWO's will be sensitive to this dynamics

Thanks for listening!

2 → N + 2 amplitudes Trans-Planckian gravitation scattering: from wee partons to Black Holes

HIGH-ENERGY SCATTERING IN QCD AND IN QUANTUM GRAVITY AND TWO-DIMENSIONAL FIELD THEORIES

L.N. LIPATOV*

We construct effective actions describing high-energy processes in QCD and in quantum gravity with intermediate particles (gluons and gravitons) having the multi-Regge kinematics. The S -matrix for these effective scalar field models contains the results of the leading logarithmic approximation and is unitary. It can be expressed in terms of correlation functions for two field theories acting in longitudinal and transverse two-dimensional subspaces.

NPB 364 (1991) 614; 161 cites in INSPIRE

Effective action and all-order gravitational eikonal at planckian energies

AMATI, CIAFALONI, VENEZIANO **NPB403 (1993)707**

Building on previous work by us and by Lipatov, we present an effective action approach to the resummation of all semiclassical (i.e. $O(\hbar^{-1})$) contributions to the scattering phase arising in high-energy gravitational collisions. By using an infrared-safe expression for Lipatov's effective action, we derive an eikonal form of the scattering matrix and check that the superstring amplitude result is reproduced at first order in the expansion parameter R^2/b^2 , where R , b are the gravitational radius and the impact parameter, respectively. If rescattering of produced gravitons is neglected, the longitudinal coordinate dependence can be explicitly factored out and exhibits the characteristics of a shock-wave metric while the transverse dynamics is described by a reduced two-dimensional effective action. Singular behaviours in the latter, signalling black hole formation, can be looked for.

The World as a Hologram

LEONARD SUSSKIND

Wee partons, by contrast, are not subject to Lorentz contraction. This implies that in the Feynman Bjorken model, the halo of wee partons eternally "floats" above the horizon at a distance of order $10^{-13}cm$ as it transversely spreads. The remaining valence partons carry the various currents which contract onto the horizon as in the Einstein Lorentz case.

By contrast, both the holographic theory and string theory require all partons to be wee. No Lorentz contraction takes place and the entire structure of the string floats on the stretched horizon. I have explained in previous articles how this behavior prevents the accumulation of arbitrarily large quantities of information near the horizon of a black hole. Thus we are led full circle back to Bekenstein's principle that black holes bound the entropy of a region of space to be proportional to its area.

***J.Math.Phys.* 36 (1995) 6377; 3242 cites**

In Acknowledgements:

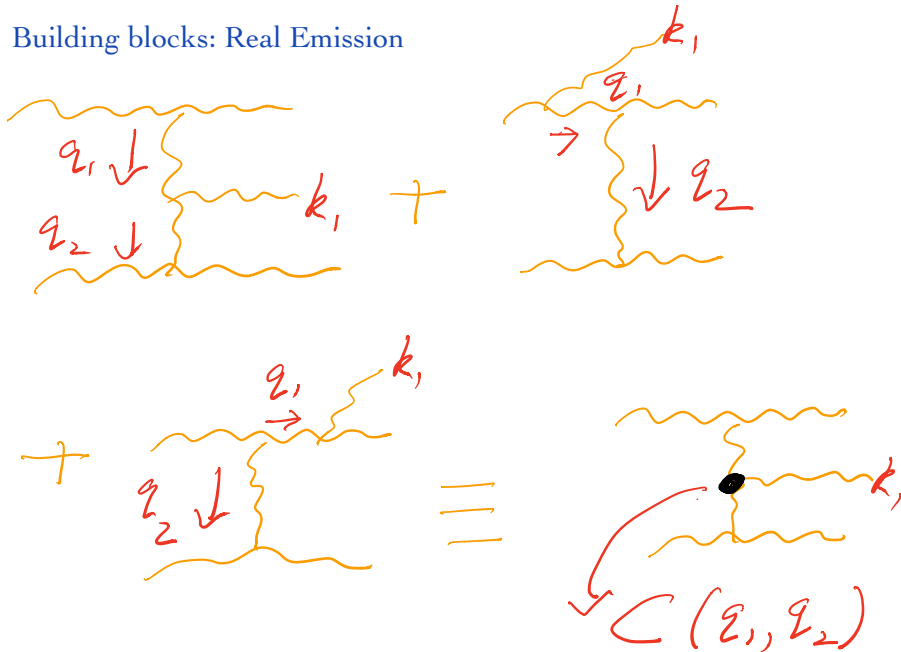
Finally I benefitted from discussions with Kenneth Wilson and Robert Perry, about boosts and renormalization fixed points in light front quantum mechanics and Lev Lipatov about high energy scattering.

These works do not explicitly discuss parton saturation
which leads to a perspective I will now discuss

2 → N + 2 amplitude in the Regge limit of QCD: the BFKL equation

To build in real and virtual corrections to all orders in α_s , first focus on one rung of 2 → N+2 ladder

Building blocks: Real Emission



Lipatov vertex

$$C(q_{i+1}, q_i) = -q_{\perp i+1} - q_{\perp i} + P_A \left(\frac{2q_i^2}{\alpha_i s} + \beta_i \right) - P_B \left(\frac{2q_{i+1}^2}{\beta_i s} + \alpha_i \right)$$

Non-local - gauge invariant!

Ward identity: $k_i \cdot C = 0$

$$A^{2 \rightarrow 3}(s, t) \sim 2s \frac{1}{t_1} C(q_2, q_1) \frac{1}{t_2}$$

2 → N + 2 amplitude in the Regge limit of QCD: the BFKL equation

Building Blocks: virtual corrections

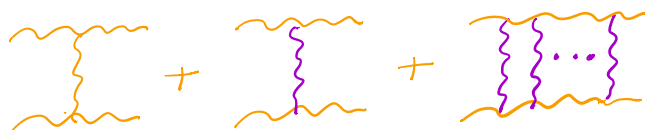


$$= \text{diagram} \quad \frac{1}{t} \rightarrow \frac{1}{t} \ln \frac{s}{-t} \alpha(t)$$

with $\alpha(t) \propto \ln \frac{-t}{\mu^2}$

Infrared cut off

Reggeization ansatz:



$$\equiv \frac{1}{t_i} \rightarrow \frac{1}{t_i} e^{\alpha(t_i)(y_{i+1} - y_i)}$$

$$\alpha(t) = \tilde{g}_s^2 \alpha^{(1)}(t) + \tilde{g}_s^4 \alpha^{(2)}(t) + \mathcal{O}(\tilde{g}_s^6)$$

Fadin, hep-ph/9807528

To this 2-loop order, this Regge trajectory can be obtained from the “cusp” anomalous dimension of the product of two Wilson lines: “Infrared factorization”

Korchemsky, Korchemskaya, hep-ph/9607229

Double log structure:
Sudakov form factor

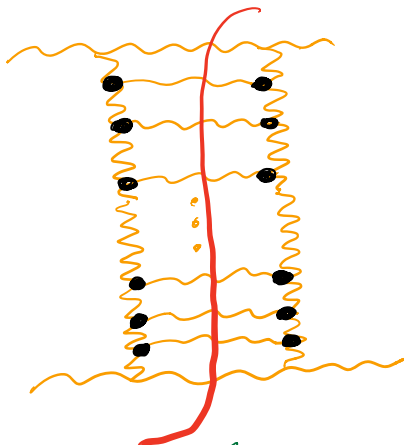
→ infrared sensitive

→ 2 → 2 amp. vanishes for $n \rightarrow 0$

2 → N + 2 amplitude in the Regge limit of QCD: the BFKL equation

Putting together real and virtual contributions in this *leading logs in x* approximation: $(\alpha_s Y)^n$

BFKL Pomeron



$$\begin{aligned} \text{Im } A(s, t) &\propto \sum_{n=0}^{\infty} (\alpha_s C_T)^{n+2} \\ &\times \int \prod_{l=1}^n \frac{dy_l}{4\pi} \prod_{j=1}^{n+1} \frac{d^2 q_{j\perp}}{(2\pi)^2} \\ &\times 2i s \prod_{l=1}^{n+1} \frac{1}{t_l t_{l'}} e^{(y_{l-1} - y_l)(\alpha(t_l) + \alpha(t_{l'}))} \\ &\times \prod_{m=1}^n (C_m C^m) [q_m, q_{m+1}] \end{aligned}$$

C_T is color factor

Phase space factors

Reggeized propagators
on both sides of cut

Product of Lipatov vertices

Inverse Laplace Transform:
 $\text{Im } A(s, t) = \oint_c \frac{d\lambda}{2\pi i} e^{\lambda y} F_\lambda(t)$

$$\begin{aligned} \sigma_{\text{tot}} &= 2 \text{Im } A(s, t=0) \\ &= s^\lambda \text{ with } \lambda = \frac{4\alpha_s N_c \ln 2}{\pi} \\ &\simeq 0.5 \text{ for } \alpha_s = 0.2 \end{aligned}$$

Real and virtual corrections
combine to cancel
Infrared divergence !

Strong violation of Froissart bound

Resummed NLO BFKL : $\lambda \approx 0.3$

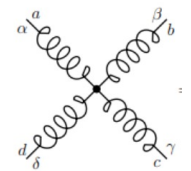
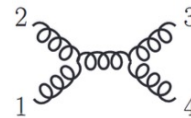
Gauge-Gravity correspondence

Double copy between QCD and Gravity amplitudes

Old idea (Kawai-Lewellyn-Tye) based on relations between closed and open string amplitudes – in "low energy" limit between Einstein & Yang-Mills amplitudes

$$M_4^{\text{tree}}(1, 2, 3, 4) = \left(\frac{\kappa}{2}\right)^2 s A_4^{\text{tree}}(1, 2, 3, 4) A_4^{\text{tree}}(1, 2, 4, 3)$$

$$\kappa = 32 \pi^2 G_N$$



Remarkable "BCJ" color-kinematics duality

Bern, Carrasco, Johansson, arXiv:0805.3993

Tree level $gg \rightarrow gg$ amplitudes (with on shell legs) can be written as

$$i\mathcal{A}_4^{\text{tree}} = g^2 \left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u} \right) \quad \text{with the s channel color factor } c_s = -2 f^{a_1 a_2 b} f^{b a_3 a_4}$$

kinematic factor $n_s = -\frac{1}{2} \left\{ \left[(\epsilon_1 \cdot \epsilon_2) p_1^\mu + 2(\epsilon_1 \cdot p_2) \epsilon_2^\mu - (1 \leftrightarrow 2) \right] \left[(\epsilon_3 \cdot \epsilon_4) p_3^\mu + 2(\epsilon_3 \cdot p_4) \epsilon_4^\mu - (3 \leftrightarrow 4) \right] \right.$

$$\left. + s \left[(\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot \epsilon_4) - (\epsilon_1 \cdot \epsilon_4)(\epsilon_2 \cdot \epsilon_3) \right] \right\}$$

Tree level gravity amplitude obtained from replacing color factors by kinematic factors

$$i\mathcal{A}_4^{\text{tree}}|_{c_i \rightarrow n_i, g \rightarrow \kappa/2} = i\mathcal{M}_4^{\text{tree}} = \left(\frac{\kappa}{2}\right)^2 \left(\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u} \right) \quad \text{Significant on-going work on extension to loop amplitudes}$$

Review: Bern et al., arXiv: 1909.01358

Evolution and decay of Black Holes and CGC

We noted previously that the BH half life is $t_{BH} = N R_S$ due to graviton loss from the leaky condensate (aka Hawking radiation)

This time scale is also inverse of $\omega = \frac{Q_S}{N}$, the energy gap of information carrying Bogoliubov Goldstone modes of the condensate
– microscopic understanding of Page time

However the semi-classical BHNP description breaks down beyond Page time and genuine quantum effects take over

A similar process occurs in the Glasma (though now with 1-d expansion)
In the bottom-up thermalization picture, the quantum break time

is estimated to be $t_{quant} \sim \frac{\alpha_S^{-3/2}}{Q_S}$

In this case, the entropy continues to grow even after until the system thermalizes into a QGP

